ALPHA Collaboration

FLAG2013

The strong coupling and V_{ub} from lattice QCD

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Saclay, 18 January 2016

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QCD

- Theory of strong interactions
- Field theory with Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2{g_0}^2}\mathrm{tr}\{F_{\mu\nu}F_{\mu\nu}\} + \sum_{f=1}^{N_\mathrm{f}} \overline{\psi}_f\{D + m_{0f}\}\psi_f$$

- Fields: gluons and quarks
- But particles: hadrons

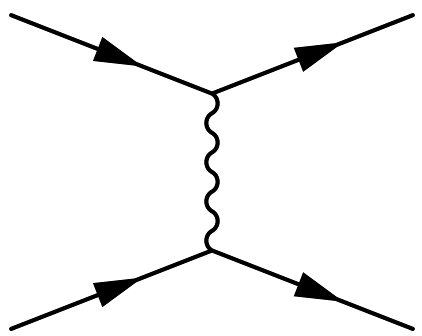
  p, n, π, K,… confinement!
- Definition of coupling is not straight forward
  (we do e.g. not want the π-π coupling)

QCD coupling

- Theorists: $\alpha_{\overline{\mathrm{MS}}}(\mu)$

  take $D = 4 - 2\epsilon$ dimensions
  subtract poles in $1/\epsilon$ … <— no physics

- for QED:
  charged particle scattering at small energy



$$\sigma = \mathrm{kinematics} \times \alpha_{\mathrm{em}}^2$$

physics!

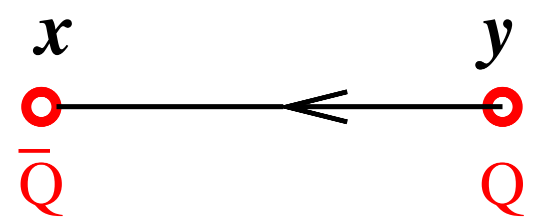
same coupling as $F_{pe}(r) = \alpha_{\mathrm{em}} \dfrac{1}{r^2}$

QCD coupling

Analogous to $$F_{pe}(r) = \alpha_{\rm em}\frac{1}{r^2}$$

quark as test charge Q with $m_Q \to \infty$

$r = |\mathbf{x}-\mathbf{y}|$



force in PT: $$F_{Q\bar{Q}}(r) = \alpha_{\overline{\rm MS}}(\mu)\frac{4}{3}\frac{1}{r^2} + \mathrm{O}(\alpha^2_{\overline{\rm MS}})$$

define:

$$\alpha_{\rm qq}(\mu) \equiv \frac{3r^2}{4}F_{Q\bar{Q}}(r)\,, \quad \mu = \frac{1}{r}$$

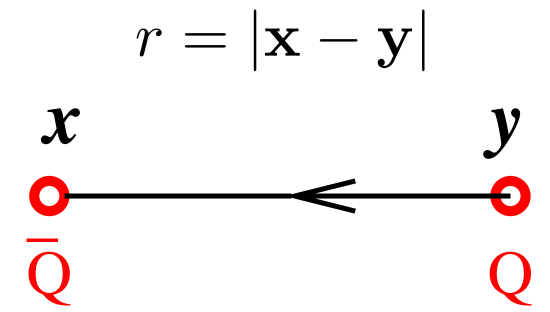
no corrections

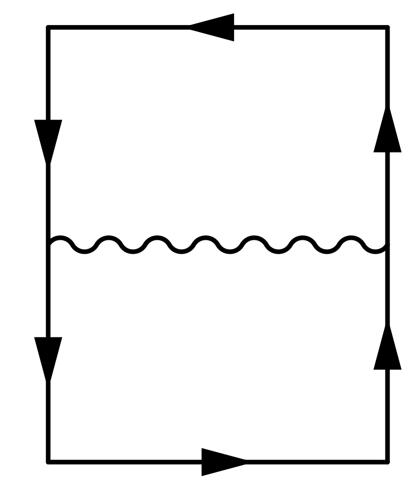
$$\alpha_{\rm qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1\alpha^2_{\overline{\rm MS}}(\mu) + \ldots$$

$$c_1 = \frac{1}{(4\pi)^2}\left\{\frac{35}{3} - 22\gamma_E - (\frac{2}{9} - \frac{4}{3}\gamma_E)N_{\rm f}\right\} = \mathrm{O}(1)$$

QCD coupling

$$\alpha_{\rm qq}(\mu) \equiv \frac{3r^2}{4} F_{Q\bar{Q}}(r)\,, \quad \mu = \frac{1}{r}$$





then

$$\alpha_{\rm qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1 \alpha^2_{\overline{\rm MS}}(\mu) + \ldots$$

always (non-perturbatively) defined physics!

perturbatively defined by such relations

makes sense for $\alpha \ll 1$

QCD coupling, energy dependence

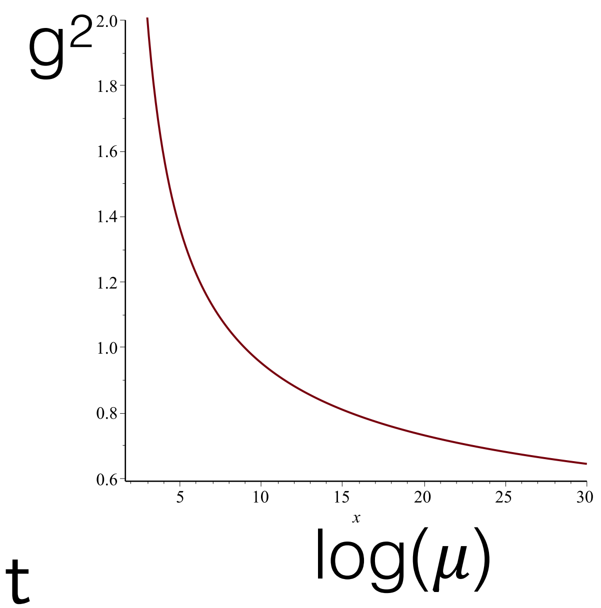
$$\text{RGE:} \quad \mu\frac{\partial\bar{g}}{\partial\mu} = \beta(\bar{g}) \qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3\left\{b_0 + b_1\bar{g}^2 + b_2\bar{g}^4 + \ldots\right\}$$

$$b_0 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_\mathrm{f}\right)$$

Asymptotic freedom

QCD coupling, energy dependence



RGE: $\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3 \left\{ b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + \ldots \right\}$$

$$b_0 = \frac{1}{(4\pi)^2}\left(11 - \tfrac{2}{3} N_{\rm f}\right)$$

Λ-parameter $(\bar{g} \equiv \bar{g}(\mu))$ = Renormalization Group Invariant

= intrinsic scale of QCD = integration constant of RGE

$$\Lambda = \mu \, (b_0 \bar{g}^2)^{-b_1/2b_0^2} \mathrm{e}^{-1/2b_0\bar{g}^2} \exp\left\{ -\int_0^{\bar{g}} \mathrm{d}g \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \right\}$$

singular behavior $\bar{g} \equiv \bar{g}(\mu)$

convergent for g -> 0

$$\bar{g} = \bar{g}_{\overline{\rm MS}} \to \Lambda = \Lambda_{\overline{\rm MS}} \,, \quad \bar{g} = \bar{g}_{\rm qq} \to \Lambda = \Lambda_{\rm qq}$$

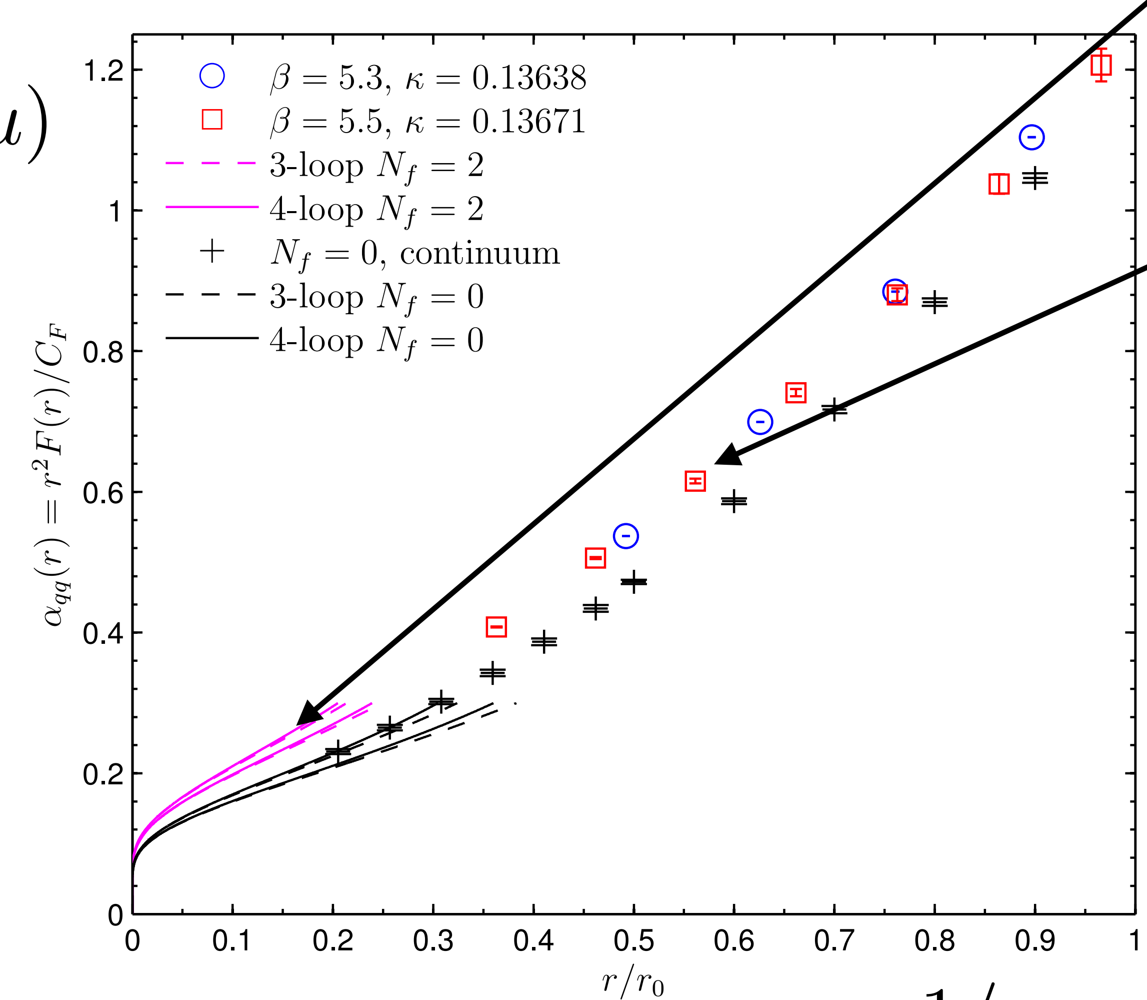
$$\Lambda_{\overline{\rm MS}} / \Lambda_{\rm qq} = \exp\left(c_1/(2b_0)\right)$$

$$\alpha_{\rm qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1 \alpha^2_{\overline{\rm MS}}(\mu) + \ldots$$

Λ is the goal, relative uncertainty: $k\alpha^n$ for n+1 - loop $\beta(g)$

QCD coupling, comparison perturbative / non-pert.

$\alpha_{\rm qq}(\mu)$



$1/\mu$

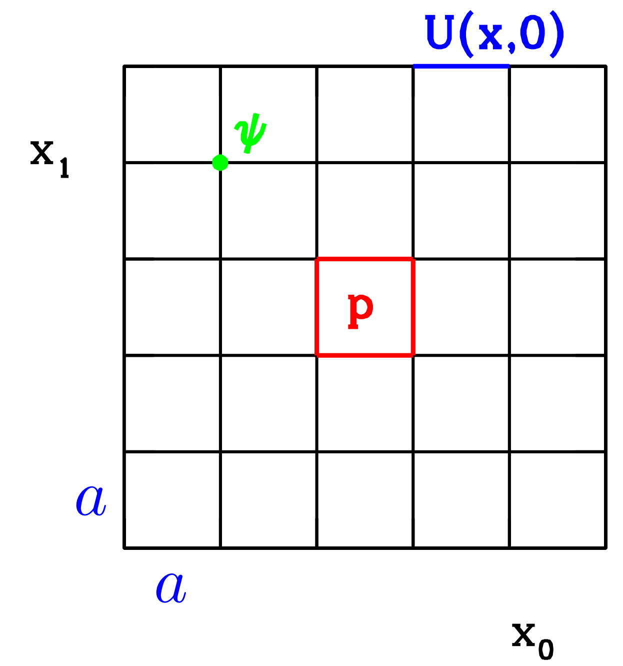
graph from [Leder & Knechtli, 2011]

$N_{\rm f} = 0$, continuum limit [Necco & S., 2001]

$N_{\rm f} = 2$, small lattice spacing [Leder & Knechtli, 2011]

- non-perturbative = lattice QCD
- let us see how this works
- remember for later: we need small r, large μ

Lattice gauge theory



- discrete space-time, spacing a, hypercubic lattice
- Quantization by Feynman path integral
- Euclidean time:

$$\mathrm{e}^{it\mathbb{H}} \;\to\; \mathrm{e}^{-t\mathbb{H}}\,; \quad \mathrm{e}^{itE_n} \;\to\; \mathrm{e}^{-tE_n}$$

- numerical treatment by MC
  “simulation”

"Simulation" of quantum theory (quantum mechanics)

Euclidean Green functions of QM

$$G_f(t_2,t_1) = \langle 0|f(\hat{q}(t_2))\, f(\hat{q}(t_1))|0\rangle$$

with

$$\hat{q}(t) = \mathrm{e}^{\hat{H}\,t}\hat{q}\mathrm{e}^{-\hat{H}\,t}\,, \qquad \hat{H} = V(\hat{q}) + \frac{\hat{p}^2}{2m}, \qquad \hat{H}|0\rangle = 0$$

Information on low-lying spectrum and matrix elements from

$$G_f(t_2,t_1) = \sum_n |\alpha_n|^2 \mathrm{e}^{-(E_n-E_0)\,|t_2-t_1|}\,,$$

$$\alpha_n = \langle n|f(\hat{q}(0))|0\rangle$$

in QCD: m_π, m_{prot},…

in QCD: f_π, …

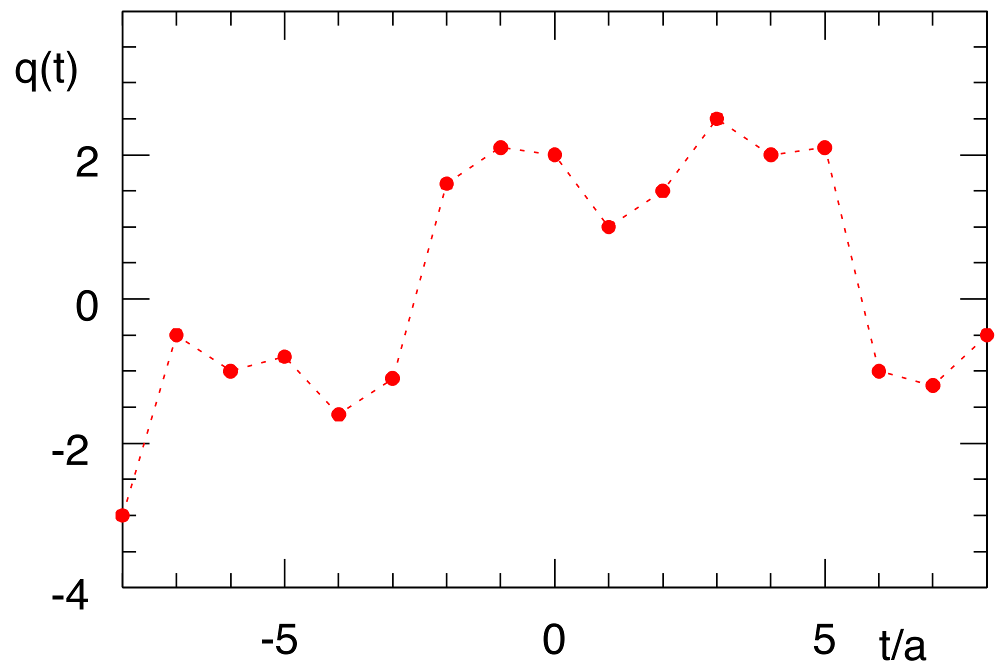
“Simulation” of quantum theory (quantum mechanics)

Feynman’s (Euclidean) path integral representation (discretised):

$$G_f(t_2,t_1) \quad = \quad \lim_{N\to\infty} \frac{\int [\prod_{i=-N/2}^{N/2} \mathrm{d}q_i]\, \mathrm{e}^{-S[q]}\, f(q(n_2\,a))\, f(q(n_1\,a))}{\int [\prod_i \mathrm{d}q_i]\, \mathrm{e}^{-S[q]}} \; + \mathrm{O}(a^2)$$

with

$$S[q] \quad = \quad \sum_i V(q_i) + \frac{m}{2}\left(\frac{q_{i+1}-q_i}{a}\right)^2$$
$$\quad = \quad \int \mathrm{d}t\,[V(q(t)) + \frac{m}{2}\dot{q}(t)^2] + \mathrm{O}(a^2)$$



For finite $N = T/a$:

Monte Carlo integration called Simulation:

- Importance sampling of $\{q_i\}$ with weight $W[q] \propto \mathrm{e}^{-S[q]}$.
- Essentially no restriction on $V(q)$.
- Arbitrarily non-linear.

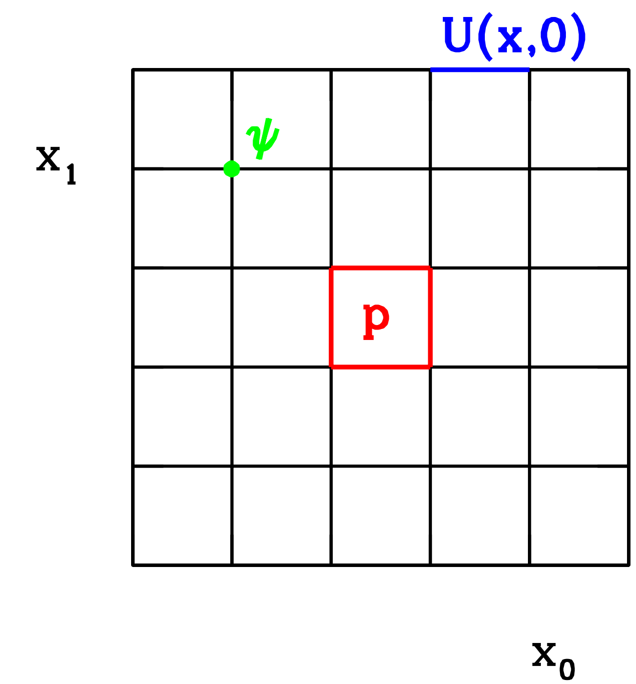
“Simulation” of QCD

lattice: $x_\mu = a n_\mu, \; n_\mu \in \mathbb{Z}, \; \mu = 0,1,2,3$

quarks: $\psi(x)$ on lattice points

gluons: $U(x,\mu) = \mathcal{P} \exp\left\{ a \int_0^1 \mathrm{d}t \, A_\mu(x + a(1-t)\hat{\mu}) \right\}$

$\in \mathrm{SU}(3)$ on links



Euclidean action: $S = S_\mathrm{G} + S_\mathrm{F}$

$$S_\mathrm{G} = \frac{1}{g_0^2} \sum_p \mathrm{tr}\, \{1 - U(p)\} \,,$$

$$S_\mathrm{F} = a^4 \sum_x \overline{\psi}(x)(D(U) + m)\psi(x)$$

$D(U)$: discretized Dirac operator

MC-evaluation of the Path Integral $\rightarrow$ statistical errors $\propto \frac{1}{\sqrt{\text{computer time}}}$

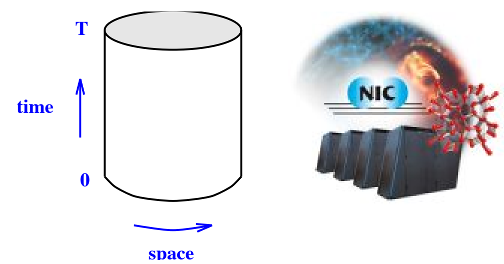
The logics of lattice QCD computations

experiments, hadrons

$$
\begin{aligned}
m_p &= 938.272\,\mathrm{MeV} \\
m_\pi &= 139.570\,\mathrm{MeV} \\
m_\mathrm{K} &= 493.7\,\mathrm{MeV} \\
m_\mathrm{D} &= 1896\,\mathrm{MeV} \\
m_\mathrm{B} &= 5279\,\mathrm{MeV}
\end{aligned}
$$

fix parameters in Lagrangian g_0, m_{0f}

- **The Lagrangian**
- **Non-perturbative formulation: lattice with spacing** a
- **Technology**



continuum limit $a \to 0$

Non-perturbative in the coupling

So far only achievable by numerical simulation

fundamental parameters
& hadronic matrix elements

$$\alpha(\mu)$$

$$m_\mathrm{u}(\mu)\,,\ m_\mathrm{s}(\mu)$$

$$m_\mathrm{c}(\mu)\,,\ m_\mathrm{b}(\mu)$$

$$F_\mathrm{B}\,,\ F_{\mathrm{B_s}}\,,\ \xi\ \ldots$$

our focus today

form factors for
B_s—> K, B—> π

Example: QCD with mass-degenerate quarks

input numbers

$$am_{\rm had} = F_{\rm had}(g_0, am_0, \frac{L}{a}) \quad \text{all dimensionless}$$

$$= F^{\infty}_{\rm had}(g_0, am_0) + \mathrm{O}(\mathrm{e}^{-m_\pi L})$$

$$\mathrm{e}^{-m_\pi a \times \frac{L}{a}}$$

output number

non-degenerate masses

determine

$$am_0 = M_{\rm phys}(g_0)$$

such that

$$\frac{F^{\infty}_{\pi}(g_0, M_{\rm phys}(g_0))}{F^{\infty}_{\rm prot}(g_0, M_{\rm phys}(g_0))} =: R^{\rm phys}_{\pi,\rm prot}(g_0) = \frac{134}{938}$$

$R_{\pi,\rm prot}$

$R_{K,\rm prot}$

$R_{D,\rm prot}$

$\ldots$

we then have the physical quark mass

then, on the physical quark mass trajectory $am_0 = M_{\rm phys}(g_0)$

$$\alpha_{\rm q\bar{q}}(\mu a = \rho \times m_{\rm prot} a) = \alpha_{\rm q\bar{q}}^{\rm cont}(\mu = \rho \times m_{\rm prot})\,(1 + {\rm O}(am_{\rm prot})^2)$$

$$am_{\rm prot} \sim {\rm e}^{-1/(2b_0 g_0^2)}$$

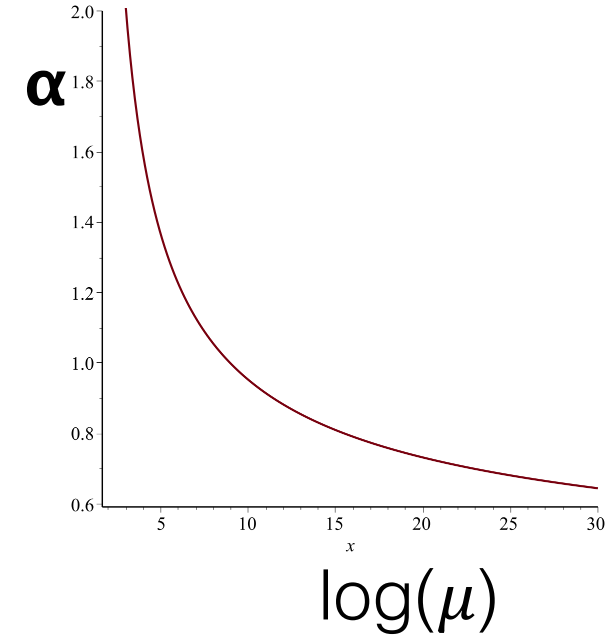
$$a \to 0 \quad \leftrightarrow \quad g_0 \to 0$$

$$$ (L/a >>1)

One gets $\alpha_{\rm q\bar{q}}^{\rm cont}(\mu)$

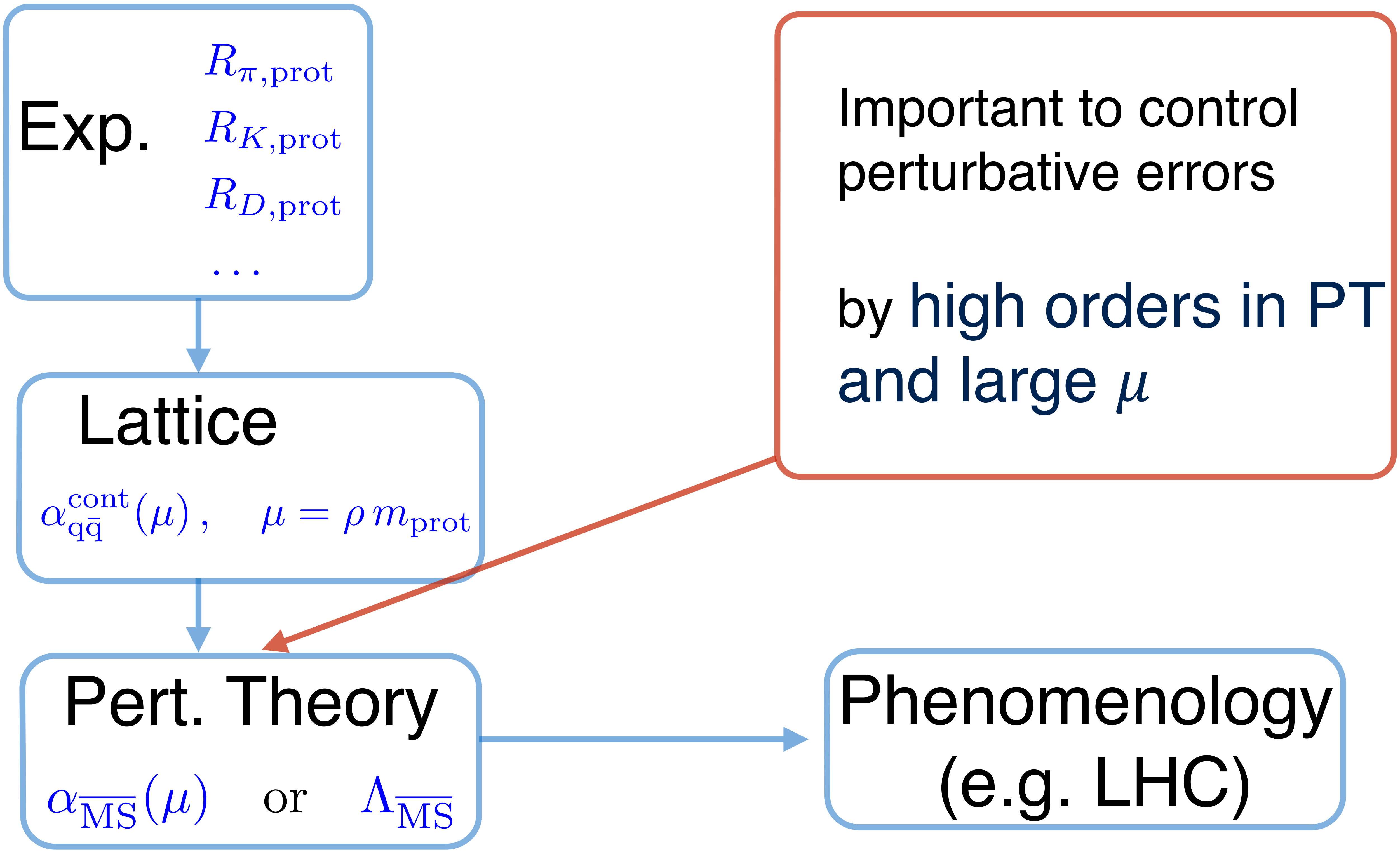
but: one wants large μ because then one has a small uncertainty

$$\frac{\Delta\Lambda}{\Lambda} \sim \{\alpha(\mu)\}^n$$



this is not easy, we come back to it later

Summary of the principle



FLAG-2 review

- review and summary of lattice results relevant for phenomenology
- averages, ranges
- somewhat critical
- here just a summary of the

$$\alpha_s$$

part

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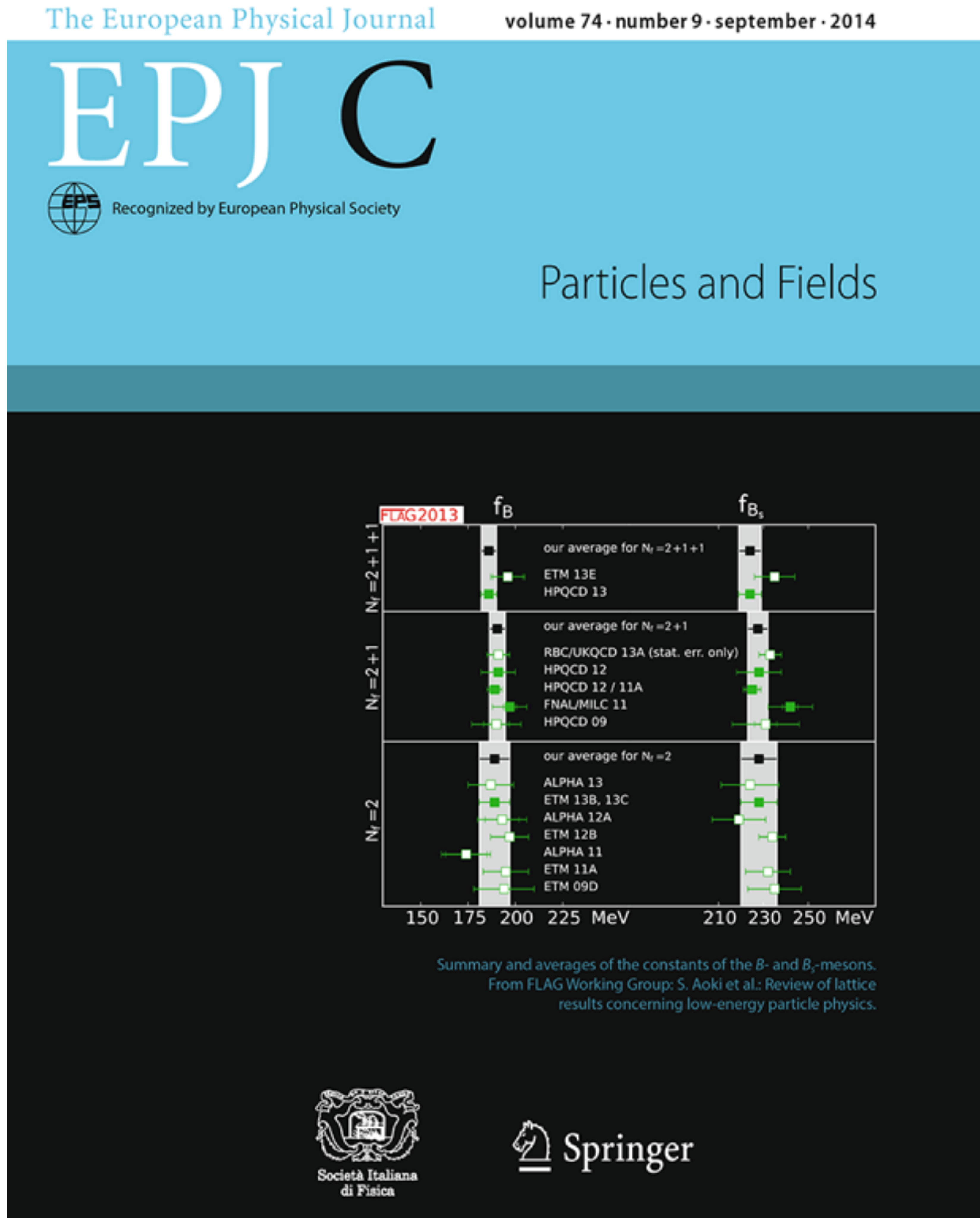

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apologies: for references please see the report

Limitations of lattice computations

- Observable with energy/momentum scale μ

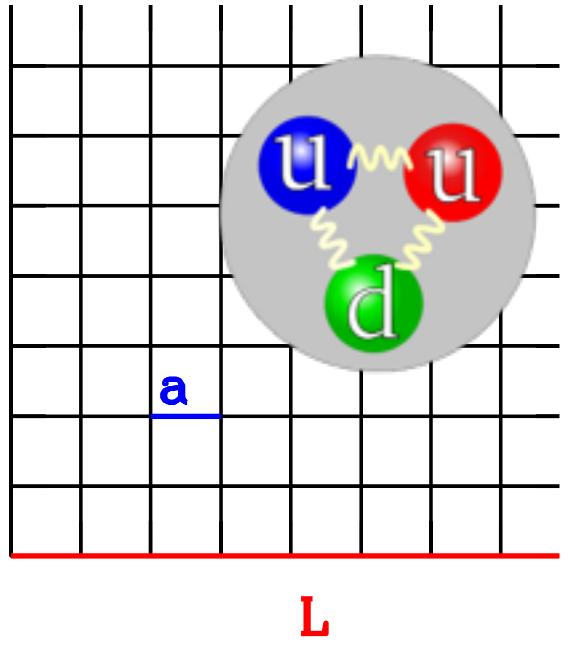
$$\mathcal{O}(\mu) \equiv \lim_{a\to 0} \mathcal{O}_{\rm lat}(a,\mu) \text{ with } \mu \text{ fixed}$$

- avoid finite size and discretization effects

$$L \gg \text{hadron size} \sim \Lambda_{\rm QCD}^{-1} \quad \text{and} \quad 1/a \gg \mu$$

or:

$$L/a \ggg \mu/\Lambda_{\rm QCD}$$



$$\mu \lll L/a \times \Lambda_{\rm QCD} \sim 5 - 20\,\text{GeV}$$

$1-3\,\text{GeV}$ at most, in conflict with $\frac{\Delta\Lambda}{\Lambda} \sim \{\alpha(\mu)\}^n$

Reliability of perturbation theory

- Observable with perturbative expansion

$$\mathcal{O} \sim \sum_{i \leq n} c_i \alpha_s^i$$

- truncation errors:

$$k\,\alpha_s^{n+1} \qquad\qquad \mathrm{O}(\exp(-\gamma/\alpha_s))$$

perturbative and non-perturbative

Quality criteria

try to make sure
the observable is at
sufficiently short distance

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- Renormalisation scale
  - ★ all points relevant in the analysis have $\alpha_{\rm eff} < 0.2$
  - ○ all points have $\alpha_{\rm eff} < 0.4$ an
  - ■ otherwise

try to make sure
the μ-dependence is as predicted
by perturbation theory

- Perturbative behaviour
  - ★ verified over a range of a factor 2 in $\alpha_{\rm eff}$ (without power corrections)
  - ○ agreement with perturbation theory over a range of a factor 1.5 in $\alpha_{\rm eff}$ (possibly fitting with power corrections)
  - ■ otherwise

try to make sure
the continuum limit can be taken

- Continuum extrapolation

  At a reference point of $\alpha_{\rm eff} = 0.3$ (or less) we require
  - ★ three lattice spacings with $\mu a < 1/2$ and full $O(a)$ improvement
    or three lattice spacings with $\mu a \leq 1/4$ and 2-loop $O(a)$ improvement
    or $\mu a \leq 1/8$ and 1-loop $O(a)$ improvement
  - ○ three lattice spacings with $\mu a < 1.5$ reaching down to $\mu a = 1$ and full $O(a)$ improvement
    or three lattice spacings with $\mu a \leq 1/4$ and 1-loop O(a) improvement
  - ■ otherwise

Current 2-point functions of heavy quarks

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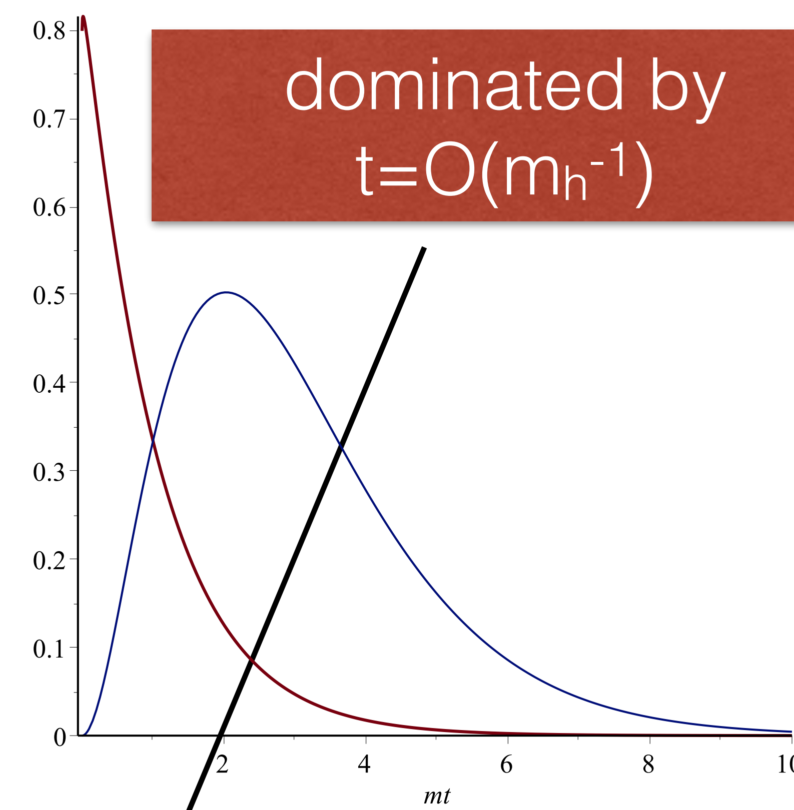
- consider a pair of heavy quarks, h,h' [HPQCD 08b, HPQCD 10]
  — mass of charm or heavier
  — 2-point function (Euclidean space)

$$G(x_0) = a^3 \sum_{\vec{x}} \langle J^\dagger(x) J(0) \rangle \qquad J(x) = i m_h \overline{\psi}_h(x) \gamma_5 \psi_{h'}(x)$$



- moments

$$G_n = a \sum_{t=-(T/2-a)}^{T/2-a} t^n \, G(t) \approx \int_{-T/2}^{T/2} t^n \, G(t) \, \mathrm{d}t$$

- have a perturbative expansion, e.g.

$$R_4 \equiv G_4/G_4^{(0)} = 1 + r_{4,1}\alpha_s + r_{4,2}\alpha_s^2 + r_{4,3}\alpha_s^3 + \ldots \,,$$

$$\alpha_s = \alpha_s(\mu = 2m_h)$$

Current 2-point functions of heavy quarks

- define effective coupling of THIS process

$$\alpha_{\text{eff}} \equiv (R_4 - 1)/r_{4,1} = \alpha_{\overline{\text{MS}}} + \text{O}(\alpha^2_{\overline{\text{MS}}})$$

Relevance of discretization errors

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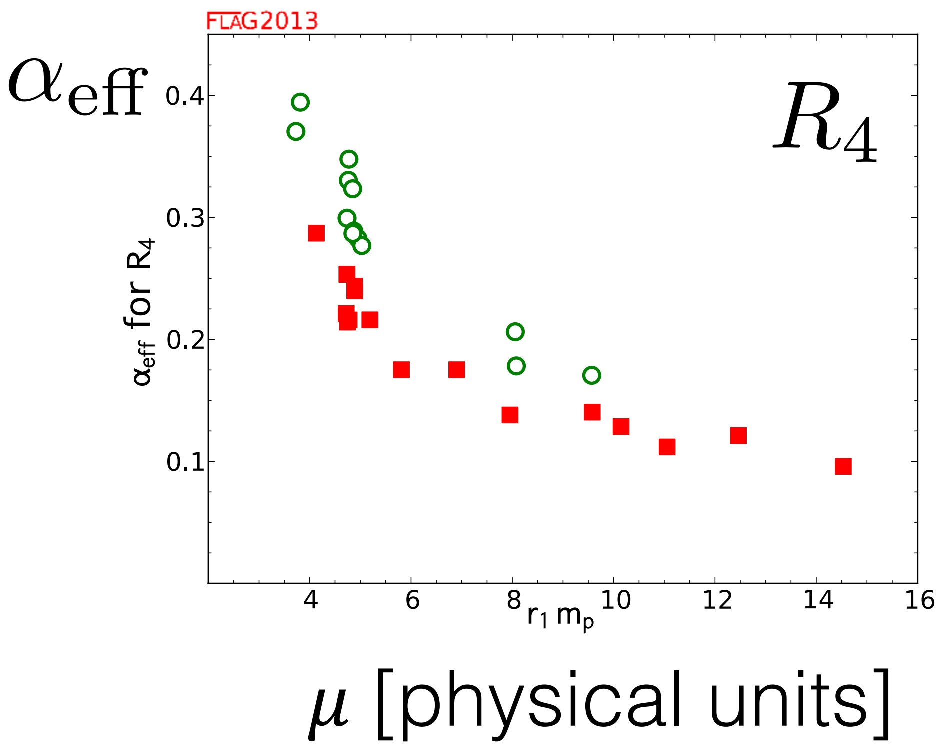
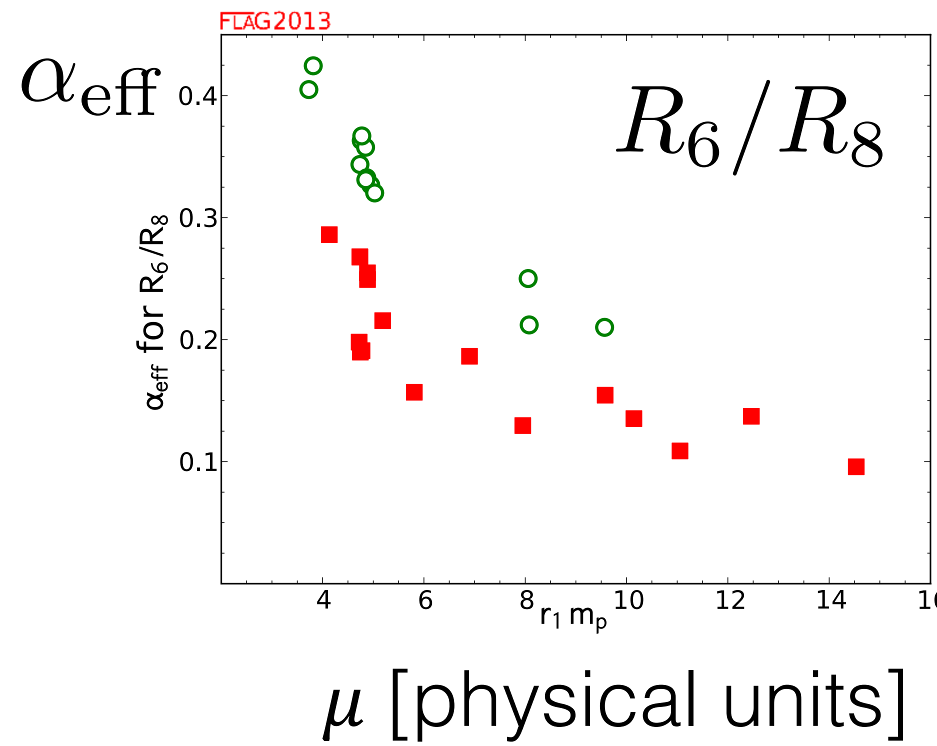




- Continuum limit: universal curves
- Vertical displacements are discretization effects
- green circles: pass criteria
- a very good understanding of discretization errors is needed to extract precise continuum numbers

criteria are relevant

Requirements in a nut-shell

$$L/a \ggg \mu/\Lambda_{\mathrm{QCD}}$$

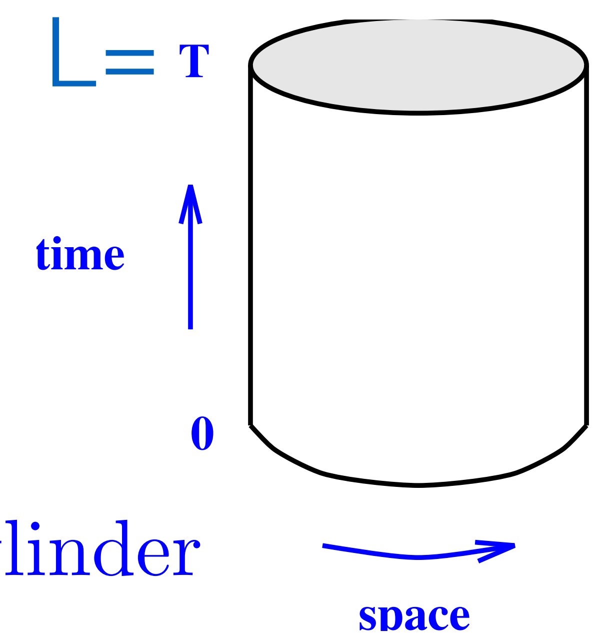
- two tricks are possible

  - Observables in finite volume $L/a \gg 1$

  - Observables at the cutoff

    (skip because of time)

Observables in finite volume



- $$\mathcal{O}(L,a) = c_1 \alpha_s(1/L) + c_2 \alpha_s^2(1/L) + \ldots \equiv c_1 \alpha_{\rm L}(1/L)$$

L^4 torus or cylinder

- iteratively connect L and L/2 needs L/a » 1, not more step scaling function:

$$\sigma(u) = \bar{g}^2(2L) \text{ when } \bar{g}^2(L) = u$$

$\Longrightarrow$ L=2^{-10} fm perturbative region, running of coupling

- idea: Lüscher, Weisz, Wolff: 2-d, O(3) sigma-model
- development and application for QCD: ALPHA Collaboration

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Step Scaling: Connecting L ⟹ 2L

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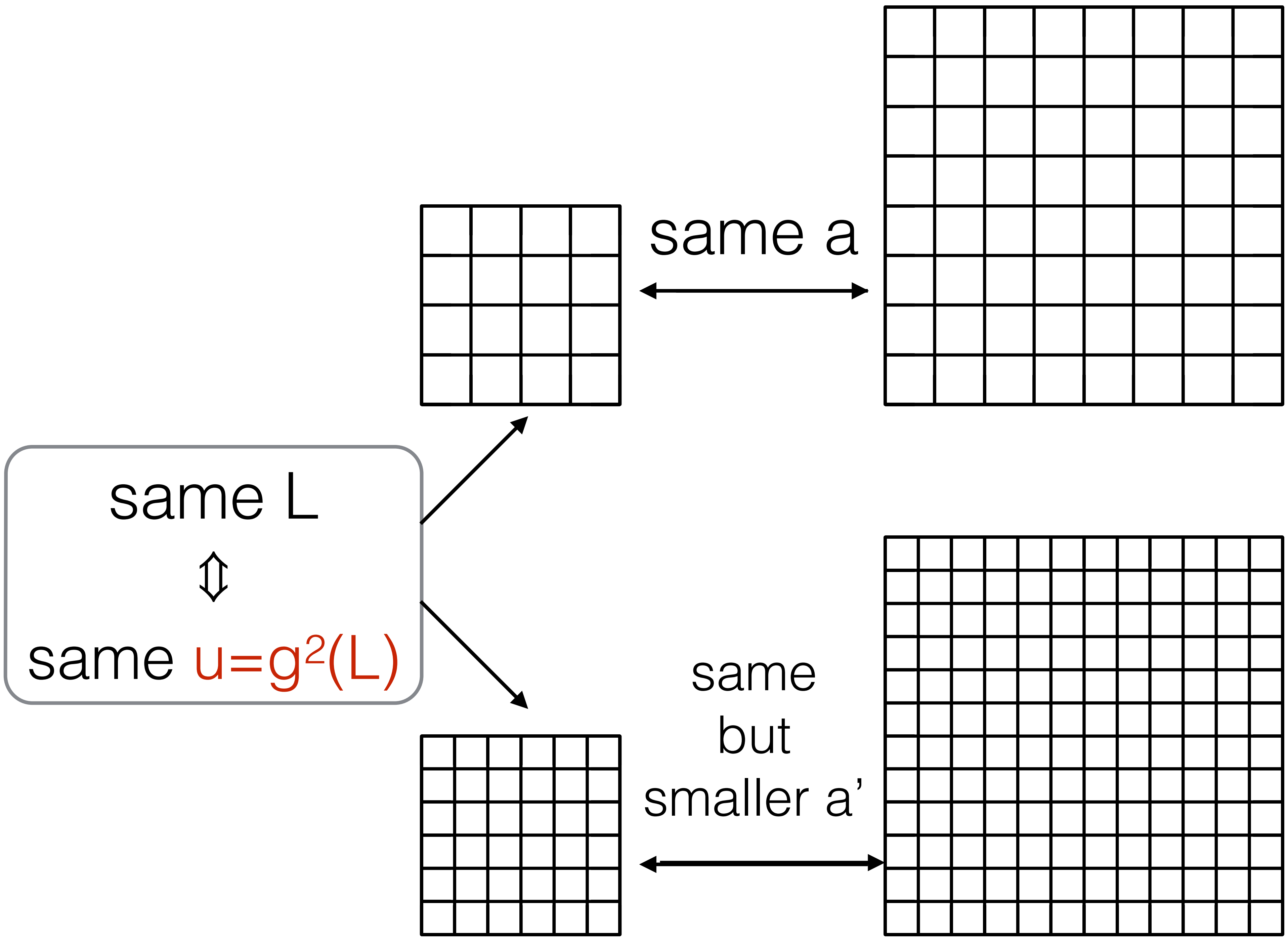

$\Sigma(u,a/L)=$
$g^2(2L,a/L)$ =
$g^2(2L,1/4)$

$\Sigma(u,a/L)=$
$g^2(2L,a'/L)$ =
$g^2(2L,1/6)$

⇓ extrapolate

$g^2(2L,0)=\sigma(u)$
cont. limit

- needs $L/a \gg 1$, not more:

$$\frac{\bar{g}^2(2L) - \bar{g}^2(L)}{\bar{g}^2(L)} \quad (= 2\ln 2\, b_0 + \ldots \;= \text{discrete } \beta\text{-function})$$

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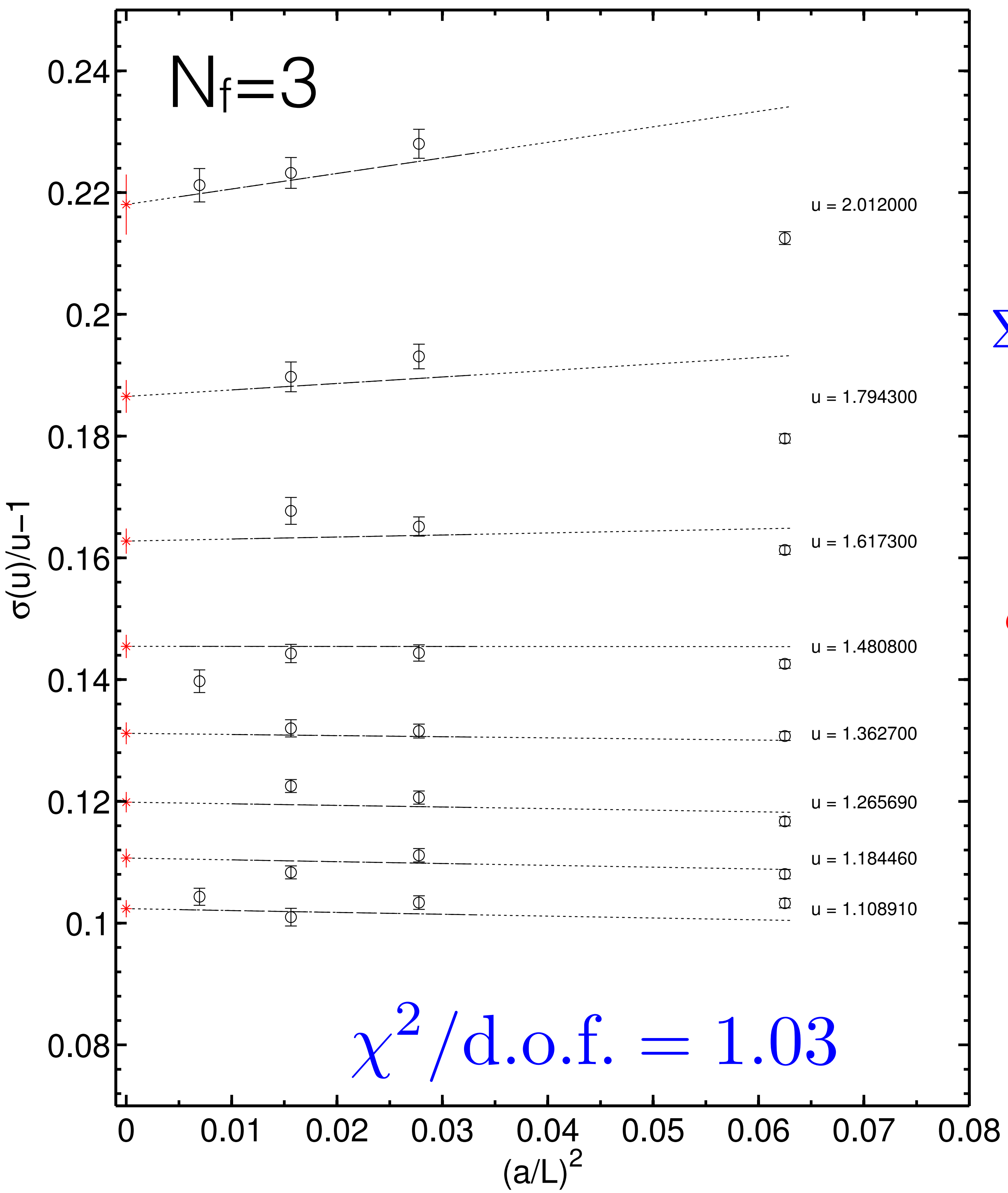

Continuum extrapolation

$$\Sigma(u, a/L) = u + s_0 u^2 + s_1 u^3 + c_1 u^4 + c_2 u^5 + (\rho_1 u^2 + \rho_2 u^3)[a/L]^2$$

c_1, c_2, ρ_1, ρ_2 fit parameters

then solve

$$\sigma(u_{n+1}) = u_n$$

$$u_0 \to u_1, \ldots u_n$$

$$\bar{g}^2(L_0) \to \bar{g}^2(2^{-1}L_0), \ldots, \bar{g}^2(2^{-n}L_0)$$

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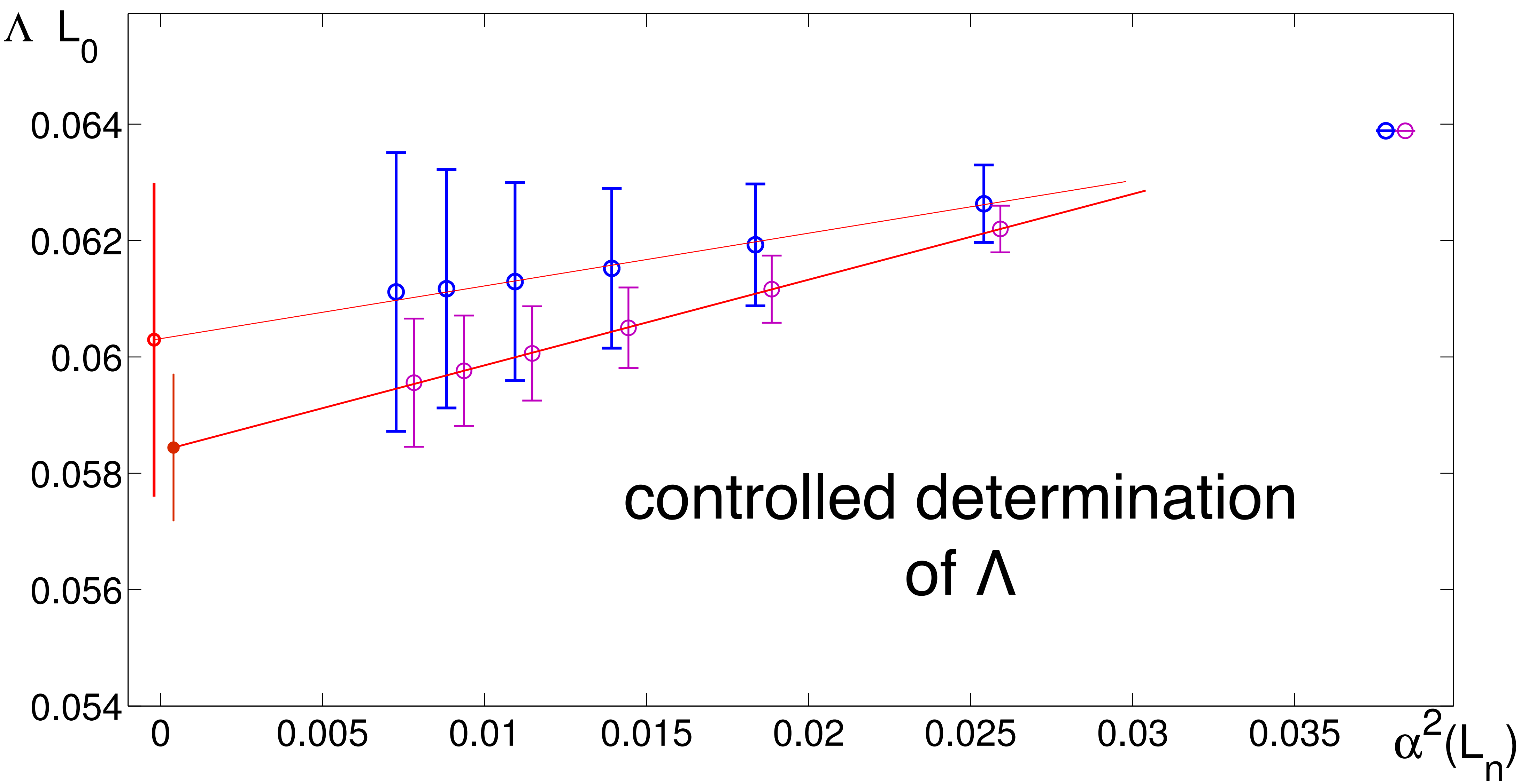
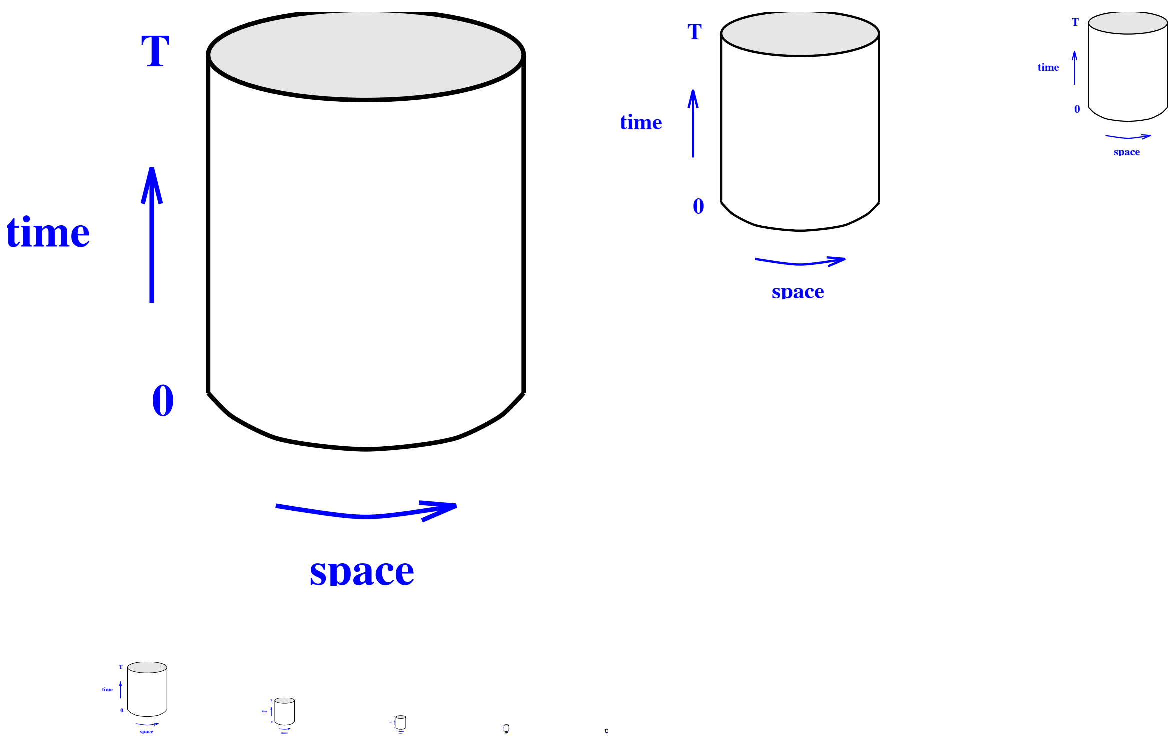
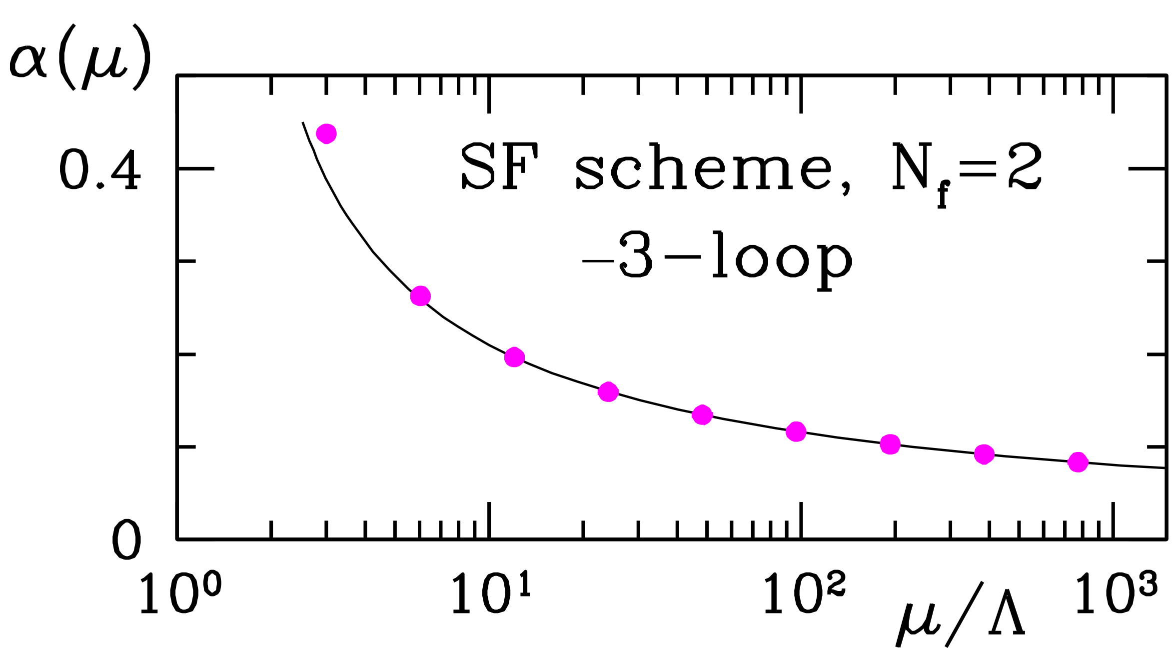
$$L_0\Lambda = 2^n \left(b_0\bar{g}^2(L_n)\right)^{-b_1/(2b_0^2)} e^{-1/(2b_0\bar{g}^2(L_n))} \times \exp\left\{-\int_0^{\bar{g}(L_n)} dx \left[\frac{1}{\beta(x)} + \frac{1}{b_0x^3} - \frac{b_1}{b_0^2x}\right]\right\}$$

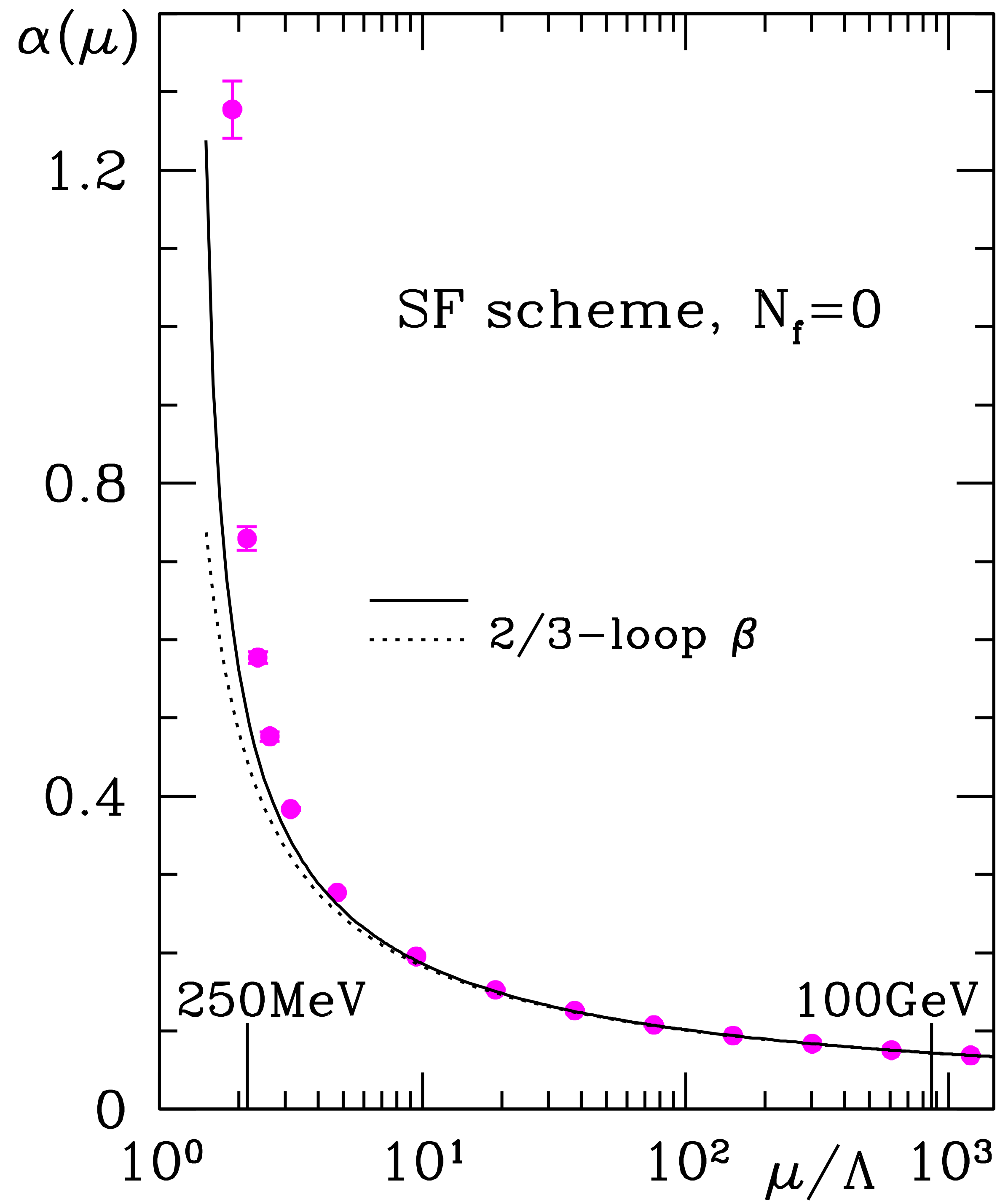


Running from Observables in finite volume





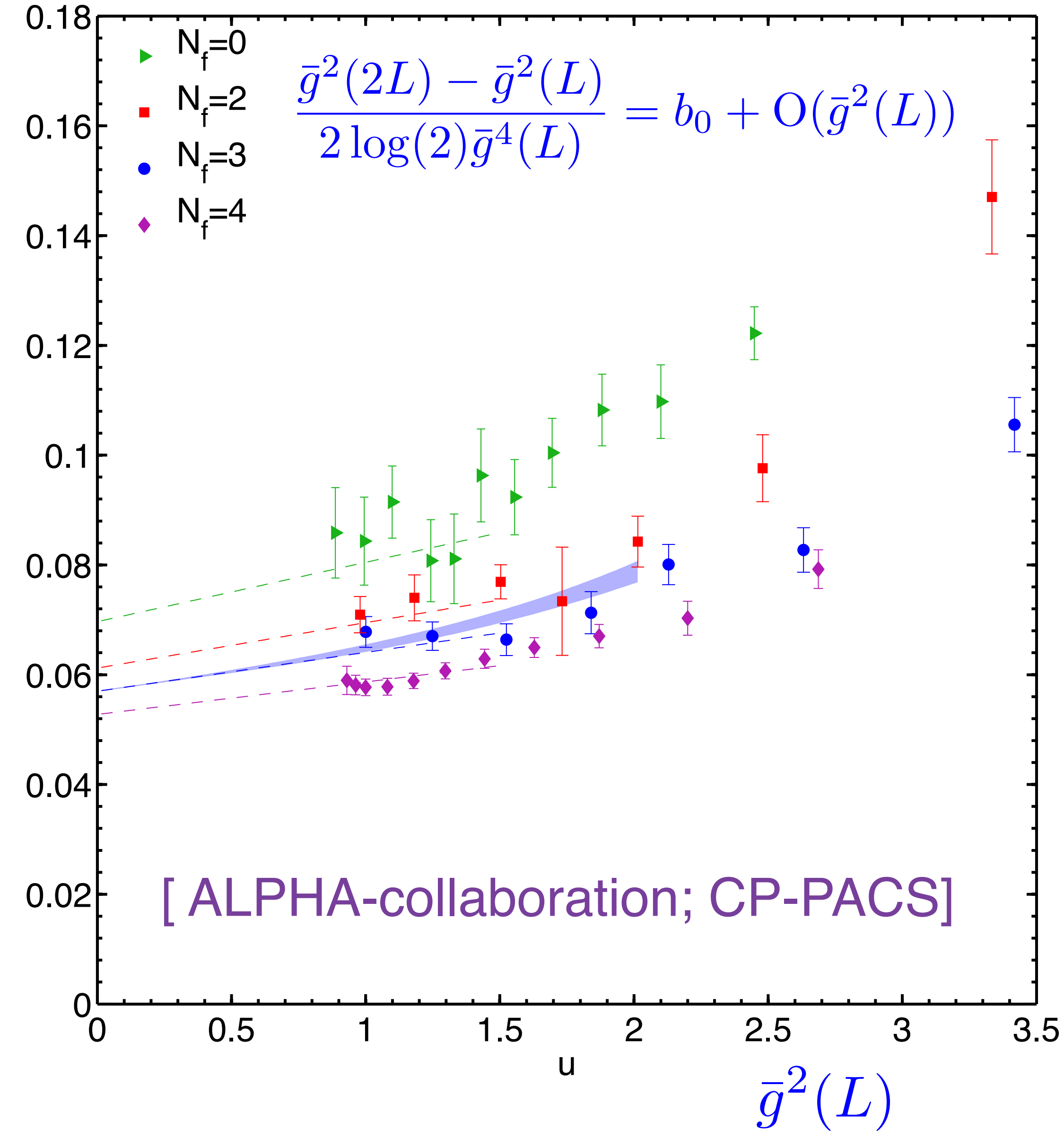
[ALPHA Collaboration, 2005]



[ALPHA Collaboration, 2001]

Observables in finite volume

- $\alpha_s(M_Z)$ without compromises
- unfortunately no high precision result for N_f=3 yet (many simulations + analysis are needed)
- N_f=3, precision computations are in progress



non-perturbative N_f, g^2 -dependence of the β-function

Back to the Review of α_s

Methods used on the lattice and main challenges

- finite L, step scaling ➡ statistical errors
- observables at the lattice spacing scale ➡ perturbative order, behavior of (non-universal) PT
- potential
- vacuum polarisation
- current two-point functions
- QCD vertices

➡ compromise between discretisation errors vs. perturbative error

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Detailed tables, e.g. α from Wilson loops

Collaboration	Ref.	N_f	publication status	renormalisation scale	perturbative behaviour	lattice spacings	scale	$\Lambda_{\overline{\rm MS}}$[MeV]	$r_0\Lambda_{\overline{\rm MS}}$
HPQCD 10A[a §]	[73]	2+1	A	○	★	★	$r_1 = 0.3133(23)$ fm	340(9)	0.812(22)
HPQCD 08A[a]	[505]	2+1	A	○	★	★	$r_1 = 0.321(5)$ fm[††]	338(12)[⋆]	0.809(29)
Maltman 08[a]	[508]	2+1	A	○	○	○	$r_1 = 0.318$ fm	352(17)[†]	0.841(40)
HPQCD 05A[a]	[504]	2+1	A	○	○	○	r_1[††]	319(17)[⋆⋆]	0.763(42)
QCDSF/UKQCD 05	[509]	2	A	★	■	★	$r_0 = 0.467(33)$ fm	261(17)(26)	0.617(40)(21)[b]
SESAM 99[c]	[510]	2	A	○	■	■	$c\bar{c}$(1S-1P)		
Wingate 95[d]	[511]	2	A	★	■	■	$c\bar{c}$(1S-1P)		
Davies 94[e]	[512]	2	A	★	■	■	Υ		
Aoki 94[f]	[513]	2	A	★	■	■	$c\bar{c}$(1S-1P)		
QCDSF/UKQCD 05	[509]	0	A	★	○	★	$r_0 = 0.467(33)$ fm	259(1)(20)	0.614(2)(5)[b]
SESAM 99[c]	[510]	0	A	★	■	■	$c\bar{c}$(1S-1P)		
Wingate 95[d]	[511]	0	A	★	■	■	$c\bar{c}$(1S-1P)		
Davies 94[e]	[512]	0	A	★	■	■	Υ		
El-Khadra 92	[514]	0	A	★	○	■	$c\bar{c}$(1S-1P)	234(10)	0.593(25)[g]

[a] The numbers for Λ have been converted from the values for $\alpha_s^{(5)}(M_Z)$.

[§] $\alpha_{\overline{\rm MS}}^{(3)}(5\text{ GeV}) = 0.2034(21)$, $\alpha_{\overline{\rm MS}}^{(5)}(M_Z) = 0.1184(6)$, only update of intermediate scale and c, b quark masses, supersedes HPQCD 08A and Maltman 08.

[†] $\alpha_{\overline{\rm MS}}^{(5)}(M_Z) = 0.1192(11)$.

[⋆] $\alpha_V^{(3)}(7.5\,\text{GeV}) = 0.2120(28)$, $\alpha_{\overline{\rm MS}}^{(5)}(M_Z) = 0.1183(8)$, supersedes HPQCD 05.

[††] Scale is originally determined from Υ mass splitting. r_1 is used as an intermediate scale. In conversion to $r_0\Lambda_{\overline{\rm MS}}$, r_0 is taken to be 0.472 fm.

[⋆⋆] $\alpha_V^{(3)}(7.5\,\text{GeV}) = 0.2082(40)$, $\alpha_{\overline{\rm MS}}^{(5)}(M_Z) = 0.1170(12)$.

[b] This supersedes [515–517]. $\alpha_{\overline{\rm MS}}^{(5)}(M_Z) = 0.112(1)(2)$. The $N_f = 2$ results were based on values for r_0/a which have later been found to be too small [59]. The effect will be of the order of 10–15%, presumably an increase in Λr_0.

[c] $\alpha_{\overline{\rm MS}}^{(5)}(M_Z) = 0.1118(17)$.

[d] $\alpha_V^{(3)}(6.48\,\text{GeV}) = 0.194(7)$ extrapolated from $N_f = 0, 2$. $\alpha_{\overline{\rm MS}}^{(5)}(M_Z) = 0.107(5)$.

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Λ-parameter for various N_f

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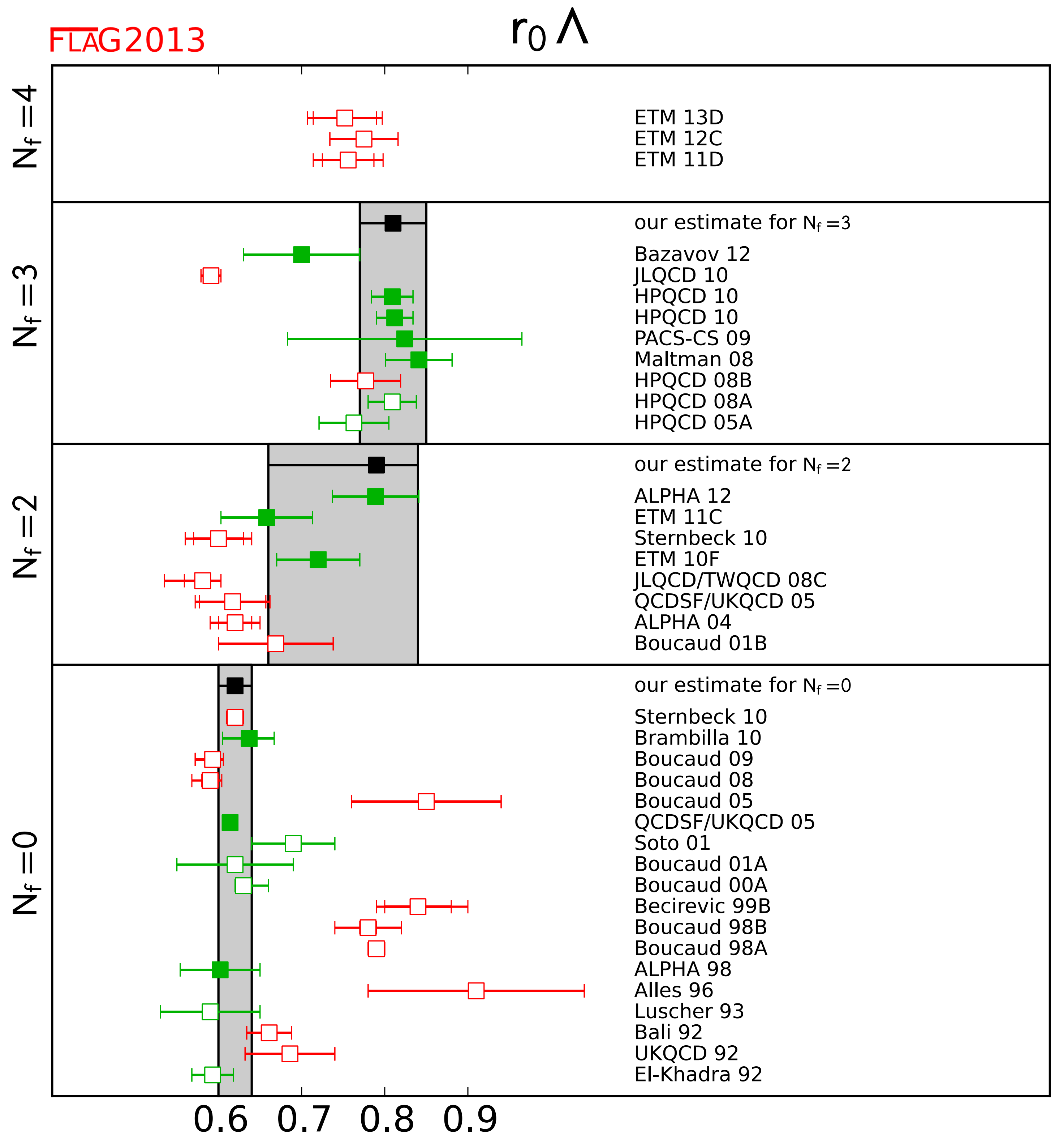

■ enter ranges /averages

□ do not enter (e.g. superseded by new computation)

□ do not enter (does not satify quality criteria)

$r_0 \approx 0.5\,\mathrm{fm}$

reference scale computed in most computations

Range of the strong coupling at M_Z

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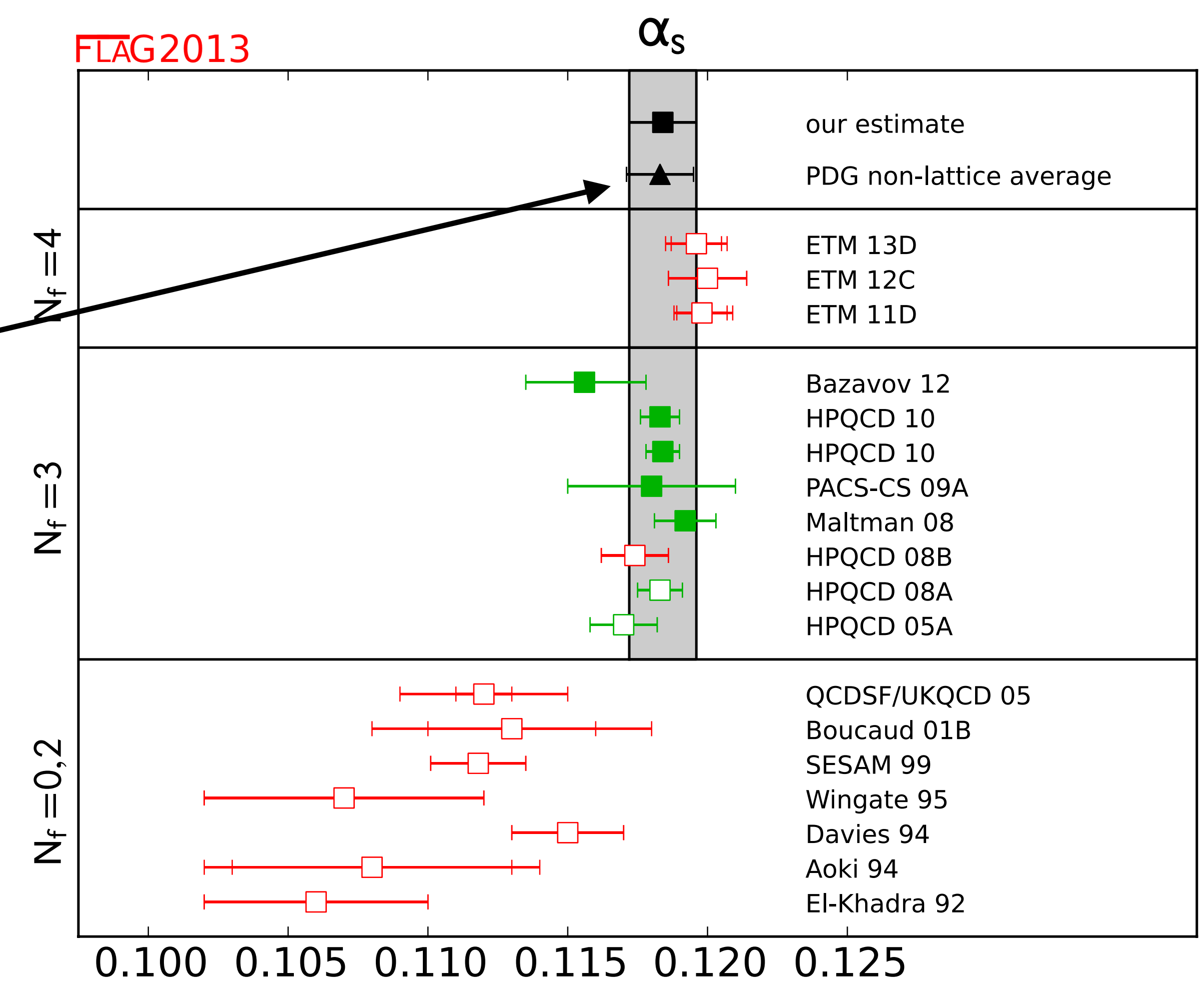
- Dominated by few computations
- Almost identical to PDG non-lattice average
  (note that there averages are formed purely
  compatibility)



- Strong mutual confirmation

  perturbative QCD = non-pert. QCD = QCD

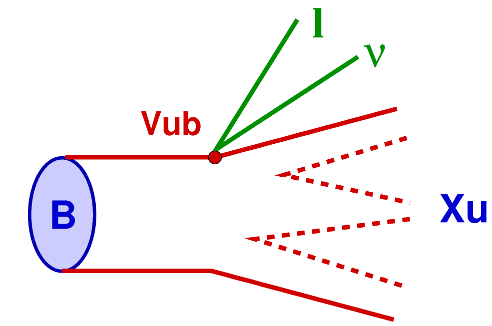
V_{ub} from exclusive B-decays

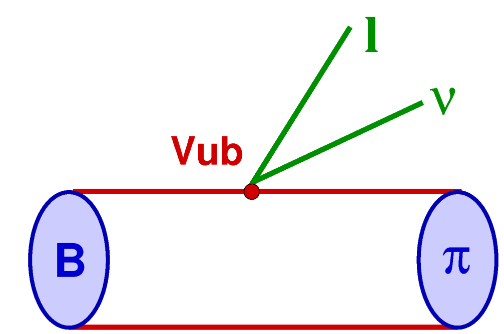
- another very interesting application of lattice QCD
- a personal view, not FLAG

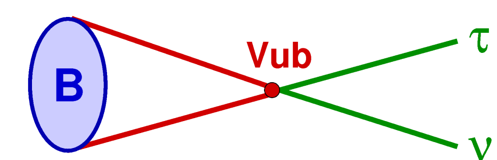
Felix Bahr[a], Debasish Banerjee[a], Fabio Bernardoni[a,b], Anosh Joseph[c], Mateusz Koren[a], Hubert Simma[a], Rainer Sommer[a]

V_{ub} puzzle

- Determination of $|V_{\mathrm{ub}}|$
- $\sim 3\sigma$ discrepancy [PDG] :
  - Inclusive $\mathrm{B} \to X_{\mathrm{u}} \ell \nu$:
    $V_{\mathrm{ub}} = (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3}$
  - Exclusive $\mathrm{B} \to \pi \ell \nu$: $V_{\mathrm{ub}} = (3.28 \pm 0.29) \times 10^{-3}$
  - from $\mathrm{B} \to \tau \nu$ via f_{B}: $V_{\mathrm{ub}} = (4.22 \pm 0.42) \times 10^{-3}$
- theoretical and experimental input needed
- This talk: Non-perturbative determination of form factors for $\mathrm{B_s} \to \mathrm{K} \ell \nu$ decay





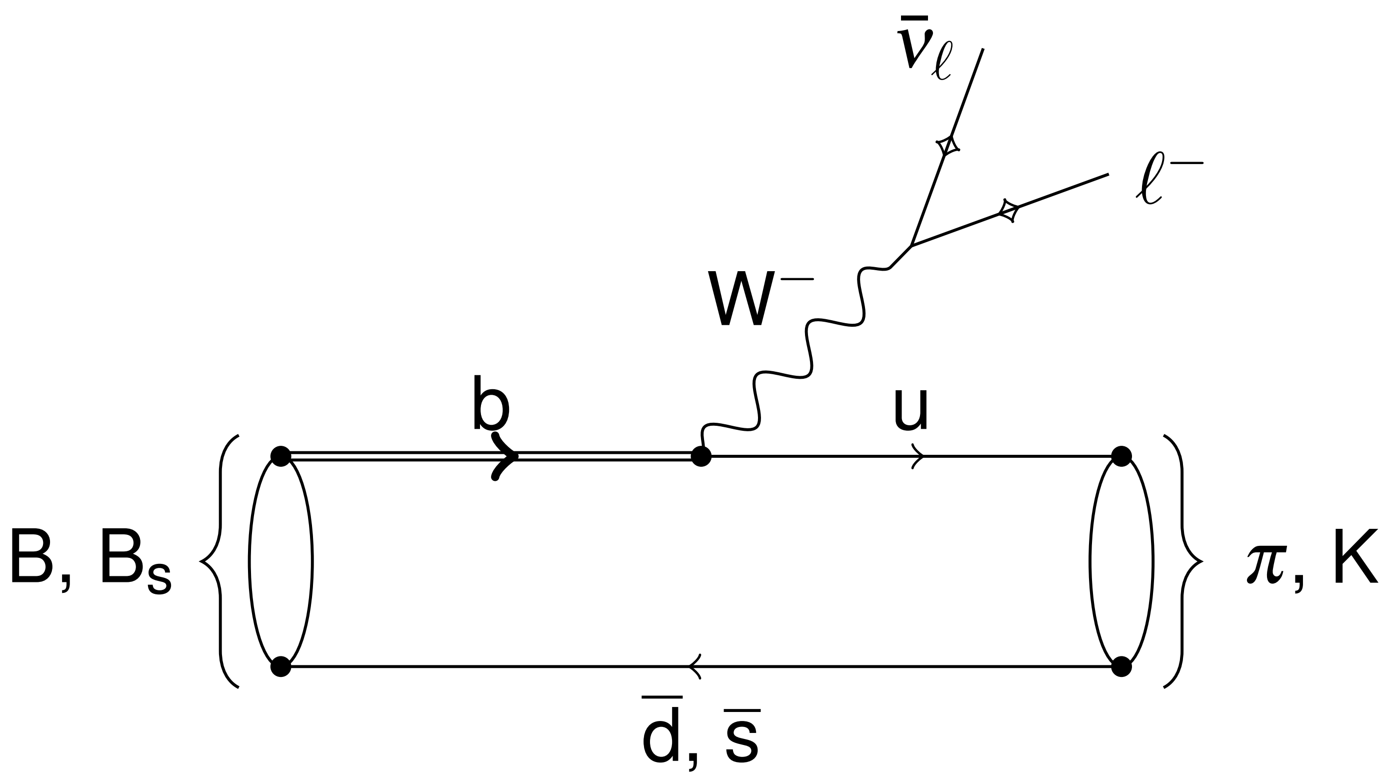


Based on a lot of complicated theory (assumptions)

e.g. HMrstCh PT

e.g. HPChPT inspired factorization of Eq. (19) allows a simultaneous chiral, continuum, and kinematic extrapolation of lattice data at arbitrary energies. Because the chi-

Semi-leptonic decays $B \to \pi \ell \nu$, $B_s \to K \ell \nu$



$B_s \to K$:

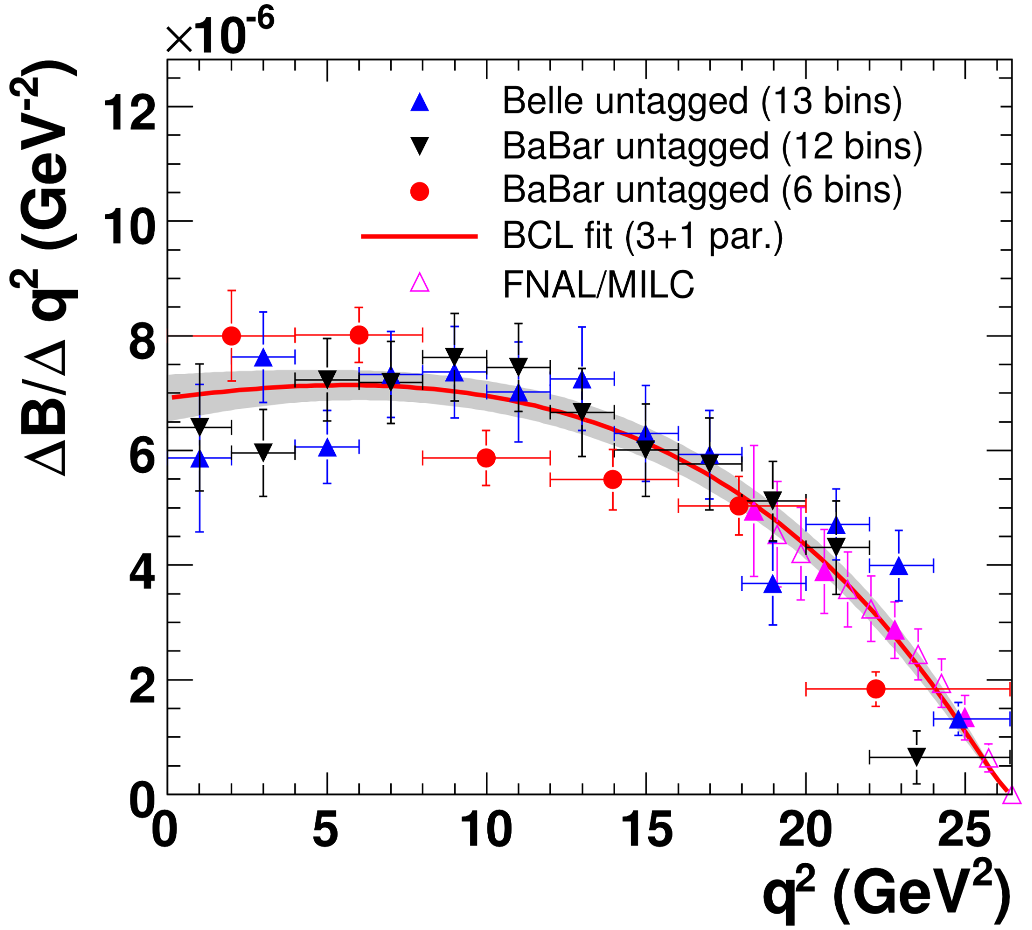
- no experimental data *yet* – predictions
- easier on the lattice (valence $m_K = m_K^{phys}$ computationally less expensive than for the π)
- not far from $B \to \pi$

$$\langle K(p_K^\mu) | V^\mu | B_s(p_{B_s}^\mu) \rangle = f_+(q^2) \left[p_{B_s}^\mu + p_K^\mu - \frac{m_{B_s}^2 - m_K^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_{B_s}^2 - m_K^2}{q^2} q^\mu$$

Experimental decay rates

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_{B_s}^3} \lambda^{3/2}(q^2) \left| f_+(q^2) \right|^2$$

$$\lambda(q^2) = (m_{B_s}^2 + m_K^2 - q^2)^2 - 4 m_{B_s}^2 m_K^2$$



- experimentally measured decay rate
- form factor $f_+(q^2)$ computed in LQCD
- $\Rightarrow$ determine V_{ub}

The essential steps

status

- obtain ground state ME's
  $\langle K|V^{\mu}(0)|B_s\rangle$
- satisfactory but improvable

- Renormalize currents and match to QCD
- unsatisfactory
  ("mostly non-perturbative" or 1-loop perturbative)

- Take the continuum limit (at each q^2)
- unsatisfactory

- Map out q^2 dependence (to make more use of experimental data)
- good, but a bit in conflict with the above

- physical light+strange quark masses
- quite good

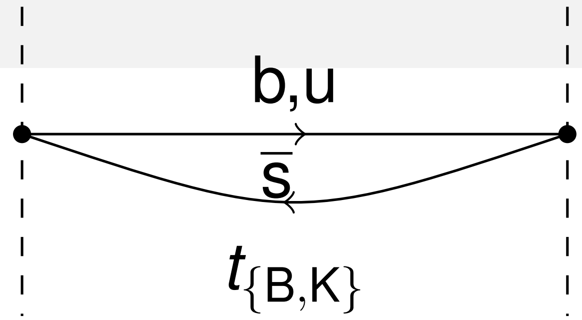
The essential steps

our contribution

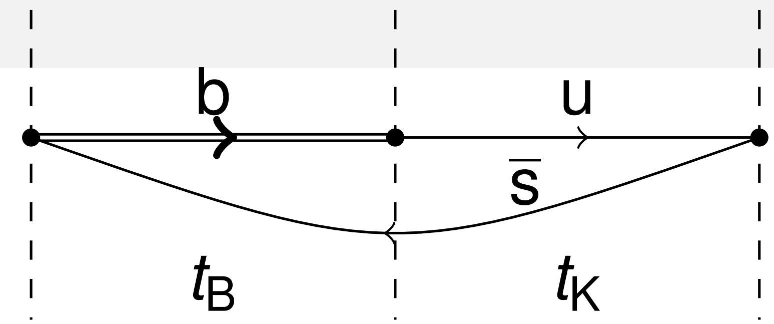
- obtain ground state ME's
  $\langle K|V^\mu(0)|B_s\rangle$
- improved methods
  - but not tested in praxis
- Renormalize currents and match to QCD
- solution in HQET
  (but no 1/m terms yet)
- Take the continuum limit
  (at each q^2)
- solution in HQET
  (but no 1/m terms yet)
- Map out q^2 dependence
  (to make more use of experimental data)
- single q^2
- physical light+strange quark masses
- "soon"

Obtaining the form factor

$\mathcal{C}^{\{B,K\}} \sim$



$\mathcal{C}^3 \sim$



Ratio – plateaux

$$\langle \mathrm{K}(p_{\mathrm{K}}^{\theta}) | V^{\mu} | \mathrm{B_s}(0) \rangle = \lim_{T, t_B, t_K \to \infty} \frac{\mathcal{C}^3_{\mu}(t_K, t_B)}{\sqrt{\mathcal{C}^K(t_K)\mathcal{C}^B(t_B)}} \mathrm{e}^{E_K t_K/2} \mathrm{e}^{E_B t_B/2} \equiv \lim_{T, t_B, t_K \to \infty} f_{\mu}^{\mathrm{ratio}}(q^2)$$

Factorising Fit

Combined fit to ground and first excited state of $\mathcal{C}^3, \mathcal{C}^B$

$$\begin{cases} \mathcal{C}^3_{\mu\, i}(t_B, t_K) & = \sum_{n,m} \beta_i^{(n)} \varphi_{\mu}^{(n,m)} \kappa^{(m)} \mathrm{e}^{-E_B^{(n)} t_B} \mathrm{e}^{-E_K^{(m)} t_K}, \qquad \varphi_{\mu}^{(1,1)} \sim f_+(q^2) \\ \mathcal{C}^B_{ij}(t_B) & = \sum_n \beta_i^{(n)} \beta_j^{(n)} \mathrm{e}^{-E_B^{(n)} t_B} \\ \mathcal{C}^K(t_K) & = \sum_m (\kappa^{(m)})^2 \mathrm{e}^{-E_K^{(m)} t_K} \end{cases}$$

- Gaussian smearing, $\psi_l^{\mathrm{sm}}(x) = (1 + \kappa\Delta)^{N_{\mathrm{it}}} \psi_l(x)$, $N_{\mathrm{it}} \leftrightarrow$ wavefunctions
- random noise sources, full time dilution

HQET expansion (because $m_b > 1/a$)

expansion in Λ/m_b, $|p|/m_b$

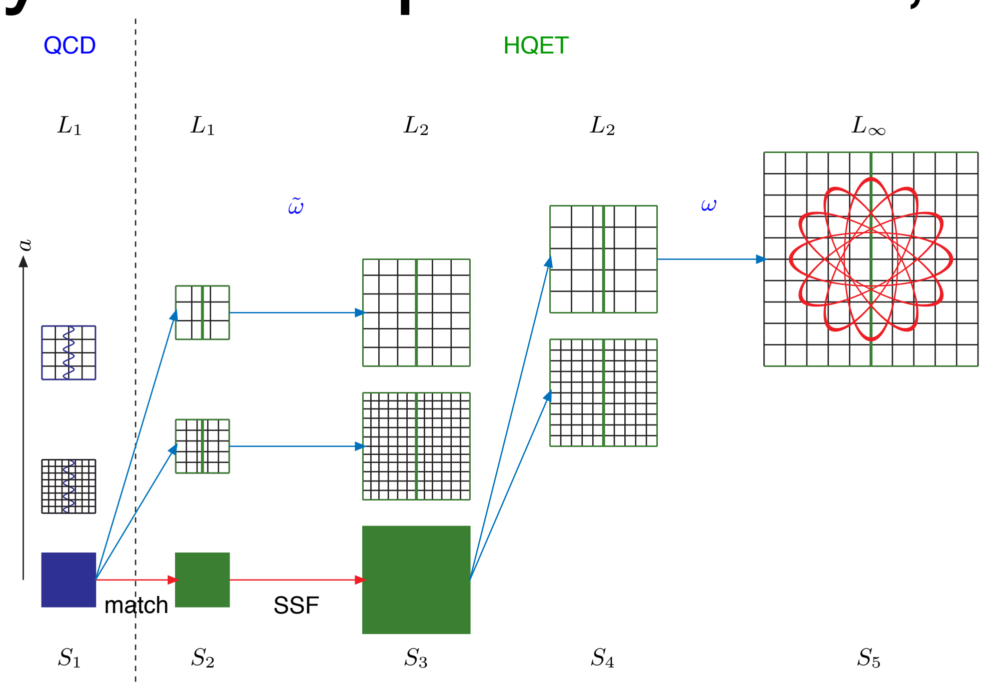
step scaling->
fully non-perturbative

$$f_+ = f_+^{\rm stat} \times [1 + {\rm O}(1/m_{\rm b})] \ ,$$

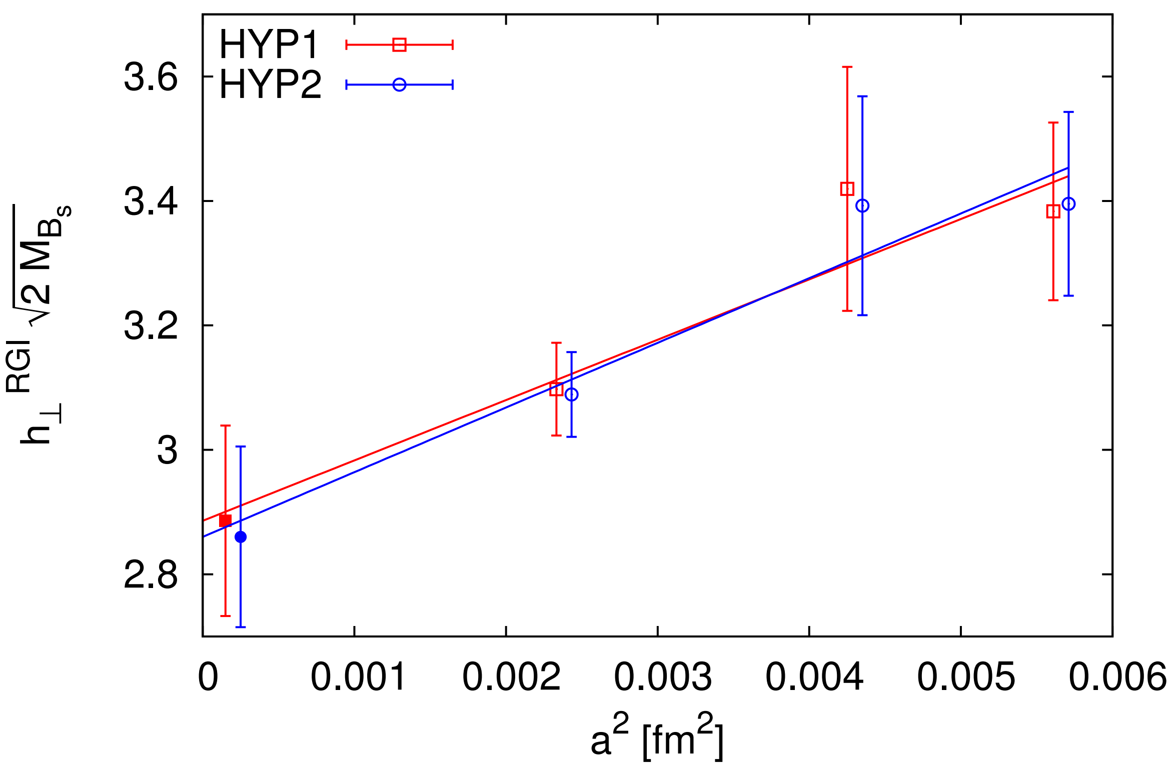
$$f_+^{\rm stat} = \sqrt{m_{\rm B_s}/2}\left((1 - \frac{E_{\rm K}}{m_{\rm B_s}})\, C_{\rm V_k}\, h_\perp^{\rm stat,RGI}(E_{\rm K}) + \frac{1}{m_{\rm B_s}}\, C_{\rm V_0}\, h_\parallel^{\rm stat,RGI}(E_{\rm K}) \right)$$

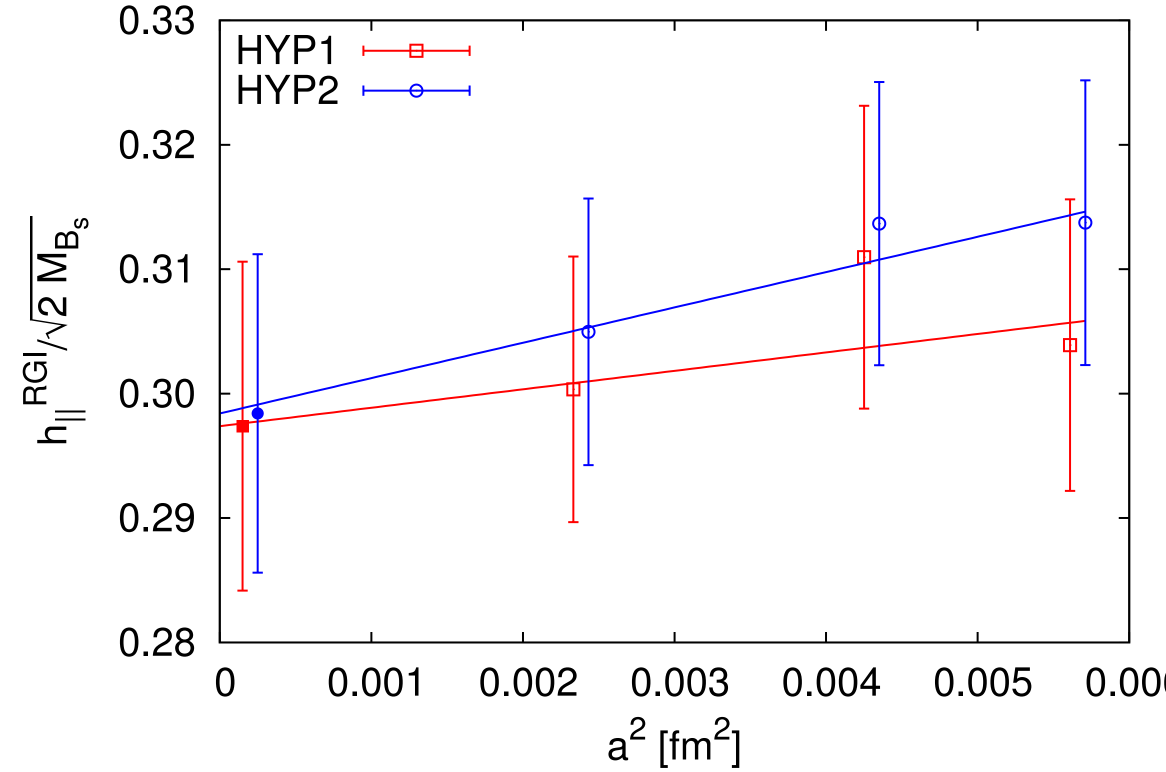
perturbative, 3-loop

To be replaced by all-non-perturbative, with 1/m terms



Continuum extrapolation

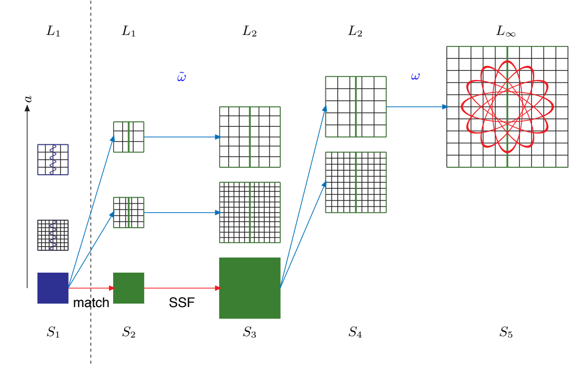




Felix Bahr[a], Debasish Banerjee[a], Fabio Bernardoni[a,b], Anosh Joseph[c], Mateusz Koren[a], Hubert Simma[a], Rainer Sommer[a]

Report-no: DESY 16-009, arxiv: Jan 18, 2016

Comparison



$$f_+(21.22\,\mathrm{GeV}^2) = 1.63(8)(6) \pm 0.24$$ ALPHA

$$f_+(21.22\,\mathrm{GeV}^2) \approx 1.65(10)$$ Flynn et al. (RBC/UKQCD)

$$f_+(21.22\,\mathrm{GeV}^2) \approx 1.80(20)$$ Bouchard et al. (HPQCD)

$$f_0(21.22\,\mathrm{GeV}^2) = 0.66(3)(1)$$ ALPHA

$$f_0(21.22\,\mathrm{GeV}^2) \approx 0.62(5)$$ Flynn et al. (RBC/UKQCD)

$$f_0(21.22\,\mathrm{GeV}^2) \approx 0.66(5)$$ Bouchard et al. (HPQCD)

Very different systematics, mutual confirmation

V_{ub} puzzle remains

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Hubert Simma[a], Rainer Sommer[a]

Summary

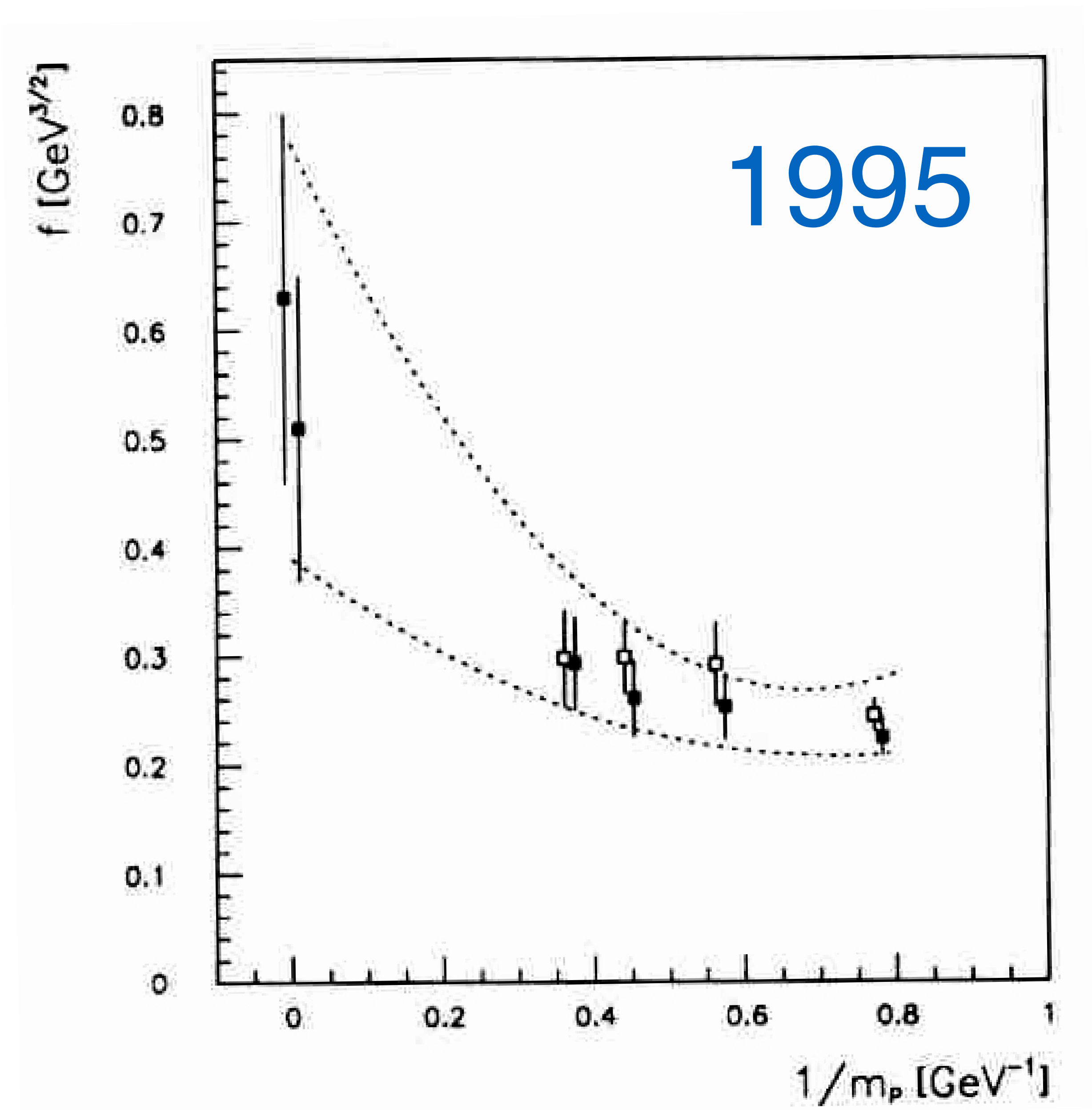
- $\alpha_{\overline{\rm MS}}$ is only a perturbative concept, ok for large μ,
  e.g. $\alpha_{\overline{\rm MS}}(m_{\rm Z})$

- lattice determinations confirm phenomenological determinations
  perturbative QCD = non-pert. QCD = QCD

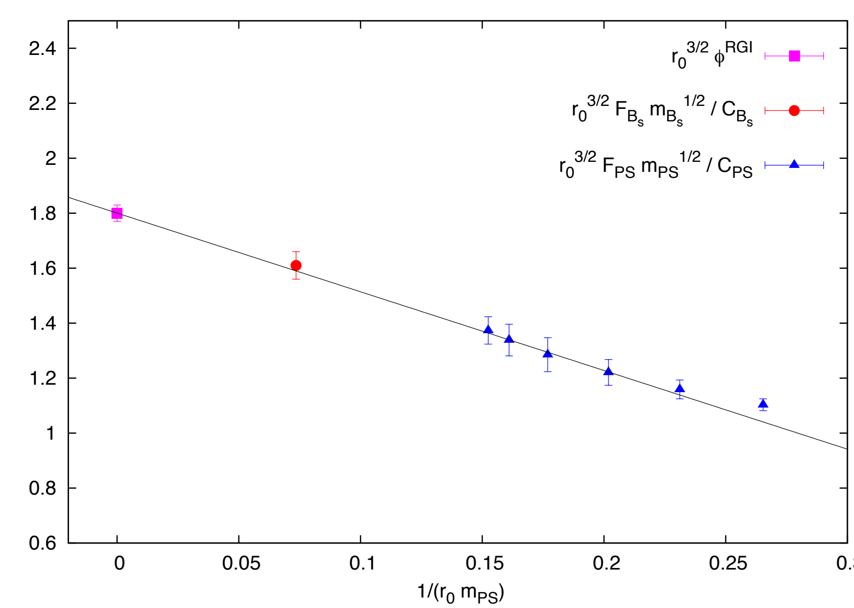
- Form factors for B-decays are challenging, but
  - overall agreement between different determinations
  - HQET approach promising
  - V_{ub} "puzzle" remains → theory for inclusive decays? → new physics?

History

- Lattice QCD has come a long way
- example of the history:



2010



continuum limit taken

transition amplitude for $\mathrm{B} \to \ell\nu$