

# Neutrino Physics with the PTOLEMY project



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# Pillars of the Cosmological Model

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- Hubble law

$d_L = (1+z) x$ ,  $x$  comoving distance



- CMB

black body distribution  $T = 2.275 \text{ }^\circ\text{K}$



- BBN

light nuclei forms at  $T = \text{MeV} - 10 \text{ keV}$



- Cosmic Neutrino Background (CMB)

direct measurement??

# CNB Relic neutrino production and decoupling

$$1 \text{ MeV} \leq T \leq m_\mu$$

$$T_\nu = T_e = T_\gamma$$

$$\nu_\alpha \nu_\beta \leftrightarrow \nu_\alpha \nu_\beta$$

$$\nu_\alpha \bar{\nu}_\beta \leftrightarrow \nu_\alpha \bar{\nu}_\beta$$

$$\nu_\alpha e^- \leftrightarrow \nu_\alpha e^-$$

$$\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$$

$$\mathcal{L}_{\text{SM}} = -2\sqrt{2}G_F \left\{ (\bar{\nu}_e \gamma^\mu L \nu_e) (\bar{e} \gamma_\mu L e) + \sum_{P,\alpha} g_P (\bar{\nu}_\alpha \gamma^\mu L \nu_\alpha) (\bar{e} \gamma_\mu P e) \right\}$$

$$P = L, R = (1 \mp \gamma_5)/2$$

$$g_L = -\frac{1}{2} + \sin^2 \theta_W \text{ and } g_R = \sin^2 \theta_W$$



# Neutrino decoupling

As the Universe expands, particle densities are diluted and temperatures fall. Weak interactions become ineffective to keep neutrinos in good thermal contact with the e.m. plasma

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Rough, but quite accurate estimate of the decoupling temperature

Rate of weak processes  $\sim$  Hubble expansion rate

$$\Gamma_w \approx \sigma_w |v| n, \quad H^2 = \frac{8\pi\rho_R}{3M_p^2} \rightarrow G_F^2 T^5 \approx \sqrt{\frac{8\pi\rho_R}{3M_p^2}} \rightarrow T_{dec}^v \approx 1 \text{ MeV}$$

Since  $\nu_e$  have both CC and NC interactions with  $e^\pm$

$$T_{dec}(\nu_e) \sim 2 \text{ MeV}$$

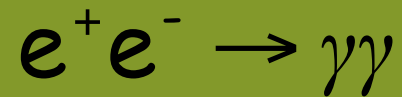
$$T_{dec}(\nu_{\mu,\tau}) \sim 3 \text{ MeV}$$



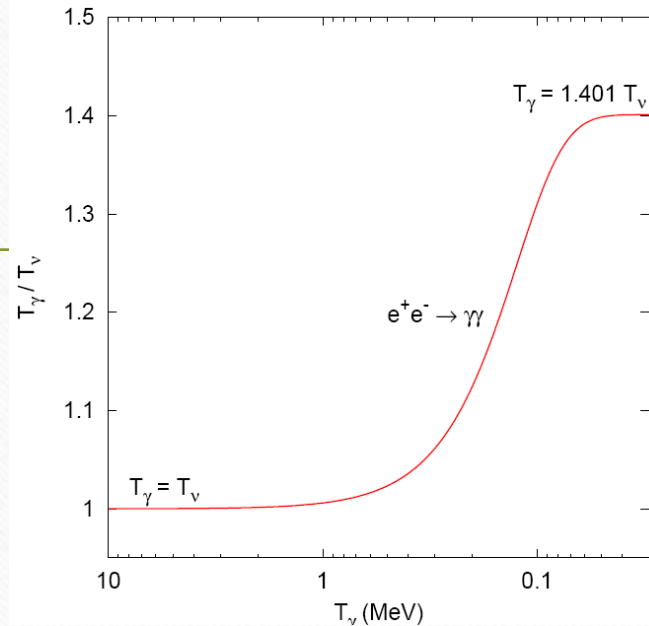
# Neutrino and Photon (CMB) temperatures

$$f_\nu(p, T) = \frac{1}{e^{p/T_\nu} + 1}$$

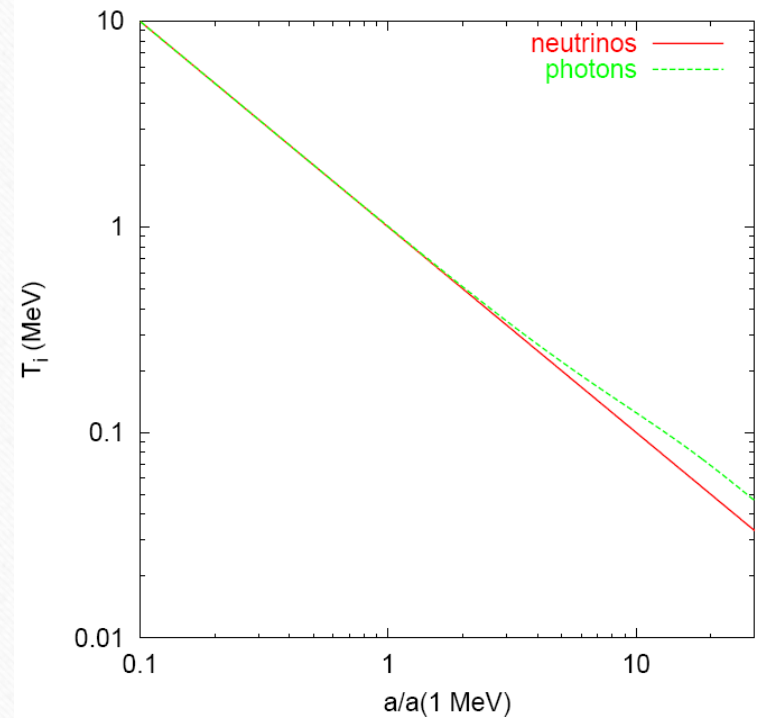
At  $T \sim m_e$ , electron-positron pairs annihilate



heating photons but not the decoupled neutrinos



$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3}$$



Neutrinos decoupled at  $T \sim \text{MeV}$ , keeping a spectrum as that of a relativistic species

$$f_\nu(p, T) = \frac{1}{e^{p/T_\nu} + 1}$$

$$n_\nu = \int \frac{d^3 p}{(2\pi)^3} f_\nu(p, T_\nu) = \frac{3}{11} n_\gamma = \frac{6\zeta(3)}{11\pi^2} T_{CMB}^3$$

At present  $112 \text{ cm}^{-3}$  per flavour

$$\rho_{\nu_i} = \int \sqrt{p^2 + m_{\nu_i}^2} \frac{d^3 p}{(2\pi)^3} f_\nu(p, T_\nu) \rightarrow \begin{cases} \frac{7\pi^2}{120} \left(\frac{4}{11}\right)^{4/3} T_{CMB}^4 & \text{Massless} \\ m_{\nu_i} n_\nu & \text{Massive } m_{\nu_i} \gg T \end{cases}$$

$$\Omega_\nu h^2 = 1.7 \times 10^{-5}$$

$$\Omega_\nu h^2 = \frac{\sum_i m_i}{94.1 \text{ eV}}$$

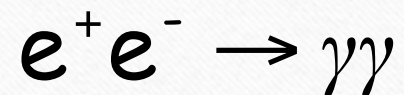


$N_{\text{eff}}$

$$\rho_R = \rho_\gamma + \rho_\nu + \rho_x = \left( 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}_\nu} \right) \rho_\gamma$$

# CNB details

At  $T \sim m_e$ ,  $e^+e^-$  pairs annihilate heating photons

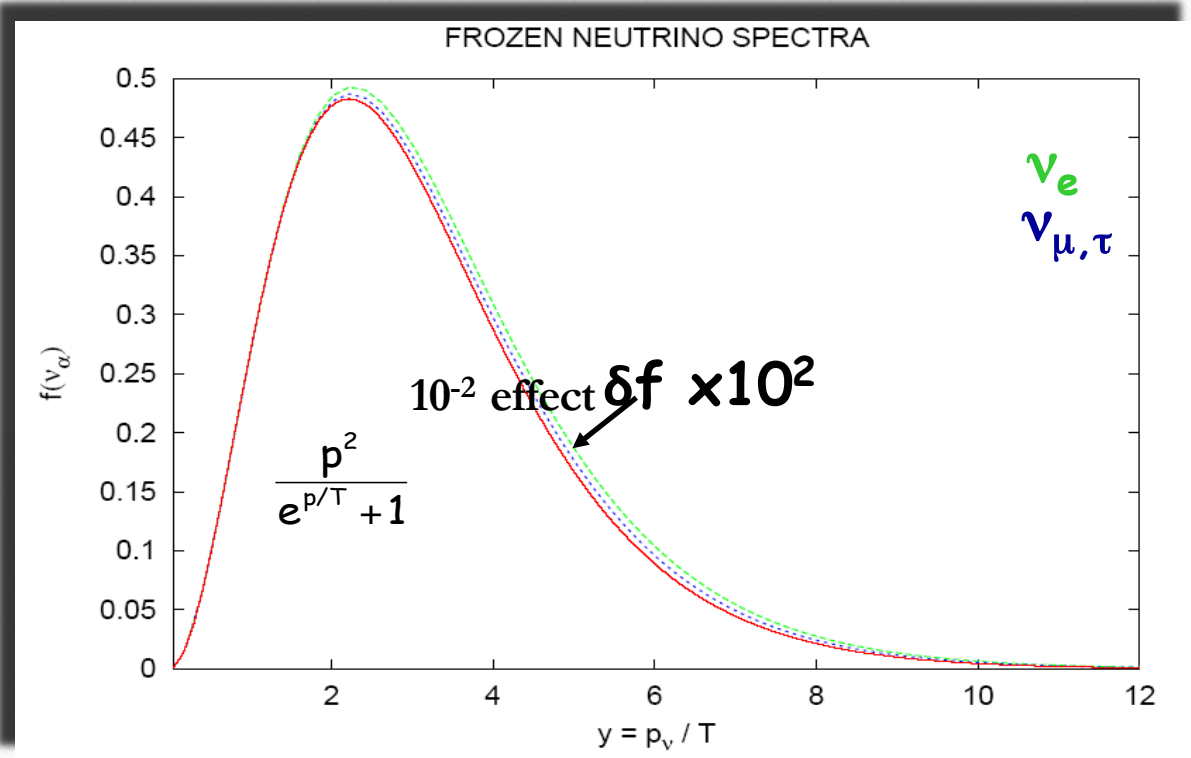
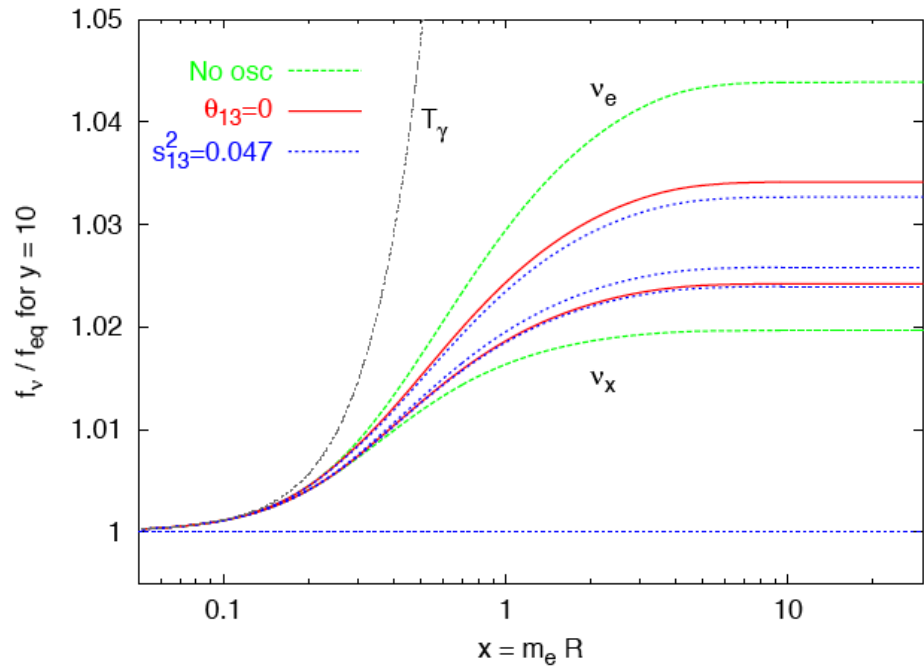


... and neutrinos. Non thermal features in  $\nu$  distribution (small effect). Oscillations slightly modify the result

$$f_\nu = f_{\text{FD}}(p, T_\nu)[1 + \delta f(p)]$$

$$(i\partial_t - Hp\partial_p)\rho = \left[ \frac{M^2}{p} - \frac{8\sqrt{2}G_F}{m_W^2} E, \rho \right] + C(\rho)$$





# Results

	$T_{fin}^\gamma / T_0^\gamma$	$\delta\rho_{\nu e}(\%)$	$\delta\rho_{\nu\mu}(\%)$	$\delta\rho_{\nu\tau}(\%)$	$N_{\text{eff}}$
Instantaneous decoupling	1.40102	0	0	0	3
<b>SM</b>	1.3978	0.94	0.43	0.43	3.046
<b>+3ν mixing (<math>\theta_{13}=0</math>)</b>	1.3978	0.73	0.52	0.52	3.046
<b>+3ν mixing (<math>\sin^2\theta_{13}=0.047</math>)</b>	1.3978	0.70	0.56	0.52	3.046

Dolgov, Hansen & Semikoz, NPB 503 (1997) 426

G.M. et al, PLB 534 (2002) 8

G.M. et al, NPB 729 (2005) 221



# CvB details

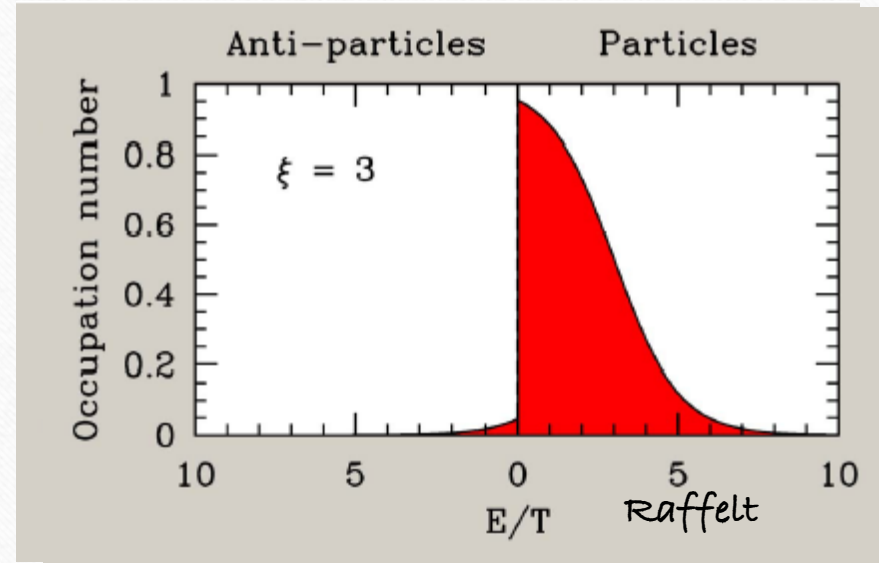
Fermi-Dirac spectrum with  
temperature  $T$  and chemical potential

$$\mu_\nu = \xi_\nu T_\nu$$

$$n_\nu \neq n_{\bar{\nu}}$$

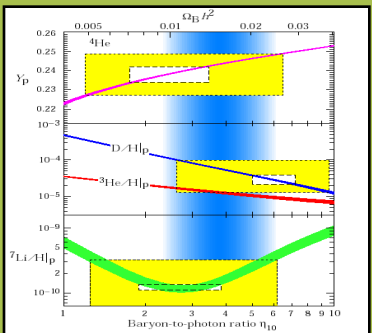
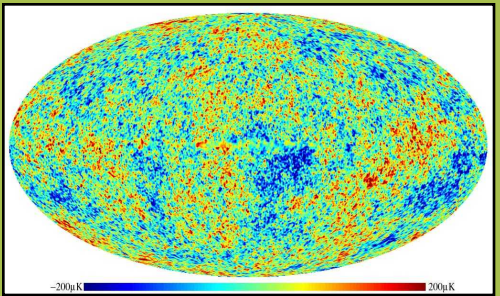
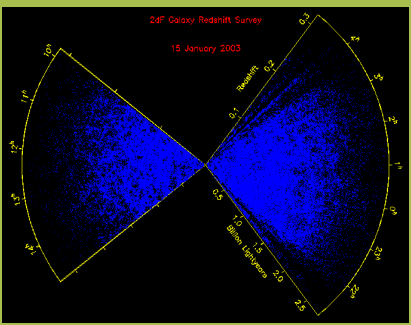


$$L_\nu = \frac{n_\nu - n_{\bar{\nu}}}{n_\gamma} = \frac{1}{12\zeta(3)} \left(\frac{T_\nu}{T_\gamma}\right)^3 \left[ \pi^2 \xi_\nu + \xi_\nu^3 \right]$$



$$\Delta\rho_\nu = \frac{15}{7} \left[ 2 \left(\frac{\xi_\nu}{\pi}\right)^2 + \left(\frac{\xi_\nu}{\pi}\right)^4 \right]$$

# CNB indirect evidences

 <p>The graph shows the yields of <math>^4\text{He}</math>, <math>\text{D}/\text{H}</math>, <math>^3\text{He}/\text{H}</math>, and <math>^7\text{Li}/\text{H}</math> as a function of the baryon-to-photon ratio <math>\eta_{10}</math>. The x-axis ranges from 1 to 10, and the y-axis shows the yield ratios. Shaded regions indicate observational constraints.</p>	 <p>A map of the Cosmic Microwave Background (CMB) showing temperature fluctuations across the sky. A color scale at the bottom indicates temperatures from -200K (blue) to 200K (red).</p>	 <p>A map of the Galaxy Redshift Survey (LSS) showing the distribution of galaxies. The map is dated 15 January 2003 and includes labels for '2dF Galaxy Redshift Survey' and 'Galaxy Redshift Survey'.</p>
<p>Primordial Nucleosynthesis BBN</p>	<p>Cosmic Microwave Background CMB</p>	<p>Formation of Large Scale Structures LSS</p>
<p><math>T \sim \text{MeV}</math></p>	<p><math>T &lt; \text{eV}</math></p>	
<p>flavor dependent</p>	<p>Flavor blind</p>	



## BBN: almost seventy years after $\alpha\beta\gamma$ seminal paper( Alpher, Bethe & Gamow 1948)

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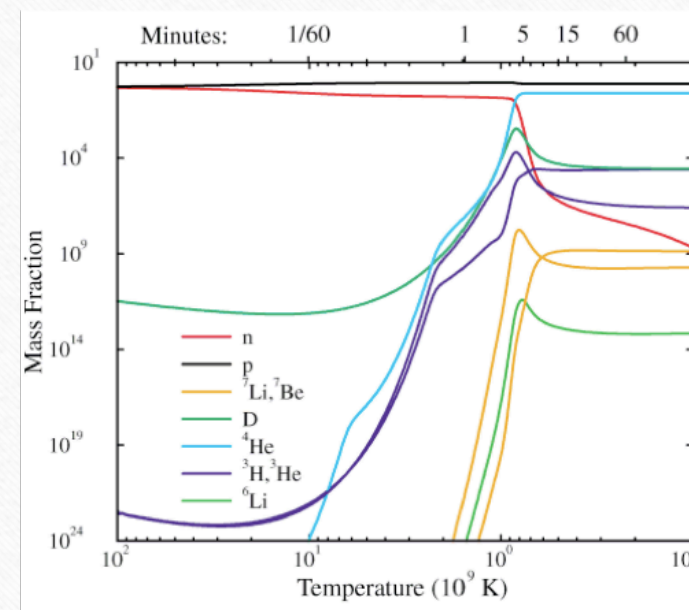
- ◆ Theory reasonably under control (per mille level for  $^4\text{He}$  (neutron lifetime), 1-2 % for  $^2\text{H}$ );
- ◆ Increased precision in nuclear reaction cross sections at low energy (underground lab's);
- ◆  $\Omega_{\text{b}}h^2$  measured by WMAP/Planck with high precision;
- ◆ Decreasingly precise data ( $^4\text{He}$ , but see later),  $^7\text{Li}$  not understood,  $^2\text{H}$  fixes  $\Omega_{\text{b}}h^2$  value in good agreement with CMB data.

# THEORY

weak rate freeze out (1 MeV);  
 $^2\text{H}$  forms at  $T \sim 0.08$  MeV;  
 nuclear chain;

Z \ N	0	1	2	3	4	5	6	7	8
0		n							
1	H	$^2\text{H}$	$^3\text{H}$						
2		$^3\text{He}$	$^4\text{He}$						
3				$^6\text{Li}$	$^7\text{Li}$	$^8\text{Li}$			
4				$^7\text{Be}$		$^9\text{Be}$			
5				$^8\text{B}$		$^{10}\text{B}$	$^{11}\text{B}$	$^{12}\text{B}$	
6						$^{11}\text{C}$	$^{12}\text{C}$	$^{13}\text{C}$	$^{14}\text{C}$
7						$^{12}\text{N}$	$^{13}\text{N}$	$^{14}\text{N}$	$^{15}\text{N}$
8							$^{14}\text{O}$	$^{15}\text{O}$	$^{16}\text{O}$

Public numerical codes: Kawano,  
 PArthENoPE, PRIMAT  
 private numerical codes: many...



Iocco et al, Phys Rept. 472, 1 (2009)



## Weak rates:

- radiative corrections  $O(\alpha)$
- finite nucleon mass  $O(T/M_N)$
- plasma effects  $O(\alpha T/m_e)$
- neutrino decoupling  $O(G_F^2 T^3 m_{Pl})$

$$N_{\text{eff}} \approx 3.046$$

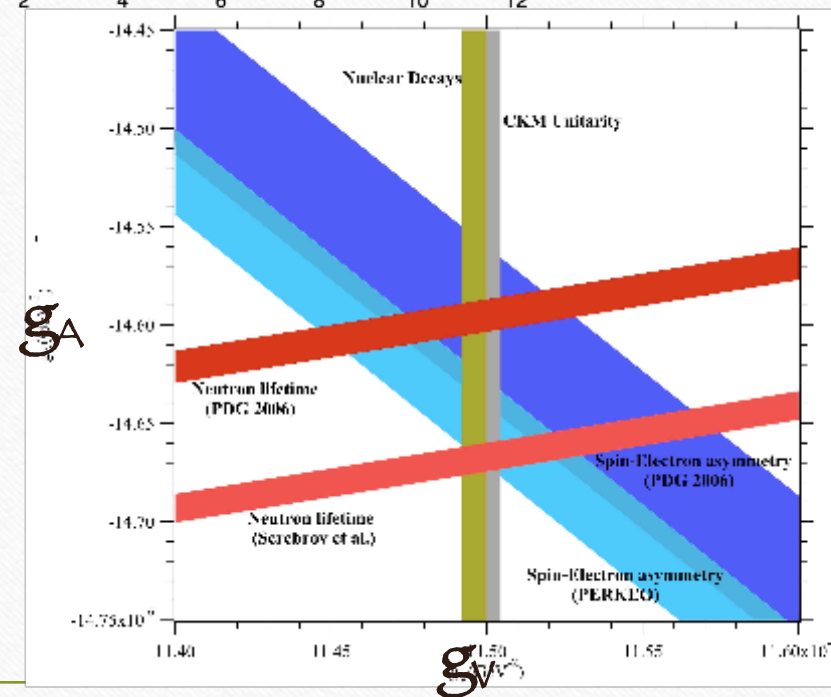
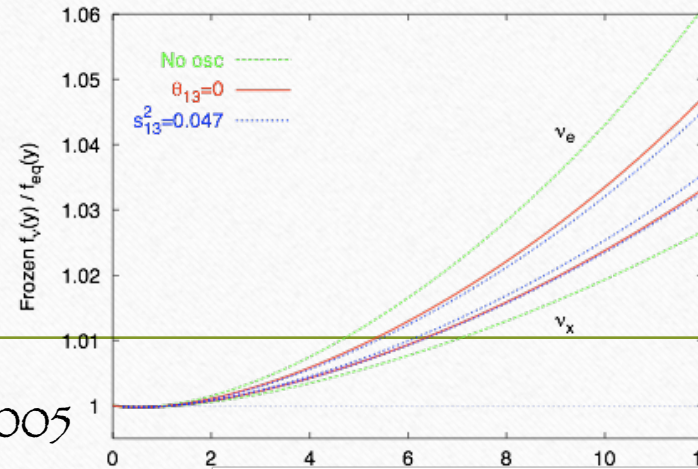
G.M. et al 2005

- Main uncertainty: neutron lifetime
- $\tau_n = 885.6 \pm 0.8$  sec (old PDG mean)
- $\tau_n = 878.5 \pm 0.8$  sec (Serebrov et al 2005)

Presently:

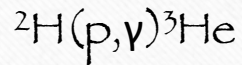
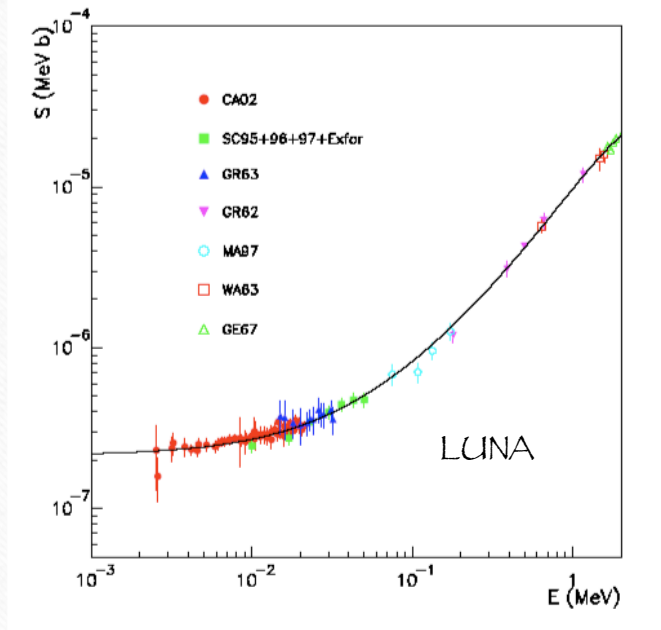
$$\tau_n = 880.3 \pm 1.1 \text{ sec}$$

$^4\text{He}$  mass fraction  $Y_P$  linearly increases with  $\tau_n$ : 0.246 - 0.249



# Nuclear rates:

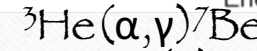
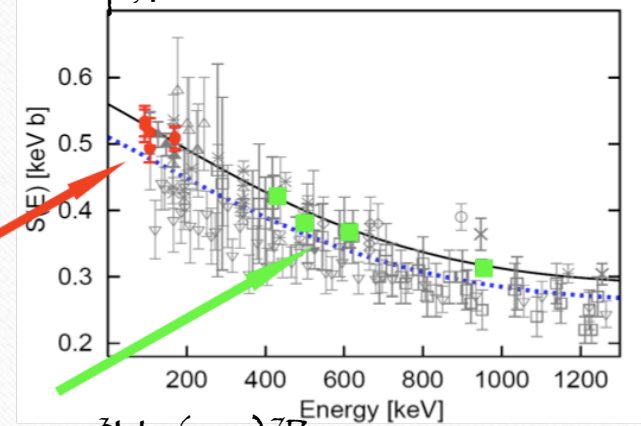
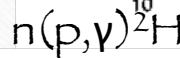
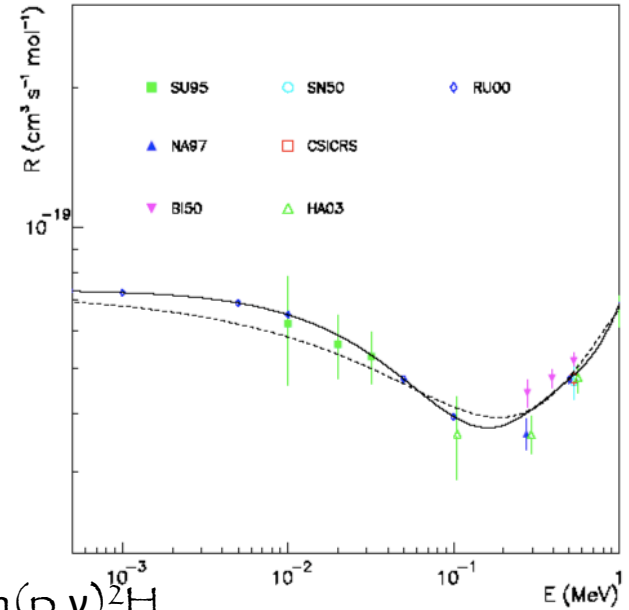
main input from experiments  
 low energy range ( $10^2$  KeV)  
 major improvement: underground  
 measurements (e.g. LUNA at LNGS)



LUNA

Weitzmann Inst.

Rupak



ERNA:  $S(0) = 0.57 \pm 0.04$  KeVb DiLeva et al 2010



# DATA

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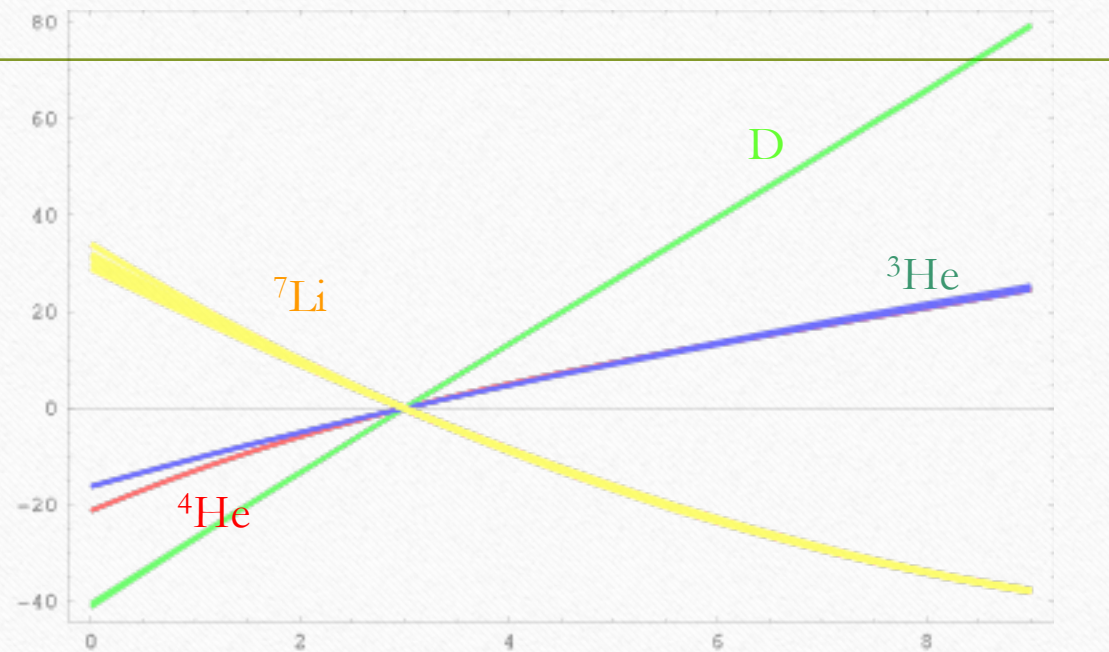
## The quest for primordiality

- ◆ Observations in systems negligibly contaminated by stellar evolution (e.g. high redshift);
- ◆ Careful account for galactic chemical evolution.

# Effect of neutrinos on BBN

1.  $N_{\text{eff}}$  fixes the expansion rate during BBN

$$\rho_R = \rho_\gamma + \rho_\nu + \rho_x = \left( 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff},\nu} \right) \rho_\gamma$$



2. Direct effect of electron neutrinos and antineutrinos  
on the n-p reactions

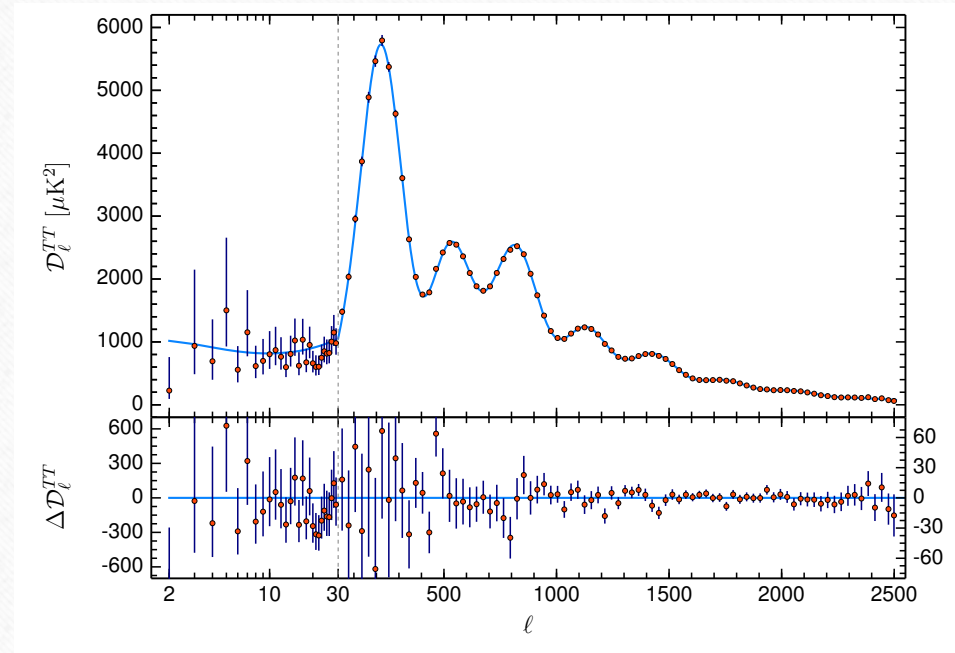




# The CMB

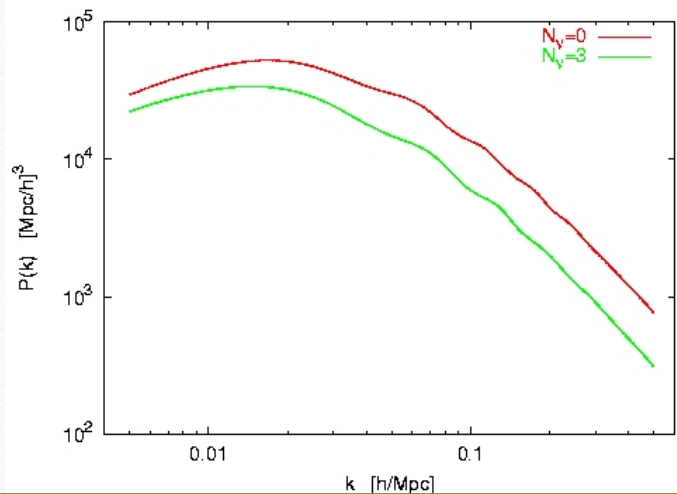
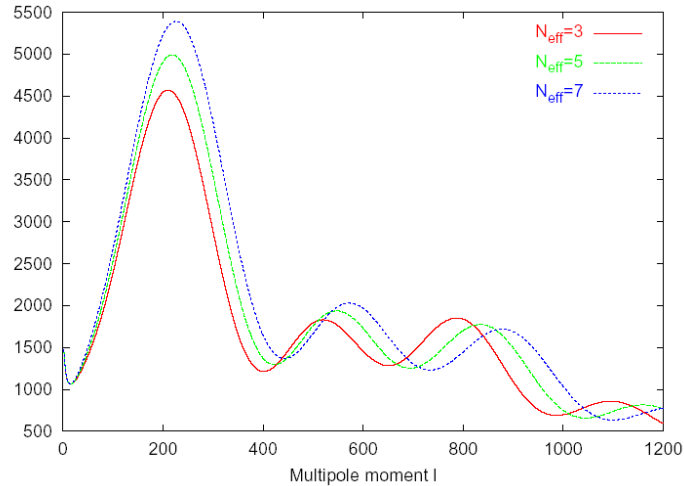
Anisotropies contain so much information about the cosmological model!

Planck 2018



# Effect of CNB on CMB and LSS

Mean effect (Sachs-Wolfe, M-R equality)+ perturbations



Perturbations

Acoustic peak and damping tail:  $N_{\text{eff}}$

Lensing potential on CMB:  $m_\nu$  larger expansion rate suppresses clustering

Large Scale Structure: suppression at small scales  
 $k > 0.1$  h Mpc $^{-1}$



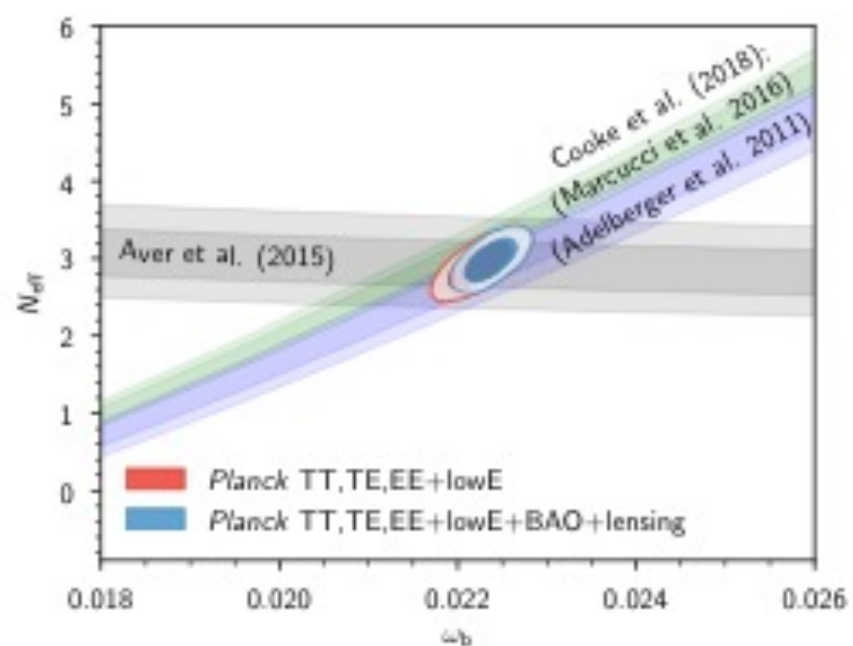
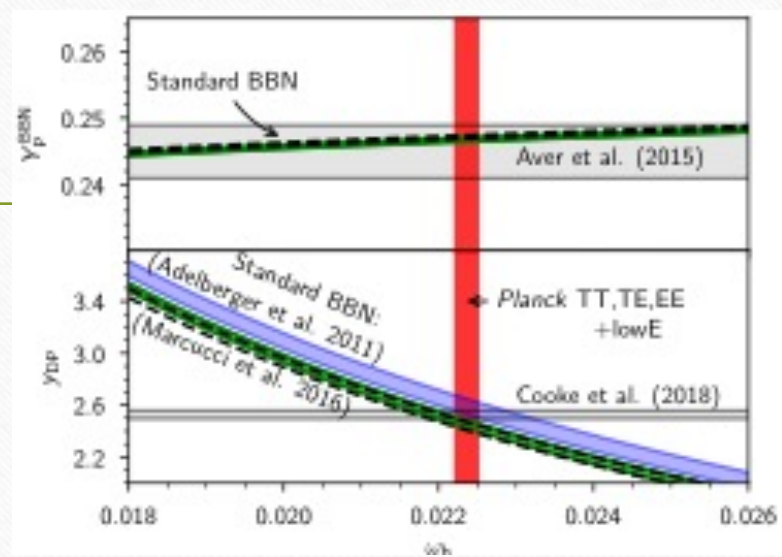
# CMB+LSS: allowed ranges for $N_{\text{eff}}$

Set of parameters:  $(\Omega_b h^2, \Omega_{\text{cdm}} h^2, h, n_s, A, b, N_{\text{eff}})$

- DATA: Planck , Flat Models
- 

$$N_{\text{eff}} = 3.11^{+0.44}_{-0.43} \quad (95 \%, \text{ TT+lowE+lensing+BAO});$$

$$N_{\text{eff}} = 2.99^{+0.34}_{-0.33} \quad (95 \%, \text{ TT,TE,EE+lowE+lensing+BAO}).$$





# Neutrino masses

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Planck 2018

$$\sum m_\nu < 0.44 \text{ eV} \quad (95 \%, \text{ TT+lowE+lensing}),$$

$$\sum m_\nu < 0.24 \text{ eV} \quad (95 \%, \text{ TT,TE,EE+lowE+lensing}).$$

# CNB direct detection

CNB: very low energy, difficult to measure directly by  $\nu$ -scattering

1. Large De Broglie wavelength  $\lambda \sim 0.1$  cm

Coherent scattering over nuclei (or macroscopic domain)

Wind force on a test body,

Cross section

$$\sigma_{\nu N} \sim 10^{-56} (m_\nu / \text{eV})^2 \text{ cm}^2 \text{ non relativistic}$$

$$\sigma_{\nu N} \sim 10^{-63} (T_\nu / \text{eV})^2 \text{ cm}^2 \text{ relativistic}$$

acceleration

$$n_\nu \beta NA/A \sigma_{\nu N} dp \sim (100/A) 10^{-51} (m_\nu / \text{eV}) \text{ cm s}^{-2}$$

Today: Cavendish torsion balances can test acceleration as small as  $10^{-13}$  cm s<sup>-2</sup> !!



## 2. Accelerators:

Too small even at LHC or beyond !

## 3. Effects linear in $G_F$ :

No go theorem (Cabibbo & Maiani, Langacker et al) effect vanishes if  
static source - background interaction

Homogeneous  $\nu$  flux on the target scale

Stodolski effect: polarized electron target experiences a torque due to helicity energy splitting in  
presence of a polarized (asymmetry) neutrino wind

$$dE \sim g_A \vec{\sigma} \cdot \vec{\beta} (n_\nu - n_{\bar{\nu}})$$

A '62 paper by S. Weinberg and  $\nu$  chemical potential

PHYSICAL REVIEW

VOLUME 128, NUMBER 3

NOVEMBER 1, 1962

## Universal Neutrino Degeneracy

STEVEN WEINBERG\*

*Imperial College of Science and Technology, London, England*

(Received March 22, 1962)

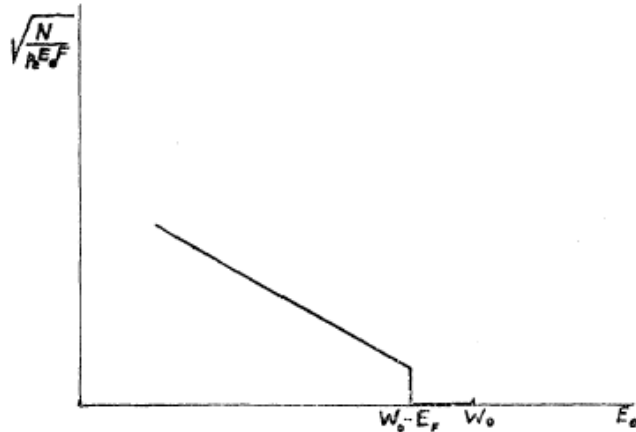


FIG. 1. Shape of the upper end of an allowed Kurie plot to be expected in a  $\beta^+$  decay if neutrinos are degenerate up to energy  $E_F$ , or in a  $\beta^-$  decay if antineutrinos are degenerate.

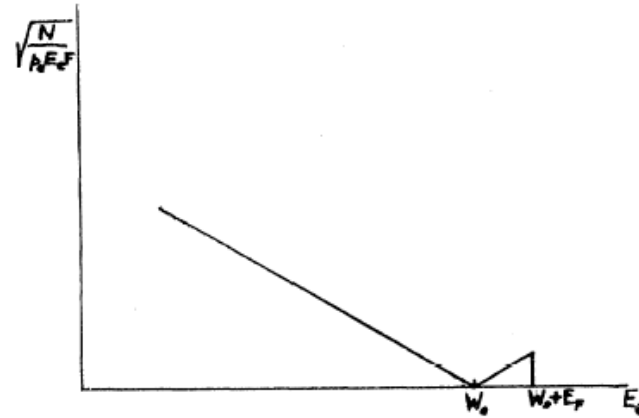


FIG. 2. Shape of the upper end of an allowed Kurie plot to be expected in a  $\beta^-$  decay if neutrinos are degenerate up to energy  $E_F$ , or in a  $\beta^+$  decay if antineutrinos are degenerate.



Neutrino-antineutrino asymmetry ( $\xi = \mu/T_\nu$ ,  $E_F(\xi)$ ) strongly constrained by Big Bang Nucleosynthesis

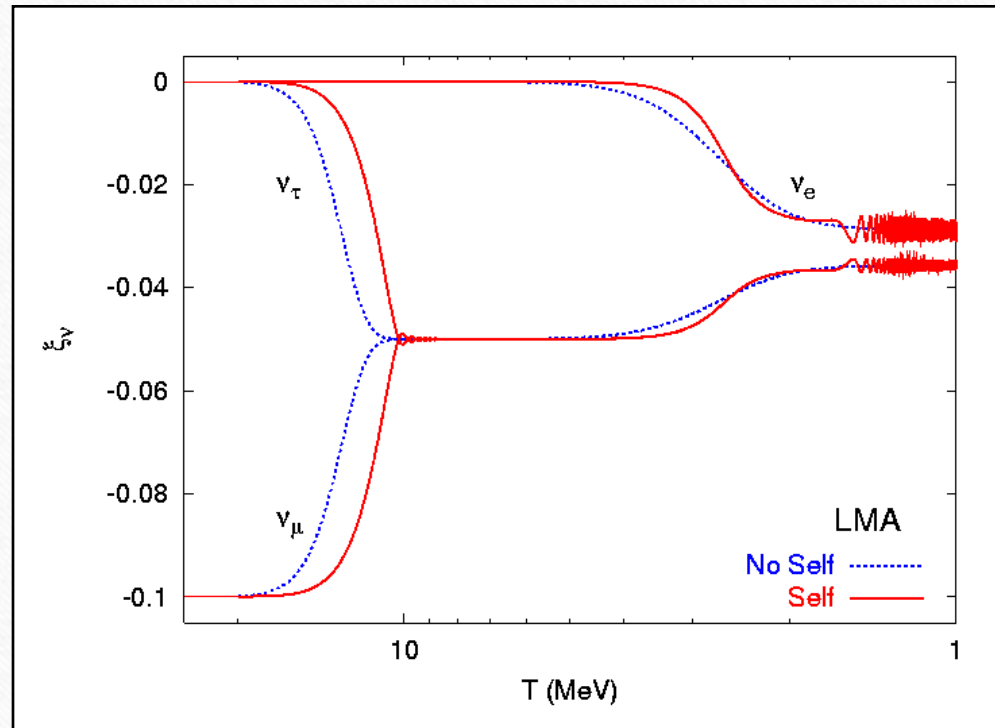
- 1) chemical potentials contribute to neutrino energy density**

$$\rho_\nu = \frac{7\pi^2}{120} \left( 3 + \sum_i \left( \frac{30\xi_i^2}{7\pi^2} + \frac{15\xi_i^4}{7\pi^4} \right) + \dots \right) T_\nu^4$$

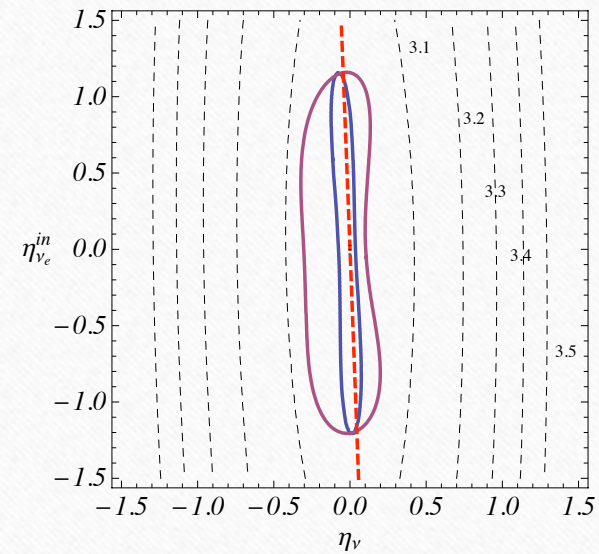
- 2) a positive electron neutrino chemical potential (more neutrinos than antineutrinos) favour  $n \rightarrow p$  processes with respect to  $p \rightarrow n$  processes.**

**Change the  $^4\text{He}$  abundance!**

Though different neutrino flavor may have different chemical potentials, they however mix under oscillations



Likelihood contours 68 & 95 c.l.



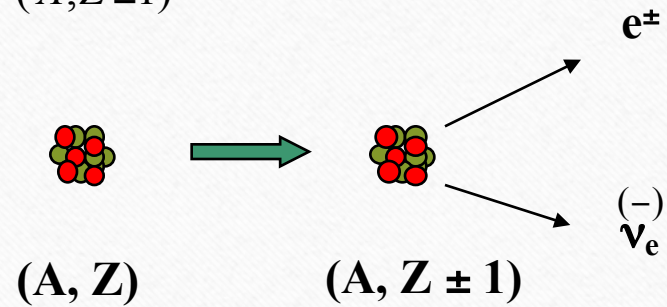
$\xi$  very small!



$$N_{(A,Z)} \rightarrow N'_{(A,Z\pm 1)} e^{\pm} \bar{\nu}_e^{(-)}$$

$$\bar{\nu}_e^{(-)} N_{(A,Z)} \rightarrow N'_{(A,Z\pm 1)} e^{\pm}$$

Beta decay



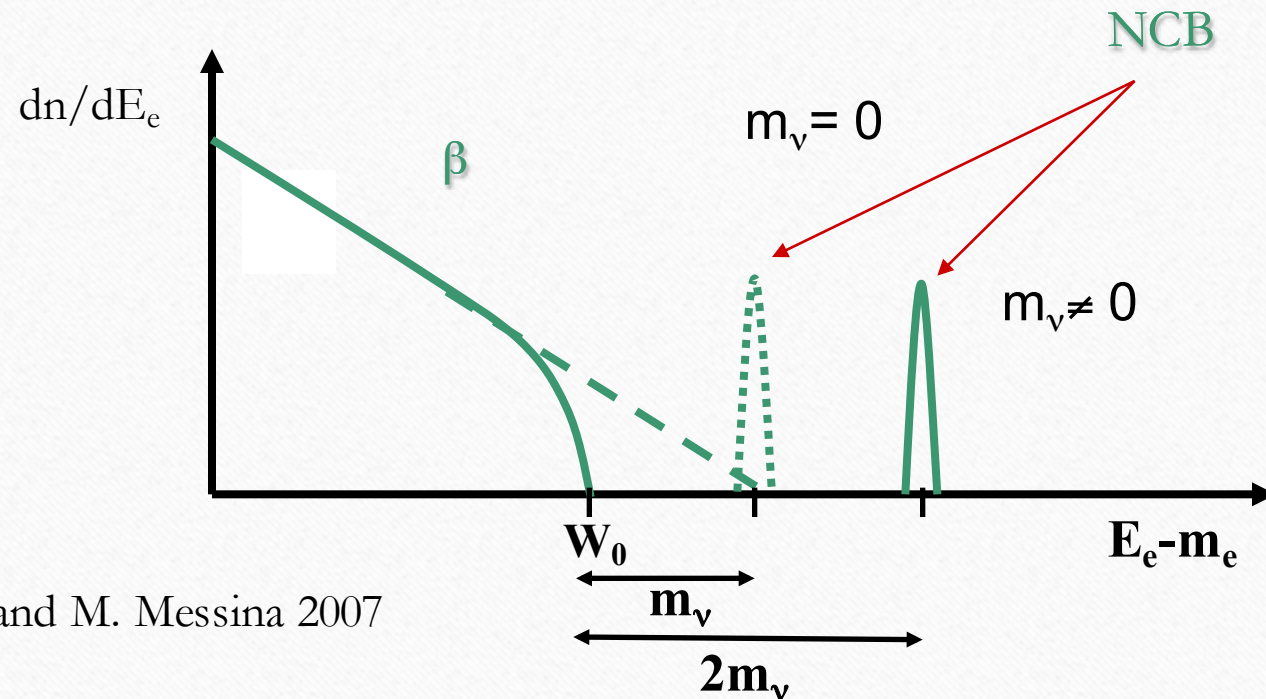
Neutrino Capture on a  
Beta Decaying Nucleus  
(NC $\beta$ )



Weinberg: if neutrinos are degenerate we could observe structures around the beta decaying nuclei endpoint  $Q$

$\nu$ 's are NOT degenerate but are massive!

$2 m_\nu$  gap in electron spectrum around  $Q$



A. Cocco, G.M. and M. Messina 2007



# Neutrino masses

## Terrestrial bounds

$$\nu_e < 2 \text{ eV (} ^3\text{H decay)}$$

$$\nu_\mu < 0.19 \text{ MeV (pion decays)}$$

$$\nu_\tau < 18.2 \text{ MeV (\tau decays)}$$

## CMB Planck 2018

$$\sum m_\nu < 0.44 \text{ eV (95 \% , TT+lowE+lensing),}$$

$$\sum m_\nu < 0.24 \text{ eV (95 \% , TT,TE,EE+lowE+lensing).}$$

## Oscillation Parameters

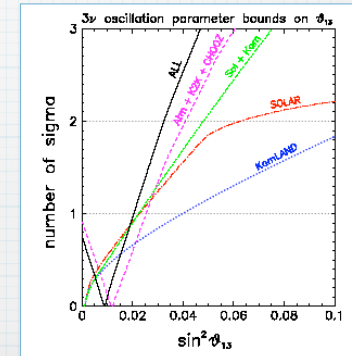
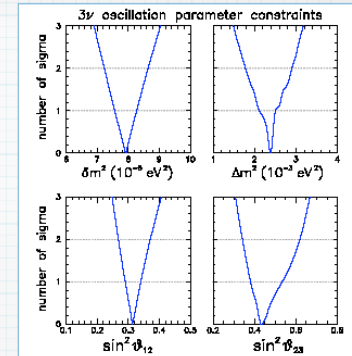
$$\delta m^2 = 7.92(1 \pm 0.09) \times 10^{-5} \text{ eV}^2$$

$$\sin^2 \theta_{12} = 0.314(1_{-0.15}^{+0.18})$$

$$\Delta m^2 = 2.6(1_{-0.15}^{+0.14}) \times 10^{-3} \text{ eV}^2$$

$$\sin^2 \theta_{23} = 0.45(1_{-0.20}^{+0.35})$$

$$\sin^2 \theta_{13} = 0.8(1_{-0.8}^{+2.3}) \times 10^{-2}$$



## Issues:

### 1. Rates

$$\lambda_\nu = \int \sigma_{\text{NCB}} v_\nu f(p_\nu) \frac{d^3 p_\nu}{(2\pi)^3}, = \frac{G_\beta^2}{2\pi^3} \int_{W_o+2m_\nu}^{\infty} p_e E_e F(Z, E_e) C(E_e, p_\nu)_\nu \cdot E_\nu p_\nu f(p_\nu) dE_e,$$

$$\lambda_\beta = \frac{G_\beta^2}{2\pi^3} \int_{m_e}^{W_o} p_e E_e F(Z, E_e) C(E_e, p_\nu)_\beta E_\nu p_\nu dE_e,$$

Nuclear form factors (shape factors) uncertainties: use beta observables

$$A = \int_{m_e}^{W_o} \frac{C(E'_e, p'_\nu)_\beta}{C(E_e, p_\nu)_\nu} \frac{p'_e}{p_e} \frac{E'_e}{E_e} \frac{F(E'_e, Z)}{F(E_e, Z)} E'_\nu p'_\nu dE'_e$$

$$\sigma_{\text{NCB}} v_\nu = \frac{2\pi^2 \ln 2}{A t_{1/2}}$$



## Cross sections times $v_\nu$ as high as $10^{-41}$ cm<sup>2</sup> c

**Table 1.** The product  $\sigma_{\text{NCB}}(v_\nu/c)$  for the best known superallowed  $0^+ \rightarrow 0^+$  transitions. Numerical values for  $Q_\beta$  and partial half-lives are taken from [33]. The value of  $f$  is calculated adopting the parametrization of the Fermi function of [28].

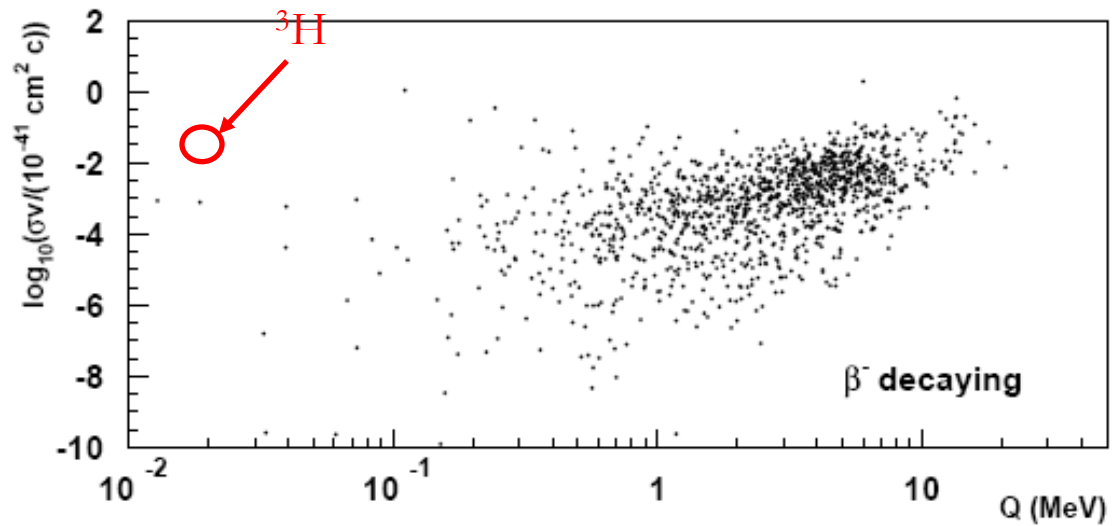
Isotope	$Q_\beta$ (keV)	Half-life (sec)	$\sigma_{\text{NCB}}(v_\nu/c)$ ( $10^{-41}$ cm <sup>2</sup> )
<sup>10</sup> C	885.87	1320.99	$5.36 \times 10^{-3}$
<sup>14</sup> O	1891.8	71.152	$1.49 \times 10^{-2}$
<sup>26m</sup> Al	3210.55	6.3502	$3.54 \times 10^{-2}$
<sup>34</sup> Cl	4469.78	1.5280	$5.90 \times 10^{-2}$
<sup>38m</sup> K	5022.4	0.92512	$7.03 \times 10^{-2}$
<sup>42</sup> Sc	5403.63	0.68143	$7.76 \times 10^{-2}$
<sup>46</sup> V	6028.71	0.42299	$9.17 \times 10^{-2}$
<sup>50</sup> Mn	6610.43	0.28371	$1.05 \times 10^{-1}$
<sup>54</sup> Co	7220.6	0.19350	$1.20 \times 10^{-1}$

A. Cocco, G.M. and M. Messina 2007

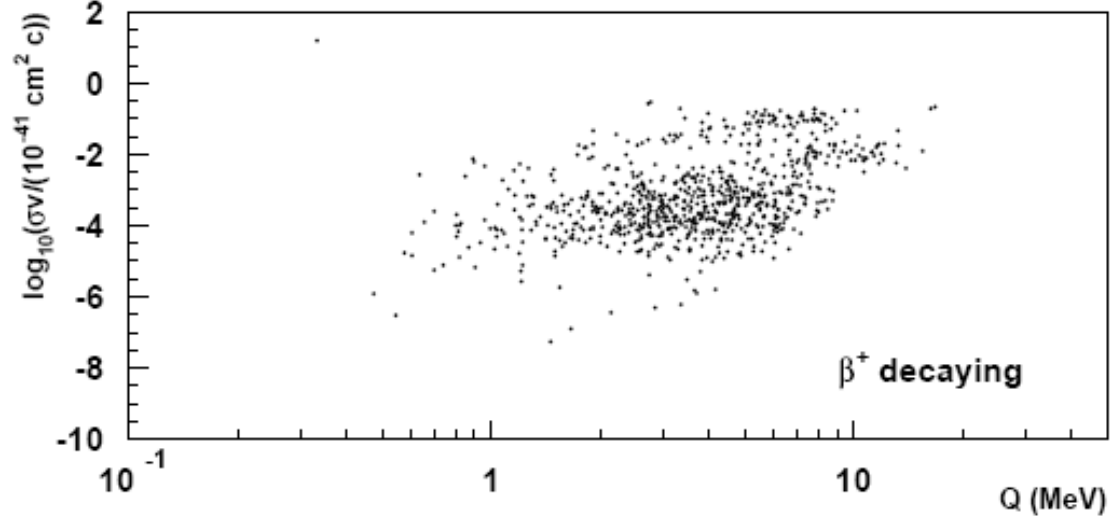
**Table 2.** Beta decaying nuclei that present the largest product of  $\sigma_{\text{NCB}}(v_\nu/c) \cdot t_{1/2}$  for low neutrino momentum and have a  $\beta^\pm$  decay branching fraction larger than 80%.

Isotope	Decay	$Q_\beta$ (keV)	Half-life (sec)	$\sigma_{\text{NCB}}(v_\nu/c)$ ( $10^{-41}$ cm <sup>2</sup> )
<sup>3</sup> H	$\beta^-$	18.591	$3.8878 \times 10^8$	$7.84 \times 10^{-4}$
<sup>63</sup> Ni	$\beta^-$	66.945	$3.1588 \times 10^9$	$1.38 \times 10^{-6}$
<sup>93</sup> Zr	$\beta^-$	60.63	$4.952 \times 10^{13}$	$2.39 \times 10^{-10}$
<sup>106</sup> Ru	$\beta^-$	39.4	$3.2278 \times 10^7$	$5.88 \times 10^{-4}$
<sup>107</sup> Pd	$\beta^-$	33	$2.0512 \times 10^{14}$	$2.58 \times 10^{-10}$
<sup>187</sup> Re	$\beta^-$	2.64	$1.3727 \times 10^{18}$	$4.32 \times 10^{-11}$
<sup>11</sup> C	$\beta^+$	960.2	$1.226 \times 10^3$	$4.66 \times 10^{-3}$
<sup>13</sup> N	$\beta^+$	1198.5	$5.99 \times 10^2$	$5.3 \times 10^{-3}$
<sup>15</sup> O	$\beta^+$	1732	$1.224 \times 10^2$	$9.75 \times 10^{-3}$
<sup>18</sup> F	$\beta^+$	633.5	$6.809 \times 10^3$	$2.63 \times 10^{-3}$
<sup>22</sup> Na	$\beta^+$	545.6	$9.07 \times 10^7$	$3.04 \times 10^{-7}$
<sup>45</sup> Ti	$\beta^+$	1040.4	$1.307 \times 10^4$	$3.87 \times 10^{-4}$

Beta decaying nuclei having  $BR(\beta^\pm) > 5\%$  selected from 14543 decays listed in the ENSDF database



1272  $\beta^-$  nuclei

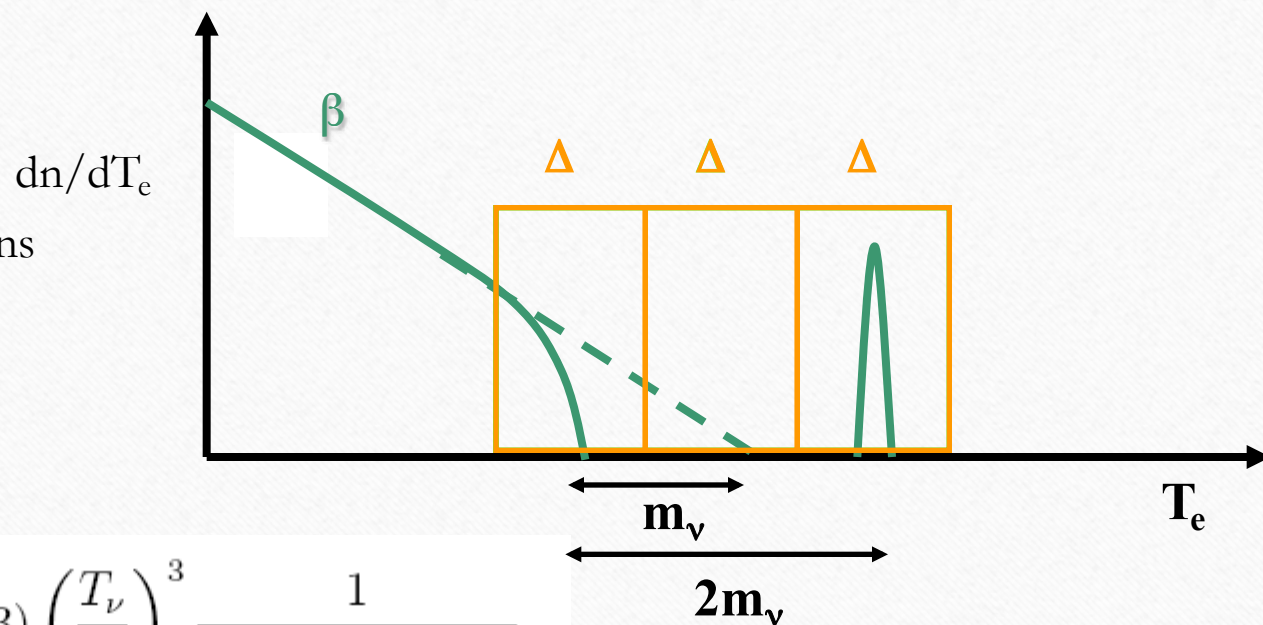


799  $\beta^+$  nuclei



## 2. Background

Observing the last energy bins  
of width  $\Delta$



$$\frac{\lambda_\nu}{\lambda_\beta(\Delta)} = \frac{9}{2} \zeta(3) \left( \frac{T_\nu}{\Delta} \right)^3 \frac{1}{(1 + 2m_\nu/\Delta)^{3/2}},$$

signal/background  $> 1$

$$\frac{9}{2} \zeta(3) \left( \frac{T_\nu}{\Delta} \right)^3 \frac{1}{(1 + 2m_\nu/\Delta)^{3/2}} \rho \geq 1,$$

$$\rho = \frac{1}{\sqrt{2\pi}} \int_{2m_\nu/\Delta - 1/2}^{2m_\nu/\Delta + 1/2} e^{-x^2/2} dx.$$

It works for  $\Delta < m_\nu$

- Clustering and  $\nu$  local density

Massive neutrinos cluster on CDM and baryonic structures. The local density at Earth (8 kpc away from the galactic center) is expected to be larger than  $56 \text{ cm}^{-3}$

$$\frac{\partial f_i}{\partial \tau} + \frac{\mathbf{p}}{am_i} \cdot \frac{\partial f_i}{\partial \mathbf{x}} - am_i \nabla \phi \cdot \frac{\partial f_i}{\partial \mathbf{p}} = 0,$$

$$\nabla^2 \phi = 4\pi G a^2 \sum_i \bar{\rho}_i(\tau) \delta_i(\mathbf{x}, \tau),$$

$$\delta_i(\mathbf{x}, \tau) \equiv \frac{\rho_i(\mathbf{x}, \tau)}{\bar{\rho}_i(\tau)} - 1, \quad \rho_i(\mathbf{x}, \tau) = \frac{m_i}{a^3} \int d^3p f_i(\mathbf{x}, \mathbf{p}, \tau),$$

Neutrinos accrete when their velocity becomes comparable with protocluster velocity dispersion ( $z < 2$ )

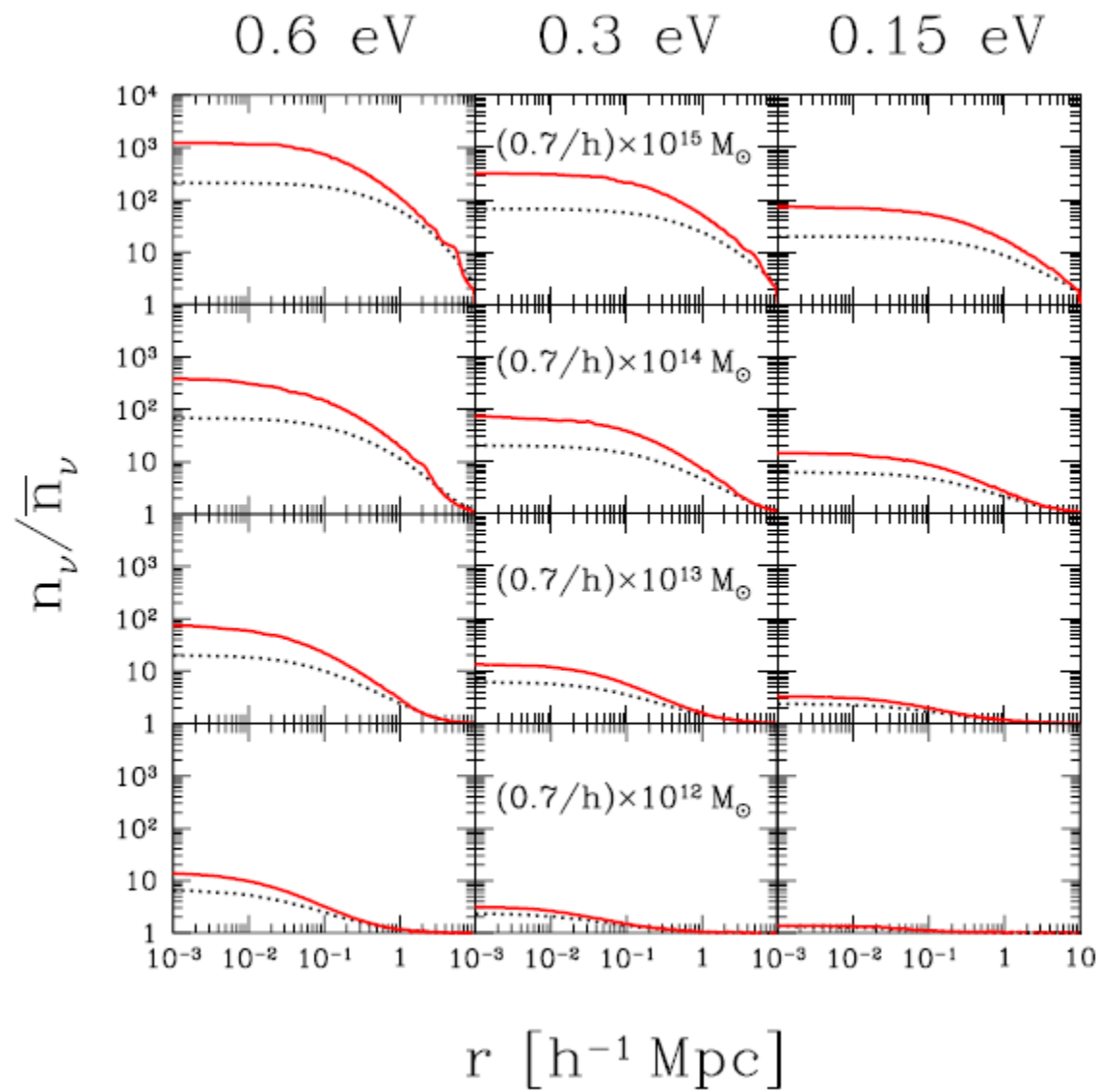
Usual assumption: Halo profile governed by CDM only

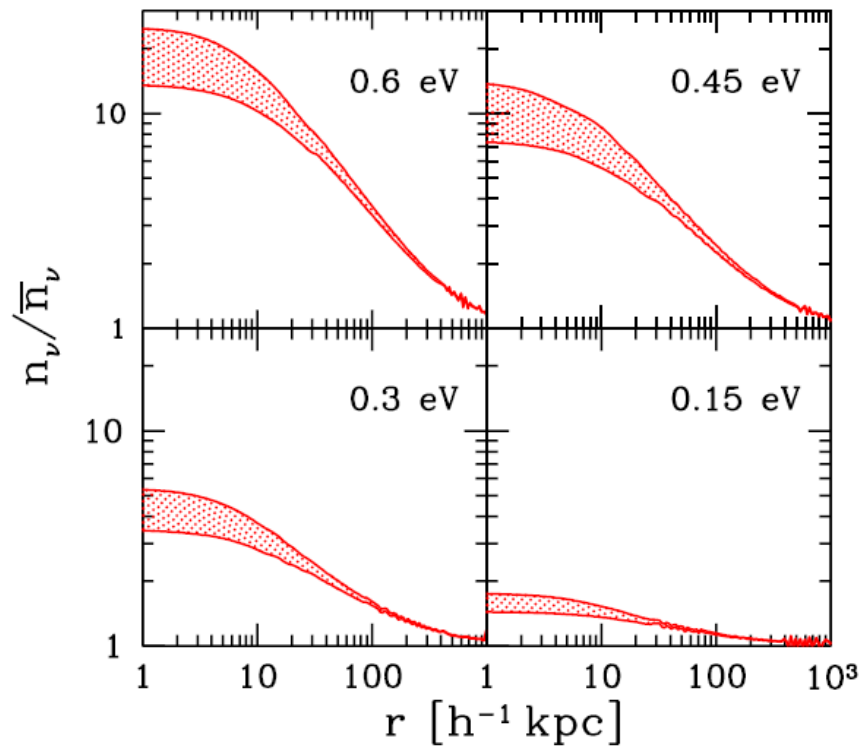
NFW universal profile

$$\rho_{\text{halo}}(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2},$$



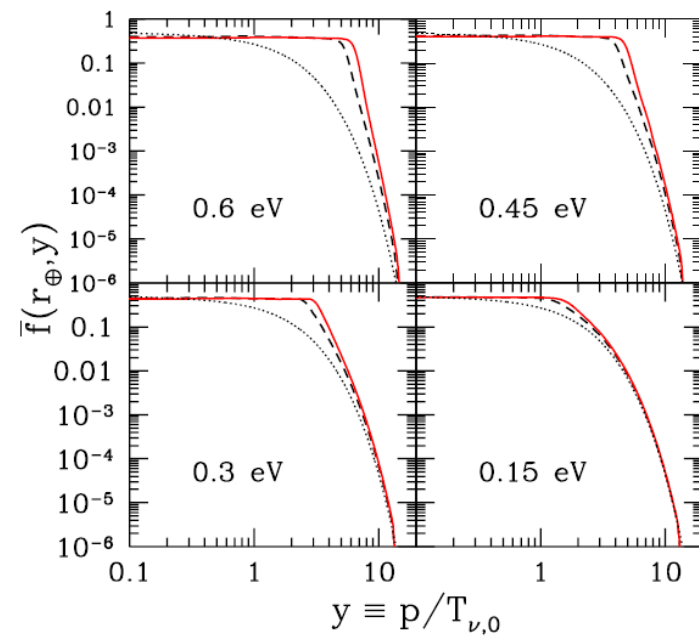
**A. Ringwald and Y. Wong 2004**  
**N-1-body simulations**





Milky Way

Top curve: NFW    Bottom curve:  
static present MW matter profile



**A. Ringwald and Y. Wong 2004**  
**N-1-body simulations**



The case of  ${}^3\text{H}$

$$\lambda_\beta = 2.85 \cdot 10^{-2} \frac{\sigma_{\text{NCB}} v_\nu / c}{10^{-45} \text{cm}^2} \text{yr}^{-1} \text{mol}^{-1}, \quad \sigma_{\text{NCB}}({}^3\text{H}) \frac{v_\nu}{c} = (7.84 \pm 0.03) \times 10^{-45} \text{cm}^2,$$

$m_\nu$ (eV)	FD (events yrs $^{-1}$ )	NFW (events yrs $^{-1}$ )	MW (events yrs $^{-1}$ )
0.6	7.5	90	150
0.3	7.5	23	33
0.15	7.5	10	12

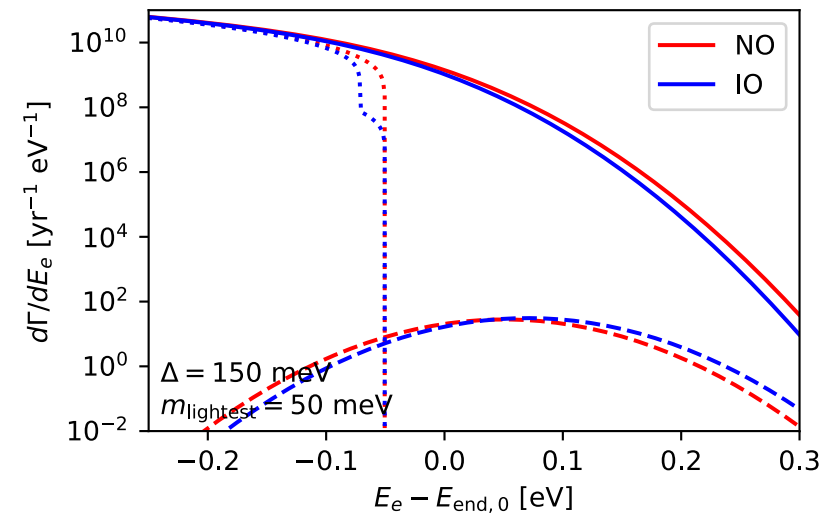
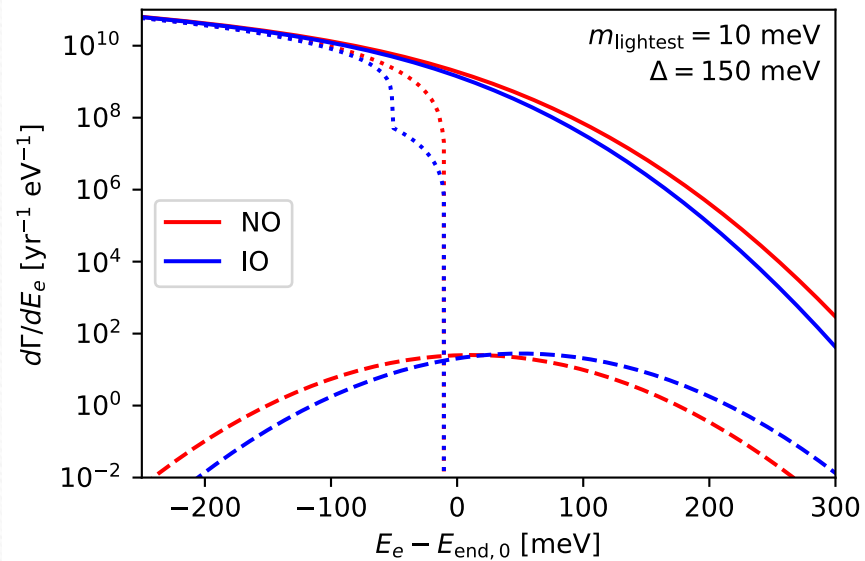
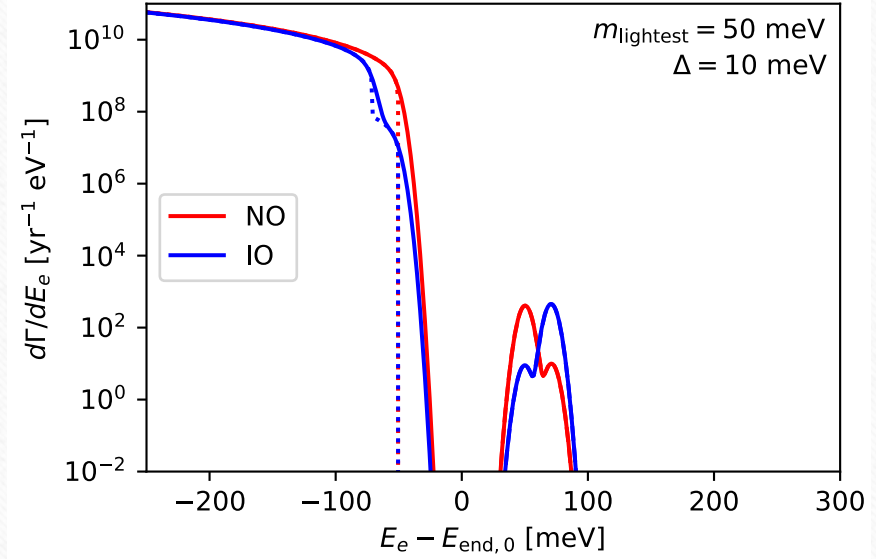
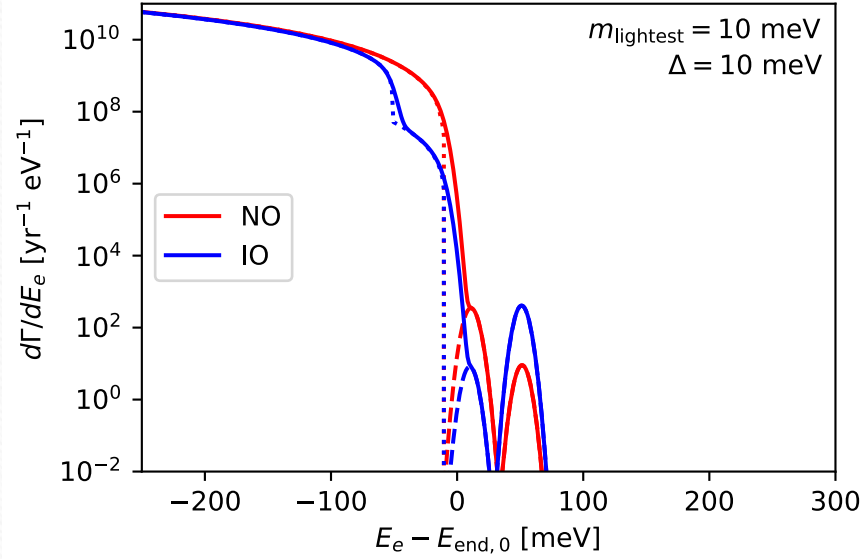
The number of NCB events per year for 100 g of  ${}^3\text{H}$

8 events yr $^{-1}$  per 100g of  ${}^3\text{H}$  (no clustering)

up to  $10^2$  events yr $^{-1}$  per 100 g of  ${}^3\text{H}$  due to clustering effect

signal/background = 3 for  $\Delta=0.2$  eV if  $m_\nu=0.7$  eV

$\Delta=0.1$  eV if  $m_\nu=0.3$  eV

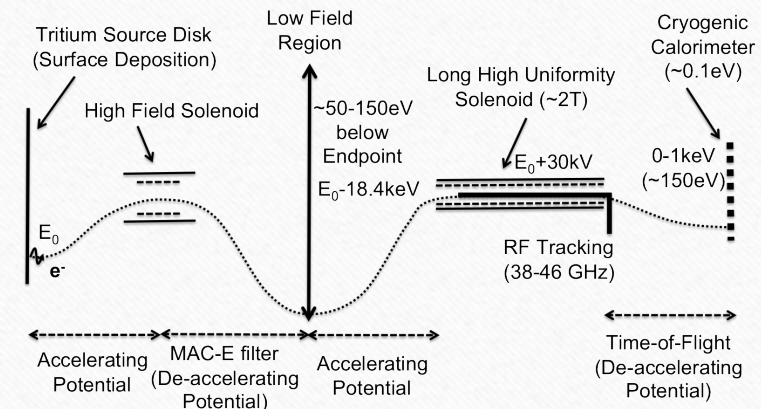
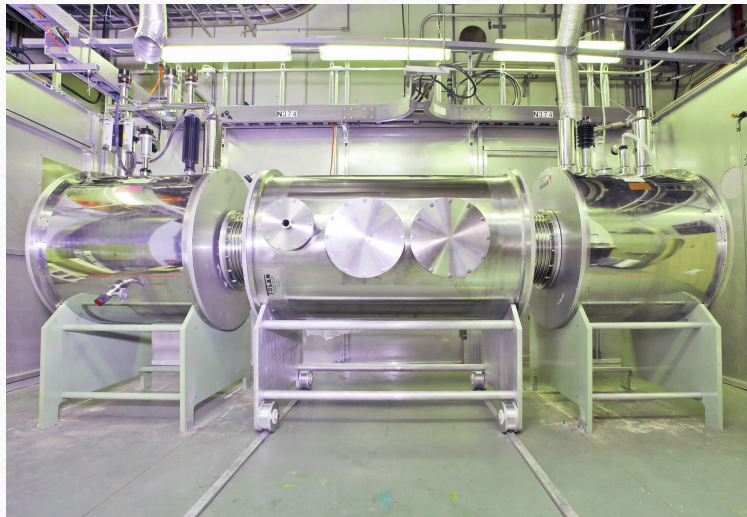




# The Ptolemy Project

Development of a Relic Neutrino Detection Experiment at PTOLEMY:  
Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield

Pontecorvo



INFN Laboratori Nazionali del Gran Sasso, Italy,

$$\frac{d\tilde{\Gamma}_{\text{CNB}}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi}(\Delta/\sqrt{8\ln 2})} \sum_{i=1}^{N_\nu} \Gamma_i \times \exp \left\{ -\frac{[E_e - (E_{\text{end}} + m_i + m_{\text{lightest}})]^2}{2(\Delta/\sqrt{8\ln 2})^2} \right\},$$

For the fiducial model, the number of expected events per energy bin is given by:

$$\hat{N}^i = N_\beta^i(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}) + N_{\text{CNB}}^i(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}). \quad (3.3)$$

The total number of events that will be measured in a bin is the sum of  $\hat{N}^i$  and a constant background:

$$\begin{aligned} \hat{N}_t^i &= \hat{N}^i + \hat{N}_b. \\ N_{\text{exp}}^i(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}) &= \hat{N}_t^i \pm \sqrt{\hat{N}_t^i}, \end{aligned} \quad (3.4)$$

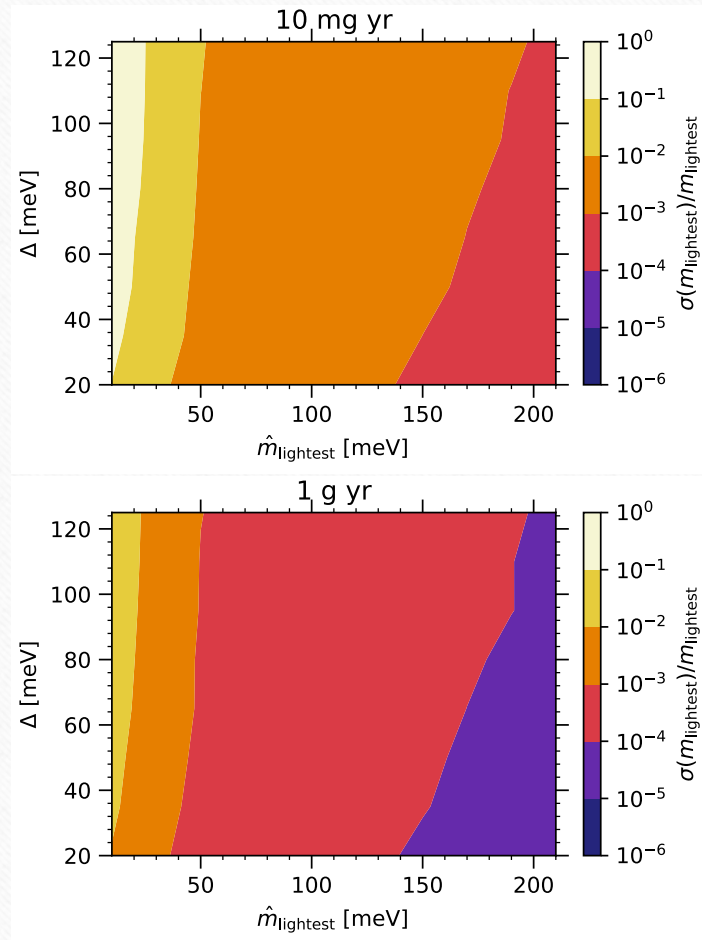
$$\begin{aligned} N_{\text{th}}^i(\boldsymbol{\theta}) &= N_b + A_\beta N_\beta^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) \\ &+ A_{\text{CNB}} N_{\text{CNB}}^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U). \end{aligned} \quad (3.6)$$

In order to perform the analysis and fit the desired parameters  $\boldsymbol{\theta}$ , we use a Gaussian  $\chi^2$  function:

$$\chi^2(\boldsymbol{\theta}) = \sum_i \left( \frac{N_{\text{exp}}^i(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}) - N_{\text{th}}^i(\boldsymbol{\theta})}{\sqrt{N_t^i}} \right)^2, \quad (3.7)$$



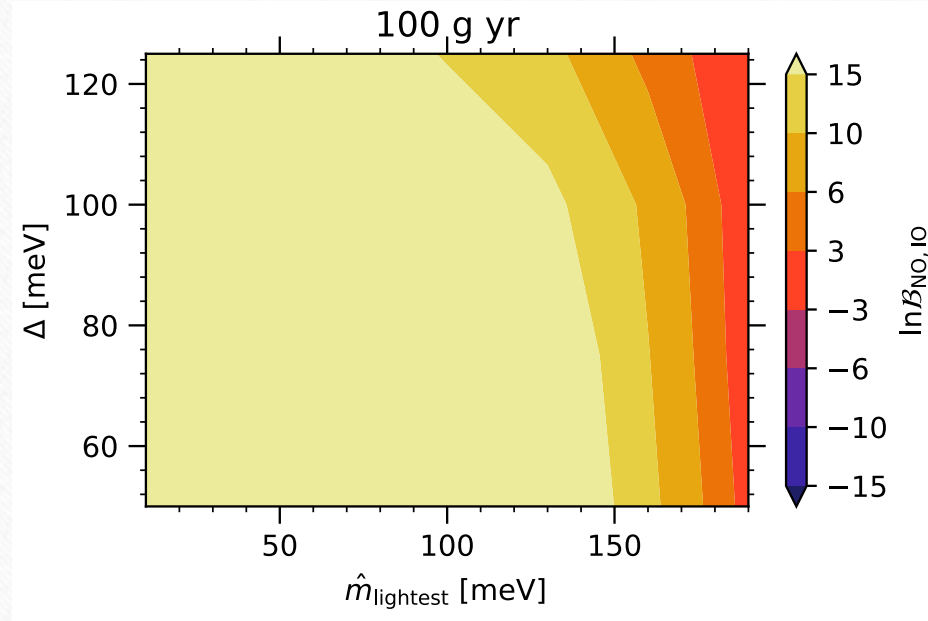
# Neutrino mass sensitivity



# Mass Ordering (Hierarchy)

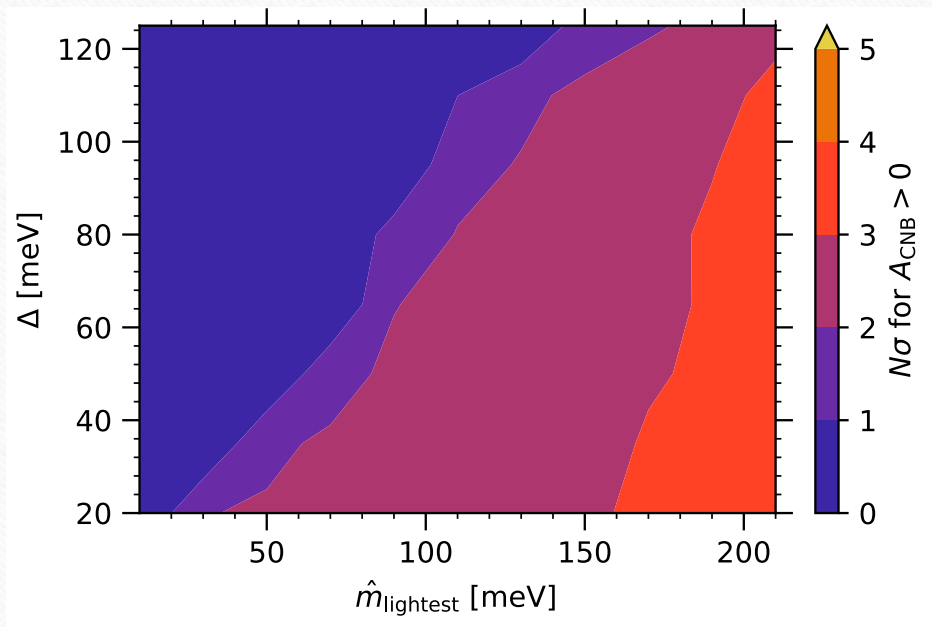
Bayesian evidence

$$\frac{\int d\Theta P(d|M1)P(M1)}{\int d\Theta P(d|M2)P(M2)}$$

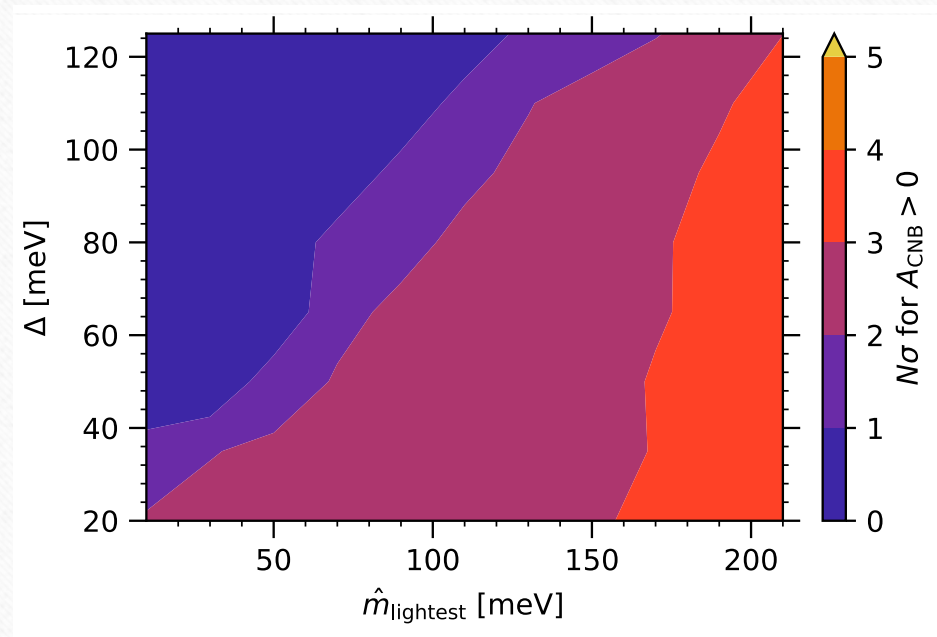




# CNB detection (100 g)



Normal ordering



Inverted ordering

# eV sterile neutrinos (100 g)

