Gluon distributions at the EIC
New insights on correlation and saturation

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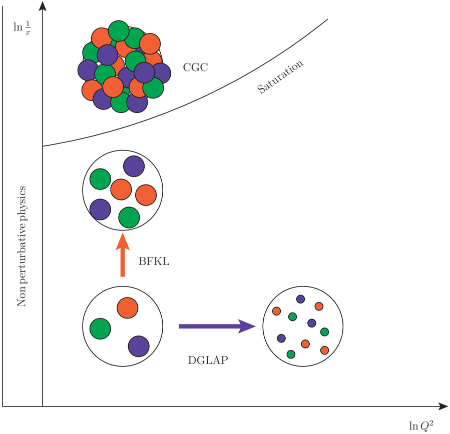
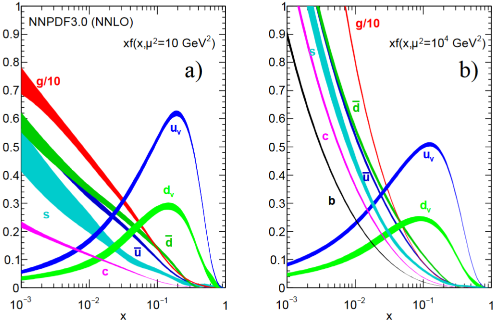
Standard gluon distributions



Parton Distribution Functions

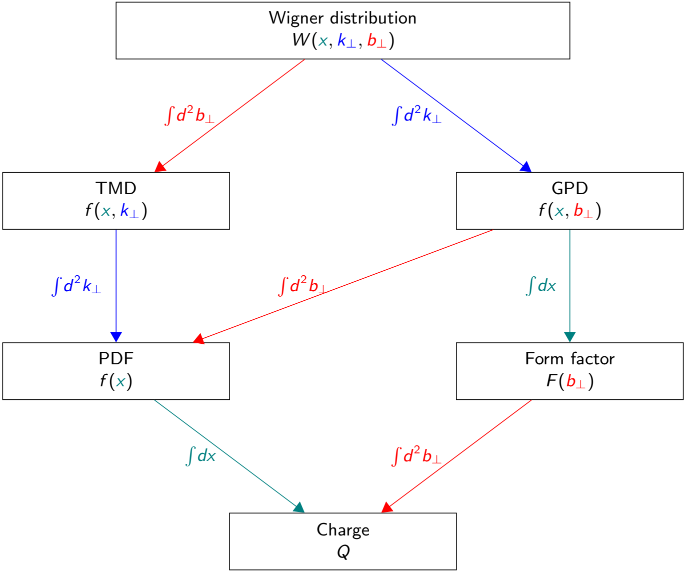
Gluon exchanges dominate at small x



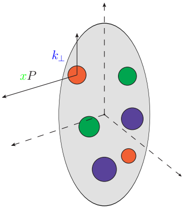
[NNLO NNPDF3.0 global analysis, taken from PDG2018]

Wigner distributions: the "Mother Distributions"

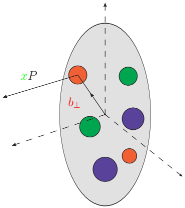
[Belitsky, Ji, Yuan, 2003], [Lorce, Pasquini, 2011]

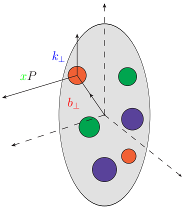


TMD



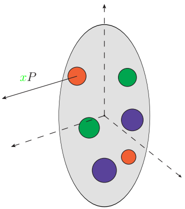
GPD



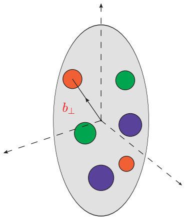


Wigner

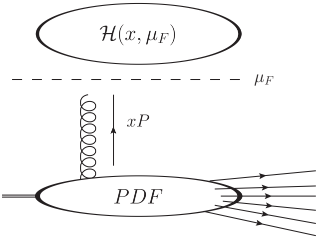
PDF

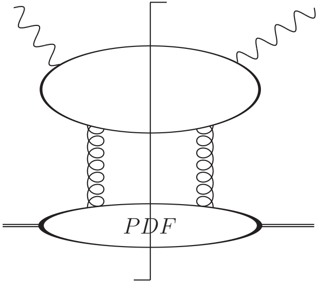


FF



Parton Distribution Functions (PDF)

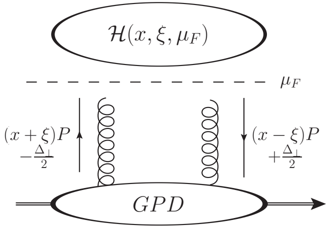


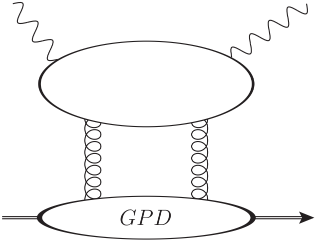


$$xg\left(x\right) \propto \int dz^{+} e^{ixP^{-}z^{+}} \left\langle P \left| F^{-i}\left(z^{+}\right) F^{-j}\left(0\right) \right| P \right\rangle$$

Example of a process involving a PDF: inclusive DIS

Generalized Parton Distributions (GPD)

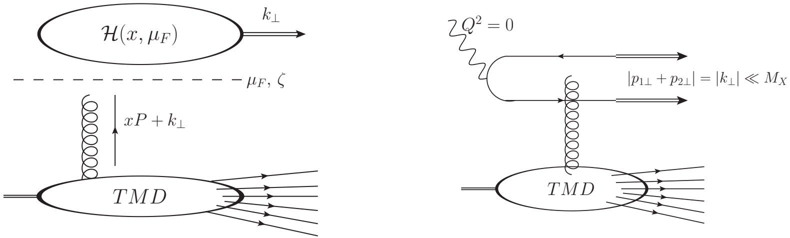




$$xG(x, \Delta) \propto \int dz^+ e^{ixP^- z^+} \left\langle P + \frac{\Delta}{2} \middle| F^{-i}(z^+) F^{-j}(0) \middle| P - \frac{\Delta}{2} \right\rangle$$

Example of a process involving a GPD: DVCS

Transverse Momentum Dependent (TMD) Distributions

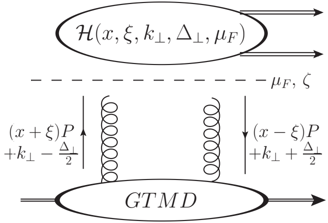


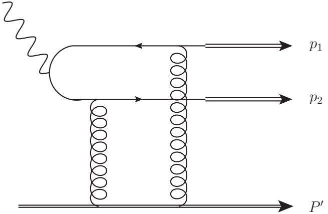
$$xF(x, k_\perp) \propto \int dz^+ d^2 z_\perp e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{-i}(z)\, W_{z,0}^{[\pm]} F^{-j}(0)\, W_{z,0}^{[\pm]} \right| P \right\rangle_{z^- = 0}$$

Example of a process involving a TMD

Photoproduction of an almost back-to-back pair of hadrons/jets

Generalized Transverse Momentum Dependent (GTMD) Distributions





$$xF\left(x, k_\perp\right) \propto \int dz^+ d^2 z_\perp e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P - \frac{\Delta}{2} \middle| F^{-i}(z)\, W_{z,0}^{[\pm]} F^{-j}(0)\, W_{z,0}^{[\pm]} \middle| P + \frac{\Delta}{2} \right\rangle$$

Example of a process involving a GTMD

Exclusive photoproduction of a dijet/dihadron

Resummation schemes

Moderate x: factorization

- Application range $Q^2 \sim s$, largest logarithm $\log Q$
- Includes leading powers of Q, all powers of $Q/\sqrt{s}$
- Standard Operator Product Expansion

$$J(z)\,J(0) \to \sum C_n(z,\mu_F)\,\mathcal{O}_n(\mu_F)$$

- Divergences in the Wilson coefficient C_n are canceled by renormalization of $\mathcal{O}_n$
- Involves standard gluon distributions

Resummation schemes

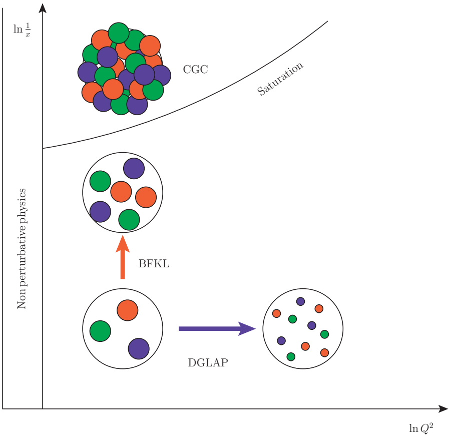
Low x: rapidity separation

- Application range $Q^2 \ll s$, largest logarithm $\log s$
- Includes all powers of Q, leading powers of $Q/\sqrt{s}$
- Low x Operator Product Expansion

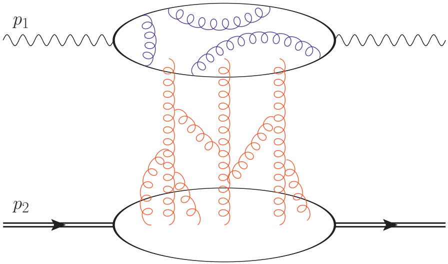
$$J(z)\,J(0) \rightarrow C_0\,(z, Y_c)\,\mathcal{O}_{tree}(Y_c) + \alpha_s C_1\,(z, Y_c)\mathcal{O}_{1-loop}(Y_c) + \ldots$$

- Spurious divergences in the n-th Wilson coefficient C_n are canceled by the rapidity evolution of $\mathcal{O}_{n-1}$ into $\mathcal{O}_n$
- Involves Wilson line operators

Large s physics, from BFKL to the CGC



Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2s} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone (Sudakov) vectors

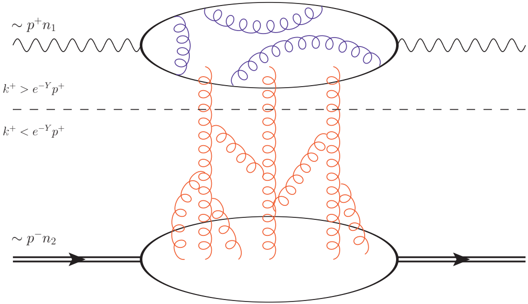
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \rightarrow (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation

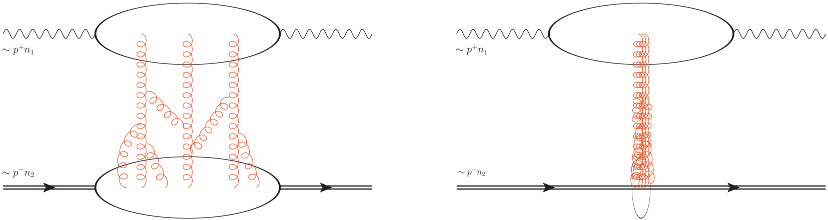


Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}) + b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k})$$

$e^\eta = e^{-Y} \ll 1$

Large longitudinal boost to the projectile frame



$$b^+(x^+, x^-, \vec{x}) \longrightarrow \frac{1}{\Lambda} b^+(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^-(x^+, x^-, \vec{x}) \longrightarrow \Lambda b^-(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

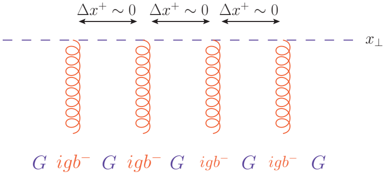
$$b^k(x^+, x^-, \vec{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad b^k(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

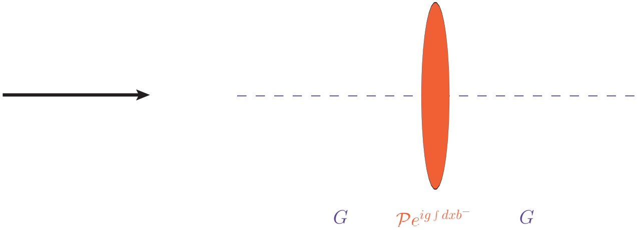
$$b^\mu(x) \to b^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O(\sqrt{\frac{m_t^2}{s}})$$

Shockwave approximation

Effective Feynman rules in the external shockwave field

The interactions with the external field can be exponentiated





Quark line in the external field in momentum space

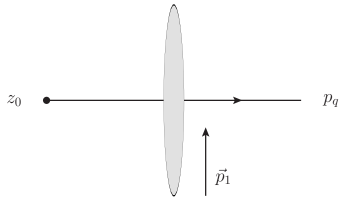
Wilson lines

$$U_i^{\eta} = \mathcal{P} \exp \left[ig \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^{+}, \vec{z}_i)\, dz_i^{+} \right], \quad \tilde{U}^{\eta}(\vec{p}) = \int d^{D-2}\vec{z}\; e^{-i(\vec{p}\cdot\vec{z})} U_{\vec{z}}^{\eta}$$

Effective quark line

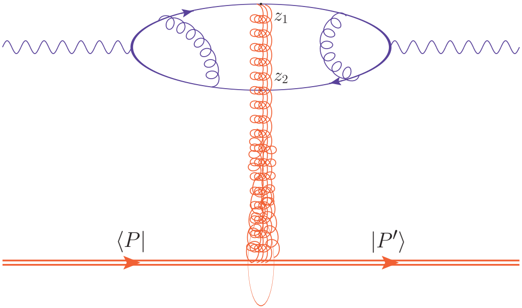
$$\bar{u}\left(p_q,\, z_0\right) = \int \frac{d^d \vec{p}_1}{(2\pi)^d} e^{i p_q^{+} z_0^{-} + i z_0^{+} \frac{(\vec{p}_q - \vec{p}_1)^2 - i0}{2 p_q^{+}} - i\left(\vec{p}_q - \vec{p}_1\right)\cdot \vec{z}_0}$$

$$\times \bar{u}_{p_q} \gamma^{+} \left[\tilde{U}_{\vec{p}_1} \theta\left(-z_0^{+}\right) + (2\pi)^d \delta\left(\vec{p}_1\right) \theta\left(z_0^{+}\right) \right] \frac{p_q^{+}\gamma^{-} + \hat{p}_{q\perp} - \hat{p}_{1\perp}}{2 p_q^{+}}$$



Exchange in t-channel of an effective off-shell particle

Factorized picture



Factorized amplitude

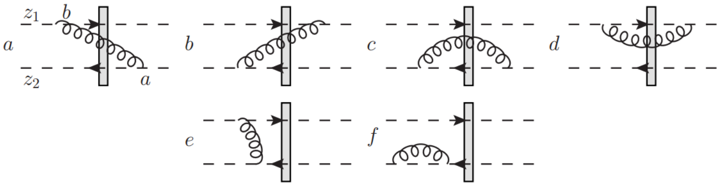
$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^\eta(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Evolution for the dipole operator

$$\mathcal{U}_{12}^{\eta+\delta\eta} - \mathcal{U}_{12}^{\eta}$$



B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta} + \mathcal{U}_{13}^{\eta} \mathcal{U}_{32}^{\eta} \right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta} \mathcal{U}_{32}^{\eta}}{\partial \eta} = \ldots$$

Evolves a dipole into a double dipole

The JIMWLK Hamiltonian

Hamiltonian formulation of the hierarchy of equations

For an operator built from n Wilson lines, the JIMWLK evolution is given at LO accuracy by

$$\frac{\partial}{\partial\eta}\left[U^{\eta}_{\vec{z}_1}...U^{\eta}_{\vec{z}_n}\right] = \sum_{i,j=1}^{n} H_{ij}\cdot\left[U^{\eta}_{\vec{z}_1}...U^{\eta}_{\vec{z}_n}\right],$$

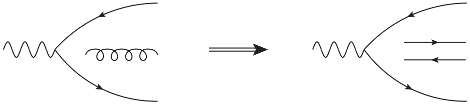
JIMWLK Hamiltonian

$$H_{ij} = \frac{\alpha_s}{2\pi^2}\int d\vec{z}_k\,\frac{\vec{z}_{ik}\cdot\vec{z}_{kj}}{\vec{z}^{\,2}_{ik}\vec{z}^{\,2}_{kj}}[T^a_{i,L}T^a_{j,L} + T^a_{i,R}T^a_{j,R} - U^{ab}_{\vec{z}_k}(T^a_{i,L}T^b_{j,R} + T^a_{j,L}T^b_{i,R})],$$

$$T^a_{j,L}U_r(z_i) = T^a_r U_r(z_i), \quad T^a_{j,R}U_r(z_i) = U_r(z_i)T^a_r$$

The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in the dipole B-JIMWLK equation



[Mueller]

⇒ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle \, \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

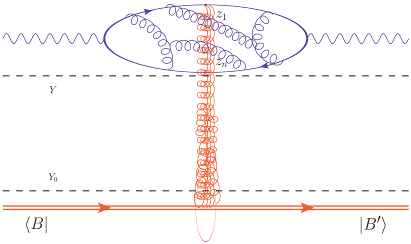
BFKL/BKP part Triple pomeron vertex

Non-linear term : one type of saturation

Non-perturbative elements are compatible with CGC-type models

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$. May require adjustment.
- Evaluate the solution at a typical projectile rapidity $\eta = Y$, or at the rapidity of the slowest gluon
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

Exclusive diffraction allows one to probe the $b_\perp$-dependence of the non-perturbative scattering amplitude

The seemingly incompatible nature of the distributions

Two different kinds of gluon distributions

Moderate x distributions	Low x distributions
GTMD, GPD, TMD, PDF...	Dipole scattering amplitude
$\langle P^{(\prime)}\vert F^{-i} W F^{-j} W \vert P\rangle$	$\langle P^{(\prime)}\vert \mathrm{tr}(U_1 U_2^\dagger)\vert P\rangle$

Link from low x to (G)TMD distributions

Gluon Wigner distributions

Naive Gluon Wigner distribution

$$xW^{ij}(x,\vec{k},\vec{b}) \equiv \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\left(\vec{b}\cdot\vec{\Delta}\right)} \int \frac{dz^+ d^2 z_\perp}{16\pi^3} e^{ixP^- z^+ - i\left(\vec{k}\cdot\vec{z}\right)}$$
$$\times \left\langle P - \frac{\Delta}{2} \middle| F^{+i}\left(-\frac{z}{2}\right) F^{+j}\left(\frac{z}{2}\right) \middle| P + \frac{\Delta}{2} \right\rangle$$

Wigner $\underset{\longleftrightarrow}{\text{Fourier}}$ GTMD

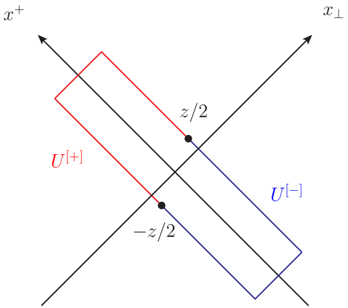
$$x\mathcal{G}^{ij}(x,\vec{k},\vec{\Delta}) \equiv \int \frac{dz^+ d^2 z_\perp}{16\pi^3} e^{ixP^- z^+ - i\left(\vec{k}\cdot\vec{z}\right)}$$
$$\times \left\langle P - \frac{\Delta}{2} \middle| F^{+i}\left(-\frac{z}{2}\right) F^{+j}\left(\frac{z}{2}\right) \middle| P + \frac{\Delta}{2} \right\rangle$$

Not gauge invariant objects!

Gluon Wigner distributions

Two ways to build gauge links

[Bomhof,Mulders, 2008], [Dominguez, Marquet, Xiao, Yuan, 2011]



Dipole distribution

$$\mathrm{Tr}[F^{+i}(-z/2)U^{[+]}F^{+j}(z/2)U^{[-]}]$$

Weizsäcker-Williams (WW) distribution

$$\mathrm{Tr}[F^{+i}(-z/2)U^{[+]}F^{+j}(z/2)U^{[+]}]$$

Staple gauge links in the shockwave formalism

[Dominguez, Marquet, Xiao, Yuan]

Consider the derivative of a path-ordered Wilson line, denoting

$$[x_1^+, x_2^+]_{\vec{x}} \equiv \mathcal{P}\exp\left[\, ig\int_{x_1^+}^{x_2^+} dx^+ b^-(x^+,\vec{x})\,\right]$$

For a given shockwave operator $U_{\vec{x}} = [-\infty, +\infty]_{\vec{x}}$

$$\partial^i U_{\vec{x}} = ig\int dx^+ [-\infty, x^+]_{\vec{x}}\, F^{+i}(x^+,\vec{x})\, [x^+, +\infty]_{\vec{x}}$$

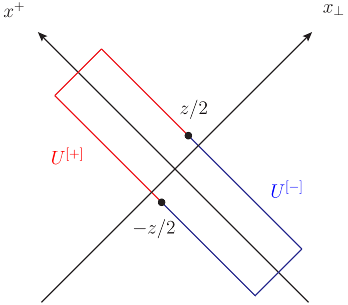
$$\partial^j U^\dagger_{\vec{x}} = -ig\int dx^+ [+\infty, x^+]_{\vec{x}}\, F^{+j}(x^+,\vec{x})\, [x^+, -\infty]_{\vec{x}}$$

$$(\partial^i U^\dagger_{\vec{x}})U_{\vec{x}} = g^2\int dx^+ [+\infty, x^+]_{\vec{x}}\, F^{+i}(x^+,\vec{x})\, [x^+, +\infty]_{\vec{x}}$$

The dipole (G)TMD: operator

Operator involved in the dipole case

$$\mathrm{Tr}[(\partial^i U_{\vec{x}_1})(\partial^j U^\dagger_{\vec{x}_2})] = g^2 \int dx_1^+ dx_2^+ \mathrm{Tr}\, F^{+i}(x_1^+, \vec{x}_1)\, [x_1^+, +\infty]_{\vec{x}_1} [+\infty, x_2^+]_{\vec{x}_2} \times F^{+j}(x_2^+, \vec{x}_2) [x_2^+, -\infty]_{\vec{x}_2} [-\infty, x_1^+]_{\vec{x}_1}$$



Dipole distribution

$$\mathrm{Tr}[F^{+i}(-z/2) U^{[+]} F^{+j}(z/2) U^{[-]}]$$

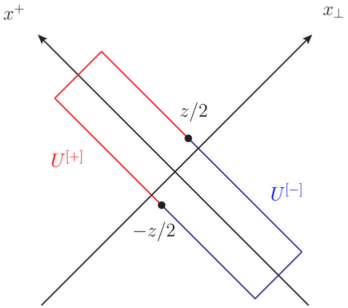
Weizsäcker-Williams (WW) distribution

$$\mathrm{Tr}[F^{+i}(-z/2) U^{[+]} F^{+j}(z/2) U^{[+]}]$$

The WW (G)TMD: operator

Operators involved in the WW case

$$\mathrm{Tr}[(\partial^i U^\dagger_{\vec{x}_1})U_{\vec{x}_1}(\partial^j U^\dagger_{\vec{x}_2})U_{\vec{x}_2}] = g^2 \int dx_1^+ dx_2^+ \mathrm{Tr}[x_2^+, +\infty]_{\vec{x}_2}[+\infty, x_1^+]_{\vec{x}_1} \times F^{+i}(x_1^+, \vec{x}_1)[x_1^+, +\infty]_{\vec{x}_1}[+\infty, x_2^+]_{\vec{x}_2} F^{+j}(x_2^+, \vec{x}_2)$$



Dipole distribution

$$\mathrm{Tr}[F^{+i}(-z/2)U^{[+]}F^{+j}(z/2)U^{[-]}]$$

Weizsäcker-Williams (WW) distribution

$$\mathrm{Tr}[F^{+i}(-z/2)U^{[+]}F^{+j}(z/2)U^{[+]}]$$

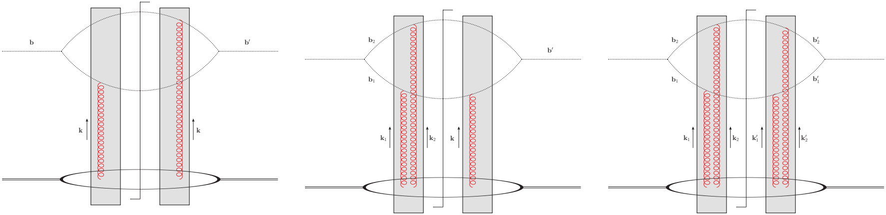
The actual low x building block is the derivative of a Wilson line

An equivalence is obtained by rewriting lines in terms of their derivatives

Inclusive low x cross section

Inclusive low x cross section = TMD cross section

[Altinoluk, RB, Kotko], [Altinoluk, RB]

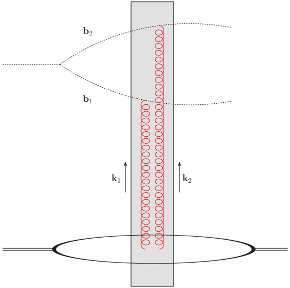


$$\sigma = \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \left| F^{-i} W F^{-j} W \right| P \right\rangle$$
$$+ \mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W F^{-k} W \right| P \right\rangle$$
$$+ \mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \right| P \right\rangle$$

Exclusive low x cross section

Exclusive low x amplitude = GTMD amplitude

[Altinoluk, RB]



$$\mathcal{H}^{ij}(k_{1\perp}, k_{2\perp}) \otimes \left\langle P' \left| F^{-i} W F^{-j} W \right| P \right\rangle$$

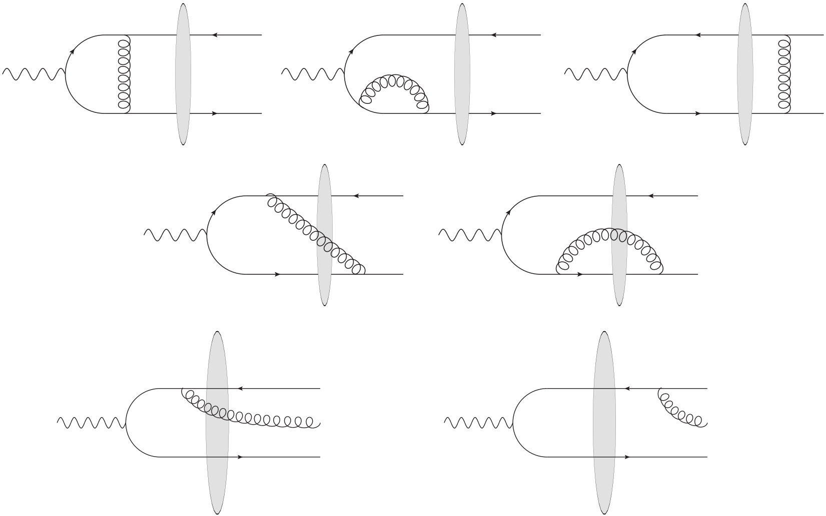
Every exclusive low x process probes a Wigner distribution!

One-loop correction to processes probing Wigner distributions

Exclusive diffractive dijet production

Exclusive diffractive dijet production

NLO corrections to this process probing Dipole Wigner are known [RB, Grabovsky, Szymanowski, Wallon (JHEP)]



Divergences

All divergences cancel: factorization holds at one loop

- Rapidity divergence $p_g^+ \to 0$ (spurious gauge pole in axial gauge)
  - Removed via JIMWLK evolution
- UV, soft divergence, collinear divergence
  - Cancels between real and virtual corrections, along with renormalization
- Soft and collinear divergence
  - Removed via a jet algorithm

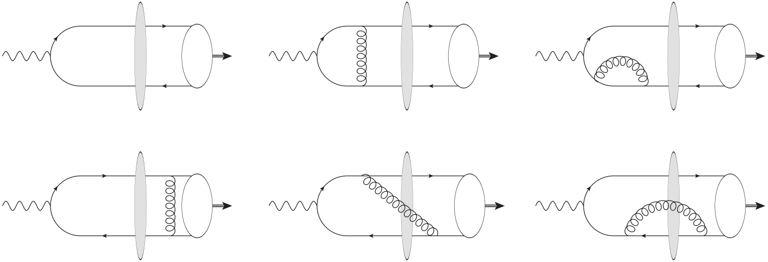
Thus the NLO cross section for this process which probes the Dipole Wigner distribution is finite

Exclusive diffractive ρ_L production:

NLO corrections to a twist 2 process

Exclusive diffractive production of a light neutral vector meson

[RB, Grabovsky, Ivanov, Szymanowski, Wallon (PRL)]



$$\mathcal{A} = -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_\parallel(x) \int \frac{d^d\vec{p}_1}{(2\pi)^d} \frac{d^d\vec{p}_2}{(2\pi)^d} \frac{d^d\vec{p}_3}{(2\pi)^d}$$
$$\times (2\pi)^{d+1} \delta\left(p_V^+ - p_\gamma^+\right) \delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3\right)$$
$$\times \left[\left(\Phi_0^\beta\left(x,\, \vec{p}_1,\, \vec{p}_2\right) + C_F \Phi_{V1}^\beta\left(x,\, \vec{p}_1,\, \vec{p}_2\right)\right) \tilde{\mathcal{U}}_{12}^\eta\, (2\pi)^d \delta(\vec{p}_3)\right.$$
$$\left. + \Phi_{V2}^\beta(x, \vec{p}_1, \vec{p}_2, \vec{p}_3)\, \tilde{\mathcal{W}}_{123}^\eta\right]$$

Probes gluon GPDs at low x, as well as twist 2 DAs

Divergences

Divergences

- Rapidity divergence $p_g^+ \to 0$ (spurious gauge pole in axial gauge)
  - Removed via JIMWLK evolution
- UV, soft divergence, collinear divergence
  - Mostly cancel each other, but requires renormalization of the operator in the vacuum-to-meson matrix element $\to$ ERBL evolution equation for the DA

We thus built a finite NLO exclusive diffractive amplitude with a Wigner distribution and a twist 2 DA

What can low x results tell us about gluon distributions?

1- Saturation effects

Saturation effects

Saturation as understood at small x

Two origins of saturation effects

- Large gluon densities $\Rightarrow$ Gluon recombination effects

  Arises from non-linearities in the evolution equation

- Large gluon densities $\Rightarrow$ Multiple gluon scattering effects

  Arises from the exponentiation of interactions $U_x = \mathcal{P} e^{ig_s \int A^-}$

A new understanding of saturation

"Saturation" effects in terms of (G)TMD distributions

- Large gluon densities ⇒ Gluon recombination effects

Arises from non-linearities in the $\ln(s)$ resummation equation

- TMD gauge links = multiple soft scattering effects

Kinematic saturation: small $k_\perp$ ⇒ Sivers effect!

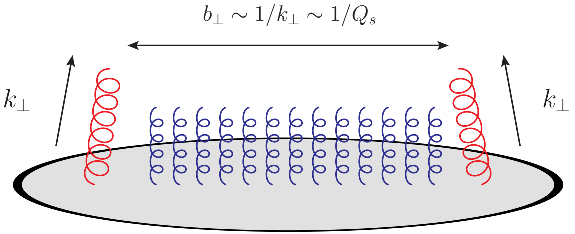
- Large gluon occupancy number ⇒ large $g_s F \sim 1$

Genuine saturation: genuine twist corrections are enhanced on a dense target

Two types of saturation are not specific to small x physics

Kinematic saturation

"Saturation" from a TMD gauge link



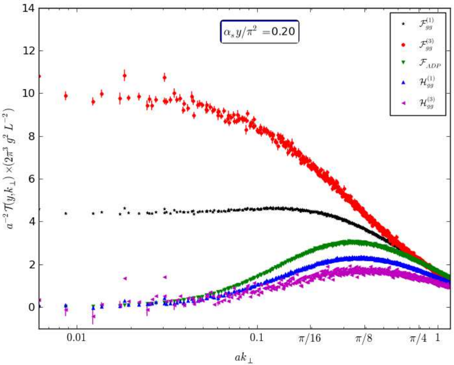
$$g_s^2 \int d^4 b\, \delta\left(b^-\right) e^{i(k\cdot b)} \left\langle P \left| F^{i-}(b)\, \mathcal{U}_{b,0}^{[\pm]} F^{j-}(0)\, \mathcal{U}_{0,b}^{[\pm]} \right| P \right\rangle$$

Expected in any process involving a gluon (G)TMD

Kinematic saturation

"Saturation" from a TMD gauge link

Link length $\sim 1/|k_\perp|$, hence effect suppressed at large $k_\perp$



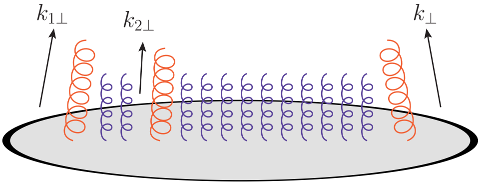
[Marquet, Petreska, Roiesnel ; Marquet, Roiesnel, Taels]

Observed by Golec-Biernat and Wüsthoff?

Genuine saturation

Saturation as an enhancement of genuine twists

Large gluon occupancy $\Rightarrow g_s F \sim 1$

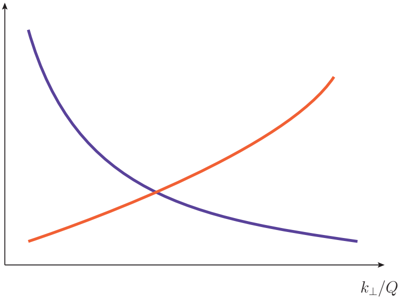


$$g_s^2 \int d^4b_1 d^4b_2 d^4b' \delta\left(b_1^-\right)\delta\left(b_2^-\right)\delta\left(b'^-\right) e^{i(k_1\cdot b_1)+i(k_2\cdot b_2)-i\left(k\cdot b'\right)}$$
$$\times \frac{\left\langle P \left| F^{i-}(b_1)\mathcal{U}^{[\pm]}_{b_1,b_2} g_s F^{j-}(b_2)\mathcal{U}^{[\pm]}_{b_2,b'} F^{k-}(b')\mathcal{U}^{[\pm]}_{b',b_1} \right| P\right\rangle}{\langle P|P\rangle}$$

Expected in any process involving dense targets

Distinguishing saturation effects

Different kinds of saturation occur in different kinematic regions



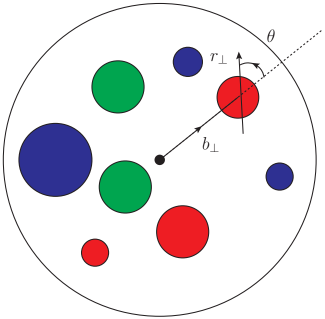
- Kinematic saturation occurs at small $k_\perp$: gauge links shrink at large $k_\perp$
- Genuine saturation is a twist-suppressed $O(k_\perp/Q)$ effect: it is suppressed at small $k_\perp$
- Color Glass Condensate results lead to genuine saturation effects being enhanced on heavy targets

What can low x results tell us about gluon distributions?

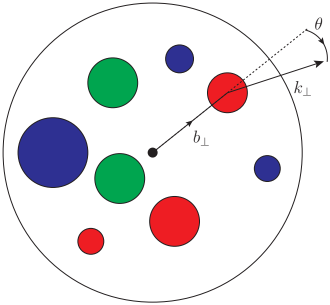
2- Cold nuclear correlation effects

Angular correlation

Cold nuclear correlation effects



CGC correlation



Wigner correlation

Dipole GTMD and elliptic flow

The dipole GTMD can produce initial state correlations
[Hatta, Xiao, Yuan ; Hagiwara, Hatta, Xiao, Yuan ; Iancu, Rezaeian]
Inspired by [Kopeliovich *et al* ; Levin, Rezaeian]

For small $k_\perp$,

$$\mathcal{G}\left(|b_\perp|, |k_\perp|\right) \simeq \mathcal{G}_0\left(|b_\perp|, |k_\perp|\right) + 2\cos\left(2\phi_{b,k}\right)\mathcal{G}_e\left(|b_\perp|, |k_\perp|\right)$$

The $\cos(2\phi)$ term (elliptic Wigner), leads to angular correlations in observed transverse momenta.
For exclusive processes, cold nuclear effects are the only expected correlation effects.

$\Rightarrow$ Dipole Wigner = initial state origin of elliptic flow in small systems?

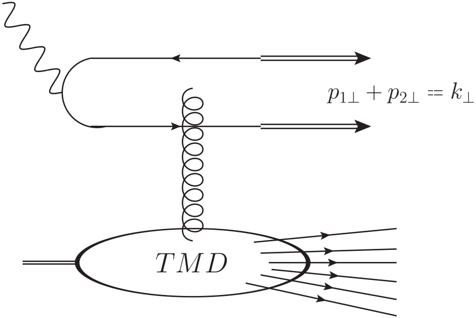
Elliptic flow from TMDs

Elliptic flow in inclusive processes can arise from polarized TMDs

[Boer, Mulders, Pisano], [Metz, Zhou], [Dominguez, Qiu, Xiao, Yuan] , [Dumitru, Skokov]

$$\left\langle P \left| F^{-i}(z) W_{z,0}^{[\pm]} F^{-j}(0) W_{z,0}^{[\pm]} \right| P \right\rangle_{z^-=0} \to \frac{\delta^{ij}}{2}\mathcal{F}(k_\perp) + \left(\frac{\boldsymbol{k}^i \boldsymbol{k}^j}{\boldsymbol{k}^2} - \frac{\delta^{ij}}{2}\right)\mathcal{H}(k_\perp)$$

$$\langle P | F W F W | P \rangle \times \mathcal{H} \Rightarrow v_0 \mathcal{F}(k_\perp) + v_2 \cos(2\phi) \mathcal{H}(k_\perp)$$



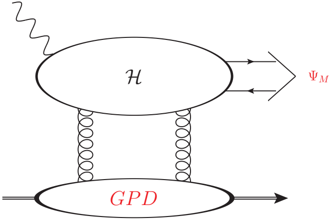
What can low x results tell us about gluon distributions?

3- Constraining GPDs where they would not factorize

Twist 3 DVMP

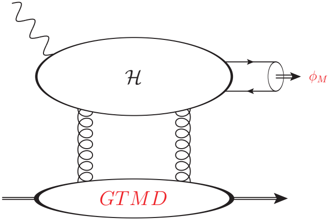
Full GPD+DA colinear factorization for DVMP breaks at twist 3

2 solutions



[Ahmad, Goldstein, Liuti]
[Goloskokov, Kroll]

Full wavefunction for the meson



[Anikin, Ivanov, Pire, Szymanowski, Wallon]
[RB *et al*]

Full GTMD for the target

Exclusive low x cross section

Exclusive low x amplitude = GTMD amplitude

$$\mathcal{H}^{ij}(k_{1\perp}, k_{2\perp}) \otimes \langle P' | F^{-i} W F^{-j} W | P \rangle$$

Twist $\sim$ power of $k_{1\perp}/Q$ or $k_{2\perp}/Q$ in $\mathcal{H}$.

Leading twist

$$\mathcal{H}^{ij}(0_\perp, 0_\perp) \otimes \int d^2k_{1\perp} d^2k_{2\perp} \langle P' | F^{-i} W F^{-j} W | P \rangle = \mathcal{H} \otimes (\mathrm{GPD})$$

Next-To-Leading twist

$$\mathcal{H} \otimes \partial(\mathrm{GPD}) \to \infty$$

By not expanding in twists, low x physics restores factorization with GTMDs where GPDs would not work. At large s and large Q, a low x description of a twist 3 process is the closest thing to constraining higher twist GPD we can get.

How can gluon distributions help up for low x analysis?

Negativity of low x cross sections

Negativity of low x cross sections

[Ducloué, Lappi, Stasto, Watanabe, Xiao, Yuan, Zaslavsky, Zhu]

- One-loop predictions for forward particle production in pA collisions lead to negative cross sections at large $p_\perp$.
- Origins: Coulomb tail in the evolution equation ; large colinear logarithms $\log Q$
- The issue was postponed to larger $p_\perp$ by using a non-local factorization scheme [Iancu, Mueller, Triantafyllopoulos], [Ducloué, Hänninen, Lappi, Zhu]
- Improving the JIMWLK evolution with colinear logarithm resummation helps a great deal [Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos]

Negativity of low x cross sections

Moderate x results for TMD evolution can help us resum colinear logarithms

- First, resum $\log(s)$ via regular JIMWLK evolution

$$\mathcal{H}^{Y} \otimes \mathcal{O}^{Y} \to \mathcal{H}^{Y_c} \otimes \mathcal{O}^{Y_c}$$

- Then rewrite the result into a TMD form

$$\mathcal{H}^{Y_c} \otimes \mathcal{O}^{Y_c} \to \sum \mathcal{H}_n^{Y_c} \otimes \mathcal{F}_n^{Y_c}$$

- Use renormalization of the TMD to resum $\log(Q)$

$$\mathcal{F}_n^{Y_c} \to \mathcal{F}_n^{Y_c}(\mu_F)$$

- Use the Collins-Soper-Sterman (CSS) framework to resum Sudakov logarithms $\log(k_\perp/Q)$

- (Use previous low x results [Iancu, Mueller, Triantafyllopoulos], [Mueller, Xiao, Yuan] in check to avoid double counting)

Conclusion

- We finally unified moderate x and low x distributions
- This allows us to use low x results to constrain gluon distributions
- We provided full NLO predictions to probe Wigner distributions
- We also reinterpreted the notion of saturation and the nuclear correlations
- It also allows us to use moderate x evolution to improve low x predictions

Weizsäcker-Williams Wigner distribution at low x

WW gluon GTMD

$$x\mathcal{G}_{ij}\left(x,\boldsymbol{K},\boldsymbol{\Delta}\right)\equiv\frac{-2}{\alpha_s}\int d^2\boldsymbol{b}_1 d^2\boldsymbol{b}_2 e^{-i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{b}_1+\boldsymbol{b}_2}{2}-i\boldsymbol{K}\cdot(\boldsymbol{b}_1-\boldsymbol{b}_2)}\left\langle \mathrm{Tr}\left[\left(\partial_i U^\dagger_{\boldsymbol{b}_1}\right)U_{\boldsymbol{b}_1}\left(\partial_j U^\dagger_{\boldsymbol{b}_2}\right)U_{\boldsymbol{b}_2}\right]\right\rangle_x$$

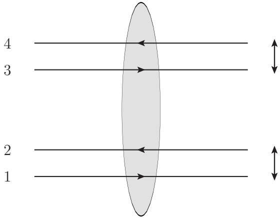
Symmetry relations:

$$\mathcal{G}_{ij}(K,\Delta)=\mathcal{G}_{ji}(-K,\Delta)=\mathcal{G}^*_{ij}(-K,-\Delta)$$

Then the decomposition is much richer than in the dipole case:

$$\mathcal{G}_{ij}(x,\mathbf{K},\boldsymbol{\Delta}) \equiv \delta_{ij}\,\mathcal{G}_1+\left(\frac{K_iK_j}{\mathbf{K}^2}-\frac{\delta_{ij}}{2}\right)\frac{\mathbf{K}^2}{M^2}\,\mathcal{G}_2+\left(\frac{\Delta_i\Delta_j}{\boldsymbol{\Delta}^2}-\frac{\delta_{ij}}{2}\right)\frac{\boldsymbol{\Delta}^2}{M^2}\,\mathcal{G}_3 +\left(\frac{K_i\Delta_j-K_j\Delta_i}{M^2}\right)\mathcal{G}_4$$

[Boer, van Daal, Mulders, Petreska]



Consider a quadrupole $\mathrm{Tr}(U_2^\dagger U_1 U_4^\dagger U_3)$ such that $1 \sim 2$, $3 \sim 4$.

$$U_2^\dagger U_1 = U^\dagger_{b_1+r_1/2} U_{b_1-r_1/2} = -r_1^i(\partial_i U^\dagger_{b_1}) U_{b_1}$$

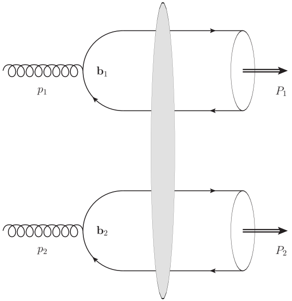
$$U_4^\dagger U_3 = U^\dagger_{b_2+r_2/2} U_{b_2-r_2/2} = -r_2^j(\partial_j U^\dagger_{b_2}) U_{b_2}$$

so
$\mathrm{Tr}(U_2^\dagger U_1 U_4^\dagger U_3) \simeq r_1^i r_2^j \mathrm{Tr}[(\partial_i U^\dagger_{b_1}) U_{b_1} (\partial_j U^\dagger_{b_2}) U_{b_2}]$

$\Rightarrow$ The Weizsäcker-Williams gluon GTMD at low-x can be probed in exclusive processes with a quadrupole made of two small dipoles at the amplitude level.

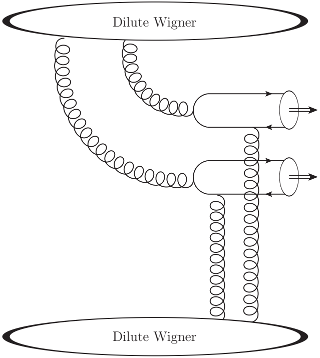
Typical WW-probing process

Production of a pair of heavy quarkonia



At leading approximation, the WW GTMD encodes the exchange of a gluon pair in the t channel. For connected diagrams to exist, we thus require the $gg \to M$ transitions to exist. Thus only C^+ quarkonia are allowed: η, χ_J mesons.

Exclusive production of a pair of η_c
[Bhattacharya, Metz, Ojha, Tsai, Zhou]

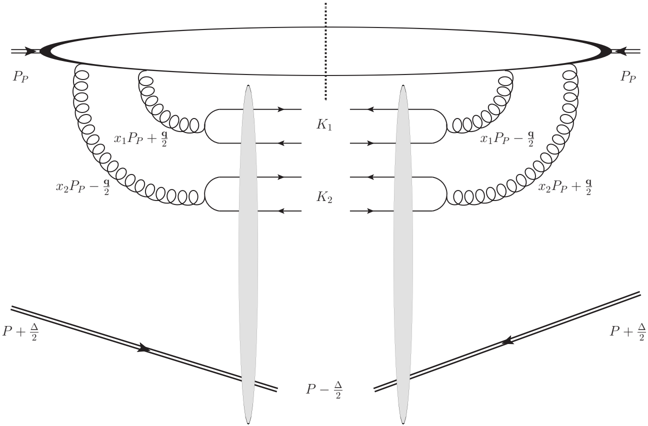


Only valid in the dilute approximation, otherwise factorization could be broken by entangled gauge links

No gauge links: does not distinguish Dipole and WW

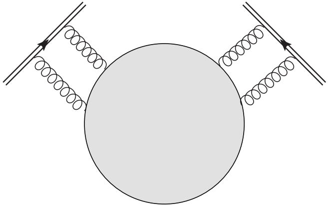
Hybrid process

Diffractive production of a forward pair of C^+ quarkonia
[RB, Hatta, Xiao, Yuan]



Forward production allows factorization to hold in the hybrid scheme, and allows to be inclusive in the projectile remnants

The Weizsäcker-Williams gluon GTMD at small x: hybrid factorization



Forward production $\Rightarrow$ collinear factorization on the projectile side

Multiple scattering: Double PDF

$$\left\langle P \left| G^{+i'} G^{+i} G^{+j'} G^{+j} \right| P \right\rangle$$

Spin decomposition

$$H^{jj'} = \frac{1}{2}\delta^{jj'} \left(\Pi_g^{kk'} H^{kk'}\right) - \frac{1}{2} i\epsilon^{jj'} \left(\Pi_{\Delta g}^{kk'} H^{kk'}\right) + \tau^{jj',ll'} \left(\Pi_{\delta g}^{kk'}\right)^{ll'} H^{kk'},$$

thus 3×3 types of double PDFs: unpolarized ($\delta^{jj'}$), longitudinally polarized ($\epsilon^{jj'}$) and linearly polarized ($\tau^{jj',mm'}$).

Full factorized cross section

$$
\begin{aligned}
&\frac{d\sigma\left(M_1, M_2\right)}{dY_1 dY_2 d^2\boldsymbol{\Delta} d^2\boldsymbol{K}} \\
&= \frac{\alpha_s^2 x_1 x_2}{16 m_1^5 m_2^5 N_c^4 \left(N_c^2-1\right)^2} \left\langle \mathcal{O}_{M_1}\left({}^{2S_1+1}L_{1J_1}^{1}\right)\right\rangle \left\langle \mathcal{O}_{M_2}\left({}^{2S_2+1}L_{2J_2}^{1}\right)\right\rangle \\
&\times \int d^2\boldsymbol{q} \left[\delta^{ii'}\delta^{jj'}\mathcal{F}_{g,g}\left(x_1, x_2, \boldsymbol{q}\right) - i\delta^{ii'}\epsilon^{jj'}\mathcal{F}_{g,\Delta g}\left(x_1, x_2, \boldsymbol{q}\right) + 2\delta^{ii'}\mathcal{F}_{g,\delta g}^{jj'}\left(x_1, x_2, \boldsymbol{q}\right)\right. \\
&-i\epsilon^{ii'}\delta^{jj'}\mathcal{F}_{\Delta g,g}\left(x_1, x_2, \boldsymbol{q}\right) - \epsilon^{ii'}\epsilon^{jj'}\mathcal{F}_{\Delta g,\Delta g}\left(x_1, x_2, \boldsymbol{q}\right) - 2i\epsilon^{ii'}\mathcal{F}_{\Delta g,\delta g}^{jj'}\left(x_1, x_2, \boldsymbol{q}\right) \\
&\left.+2\delta^{jj'}\mathcal{F}_{\delta g,g}^{ii'}\left(x_1, x_2, \boldsymbol{q}\right) - 2i\epsilon^{jj'}\mathcal{F}_{\delta g,\Delta g}^{ii'}\left(x_1, x_2, \boldsymbol{q}\right) + 4\mathcal{F}_{\delta g,\delta g}^{ii',jj'}\left(x_1, x_2, \boldsymbol{q}\right)\right] \\
&\times \Pi_1^{ii',kk'}\left(M_1\right)\Pi_2^{jj',\ell\ell'}\left(M_2\right) x\mathcal{G}^{k\ell}\left(\boldsymbol{K}-\frac{\boldsymbol{q}}{2}, \boldsymbol{\Delta}\right) x\mathcal{G}^{k'\ell'*}\left(\boldsymbol{K}+\frac{\boldsymbol{q}}{2}, \boldsymbol{\Delta}\right),
\end{aligned}
$$

Let us assume $|\mathbf{q}| \ll |\mathbf{K}|$

Then the dependence on q disappears in the hard parts and in the GTMD

$\Rightarrow$ we absorb it into the double PDFs.

Thus what we actually need are Integrated double PDFs which we define as

$$\mathcal{F}_{a_1,a_2}(x_1, x_2) \equiv \int d^2\mathbf{q}\, \mathcal{F}_{a_1,a_2}(x_1, x_2, \mathbf{q})$$

Note that single-linearly polarized double PDFs integrate to 0
Indeed the only symmetric and traceless tensor built with only one transverse scale is given by

$$\left(\frac{q^m q^n}{\mathbf{q}^2} - \frac{\delta^{mn}}{2}\right)$$

which integrates to 0.

Full factorized cross section

$$\frac{d\sigma\left(M_1, M_2\right)}{dY_1 dY_2 d^2\boldsymbol{\Delta} d^2\boldsymbol{K}}$$
$$\simeq \frac{\alpha_s^2 x_1 x_2}{16 m_1^5 m_2^5 N_c^4 \left(N_c^2-1\right)^2} \left\langle \mathcal{O}_{M_1}\left({}^{2S_1+1}L^1_{1J_1}\right)\right\rangle \left\langle \mathcal{O}_{M_2}\left({}^{2S_2+1}L^1_{2J_2}\right)\right\rangle$$
$$\times \Big[\delta^{ii'}\delta^{jj'}\mathcal{F}_{g,g}\left(x_1, x_2\right) - i\delta^{ii'}\epsilon^{jj'}\mathcal{F}_{g,\Delta g}\left(x_1, x_2\right) + 2\delta^{ii'}\mathcal{F}^{jj'}_{g,\delta g}\left(x_1, x_2\right)$$
$$- i\epsilon^{ii'}\delta^{jj'}\mathcal{F}_{\Delta g,g}\left(x_1, x_2\right) - \epsilon^{ii'}\epsilon^{jj'}\mathcal{F}_{\Delta g,\Delta g}\left(x_1, x_2\right) - 2i\epsilon^{ii'}\mathcal{F}^{jj'}_{\Delta g,\delta g}\left(x_1, x_2\right)$$
$$+ 2\delta^{jj'}\mathcal{F}^{ii'}_{\delta g,g}\left(x_1, x_2\right) - 2i\epsilon^{jj'}\mathcal{F}^{ii'}_{\delta g,\Delta g}\left(x_1, x_2\right) + 4\mathcal{F}^{ii',jj'}_{\delta g,\delta g}\left(x_1, x_2\right)\Big]$$
$$\times \Pi_1^{ii',kk'}\left(M_1\right)\Pi_2^{jj',\ell\ell'}\left(M_2\right) x\mathcal{G}^{k\ell}\left(\boldsymbol{K}, \boldsymbol{\Delta}\right) x\mathcal{G}^{k'\ell'*}\left(\boldsymbol{K}, \boldsymbol{\Delta}\right),$$

Simplifications

Getting rid of the longitudinally polarized integrated double PDFs

Defining φ as the angle between $\mathbf{\Delta}$ and $\mathbf{K}$, we will actually observe the angular averaged cross section:

$$\int \frac{d\varphi}{2\pi} \frac{d\sigma\left(M_1, M_2\right)}{dY_1 dY_2 d\mathbf{\Delta}^2 d^2\mathbf{K}}$$

$$\simeq \frac{\alpha_s^2 x_1 x_2}{8 m_1^5 m_2^5 N_c^4 \left(N_c^2-1\right)^2} \left\langle \mathcal{O}_{M_1}\left({}^{2S_1+1}L^1_{1J_1}\right)\right\rangle \left\langle \mathcal{O}_{M_2}\left({}^{2S_2+1}L^1_{2J_2}\right)\right\rangle$$

$$\times \Big[\delta^{ii'}\delta^{jj'}\mathcal{F}_{g,g}\left(x_1,x_2\right) - i\delta^{ii'}\epsilon^{jj'}\mathcal{F}_{g,\Delta g}\left(x_1,x_2\right) + 2\delta^{ii'}\mathcal{F}^{jj'}_{g,\delta g}\left(x_1,x_2\right)$$

$$-i\epsilon^{ii'}\delta^{jj'}\mathcal{F}_{\Delta g,g}\left(x_1,x_2\right) - \epsilon^{ii'}\epsilon^{jj'}\mathcal{F}_{\Delta g,\Delta g}\left(x_1,x_2\right) - 2i\epsilon^{ii'}\mathcal{F}^{jj'}_{\Delta g,\delta g}\left(x_1,x_2\right)$$

$$+2\delta^{jj'}\mathcal{F}^{ii'}_{\delta g,g}\left(x_1,x_2\right) - 2i\epsilon^{jj'}\mathcal{F}^{ii'}_{\delta g,\Delta g}\left(x_1,x_2\right) + 4\mathcal{F}^{ii',jj'}_{\delta g,\delta g}\left(x_1,x_2\right)\Big]$$

$$\times \Pi_1^{ii',kk'}\left(M_1\right)\Pi_2^{jj',\ell\ell'}\left(M_2\right) x\mathcal{G}^{k\ell}\left(\mathbf{K},\mathbf{\Delta}\right) x\mathcal{G}^{k'\ell'*}\left(\mathbf{K},\mathbf{\Delta}\right),$$

Simplified and averaged generic cross section

$$\int \frac{d\varphi}{2\pi} \frac{d\sigma\left(M_1, M_2\right)}{dY_1 dY_2 d\boldsymbol{\Delta}^2 d^2\boldsymbol{K}}$$
$$\simeq \frac{\alpha_s^2 x_1 x_2}{8 m_1^5 m_2^5 N_c^4 \left(N_c^2 - 1\right)^2} \left\langle \mathcal{O}_{M_1}\left({}^{2S_1+1}L^1_{1J_1}\right)\right\rangle \left\langle \mathcal{O}_{M_2}\left({}^{2S_2+1}L^1_{2J_2}\right)\right\rangle$$
$$\times \left[\delta^{ii'}\delta^{jj'}\mathcal{F}_{g,g}\left(x_1, x_2\right) + 4\mathcal{F}^{ii',jj'}_{\delta g,\delta g}\left(x_1, x_2\right)\right]$$
$$\times \Pi_1^{ii',kk'}\left(M_1\right)\Pi_2^{jj',\ell\ell'}\left(M_2\right) x\mathcal{G}^{k\ell}\left(\boldsymbol{K}, \boldsymbol{\Delta}\right) x\mathcal{G}^{k'\ell'*}\left(\boldsymbol{K}, \boldsymbol{\Delta}\right),$$

Perturbative parts

$$\Pi_1^{ii',kk'}(\eta) = \delta^{ii'}\delta^{kk'} - \delta^{ik'}\delta^{i'k},$$

$$\Pi_1^{ii',kk'}(\chi_0) = 3\frac{\delta^{ik}\delta^{i'k'}}{m_1^2}$$

$$\Pi_1^{ii',kk'}(\chi_1) = \frac{\delta^{ii'}}{2m_1^4}\left(\boldsymbol{K}^k + \frac{\boldsymbol{\Delta}^k}{2}\right)\left(\boldsymbol{K}^{k'} + \frac{\boldsymbol{\Delta}^{k'}}{2}\right),$$

$$\Pi_1^{ii',kk'}(\chi_2) = \frac{2}{m_1^2}\left[\delta^{ii'}\delta^{kk'} - \delta^{ik}\delta^{i'k'} + \delta^{i'k}\delta^{ik'} + \frac{\delta^{ii'}}{4m_1^2}\left(\boldsymbol{K}^k + \frac{\boldsymbol{\Delta}^k}{2}\right)\left(\boldsymbol{K}^{k'} + \frac{\boldsymbol{\Delta}^{k'}}{2}\right)\right]$$

A few cross sections - 1

$$(\chi_1\chi_1)$$

$$\frac{\alpha_s^4 x^2 \boldsymbol{K}^4}{32 m_1^9 m_2^9 N_c^4 \left(N_c^2-1\right)^2} x_1 x_2 \mathcal{F}_{g,g}\left(x_1,x_2\right) \left\langle \mathcal{O}_{\chi_{f_1 1}}\left({}^3P_1^1\right)\right\rangle \left\langle \mathcal{O}_{\chi_{f_2 1}}\left({}^3P_1^1\right)\right\rangle$$
$$\times\left[\left(\mathcal{G}_1+\frac{\boldsymbol{K}^2}{2M^2}\mathcal{G}_2\right)^2-\frac{\boldsymbol{\Delta}^2}{2\boldsymbol{K}^2}\left(\mathcal{G}_1+\frac{\boldsymbol{K}^2}{2M^2}\mathcal{G}_2\right)\left(\mathcal{G}_1+2\frac{\boldsymbol{K}^2}{M^2}\mathcal{G}_4\right)\right]$$

$$(\chi_1\chi_0)$$

$$\frac{3\alpha_s^4 x^2 \boldsymbol{K}^2}{32 m_1^9 m_2^7 N_c^4 \left(N_c^2-1\right)^2} x_1 x_2 \mathcal{F}_{g,g}\left(x_1,x_2\right) \left\langle \mathcal{O}_{\chi_1}\left({}^3P_1^1\right)\right\rangle \left\langle \mathcal{O}_{\chi_0}\left({}^3P_0^1\right)\right\rangle$$
$$\times\left[\left(\mathcal{G}_1+\frac{\boldsymbol{K}^2}{2M^2}\mathcal{G}_2\right)^2+\frac{\boldsymbol{\Delta}^2}{4\boldsymbol{K}^2}\left(\mathcal{G}_1^2+\frac{\boldsymbol{K}^4}{4M^4}\left(\mathcal{G}_2^2-8\mathcal{G}_2\mathcal{G}_4+8\mathcal{G}_4^2\right)\right)\right]$$

A few cross sections

$$(\chi_0\chi_0)$$

$$\frac{9\alpha_s^4 x^2}{16 m_1^7 m_2^7 N_c^4 \left(N_c^2-1\right)^2} \left\langle \mathcal{O}_{\chi_{f_1 0}} \left({}^3P_0^1\right) \right\rangle \left\langle \mathcal{O}_{\chi_{f_2 0}} \left({}^3P_0^1\right) \right\rangle$$

$$\times \left\{ x_1 x_2 \mathcal{F}_{g,g}\left(x_1,x_2\right) \left(\mathcal{G}_1^2 + \frac{\boldsymbol{K}^4}{4M^4}\mathcal{G}_2^2 + \frac{\boldsymbol{K}^2\boldsymbol{\Delta}^2}{2M^4}\mathcal{G}_4^2 \right) \right.$$

$$\left. +4 x_1 x_2 \mathcal{H}_{\delta g,\delta g}\left(x_1,x_2\right) \left[\mathcal{G}_1^2 - 2\frac{\boldsymbol{\Delta}^2}{M^2}\left(\mathcal{G}_1\mathcal{G}_3 - \frac{\boldsymbol{K}^2}{4M^2}\mathcal{G}_4^2 \right) \right] \right\}$$

$$(\chi_0\eta)$$

$$\frac{3\alpha_s^4 x^2}{16 m_1^7 m_2^5 N_c^4 \left(N_c^2-1\right)^2} \left\langle \mathcal{O}_{\chi_0} \left({}^3P_0^1\right) \right\rangle \left\langle \mathcal{O}_{\eta} \left({}^1S_0^1\right) \right\rangle$$

$$\left\{ x_1 x_2 \mathcal{F}_{g,g}\left(x_1,x_2\right) \left(\mathcal{G}_1^2 + \frac{\boldsymbol{K}^4}{4M^4}\mathcal{G}_2^2 + \frac{\boldsymbol{K}^2\boldsymbol{\Delta}^2}{2M^4}\mathcal{G}_4^2 \right) \right.$$

$$\left. -4 x_1 x_2 \mathcal{H}_{\delta g,\delta g}\left(x_1,x_2\right) \left[\mathcal{G}_1^2 - 2\frac{\boldsymbol{\Delta}^2}{M^2}\left(\mathcal{G}_1\mathcal{G}_3 - \frac{\boldsymbol{K}^2}{4M^2}\mathcal{G}_4^2 \right) \right] \right\}$$

Issues and solutions

- Possibly large pollution from NRQCD octet contributions

  Consider the fully exclusive hybrid case: GPD+Wigner

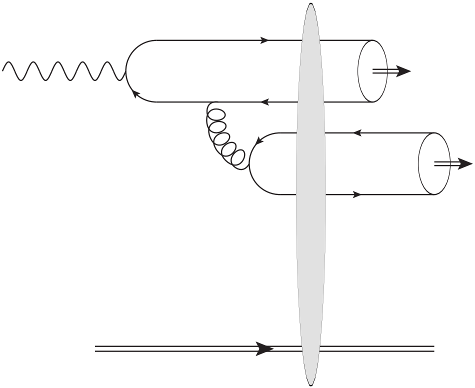
- $\frac{k_\perp}{m}$ corrections to the hard part

  Resum (kinematic) twists

- Requires two C^+ quarkonia: experimentally challenging

  No solution: consider other processes.

Suggestion of a theoretically cleaner and experimentally easier process (credits to [Feng Yuan])



No required hybrid factorization ansatz ([RB, Hatta, Xiao, Yuan]) nor dilute kinematics ([Metz *et al*])

J/psi and a C^+ quarkonium are experimentally easier to observe than double C^+ quarkonia.

Production of a transverse light vector meson

Non-forward and non-dilute extension of
[Anikin, Besse, Ivanov, Pire, Szymanowski, Wallon]

Previous works [Anikin, Besse, Ivanov, Pire, Szymanowski, Wallon]

- Full $\gamma_T \to \rho_T$ impact factor, but
  - Linear BFKL regime only
  - Forward $t = 0$ case only
  - Hence No $\gamma_L^* \to \rho_T$ transition allowed
- Proved the equivalence between two major schemes for collinear factorization at twist 3, but in a process-dependent way
- Required interesting algebra to restore QCD gauge invariance, but no deep understanding for the origin of invariance breaking in the first place

Light Cone Collinear Factorization

The Light Cone Collinear Factorization approach

Momentum factorization

- Define a Sudakov vector n such that $p \cdot n = 1$ and write $d^4 p_q = \int dx \, d^4 p_q \; \delta(x - p_q \cdot n)$.
- Taylor expansion of the hard part $H(p_q)$ along the collinear direction xp:

$$H(p_q) e^{-i p_q \cdot z} S(z) = H(xp) e^{-i p_q \cdot z} S(z) \quad + \quad \left. \frac{\partial H(p_q)}{\partial p_q^\mu} \right|_{p_q = xp} (p_q - x\, p)^\mu e^{-i p_q \cdot z} S(z) + \ldots$$

- $p_q^\mu \stackrel{IbP}{\longrightarrow}$ derivative of the soft term: $\int d^4 z \; e^{-i\ell \cdot z} \langle \rho(p) | \psi(0) \, i \; \overleftrightarrow{\partial_\mu} \, \bar{\psi}(z) | 0 \rangle$
- Standard derivative $\Rightarrow$ need for 3-body contributions to combine into a covariant derivative.

Required DAs for ρ_T production at twist 3 in LCCF

- 2-body DAs

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_\mu \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho \left[\varphi_1(x) \left(\varepsilon_\rho^* \cdot n \right) p_\mu + \varphi_3(x) \varepsilon_{\rho T \mu}^* \right]$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho i \varphi_A(x) \epsilon_{\mu\lambda\beta\delta} \varepsilon_{\rho T}^{\lambda *} p^\beta n^\delta$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_\mu i \overleftrightarrow{\partial}_\alpha \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho \varphi_{1T}(x) p_\mu \varepsilon_{\rho T \alpha}^*$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) \gamma_5 \gamma_\mu i \overleftrightarrow{\partial}_\alpha \psi(0) \right| 0 \right\rangle \to m_\rho f_\rho i \varphi_{AT}(x) p_\mu \epsilon_{\alpha\lambda\beta\delta} \varepsilon_{\rho T}^{*\lambda} p^\beta n^\delta$$

- 3-body DAs

$$\left\langle \rho(p) \left| \bar{\psi}(z_1) \gamma_\mu g A_\alpha(z_2) \psi(0) \right| 0 \right\rangle \to m_\rho f_3^V B(x_1, x_2) p_\mu \varepsilon_{\rho T \alpha}^*$$

$$\left\langle \rho(p) \left| \bar{\psi}(z_1) \gamma_5 \gamma_\mu g A_\alpha(z_2) \psi(0) \right| 0 \right\rangle \to m_\rho f_3^A i D(x_1, x_2) p_\mu \epsilon_{\alpha\lambda\beta\delta} \varepsilon_{\rho T}^{*\lambda} p^\beta n^\delta$$

7 required DAs

- Equations of motion: Dirac equation

$$\left\langle \left(i\hat{D}\psi_\alpha\right)(0)\,\bar{\psi}_\beta(z)\right\rangle = 0, \quad \left\langle \psi_\alpha(0)\left(i\hat{D}\bar{\psi}_\beta\right)(z)\right\rangle = 0$$

- Leads to two equations

$$x_1\varphi_3(x_1) + \bar{x}_1\varphi_A(x_1) + \varphi_{1T}(x_1) + \varphi_{AT}(x_1) + \int dx_2\left[\zeta_3^V B(x_1, x_2) + \zeta_3^A D(x_1, x_2)\right] = 0$$

$$\bar{x}_1\varphi_3(x_1) - x_1\varphi_A(x_1) - \varphi_{1T}(x_1) + \varphi_{AT}(x_1) - \int dx_2\left[\zeta_3^V B(x_2, x_1) - \zeta_3^A D(x_2, x_1)\right] = 0$$

7-2 required DAs

Minimal set of DAs

7-2 required DAs

- *n*-independence. *n* appeared in three constraints:
  - Lighcone direction of the separation z: $z = \lambda n$
  - Definition of the transverse polarization $\varepsilon_\rho \cdot n = 0$
  - Chosen gauge $n \cdot A = 0$
- Leads to 2 additional constraints for the DAs, plus the gauge invariance condition.

7-4 required DAs

$\varphi(x)$ ← 2-body twist 2 correlator
$B(x_1, x_2)$ ← 3-body genuine twist 3 vector correlator
$D(x_1, x_2)$ ← 3-body genuine twist 3 axial correlator

Covariant Collinear Factorization

- Work directly on the operators, with gauge invariant light ray operators
- 2-body correlators

$$\left\langle \rho(p) \left| \bar{\psi}(z) [z,0] \gamma^{\mu} \psi(0) \right| 0 \right\rangle \rightarrow f_{\rho} m_{\rho} \left[-ip^{\mu} \left(\varepsilon_{\rho}^{*} \cdot z \right) h(x) + \varepsilon_{\rho}^{\mu *} g_{\perp}^{(v)}(x) \right]$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) [z,0] \gamma_{5} \gamma_{\mu} \psi(0) \right| 0 \right\rangle \rightarrow \frac{1}{4} f_{\rho} m_{\rho} \epsilon_{\mu\alpha\beta\delta} \varepsilon_{\rho\perp}^{\alpha} p^{\beta} z^{\delta} g_{\perp}^{(a)}(x)$$

- 3-body correlators

$$\left\langle \rho(p) \left| \bar{\psi}(z) [z,tz] \gamma_{\alpha} g G_{\mu\nu}(tz) [tz,0] \psi(0) \right| 0 \right\rangle$$
$$\rightarrow -i m_{\rho} f_{3\rho}^{V} p_{\alpha} \left(p_{\mu} \varepsilon_{\rho\perp\nu}^{*} - p_{\nu} \varepsilon_{\rho\perp\mu}^{*} \right) V(x_1, x_2)$$

$$\left\langle \rho(p) \left| \bar{\psi}(z) [z,tz] \gamma_{\alpha} \gamma_{5} g \tilde{G}_{\mu\nu}(tz) [tz,0] \psi(0) \right| 0 \right\rangle$$
$$\rightarrow -m_{\rho} f_{3\rho}^{A} p_{\alpha} \left(p_{\mu} \varepsilon_{\rho\perp\nu}^{*} - p_{\nu} \varepsilon_{\rho\perp\mu}^{*} \right) A(x_1, x_2)$$

- Equations of motions ⇒ only 3 DAs are required

Matching at twist 3 accuracy

LCCF	CCF
$\varphi_3(x)$	$g_\perp^{(v)}(x)$
$\varphi_1^T(x)$	$\tilde{h}(x) - h(x)$
$\varphi_A(x)$	$-\frac{1}{4}\frac{\partial g_\perp^{(a)}}{\partial x}(x)$
$\varphi_A^T(x)$	$-\frac{1}{4}g_\perp^{(a)}(x)$
$B(x_q, x_{\bar{q}}; x_g)$	$\frac{-V\left(x_q, x_{\bar{q}}; x_g\right)}{1-x_q-x_{\bar{q}}}$
$D(x_q, x_{\bar{q}})$	$\frac{-A\left(x_q, x_{\bar{q}}; x_g\right)}{1-x_q-x_{\bar{q}}}$

A process-specific comparison was done previously [Anikin, Ivanov, Pire, Szymanowski, Wallon]
A completely generic proof exists [RB *et al*, to be published].

Effective CGC Feynman rules for fields

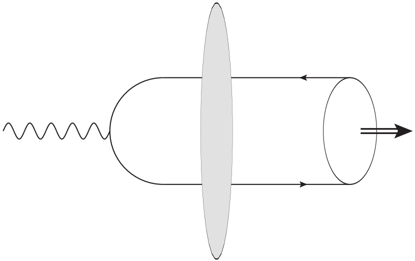
The recursion to exponentiate slow gluon scatterings into a Wilson line only starts at order g_s

$$A^{\mu}_{eff}(z_0)|_{z_0^+<0} = A^{\mu}(z_0) - 2i\int d^D z_3\,\delta(z_3^+)\,G^{\mu}_{\sigma_\perp}(z_{30})\left(U^{ba}_{\vec{z}_3} - \delta^{ba}\right)F^{+\sigma_\perp}(z_3)$$

$$\overline{\psi}_{eff}(z_0)|_{z_0^+<0} = \bar{\psi}(z_0) + \int d^D z_1\,\delta(z_1^+)\,\overline{\psi}(z_1)\,(U_{\vec{z}_1} - 1)\gamma^+ G(z_{10})$$

$$\psi_{eff}(z_0)|_{z_0^+<0} = \psi(z_0) - \int d^D z_2\delta(z_2^+)\,G(z_{02})\,\gamma^+\psi(z_2)\left(U^{\dagger}_{\vec{z}_2} - 1\right)$$

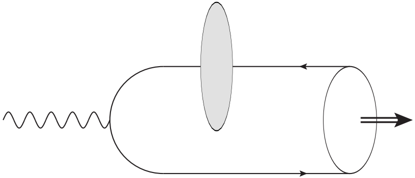
Natural 2-body CGC diagram



$$\int d^2\vec{z}_1 d^2\vec{z}_2\ \Phi^{2b}_{q\bar{q}}(\vec{z}_1,\vec{z}_2)\,\mathrm{Tr}[(U_1-1)(U_2^\dagger-1)]\,\langle\rho\,|\bar{\psi}\psi|\,0\rangle$$

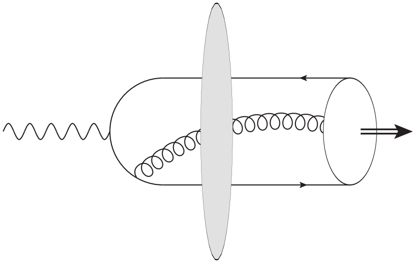
Contains monopole contributions

Antiquark monopole 2-body diagram



$$\int d^2\vec{z}_2\ \Phi_{\bar{q}}^{2b}(\vec{z}_2)\,\mathrm{Tr}[(U_2^\dagger - 1)]\left\langle \rho \left| \bar{\psi}\psi \right| 0 \right\rangle$$

Natural 3-body CGC diagram

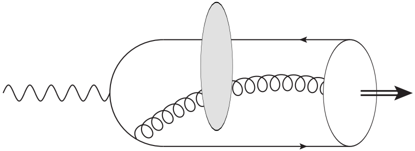


$$\int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\ \Phi^{3b}_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\,\mathrm{Tr}[(U_1-1)t^b(U_2^\dagger-1)t^a](U_3^{ab}-\delta^{ab})\,\langle\rho\,|\bar{\psi}A\psi|\,0\rangle$$

Contains dipole and monopole contributions

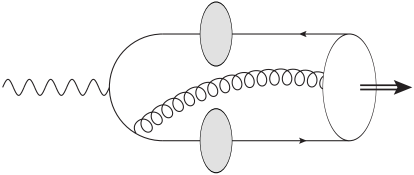
Double-dipole term even at tree level $\Rightarrow$ Great sensitivity to saturation

3-body $(\bar{q}g)$-dipole diagram



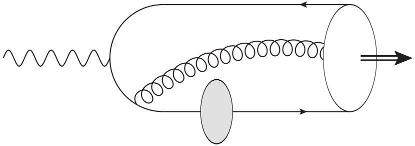
$$\mathcal{A}_{\bar{q}g}^{3b} = \int d^2\vec{z}_2 d^2\vec{z}_3 \; \Phi_{\bar{q}g}^{3b}(\vec{z}_2, \vec{z}_3)\, \mathrm{Tr}[t^b(U_2^\dagger - 1)t^a](U_3^{ab} - \delta^{ab}) \left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$

3-body $(q\bar{q})$-dipole diagram



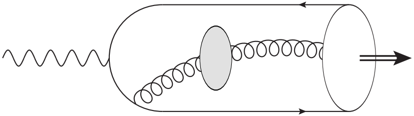
$$\mathcal{A}_{q\bar{q}}^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 \; \Phi_{q\bar{q}}^{3b}(\vec{z}_1, \vec{z}_2) \, \mathrm{Tr}[(U_1 - 1)t^b(U_2^\dagger - 1)t^a]\delta^{ab} \left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$

3-body (q)-monopole diagram



$$\mathcal{A}_q^{3b} = \int d^2\vec{z}_1 \Phi_q^{3b}(\vec{z}_1)\,\mathrm{Tr}[(U_1 - 1)t^b t^a]\delta^{ab}\left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$

3-body (g)-monopole diagram



$$\mathcal{A}_g^{3b} = \int d^2\vec{z}_3\, \Phi_g^{3b}(\vec{z}_3)\, \mathrm{Tr}[t^b t^a](U_3^{ab} - \delta^{ab}) \left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$

Cancelling the 2-body monopoles

Antiquark monopole part of the natural CGC diagram

- Monopole part of the quark line

$$\overline{\psi}_{eff}\left(z_0\right)|_{z_0^+<0} = \bar{\psi}\left(z_0\right) + \int d^D z_1\, \delta\left(z_1^+\right) \overline{\psi}\left(z_1\right)\left(U_{\vec{z}_1} - 1\right)\gamma^+ G\left(z_{10}\right)$$

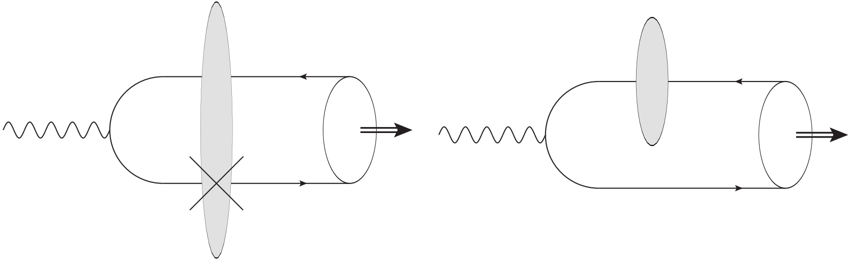
- Simple algebra allows one to get

$$\int d^D z_1 \int \frac{d^D q}{(2\pi)^D} \delta\left(z_1^+\right) \left(\frac{-i\bar{\psi}\left(z_1\right)}{\left(q^- - \frac{\vec{q}^2 - i0}{2q^+}\right)} + \frac{\bar{\psi}\left(z_1\right)\overleftarrow{\partial}\gamma^\mu\gamma^+}{2q^+\left(q^- - \frac{\vec{q}^2 - i0}{2q^+}\right)} \right) e^{-i(q\cdot z_{10})}$$

- Thus one term contributes to a 2-body monopole contribution, and (Dirac equation) the other term contributes to a 3-body monopole contribution.

Sum of the 2-body antiquark monopoles

Using the fact that the non-perturbative collinear matrix elements do not depend on z^+ variables at twist 3 accuracy ...[censored technicalities]... we get the sum between the natural 2-body antiquark monopole diagram and the 2-body antiquark monopole part of the natural CGC diagram



$$\frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - \frac{\vec{q}^2}{2q^+}} \quad - \quad \frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - 0}$$

Sum of the 2-body antiquark monopoles

Using the fact that the non-perturbative collinear matrix elements are at most linear in $z_\perp$, the sum cancels iff

$$\left.\frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - \frac{\vec{q}^2}{2q^+}} - \frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)}}\right|_{\vec{q}=\vec{0}} = 0$$

$$\frac{\partial}{\partial q_\perp^\mu}\left.\left(\frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - \frac{\vec{q}^2}{2q^+}} - \frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)}}\right)\right|_{\vec{q}=\vec{0}} = 0$$

Cancelling the 3-body unnatural dipoles, and monopoles

"Unnatural" 3-body diagrams

$$\Phi_{qg}(\vec{z}_1,\vec{z}_3)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle = \int d^2\vec{z}_2\, \Phi_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle + \mathrm{Twist}\,4$$

$$\Phi_{\bar{q}g}(\vec{z}_2,\vec{z}_3)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle = \int d^2\vec{z}_1\, \Phi_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle + \mathrm{Twist}\,4$$

$$\Phi_{q\bar{q}}(\vec{z}_1,\vec{z}_2)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle = \int d^2\vec{z}_3\, \Phi_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle + \mathrm{Twist}\,4$$

$$\Phi_{g}(\vec{z}_3)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle = \int d^2\vec{z}_1 d^2\vec{z}_2\, \Phi_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle + \mathrm{Twist}\,4$$

$$\Phi_{q}(\vec{z}_1)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle = \int d^2\vec{z}_2 d^2\vec{z}_3\, \Phi_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle + \mathrm{Twist}\,4$$

Hence the 3-body total from 3-body diagrams

$$\mathcal{A}_3^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\, \Phi^{3b}_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\left\langle \rho\left|\bar{\psi}A\psi\right|0\right\rangle$$
$$\times\left[\mathrm{Tr}(U_1 t^b U_2^\dagger t^a)U_3^{ab} - \mathrm{Tr}(t^b U_2^\dagger t^a \delta^{ab})\right]$$

Total from 3-body diagrams

$$\mathcal{A}^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3 \, \Phi^{3b}_{q\bar{q}g}(\vec{z}_1, \vec{z}_2, \vec{z}_3) \left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$
$$\times \left[\mathrm{Tr}(U_1 t^b U_2^\dagger t^a) U_3^{ab} - \mathrm{Tr}(t^b U_2^\dagger t^a \delta^{ab}) \right]$$

"3-body" antiquark monopole from the natural 2-body diagram

$$\Phi_2^{3b}(\vec{z}_2) = \int d^2\vec{z}_1 d^2\vec{z}_3 \, \Phi_{q\bar{q}g}(\vec{z}_1, \vec{z}_2, \vec{z}_3) + \mathrm{Twist}\,4$$

Sums up to a gauge invariant amplitude

$$\mathcal{A}^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3 \, \Phi^{3b}_{q\bar{q}g}(\vec{z}_1, \vec{z}_2, \vec{z}_3)$$
$$\times \left[\mathrm{Tr}(U_1 t^b U_2^\dagger t^a) U_3^{ab} - C_F \right] \left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$

Final amplitude

$$\mathcal{A} = \int d^2\vec{z}_1 d^2\vec{z}_2\, \Phi^{2b}_{q\bar{q}}(\vec{z}_1,\vec{z}_2)\left[\mathrm{Tr}\left(U_1 U_2^\dagger\right) - N_c\right] + \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\, \Phi^{3b}_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\left[\mathrm{Tr}\left(U_1 t^b U_2^\dagger t^a\right) U_3^{ab} - C_F\right]$$

Expansion in Reggeons in the dilute limit: (Reggeon momenta q_1, q_2)

$$\Phi_{BFKL} = \int d^2\vec{z}_1 d^2\vec{z}_2\, \Phi^{2b}_{q\bar{q}}(\vec{z}_1,\vec{z}_2)\left(e^{i(\vec{q}_1\cdot\vec{z}_2)} - e^{i(\vec{q}_1\cdot\vec{z}_1)}\right)\left(e^{i(\vec{q}_2\cdot\vec{z}_1)} - e^{i(\vec{q}_2\cdot\vec{z}_2)}\right)$$
$$- \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3 \Phi^{3b}_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\left[N_c\left(e^{i(\vec{q}_1\cdot\vec{z}_3)} - e^{i(\vec{q}_1\cdot\vec{z}_1)}\right)\left(e^{i(\vec{q}_2\cdot\vec{z}_3)} - e^{i(\vec{q}_2\cdot\vec{z}_2)}\right)\right.$$
$$\left.-\left(\frac{N_c^2-1}{2N_c}\right)\left(e^{i(\vec{q}_1\cdot\vec{z}_2)} - e^{i(\vec{q}_1\cdot\vec{z}_1)}\right)\left(e^{i(\vec{q}_2\cdot\vec{z}_1)} - e^{i(\vec{q}_2\cdot\vec{z}_2)}\right)\right]$$

Obviously gauge invariant in the BFKL sense: $\Phi_{BFKL} = 0$ for $q_1 = 0$ or $q_2 = 0$.

In the dilute, forward limit, our result matches the previous BFKL results