Two neutron transfer in Sn isotopes

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Introduction and Outline

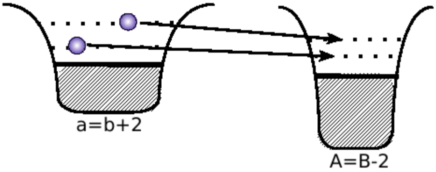
This talk will be devoted to two particle transfer reactions as the specific probe to study pairing correlations. Emphasis will be made in the connection between structure aspects and the resulting two particle transfer cross sections.

Outline:

- Reaction mechanism : two particle transfer in second order DWBA
- $^{A}\text{Sn}(p, t)^{A-2}\text{Sn}$ reactions: transition between pairing vibrational (closed shell) to pairing rotational (superfluid) regimes in the tin isotopic chain.

- Two valence nucleons go from core b of nucleus a to core A of nucleus B
- Probing two particle correlations.
- Investigating structure properties such as pairing and superfluidity in a finite fermion system (the atomic nucleus).

- Get absolute values as well as the angular distribution for the cross sections in second order DWBA.



$A(a,b)B$

Examples:

$^{112}Sn(p,t)^{110}Sn$

$^{1}H(^{11}Li,^{9}Li)^{3}H$

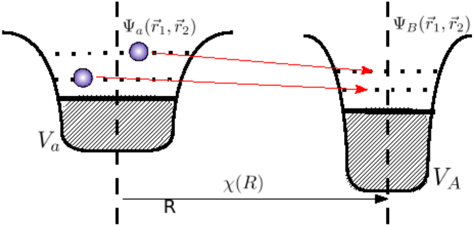
Reaction mechanism:

second order DWBA

Elements of the calculation

$\Psi_a(\vec{r}_1, \vec{r}_2)$, $\Psi_B(\vec{r}_1, \vec{r}_2)$: internal wave functions of the transferred nucleons in each nucleus

$\chi(R)$: distorted wave describing the relative motion in the optical potential $U(R) = V(R) + iW(R)$ $\left(\frac{P_R^2}{2\mu} + U(R)\right)\chi(R) = E_{CM}\chi(R)$



V_A, V_a: mean field potentials of the two nuclei

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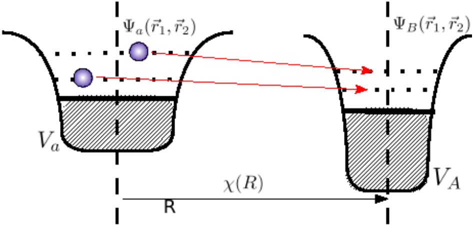

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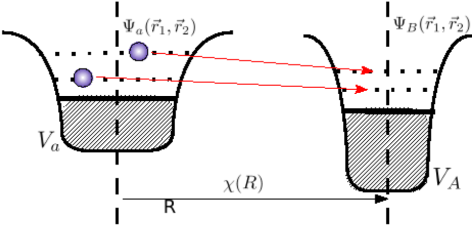

V_A, V_a: mean field potentials of the two nuclei

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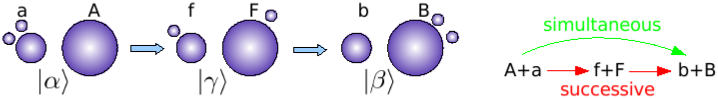
it is a single particle potential!!

Simultaneous and Successive contributions



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$$|\beta\rangle = \phi_b(\xi_b)\phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2)\times \chi_{bB}(\mathbf{r}_{bB})$$

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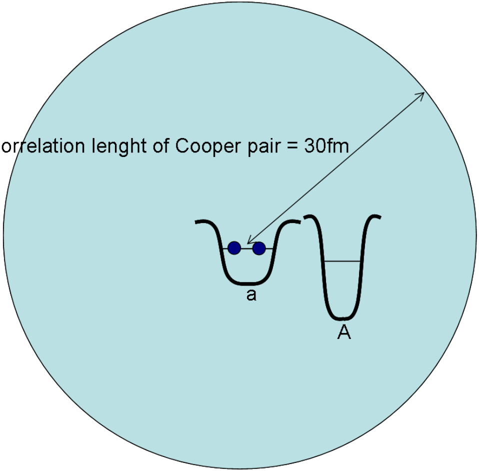

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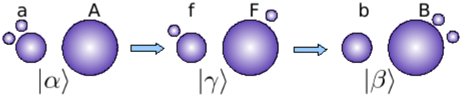

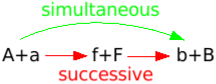

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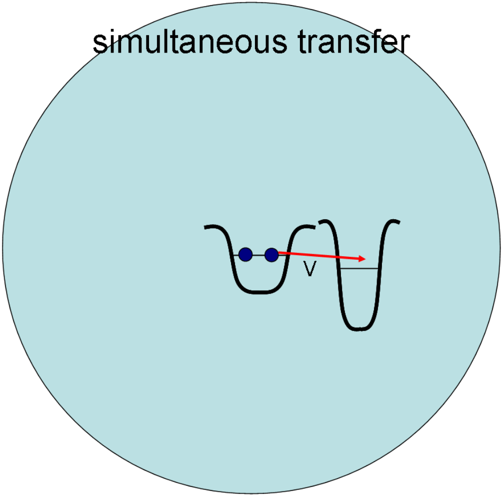

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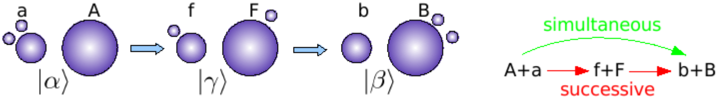

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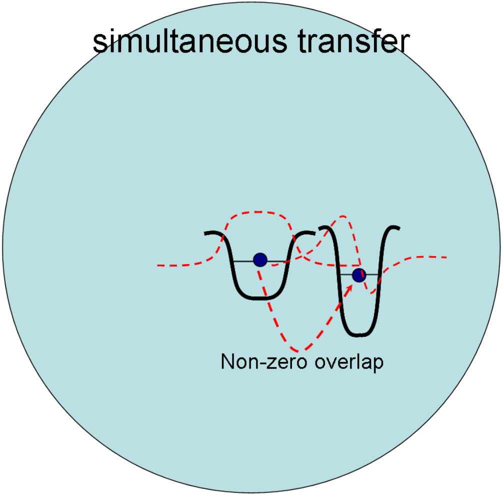

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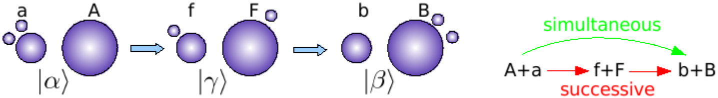

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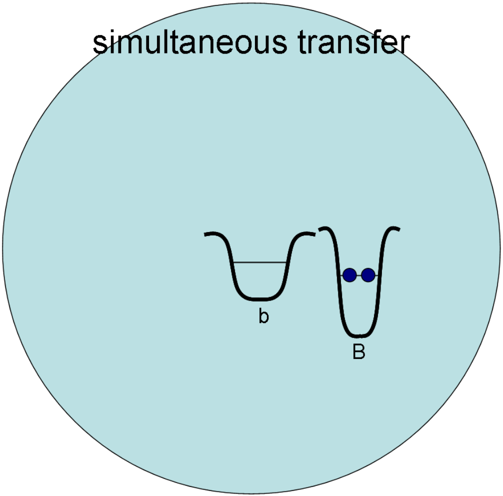

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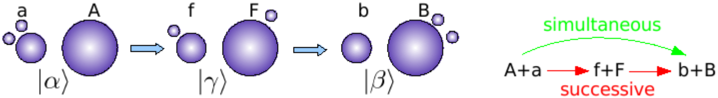

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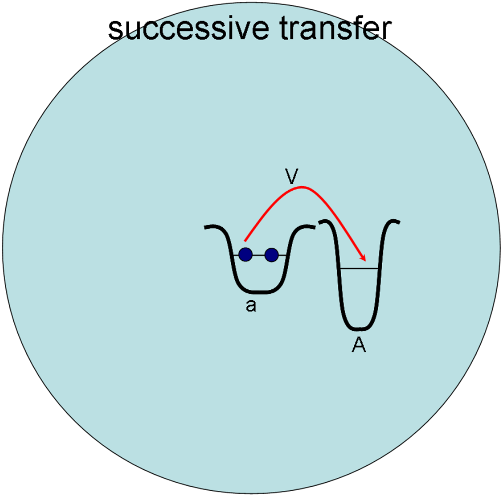

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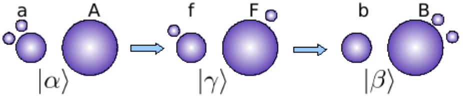

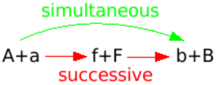

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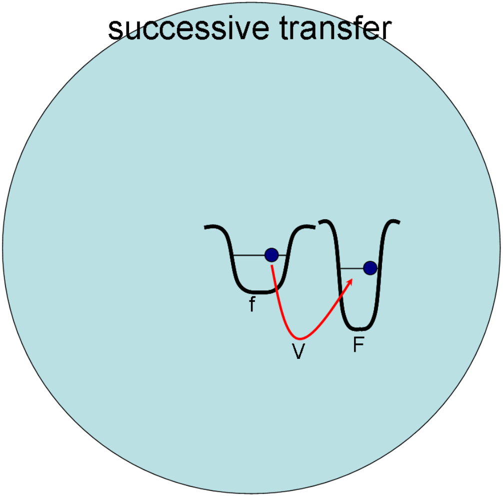

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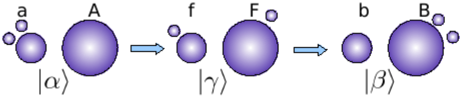

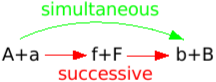

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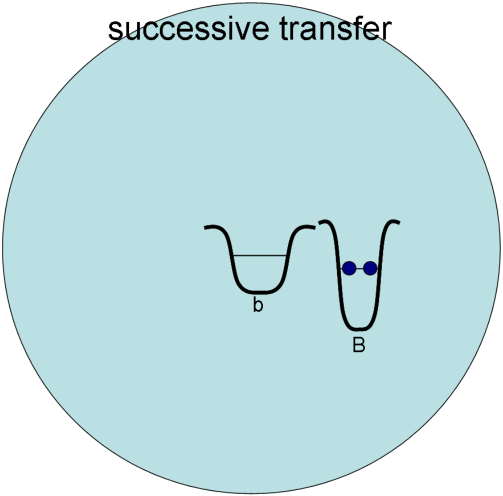

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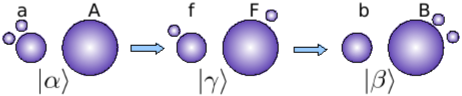

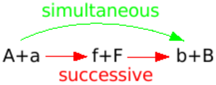

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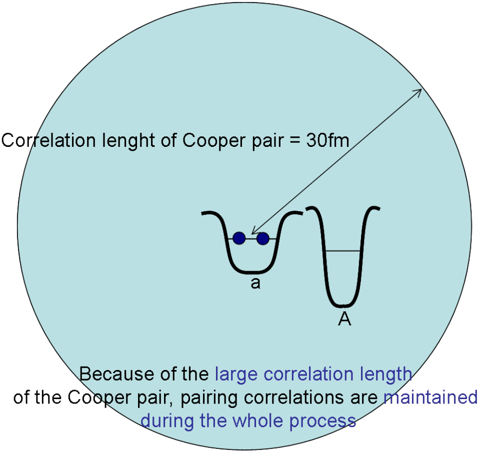
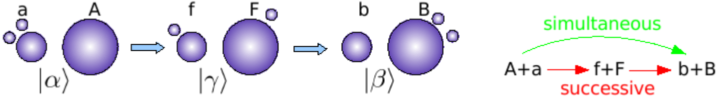


Some details of the calculation of the differential cross section for two–nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi\hbar^2)^2} \frac{k_f}{k_i} |T_{2NT}|^2$$

Simultaneous transfer

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_0 \chi^{(-)*}_{bB}(\mathbf{r}_{bB})$$

$$\times\, v(\mathbf{r}_{b1}) [\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \Psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]^{\Lambda}_{\mu} \chi^{(+)}_{aA}(\mathbf{r}_{aA})$$

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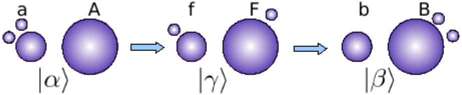

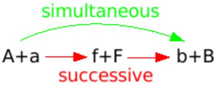

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Successive transfer

$$T^{(2)}_{succ}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1\sigma_2 \\ \sigma'_1\sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_{0}$$

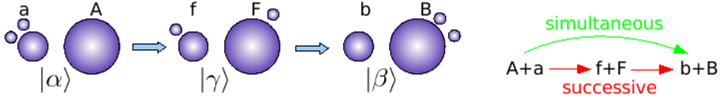
$$\times \chi^{(-)*}_{bB}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]^{K}_{M}$$

$$\times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^{K}_{M}$$

$$\times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^{\Lambda}_{\mu} \chi^{(+)}_{aA}(\mathbf{r}'_{aA})$$

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two–nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

Non–orthogonality term

$$T^{(2)}_{NO}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1\sigma_2 \\ \sigma_1'\sigma_2'}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_0$$
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$$\times \int d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^K_M$$
$$\times [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^{\Lambda}_{\mu} \chi^{(+)}_{aA}(\mathbf{r}'_{aA})$$

Cancellation of simultaneous and non-orthogonal contributions

very schematically, the first order (*simultaneous*) contribution is

$$T^{(1)} = \langle\beta|V|\alpha\rangle,$$

while the second order contribution can be separated in a *successive* and a *non-orthogonality* term

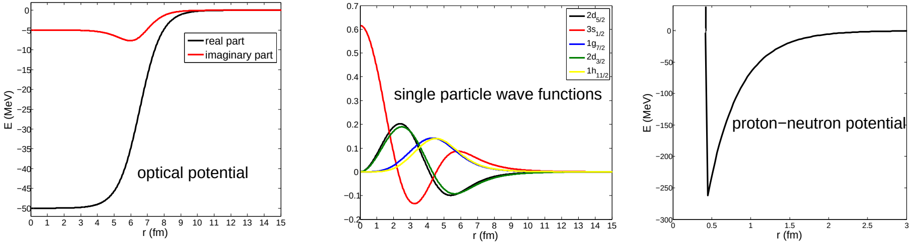
$$\begin{aligned} T^{(2)} &= T^{(2)}_{succ} + T^{(2)}_{NO} \\ &= \sum_\gamma \langle\beta|V|\gamma\rangle G \langle\gamma|V|\alpha\rangle - \sum_\gamma \langle\beta|\gamma\rangle\langle\gamma|V|\alpha\rangle. \end{aligned}$$

If we sum over a complete basis of intermediate states γ, we can apply the closure condition and $T^{(2)}_{NO}$ exactly cancels $T^{(1)}$

the transition potential being single particle, two-nucleon transfer is a second order process.

Ingredients of the calculation

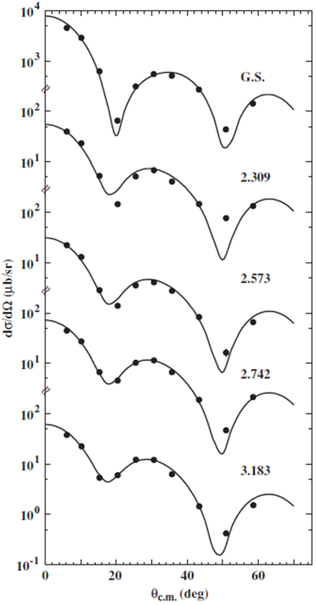
Structure input for, e.g., the ^{112}Sn(p,t)^{110}Sn reaction:



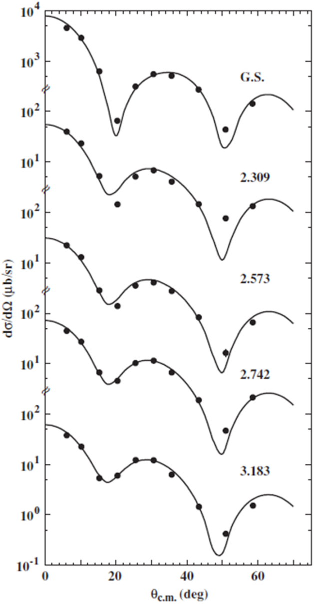
plus the B_j spectroscopic amplitudes needed to define the two–neutron wavefunction:

$$\Phi(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_j B_j \left[\psi^j(\mathbf{r}_1, \sigma_1)\psi^j(\mathbf{r}_2, \sigma_2)\right]^0_0$$

“Standard procedure”: first order DWBA



^{112}Sn(p,t)^{110}Sn reaction, $E_p = 26$ MeV (Guazzoni *et al.* PRC **74** 054605 (2006)) with first order DWBA one obtains the angular distribution of the angular differential cross section



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absolute normalization ⇒ relative cross sections

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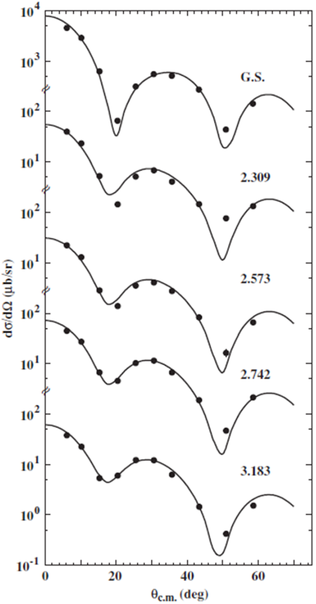

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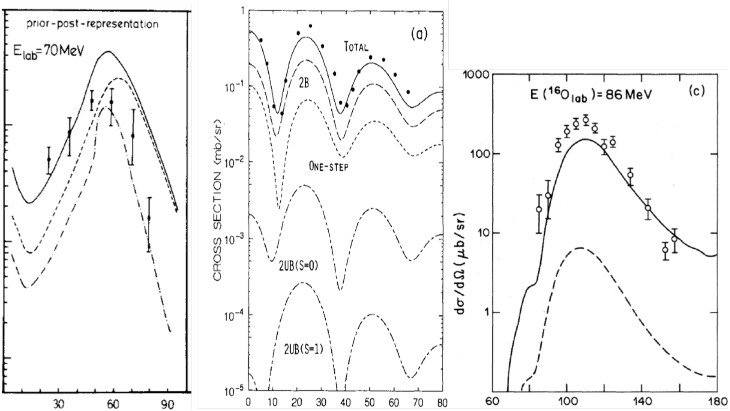
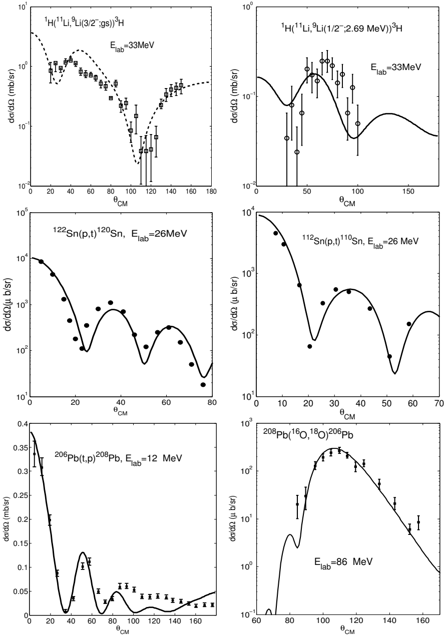
Give up absolute cross sections!!

Early results with second order DWBA



Götz, Ichimura, Broglia and Winther, Phys. Rep. **16** (1975)
Igarashi, Kubo and Yagi, Phys. Rep. **199**(1991)1
Bayman and Chen, PRC **26** (1982)1509, respectively

Examples of calculations



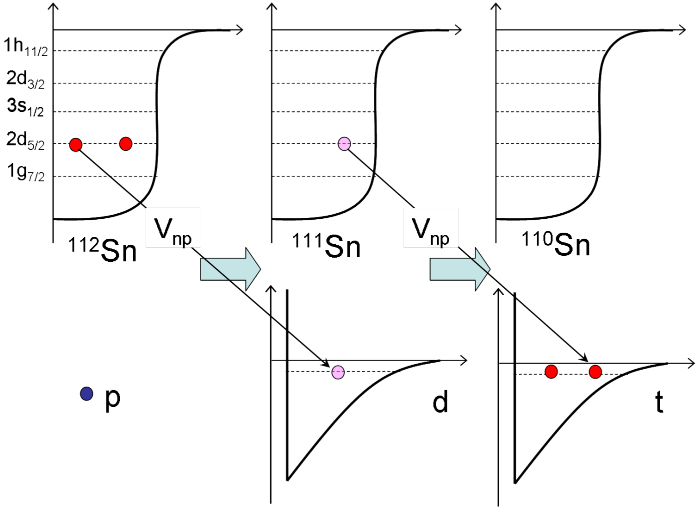
good results obtained for halo nuclei,
population of excited states,
superfluid nuclei,
normal nuclei (pairing vibrations),
heavy ion reactions...
Potel *et al.*, arXiv:0906.4298.

From pairing vibrations

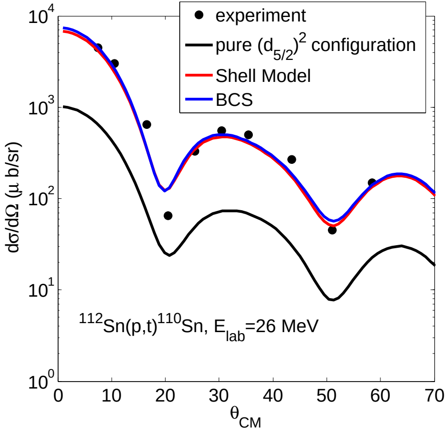
to pairing rotations:

Tin isotopic chain

$^{112}Sn(p,t)^{110}Sn$, reaction mechanism



$^{112}Sn(p,t)^{110}Sn$, results



enhancement factor with respect to the transfer of uncorrelated neutrons:
$\varepsilon = 20.6$

Experimental data and shell model wavefunction from Guazzoni *et al.* PRC **74** 054605 (2006)

Pairing correlations and two–particle transfer cross sections

two–particle transfer transition strength: $|\langle \Psi_{A-2}|P|\Psi_A\rangle|^2$

In superfluid nuclei (open shell):

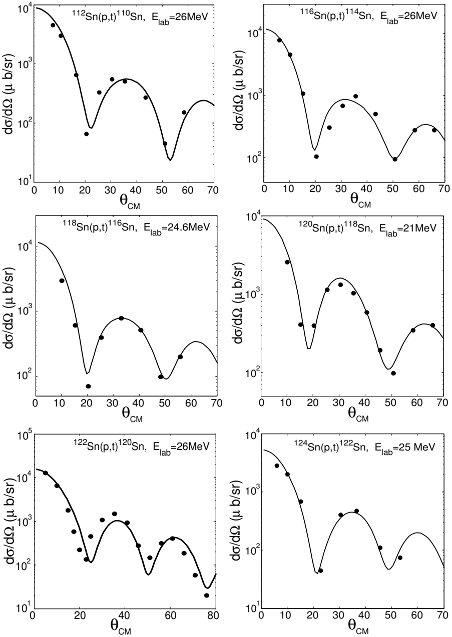
$$\langle \Psi_{A-2}|P|\Psi_A\rangle \approx \langle BCS|P|BCS\rangle = \alpha_0 = \sum_{\nu>0} U_\nu V_\nu = \Delta/G$$

$$d\sigma/d\Omega(A, g.s \rightarrow A+2, g.s.) \sim \alpha_0^2$$

In normal nuclei (closed shell), $\Delta = \alpha_0 = 0$:

$$d\sigma/d\Omega \sim \langle (\alpha - \alpha_0)^2 \rangle = \left[\langle \Psi_A|P^\dagger P|\Psi_A\rangle - \langle \Psi_A|PP^\dagger|\Psi_A\rangle \right]/2$$

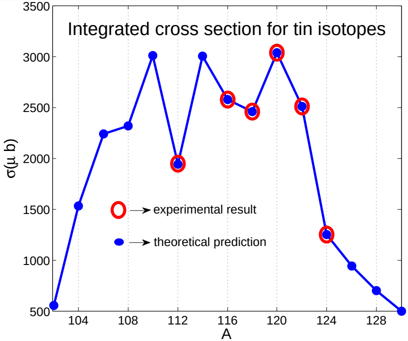
^{A}Sn(p,t)$^{A-2}$Sn, results

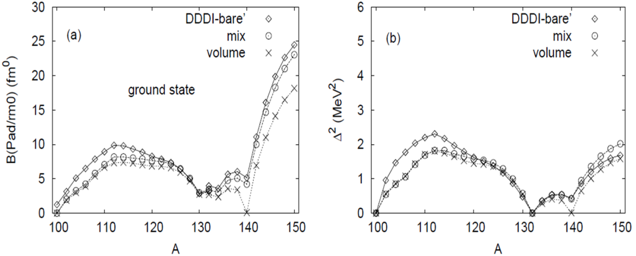


Comparison with the experimental data available so far for superfluid tin isotopes

Potel *et al.*, PRL **107**, 092501 (2011)

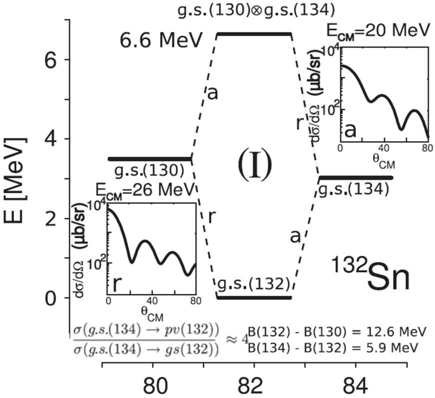
^{A}Sn(p,t)$^{A-2}$Sn, superfluid isotopic chain





Shimoyama and Matsuo, nucl-th/1106.1715

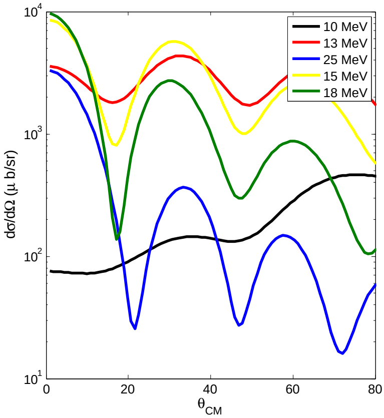
$^{134,132}Sn(p,t)^{132,130}Sn$, pairing vibrations

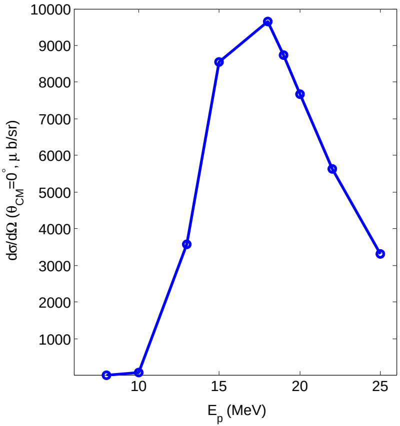


$^{132}Sn(p,t)^{130}Sn$ and $^{134}Sn(p,t)^{132}Sn$ reactions can probe the predicted pairing vibrations of the exotic double magic nucleus ^{132}Sn

Potel *et al.*, PRL **107**, 092501 (2011)

$^{132}Sn(p,t)^{130}Sn$ cross sections





Conclusions

- Second order DWBA has proven to be a valuable reaction formalism to obtain reliable absolute values,along with angular distributions, for the two particle transfer nuclear reactions angular differential cross sections.
- Two nucleon transfer reactions are an ideal tool to probe two neutrons correlations in nuclei.
- We have studied the transition between pairing vibrational (closed shell) to pairing rotational (superfluid) regimes in the tin isotopic chain.
- We hope that the predictions made for reactions with exotic beams such as ^{132}Sn(p,t)^{130}Sn will stimulate future experiments!.

Thank You!