

*A new analysis of old Saclay data reveals a breaking of  
axial symmetry in many nuclei.*

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*Collaboration supported by EU-project CHANDA*

*Level densities - n-tof data*

*Quadrupole observables in even nuclei*

*Splitting of giant dipole resonances*

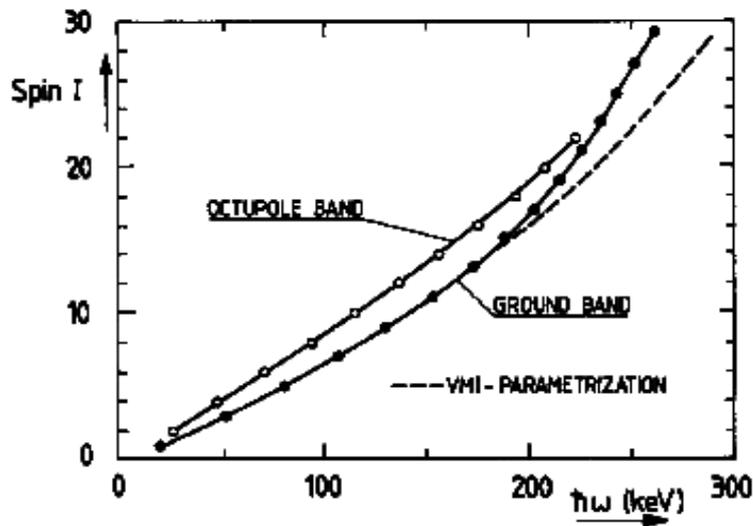
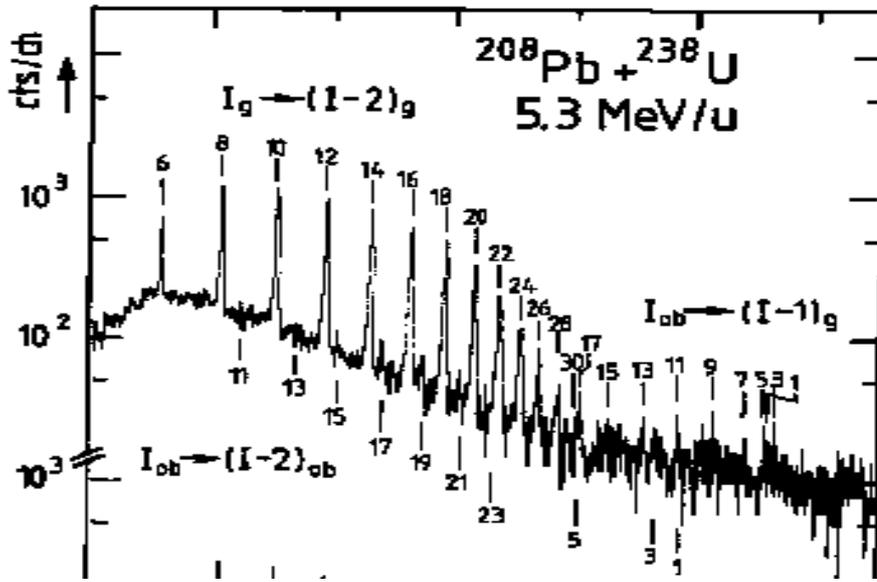
*Consequences for their interpretation*

*Photon strength and n-capture*

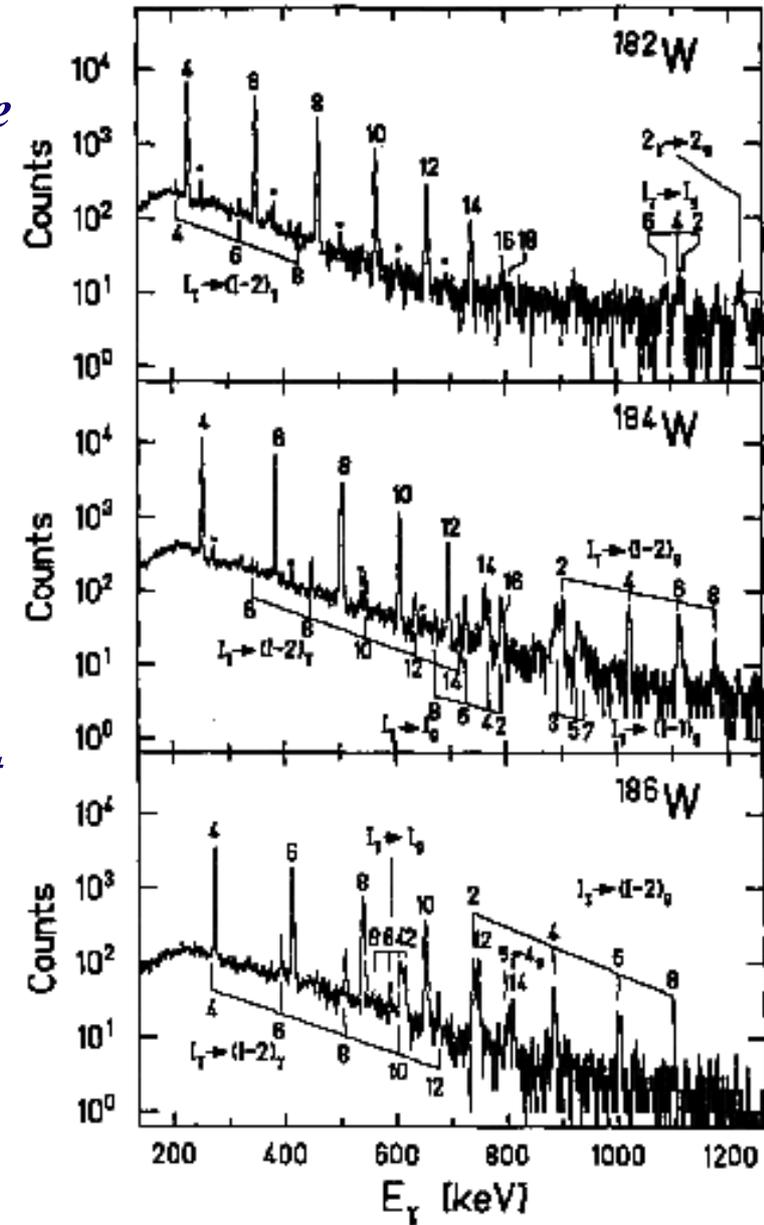
*Rotor models and microscopic calculations*

*Coulomb excitation of actinides and also of 'intermediate' nuclei seems to indicate*

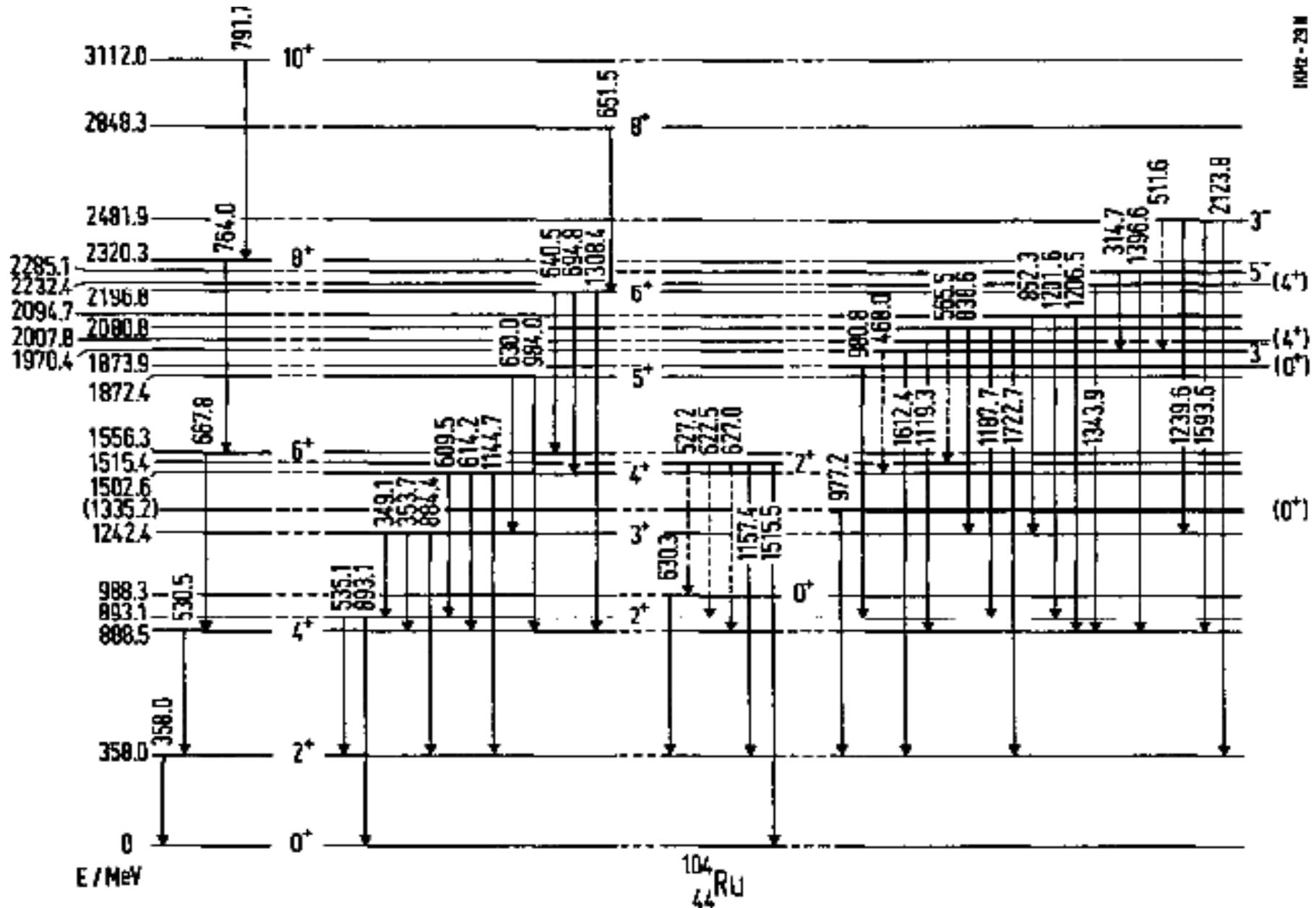
*rigid rotation of axial shape*



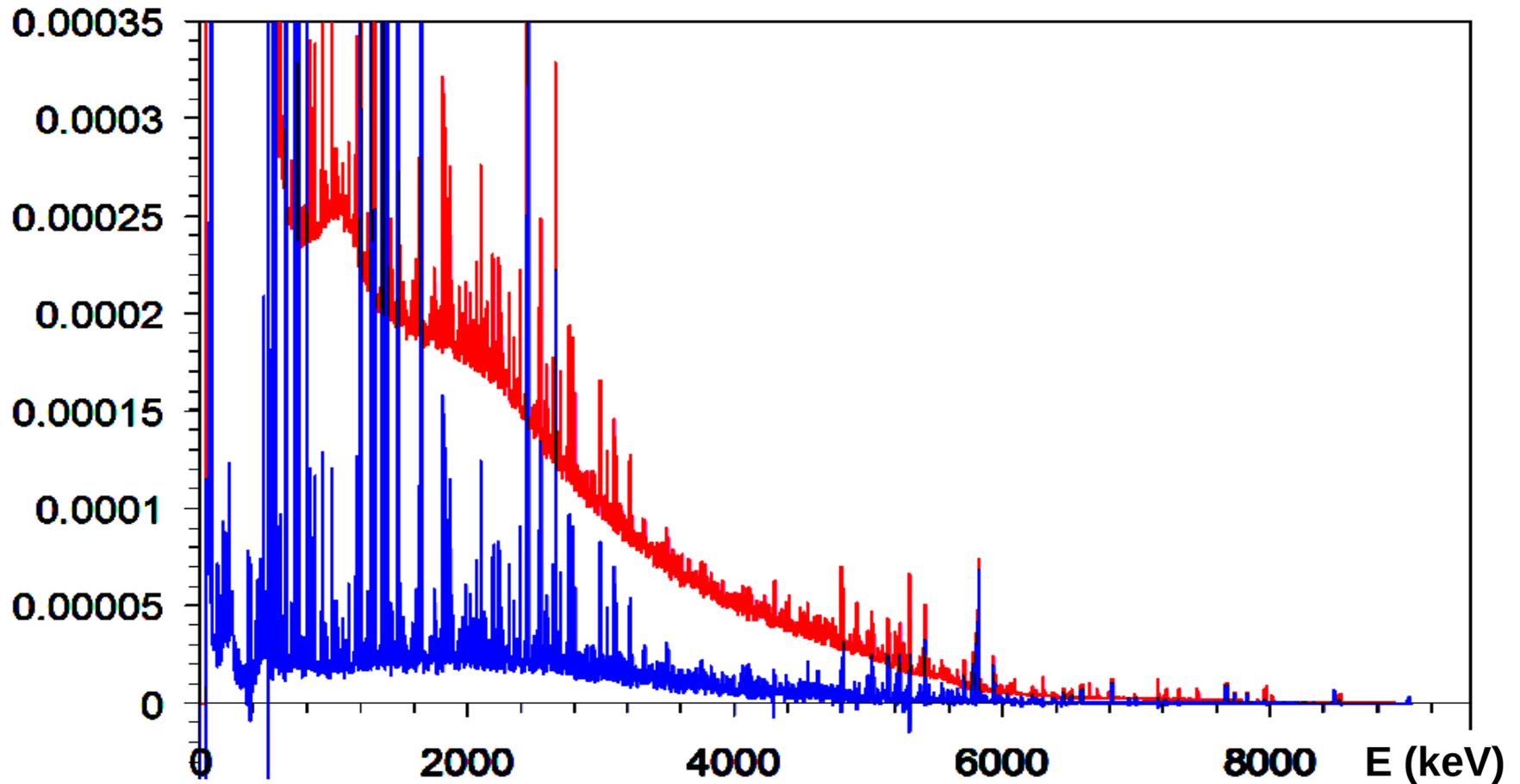
*but in general the situation is different*



*Complex Coulex data with  $^{104}\text{Ru}$  and  $^{208}\text{Pb}$ -beams indicate broken axial symmetry already at low  $E_x$*



## *Improved data analysis: Ge-detector spectra from $^{113}\text{Cd}(n_{\text{therm}}, \gamma)$*



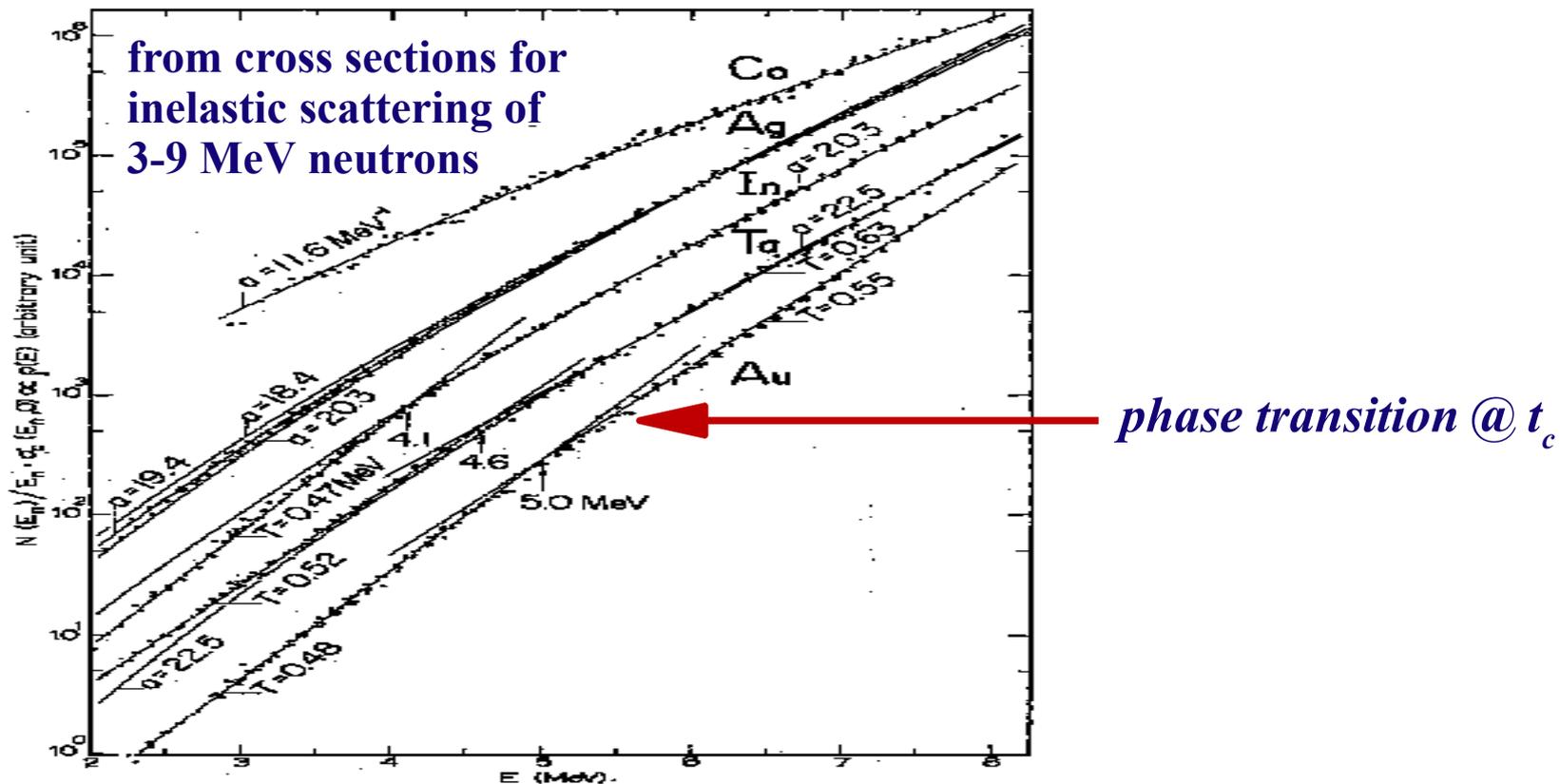
*The deconvoluted, efficiency corrected spectrum shows high level density in quasi-continuous broad distribution peaking at 1 to 3 MeV*

*Level densities  $\rho(E)$  in heavy nuclei indicate a phase transition between*

*a Fermi gas above  $t_c = \Delta_0 \cdot e^C / \pi = 0.567 \cdot \Delta_0$  (with Euler' constant C)*

*and below a regime influenced by pairing and shell effects,*

*approximated by an exponential.*



## *Fermi gas model*

$$\omega_{qp}(E_x) = \frac{\sqrt{\pi} \cdot \exp(2\sqrt{\tilde{a}(E_x - E_{bs})})}{12 \tilde{a}^{1/4} (E_x - E_{bs})^{5/4}}$$

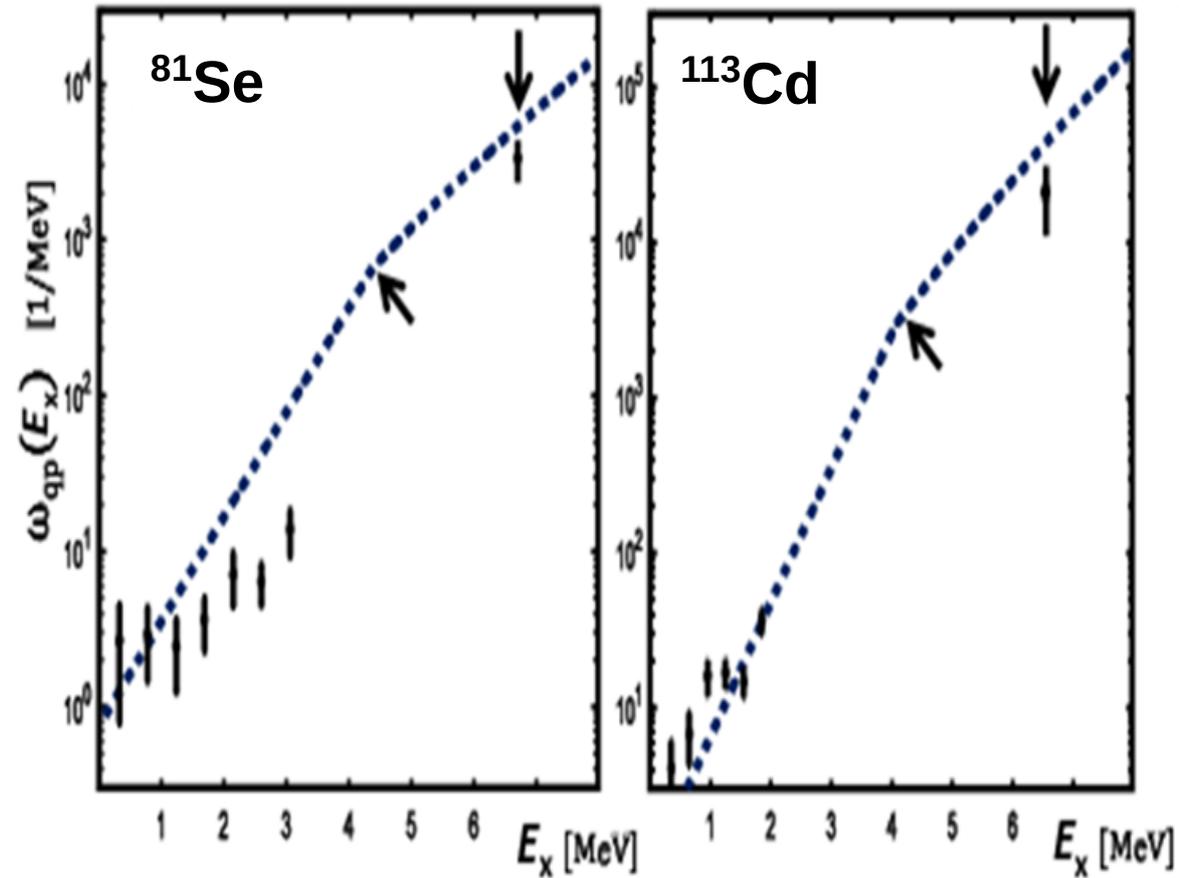
## *Constant temperature model*

$$\omega_{qp}(E_x) = \omega_{qp}(0) \exp\left(\frac{E_x}{T_\alpha}\right)$$

*The two-component level density approach has to be complemented by collective rotational enhancement, which is simple for low spins:*

$$\rho \rightarrow \frac{2J+1}{2 \cdot 4} \omega_{qp}(E_x) \quad \text{if axial symmetry is not required.}$$

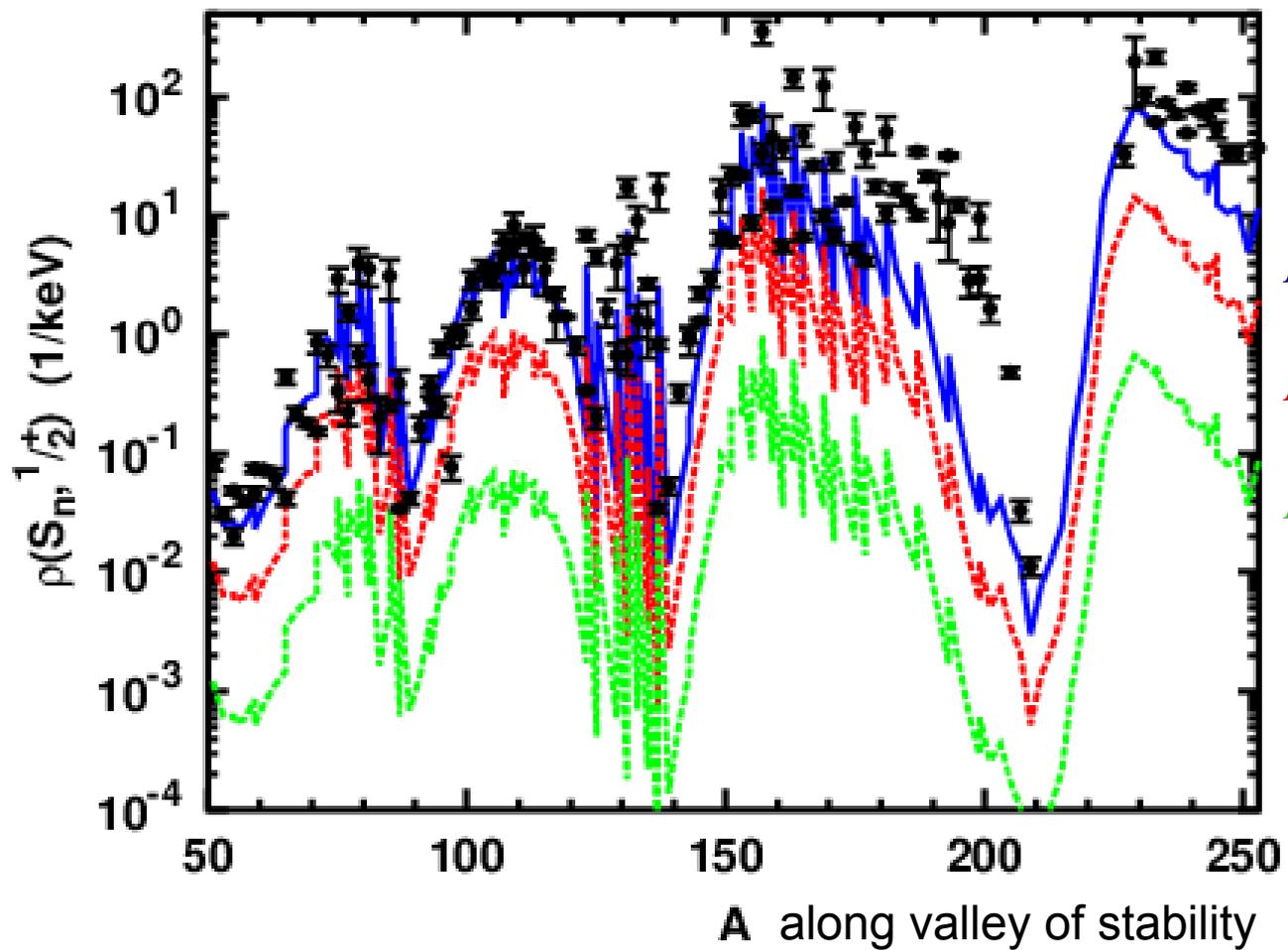
*Without fit we get good agreement for states with  $J = 1/2$  and then also for capture resonances into  $J=0$  targets. Values for  $\Delta$  and  $\tilde{a}$  taken from nuclear matter;  $E_{bs}$  from LD-mass fit.*



*Level densities  $\rho(E, I)$  in heavy nuclei result from collective enhancement (group theory)*

*of intrinsic state density  $\omega(E)$ ; account for broken axially allows to use  $\tilde{a} = \tilde{a}_{nm} = A/15$*

*Accurate data stem from n-capture resonances just above  $S_n$ :*



*prediction for broken axially*  
*prediction assuming axially*  
*prediction for spherical case,*  
*absolute scale,*  
*no parameters adjusted.*

# Rotational enhancement of nuclear level density vs. symmetry class

The intrinsic *quasi-particle state density* in a finite nucleus  $\omega_{qp}(E_x)$  is not yet the observable density of nuclear levels with *well defined spin*  $\rho(E_x, J, \pi)$ .

To fix  $J$  the underlying collective symmetry has to be determined by group theory:

1. *spherical*  $\Rightarrow$  *only q-p states*  $\rho(E_x, J, \pi) \rightarrow \frac{2J+1}{2 \cdot \sqrt{8\pi} \sigma^3} \omega_{qp}(E_x)$  ↙ small J limit

2. *axial*  $\Rightarrow$  *q-p states & rotation  $\perp$  axis*  $\rho \rightarrow \frac{2J+1}{2 \cdot \sqrt{8\pi} \sigma} \omega_{qp}(E_x)$

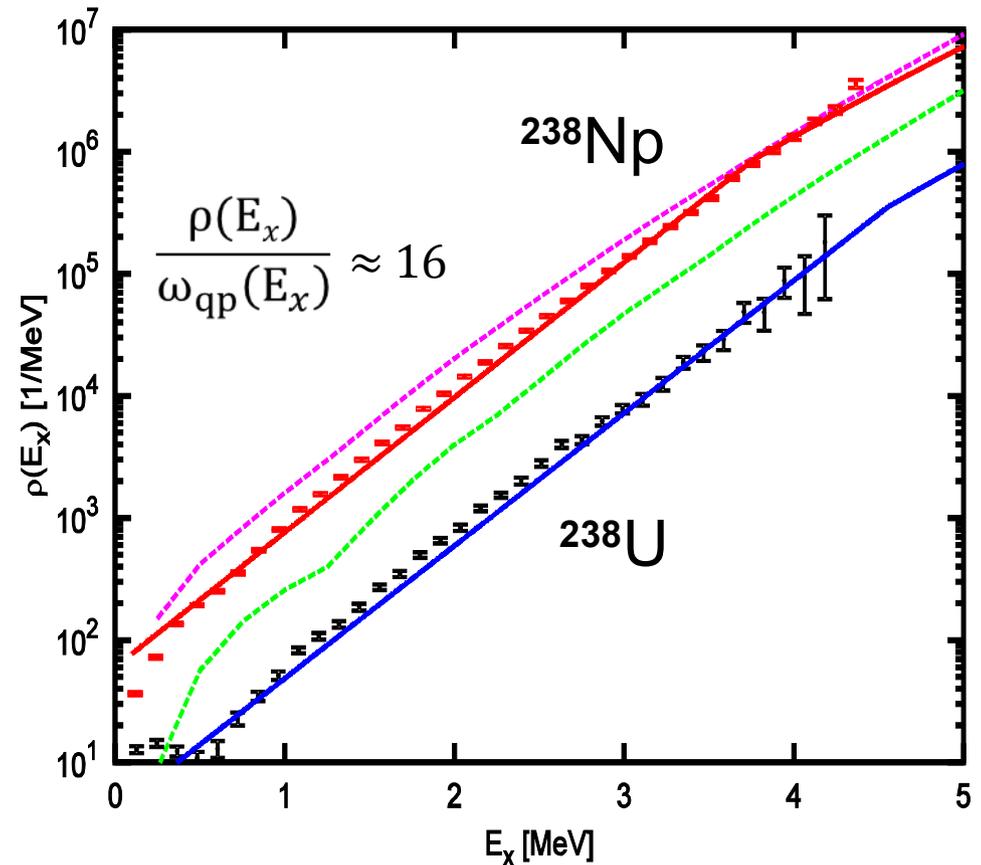
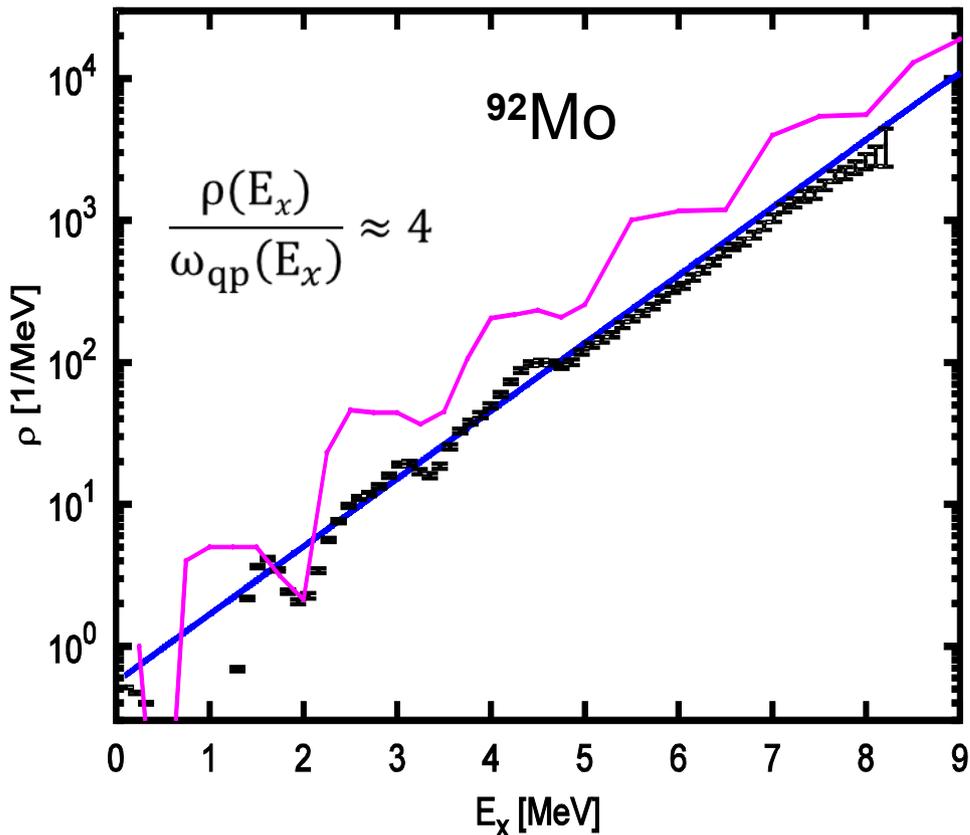
3. non-axial (*triax*)  $\Rightarrow$  *q-p states & rotation about any axis*  $\rho \rightarrow \frac{2J+1}{2 \cdot 4} \omega_{qp}(E_x)$

4. *no reflection symmetry*  $\Rightarrow$  *q-p states & octupole deform.*  $\rho \rightarrow \frac{2J+1}{2} \omega_{qp}(E_x)$

Thomas-Fermi Model  $\Rightarrow \sigma^2 = \frac{\tilde{a} \cdot t}{11} A^{2/3} \approx \frac{A^{5/3}}{143} \cdot t$  ↖ 1 parity

*Collective enhancement for “total” level density  $\rho(E_x, I) = \sum_{I=0}^{\infty} \rho(E_x, I)$*

$$\frac{\rho(E_x)}{\omega_{qp}(E_x)} = \frac{3}{4} \cdot \sum_{I=0}^{\infty} (2I+1) \cdot e^{-E_{yr}(I)/T_{eff}} \approx 3 \cdot \frac{T_{eff}}{1\text{MeV}} \cdot \sqrt{\frac{\pi \cdot \mathfrak{I}_{app}}{1\text{eV} \cdot \hbar^2}}$$



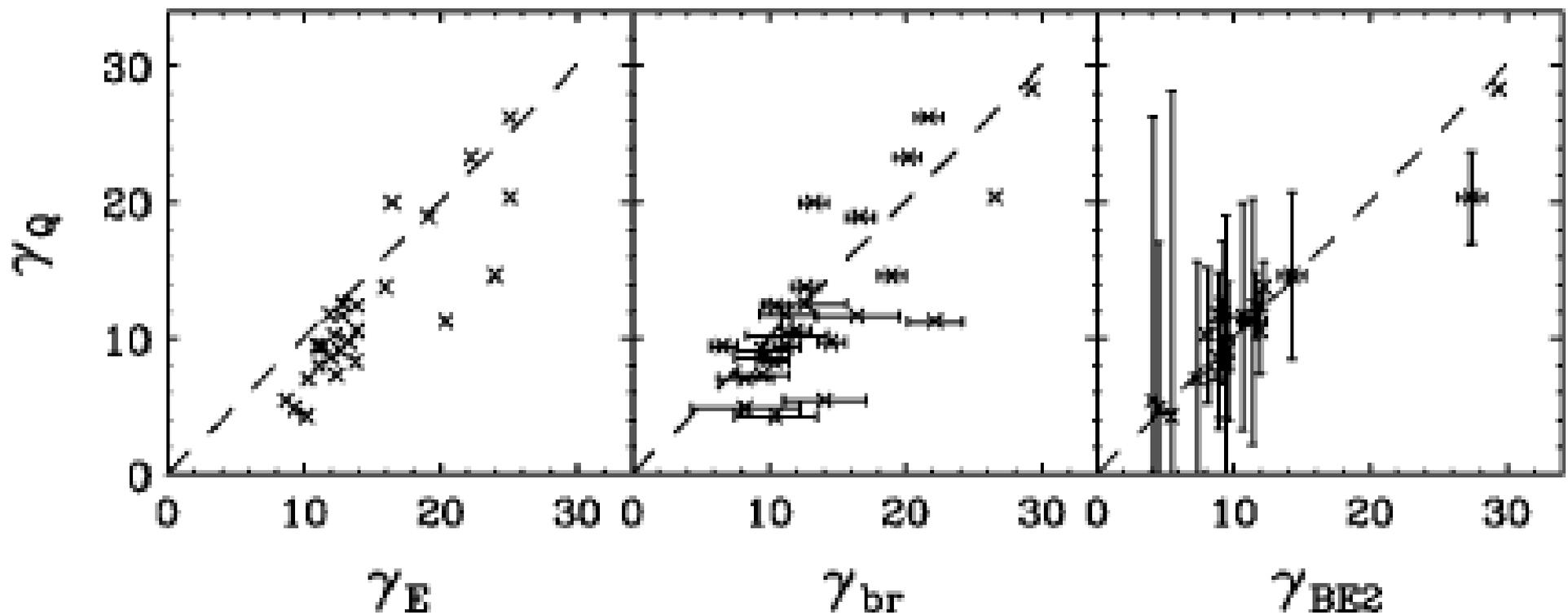
*axiality broken, no parameters adjusted => good agreement on absolute scale.*

## *Comparison of empirical $\gamma$ -values for nuclei with $50 < Z < 82$*

*The three panels compare  $\gamma_Q$  obtained from IBA-1 fits to the data with*

*$\gamma_E$ ,  $\gamma_{br}$ , and  $\gamma_{BE2}$  values obtained from the Davydov model relating  $\gamma$  to the empirical energy ratio, branching ratio, and  $B(E2)$  ratios, respectively.*

*The uncertainties in  $\gamma_Q$  are the same in each of these panels and shown in only one of them*



*The IBA-1 suggests that axial asymmetry arises from  $\gamma$ -softness*

# *New view on heavy nuclei – avoiding axial symmetry postulate*

*by: newly applied theoretical tools and*

*HFB + GCM, projection on  $I \Rightarrow \beta, \gamma$*

*$Q^3 \cdot \cos(3\gamma) \Rightarrow$  rotation invariant*

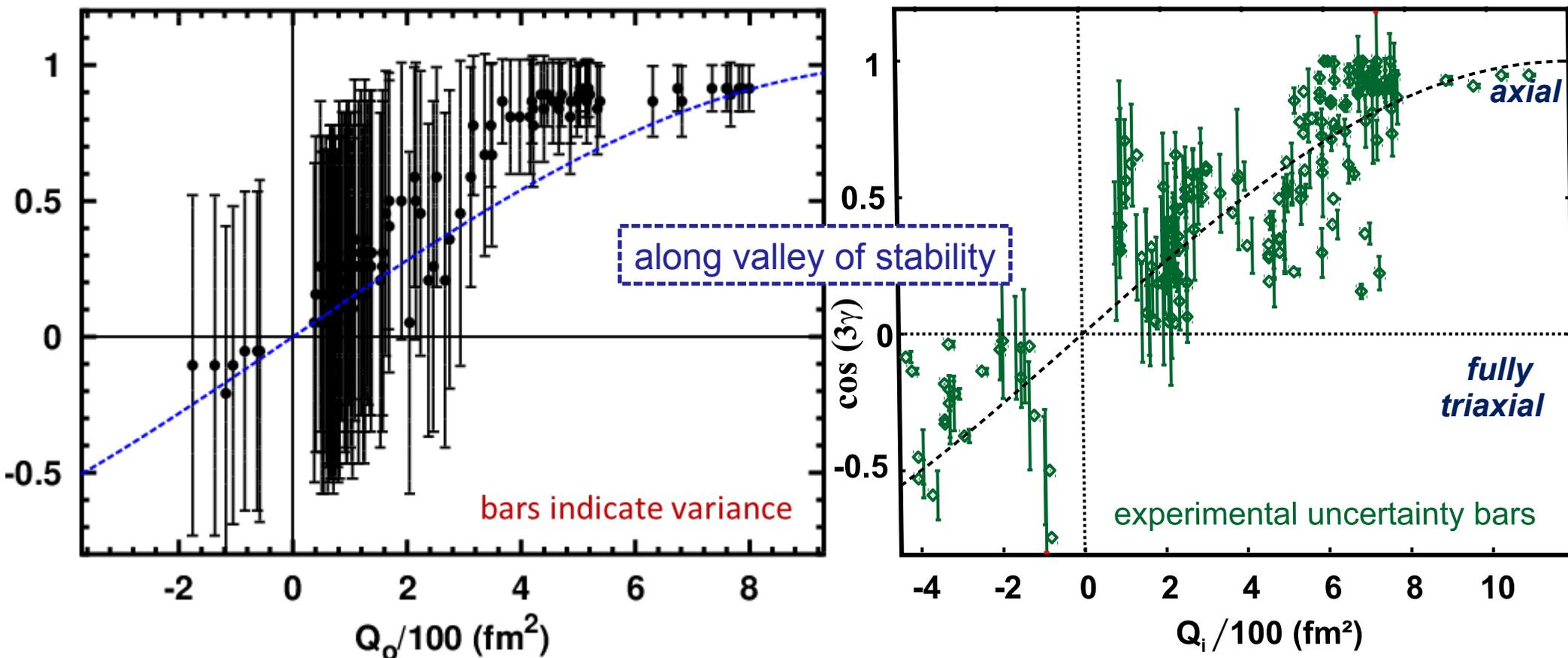
*indicator for **axiality***

*experimental observations:*

*level energies*

*decay properties and  $Q$ -moments*

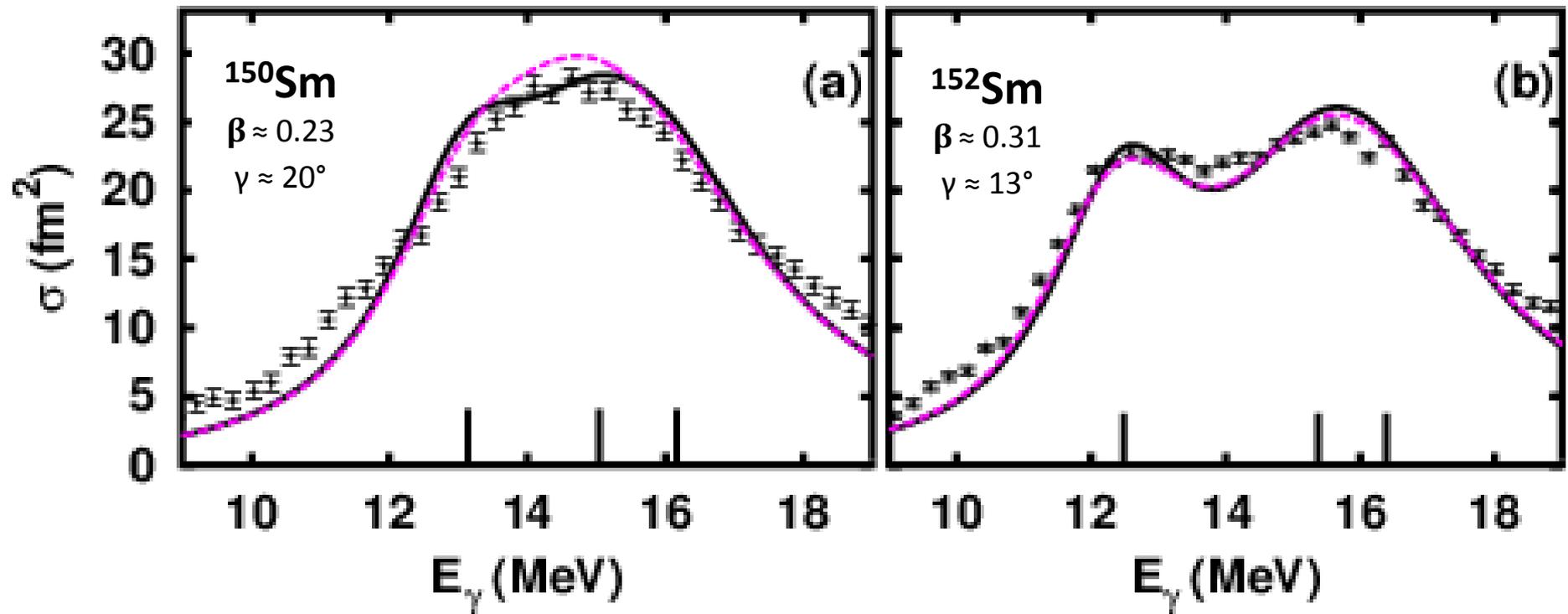
*collectively enhanced level density*



*Giant dipole resonances as sensitive to deformation should also recognize it !*

## *IVGDR's in neighboring nuclei indicate axial symmetry breaking*

*energies from LDM and widths from surface dissipation model, incl. shape sampling*



*with deformation-parameters  $\beta, \gamma$  from QHFB/GCM*

*and global fixing of the width  $\Gamma = c_w \cdot E_r^{1.6}$*

*2-pole fit seems impossible for  $^{150}\text{Sm}$  but may be possible for  $^{152}\text{Sm}$*

$^{142}\text{Nd}$  to  $^{150}\text{Nd}$  in comparison to the sum of three Lorentzians (TLO, dashed blue).

The drawn magenta curves show the effect of shape sampling using variances calculated by HFB.

The parameters for central energy and width are the same as for all other nuclides with  $A > 70$

$$E_0 = \frac{\hbar c}{R_0} \sqrt{\frac{8J}{m_{eff}} \cdot \frac{A^2}{4NZ} \left[ 1 + u - \epsilon \cdot \frac{1 + \epsilon + 3u}{1 + \epsilon + u} \right]^{-1/2}}$$

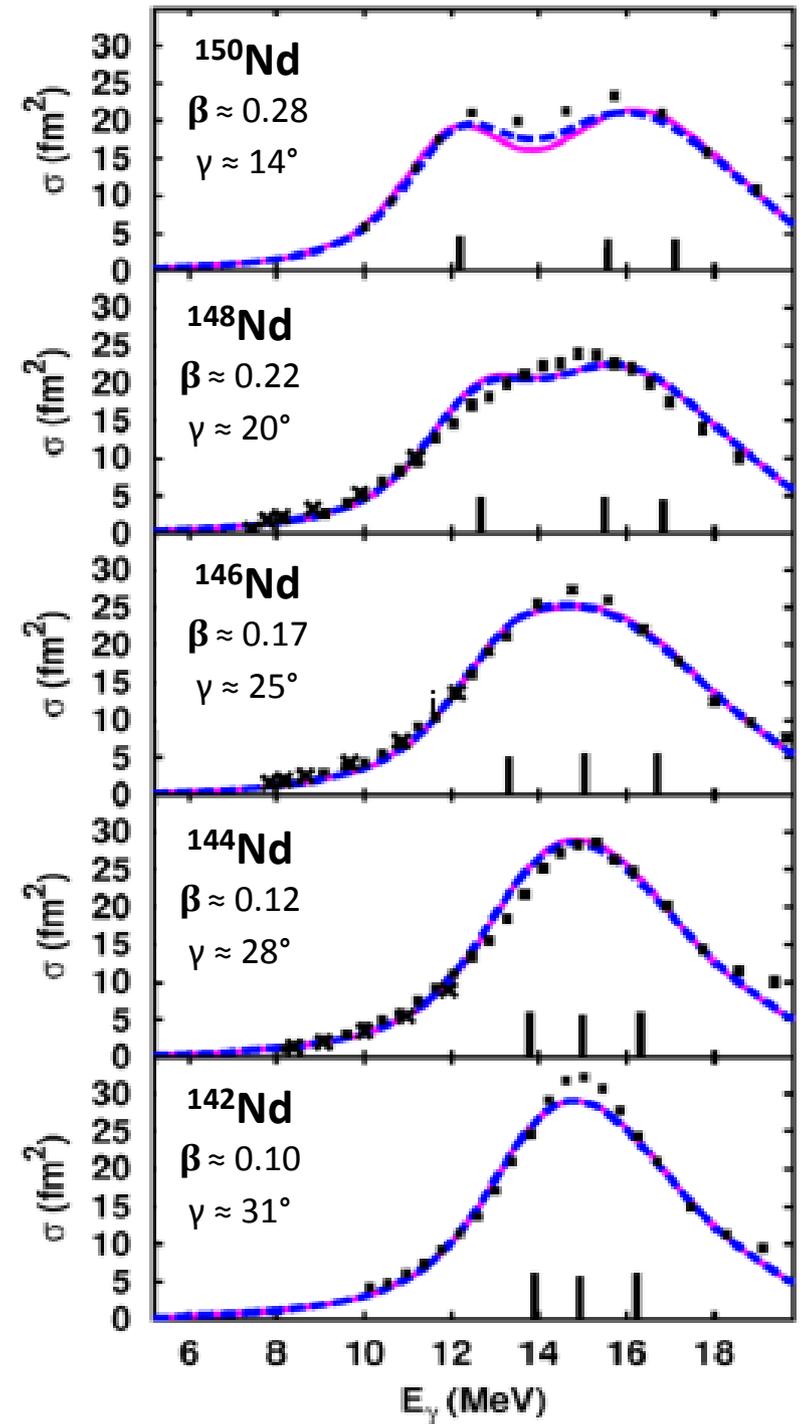
$$\epsilon = 0.0768, u = (1 - \epsilon) \cdot A^{-1/3} \cdot \frac{3J}{Q}$$

$$E_i = \frac{\omega_i}{\omega_0} \cdot E_0 \quad \text{and} \quad \Gamma_i = c_w E_i^{1.6}$$

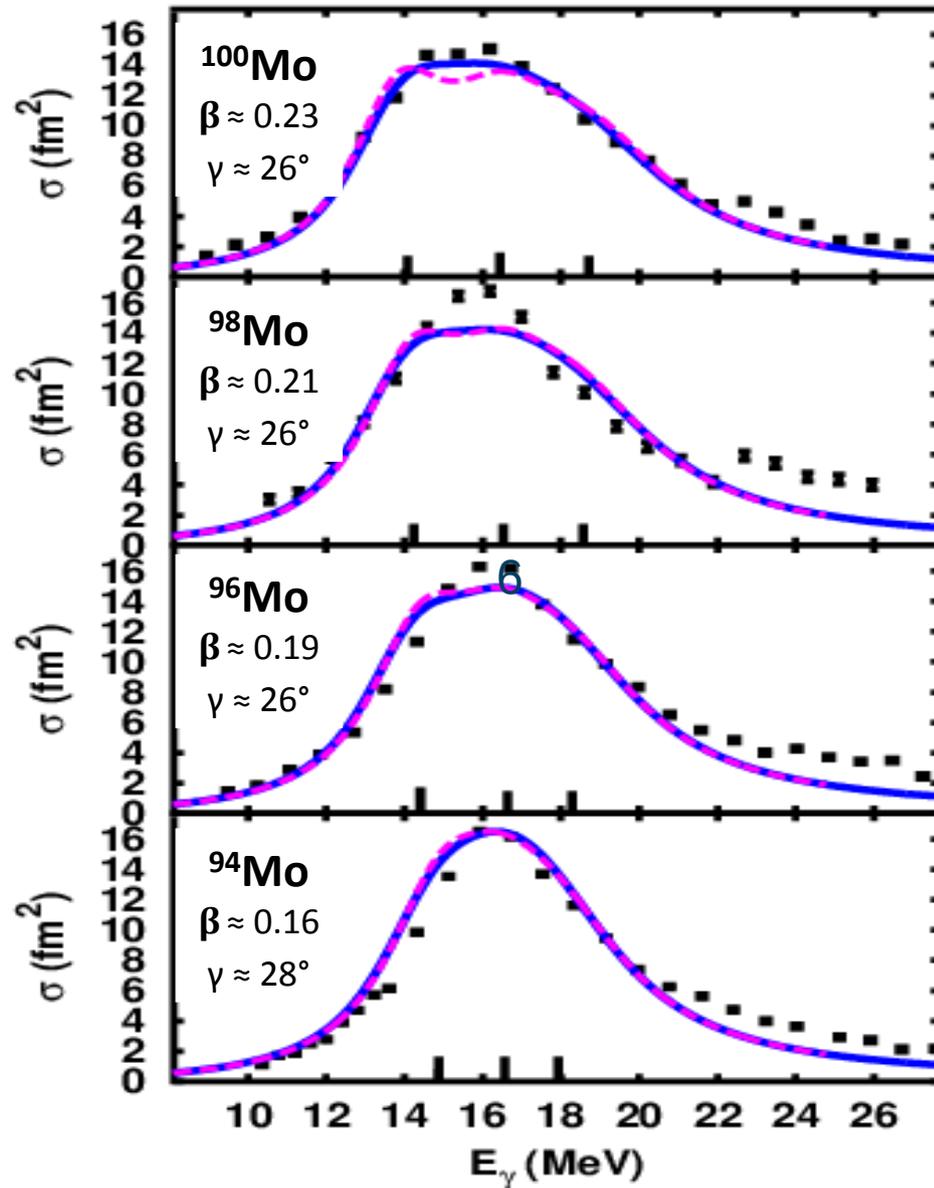
Only the 2 parameters  $m_{eff} = 800 \text{ MeV}$  and  $c_w = 0.045$  (3)

are adjusted – globally valid for all nuclides with  $A > 70$ .

The  $\omega_i$  are taken from the HFB/GCM calculations and symmetry energy  $J = 32.7 \text{ MeV}$  and surface stiffness  $Q = 29.2 \text{ MeV}$  are from droplet model fits to masses.

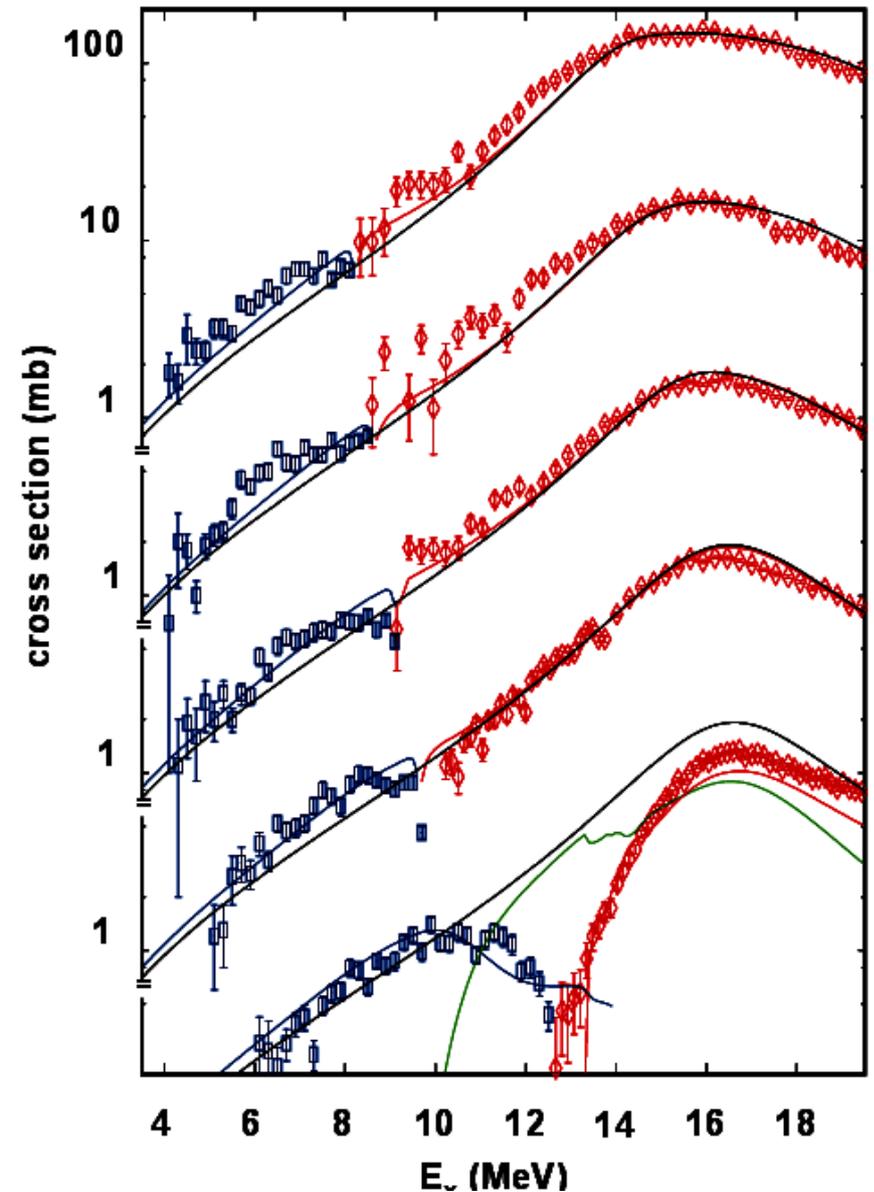


*IVGDR's from  $(\gamma, n)$  and TLO fits with deformation parameters from HFB/GCM incl. shape sampling*



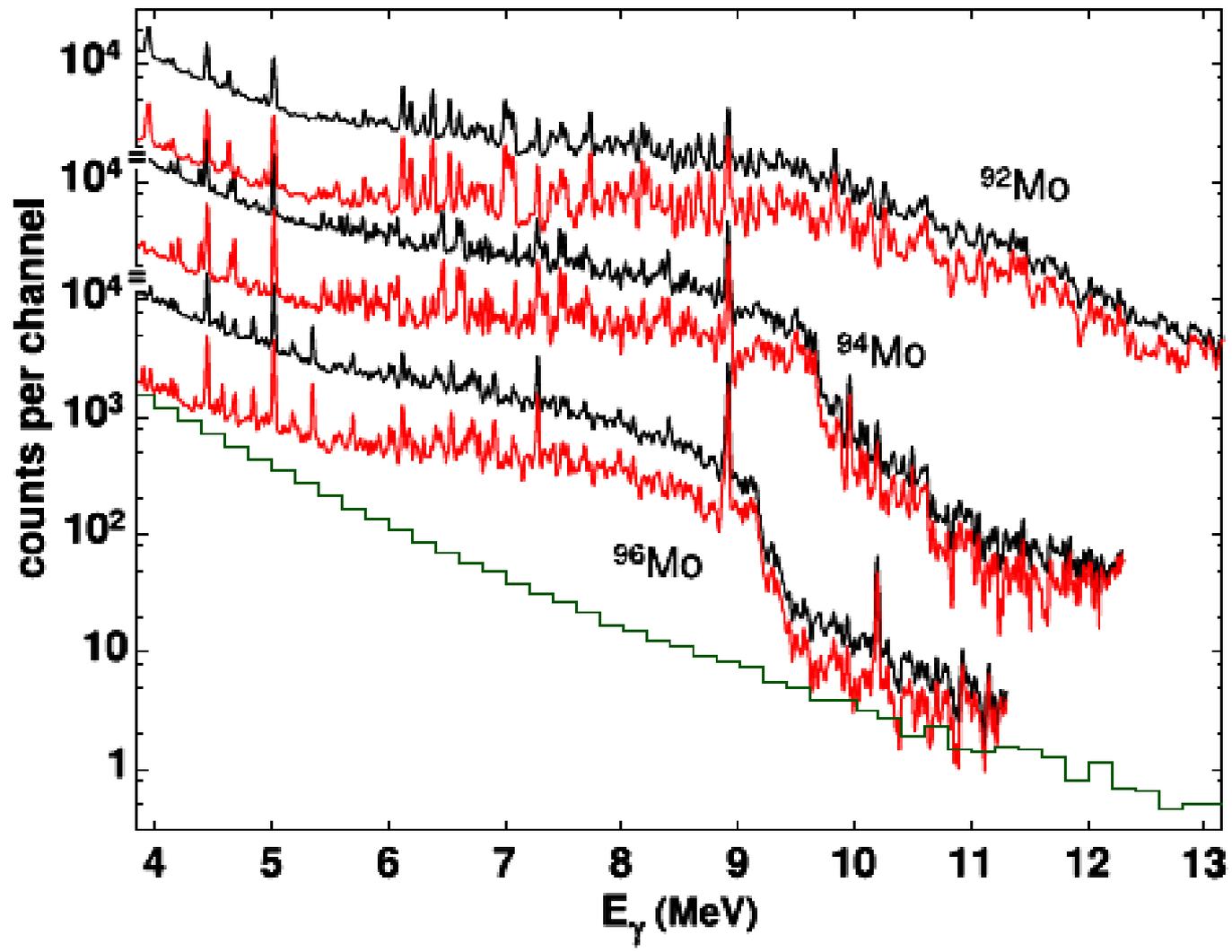
Beil et al., Nucl. Phys. A 227, 427 (1974)

*combined to photon scattering  $(\gamma, \gamma')$*

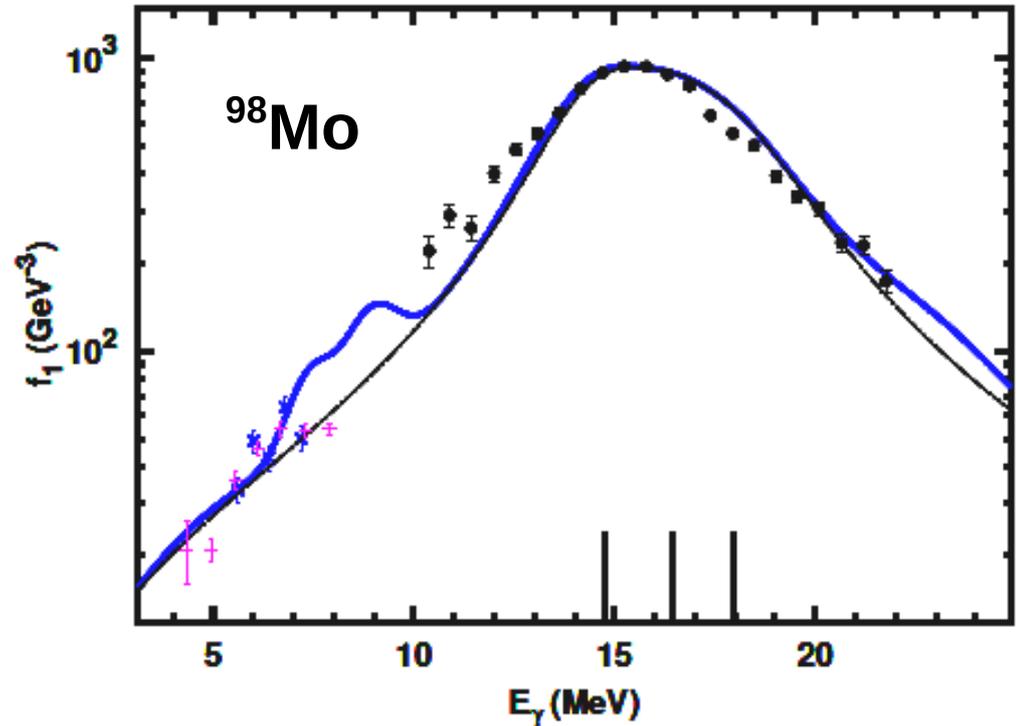
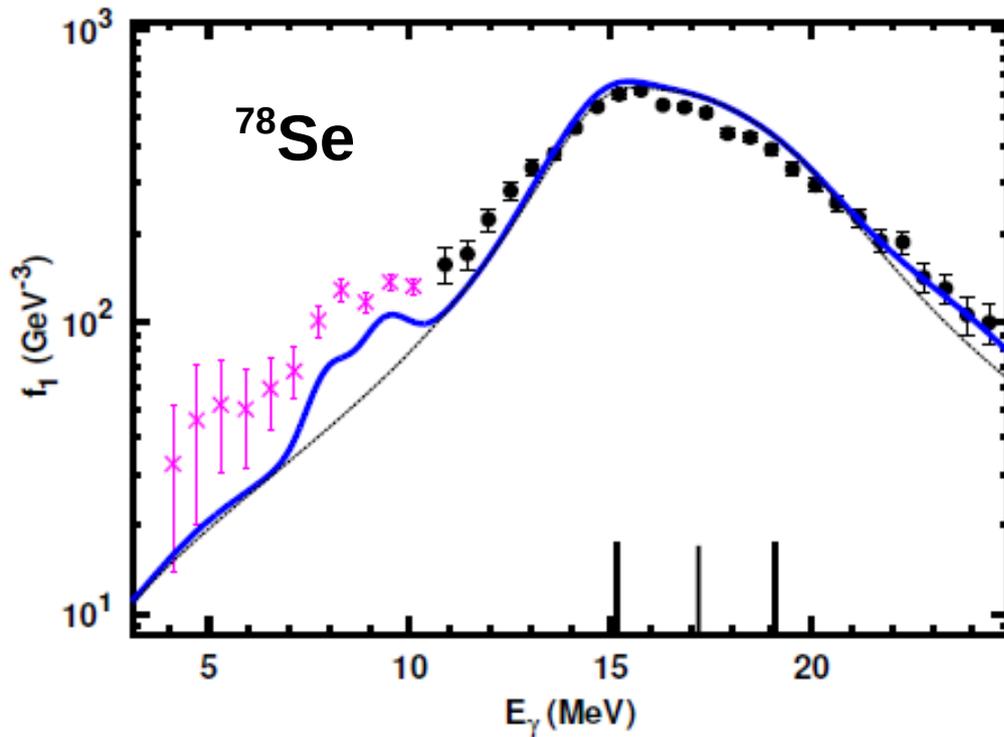


In  $^{92}\text{Mo}$  the  $(\gamma, p)$  channel has to be included from Hauser-Feshbach calc.

Erhard et al, Phys. Rev. C 81, 034319 (2010)

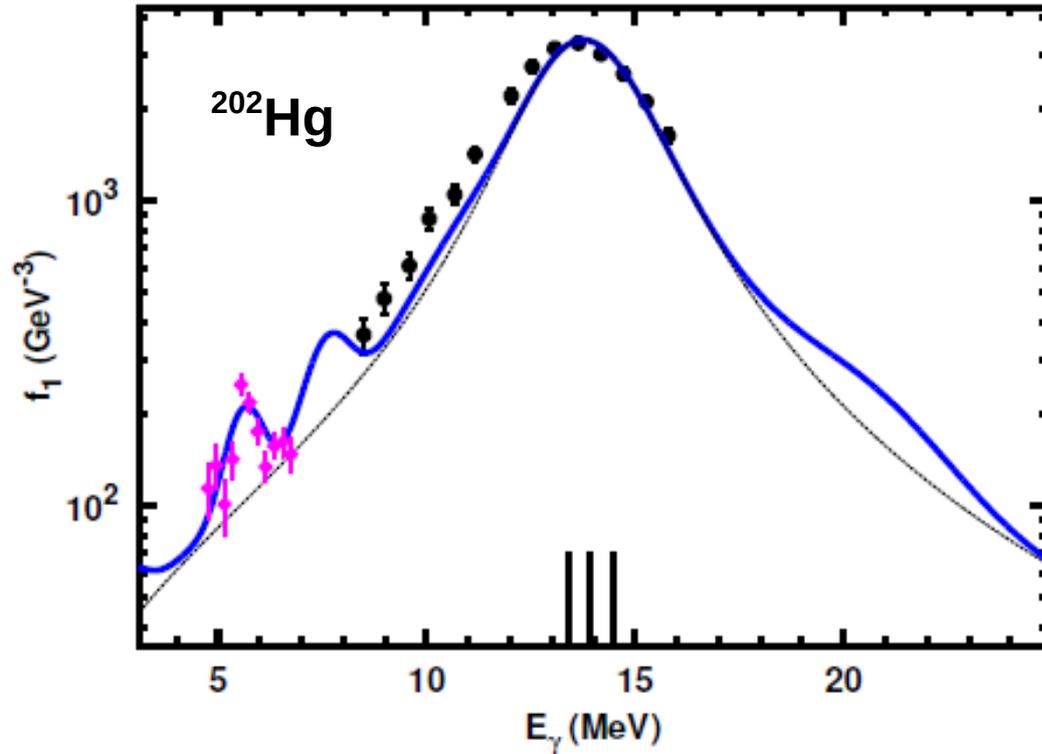


*HFB/GCM for these nuclei indicates broken axially --  
in agreement to photo-neutron data.*

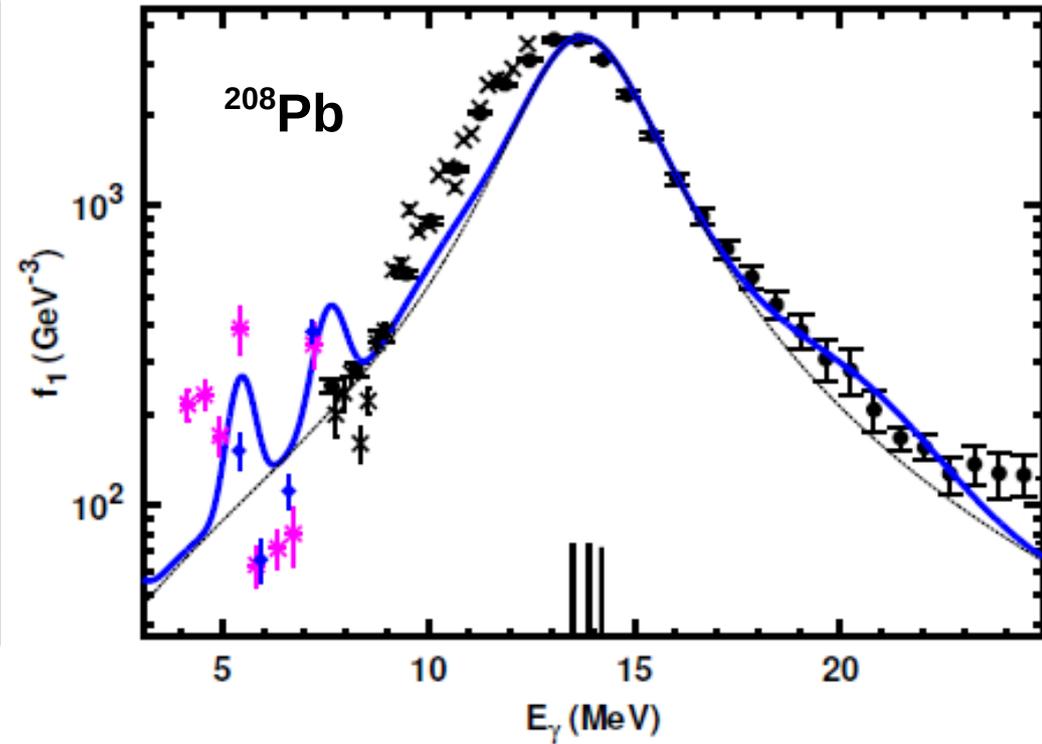


*The low energy strength was obtained from absolute scale photon scattering data,  
partly observed at Duke with a quasi-monochromatic beam from laser back-scattering.*

## *Data near shell closure*

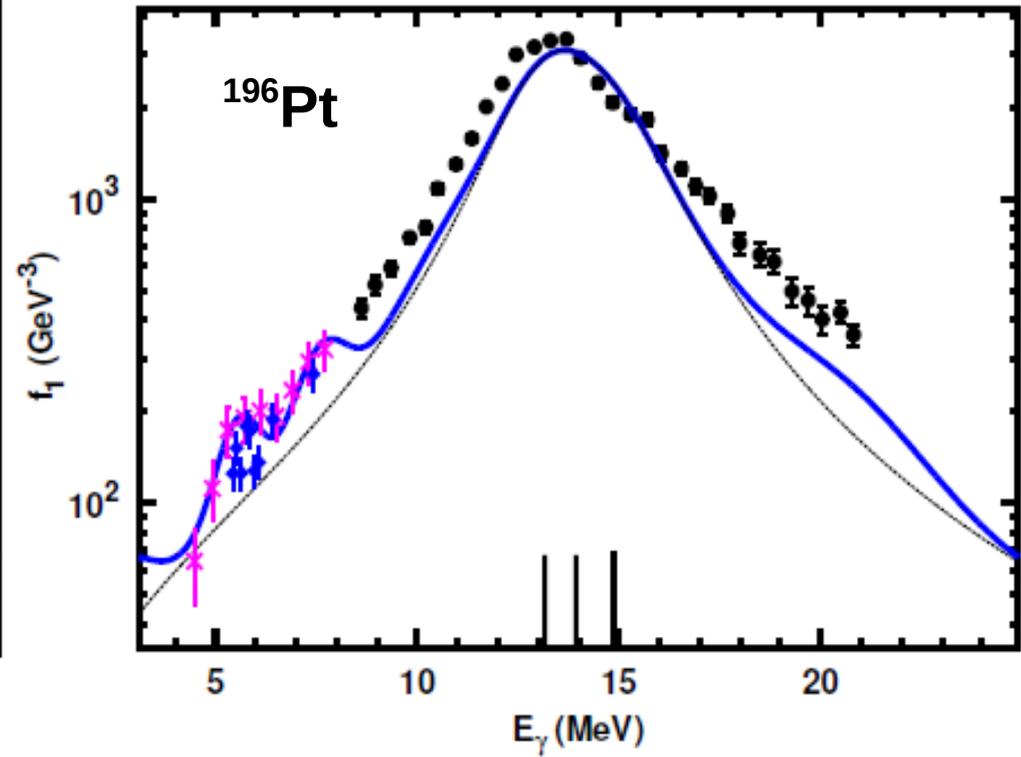
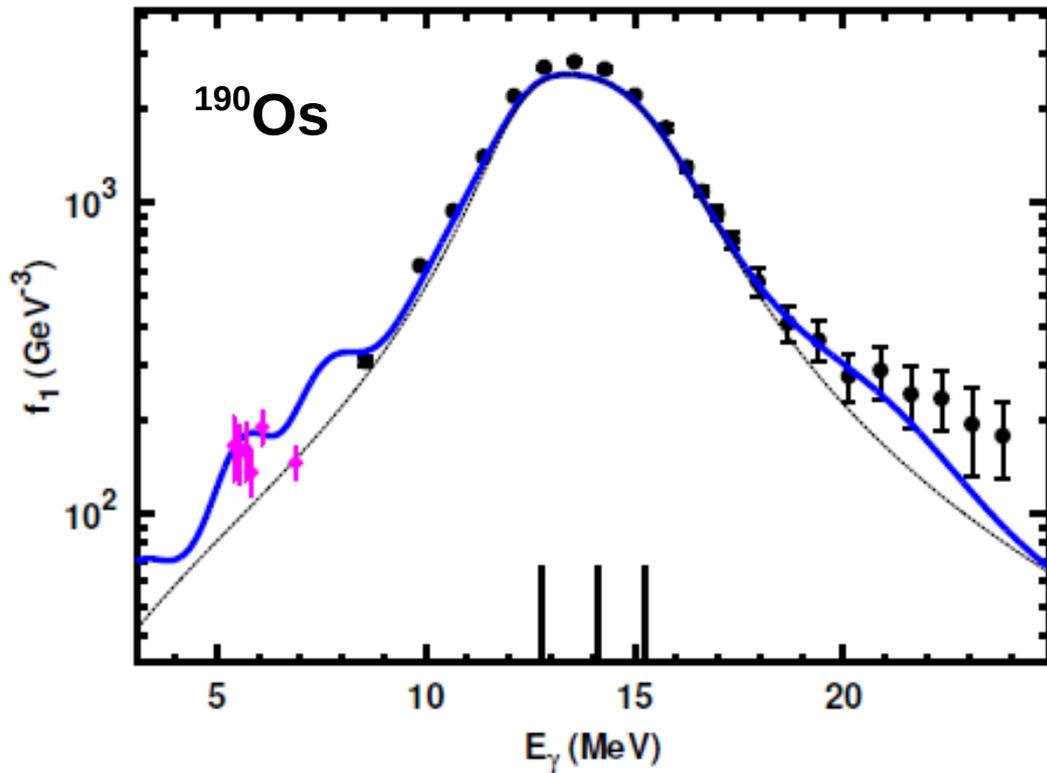


*These old data for  $^{202}\text{Hg}$  obtained at Urbana with low energy resolution demonstrate a well localized enhancement near 5 MeV.*



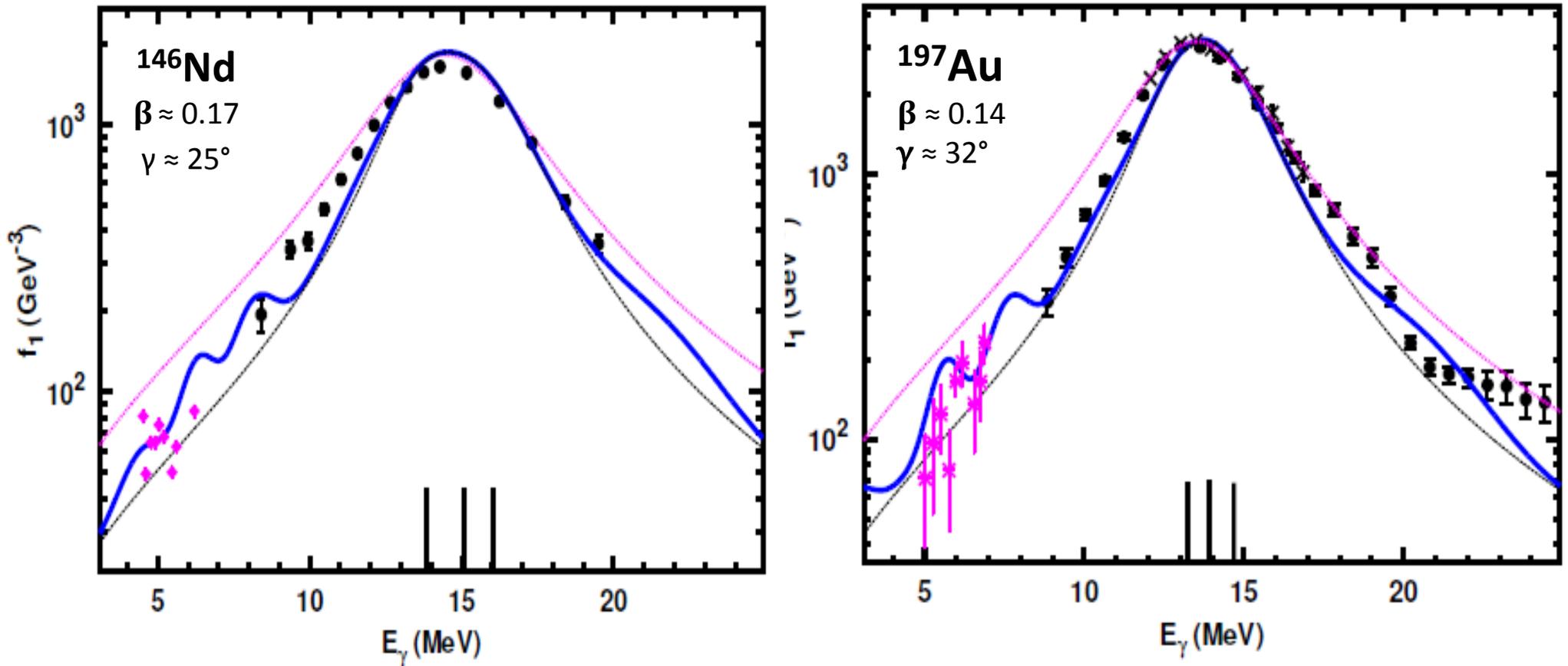
*Data for  $^{208}\text{Pb}$  show single peaks indicating Porter-Thomas fluctuations and at 5.2 MeV a strong one, identified with a neutron p-h-state.*

*Data for nuclei often assumed to be oblate*



*Oblateness is usually seen in near magic nuclei with small  $Q$*

## *TLO should not be replaced by SLO (single Lorentzian)*

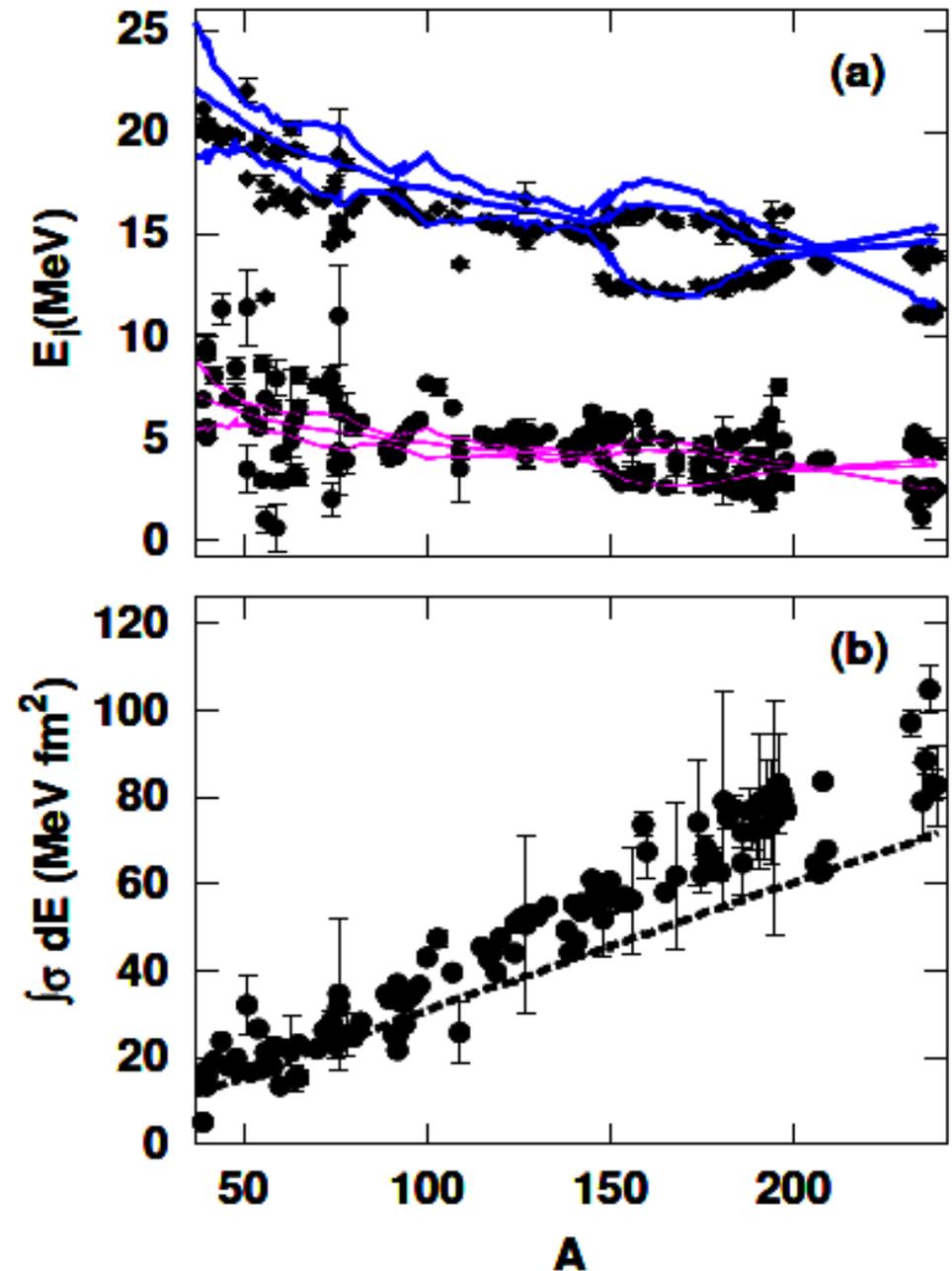


*The dotted red curve shows the fit made by **Plujko et al.**  
It overpredicts the width and the integral considerably  
and thus the strength at low energy by a factor of  $\approx 3$ .*

*Strength functions shown in RIPL-3, [Pluiko et al.] were obtained from individual fits assuming axially or spherically.*

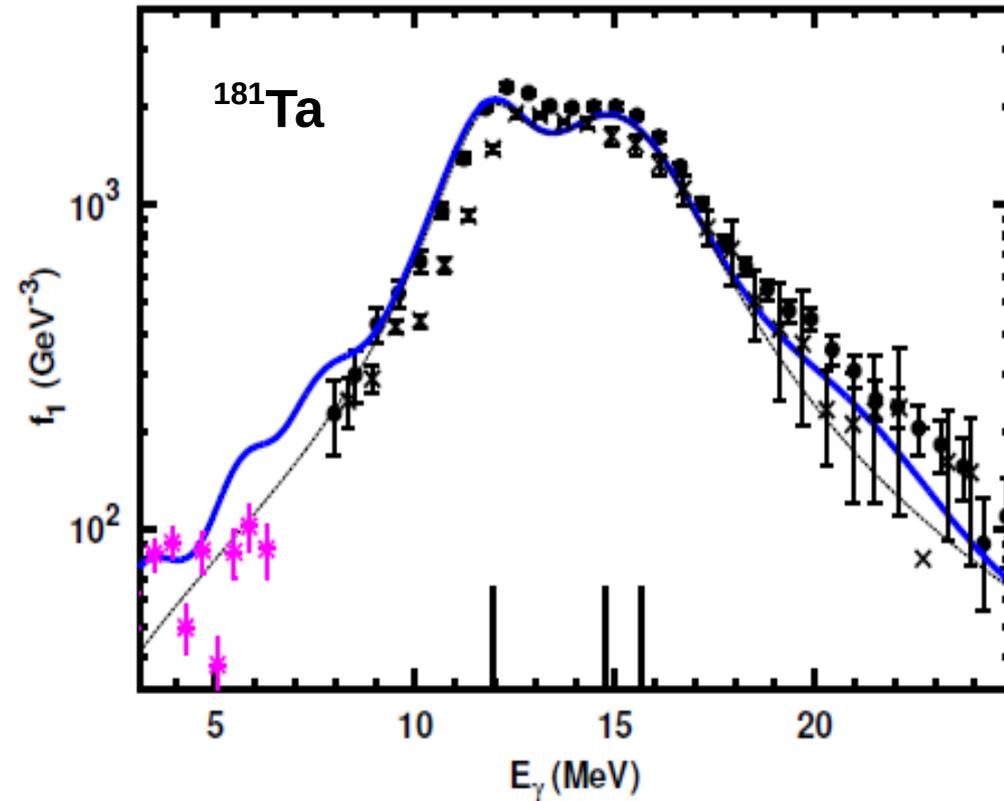
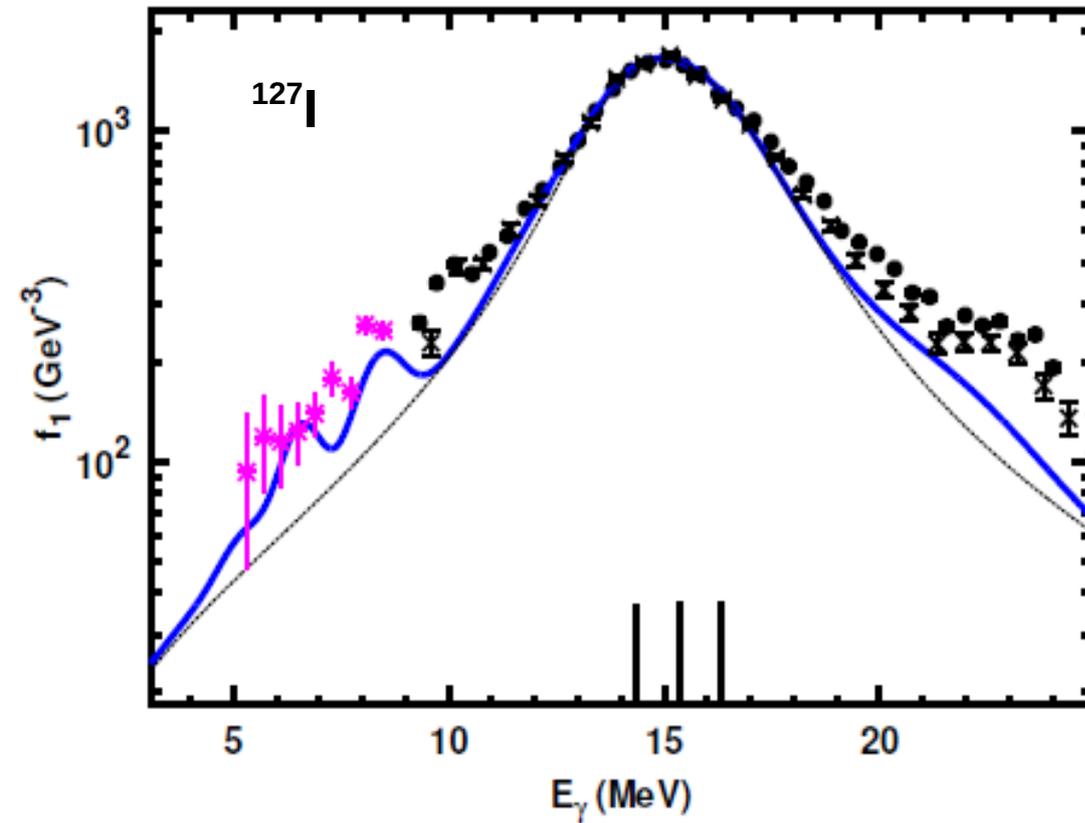
*The top figure shows pole energies, which are similar to TLO-predictions but a clear difference in widths is seen.*

*The bottom figure indicates the difference to TRK-sum-rule, which is integrated in TLO.*



From RIPL-3 [Pluiko et al.].

*Data for odd nuclei indicate: TLO can be applied as well.*

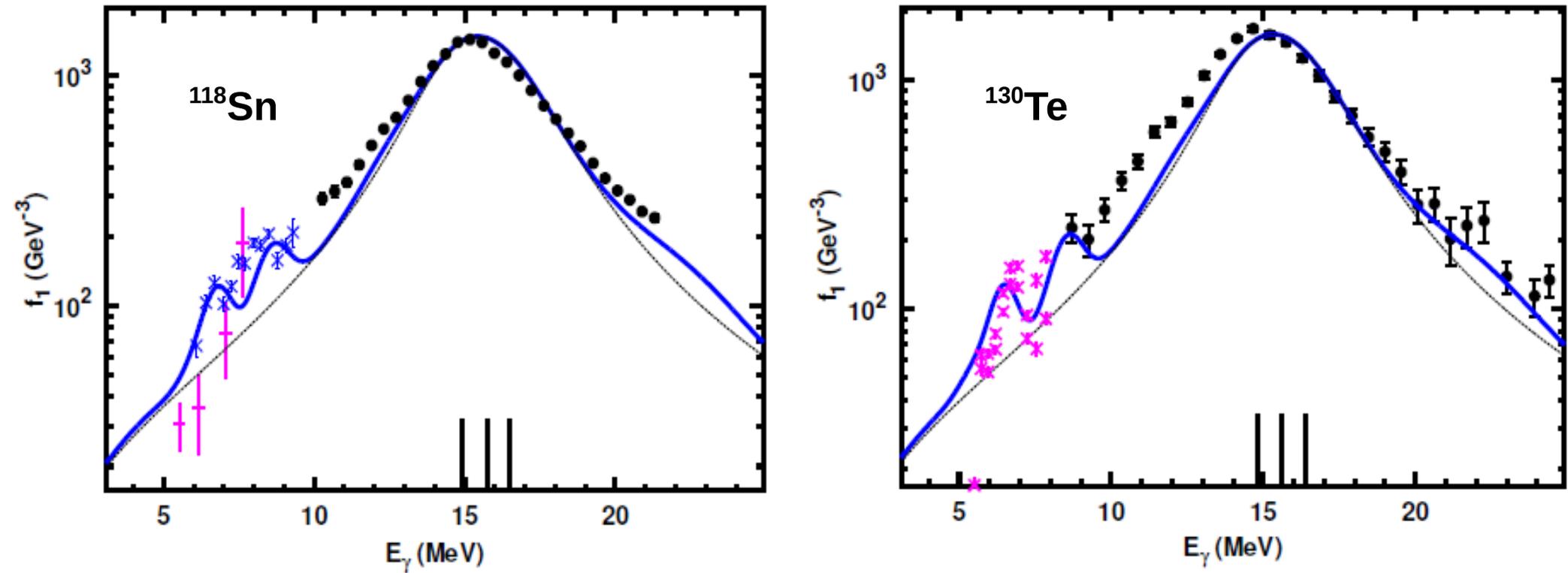


*The strength observed corresponds to the cross section summed over a spin multiplet*

with  $m = \min(2\lambda + 1, 2J_0 + 1)$ :

$$g_{eff} = \sum_{r=1, m} \frac{2J_r + 1}{2J_0 + 1} = 2\lambda + 1$$

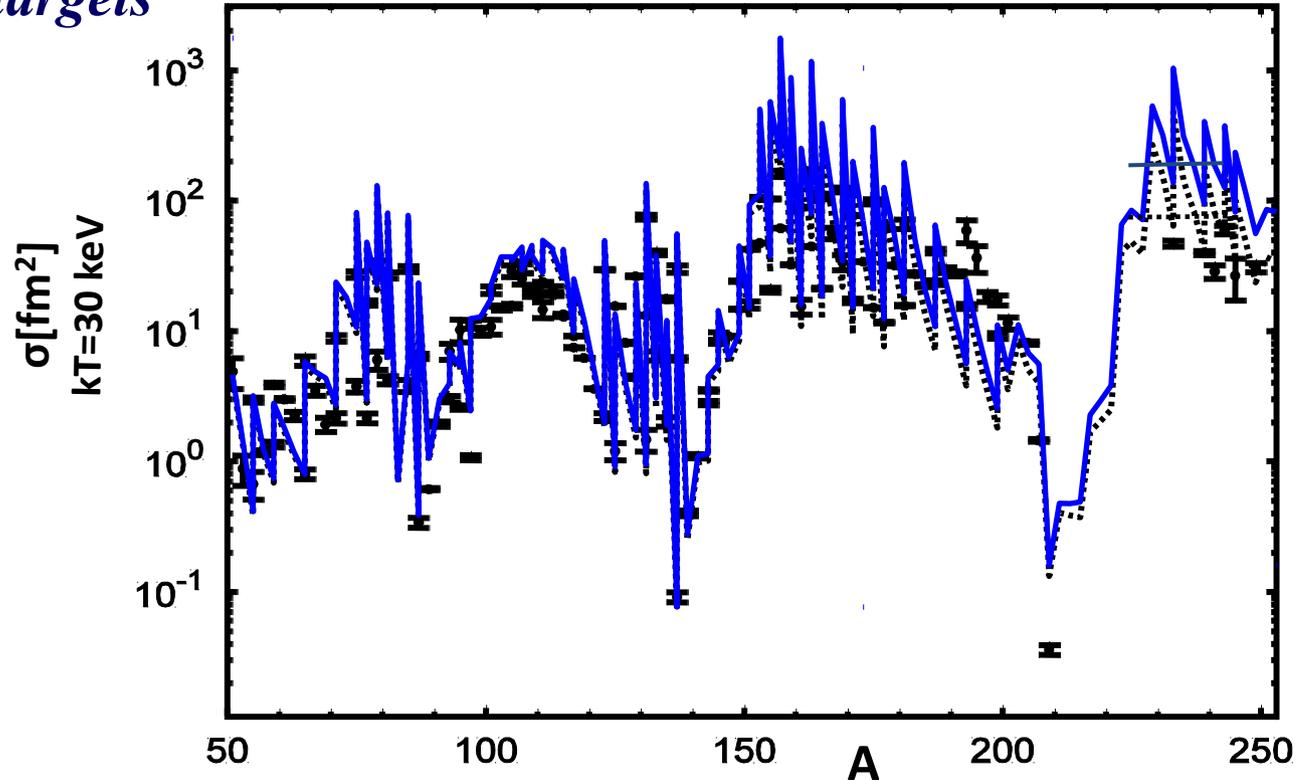
*These ‘vibrators’ may well be triaxial nuclei and are treated as such in TLO.*



*The data for low energy in these nuclei indicate a separation between 2 pygmy modes and TLO is complemented by adding this to the IVGDR Lorentzians as indicated in blue.*

*Maxwellian average capture cross-sections, at stellar temperatures of  $3 \cdot 10^8$  K,*

*for  $J=0$  targets*



*TLO + minor strength  
only E1 from TLO*

*Good agreement for  $>130$  nuclei on absolute scale calculated from global predictions for average level densities  $\rho(E_r)$ , obtained by admitting **broken axially** and **photon widths for radiative neutron capture from an extrapolation of TLO-fits to IVGDR's***  
*=> simultaneous test of broken axially for photon strength and the level density prediction*

***Broken axial symmetry indicated by experimental data on:***

***(a) level densities, esp. for low spins near  $S_n$***

***(b) level energies and transitions,***

***(c) splitting of giant dipole resonances, resulting in:***

*if global width is scaled triaxially only one parameter  $c_w$  is needed for all heavy nuclei*

*TRK sum rule agrees to CENS-data, when neutron efficiency is reduced by 10%*

*GDR pole energies  $E_0$  agree well to LDM prediction, when using theoretical def' values*

***(d) n-capture cross sections, if TLO is extrapolated to low energies***

***Theoretical models assume axially very often, but:***

***(a) rigid 3-ax rotor does not***

***(b) cranking of 3-ax body is possible***

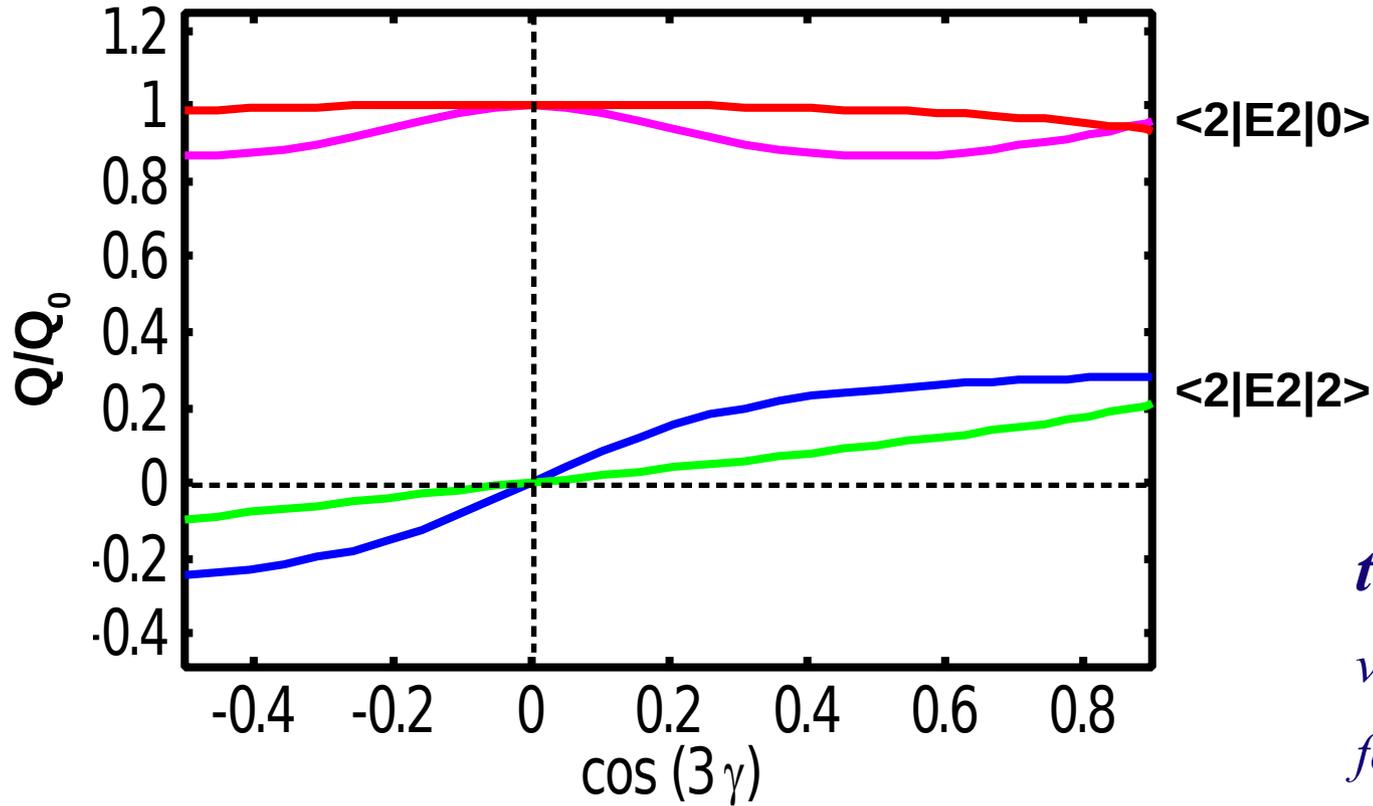
***(c) HF-variation after projection enhances broken axially***

***(d) QHFB+GCM (GognyDIS) creates triaxiality; combined to LDM for TLO***

***(e) RPA+OM predicts GDR in  $^{208}\text{Pb}$  with 3 MeV width; scaled for TLO***

***(f) RPA+QHFB produce GDR with 1 or 2 poles plus fragments***

### *3-axial rotor, rigid & with cranking*



*$\cos(3\gamma) \Rightarrow$  indicator for axiality*

*$Q$  is especially sensitive to it*

*the two models make very similar predictions for the two observables  $Q(2^+)$  and  $B(E2, 0^+ \rightarrow 2^+)$ ; this does not help to find best approach to treat axial symmetry breaking*

## *Shell model + RPA*

*schematic calculation,*

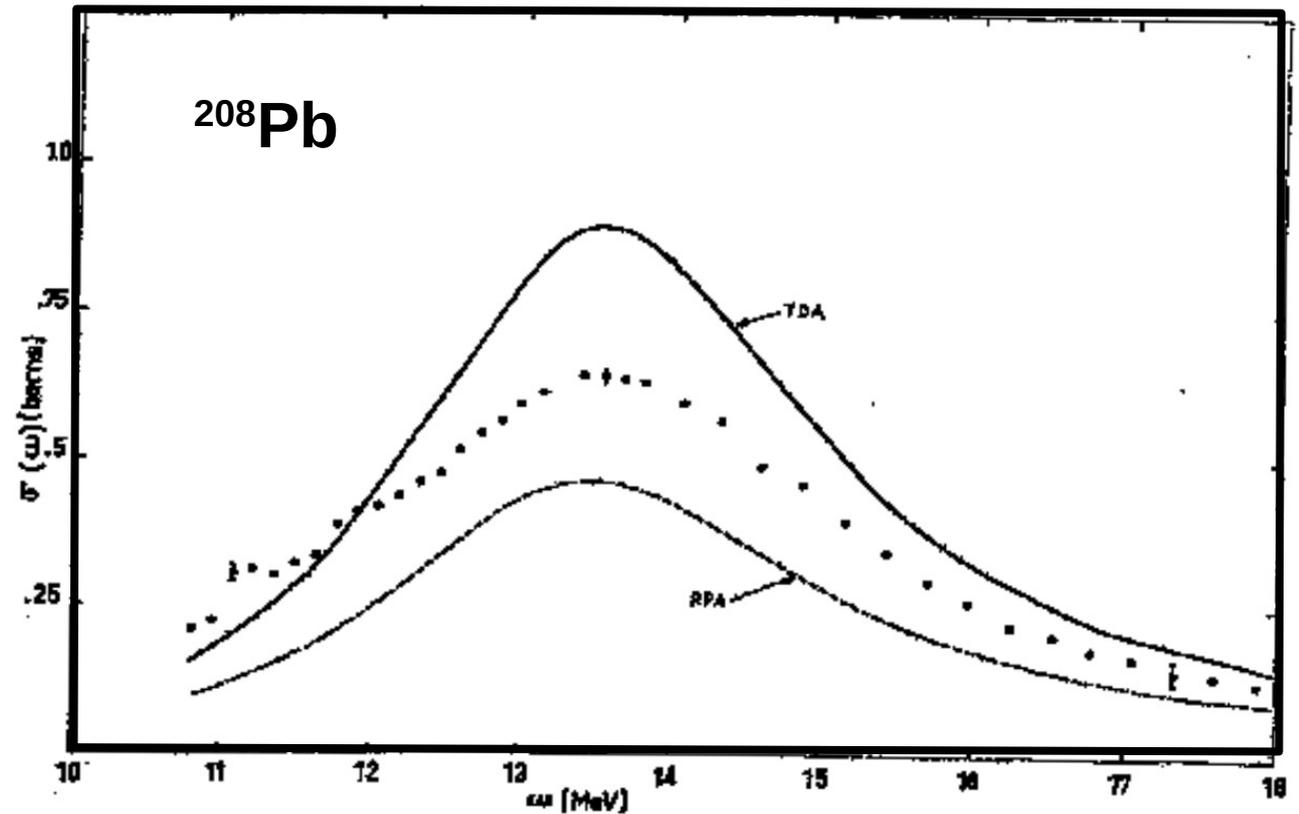
*for  $^{208}\text{Pb}$ ,  $E_r$  adjusted,*

*strength integral depends*

*on gs-corr. (RPA),*

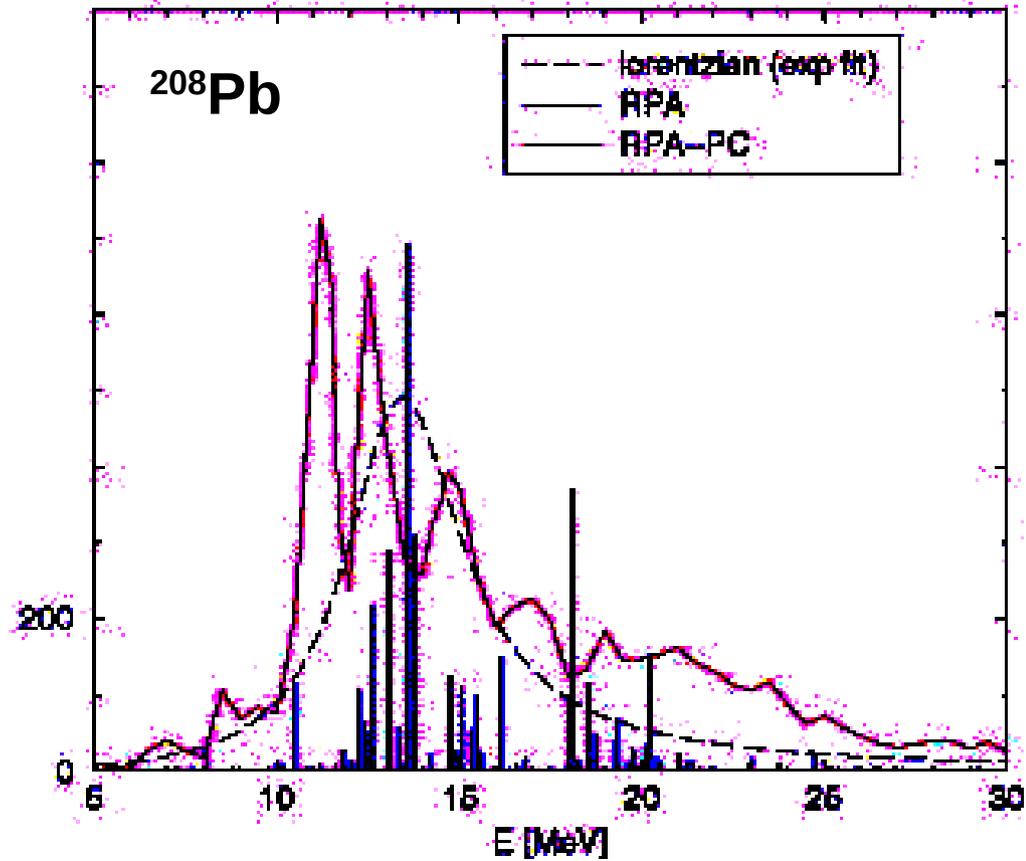
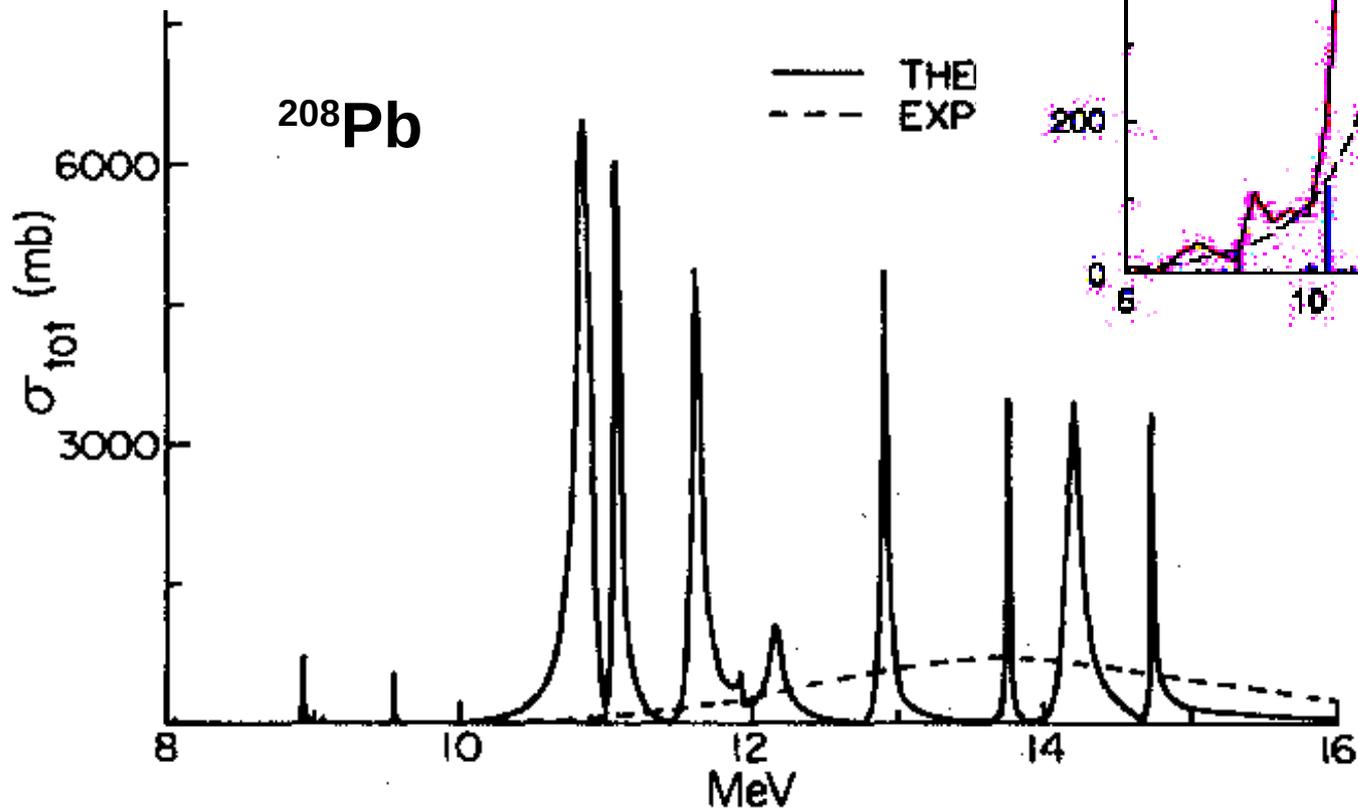
*width is used by TLO*

*after scaling by  $(E/E_{208})^{1.6}$*



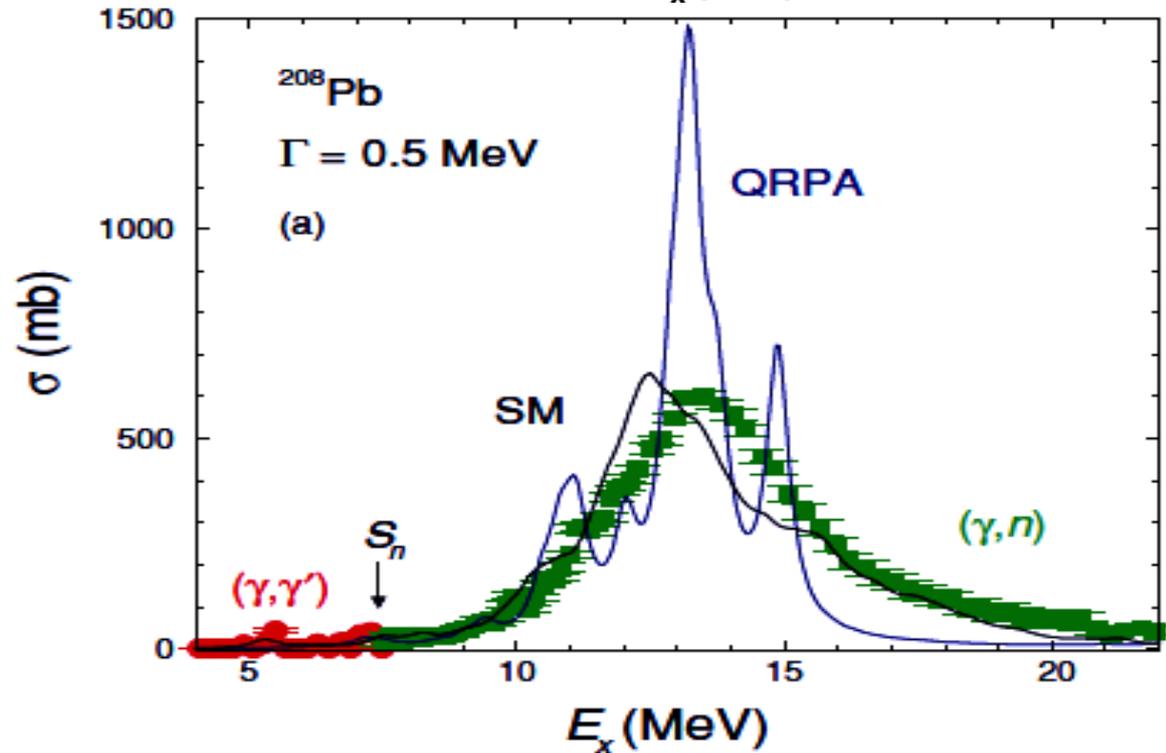
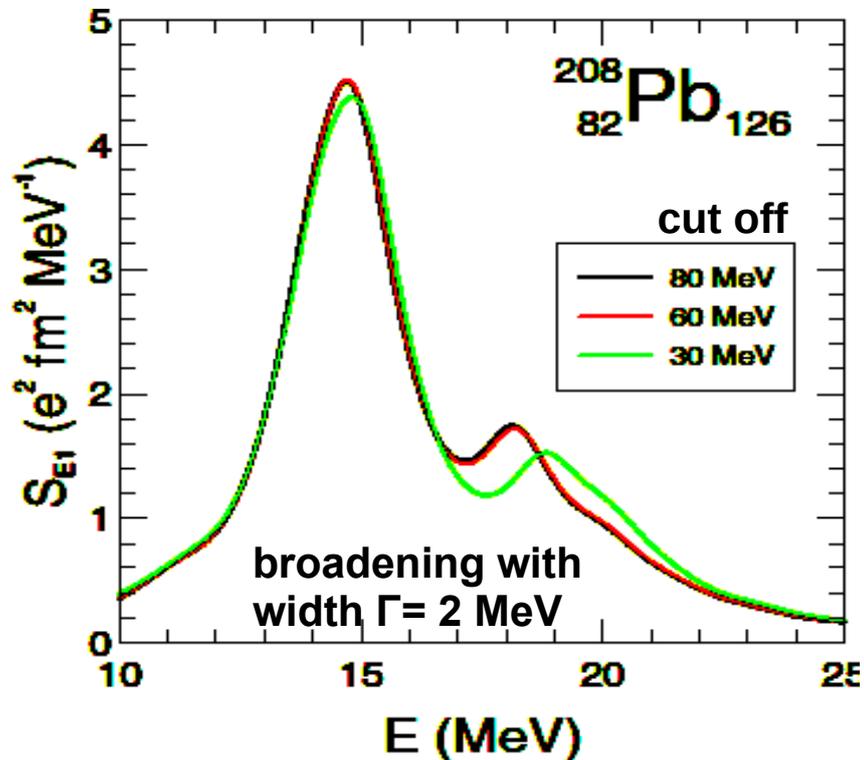
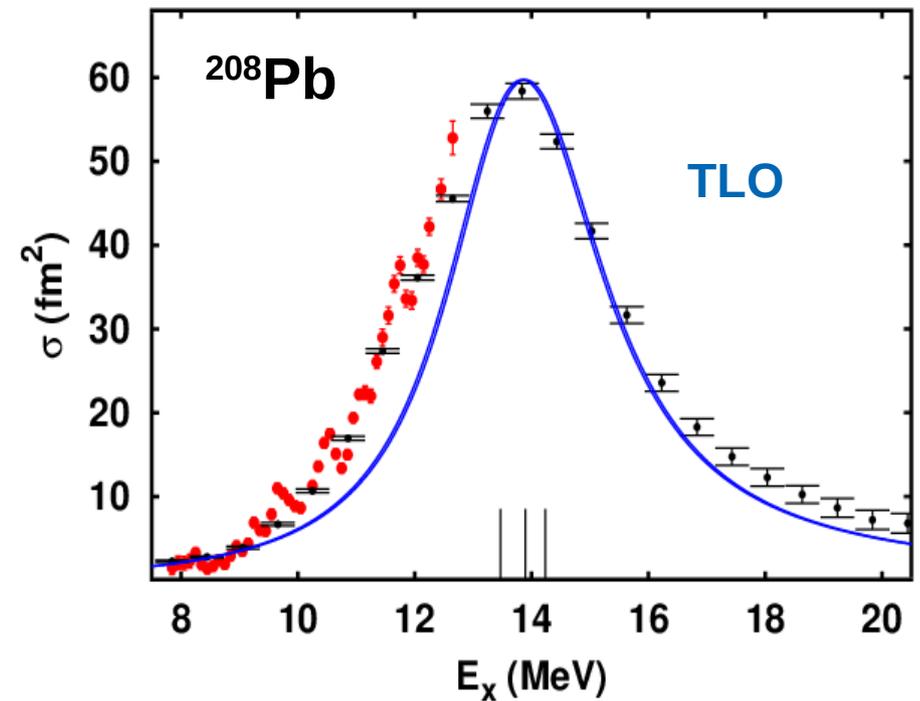
**QRPA-HFB** (*Hartree-Fock-Bogolyubov*)

*calcul's show distinct fragmentation with spreading clearly exceeding escape widths; often reduced by phonon coupling or smeared by additional broadening*



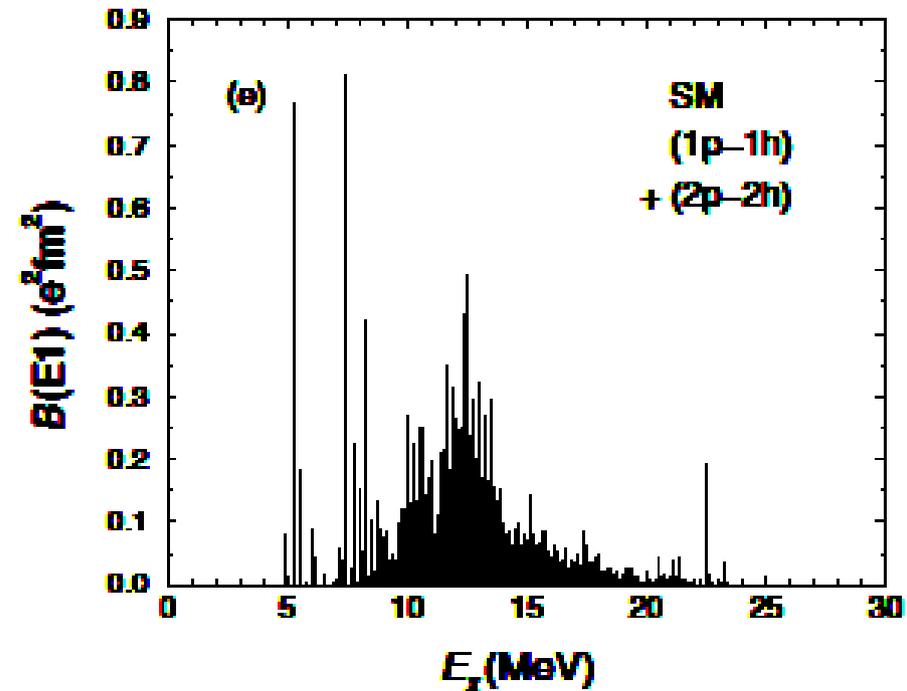
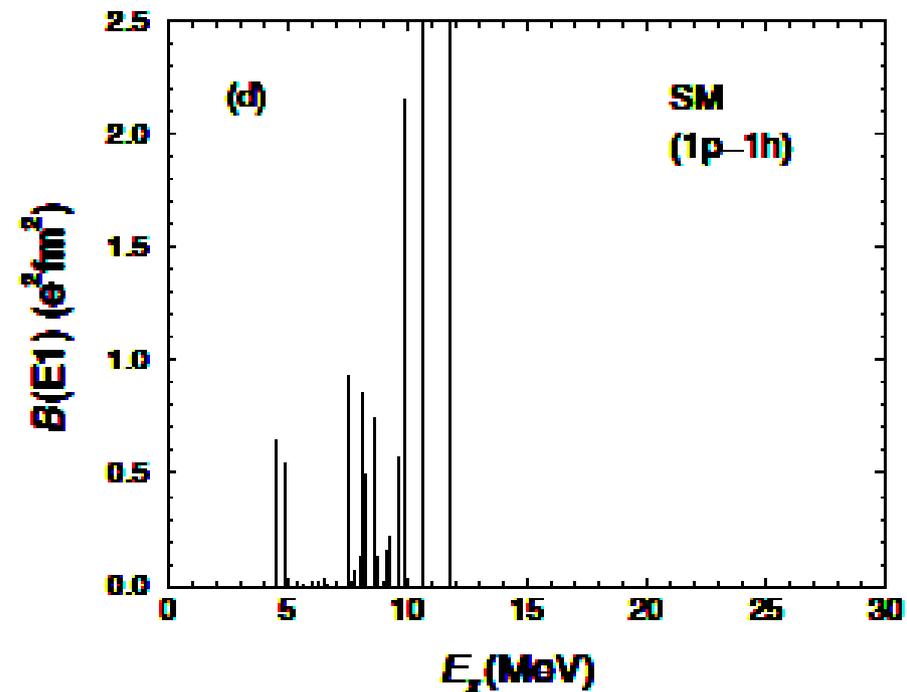
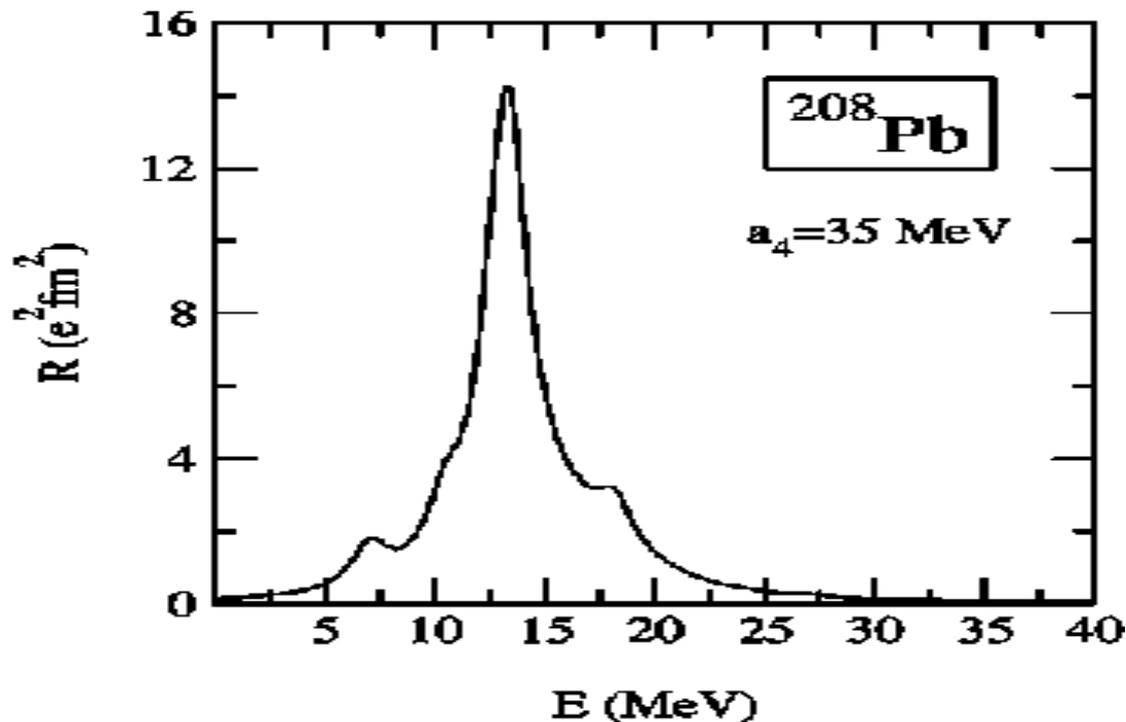
*HFB-QRPA-calc's show distinct fragmentation (p-h, Landau damping), many apply additional broadening (2p-2h); experimental data show much less:*

(Van de Vyver et al., Z.Phys. A 284, 91 (1978))



*HFB-QRPA-calcul's show distinct fragmentation, indicating strong spreading covariant (relativistic with meson coupling) or shell model calculations*

*show less of it*

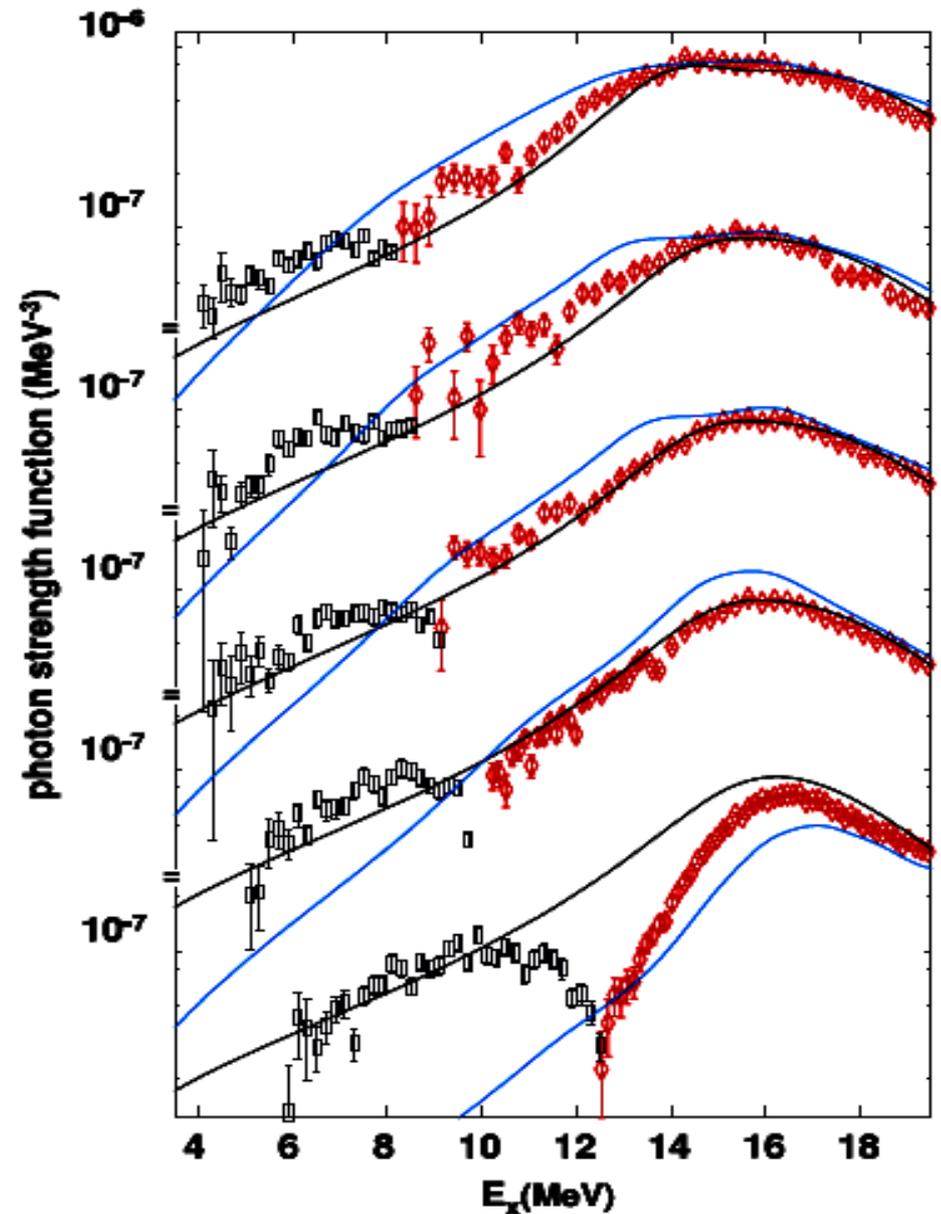
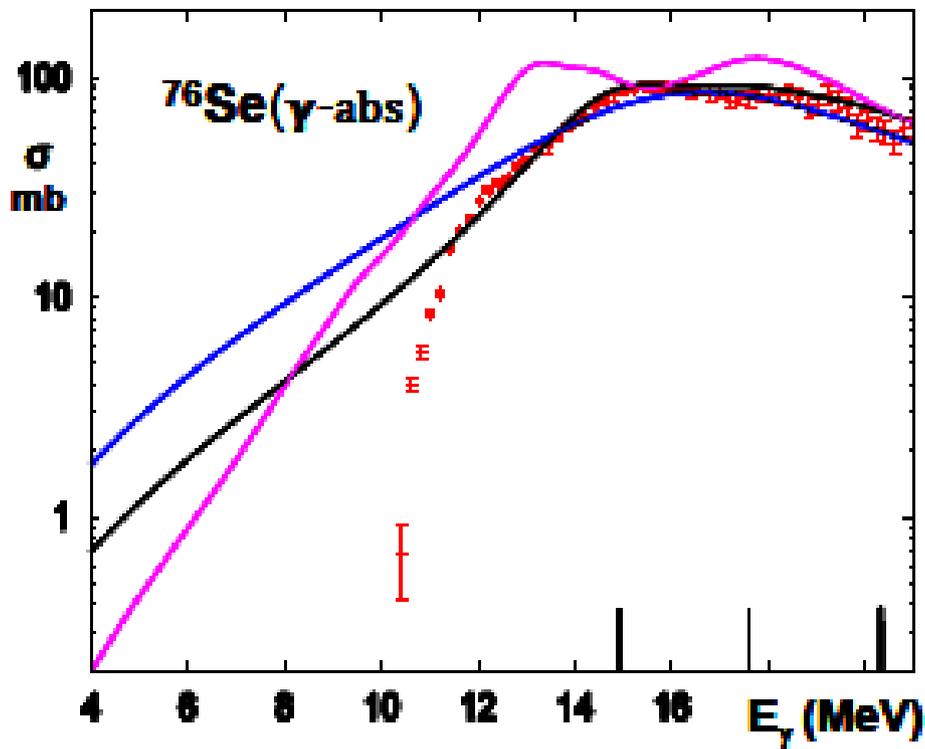


Brown, Phys. Rev. Lett. **85**, 5300 (2000).

Schwengner et al., PRC **81**, 054315 (2010)

Niksic, Vretenar, and Ring, Phys. Rev. C **66**, 064302 (2002).

*HF-RPA-calcul's often show less strength in the tail region, which is of importance for radiative neutron capture*



Probably, the rareness of predicted *triaxiality*

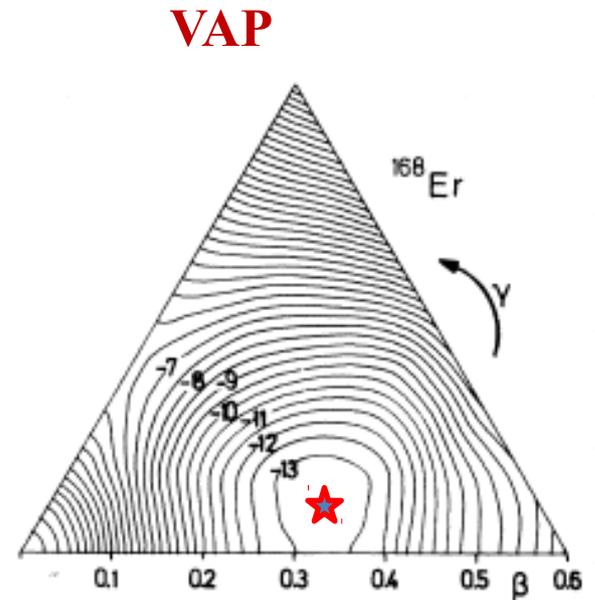
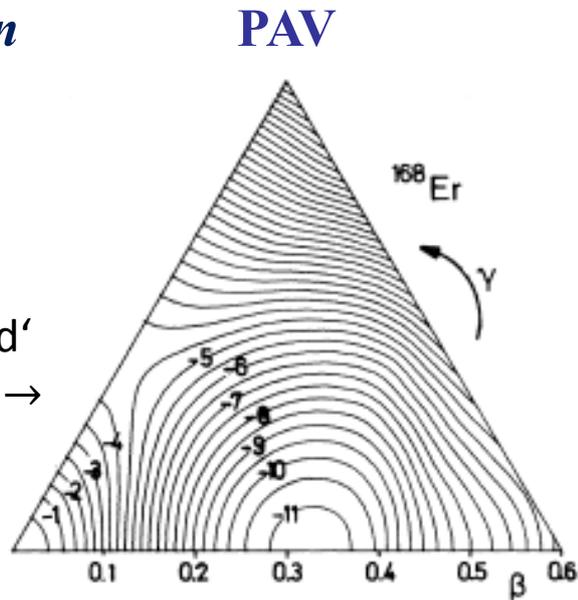
(e.g. in Hartree-Fock-Bogolyubov calculations)

results from **not** performing the

*angular momentum projection*

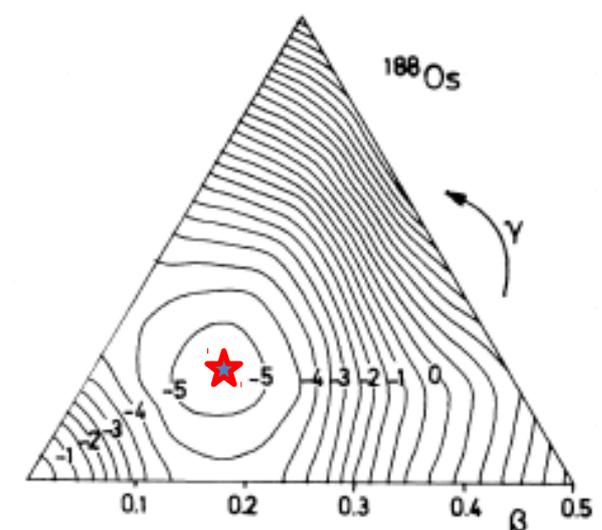
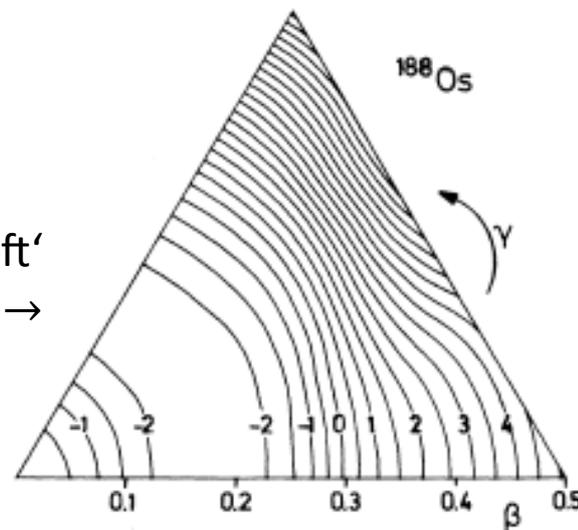
*after* the variation.

'axially deformed'  
nucleus →



Often the VAP, required by  
quantum mechanics,  
is not regarded.

'-soft'  
nucleus →



## ***Conclusions:***

***Many experimental facts indicate broken axial symmetry for heavy nuclei :***

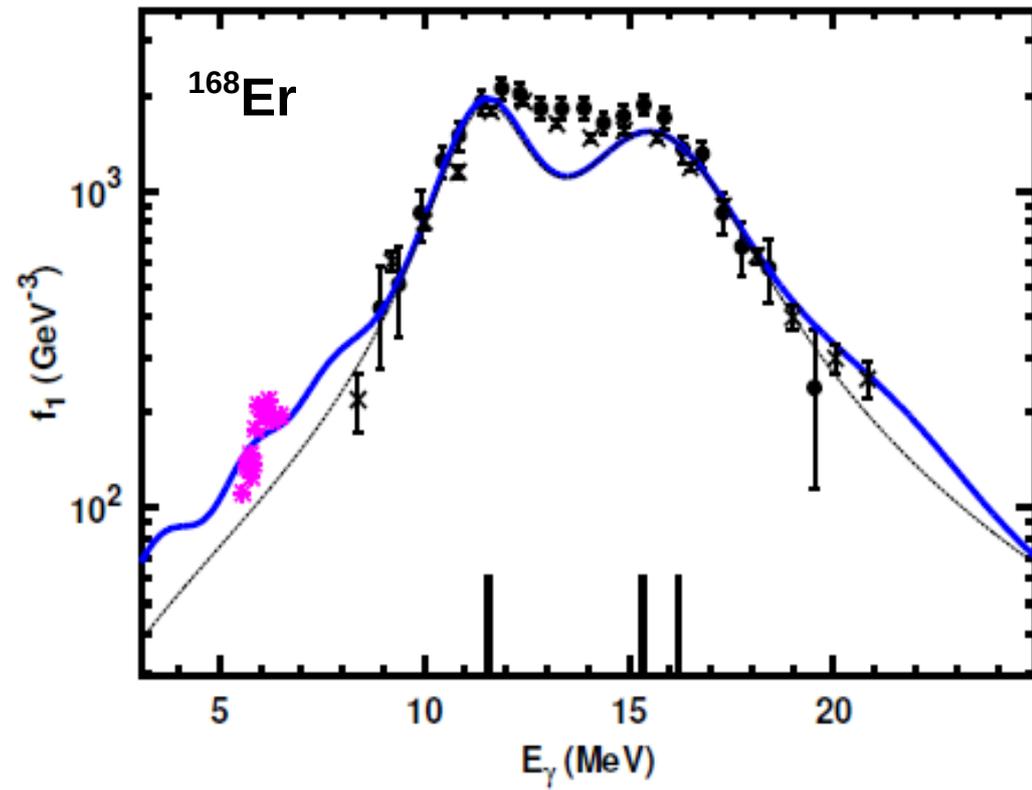
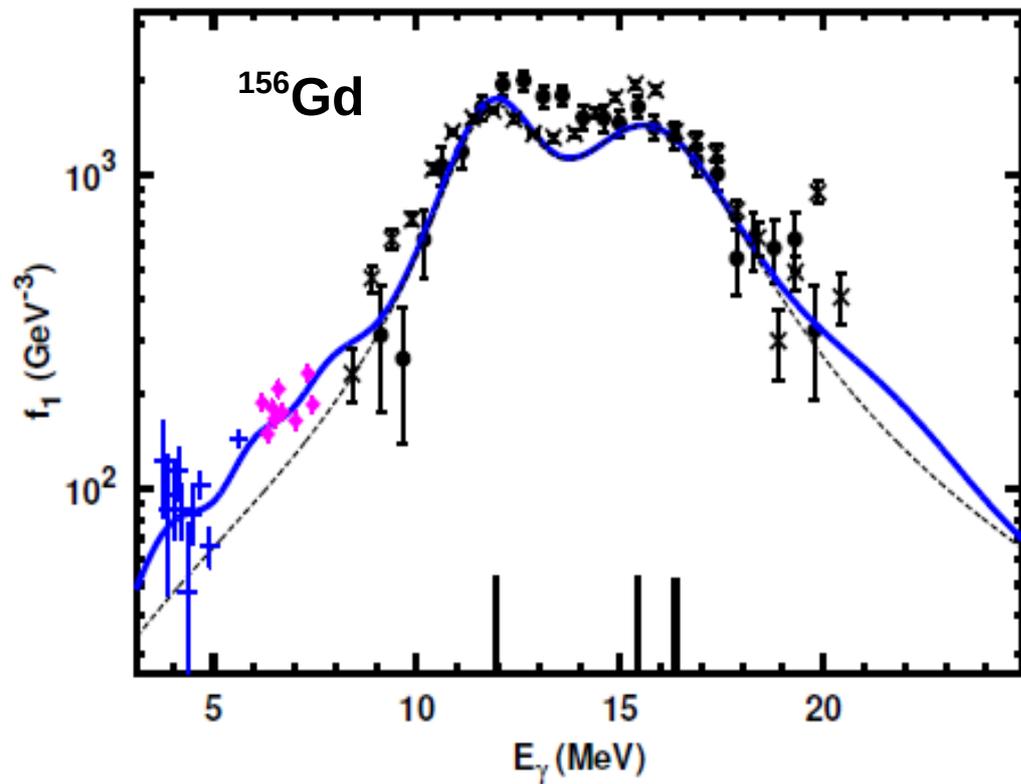
- 1. Level densities predicted on absolute scale***
- 2. Level sequences and transition rates***
- 3. Coulomb reorientation and multiple excitation***
- 4. Triple split of the giant dipole resonances has interesting consequences for their interpretation***
- 5. Neutron capture cross sections (via 1 & 4)***

***Theoretical calculations may impose triaxiality as property of a rotor, but many assume axially and predict level densities, photon strength or GDR shapes.***

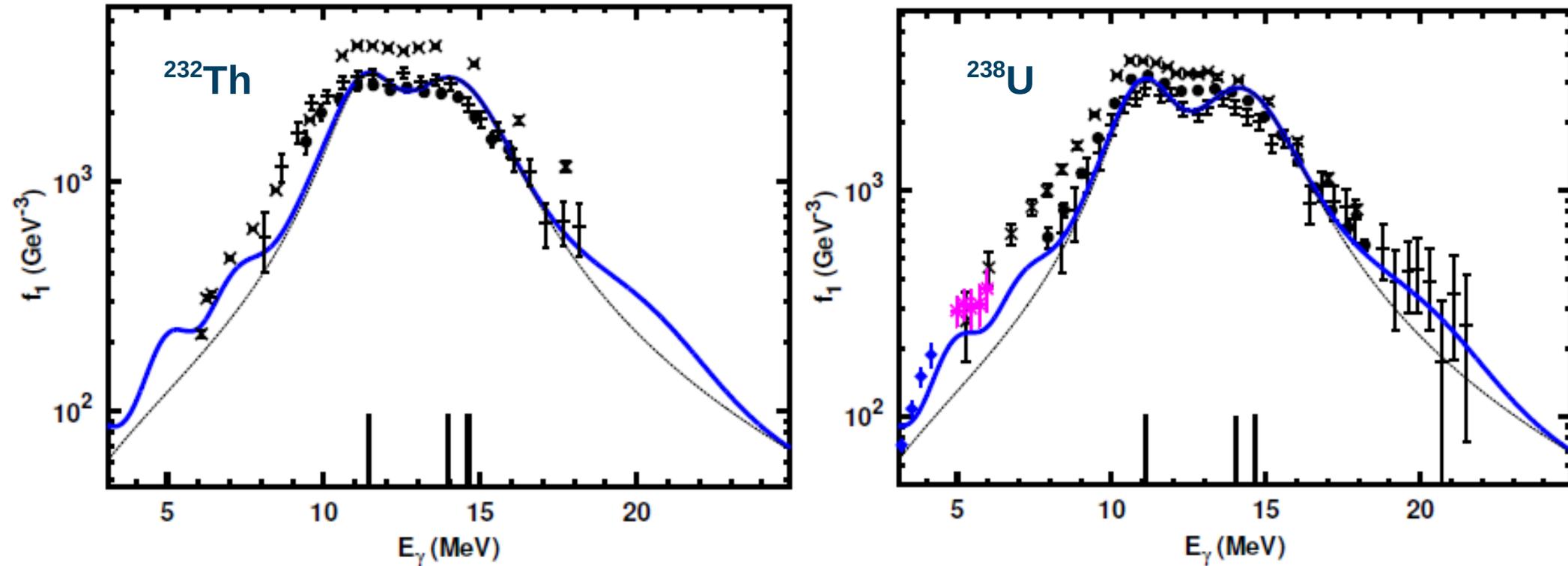
***Axial symmetry breaking is found by***

- 1. Angular momentum projection before the Hartree-Fock-Bogolyubov-variation***
  - 2. HFB calculations with mapping onto a 5D collective quadrupole Hamiltonian (GCM)***
  - 3. Jahn-Teller effect: symmetric configurations do not always have the lowest energy***
- All heavy nuclei are triaxial, some are more deformed and less triaxial than others***

*Data and TLO for these nuclei indicate: The top peak can be the smaller one, although it represents 2 components with equal integral but increased width. This has led to some confusion in older RIPL's.*



*These actinide data show a clear disaccord between different experiments !*



*The agreement between experiment and TLO is important with respect to the disagreeing data obtained at Livermore [Caldwell et al., 1980]. These cross sections for  $^{232}\text{Th}$  and  $^{238}\text{U}$  are exceptionally large in the sense, that an analysis with TLO indicates an overshoot of 30% as compared to the TRK sum.*

*Thomas-Fermi (ETFSI) method  
used to calculate nuclear masses  
(randomly selected in valley of stability).*

*When triaxiality is admitted in the  
calculations, ground state energy is  
lowered by less than 0.5 MeV.*

*But axial symmetry is broken anyhow.*

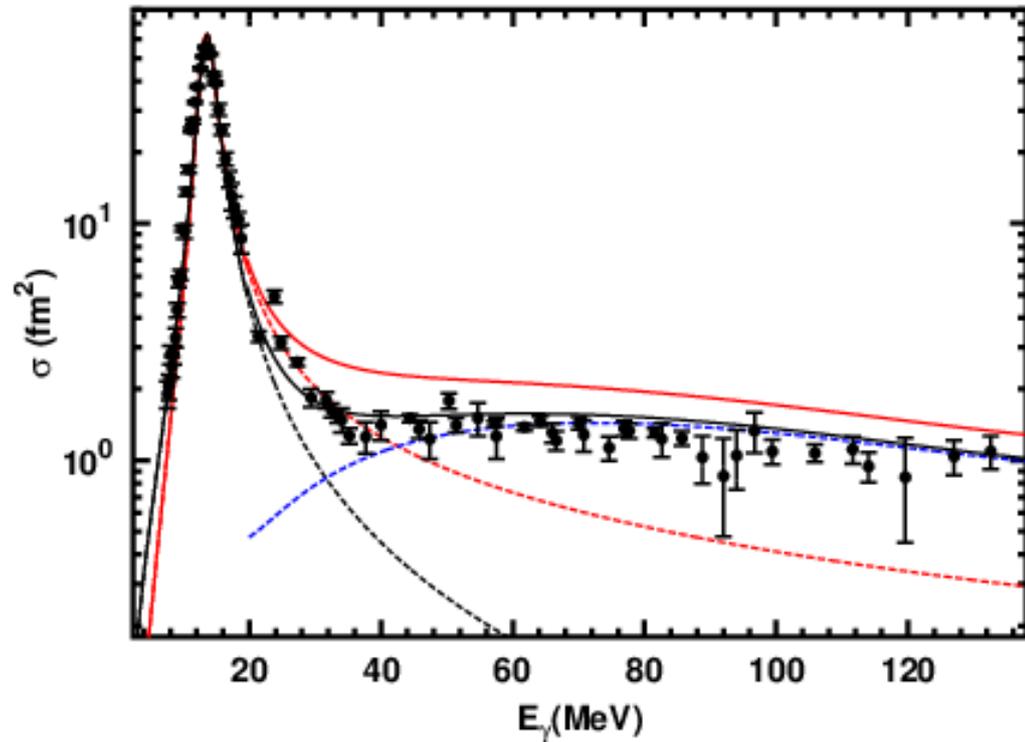
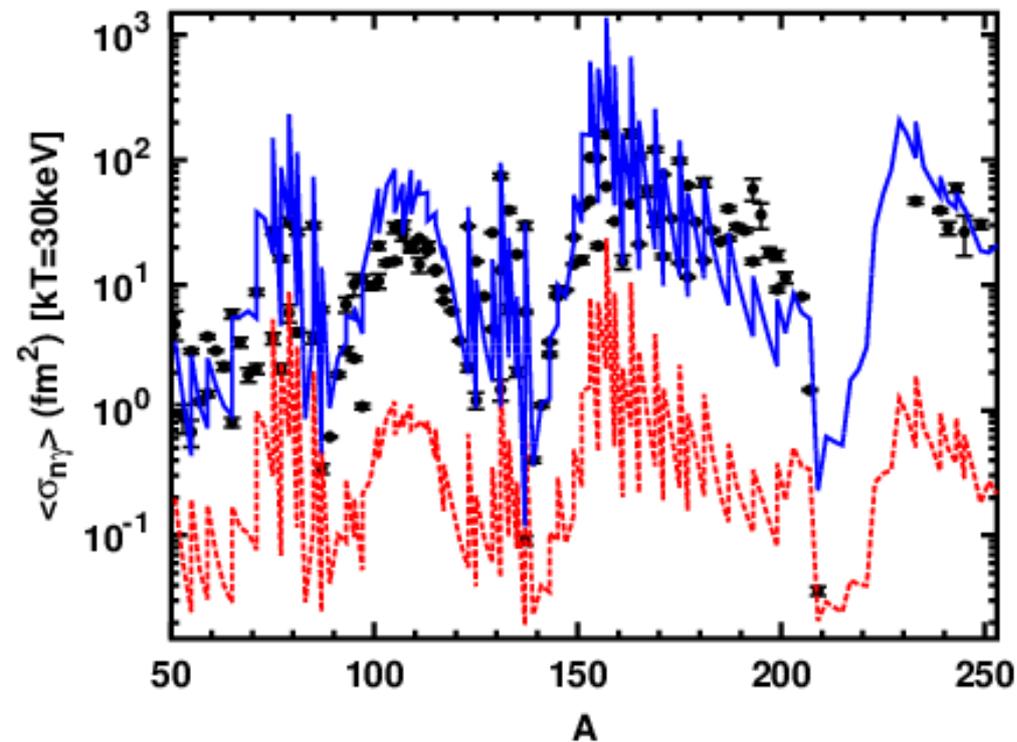
*And it is also broken if triaxiality is only  
dynamic.*

$Z$	$A$	$c$	$h$	$\epsilon(\gamma)$	$\Delta E_{\text{triax}}$
30	62	0.89	0.04	1.00 (0.0°)	0.0 MeV
32	74	1.16	-0.01	1.05 (9.2°)	-0.1
42	106	1.23	0.01	1.05 (5.8°)	-0.2
56	132	1.11	-0.04	1.05 (16°)	-0.2
58	134	1.11	0.0	1.04 (11°)	-0.2
62	138	1.16	0.02	1.05 (8.2°)	-0.4
68	168	1.18	0.06	1.04 (5.2°)	-0.2
74	186	1.09	0.20	1.05 (7.3°)	-0.3
76	188	1.06	0.22	1.04 (7.0°)	-0.4
76	192	1.04	0.24	1.04 (7.8°)	-0.3
88	222	1.21	-0.27	1.03 (20°)	-0.3
90	233	1.25	-0.21	1.06 (16°)	-0.5
92	236	1.25	-0.21	1.03 (8.3°)	-0.6

## *GDR's, their widths $\Gamma_i$ and low & high energy tail*

*As proposed 1983 by Kadenskii, Markushev and Furmann for n-capture resonances  $\Gamma_i$  vary with  $E_i$ .*

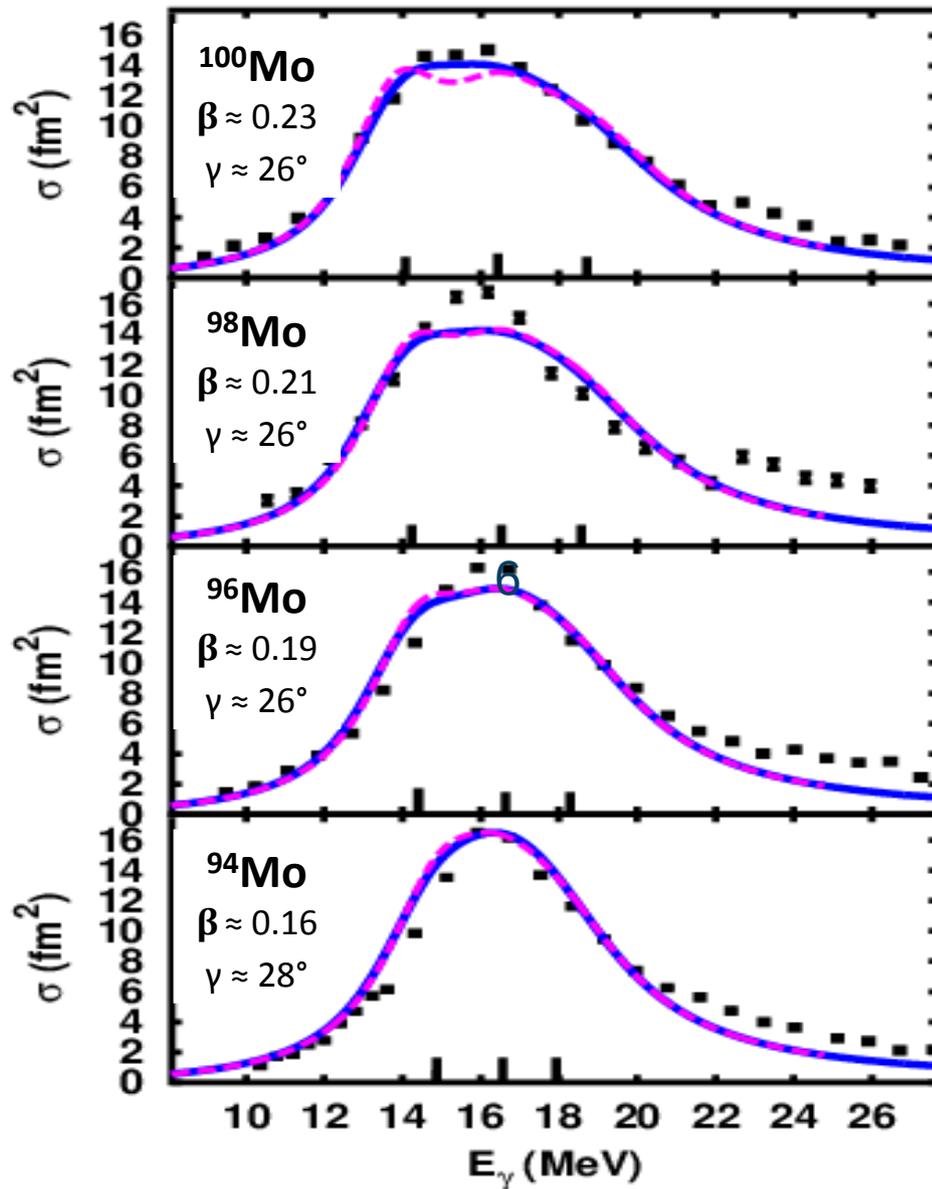
*A false application often labelled **KMF** proposes to apply this to GDR's with a dependence of  $\Gamma_i$  on  $E_\gamma$ ; this results in a **low** prediction for  $\sigma(n,\gamma)$ , if the TLO fit is used [left panel] - and a **surplus** above the GDR, where one sees effect of quasi-deuteron break up, calculated 1991 by Chadwick et al [right panel].*



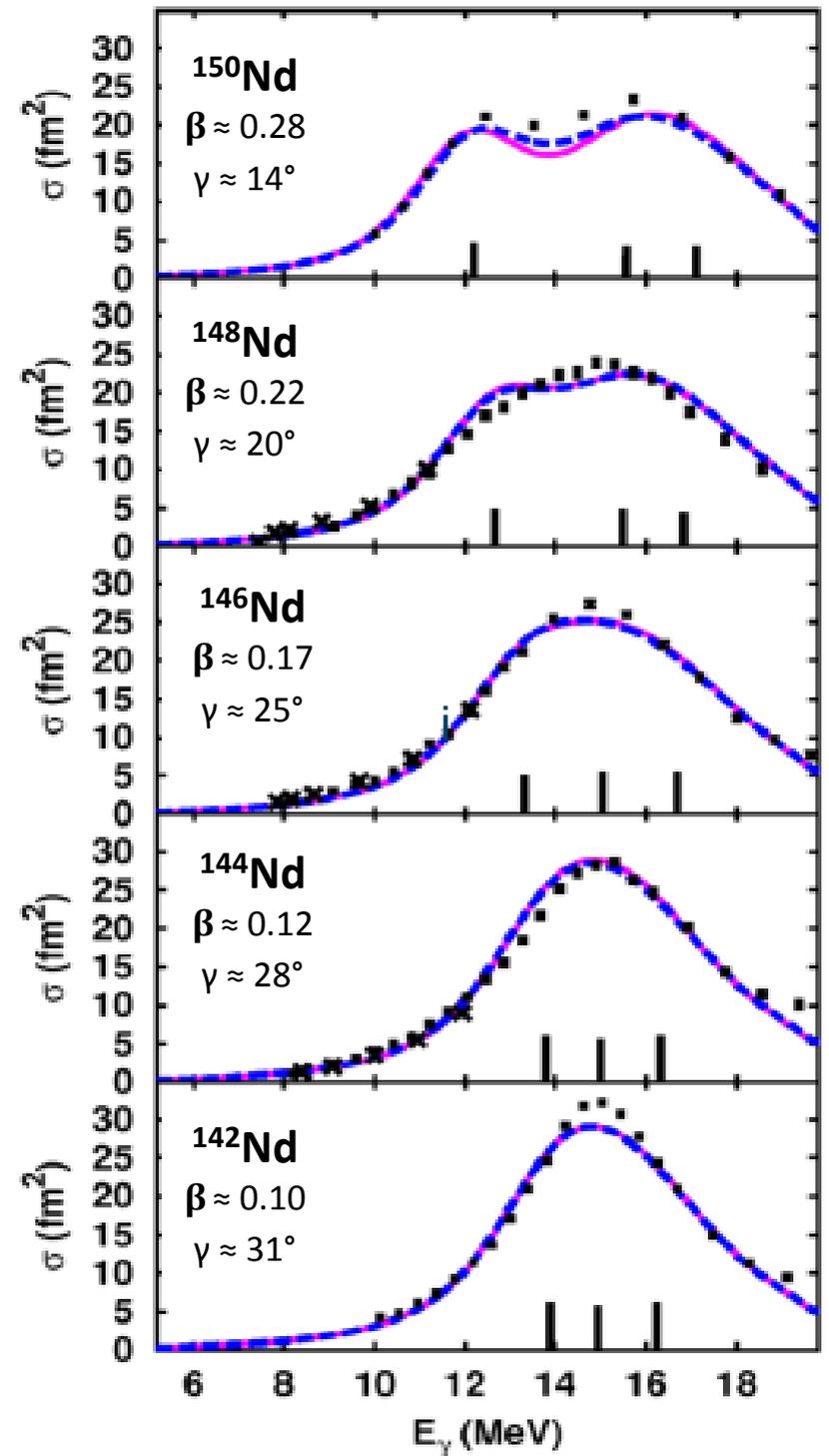
# *IVGDR's and deformation*

*parameters from HFB/GCM*

*incl. shape sampling*

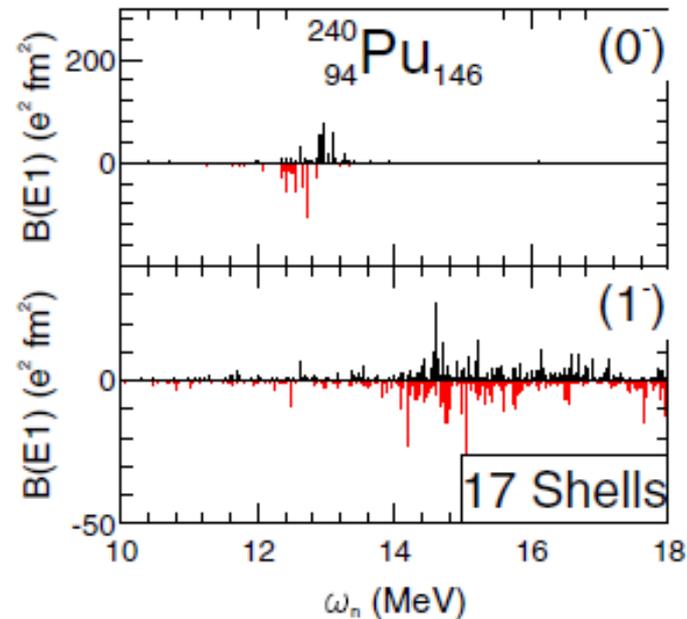
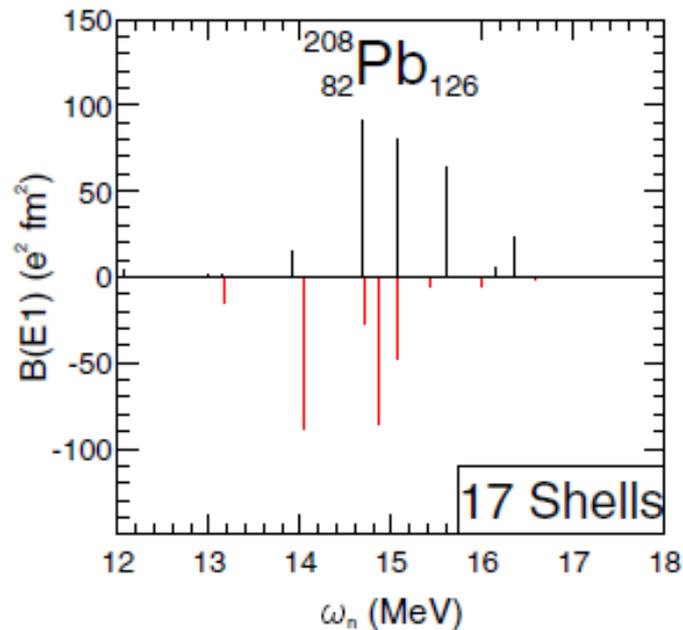
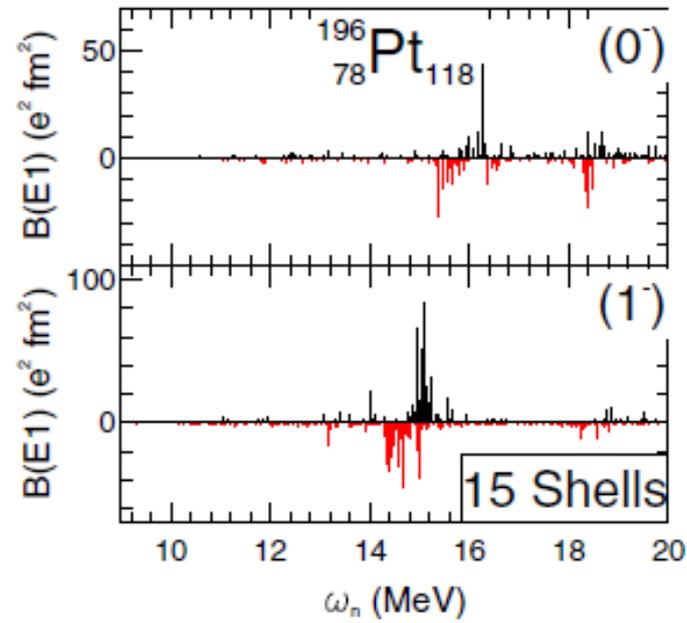
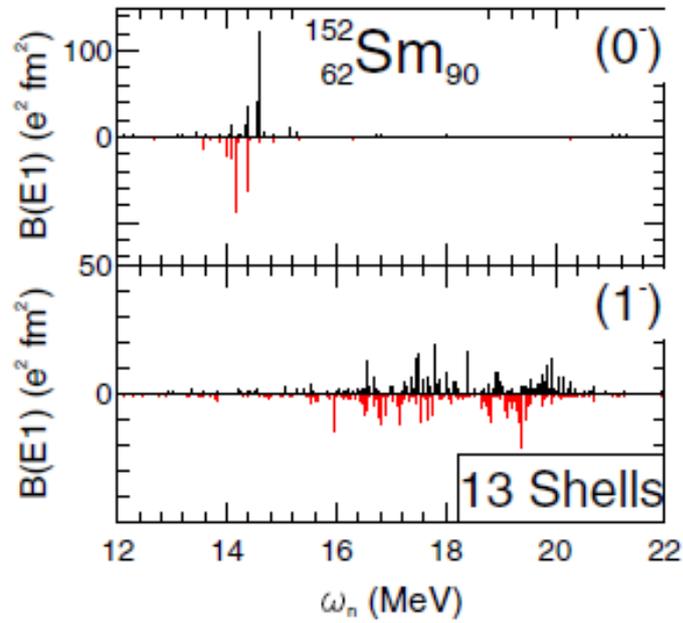


Beil *et al.*, Nucl. Phys. A 227, 427 (1974)



Carlos *et al.*, Nucl. Phys. A172, 437 (1971)

*At CEA/DAM an axially symmetric-deformed HFB+QRP is used and a constant width (2.5 MeV) and an energy shift  $\Delta=2$  MeV are adjusted on experimental data.*



*The calculations performed at CEA-DAM are using large quantities of cpu-time.*

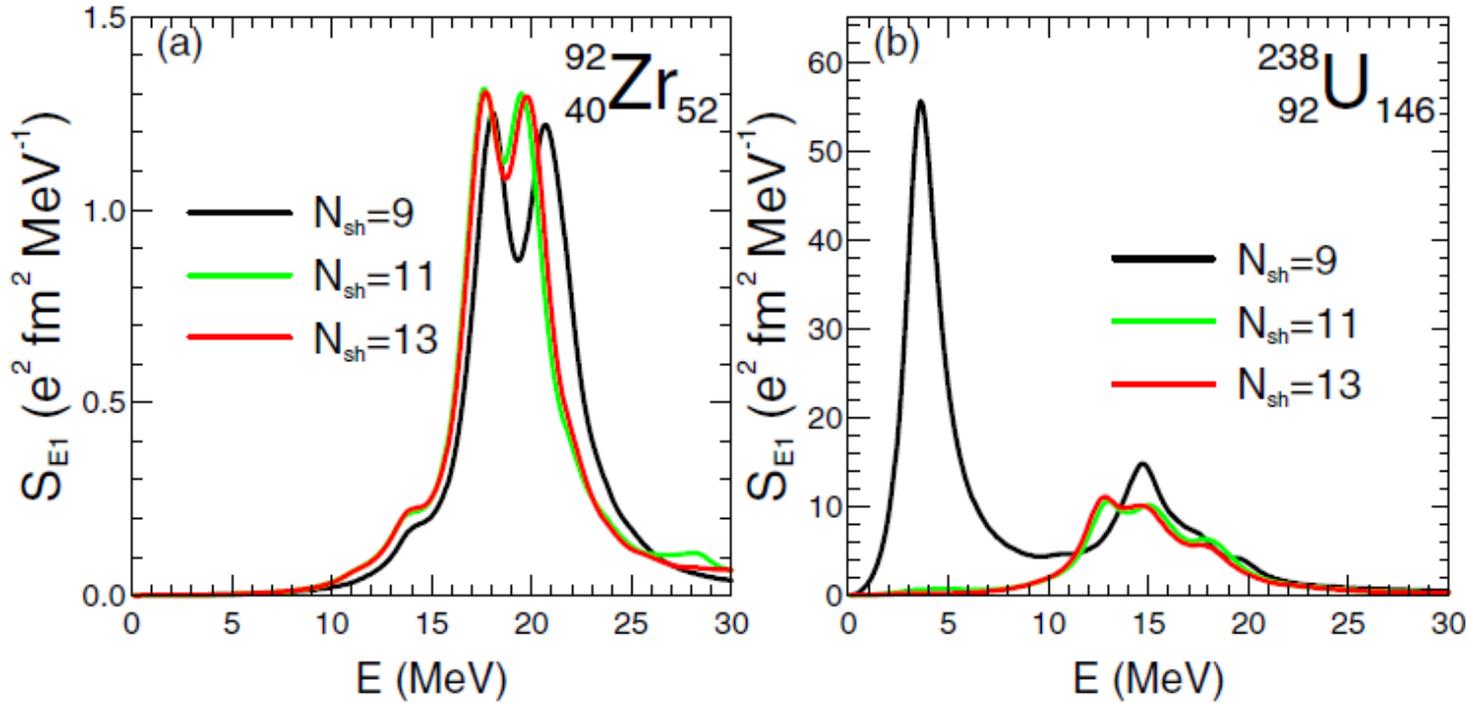


TABLE I. Average computation time for a  $K^\pi = 0^-$  of one nucleus for several energy cutoff and basis size combinations using 1024 cpus.

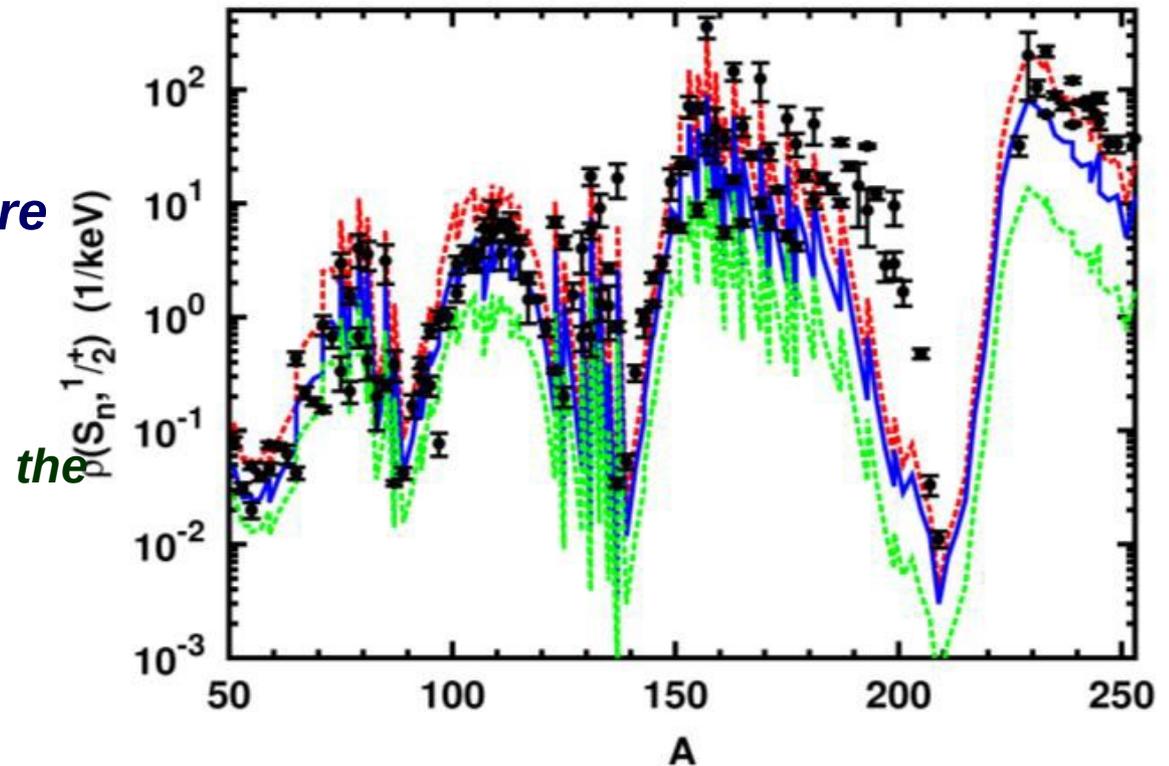
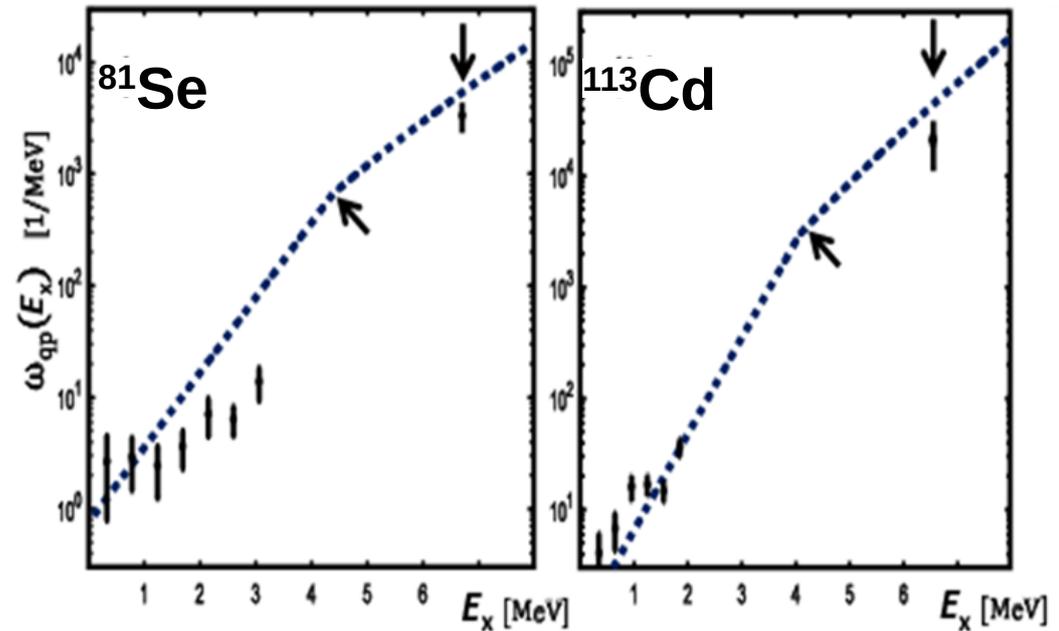
$N_{\text{sh}}$	No cut	$\varepsilon_c = 100 \text{ MeV}$	$\varepsilon_c = 60 \text{ MeV}$	$\varepsilon_c = 30 \text{ MeV}$
9	5 min	5 min	4 min	38 s
11	2 h	2 h	1 h	5 min
13	42 h	26 h	6 h	30 min
15	21 d	8 d	30 h	2 h
17	286 d	63 d	7 d	8 h

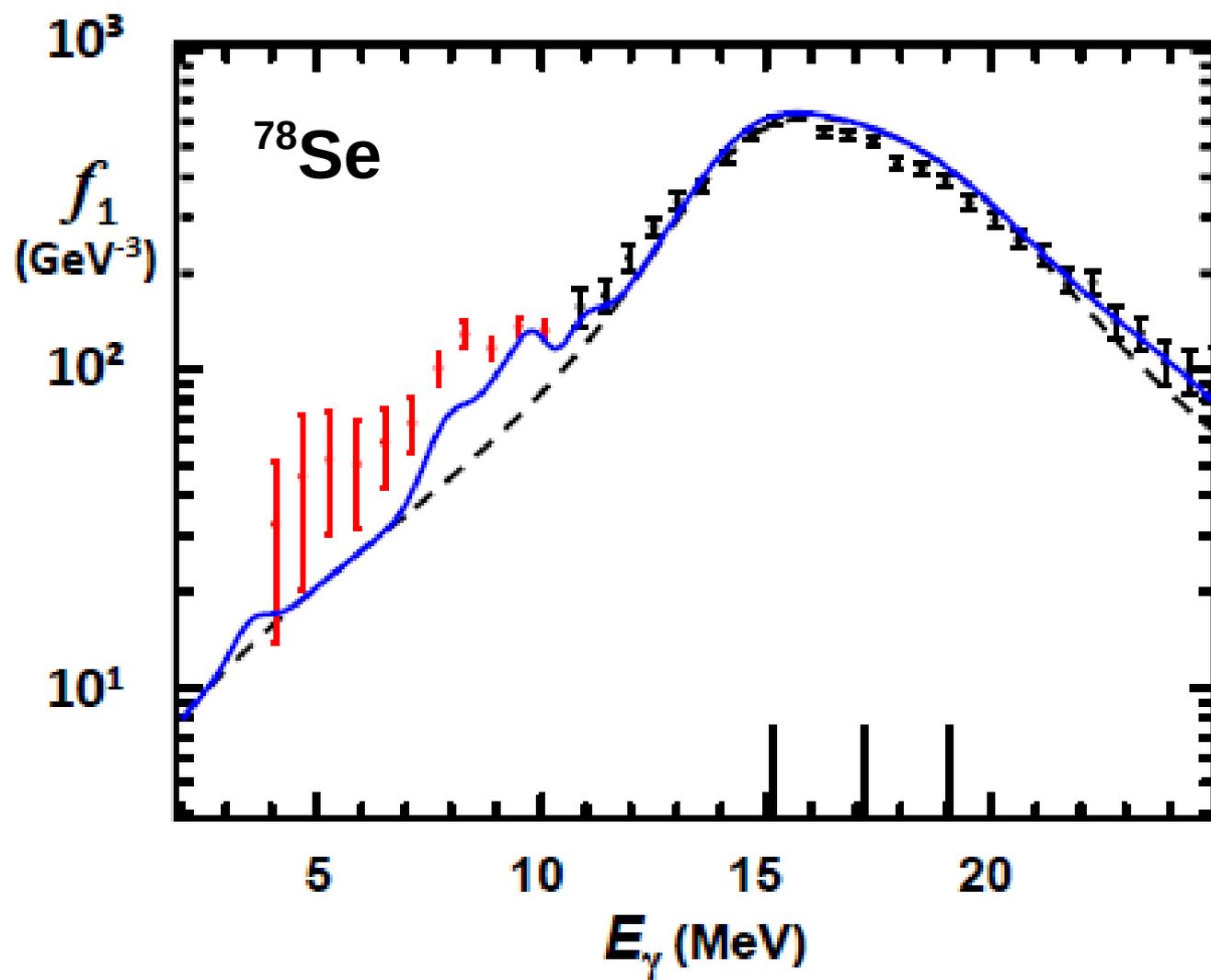
*This two-component level density approach has to be complemented by collective rotational enhancement which is very simple for low spins if axial symmetry is not required:*

$$\rho \rightarrow \frac{2J+1}{2 \cdot 4} \omega_{\text{qp}}(E_x)$$

*We get good agreement for states with  $J = \frac{1}{2}$  and then also for capture resonances into  $J=0$  targets.*

*The green curve corresponds to assumption of axially.*





# Rotational enhancement of nuclear level density vs. symmetry class

The intrinsic quasi-particle state density in a finite nucleus  $\omega_{qp}(E_x)$  is not yet the observable density of nuclear levels with well defined spin  $\rho(E_x, J = I_{rot} + j, \pi)$ .

To fix  $J$  the underlying collective symmetry has to be introduced (group theory)

1. *spherical*  $\Rightarrow$  only q-p states  $\rho(E_x, J, \pi) \rightarrow \frac{2J+1}{2 \cdot \sqrt{8\pi} \sigma^3} \omega_{qp}(E_x)$  ↙ small J limit

2. *axial*  $\Rightarrow$  q-p states & rotation  $\perp$  axis  $\rho \rightarrow \frac{2J+1}{2 \cdot \sqrt{8\pi} \sigma} \omega_{qp}(E_x)$

3. non-axial (triax)  $\Rightarrow$  q-p states & rotation about any axis  $\rho \rightarrow \frac{2J+1}{2 \cdot 4} \omega_{qp}(E_x)$

4. *no reflection symmetry*  $\Rightarrow$  q-p states & octupole deform.  $\rho \rightarrow \frac{2J+1}{2} \omega_{qp}(E_x)$

Thomas-Fermi Model  $\Rightarrow \sigma^2 = \frac{\bar{a} \cdot t}{11} A^{2/3} \approx \frac{A^{5/3}}{143} \cdot t$  ↖ 1 parity

# *New view on heavy nuclei – avoiding axial symmetry postulate*

*by: newly applied theoretical tools and*

*HFB + GCM, projection on  $I \Rightarrow \beta, \gamma$*

*$Q^3 \cdot \cos(3\gamma) \Rightarrow$  rotation invariant*

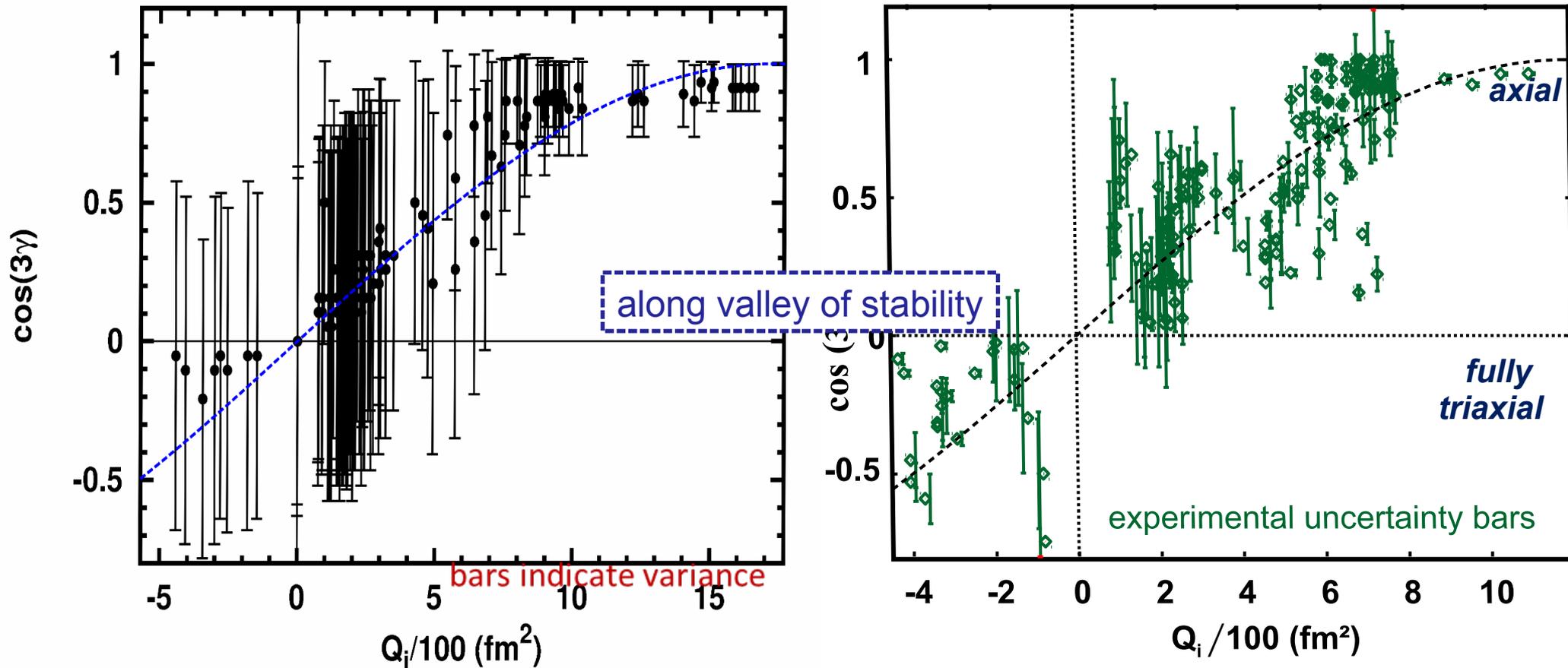
*indicator for **axiality***

*experimental observations:*

*level energies*

*decay properties and  $Q$ -moments*

*collectively enhanced level density*



*Giant dipole resonances as sensitive to deformation should also recognize it !*