Comments on polarized parton densities, on the validity of angular momentum sum rules and on the definition of parton angular momentum

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Outline

1) Controversial issues concerning the polarized parton densities: Strange quarks and Higher Twist.

2) Validity of the Transverse Angular Momentum Sum Rule.

3) The controversy about defining quark and gluon angular momenta.

The strange quark polarization

Now exist two NLO analyses of *combined* DIS and SIDIS world data.

DSSV: PR D80 (2009) 034030
LSS: arXiv:1010.0547

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Puzzling difference of $\Delta s(x) + \Delta\bar{s}(x)$ between *inclusive* DIS and *combined* DIS + SIDIS results.

Some red herrings in papers on Polarized DIS

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Nonsense!

$$
\begin{aligned}
g_1(x,Q^2)_{LT} \quad &= \quad \frac{1}{2}\sum_{flavors} e_q^2 \Big\{ [\Delta q(x,Q^2) + \Delta \bar{q}(x,Q^2)] \\
&+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \{\Delta C_q(x/y)\,[\Delta q(y,Q^2) + \Delta \bar{q}(y,Q^2)] \\
&+ \quad \Delta C_G(x/y)\,\Delta G(y,Q^2)\}\Big\}
\end{aligned}
$$

Inclusive DIS determines ONLY the sum of quark and antiquark densities.

.....Inclusive polarized DIS gives no information about the *separate* sea quark densities.....

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True!

But it does determine unambiguously $\Delta s(x)+\Delta\bar{s}(x)$

The controversy

ALL inclusive DIS analyses give negative values for $\Delta s(x) + \Delta\bar{s}(x)$

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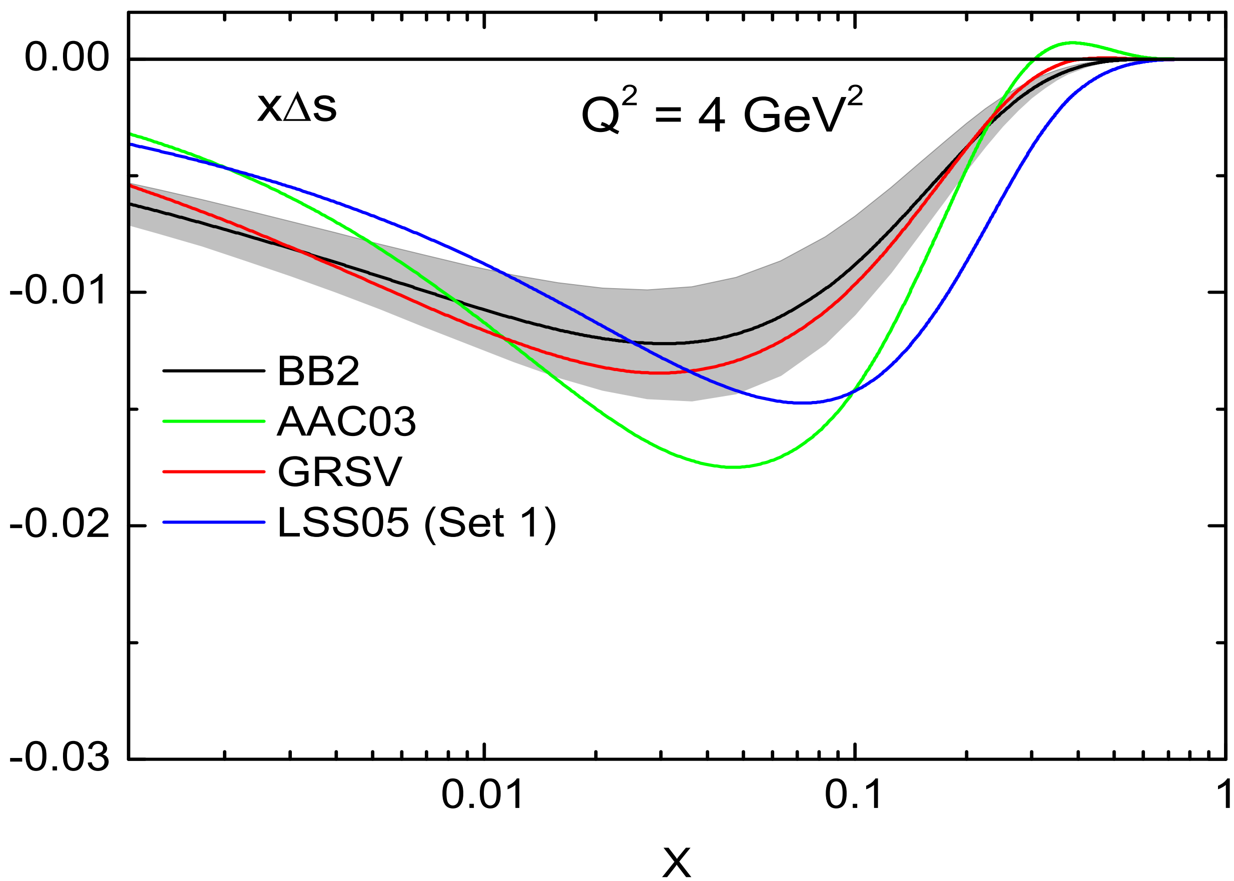
ALL SIDIS, or combined DIS and SIDIS analyses, in LO and in NLO , give either positive or sign-changing results for $\Delta s(x)+\Delta\bar{s}(x)$.

The DIS situation


0.00
-0.01
-0.02
-0.03
xΔs
Q2 = 4 GeV2
BB2
AAC03
GRSV
LSS05 (Set 1)
0.01
0.1
1
X

Constraint on positive values from SU(3) flavour

EL and D.B. Stamenov: PR D67 (2003) 037503

Define

$$\delta s(Q^2) \equiv \int_0^1 dx\,[\Delta s(x,Q^2) + \Delta\bar{s}(x,Q^2)]$$

$$\Gamma_1^p(Q^2) \equiv \int_0^1 dx\, g_1^p(x,Q^2) = \frac{1}{6}\left[\frac{1}{2}a_3 + \frac{5}{6}a_8 + 2\delta s(Q^2)\right]$$

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Rewrite as

$$a_8 = \frac{6}{5}\left[6\Gamma_1^p(Q^2) - \frac{1}{2}a_3 - 2\delta s(Q^2)\right]$$

Feed in

$$\Gamma_1^p(Q^2 = 5) = 0.118 \pm 0.004(stat) \pm 0.007(syst)$$

$$\Gamma_1^p(Q^2 = 3) = 0.133 \pm 0.003(stat) \pm 0.009(syst)$$

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Then, if $\delta s \geq 0$ find

$$a_8 \leq 0.089 \pm 0.058 \qquad a_8 \leq 0.197 \pm 0.068$$

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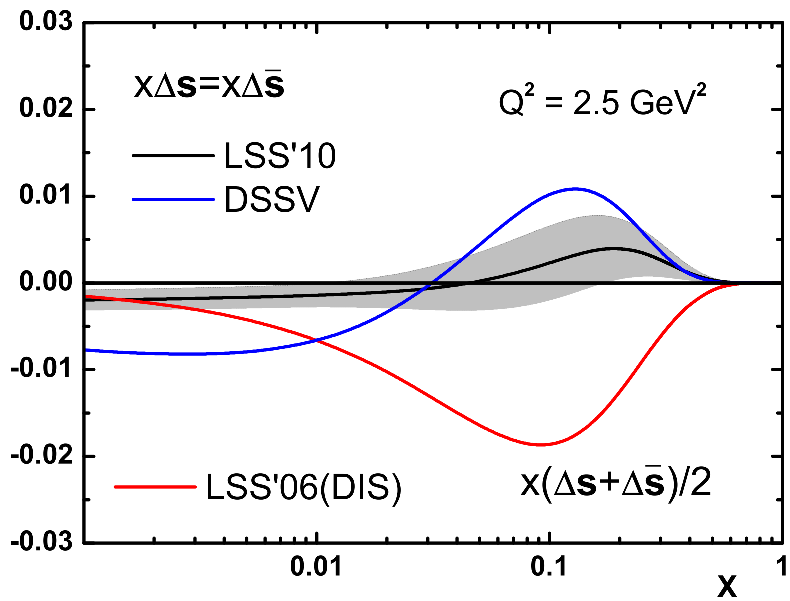
But $SU(3)_F$ seems good for hyperon decays viz. Fermilab KTev $\Xi^0 \to \Sigma^+ e\bar{\nu}$. Expect

$$a_8 = 0.585 \pm 0.025 \quad \text{i.e.} \quad 0.47 \leq a_8 \leq 0.70$$

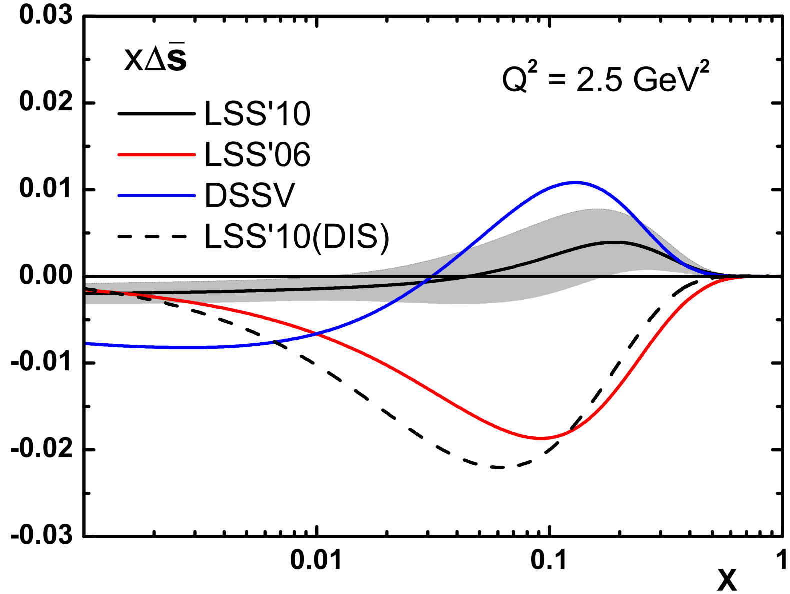
Thus $\delta s \geq 0$ implies huge breaking of $SU(3)_F$!

Combined DIS and SIDIS analysis assuming $\Delta s = \Delta\bar{s}$ and using DSS fragmentation functions



Note: DSSV use $\alpha_{\bar{s}} = \alpha_{\bar{d}}$ and find $= 0.16$
LSS find: $\alpha_{\bar{s}} = 0.05 \pm 0.02 \quad \alpha_{\bar{d}} = 0.55 \pm 0.12$

Redo DIS including term $(1 + \gamma x)$ to permit sign change.



Δs is a serious problem!

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1) Maybe we don't understand connection between DIS and SIDIS... a horrible thought which I will ignore.

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3) Maybe $\Delta s \neq \Delta \bar{s}$

A little exercise taking $\Delta s \neq \Delta \bar{s}$

Define, at $x = 0.1$ and $Q^2 = 2.5$,

$$\begin{aligned}
\Delta^h_{exact} &\equiv x\Delta s\, D^h_s + x\Delta\bar{s}\, D^h_{\bar{s}} \\
&= \frac{x}{2}[\Delta s + \Delta\bar{s}][D^h_s + D^h_{\bar{s}}] + \frac{x}{2}[\Delta s - \Delta\bar{s}][D^h_s - D^h_{\bar{s}}]
\end{aligned}$$

where

$$D^h_q = \int_{0.2}^{0.85} dz D^h_q(z)$$

DSS: Pions

$$D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^-}$$

$$\Delta^{\pi}_{exact} = x[\Delta s + \Delta\bar{s}][D_s^{\pi}] = 0.0008 \pm 0.0017 (\text{COMPASS})$$

Is this compatible with

$$\Delta^{\pi}_{exact} = x[\Delta s + \Delta\bar{s}]_{DIS}[D_s^{\pi}] = -0.0072\,?$$

Marginally

DSS: Kaons

$$D_s^{K^+} = D_{\bar{s}}^{K^-} \qquad D_s^{K^-} = D_{\bar{s}}^{K^+}$$

$$\Delta_{COMPASS}^{K^+} = \Delta_{COMPASS}^{K^-} = 0.0013 \pm 0.0026$$

Is this compatible with

$$\Delta_{exact}^{K} = \frac{x}{2}[\Delta s + \Delta\bar{s}]_{DIS}[D_s^K + D_{\bar{s}}^K] + \frac{x}{2}[\Delta s - \Delta\bar{s}][D_s^K - D_{\bar{s}}^K]\,?$$

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Is this compatible with

$$\Delta^{K}_{exact} = \frac{x}{2}[\Delta s + \Delta\bar{s}]_{DIS}[D_s^K + D_{\bar{s}}^K] + \frac{x}{2}[\Delta s - \Delta\bar{s}][D_s^K - D_{\bar{s}}^K]\ ?$$

NO! Find

$$\begin{aligned}\frac{x}{2}[\Delta s - \Delta\bar{s}] &= -0.207 \pm 0.005 \text{ for } K^+ \\ &= 0.212 \pm 0.005 \text{ for } K^-\end{aligned}$$

Strange quark summary

There is a serious contradiction.

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But it could be a signal of failure to understand the connection between DIS and SIDIS

The controversy about Higher Twist

Following Operator Product Expansion (OPE), LSS use

$$\begin{aligned} g_1(x,Q^2)_{exp} &= g_1(x,Q^2)_{LT} + g_1(x,Q^2)_{TMC} + g_1(x,Q^2)_{HT} \\ &= g_1(x,Q^2)_{LT} + g_1(x,Q^2)_{TMC} + \frac{h(x)}{Q^2} \end{aligned}$$

Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).

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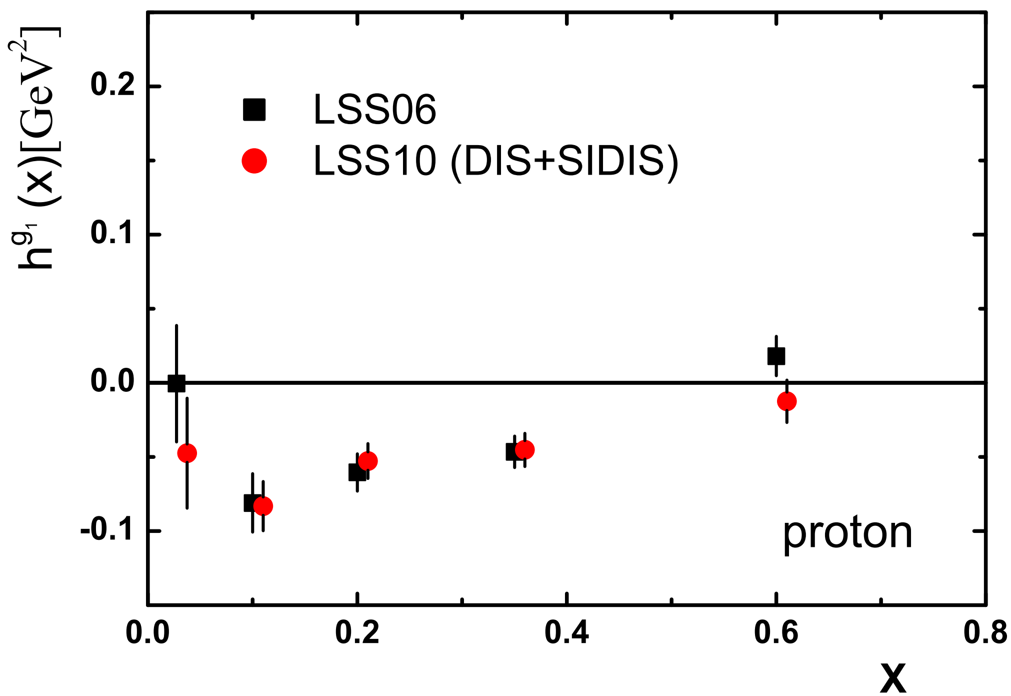
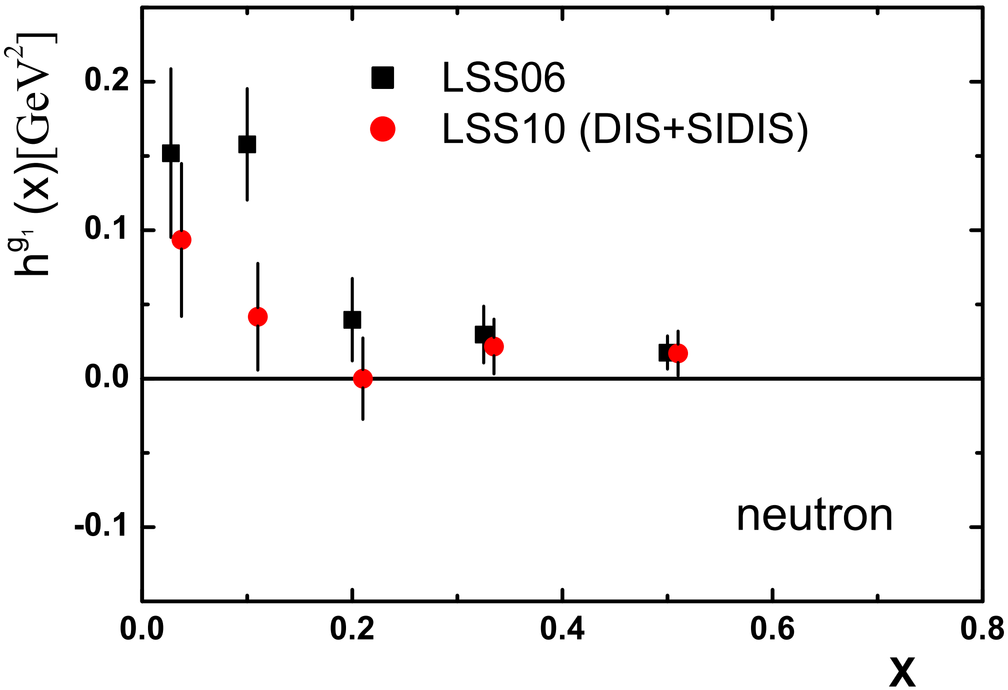
Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).
Possible slow scale i.e. Q^2 dependence in $h(x)$, the precise form of which is unknown, neglected compared to $1/Q^2$ variation.

We find significant HT contribution



Very important for CLAS data.



$h^{g_1}(x)[GeV^2]$
0.2
0.1
0.0
-0.1
LSS06
LSS10 (DIS+SIDIS)
neutron
0.0
0.2
0.4
0.6
0.8
x

Blümlein and Böttcher (BB) (arXiv:1005.3113 v1)
disagree

They use

$$g_1(x,Q^2)_{exp} = g_1(x,Q^2)_{LT}\Big[1+\frac{C(x)}{Q^2}\Big]$$

where any Q^2 dependence in $C(x)$ is neglected.

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where any Q^2 dependence in $C(x)$ is neglected.

BB find no evidence for HT i.e.their $C(x)$ for protons and neutrons is compatible with zero.

Thus

$$C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}}$$

If legitimate to neglect the scale dependence in $h(x)$ then $C(x)$ must vary considerably with Q^2, contradicting the use of $C(x)$ as Q^2-independent.

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Two approaches incompatible and their results incommensurate. One of the two methods (or perhaps both) has to be incorrect.

Since LSS formulation is closer in structure to the OPE we believe it to be the correct way to implement HT corrections.

Another problem: BB utilize above for proton and deuteron data and extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D} C_d - C_p$$

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This is incorrect. The correct relation should be

$$\begin{aligned} C_n(x) &= \frac{1}{g_{1n}(x,Q^2)_{LT}} \Big[\frac{2}{1 - 1.5\omega_D} g_{1d}(x,Q^2)_{LT} C_d(x) \\ &- g_{1p}(x,Q^2)_{LT} C_p(x) \Big] \end{aligned}$$

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Dangerous, since $g_{1n}(x, Q^2)_{LT}$ has a zero!

LSS Letter to BB—no response—so
(arXiv:1007.4781) "Comments on BB paper"

followed by Version 2 of BB, abandoning factorized form for HT

"We prefer the additive case, since the twist-2 scaling violations of $g_1(X, Q^2)$ do not influence $C_{p,d,n}(x)$."

No reference to LSS

Claim no evidence for HT, but central values essentially same as LSS. BB use only statistical errors, but, more important, define error bars by $\Delta\chi^2 = 9.3$.

LSS method agrees with approach to HT of moments.

$$\bar{h}^N \equiv \int_{0.0045}^{0.75} dx\, h^N(x) \qquad N = p, n$$

$$\bar{h}^p = (-0.028 \pm 0.005) GeV^2 \quad \bar{h}^n = (0.018 \pm 0.008) GeV^2$$

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$$\overline{h}^p + \overline{h}^n = (-0.013 \pm 0.009) GeV^2$$

$$|\overline{h}^p + \overline{h}^n| < |\overline{h}^p - \overline{h}^n|$$

Agrees $1/N_C$ expansion.

Higher Twist: summary

There is no real contradiction. With the revised BB analysis, the difference is entirely due to definition of error bars.

The transverse angular momentum sum rule

Propagation of a myth that one cannot have a sum rule for a transversely polarized nucleon.

Derivation of a sum rule begins with expression for expectation value of angular momentum operators in nucleon state specified by 4-momentum P^μ and covariant spin vector S^μ

$$\langle P, S \,|\, J_i \,|\, P, S \rangle \qquad i = 1, 2, 3$$

i.e. show dependence of matrix element on variables P^μ and S^μ

BLT [B.L.G.Bakker, E.L. and T.L.Trueman: PR D70 (2004) 114001] demonstrated:standard expression in literature (Jaffe-Manohar) is correct for nucleons polarized longitudinally, but **incorrect** for the transversely polarized case.

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This is origin of incorrect claim that there cannot exist an angular momentum sum rule for a transversely polarized nucleon.

Use canonical spin states $|P, \boldsymbol{s}\rangle$ i.e. 'boost' states where the spin state is specified by $\boldsymbol{s}$, which is twice the spin eigenvector in the *rest* frame ($\boldsymbol{s}^2 = 1$).

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Correct result for the forward matrix elements of the angular momentum is

$$\langle P', \boldsymbol{s}|J_i|P, \boldsymbol{s}\rangle = 2P^0(2\pi)^3 \left[\frac{1}{2}\, s_i + i(\boldsymbol{P} \times \boldsymbol{\nabla}_{\boldsymbol{P}})_i\right] \delta^3(\boldsymbol{P'}-\boldsymbol{P}).$$

and there is then no problem with the transverse case —it is quite analogous to the longitudinal case.

Jaffe-Manohar result has

$$\frac{1}{4Mp^0}\left\{(3p_0^2 - M^2)s_i - \frac{3p_0 + M}{p_0 + M}(\boldsymbol{p}\cdot\boldsymbol{s})p_i\right\} \quad \text{instead of} \quad \frac{1}{2}\,s_i$$

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For longitudinal case exactly same as BLT, but for transverse case, for $p_0 \gg m$

$$\text{J-M} \to \frac{3p_0}{4m} \to \infty$$

so no transverse sum rule.

Jaffe-Manohar acknowledge error!

Thus BLT were able to derive a sum rule relating the transverse spin of the nucleon to the **transverse polarized quark densities $\Delta_T q(x)$ and the transverse orbital angular momentum carried by quarks and gluons,** namely

$$\frac{1}{2} = \frac{1}{2} \sum_{\text{flavours}\, f} \left\{ \int dx \, [\, \Delta_T q^f(x) + \Delta_T \bar{q}^f \,] + \sum_{a = q, \bar{q}, G} \langle L_{\boldsymbol{s}_T} \rangle^a \right\}$$

where $L_{\boldsymbol{s}_T}$ is the component of $\boldsymbol{L}$ along $\boldsymbol{s}_T$.

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where $L_{\boldsymbol{s}_T}$ is the component of $\boldsymbol{L}$ along $\boldsymbol{s}_T$.

NB **sum of quark and antiquark transversity densities, in contrast to the case of the tensor charge of the nucleon, where the difference appears.**

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We shall show that the identification of terms on the RHS is correct.

Instant-form Fock expansion

Show instant-form Fock expansion applied to the expression for the quark correlator, gives standard connection between parton densities and the wave-functions appearing in the expansion.

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For simplicity give the proof only for unpolarized quark density $q(x)$, but the argument holds also for the polarized densities $\Delta q(x)$ and $\Delta_T q(x)$.

The sophisticated expression for the quark correlator $\Phi_{\alpha\beta}(k; P, S)$, for a given flavour, integrated over k with the constraint $x = k^+/P^+$ yields

$$\Phi\alpha\beta(x) = P^+ \int \frac{d\xi^-}{2\pi}\, e^{ixP^+\xi^-}\, \langle P,\, S\,|\,\bar{\psi}_\beta(0)\,\psi_\alpha(0, \xi^-, \mathbf{0}_\perp)\,|\,P,\, S\rangle$$

where

$$P^\pm = \frac{1}{\sqrt{2}}\,(P_0 \pm P_z)$$

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where

$$P^{\pm} = \frac{1}{\sqrt{2}} \, (P_0 \pm P_z)$$

At leading twist $\Phi\alpha\beta(x)$ expressed in terms of three LT quark distribution functions,

$$\Phi(x) = \frac{1}{2} \, \not{P} \{ q(x) - 2\,\lambda\,\Delta q(x)\gamma_5 + \Delta_T q(x)\gamma_5 \, \not{S}_\perp \}$$

where $\lambda = \pm 1/2$ is the nucleon helicity.

However, nucleon, mass m, is moving **fast** along OZ so

$$P^\mu \approx (P, 0, 0, P)$$

so that

$$Tr[\gamma^0 \Phi(x)] \approx Tr[\gamma^3 \Phi(x)]$$

so that

$$Tr[\gamma^+ \Phi(x)] \approx \frac{P^+}{P_0} Tr[\gamma^0 \Phi(x)] \approx \sqrt{2}\, Tr[\gamma^0 \Phi(x)].$$

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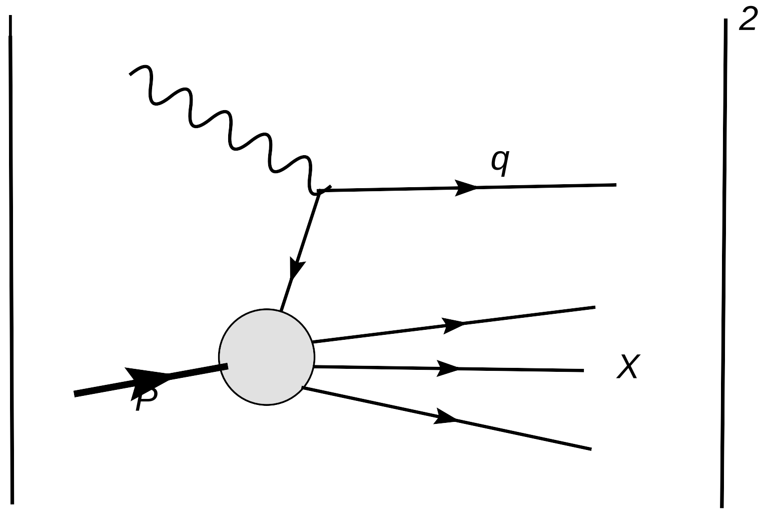
Thus we may take

$$q(x) = \frac{1}{2\,P_0} Tr[\gamma^0 \Phi(x)],$$

Using translation invariance, final expression is

$$q(x) = \frac{1}{\sqrt{2}} \sum_{X,\alpha} |\langle X \,|\, \psi_\alpha(0) \,|\, P,\, S\rangle|^2 \, \delta[P_X^+ - (1-x)P^+]$$

This corresponds to intuitive definition of quark density!



Matrix element involves field at only one time, so evaluate in Interaction Picture i.e use *free-field* expansion

$$
\begin{aligned}
\psi_\alpha(0) &= \sum_\lambda \int \frac{d^3\boldsymbol{p}}{(2\pi)^3\, 2E_p}\, b(\boldsymbol{p},\lambda) u_\alpha(\boldsymbol{p},\lambda) \\
&+ \sum_\lambda \int \frac{d^3\bar{\boldsymbol{p}}}{(2\pi)^3\, 2E_{\bar{p}}}\, d^\dagger(\bar{\boldsymbol{p}},\lambda) u_\alpha(\bar{\boldsymbol{p}},\lambda)
\end{aligned}
$$

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$$\psi_\alpha(0) = \sum_\lambda \int \frac{d^3\boldsymbol{p}}{(2\pi)^3\, 2E_p}\, b(\boldsymbol{p},\lambda) u_\alpha(\boldsymbol{p},\lambda) + \sum_\lambda \int \frac{d^3\bar{\boldsymbol{p}}}{(2\pi)^3\, 2E_{\bar{p}}}\, d^\dagger(\bar{\boldsymbol{p}},\lambda) u_\alpha(\bar{\boldsymbol{p}},\lambda)$$

where we use $\bar{\boldsymbol{p}}$ to emphasize that $d^\dagger$ creates an antiquark. Note that

$$E_p = \sqrt{\boldsymbol{p}^2 + m_q^2} \qquad \text{and} \qquad E_{\bar{p}} = \sqrt{\bar{\boldsymbol{p}}^2 + m_q^2}$$

The nucleon, $\boldsymbol{P} = (0, 0, P)$, expanded as superposition of n-parton Fock states

$$
\begin{aligned}
|\boldsymbol{P}, m\rangle \; &= \; [(2\pi)^3 2P_0]^{1/2} \sum_n \sum_\mu \int \frac{d^3\boldsymbol{k}_1}{\sqrt{(2\pi)^3 2k_1^0}} \cdots \frac{d^3\boldsymbol{k}_n}{\sqrt{(2\pi)^3 2k_n^0}} \\
&\times \psi_{\boldsymbol{P},m}(\boldsymbol{k}_1, \mu_1, \ldots \boldsymbol{k}_n, \mu_n) \\
&\times \delta^{(3)}(\boldsymbol{P} - \boldsymbol{k}_1 - \ldots - \boldsymbol{k}_n) |\boldsymbol{k}_1, \mu_1, \ldots \boldsymbol{k}_n, \mu_n\rangle .
\end{aligned}
$$

where μ_i denotes helicity.

The nucleon, $\boldsymbol{P} = (0,0,P)$, expanded as superposition of n-parton Fock states

$$|\boldsymbol{P}, m\rangle \;=\; [(2\pi)^3 2P_0]^{1/2} \sum_n \sum_\mu \int \frac{d^3\boldsymbol{k}_1}{\sqrt{(2\pi)^3 2k_1^0}} \cdots \frac{d^3\boldsymbol{k}_n}{\sqrt{(2\pi)^3 2k_n^0}}$$
$$\times \psi_{\boldsymbol{P},m}(\boldsymbol{k}_1, \mu_1, ...\boldsymbol{k}_n, \mu_n)$$
$$\times \delta^{(3)}(\boldsymbol{P} - \boldsymbol{k}_1 - ... - \boldsymbol{k}_n)|\boldsymbol{k}_1, \mu_1, ...\boldsymbol{k}_n, \mu_n\rangle$$

where μ_i denotes helicity.
$\psi_{\boldsymbol{P},m}$ is the partonic wave function of nucleon normalized so that

$$\sum_{\{\sigma\}} \int d^3\boldsymbol{k}_1..d^3\boldsymbol{k}_n |\psi_{\boldsymbol{P},m}(\boldsymbol{k}_1, \mu_1, ...\boldsymbol{k}_n, \mu_n)|^2 \delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}_1 - ... - \boldsymbol{k}_n) = \mathcal{P}_n$$

with $\mathcal{P}_n$ the probability of the n-parton state.

Consider contribution to the matrix element $\langle\, X\, |\psi_\alpha(0)|\, P\,\rangle$ when the nucleon is represented by Fock state with n constituents

$$|\, \boldsymbol{k}_1, \mu_1; ------\boldsymbol{k}_n, \mu_n\rangle \quad \text{with} \quad \sum_j \boldsymbol{k}_{j_T} = 0 \quad \text{and} \quad \sum_j k_{j_z} = P$$

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Would imply that $q(x)$ is not given simply by the modulus squared of a wave-function, as it ought to be. Sometimes said that the latter form is a miracle of the Light Cone expansion.
Show that this is not quite a miracle and that there is no problem also in the Instant form.

Focus on the $d^\dagger$ term in $\psi_\alpha(0)$

$$\langle\, X\,|d^\dagger(\bar{\boldsymbol{p}},\lambda)|\,\boldsymbol{k}_1,\mu_1;- - - - - -\boldsymbol{k}_n,\mu_n\rangle \;\; = \\ \langle\, X\,|\,\bar{\boldsymbol{p}},\lambda:\,\boldsymbol{k}_1,\mu_1;- - - - - -\boldsymbol{k}_n,\mu_n\rangle$$

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Aside from combinatoric factors this implies

$$| X \rangle = | \bar{\boldsymbol{p}}, \lambda : \boldsymbol{k}_1, \mu_1; - - - - - \boldsymbol{k}_n, \mu_n \rangle$$

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For this state

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Its energy is

$$\begin{aligned} E_X \;&=\; \sum_j \sqrt{\boldsymbol{k}_j^2 + m_q^2} + E_{\bar{p}} \\ &\geq \sum_j k_{j_z} + E_{\bar{p}} \\ &\geq P + E_{\bar{p}} \end{aligned}$$

This implies

$$P_X^+ > P^+ + \bar{p}^+ > P^+$$

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Thus the identification of the wave functions used by BLT with the standard parton densities is correct

Summary of Transverse Angular Momentum Sum Rule

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The simplification of using instant from wave functions does not affect the partonic interpretation of the terms in the sum rule.

EXTRA SLIDES

A peculiarity concerning the parton orbital angular momentum in a transversely polarized nucleon

In this section we shall draw attention to a peculiarity regarding the quark and gluon *orbital* angular momentum inside a transversely polarized nucleon, which does *not* depend on the precise definition of quark *vs* gluon angular momentum.

The shortest and most direct way to obtain the correct expression for the angular momentum matrix element $\langle P', \boldsymbol{s} | J_i | P, \boldsymbol{s} \rangle$ is actually from consideration of the effect of rotations on a state vector.

But if one uses the traditional approach, via the energy momentum tensor $T^{\mu\nu}$, then to begin with the matrix element of M^{jk}, where

$$J_{(i=x,y,z)} = \epsilon_{ijk} M^{jk} \qquad (1)$$

depends on some of the scalar functions appearing in the matrix element of $T^{\mu\nu}$. Using Ji's notation A and B for these, one has

$$\begin{aligned} \langle P, \boldsymbol{s}|M^{ij}|P, \boldsymbol{s}\rangle &= \frac{A}{2M(P_0 + M)}[P^i(\boldsymbol{P} \times \boldsymbol{s})^j - P^j(\boldsymbol{P} \times \boldsymbol{s})^i] \\ &+ \frac{1/2(A+B)}{M}\epsilon^{ij\alpha\beta} S_\alpha P_\beta \end{aligned}$$

Using the fact that T^{00} is the energy density one can show that the energy of the nucleon fixes the value of A so that

$$A(\text{nucleon}) = 1. \tag{2}$$

Applying Eqn. (2) to the case of a longitudinally polarized nucleon with helicity 1/2 implies that $1/2(A+B) = 1/2$. It follows that

$$B(\text{nucleon}) = 0 \tag{3}$$

Suppose now that we split A and B into quark (meaning a sum over flavours of quarks and antiquarks) and gluon pieces:

$$A(\text{nucleon}) = A_q + A_g = 1 \qquad (4)$$

$$B(\text{nucleon}) = B_q + B_g = 0 \qquad (5)$$

Then the quark and gluon components of M_{ij} will be given by

$$
\begin{aligned}
\langle\, P, \boldsymbol{s}|M^{ij}_{q,g}|P, \boldsymbol{s}\rangle &= \frac{A_{q,g}}{2M(P_0+M)}[P^i(\boldsymbol{P}\times\boldsymbol{s})^j - P^j(\boldsymbol{P}\times\boldsymbol{s})^i] \\
&+ \frac{1/2(A+B)_{q,g}}{M}\epsilon^{ij\alpha\beta}S_\alpha P_\beta \qquad (6)
\end{aligned}
$$

For a transversely polarized nucleon, say along OX, with $\boldsymbol{s}_T = (1,0,0)$ and $\boldsymbol{P} = (0,0,P)$ this becomes

$$
\begin{aligned}
\langle P, \boldsymbol{s}_T|J^q_x|P, \boldsymbol{s}_T\rangle &= \frac{1}{2M}[(M-P_0)A_q + 2P_0\,1/2(A+B)_q] \\
&= \frac{1}{2M}[MA_q + P_0\,B_q] \qquad (7)
\end{aligned}
$$

Thus for a fast moving nucleon, as $P_0 \to \infty$, J_x^q, and similarly J_x^g, becomes infinite. Of course J_x^{nucleon} is finite, since, as mentioned above, $B_q + B_g = 0$.

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Examination of the Fock expansion shows that the P_0 term can only come from the orbital angular momentum. This is supported by a purely classical picture where the orbital angular momentum is generated by the quark rotating about the CM of the nucleon.

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Thus irrespective of how the angular momentum is split into quark and gluon components, the separate quark and gluon *orbital* angular momentum in a fast moving transversely polarized nucleon becomes infinite. Their sum, however, is finite.

The controversy concerning the definition of quark and gluon angular momentum

- Controversy in QCD : how to split the total angular momentum into separate quark and gluon components
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- Different results for momentum and angular momentum carried by quarks and gluons e.g. as $\mu^2 \to \infty$

Since problem already arises in QED, will illustrate via QED

Since problem already arises for **linear** momentum will discuss that.

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Ji: $\frac{16}{16+3n_f} \simeq 1/2 \qquad$ for $n_f = 5$

Chen et al: $\frac{8}{8+6n_f} \simeq 1/5 \qquad$ for $n_f = 5$!

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Impossible. Cannot be checked!

The momentum operator in gauge-invariant theories

Theory invariant under translations; Noether construction, from classical Lagrangian; canonical e-m density $t^{\mu\nu}_{can}(x)$. A conserved density, generally not symmetric under $\mu \leftrightarrow \nu$.

$$\partial_\mu t^{\mu\nu}_{can}(x) = 0$$

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Canonical total linear momentum operator P^j_C

$$P^j_C = \int d^3x \; t^{0j}_{can}(x)$$

independent of time.

Canonical momentum operator as generator of translations

Classically : P_C^j generates spatial translations.
Quantum theory: check correct commutation relations with fields i.e. for any field $\phi(x)$

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Crucial when discussing division of total momentum into contributions from different fields .

The Bellinfante e-m density

Construct from $t^{\mu\nu}_{can}(x)$ and the Lagrangian, the conserved, symmetric, Bellinfante density $t^{\mu\nu}_{bel}(x)$, which may be gauge invariant.

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Differs from $t^{\mu\nu}_{can}(x)$ by a divergence term:

$$t^{\mu\,\nu}_{bel}(x) = t^{\mu\,\nu}_{can}(x) + \frac{1}{2}\partial_\rho[H^{\rho\mu\nu} - H^{\mu\rho\nu} - H^{\nu\rho\nu}]$$

where $H^{\rho\mu\nu} = -H^{\rho\nu\mu}$ and is a local operator.

Then P_B^j defined by

$$P_B^j \equiv \int d^3x \; t_{bel}^{0j}(x)$$

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For a classical *c-number* field it is meaningful to argue that the field vanishes at infinity. Much less obvious what this means for a quantum operator.

Quantum theory

Consistency of operator equations requires OK to neglect spatial integrals of divergences of local operators.

Thus, for **total** system it is OK to state $P_B^j = P_C^j$.

Non-gauge invariance of the QED momentum operator

Can prove following result:

Theorem : Consider a theory which is invariant under local c-number gauge transformations. Let P^μ be the total momentum operator, defined as the generator of space-time translations. **Then P^μ cannot be a gauge invariant operator.**

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Does that matter?

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Can show that the matrix element of P_C^j between any normalizable physical states, is unaffected by gauge changes in the operator.

The problem of defining separate quark and gluon momenta

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(1) General how to define the separate momenta for a system of interacting particles.

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Can only discuss the general question in this talk.

Interacting particles: the general problem

System of interacting particles E and F. Split the total momentum into two pieces

$$P^j = P_E^j + P_F^j$$

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Crucial: above equation, as it stands, is totally misleading, and should be written

$$P^j = P_E^j(t) + P_F^j(t)$$

to reflect the fact that the particles exchange momentum as a result of their interaction.

Criterion for identifying $P_{E,F}$ as the momentum associated with particles E, F respectively?????

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but no way we can check this, since $P_E^j(t)$ depends on t and, without solving the theory, can only compute equal time commutators .
Suggest: **minimal requirement for identifying an operator P_E^j with the momentum carried by E , is to demand that at equal times the analogue of this holds**

$$\boldsymbol{i[P_E^j(t)\,, \phi_E(t,x)] = \partial^j \phi_E(t,x).}$$

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But if split P_C into $P_{CE} + P_{CF}$ and P_B into $P_{BE} + P_{BF}$, then the integrands of P_{CE} and P_{BE} do *not* differ by a spatial divergence, and hence P_{CE} and P_{BE} do not generate the same transformation on $\phi_E(x)$.

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Example:Could identify

$$t^{0j}_{can}(\text{electron}) = \frac{i}{2}\,\bar{\psi}\gamma^0 \overleftrightarrow{\partial}^j\,\psi$$

and

$$t^{0j}_{bel}(\text{electron}) = \frac{i}{4}\,\bar{\psi}(\gamma^0 \overleftrightarrow{D}^j + \gamma^j \overleftrightarrow{D}^0)\,\psi$$

and these do not differ by a spatial divergence.

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This disagrees with both Ji, Chen et al, and Matsuda.