Multi-fermion systems with contact theories

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Roadmap

Emergent four-body parameter in universal two-species bosonic systems PLB 408 (2021)

Multi-fermion systems with contact theories, PLB 816 (2021)

Triple-X and beyond: PRD 103 (2021)

- Contact theories

  ⇕ EFT

- Universality

- Hadrons: minimal theory for systems hard to be created experimentally
- Atoms: building new universal systems
- Nuclei: challenges of contact EFT in the many-body sector

Approaches to theoretical nuclear physics:

(100 ys of nuclear physics)

+ Understanding of the **mechanisms** of nuclear proprieties;

+ Support to **experiments**;

+ **Precision description** of nuclear observables;

+ …

Universality

Universality: a simple concept hard to be defined

Systems in the same universality class behave similarly.

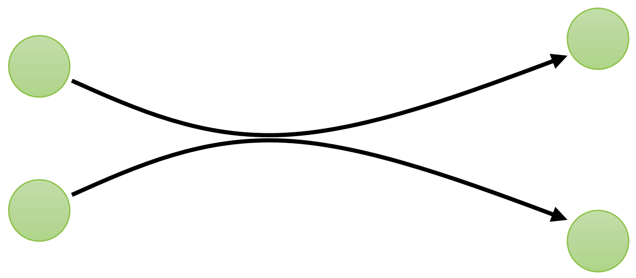
They share the same qualitative **observables**:

- Particle **statistics;** (Fermions, bosons)
- **Critical exponents;**
- **Scattering parameters;**
- Number and nature of **quantum states**
  (same resonances, bound, and virtual states).

Despite having different **typical size** and **microscopic structure.**

An example of universality – unitarity (2-body only)

Unitarity: **The size of a nonrelativistic quantum two-body system** much larger than the **range** of the **interaction** between particles.



S-wave nonrelativistic scattering;
k: center of mass collision momentum;
δ_0: phase shift in $L = 0$ partial wave.

Effective range expansion:

$$k\,\cot(\delta_0) = -\frac{1}{a_0} + \frac{1}{2}\,r_0\,k^2 + O(k^4)$$

Unitarity / unitary limit: $a_0 \to \infty$, $r_0 \to 0$, (all the other scattering parameters vanish).

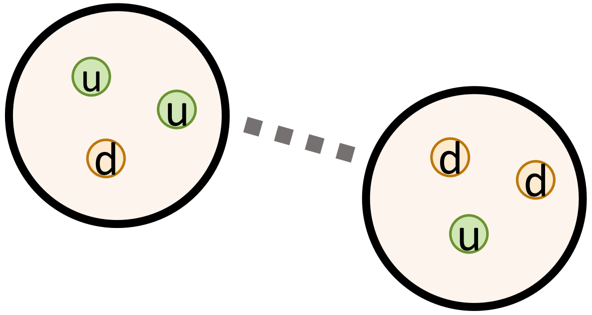
An example of universality – unitarity (2-body only)

(the **size of a nonrelativistic quantum two-body system** is much larger than the **interaction range** between particles)

Systems close to the **Unitary limit** can be found in

- **Atomic physics** (Feshbach resonances, $^{6}\mathrm{Li} - {}^{6}\mathrm{Li}$, $^{40}\mathrm{K} - {}^{40}\mathrm{K}$ atoms)
- **Nuclear physics** ($n - p$ interaction)
- **Hypernuclei** ($\Lambda - n$ interaction)
- **Hadronic physics** ($X(3872)$ Particles)

There are no scales



Atoms (experiments):
C.A. Regal (2003)
M.W. Zwierlein (2003)
M. E. Gehm (2003)
J. T. Stewart (2007)

Nuclei (theory):
U. van Kolck (1999)
S. König (2017)

Hypernuclei (theory):
H.-W. Hammer (2001)
L.C. (2018)

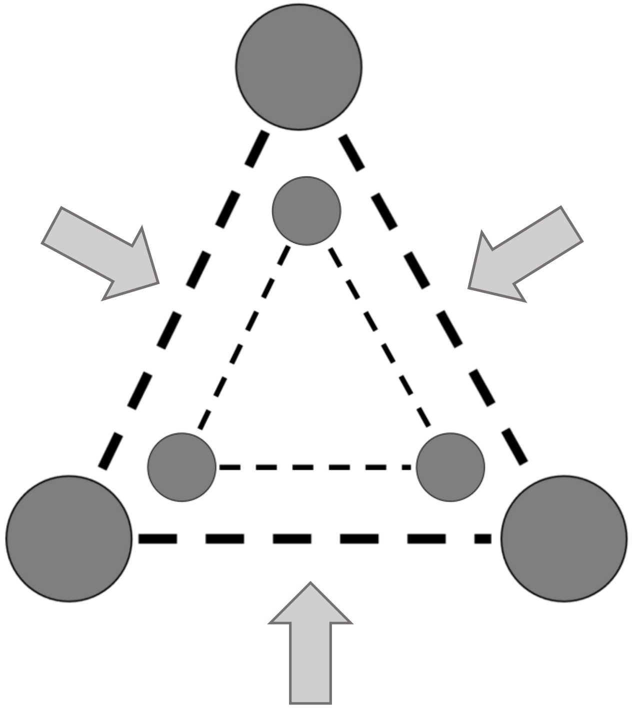
Hadrons (theory):
E. Braaten et al (2003)

Discrete scale invariance: 3+ body

L. H. Thomas (1935)
G. Skorniakov and
K. Ter-Martirosian (1957)

One of the most known and fascinating consequence of unitarity is the

Thomas Collapse / Efimov Effect



In the **unitary limit** a system of **3 bosons/distinguishable particles collapses**

$$r_0 \to 0 \implies E_3 \propto -\left|\frac{1}{r_0^2}\right|$$

A repulsion is needed to **stabilize the system to a finite energy $\boldsymbol{E_3}$**.
$\boldsymbol{E_3}$ **breaks the scale invariance** of the system!
i.e. you have to choose the scale of your system (K, eV, MeV …)

Simple and intuitive: Contact theory

- Treat **particles as degrees of freedom** (elementry particles)
- They can interact only **short-range**

  (Short range structure is irrelevant: no quark structure)

  (Long range interactions are negligible: no pion exchange)

- Works for a limited set of energies

- Easy to understand
- Clear limitations
- Expandible
- Minimal inputs required
- Universally transposable

Simple and intuitive: Contact theory

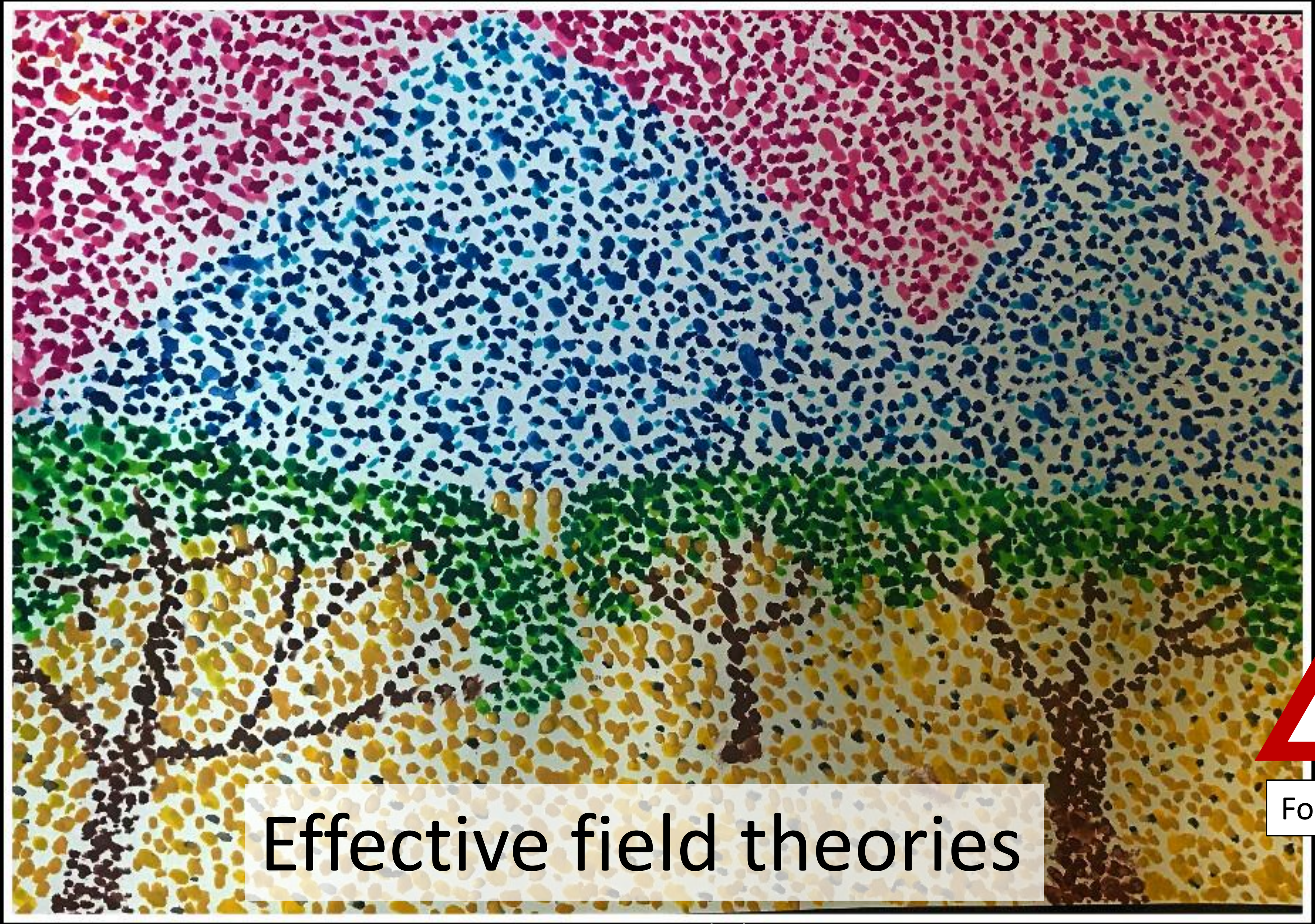
- Treat **particles as degrees of freedom** (elementary particles)
- They can interact only **short-range**

  (Short range structure is irrelevant: no quark structure)

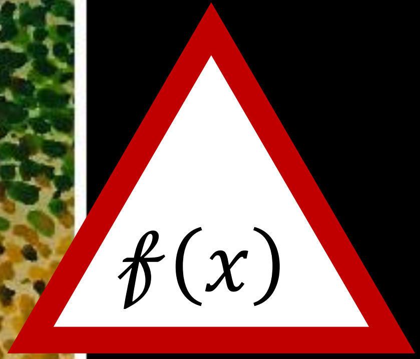
  (Long range interactions are negligible: no pion exchange)

- Works for a limited set of energies
- Tricky to be properly implemented
- Clear limitations only in the known cases
- Not trivial to be practically expanded beyond 1st order
- Minimal inputs required at the first orders

- Easy to understand
- Clear limitations
- Expandible
- Minimal inputs required
- Universally transposable



Effective field theories



Formulas ahead

A complete theory

$r_{ij} = r_i - r_j$

$$V(r_{ij}) = \delta(r_{ij})$$

Contact theory formally:

$$L = N^\dagger \left(i\partial_0 + \frac{\hbar^2}{2m} \nabla^2 \right) N \; - C_0 \, N^\dagger N^\dagger N N$$

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Contact theory formally:

$$L = N^\dagger \left(i\partial_0 + \frac{\hbar^2}{2m} \nabla^2 \right) N - C_0 N^\dagger N^\dagger N N$$

$$L^{N^{>0}LO} = C_2 \left(N^\dagger \nabla^2 N N^\dagger N + h.c. \right) + C_{11} \left(N^\dagger \vec{\nabla} N N^\dagger \vec{\nabla} N \right) + C_4 \left(N^\dagger \nabla^4 N N^\dagger N + h.c. \right) + \ldots D_0 \left(N^\dagger N^\dagger N^\dagger N N N \right) + E_0 \left(N^\dagger N^\dagger N^\dagger N^\dagger N N N N \right) + \ldots$$

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$$\begin{aligned} L^{N^{>0}LO} = \quad & C_2 \left(N^\dagger \nabla^2 N\, N^\dagger N + h.c. \right) + C_{11} \left(N^\dagger \vec{\nabla} N\, N^\dagger \vec{\nabla} N \right) + \\ & C_4 \left(N^\dagger \nabla^4 N\, N^\dagger N + h.c. \right) + \ \dots \\ & D_0 \left(N^\dagger N^\dagger N^\dagger N N N \right) + E_0 \left(N^\dagger N^\dagger N^\dagger N^\dagger N N N N \right) + \ \dots \end{aligned}$$

Including all the derivative/many-body operators one can **express any interaction**

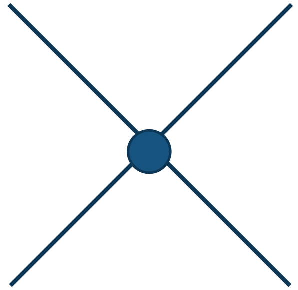
A complete theory

$r_{ij} = r_i - r_j$

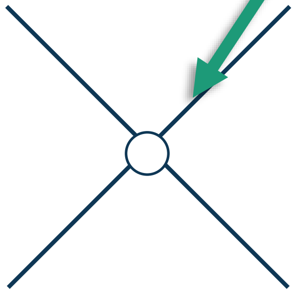
$$V(r_{ij}) = \delta(r_{ij})$$

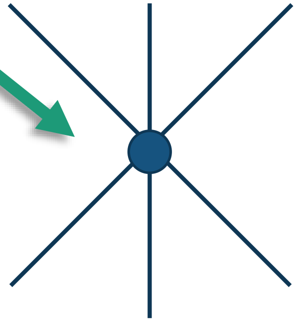
Contact theory formally:

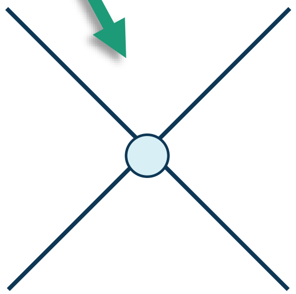
$$L = N^{\dagger}\left(i\partial_0 + \frac{\hbar^2}{2m}\,\nabla^2\right)N \;-\; C_0\,N^{\dagger}N^{\dagger}NN$$



$$L^{N^{>0}LO} = \quad C_2\left(N^{\dagger}\nabla^2 N\,N^{\dagger}N + h.c.\right) + C_{11}\left(N^{\dagger}\vec{\nabla}\,N\,N^{\dagger}\vec{\nabla}N\right) +$$
$$C_4\left(N^{\dagger}\nabla^4 N\,N^{\dagger}N + h.c.\right) + \;\ldots$$
$$D_0\left(N^{\dagger}N^{\dagger}N^{\dagger}NNN\right) + E_0\left(N^{\dagger}N^{\dagger}N^{\dagger}N^{\dagger}NNNN\right) + \;\ldots$$

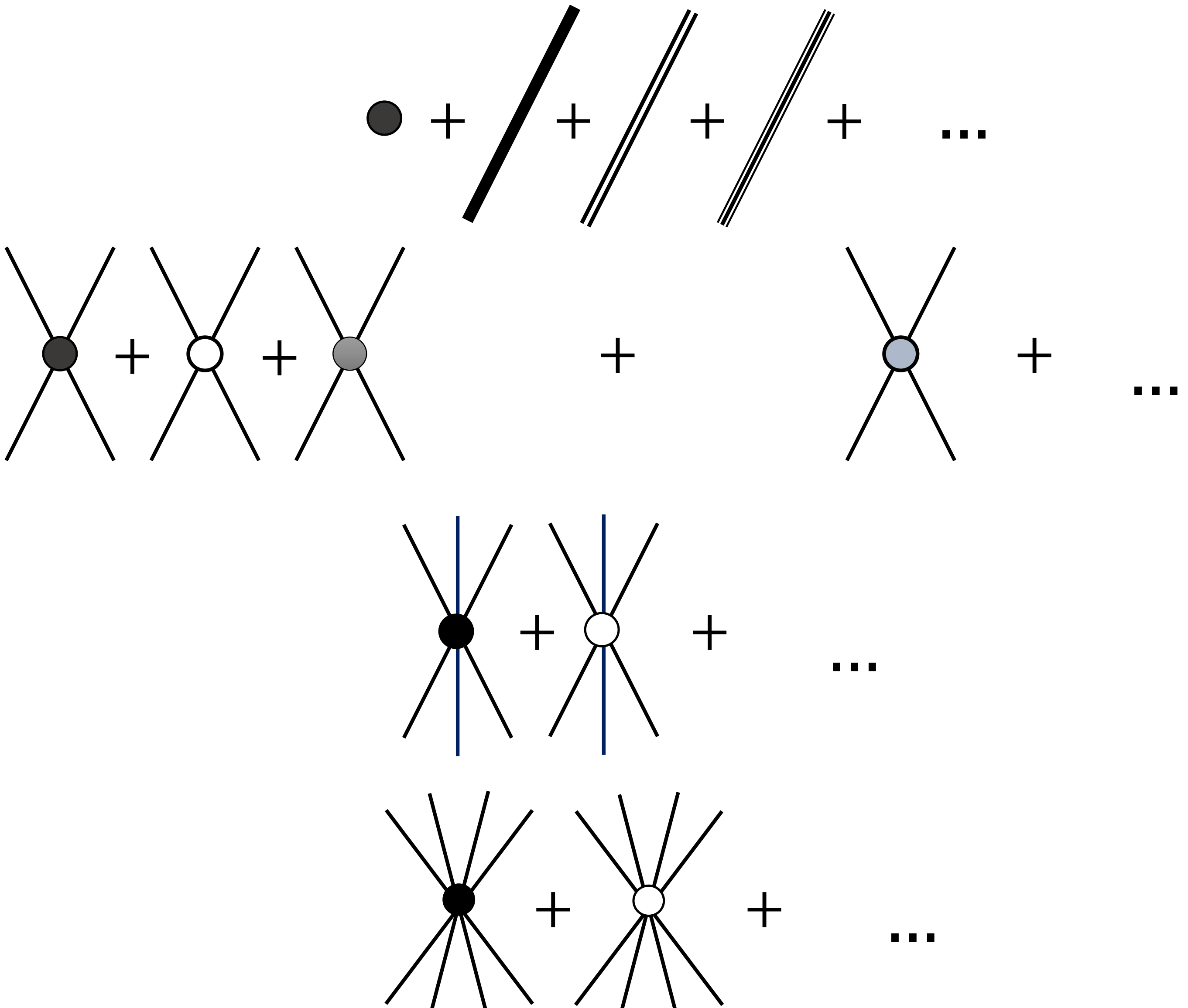






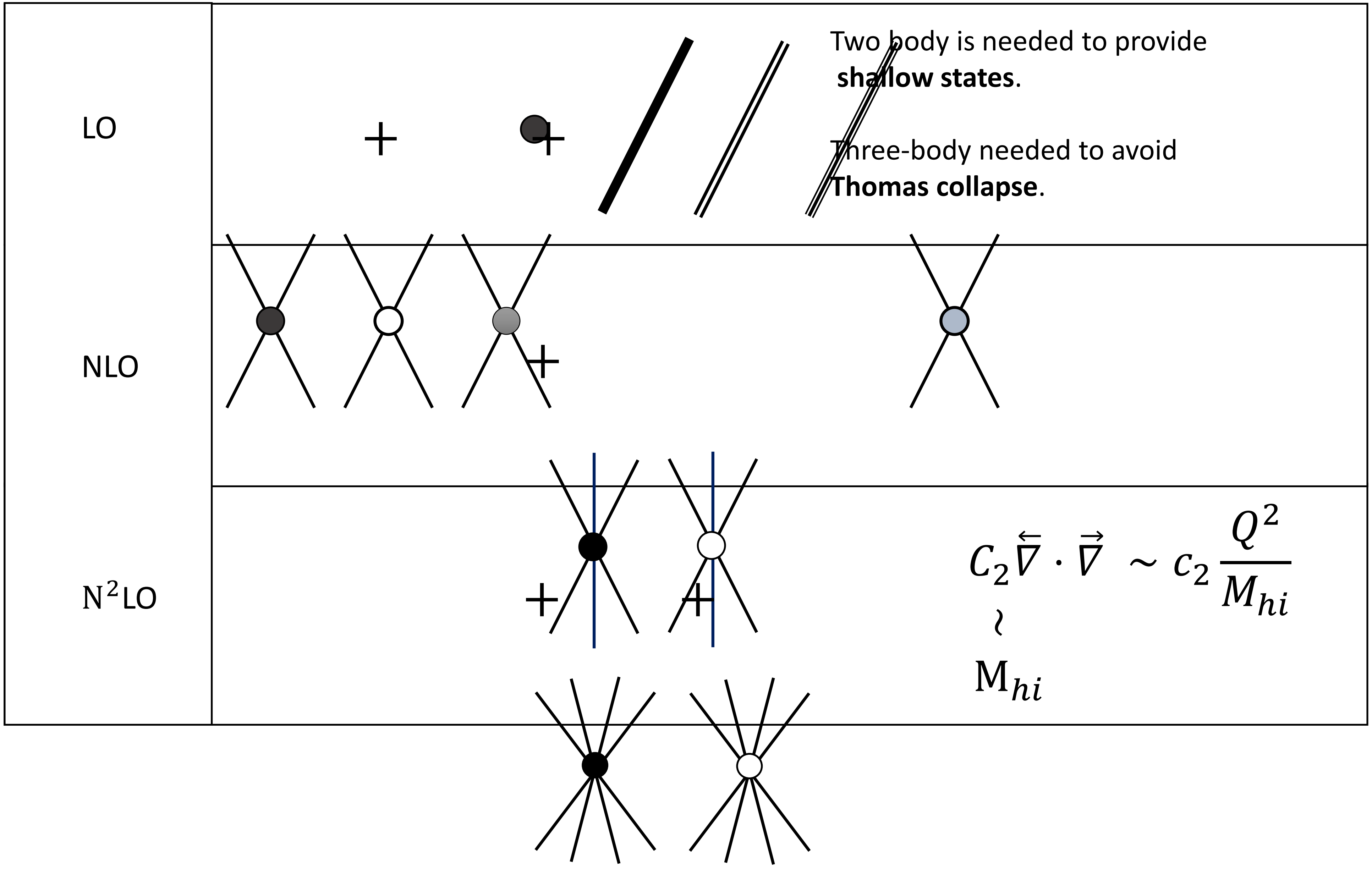
Including all the derivative/many-body operators one can **express any interaction**

Pionless EFT powercounting



U. Van Kolck - Progress in Particle and Nuclear Physics 43, 1999, 337-418

Pionless EFT powercounting

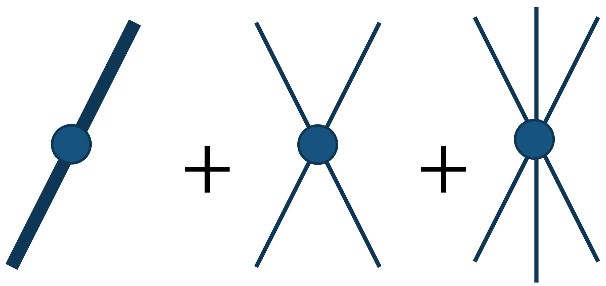


Pionless EFT powercounting

In the nuclear case: $\Gamma_{NN} = \frac{Q}{m_\pi} = 0.5 \sim 0.8$

Momentumless 2-3 body

LO



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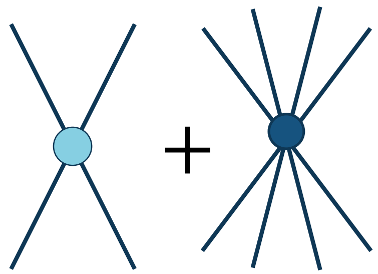
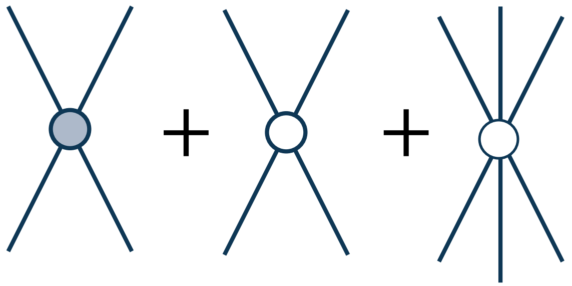
Momentum dependent / 4-body

NLO



$O(\Gamma)$

N^2LO



$O(\Gamma^2)$

$O(\Gamma^{\geq 3})$

G.P. Lepage, How to renormalize the Schrödinger equation (1997)
U. van Kolck, Nucl.Phys. A645 273-302 (1999)
J.-W. Chen, et al. Nucl.Phys. A653 (1999)
S. König, H. W. Grießhammer, H. W. Hammer, and U. van Kolck, J. Phys. G43, 055106 (2016)
B. Bazak, PRL 122.143001 (2019)

Contact Renormalizability

ç

The **Lagrangian** can be transformed into a **Hamiltonian** that may be used in **many-body calculations**

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\, \delta\left(\vec{r_i},\vec{r_j}\right) + \sum_{ijk} D_0\, \delta\left(\vec{r_i},\vec{r_j},\vec{r_k}\right)$$

Regularize the interaction to smear the contact interactions

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C^\lambda\, e^{-\frac{\lambda^2 r_{ij}^2}{4}} + \sum_{ijk} D^\lambda \sum_{cyc}\left[e^{-\frac{\lambda^2\left(r_{ij}^2+r_{ik}^2\right)}{4}}\right]$$

Renormalization fixes the dependence of C_λ and D_λ to observables
C^λ and D^λ fitted on **two- and three-body observables**.

If $\lambda \to \infty$ any observable becomes λ independent

Personal consideration

Approaches to Renormalizability approach

Puristic : the cut-off is a **mathematical entity** and should be taken to infinity.
Cut-off dependence -> intrinsic problems in the theory.

Realistic : the cut-off take a **physical connotation**.
each theory has a right cut-off to be used, usually at the break-down scale.
break-down scale: energy of the lightest not-included particle in the theory.
e.g pion exchange in pionless theory.

Practical : A theory **should be renormalizable**, but practically you use a physical **finite cut-off**.
The cut-off can represent a physical scale or have some intrinsic advantages.
It is usually in agreement with the Puristic approach up to $\sim Q/\lambda$

Q – typical system momentum
λ – cut-off

Duality

universality

Unitary limit: $a_0 = \infty$

$r_0 = 0$

Finite three-body scale: $0 > E_3 > -\infty$

(contact) EFT

(nonrelativistic)

LO

$$\mathcal{L} = N^\dagger \left(\partial_0 + \frac{\nabla^2}{2m} \right) N + $$
$$+ C_0 N^\dagger N^\dagger N N + D_0 N^\dagger N^\dagger N^\dagger N N N$$

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However, no physical system is perfectly in the unitary limit

S. König (2016)

$\mathrm{N}^n\mathrm{LO}$

Physical systems can be close to the limit:
e.g. $|a_{n-n}| = (|-23.|\ \mathrm{fm}) \gg (\mathrm{r}_0 \sim 2.7\ \mathrm{fm})$

Deviation from the universal limit
are needed to predict physical phenomena.

Effective field theory **powercounting**

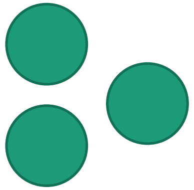
i.e. subleading perturbative corrections
define the specific physical system.

Examples of universality

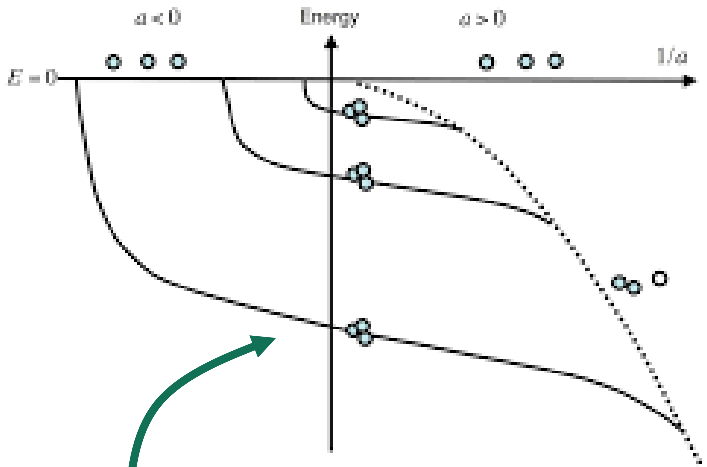
When a E_3 scale is introduced (maintaining the unitary limit):

V. Efimov (1970)

- A **tower of states** appears with universal ratios between them
- All the observables are related **to the new scale only**

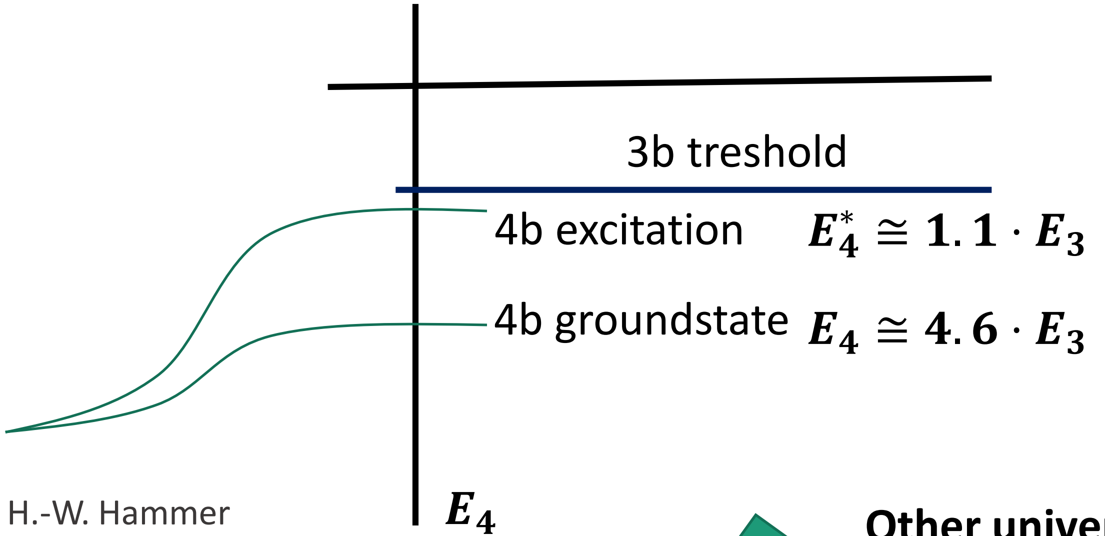


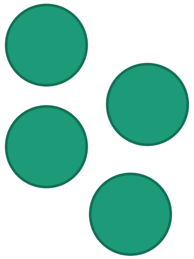
Efimov states



Groundstate fixed by the 3b force

Rajat K. Bhaduri,
American Journal of Physics **79**, 274 (2011)

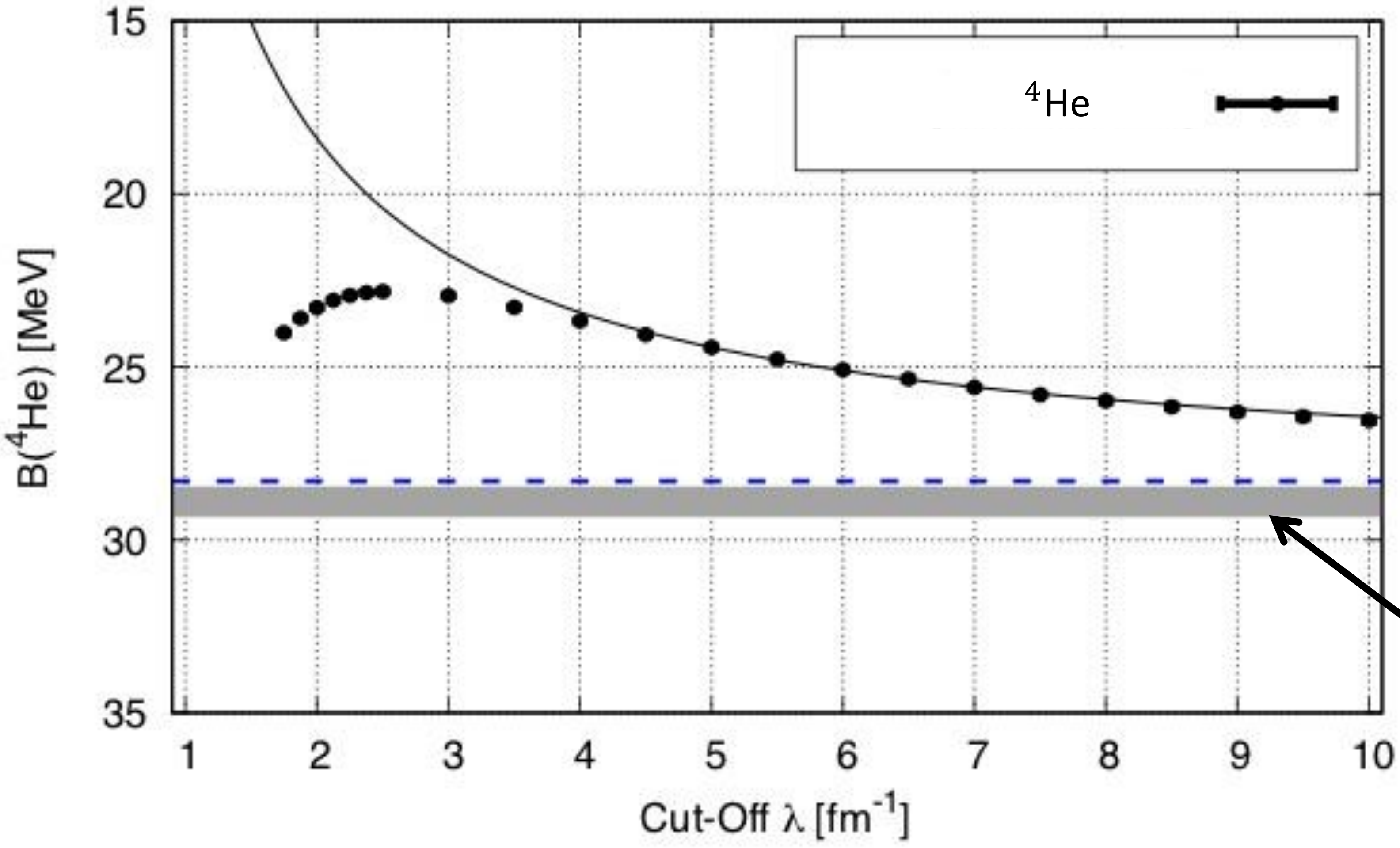




Eric Braaten, H.-W. Hammer
Phys.Rept.**428**: 259-390, 2006
J. Carlson, S. Gandolfi, U. van Kolck, and S. A. Vitiello
Phys. Rev. Lett. **119**, 223002 (2017)
P. F.Bedaque et al. Nuc. Phys. A Volume 714, 589-610
A.C.Phillips Nuc. Phys. A 107, 209-216

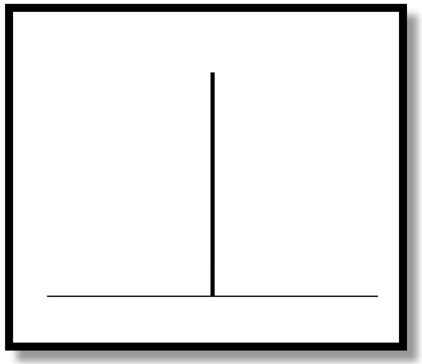
Other universality effects>
Tjon line (nuclear physics)
Bosonic drops: $E_N \propto E_3$
Phillips line

Examples of universality



$$E_\lambda \propto \frac{1}{\lambda}$$

Contact limit

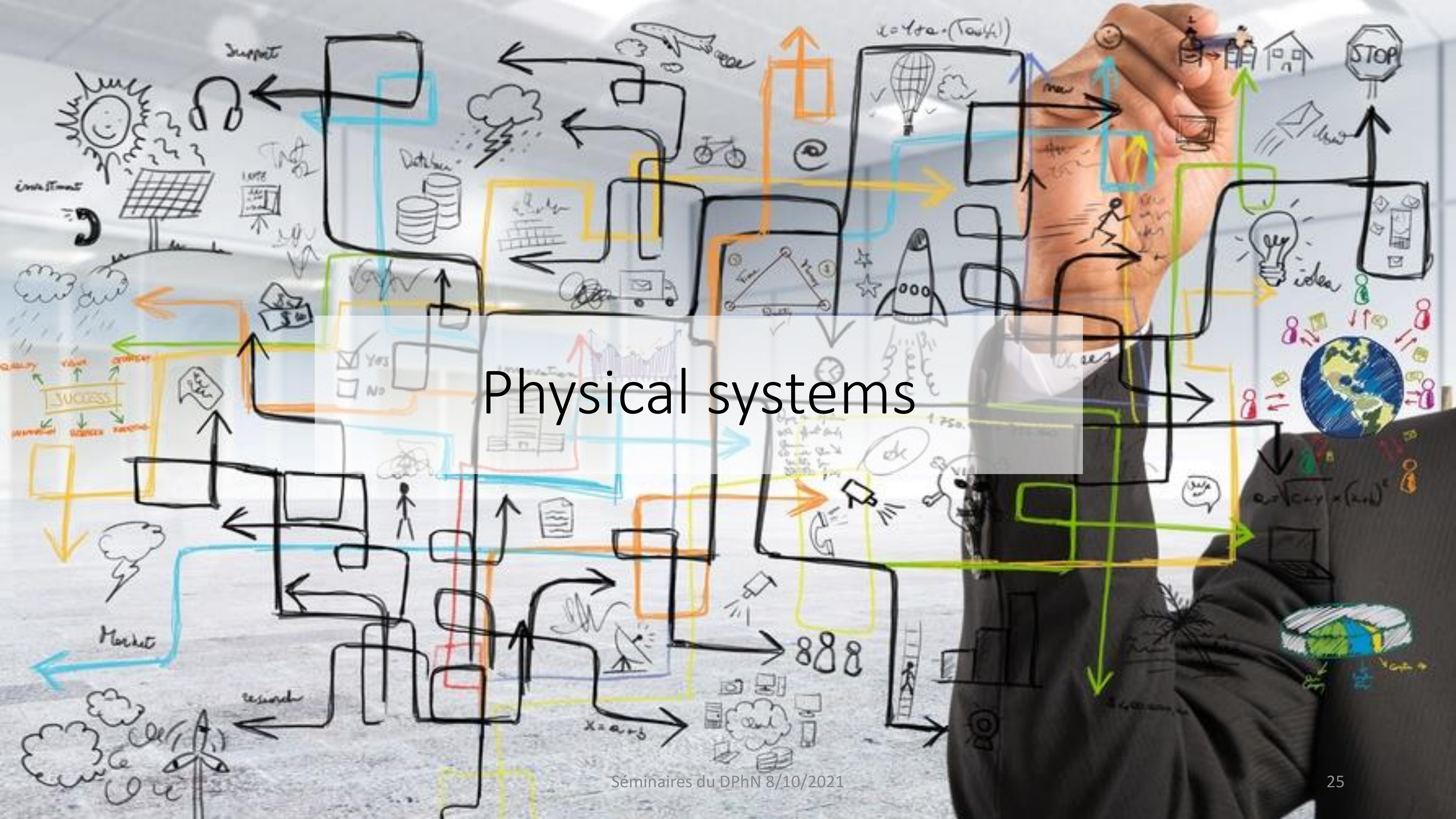


Experimental data

Extrapolation

Calculations done with **few-body stochastic variational diagonalization method**: Y. Suzuki, K. Varga (2003)

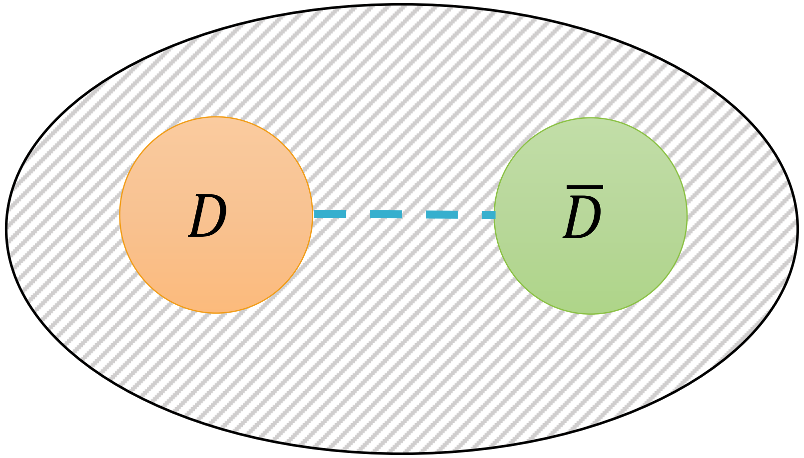
L. C., N. Barnea, and A. Gal Phys. Rev. Lett. **121**, 102502



Physical systems

Hadronic molecules e.g. $X(3872)$

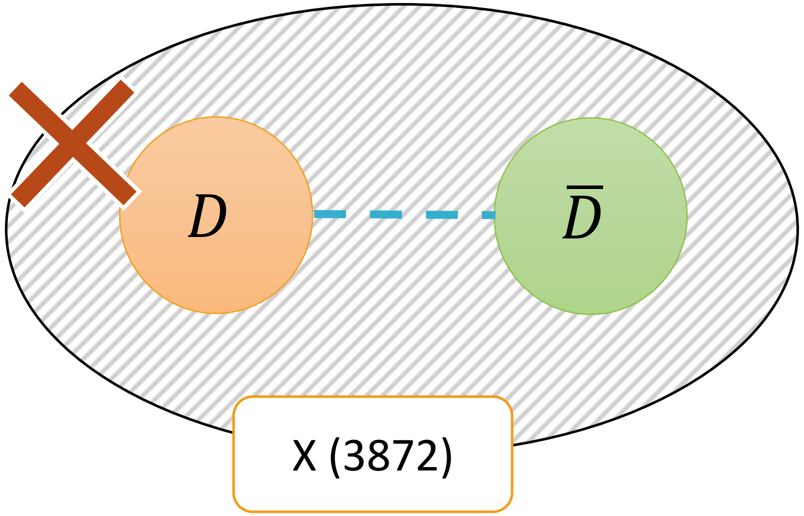
Belle collaboration (2003)
LHCb (2013)



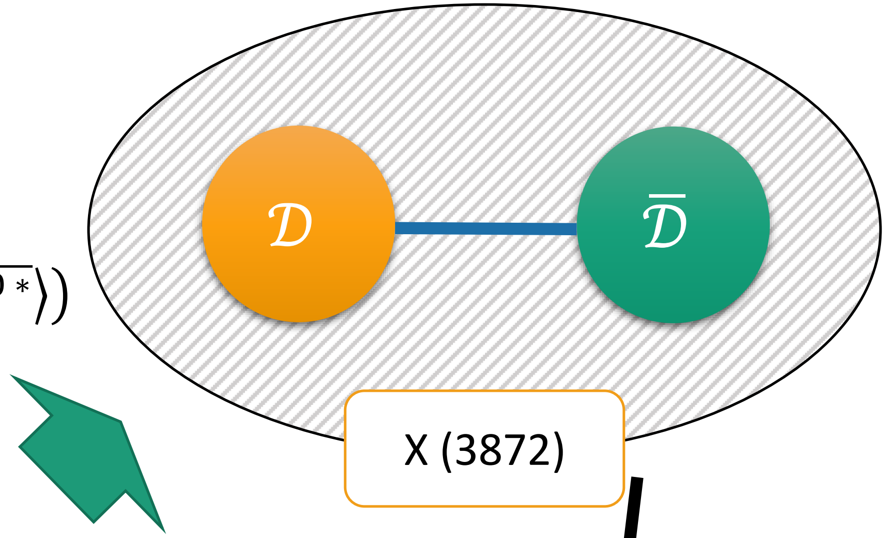
Hadronic molecules
e.g. $X(3872)$

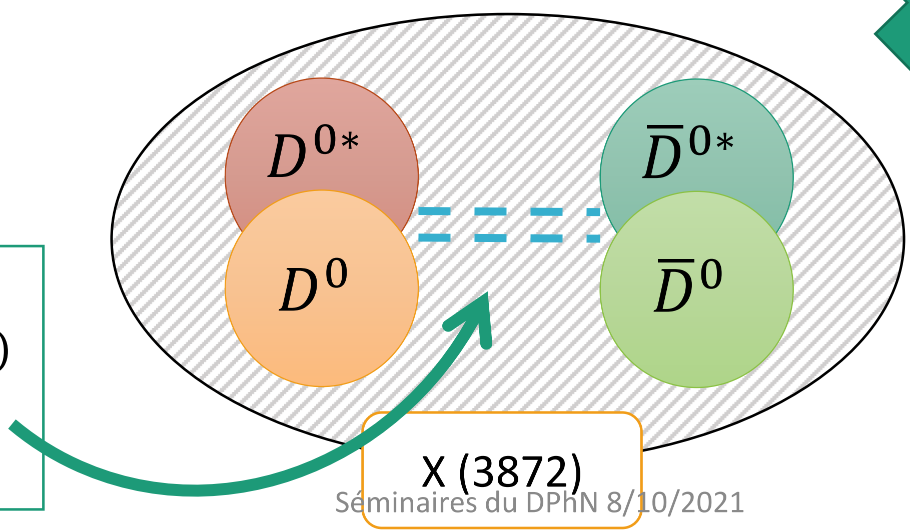
X(3872):
Boson, $J^{PC} = 1^{++}$
Mass ~ 3872.68 MeV
No charge

Belle collaboration (2003)
LHCb (2013)



$$\Psi_X = \phi(r)\frac{1}{\sqrt{2}}\left(|D^{0*}\,\overline{D^0}\rangle + |D^0\,\overline{D^{0*}}\rangle\right)$$





Interaction between states:
$D^{0*} - \overline{D^0}$, $D^0 - \overline{D^{0*}}$ ($J^\pi = 0^+$)
And / or
$D^{0*} - \overline{D^{0*}}$ ($J^\pi = 2^+$)

$B(\mathfrak{D} - \bar{\mathfrak{D}}) \sim 0 \rightarrow X(3872)$
(Unitary limit!)

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$\mathcal{D} - \bar{\mathcal{D}}$ interaction is **unitary**

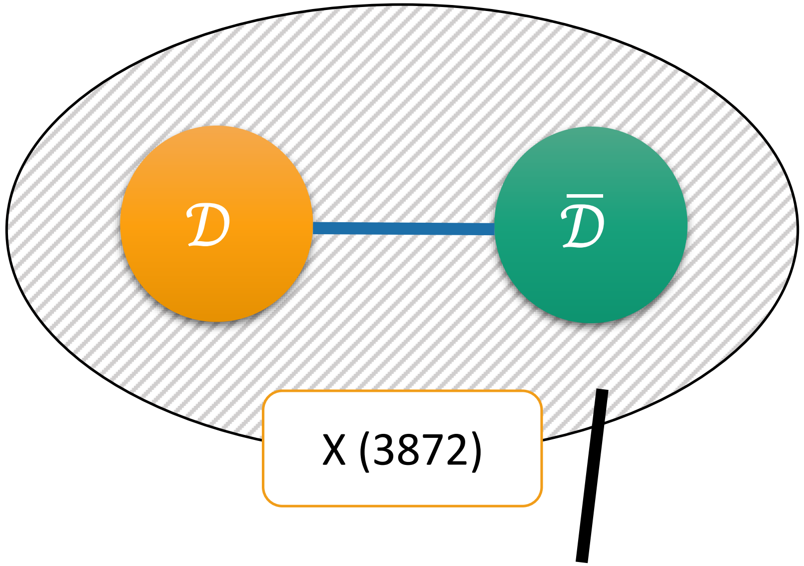
It can be expressed with a **contact theory**

$\nexists$ 3b experiment to fix the three-body scale

We use a **physical cut-off**
(we already know that the theory is renormalizable from **nuclear physics**)

The **range of the interaction** (our cut-off) is between 1 and 2 fm

We **predict** the 2-, 3-, and 4-X systems
(4-, 6-, and 8-body)



$B(\mathfrak{D} - \bar{\mathfrak{D}}) \sim 0 \rightarrow X(3872)$
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We can use contact EFT for this interaction!
We can't fix a three-body scale/system:
we have to take a practical approach

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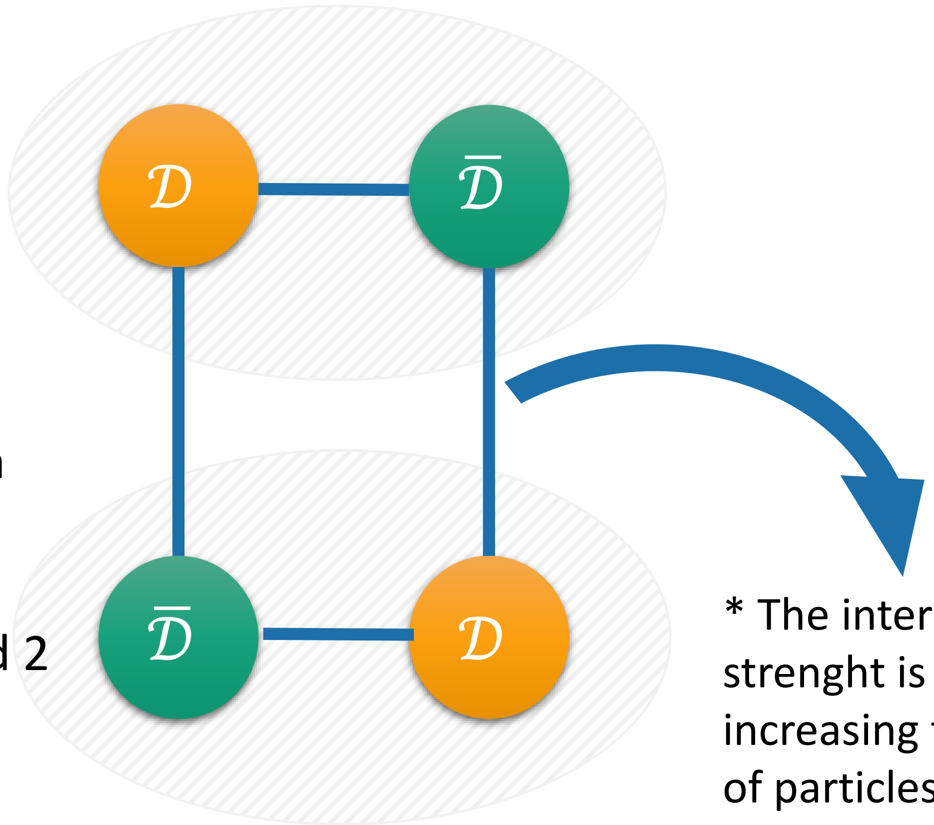

* The interaction strenght is readjusted increasing the number of particles

E. Braaten and M. Kusunoki (2004);
J. Nieves and M. P. Valderrama (2012);

Hadronic molecules e.g. $X(3872)$

Belle experiment (2003)
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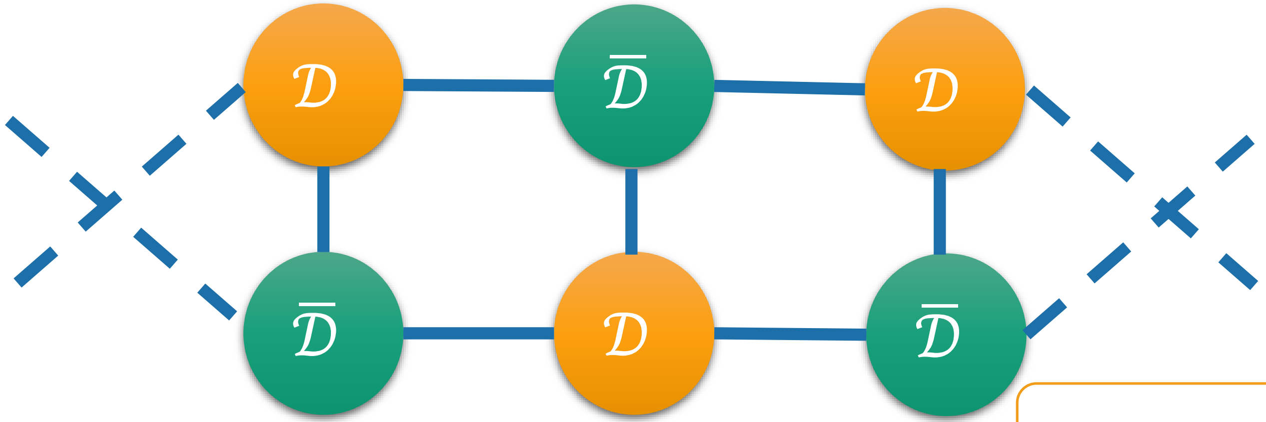
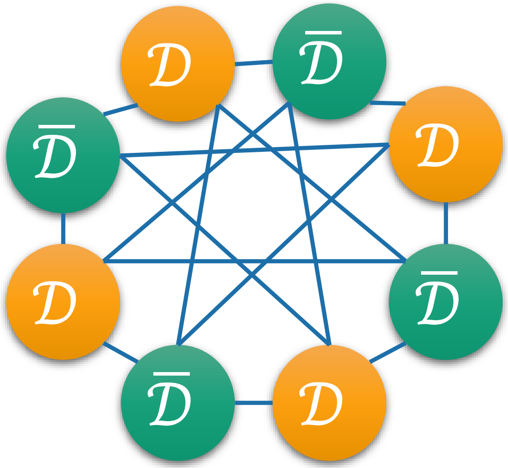


3 X



4 X

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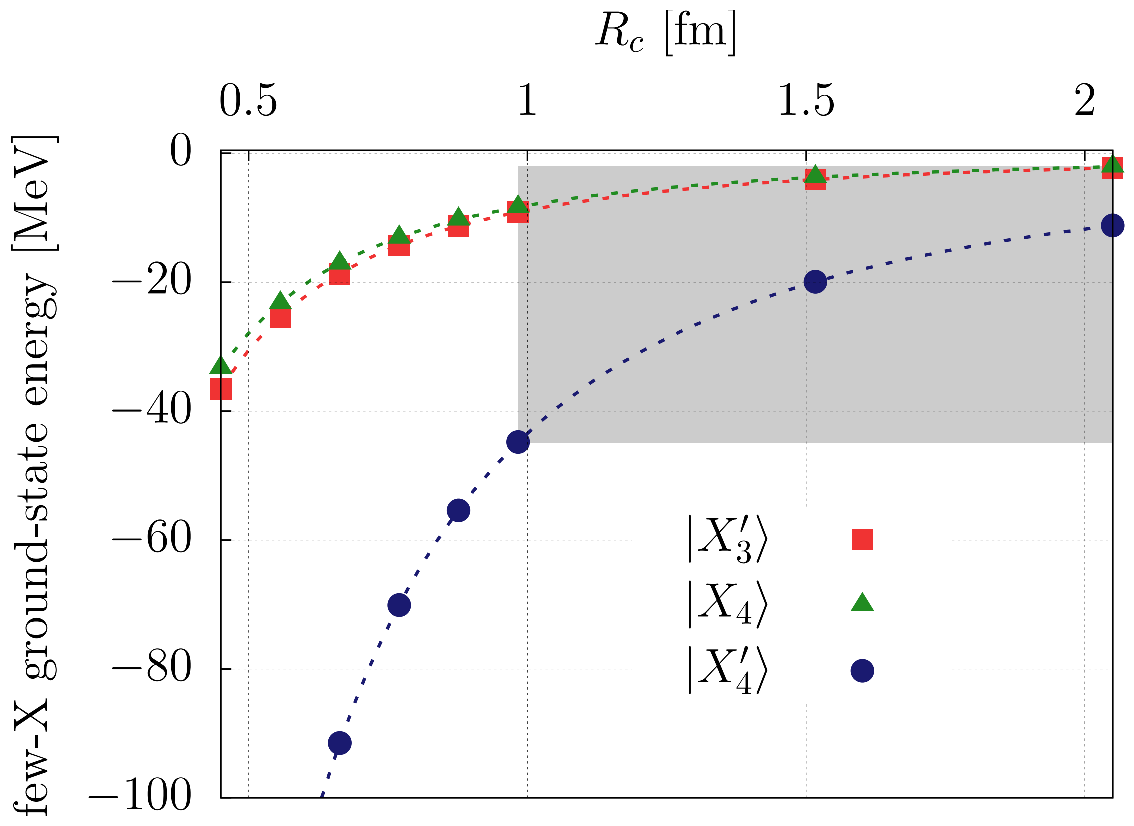
Partial Conclusions

With a simple theory and only knowing

- That $\mathcal{D} - \bar{\mathcal{D}}$ interaction is universal
- The range of such interaction

Predict **bound** $3X$, $4X$
(qualitative prediction)

$2X$ is **uncertain**
(bound only for certain cut-offs)

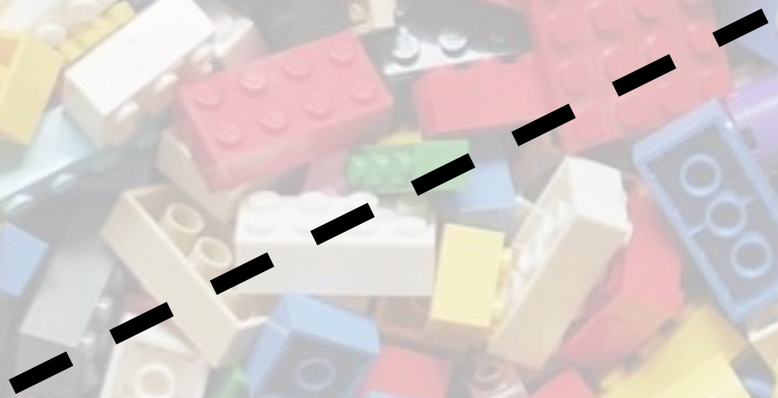


L.C., J. Kirscher, M Pavon Valderrama Phys. Rev. D., e-Print: 2008.12268

This might describe a Brunnian system!

Building a new system in a universality class

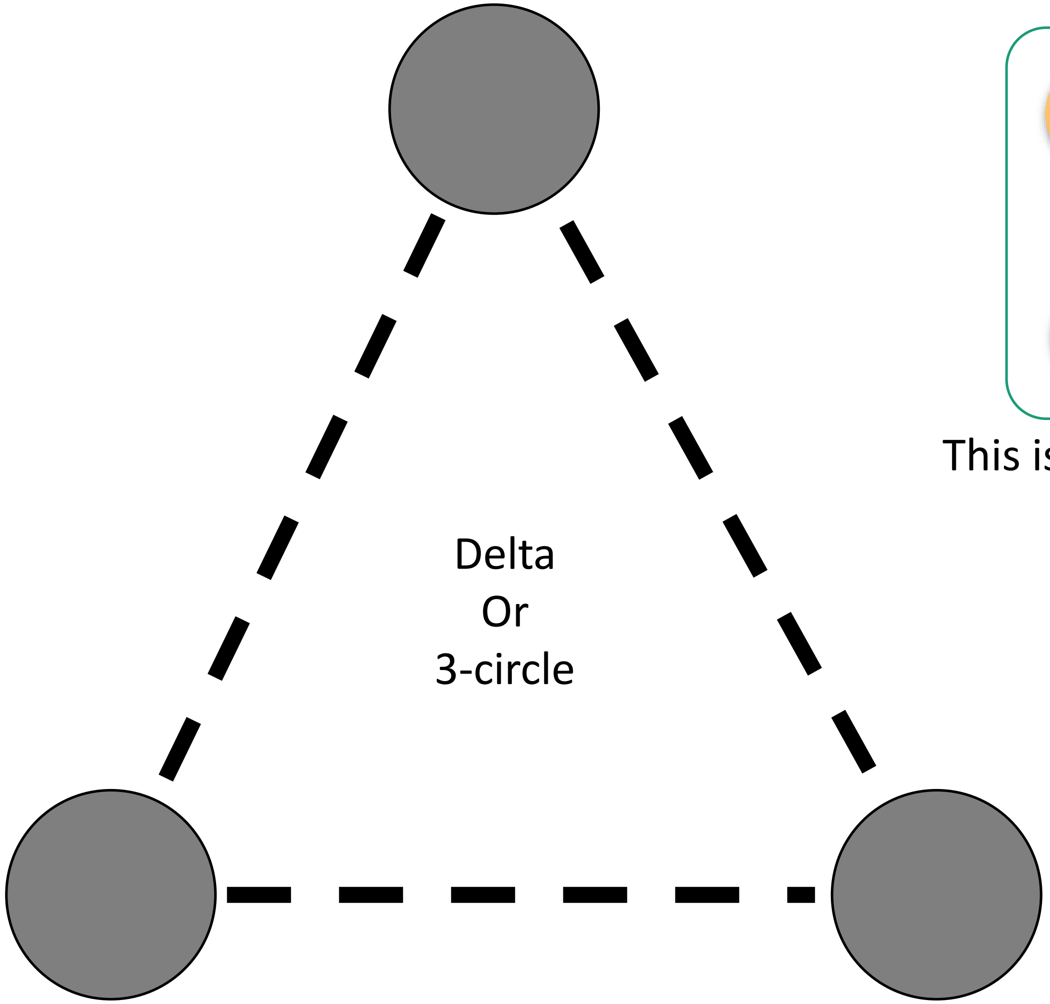
Unitary interactions

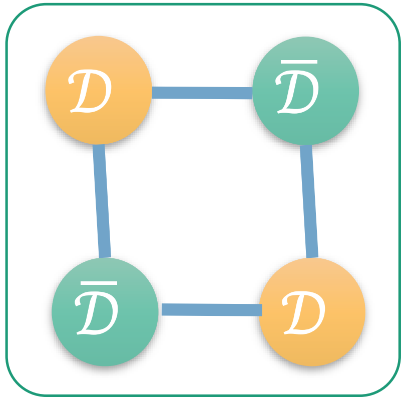


Particles

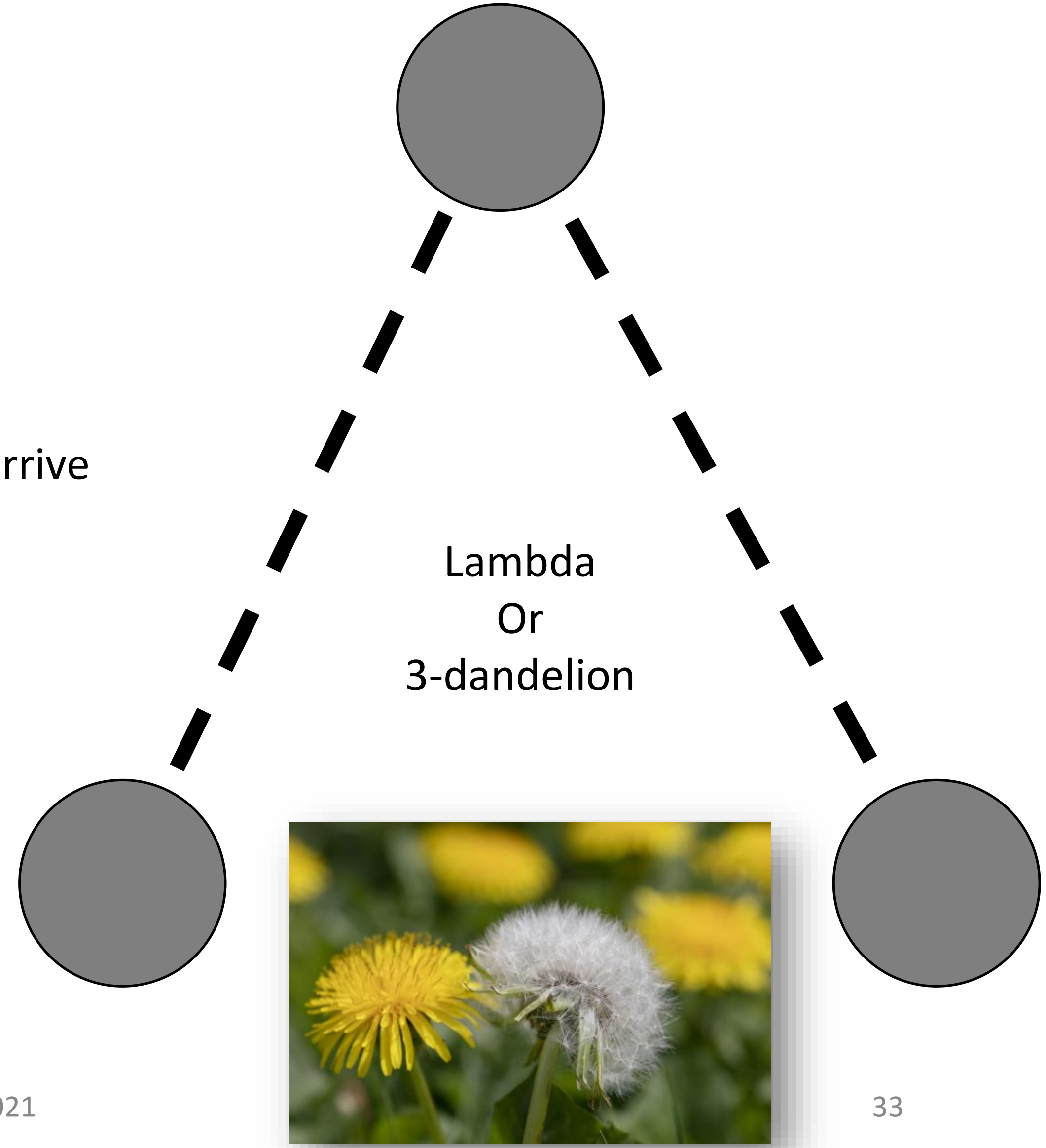
Efimovian U.C. and distinguishable particles

Dashed lines represent **contact unitary interactions**



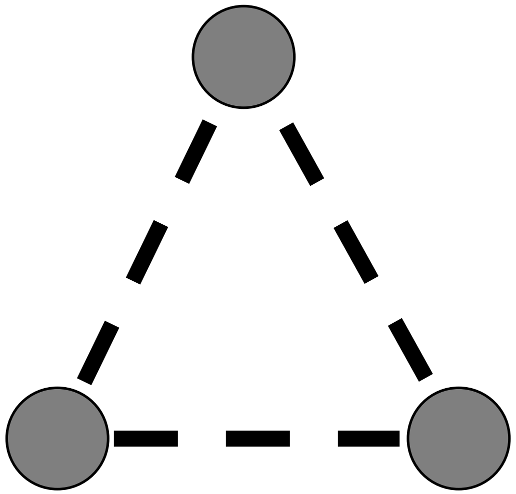


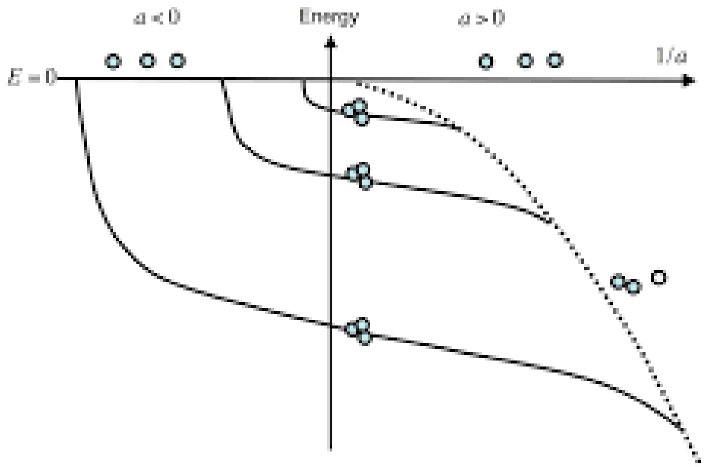
This is where I want to arrive

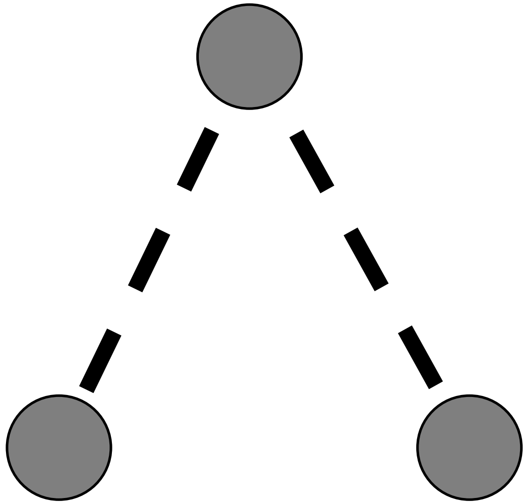


Efimovian U.C. and distinguishable particles

Dashed lines represent **contact unitary interactions**







It is Efimovian

The **Efimov factor is 22.6**
The n-excited state energy is

$$E_n = (22.6)^{2\,n}\, E_0$$

Where E_0 is fixed by the three-body coupling constant

It is also Efimovian

The **Efimov factor is 1986.1**
The n-excited state energy is

$$E_n = (1986.1)^{2\,n}\, E_0$$

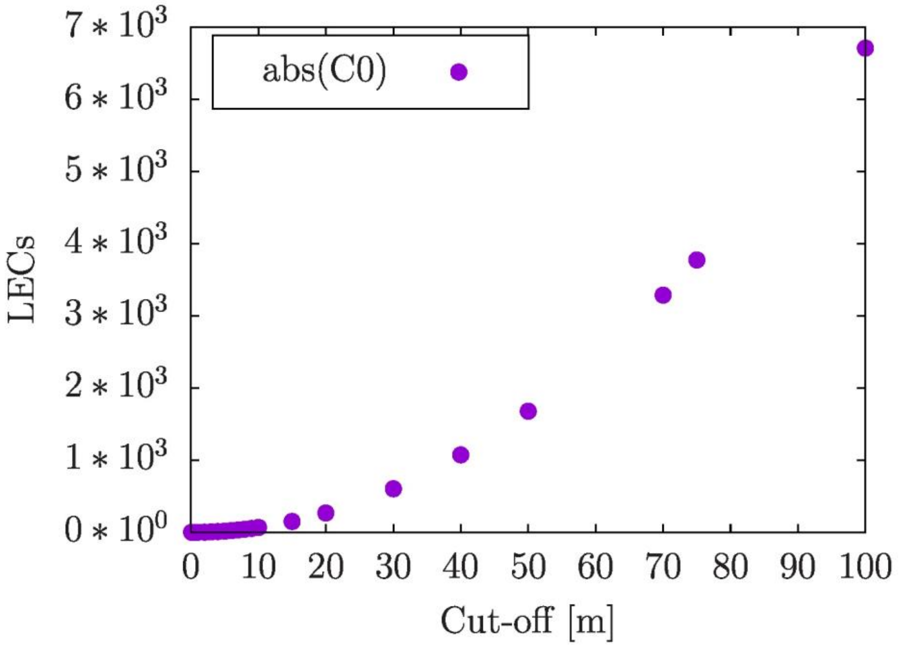
Where E_0 is fixed by the three-body coupling constant

M. T. Yamashita, T. Frederico, and L. Tomio,
AIP Conference Proceedings 884, 129 (2007)

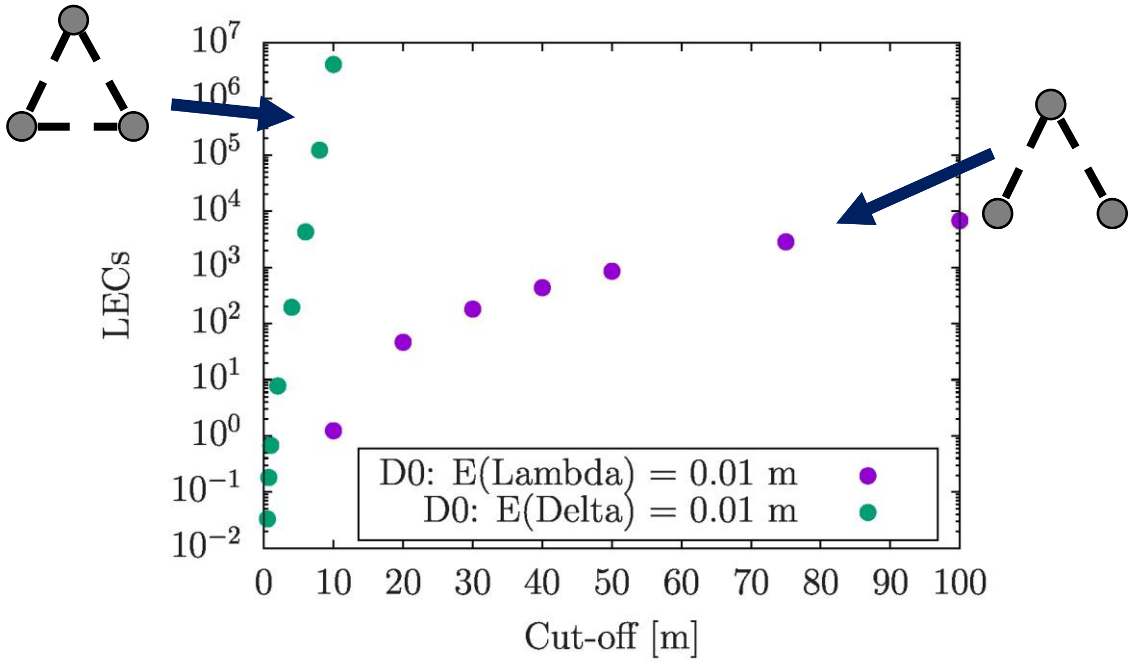
Efimovian U.C. and distinguishable particles

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C^\lambda \, e^{-\frac{\lambda^2 r_{ij}^2}{4}} + \sum_{ijk} D^\lambda \sum_{cyc} \left[e^{-\frac{\lambda^2 \left(r_{ij}^2 + r_{ik}^2\right)}{4}} \right]$$

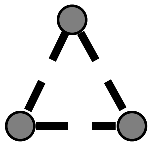
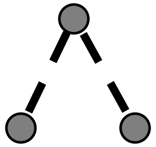
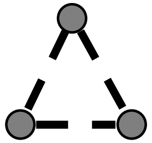
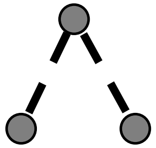
C0 = C^λ fitted to have a unitary two-body system
Regardless the used cut-off.



D0 = D^λ is fitted to have a finite 3-dandelion/circle energy (m is the mass of the particles)

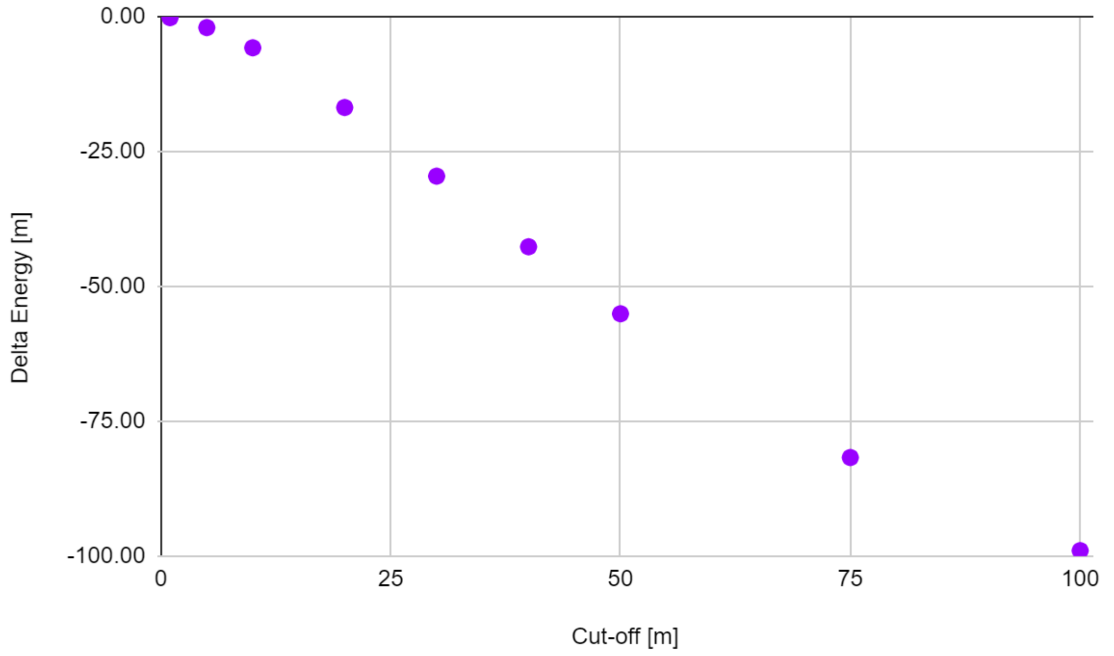


Renormalization

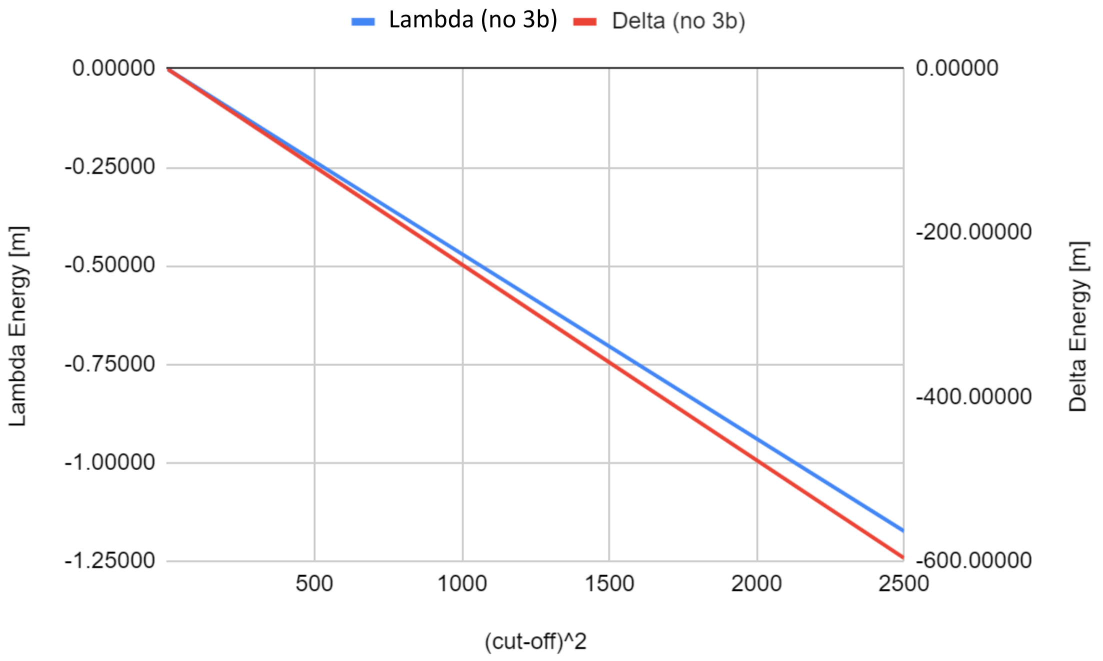
If I use the 3B repulsion that stabilize → On ↓			NO REPULSION AT ALL
	System properly **renormalized** $E_{\Delta}^{\lambda} = 0.01$	Too soft repulsion: $\boldsymbol{E_{\Delta}^{\lambda} \propto \lambda}$ (empirical relation)	Thomas collapse: $\boldsymbol{E_{\Delta}^{\lambda} \propto \lambda^2}$
	Too much repulsion: **System unbound**	System properly **renormalized** $E_{\Lambda}^{\lambda} = 0.01$	Thomas collapse: $\boldsymbol{E_{\Lambda}^{\lambda} \propto \lambda^2}$

Bad renormalization

Using D_Λ^λ that fits $\mathrm{E}^\lambda(\Lambda) = 10^{-2}\ m$ on Δ



Using $D^\lambda = 0$ on Λ and Δ $\left(E^\lambda \propto \lambda^2\right)$

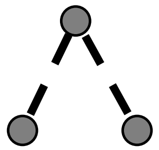


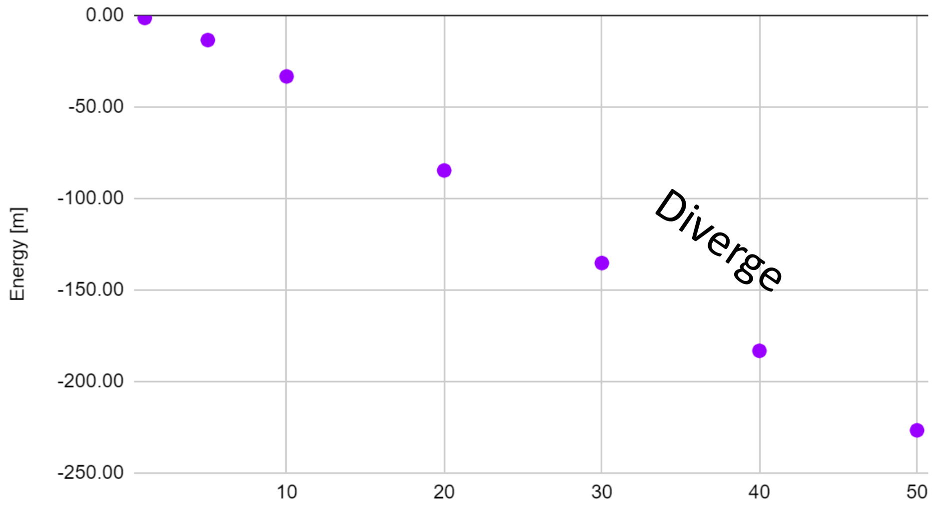
Using D_Δ^λ that renormalizes Δ on Λ is too repulsive and no boundstate is found

Standard 4-b system: Complete

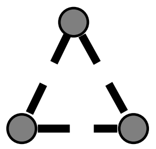
Which three-body force and behaviour does stabilize this system?

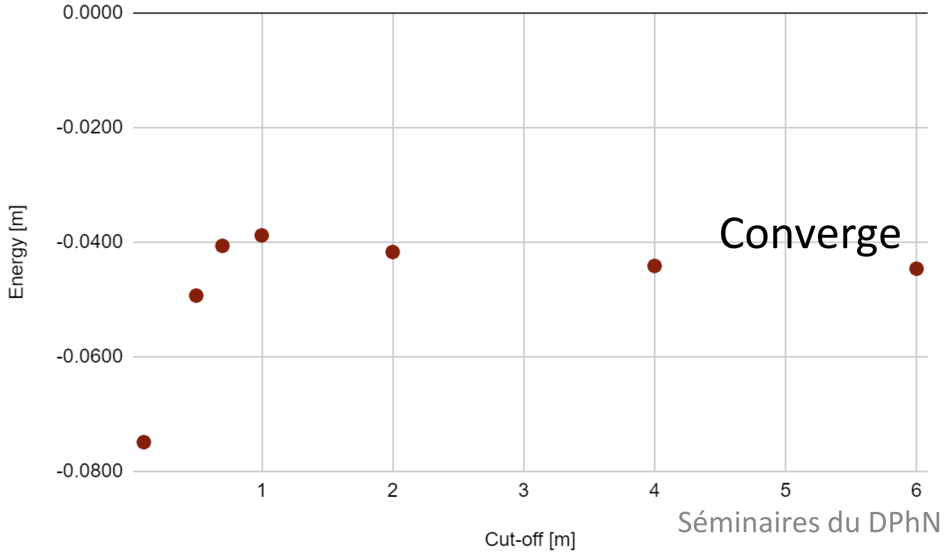
Using D_{Λ}^{λ}
That stabilizes

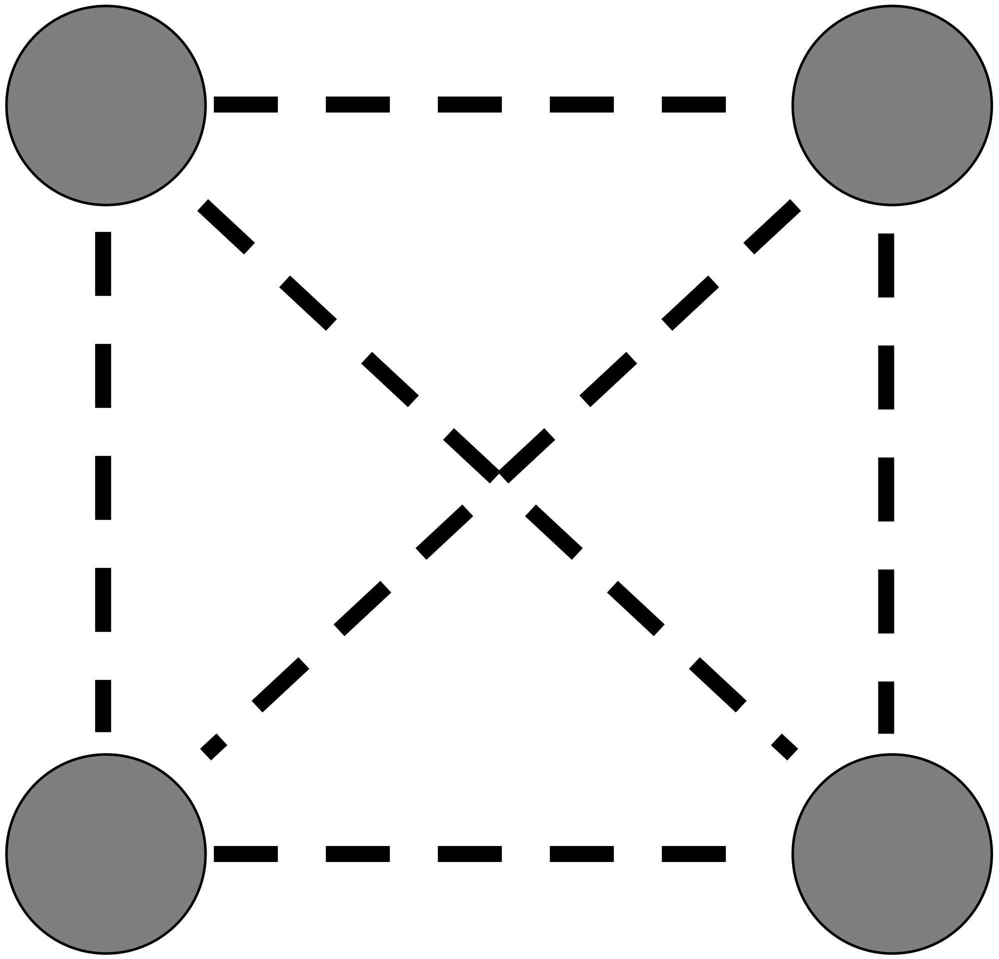




Using D_{Δ}^{λ}
That stabilizes



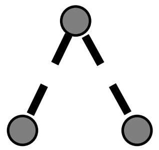


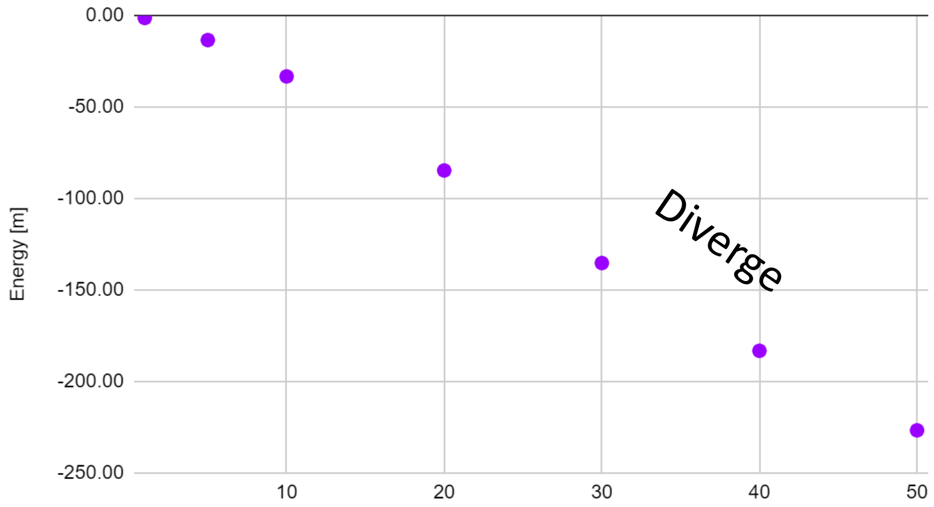


Standard 4-b system: Complete

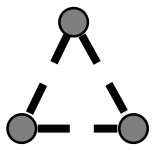
Which three-body force and behaviour does stabilize this system?

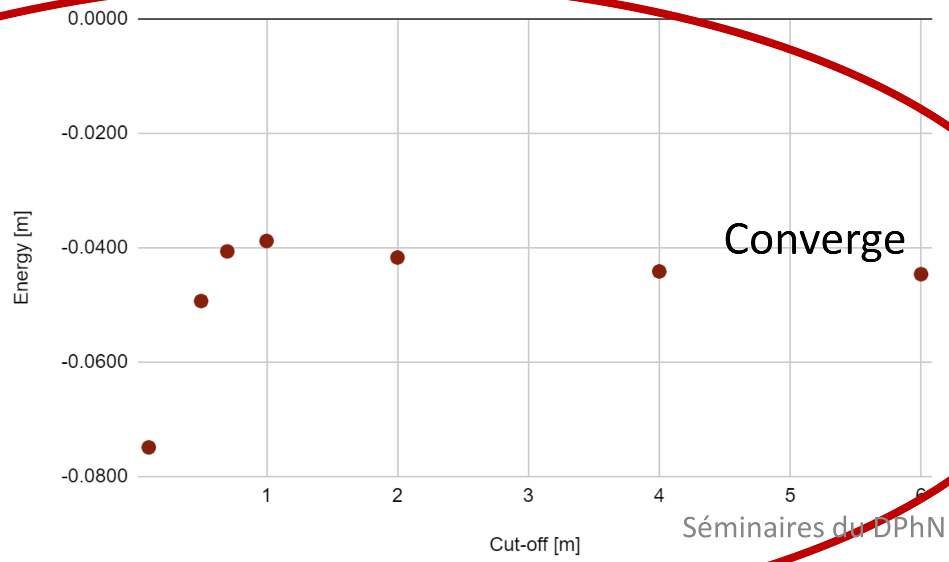
Using D_{Λ}^{λ}
That stabilizes

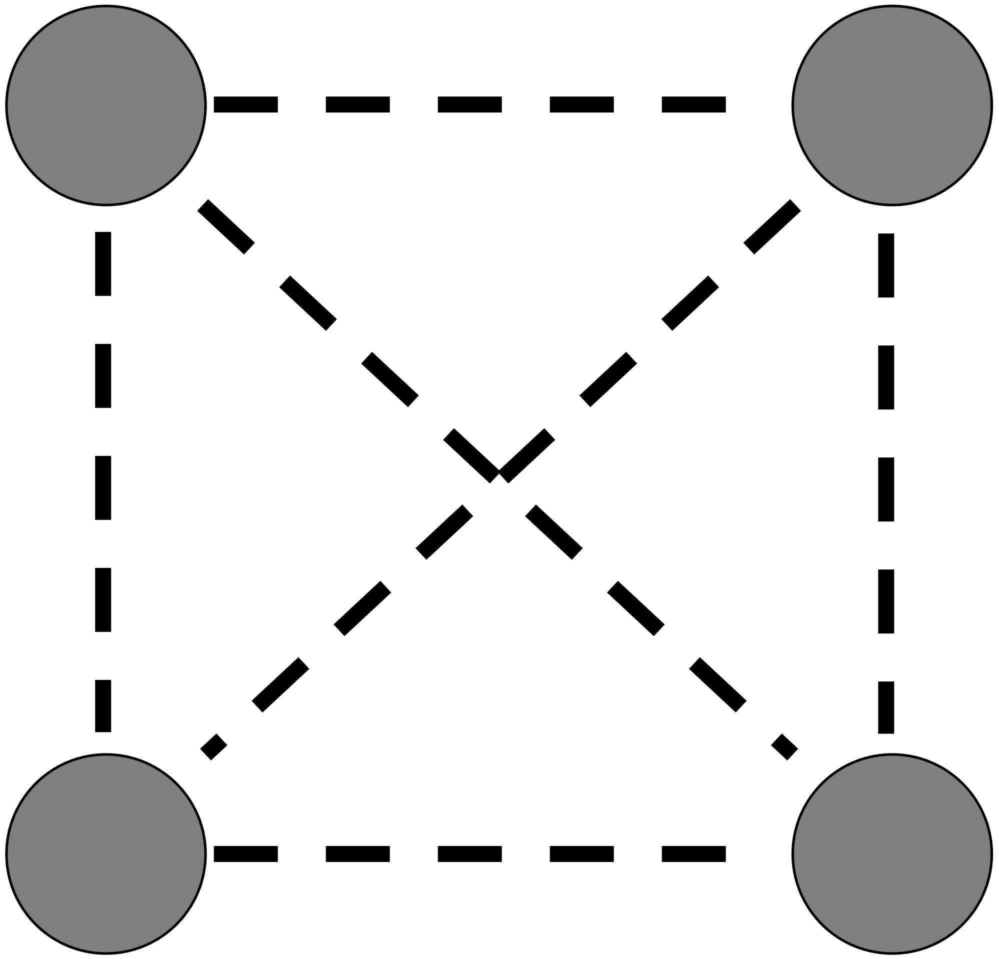




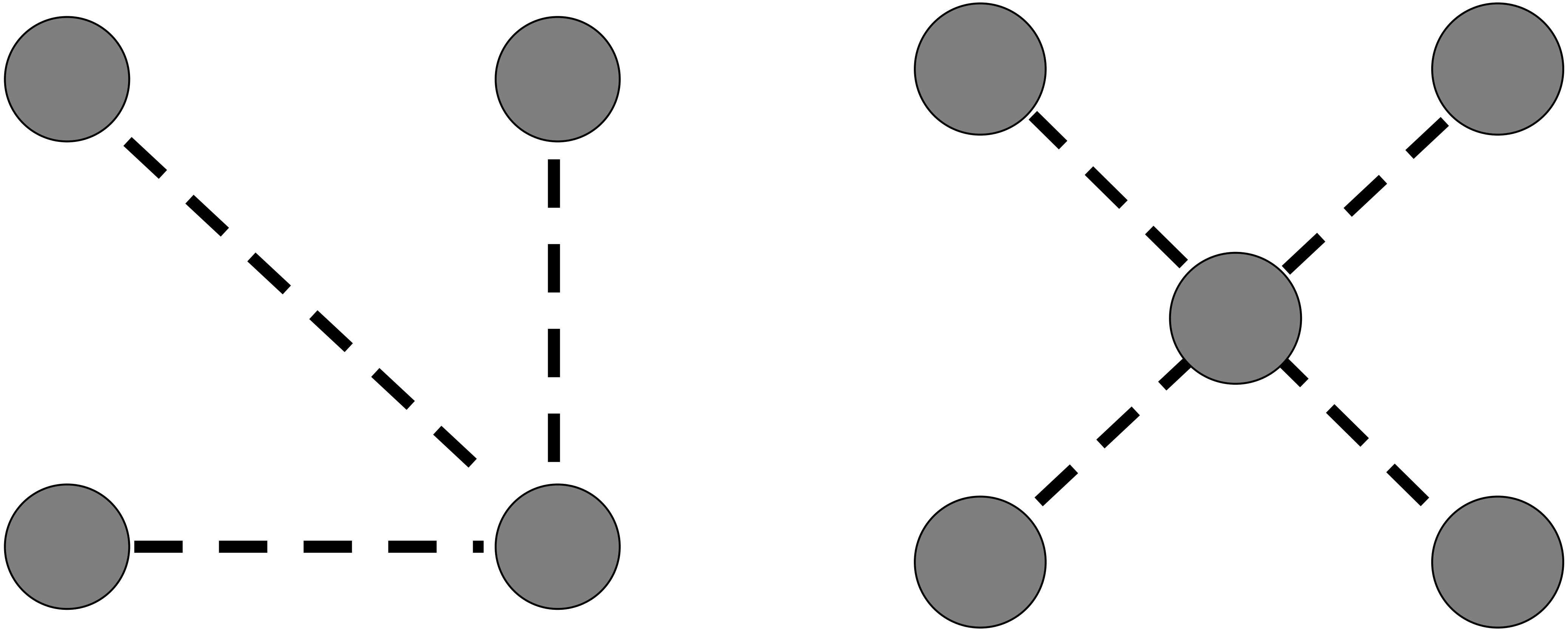
Using D_{Δ}^{λ}
That stabilizes



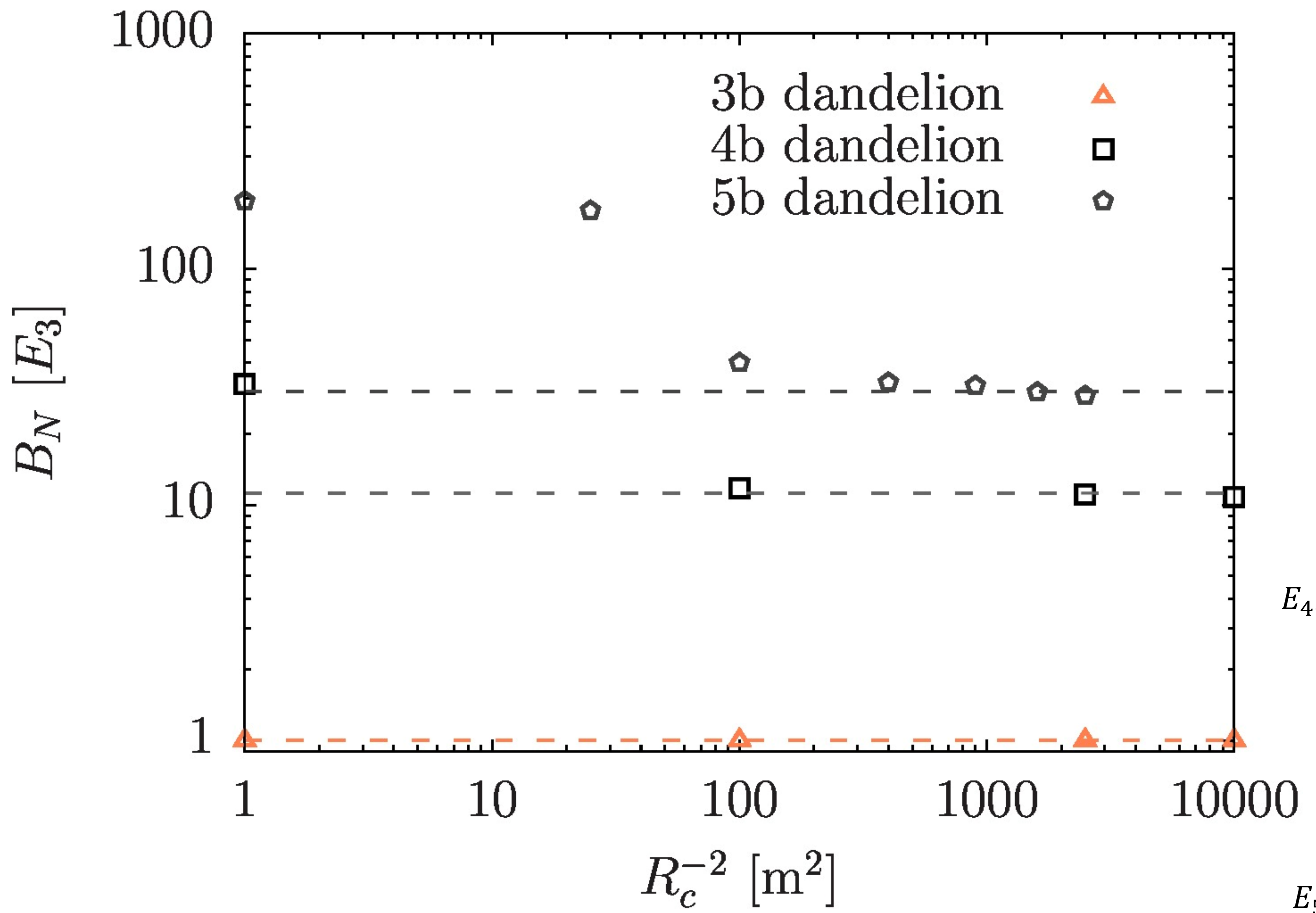




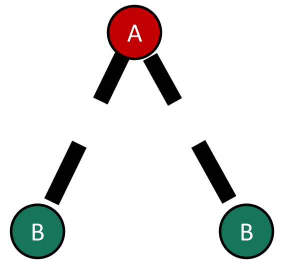
4- /5-Dandelion



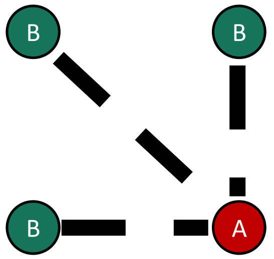
Dandelion convergence



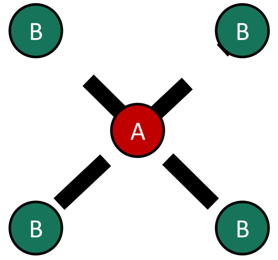
$$R_c = \frac{1}{Cut - off}$$



E_Λ

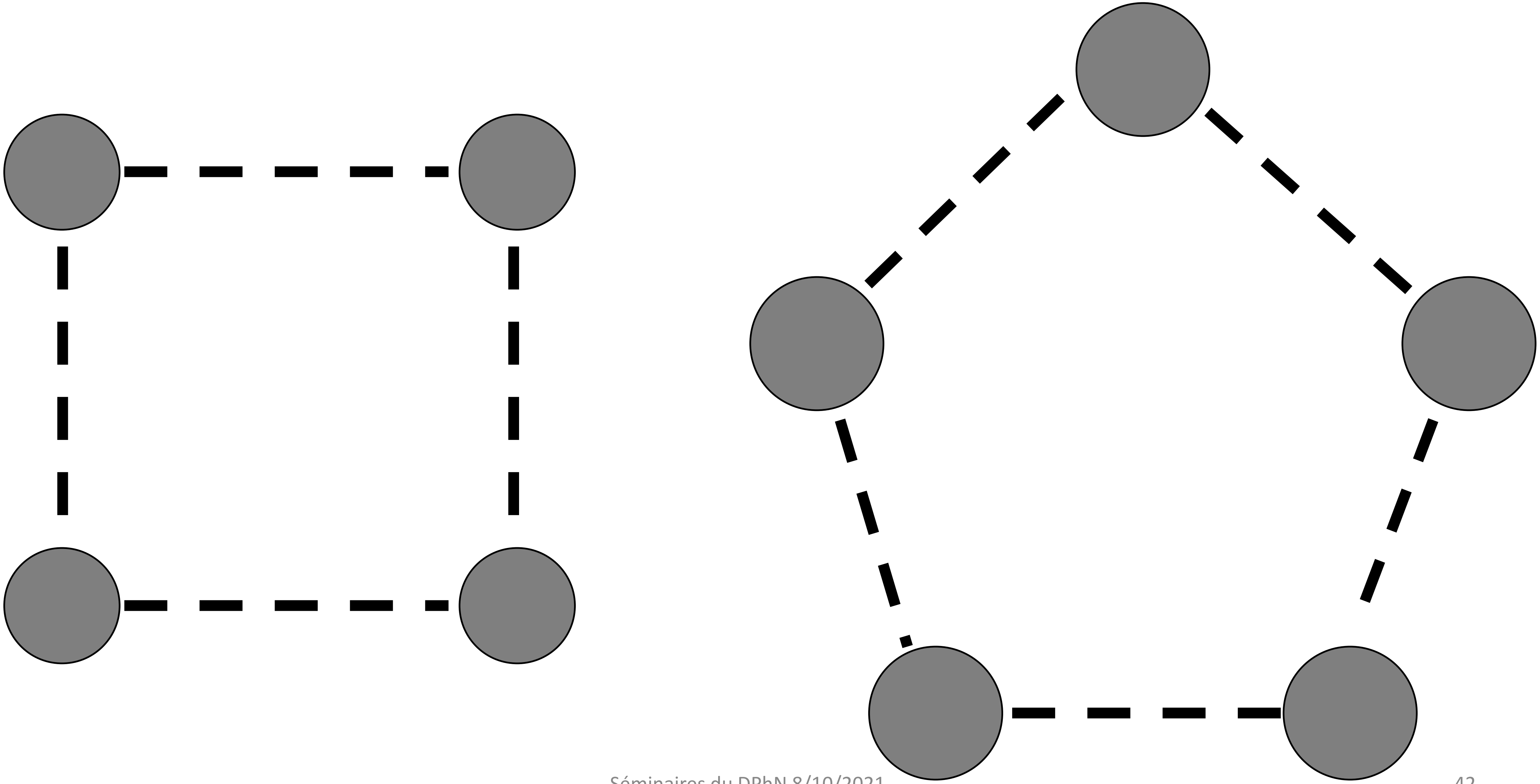


$E_{4-dandelion} = 11(1)E_\Lambda$



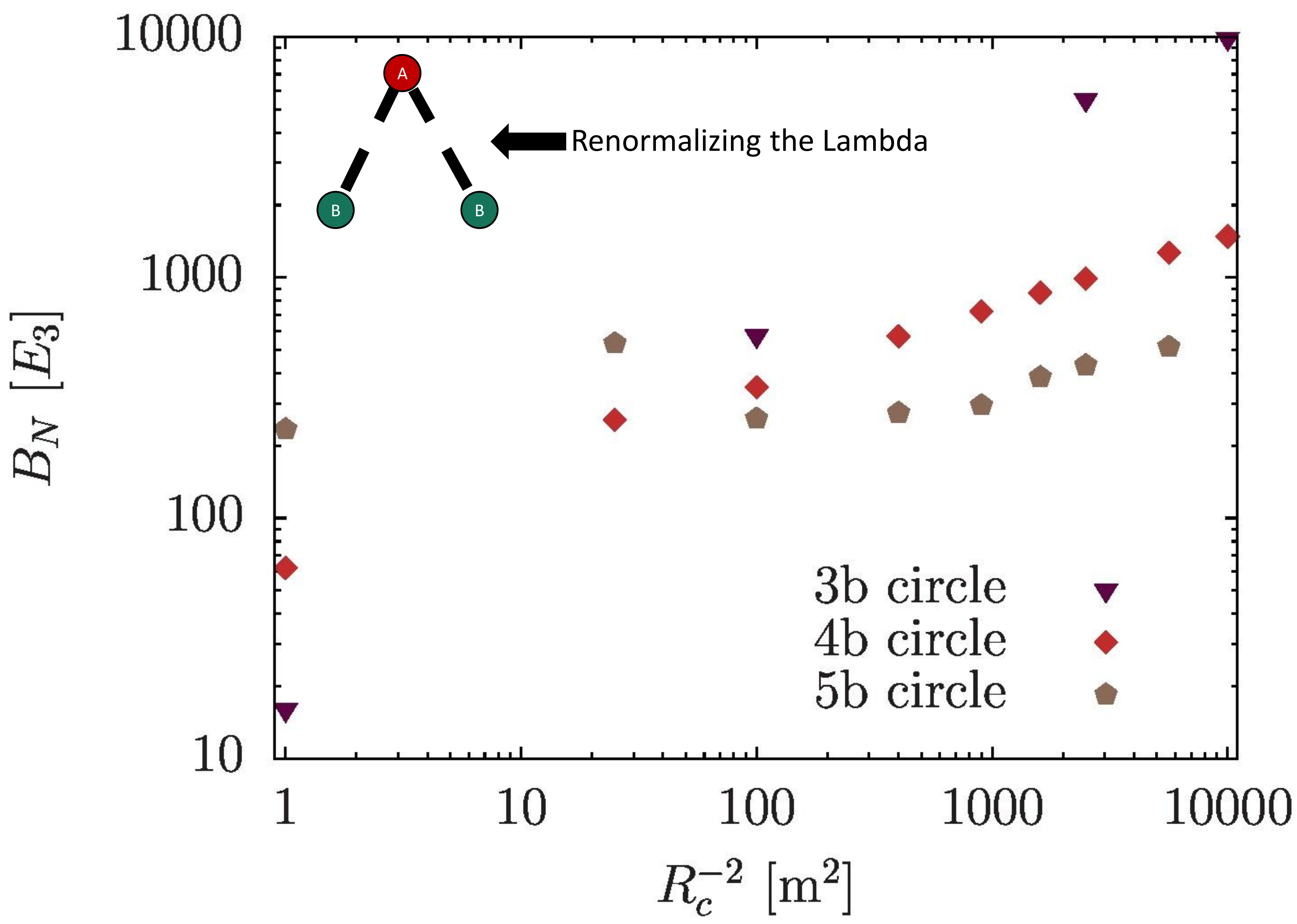
$E_{5-dandelion} = 30\ E_\Lambda$

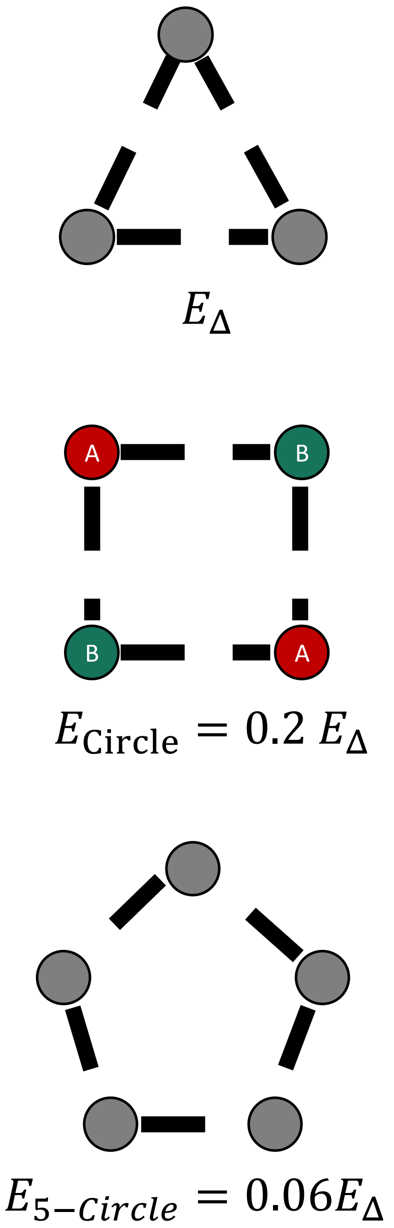
4- / 5-Circle



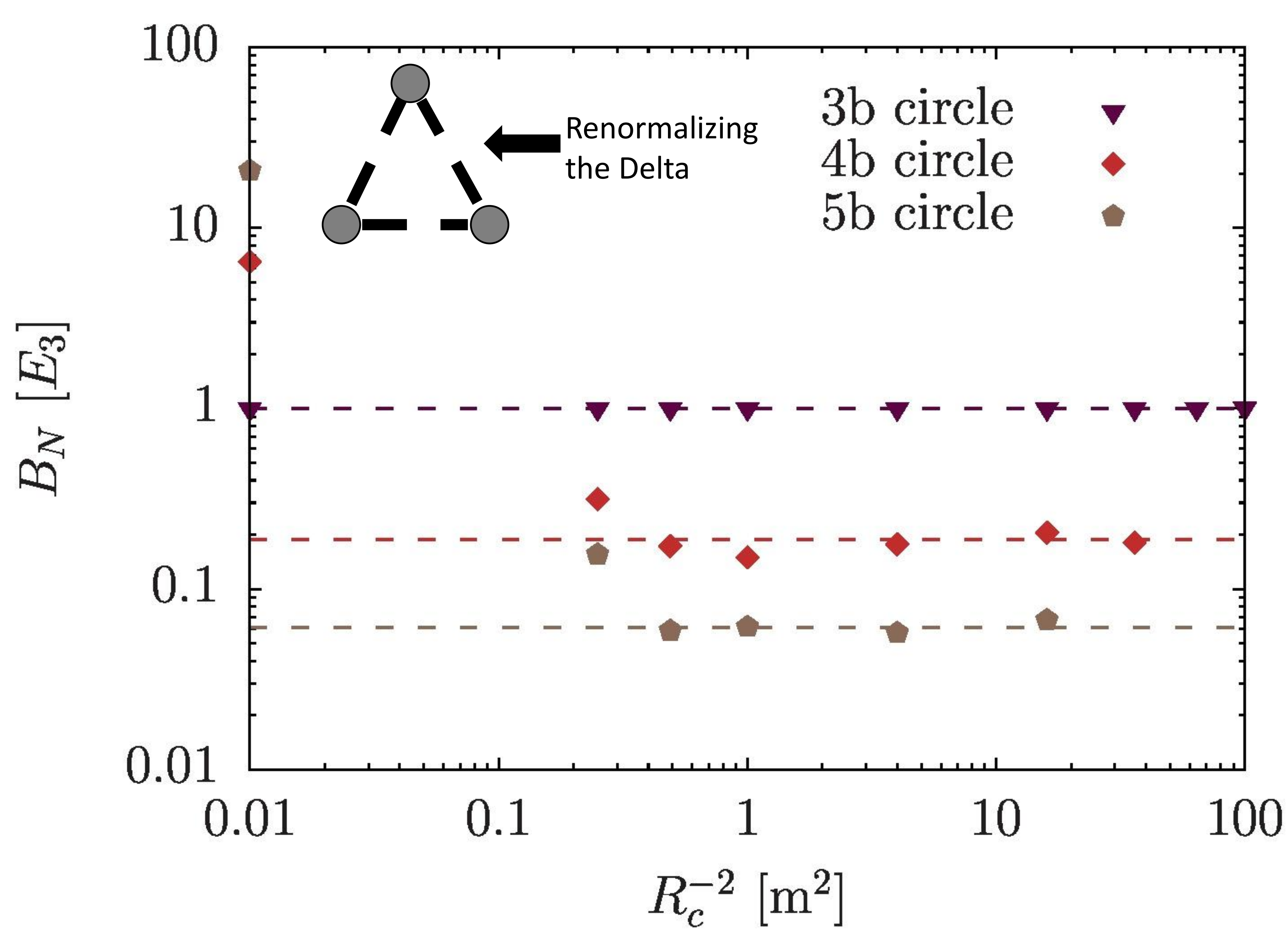
Circle divergence

$$R_c = \frac{1}{Cut - off}$$

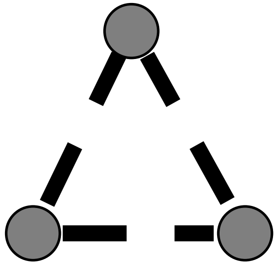




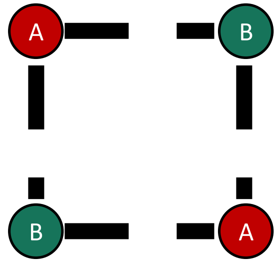
Circle convergence



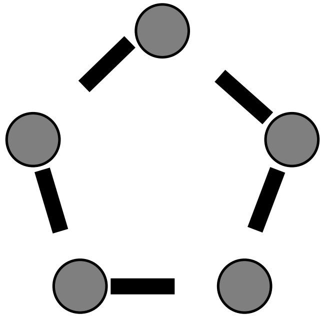
$$R_c = \frac{1}{Cut-off}$$



E_Δ

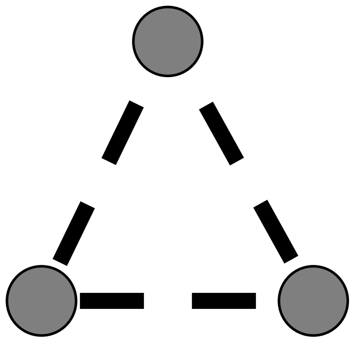


$E_{\text{Circle}} = 0.2\, E_\Delta$

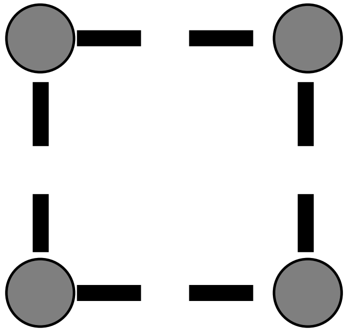


$E_{5-Circle} = 0.06 E_\Delta$

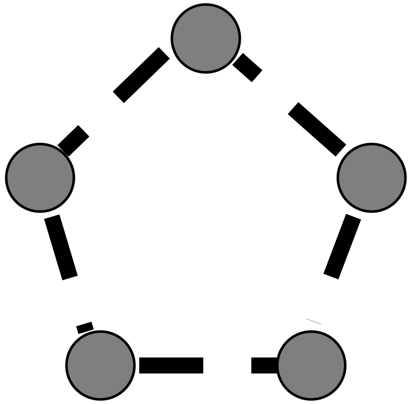
«Delta» Efimov factor 22.6



E_{Δ}

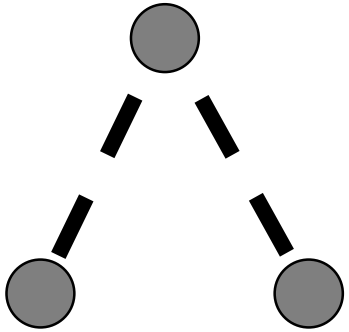


$E_{\text{Circle}} = 0.2\ E_{\Delta}$

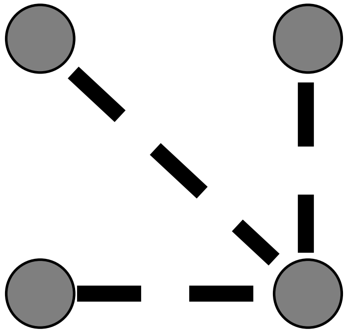


$E_{5-Circle} = 0.06 E_{\Delta}$

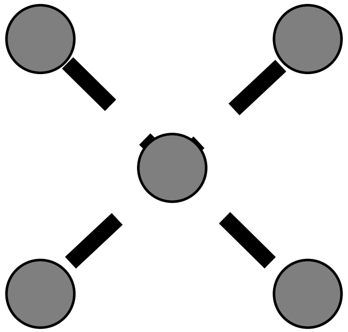
«Lambda» Efimov factor 1986.1



E_{Λ}



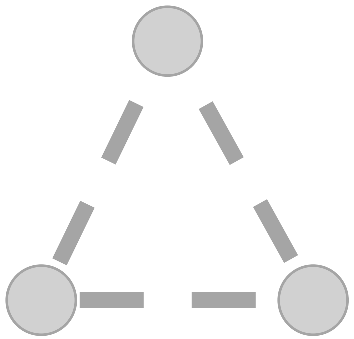
$E_{4-dandelion} = 11(1) E_{\Lambda}$



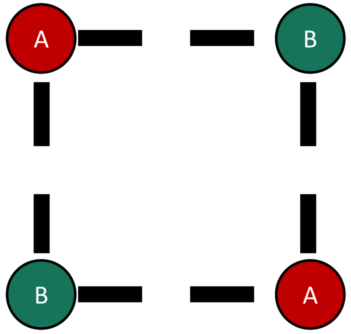
$E_{5-dandelion} = 30\ E_{\Lambda}$

«Delta» Efimov factor 22.6

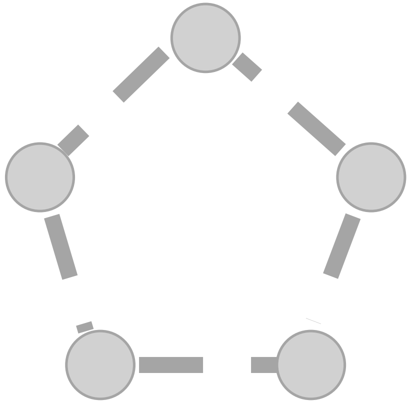
«Lambda» Efimov factor 1986.1



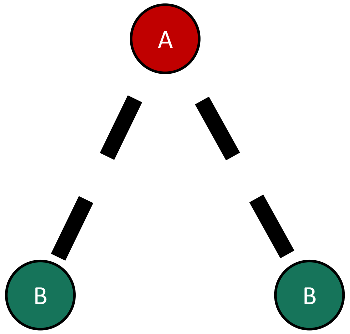
E_Δ



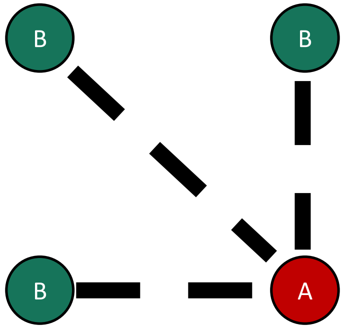
$E_{\text{Circle}} = 0.2\ E_\Delta$



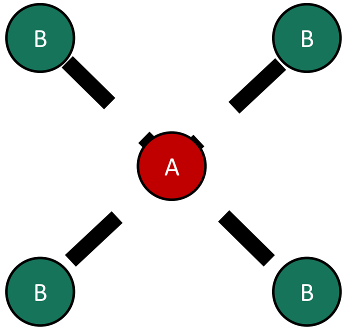
$E_{5-Circle} = 0.06 E_\Delta$



E_Λ

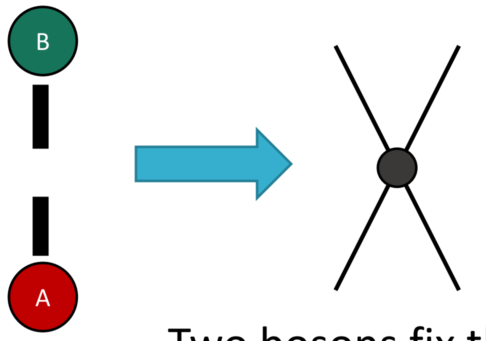


$E_{4-dandelion} = 11(1) E_\Lambda$

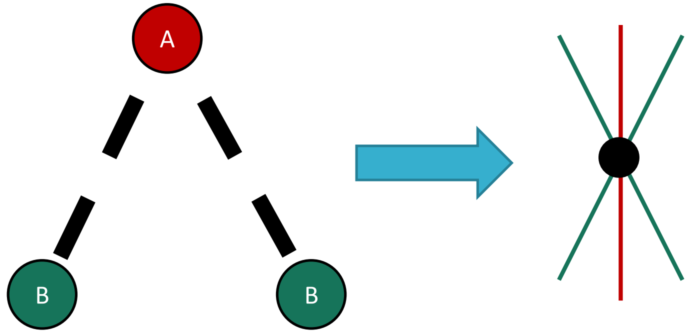


$E_{5-dandelion} = 30\ E_\Lambda$

Appearance of a 4-body scale

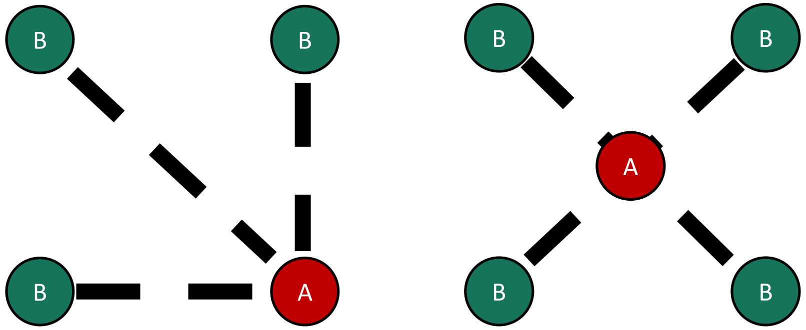


Two bosons fix the two-body vertex



Lambda system fixes the three-body vertex

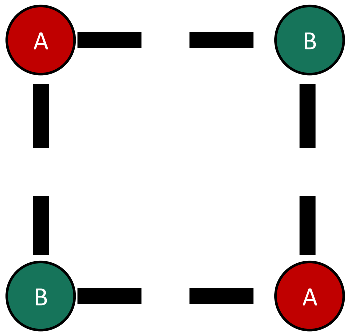
This sufficies to define the 3- and 4-dandelions



The 4-circle needs a different 3-body force:
A much more repulsive counterterm.

We can not redefine the «ABB/AAB» 3-body force!
(it would spoil the Lambda renormalization!)

We need a four-body force that acts only on AABB bosons/system!



Physical system realization

Phys. Rev. D **103**, 056001

Atomic molecules with Feshbach resonances:

$^{85}\mathrm{Ru} - {}^{87}\mathrm{Ru}$ [S. B. Papp and C. E. Wieman 2018]
$^{23}\mathrm{Na} - {}^{39}\mathrm{K}$ [Torsten Hartmann et al. 2019]

They are **mass imbalanced** systems
(Efimov ratios are different).

The **scattering length** in-between same species
Is much smaller than the interspecies one.

The **three-body system is unknown**.
(it should be compared with the same-species scattering length)

Experiments are done with **many molecules**
($10^7 - 10^9$ atoms).

Hadronic systems:

$D^0 - \bar{D}^0 = \mathrm{X}(3872)$ [S. K. Choi et al. (Belle col.) 2003]

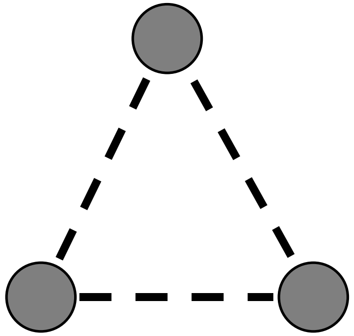
D^0 and $\bar{D}^0$ **have an excitation**: $D^{0\,*}$ and $\bar{D}^{0\,*}$
So the problem becomes a **coupled channel** problem
(it has further complications).

Very **hard to be created** in laboratory:
done up to 4 hadrons.

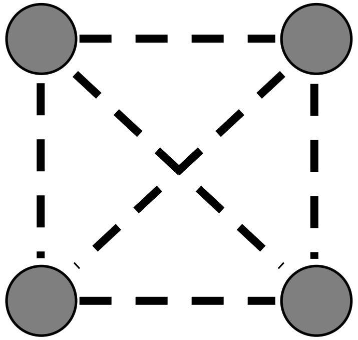
The **three-body system is unknown**.

«Delta» Efimov factor 22.6 «Lambda» Efimov factor 1986.1

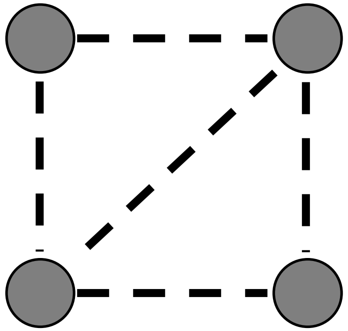
Preliminary



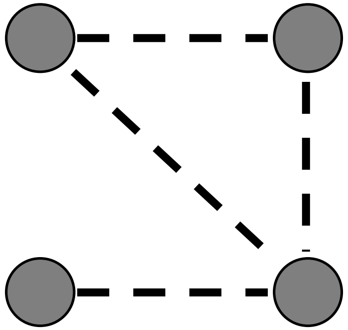
E_Δ



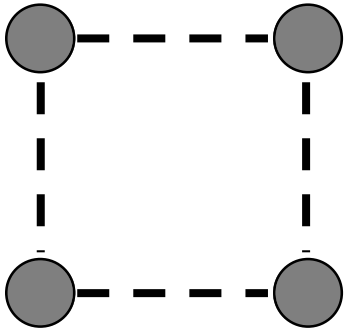
$E_{\text{Complete}} = 4.4(1)\ E_\Delta$



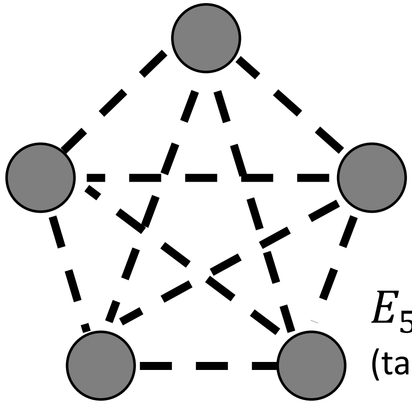
$E_{\text{Pacman}} = 1.8(1)\ E_\Delta$



$E_{\text{Stargate}} = 1.0\ E_\Delta$

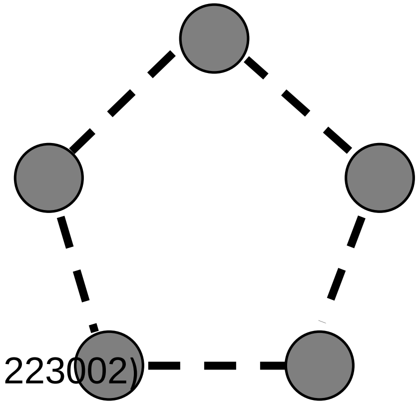


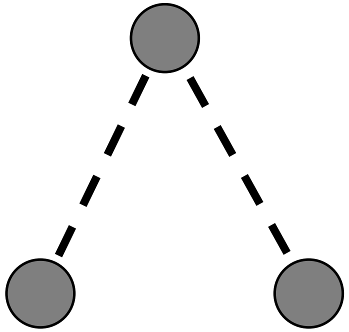
$E_{\text{Circle}} = 0.2\ E_\Delta$



$E_{5-Complete} \sim 10\ E_\Delta$
(taken from *Phys.Rev.Lett.* 119 (2017) 22, 223002)

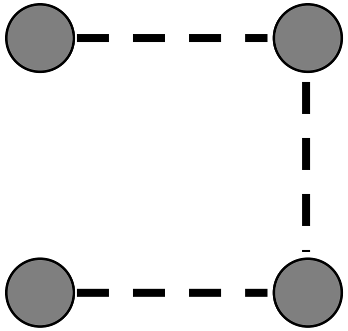
$E_{5-Circle} = 0.06 E_\Delta$



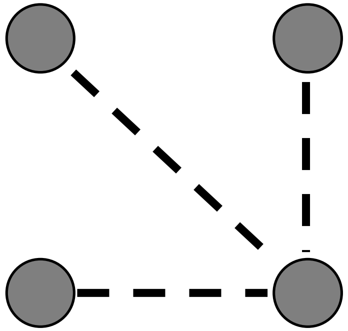


E_Λ

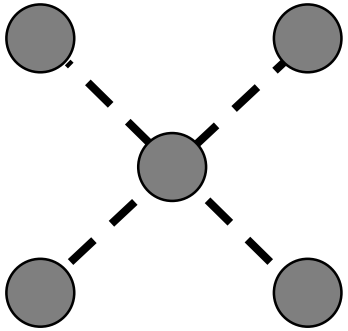
All simulations are done with SVM



$E_{snake} = 8(1) E_\Lambda$



$E_{4-dandelion} = 11(1) E_\Lambda$



$E_{5-dandelion} = 30\ E_\Lambda$

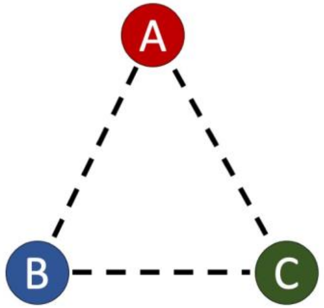
Partial Conclusions

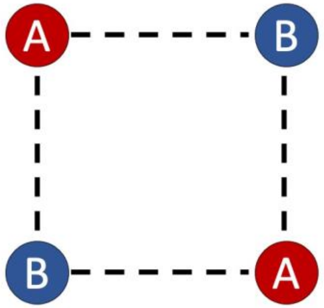
- If there is at least **one close loop**, the system universality class is the «**Delta**» one.
  In this case, the energy for few-bodies can be calculated **approximatively as the number of deltas in the system**.

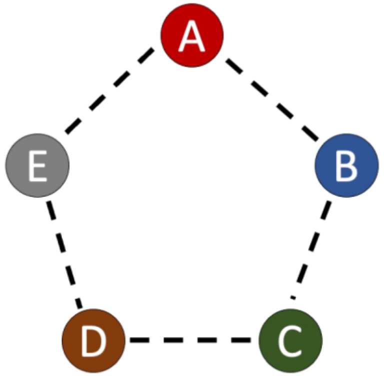
- It follows that a **four-body scale** appears in 2-specie bosonic 4-circle (AABB).
  Many questions on this scale and renormalization are still to be answered.

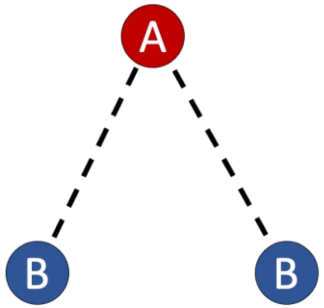
- **Renormalizability** depends also on the **geometry**, not only on the **kind of particle present**.

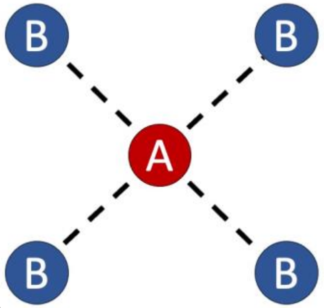
- This can be, in principle, reproduced in **atomic systems.**

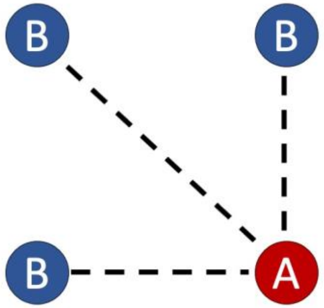






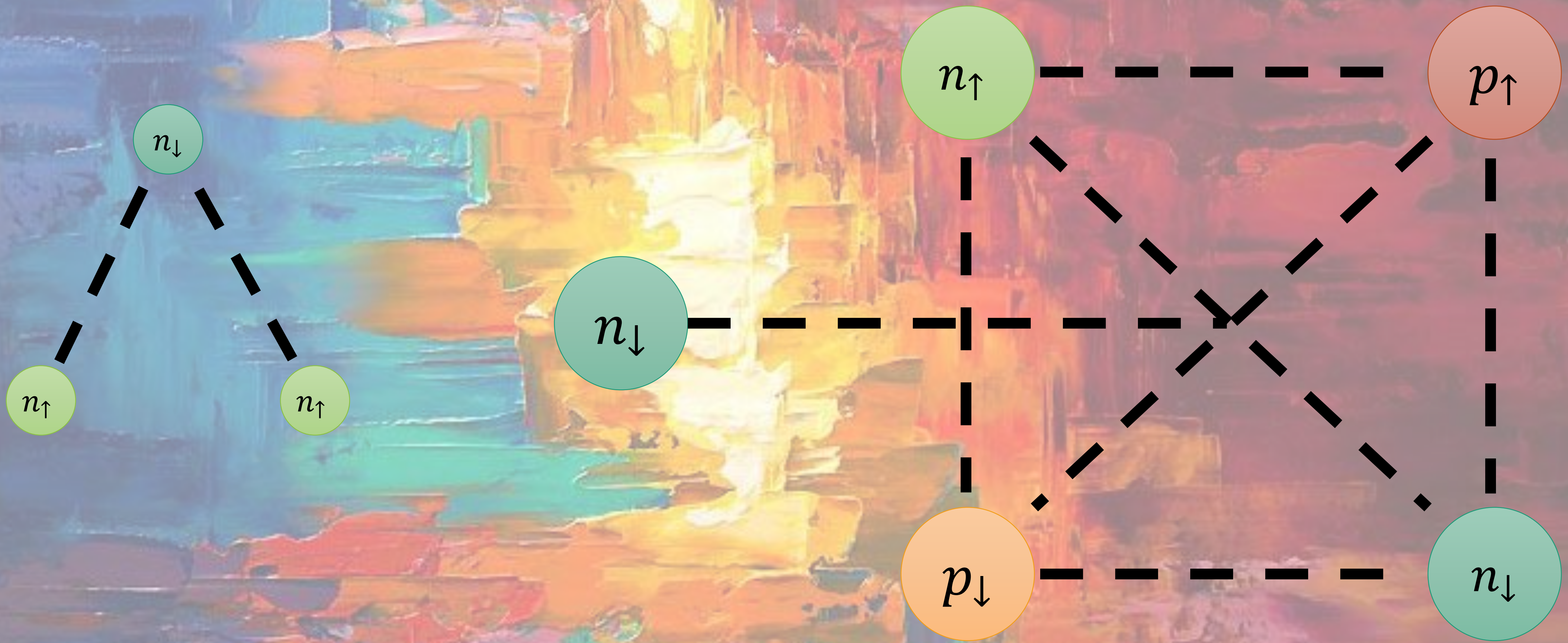




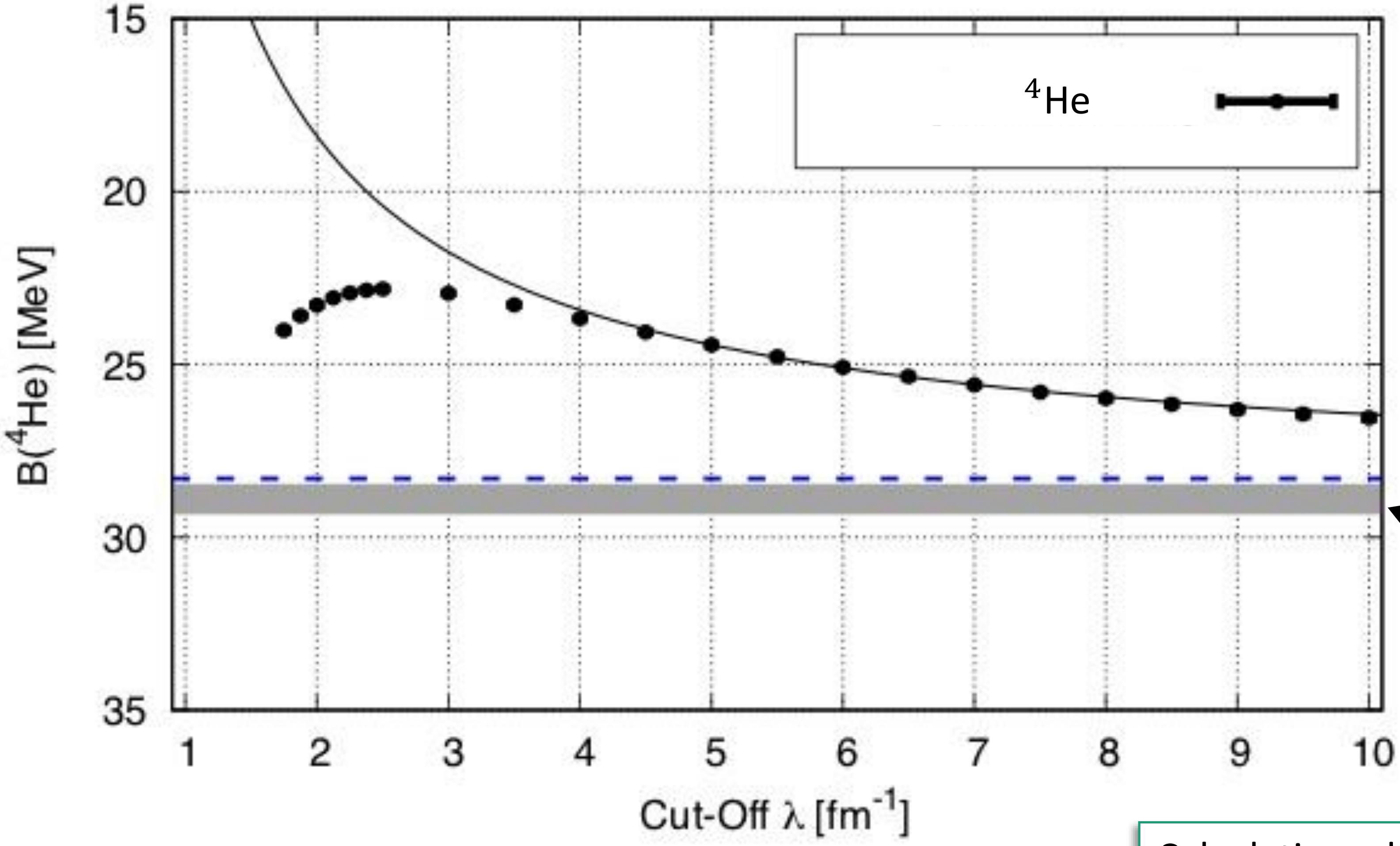


Nuclear systems and P-wave

Nuclear systems and P-wave



A practical example: few nucleons



LO pionless EFT theory fitted on **two- and three-body** observables **predicts well ^{4}He energy**!

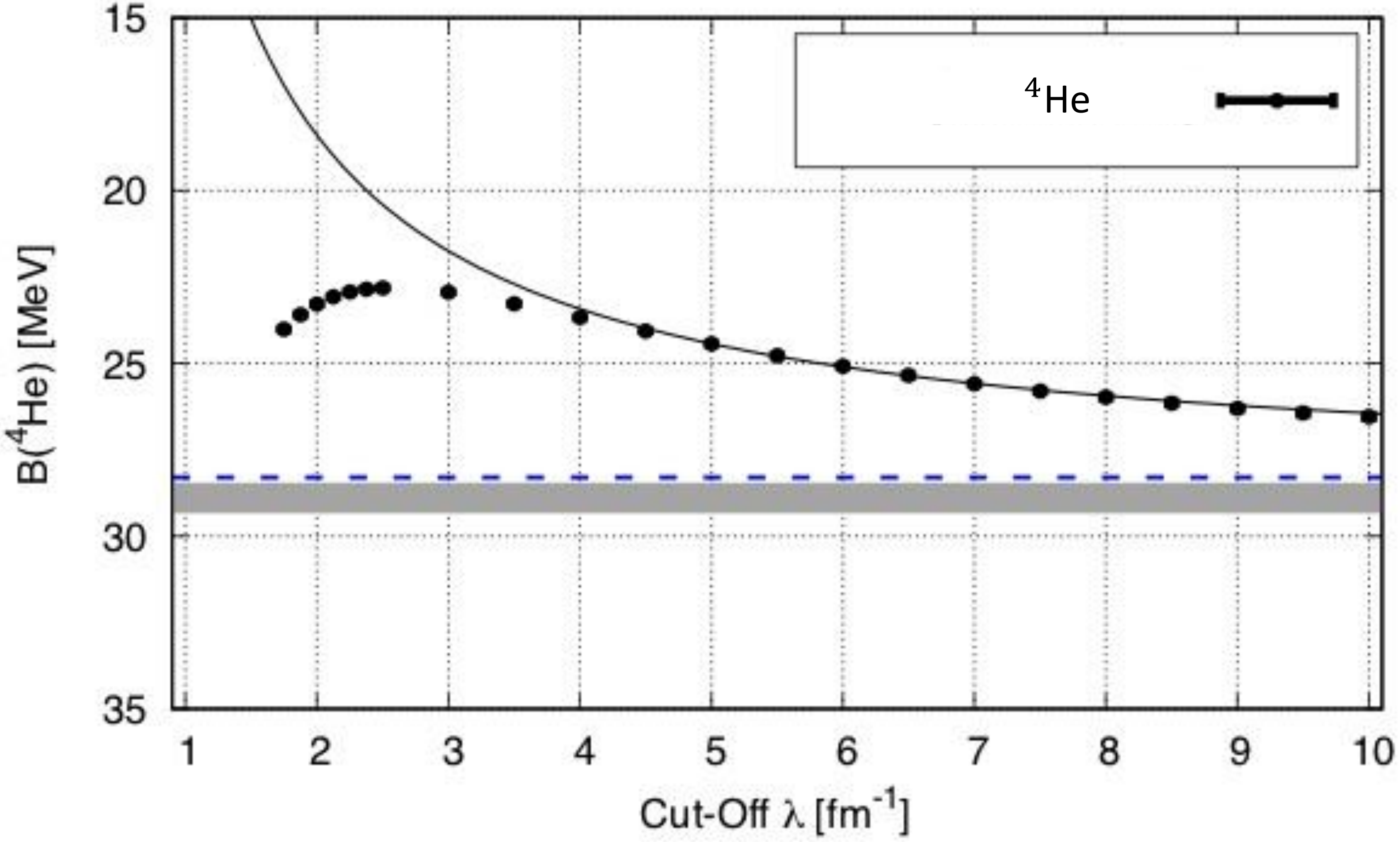
Fitted on: $a_{n-n} = -18.63$ fm
$B_d = -2.22$ MeV
$B_t = -8.48$ MeV

- - - - Experimental data: 28.3 MeV
▬ Extrapolation: 29.2±0.5 MeV

Calculations done with **few-body stochastic variational diagonalization method**: Y. Suzuki, K. Varga (2003)

L. C., N. Barnea, and A. Gal Phys. Rev. Lett. **121**, 102502

A practical example: few nucleons



Everything works great with Pionless EFT up to 4-nucleons.

^{16}O - Monte Carlo calculation

Phys.Lett.B 772 (2017) 839-848

S-wave system

Λ $[fm^{-1}]$	^4He Energy [MeV]
2	-23.17(2)
4	-23.63(3)
6	-24.06(2)
8	-26.04(5)
∞	$-30^{0.3(sys)}_{2.0(stat)}$
Exp	-28.296

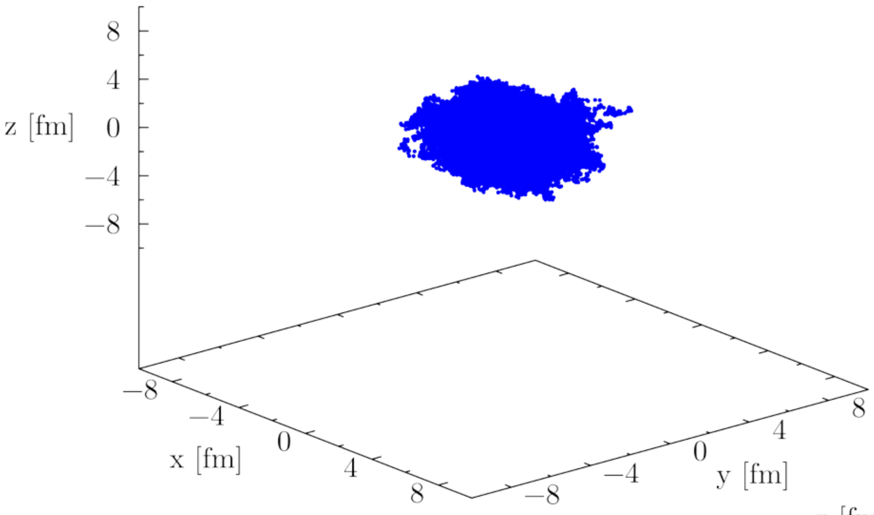
P-wave system

Λ $[fm^{-1}]$	^{16}O Energy [MeV]	4α treshold [MeV]
2	-97.19(6)	-92.68(8)
4	-92.23(14)	-94.52(9)
6	-97.51(14)	-100.24(8)
8	-100.97(20)	-104.2(2)
∞	$-115^{1(sys)}_{8(stat)}$	$-120^{1(sys)}_{8(stat)}$

- All the errors shown are statistical errors from Monte Carlo method.

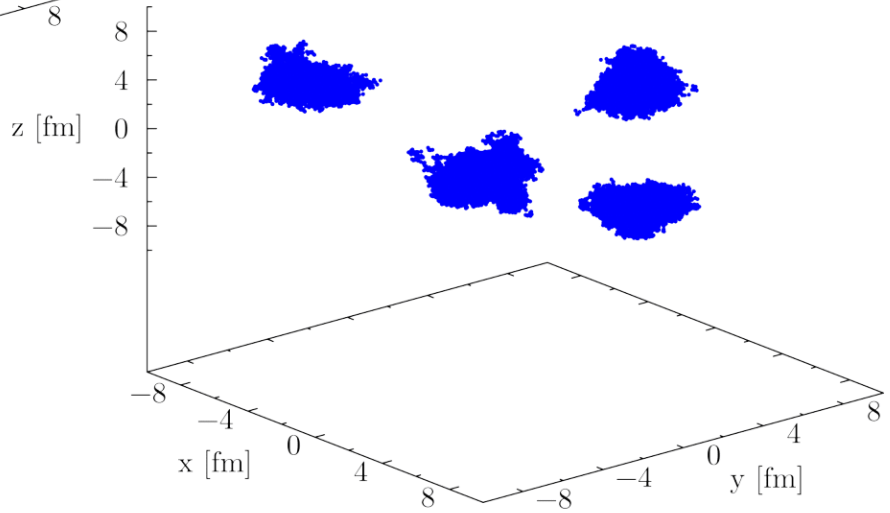
Be(^{16}O) ~ 127 MeV
Be(4α) ~ 113 MeV
It is only 10% difference!

Oxygen density ($m_\pi = 140$ MeV)



$\Lambda = 2\ \text{fm}^{-1}$

$\Lambda = 8\ \text{fm}^{-1}$



More evidences

5He 6He

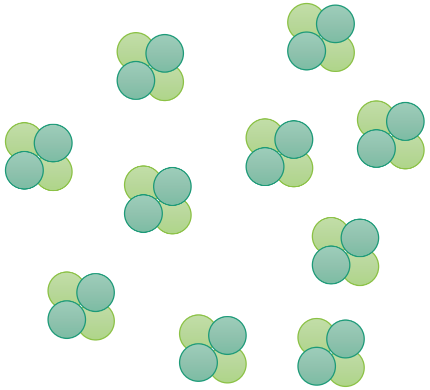
J. Kirscher, H. W. Grießhammer, D. Shukla, H. M. Hofmann: arXiv:0909.5606

Breaks in $\alpha + n$ and $\alpha + n + n$

40Ca

QMC calculation suggests the breaking in:

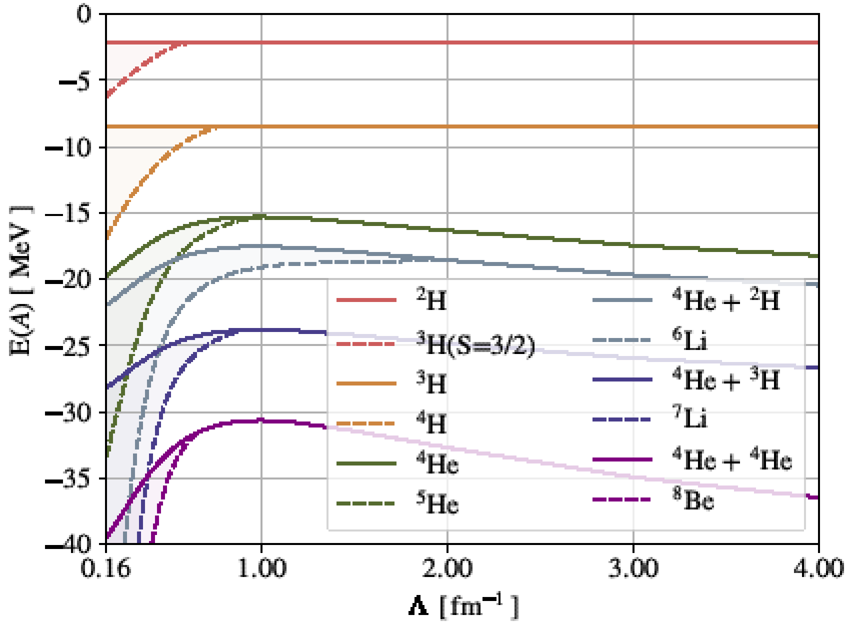
Breaks in $\alpha + \alpha + \alpha + \ \ldots$



7Li 8Be

Our calculations in SU(4) symmetry

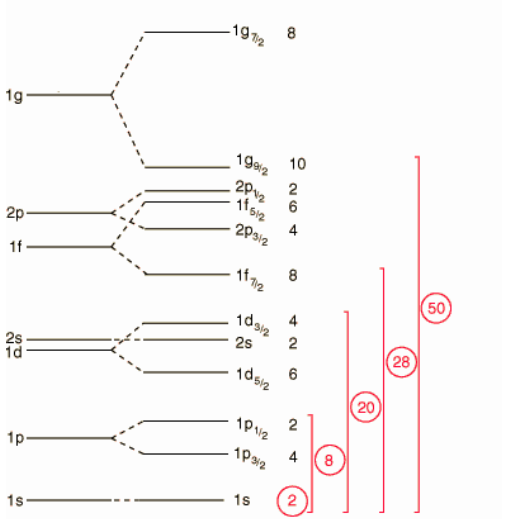
Breaks in $\alpha + d$ and $\alpha + \alpha$



P-wave systems

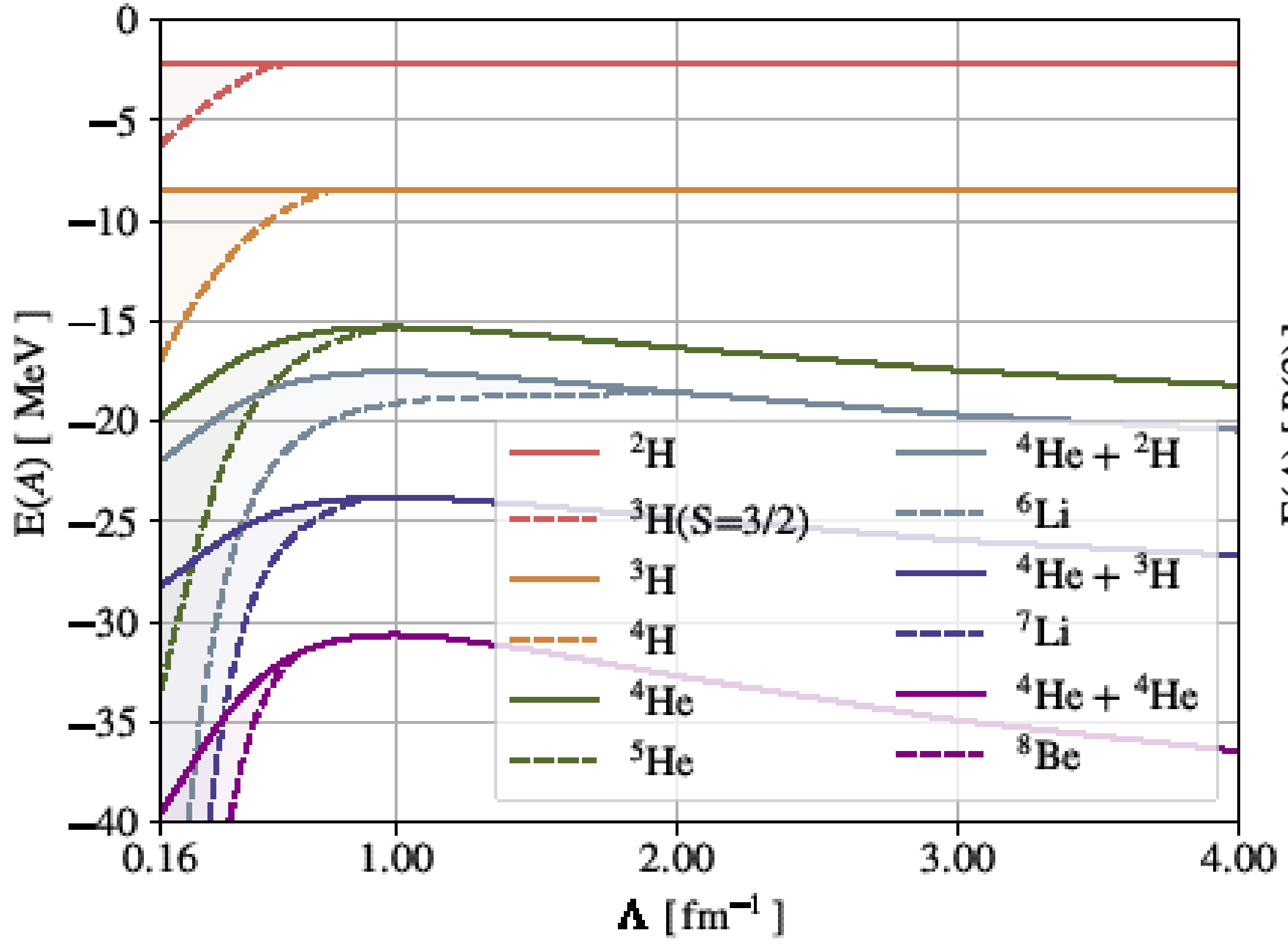
In a shell model representation

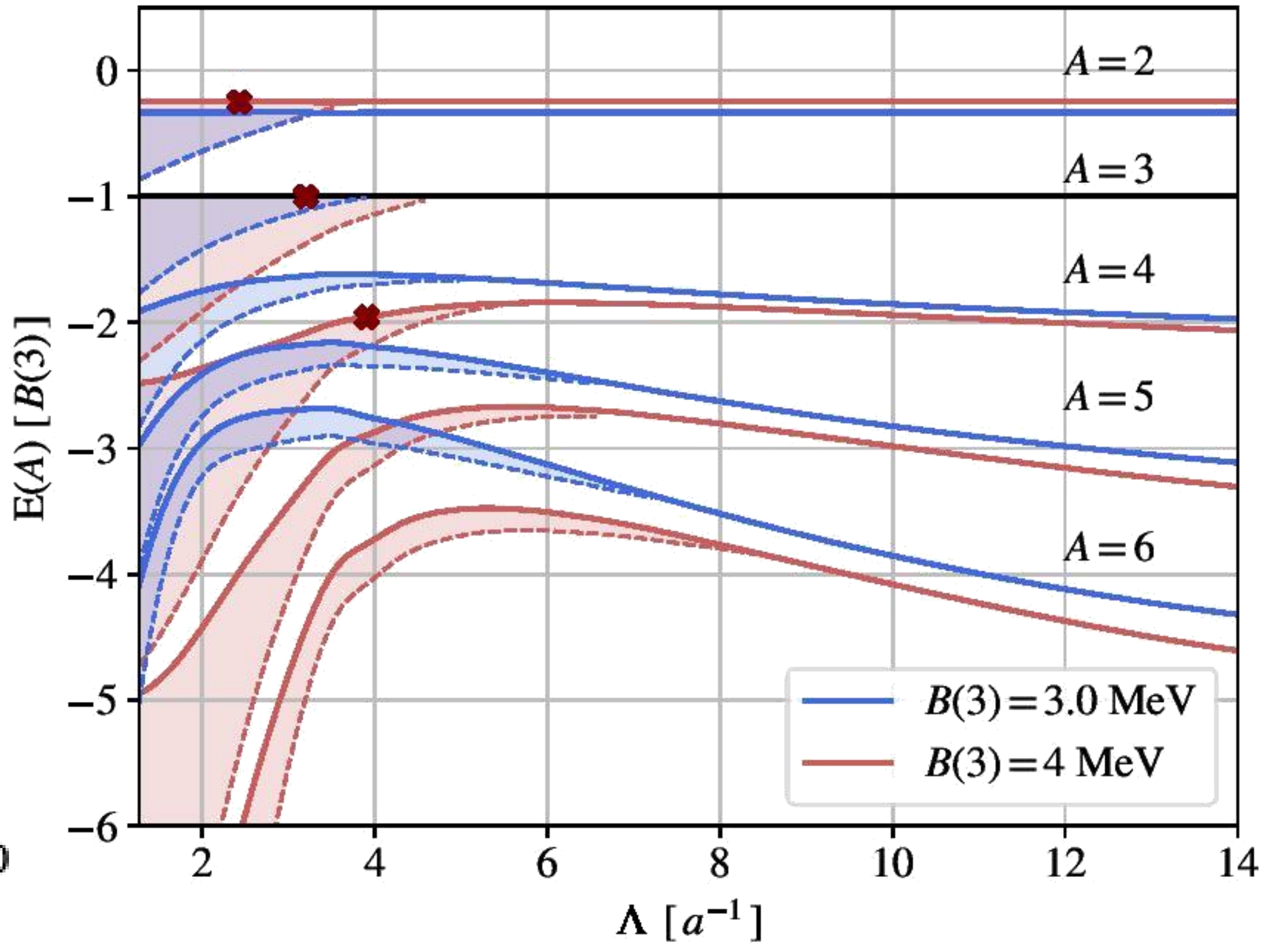
Relation between shell model and magic numbers



Will they ever bind?

Long story short: no





Multi-fermion systems with contact theories, PLB 816 (2021)

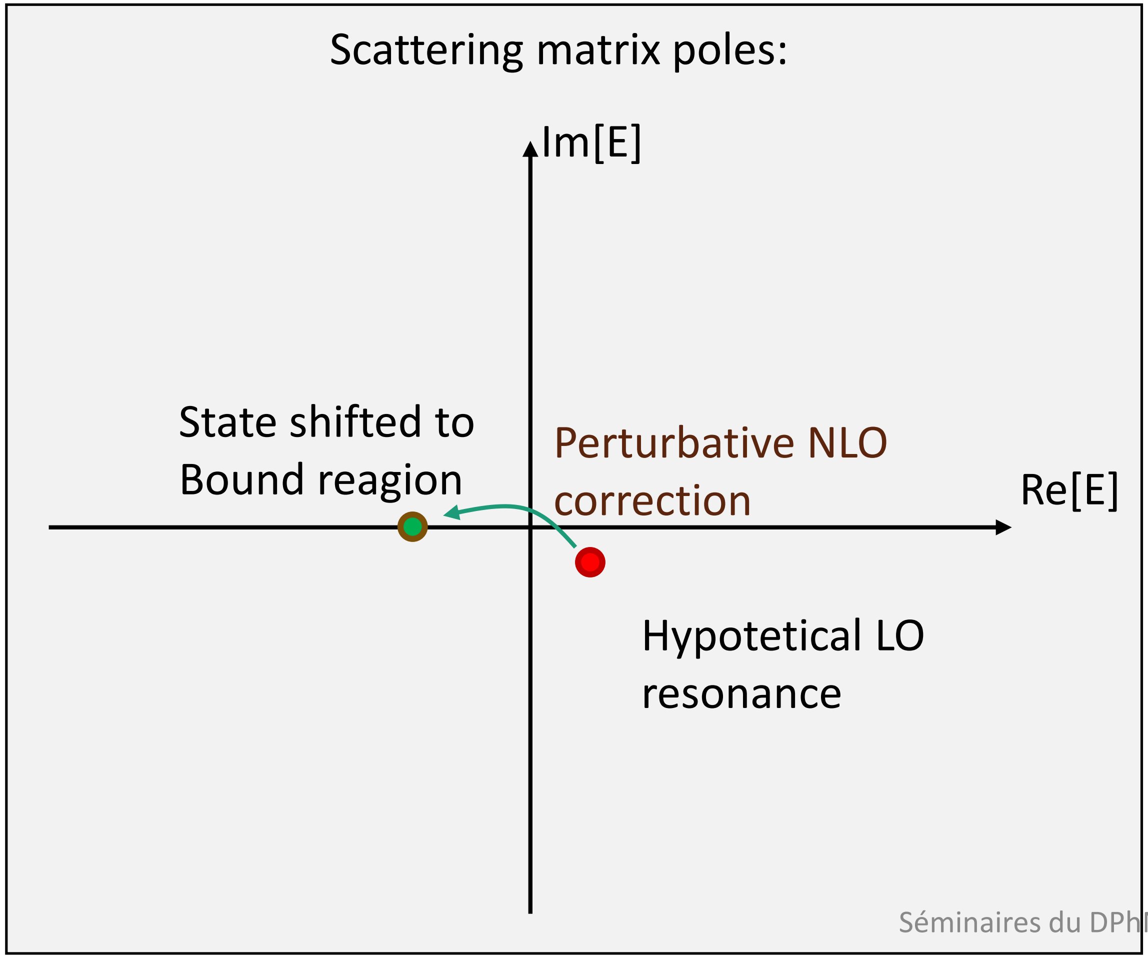
Resonance

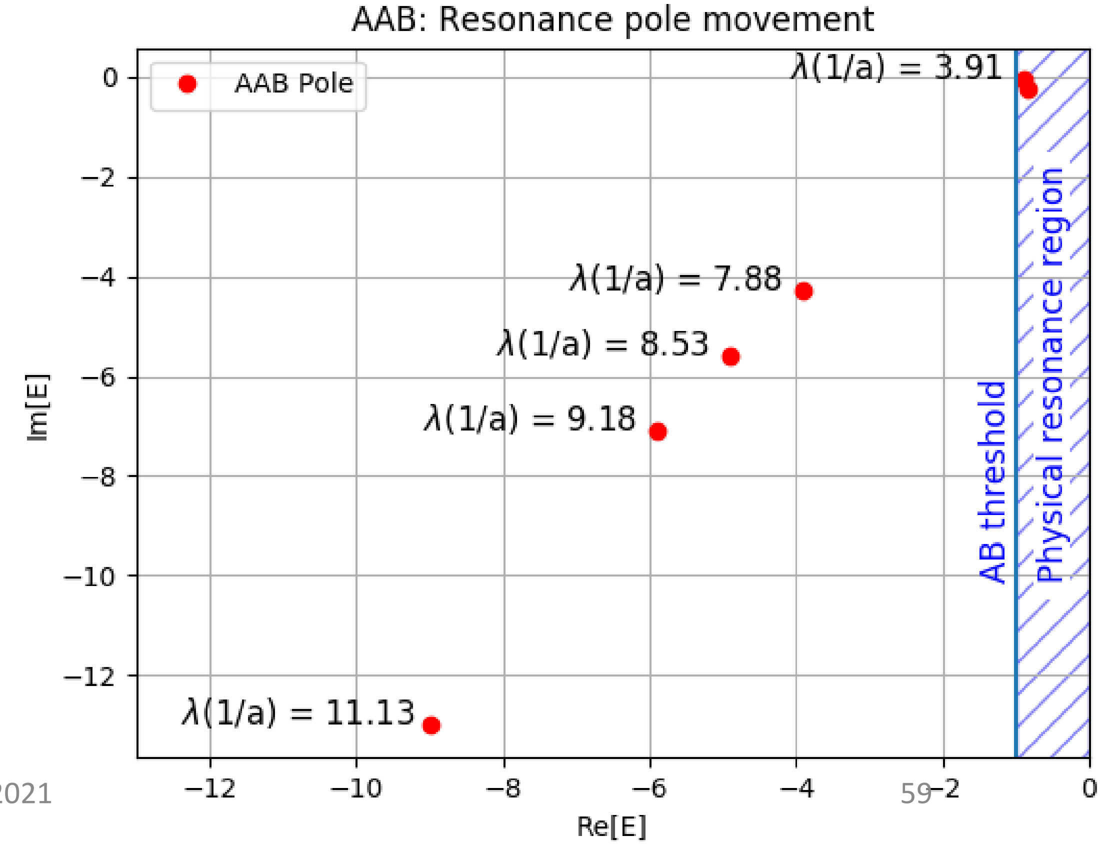
One little step further is necessary:

If a resonance is close to the **threshold**, it might be possible to move it with a **subleading correction** (there is no proof this is possible, nor proof this is not possible)

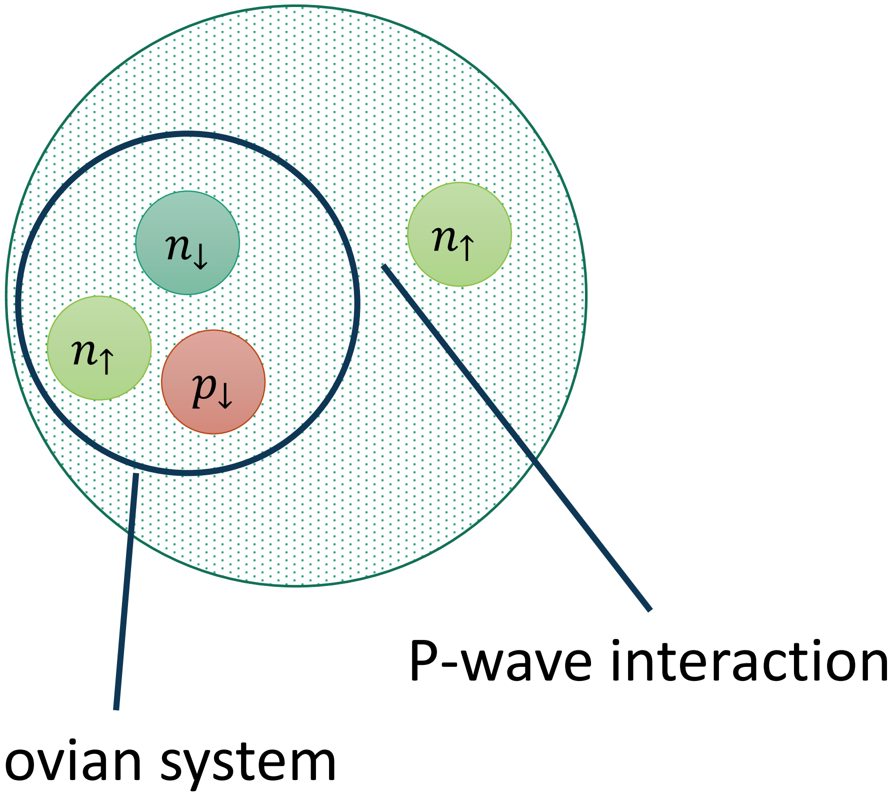
Known **three-fermion** case:
No physical resonance is found.

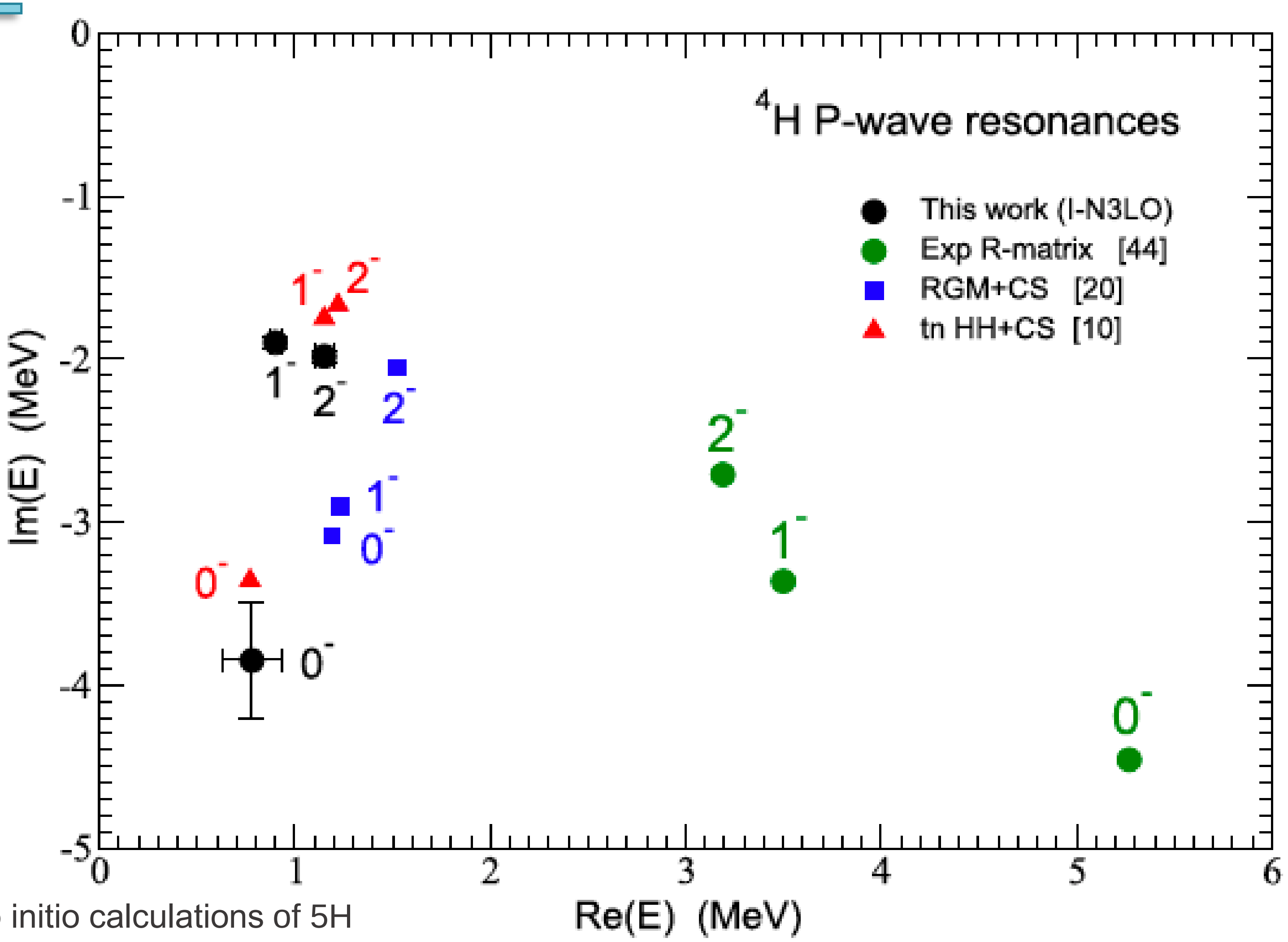
No scale invariance breaking,
Three-body force might change picture.





4H resonance:
the minimal nuclear system with an Efimovian component

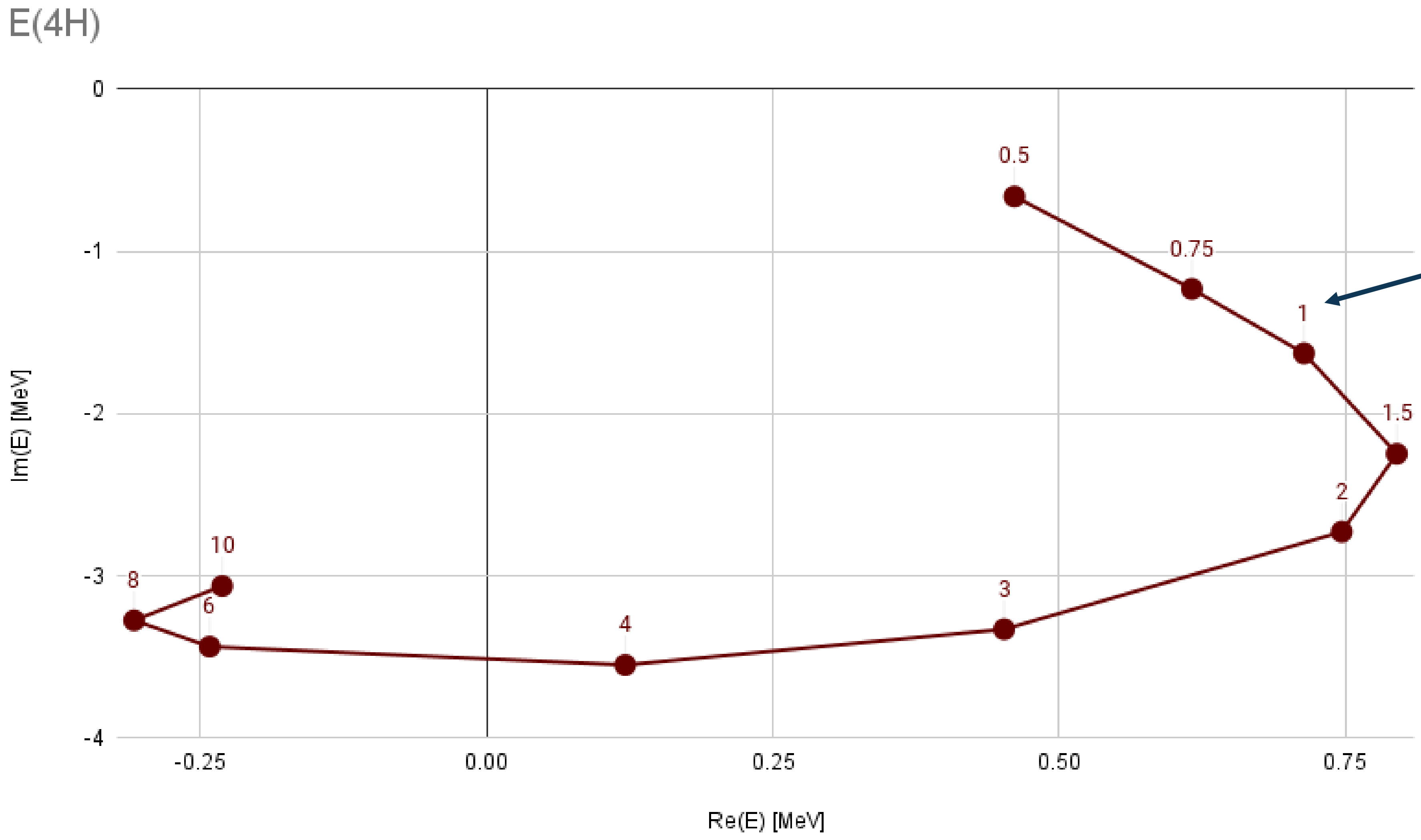




R. Lazauskas, E. Hiyama, and J. Carbonell, “Ab initio calculations of 5H resonant states,”Phys. Lett. B, vol. 791, pp. 335–341, 2019.

Contact EFT: a sub-threshold resonance is present

Preliminary



Cut-off in fm^{-1}

In preparation with

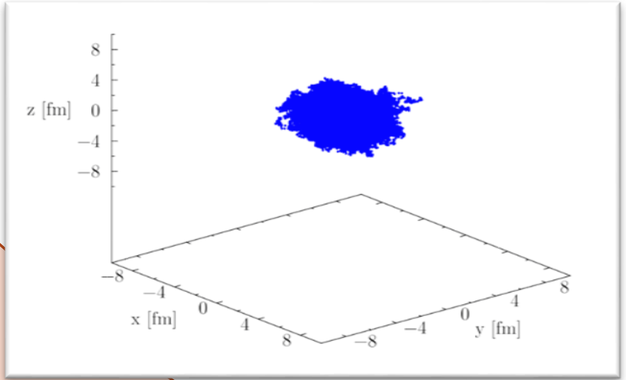
Martin Schäfer
Johannes Kirscher
Jaume Carbonell
Rimantas Lazauskas

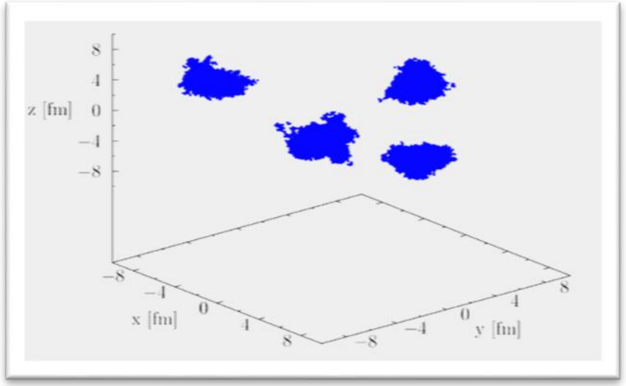
Contact EFT: a sub-threshold resonance is present

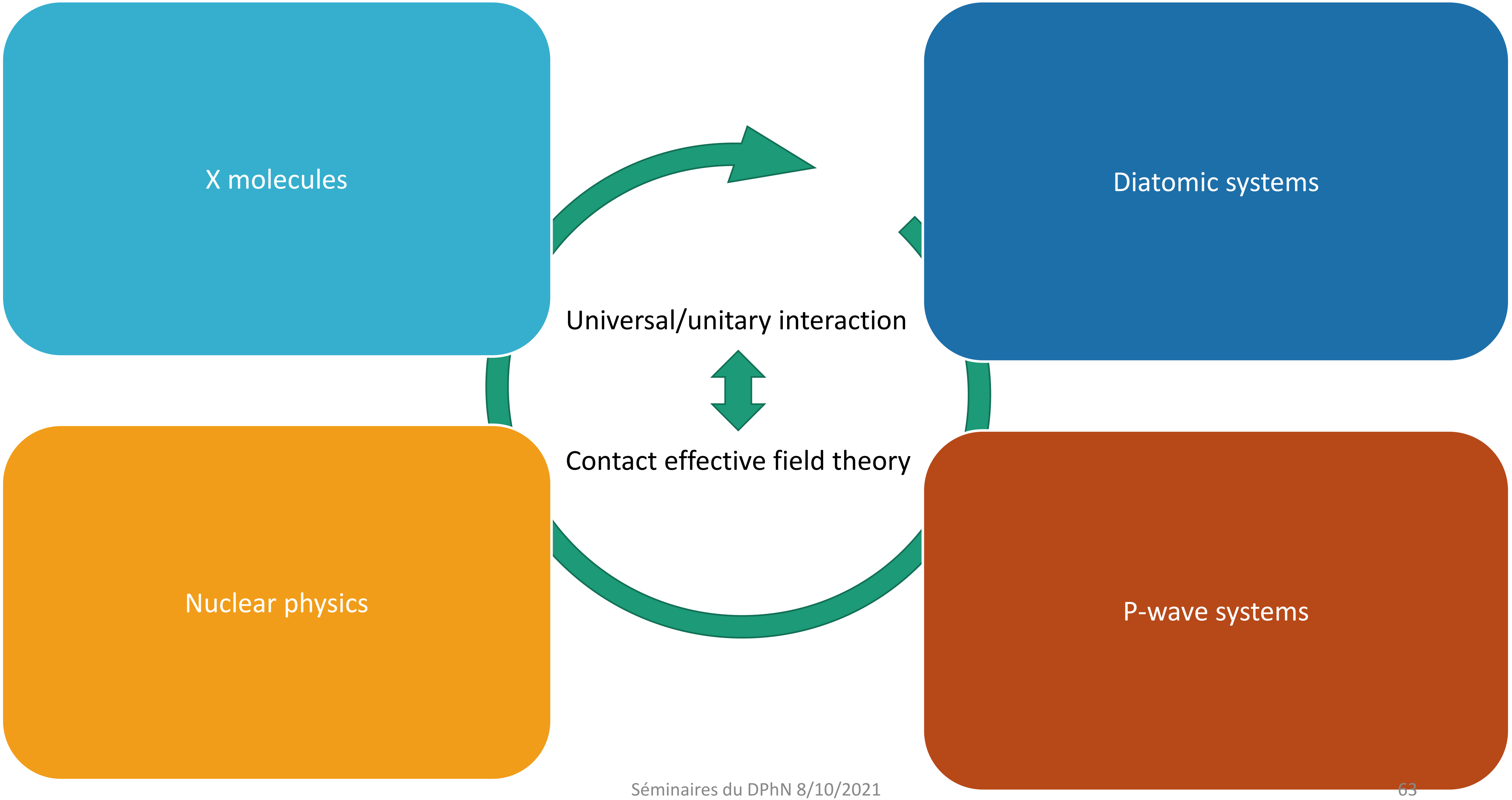
Contact theory → everything fine in **S-wave**
→ **no P-wave boundstates**

A resonance is found in ^{4}H → many-body **P-shell poles can be created**
(not in the correct physical position)

Can the resonant pole be moved to the bounded region with a **perturbative** NLO insertion?






X molecules
Diatomic systems
Universal/unitary interaction
Contact effective field theory
Nuclear physics
P-wave systems


Thank you for your patience