Nuclear matter equation of state and in-medium nucleon properties

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Outline

- Introduction
  - ⇢ *what* is nuclear matter

    definitions, limits of validity, EOS
  - ⇢ *why* we study it

    applications and constraints
  - ⇢ *how* we study it

    phenomenological vs. ab-initio approaches
- Self-consistent Green's functions at finite temperature
- The need for three-body forces
- Results I : equation of state
- Results II : in-medium single-particle properties
- Conclusions and current plans

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Nuclear matter

- strongly interacting nucleons (symmetric/pure neutron matter)
- spin-unpolarized
- homogeneous system
- thermodynamic limit

$$N, \mathcal{V} \to \infty$$

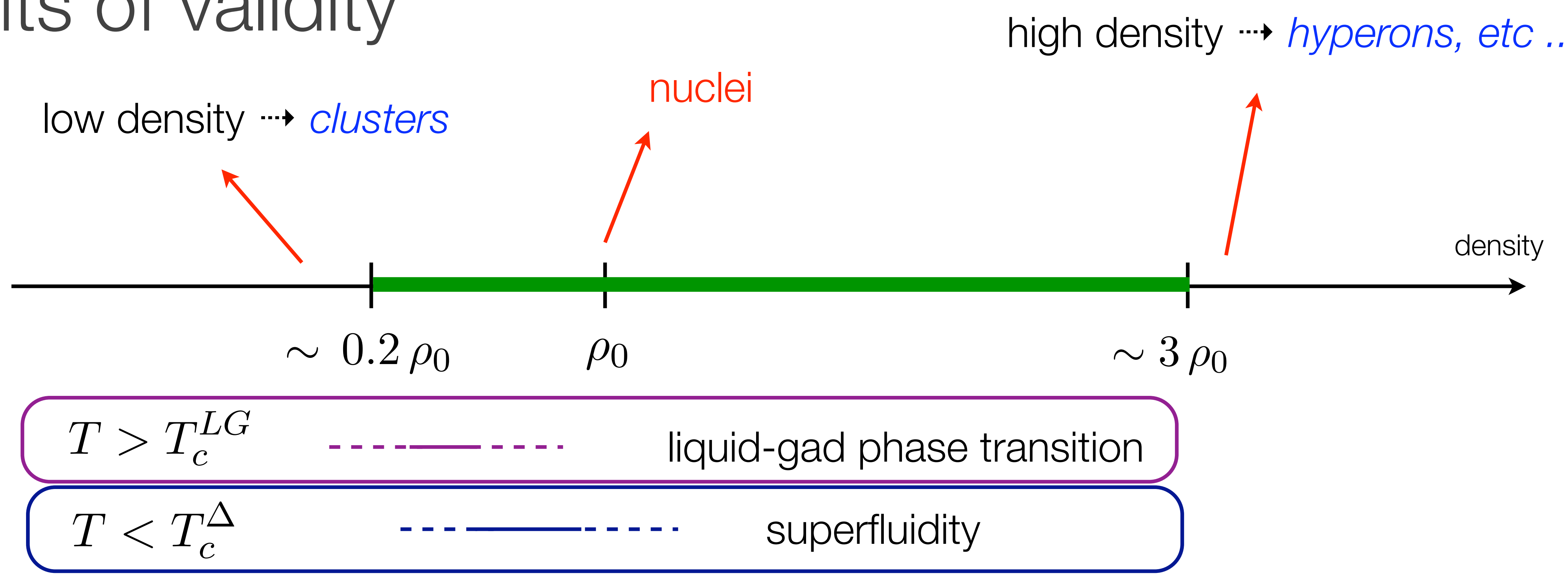
$$\rho = \frac{N}{\mathcal{V}}$$

⇢ Weizsäcker semi-empirical mass formula

$$E(N_p, N_n) = E_B\, N + E_{\text{surf}}\, N^{2/3} + E_{\text{Coul}}\, N_p^2\, N^{-1/3} + E_{\text{Pauli}}\, (N_n - N_p)^2/N$$

energy per particle in symmetric nuclear matter at *saturation density* $\boxed{\rho_0}$

Limits of validity



Challenges

- nuclear matter as a *thermodynamic ensemble*

  ⇢ equation of state

- nuclear matter as a system of *interacting nucleons*

  ⇢ modified single-particle properties

An example of EoS

- equation of state ⇢ $\boxed{P = P(\rho,\, T)}$

  ... or free energy $F = E - T\,S$

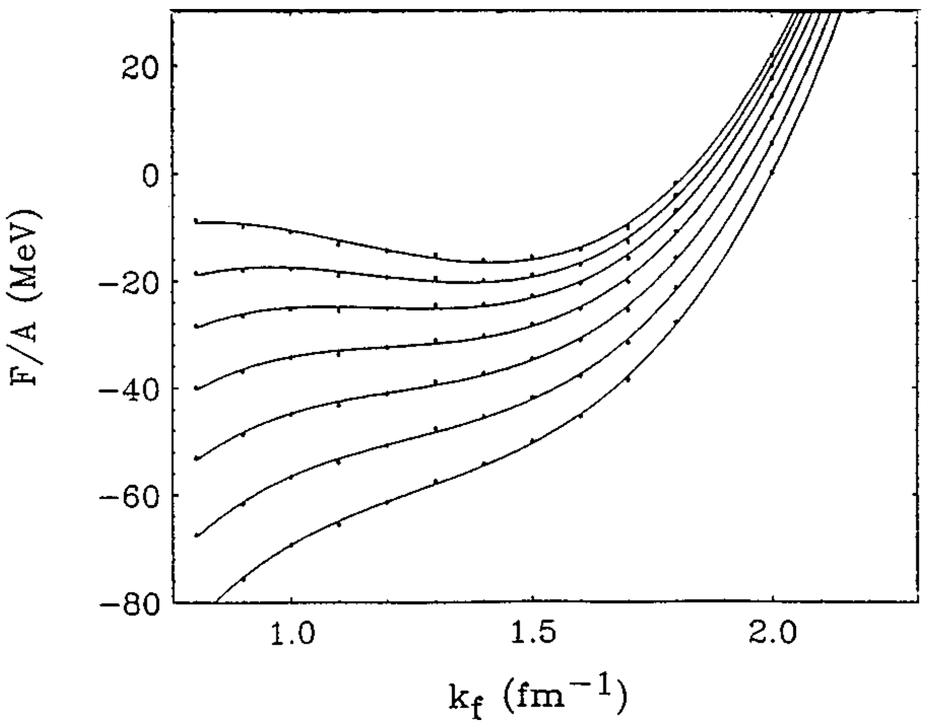
- problem of thermodynamic consistency

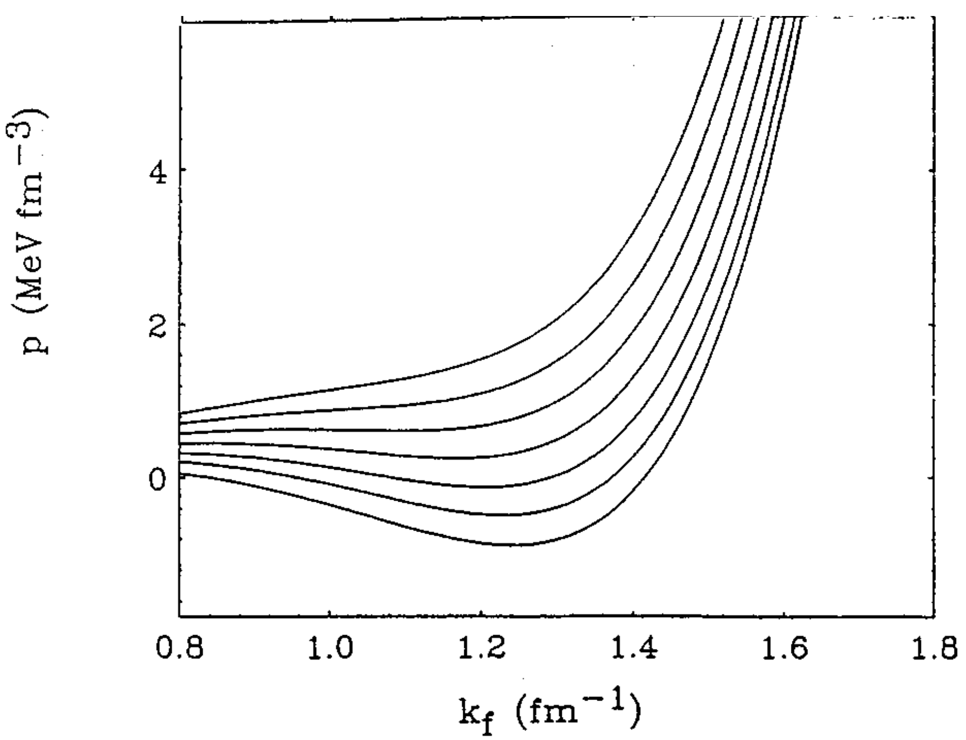
chemical potential

$$\mu \quad \longleftrightarrow \quad \mu' = \frac{\partial\, F}{\partial\, N} = \rho\, \frac{\partial\, (F/N)}{\partial\, \rho} + \frac{F}{N}$$

pressure

$$P = -\frac{\Omega}{\mathcal{V}} \quad \longleftrightarrow \quad P' = \rho^2\, \frac{\partial\, (F/N)}{\partial\, \rho}$$





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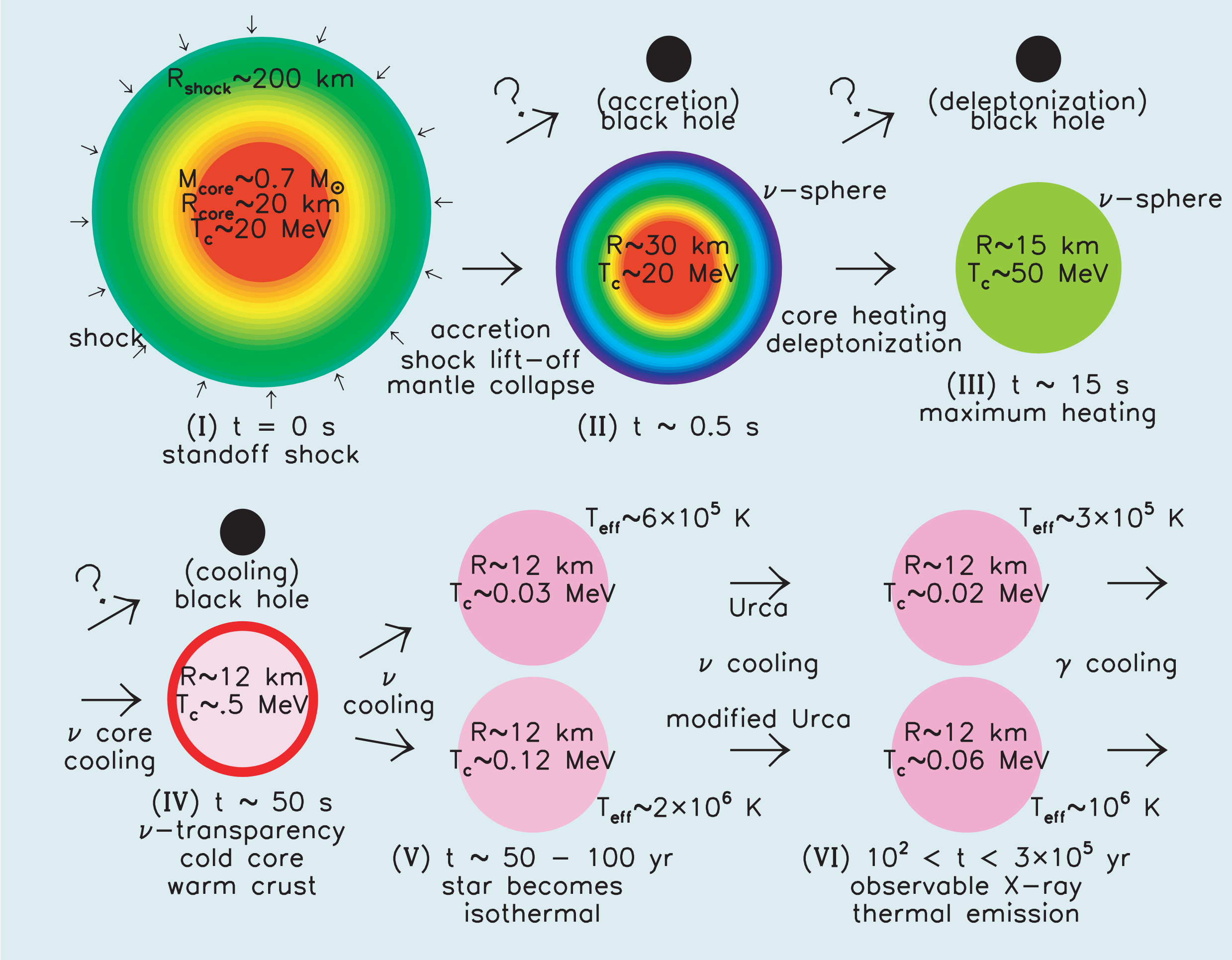
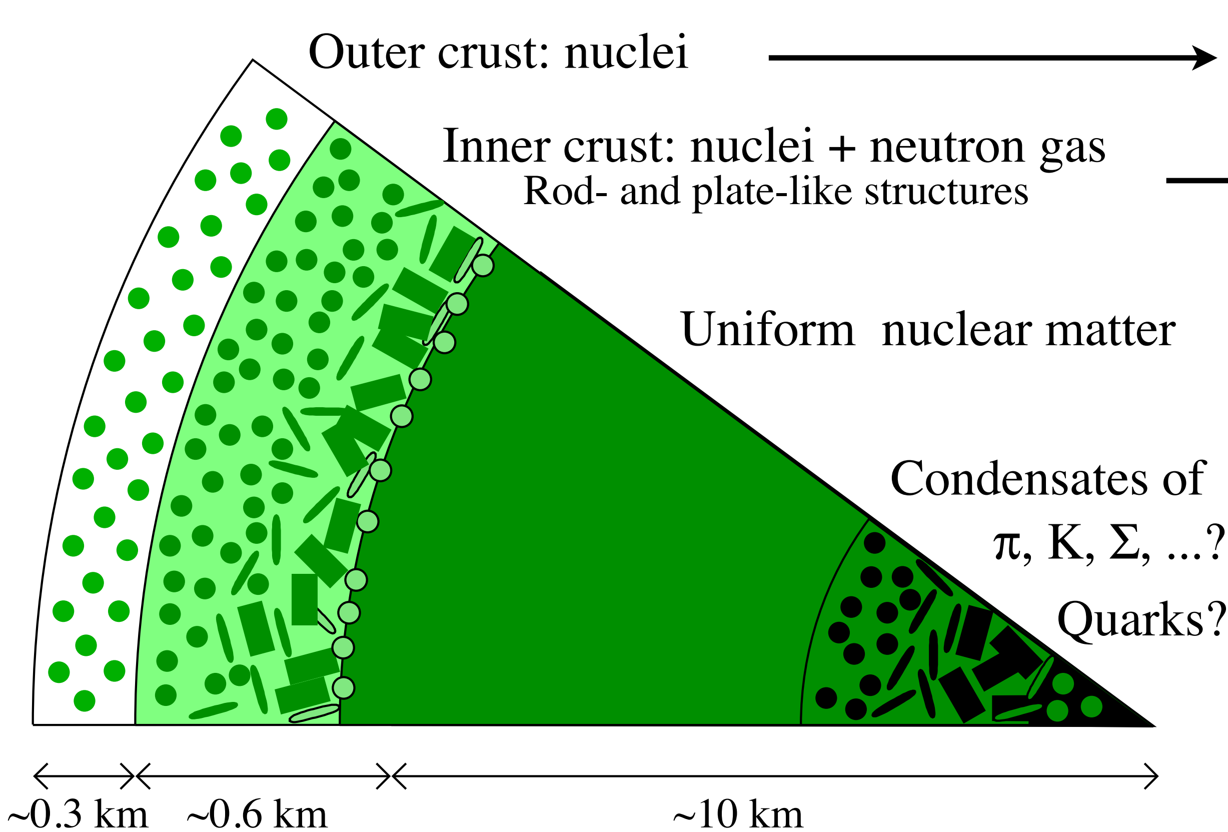


Applications and constraints: neutron stars

- densest massive objects in the universe

  M ~ 1-2 $M_\odot$ R ~ 10-15 km

- first observation in 1967 (Bell, Hewish)
- about 2000 NS observed so far...



atmosphere (H, He)

(^{56}Fe, e$^-$)

(^{56}Fe, ^{86}Kr, ^{84}Se, ^{82}Ge, ^{80}Zn, ^{124}Mo, ^{122}Zr, ^{120}Sr)

(n, p, e$^-$)

99% of the mass

[Lattimer, Prakash, Science 304 (2004)]

Cooling of neutron stars

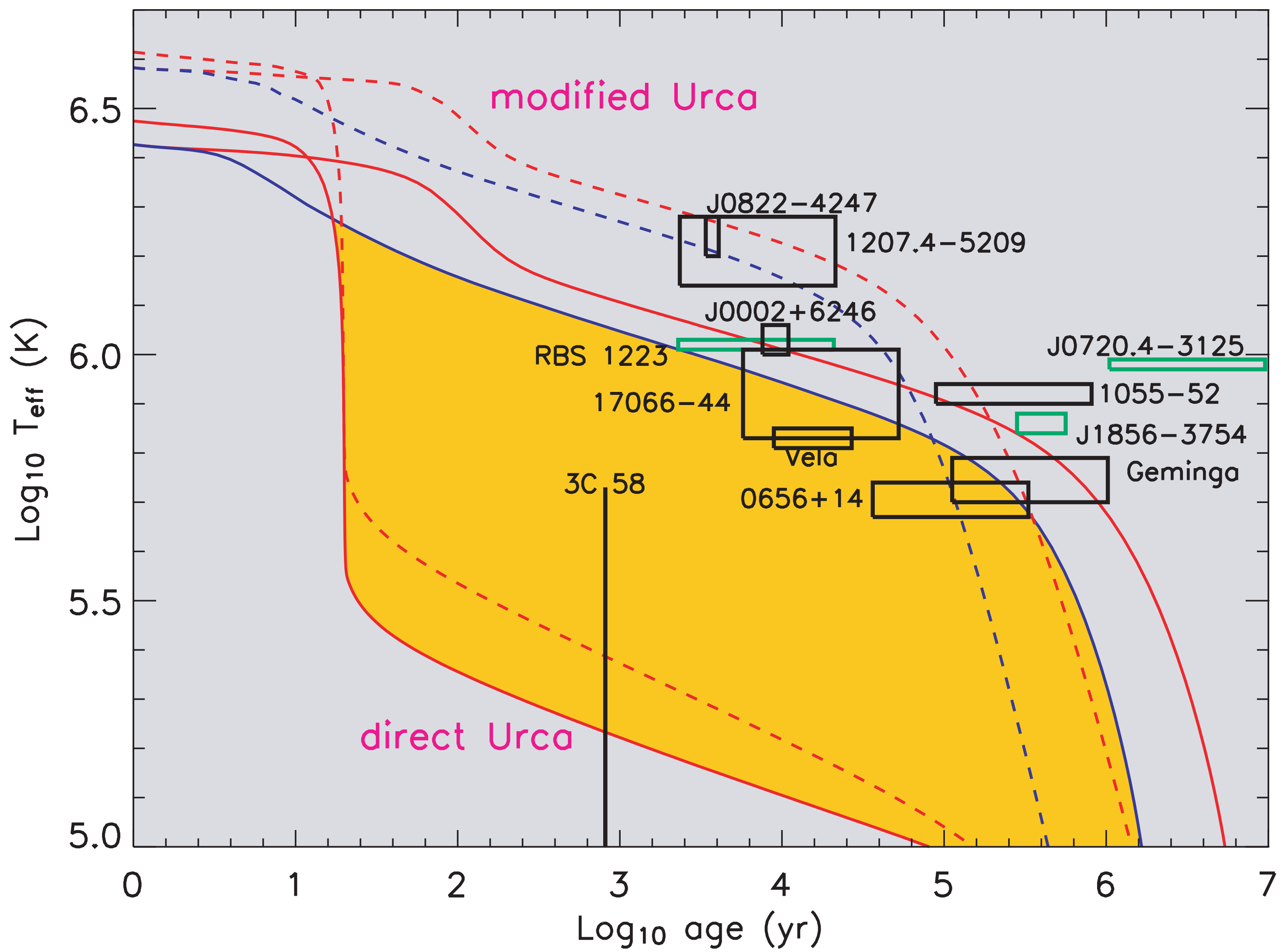
- direct Urca cooling

$$n \rightarrow p + e^- + \bar{\nu}_e, \quad p \rightarrow n + e^+ + \nu_e$$

- modified Urca cooling

$$n + (n, p) \rightarrow p + (n, p) + e^- + \bar{\nu}_e.$$
$$p + (n, p) \rightarrow n + (n, p) + e^+ + \nu_e$$

much less efficient (< 10^{-4})



- neutrino emissivities depend on the in-medium nucleon properties, in particular of the superfluid phase

Mass-radius relation in neutron stars

- Tolman-Oppenheimer-Volkov eq.

  (hydrostatic equilibrium)

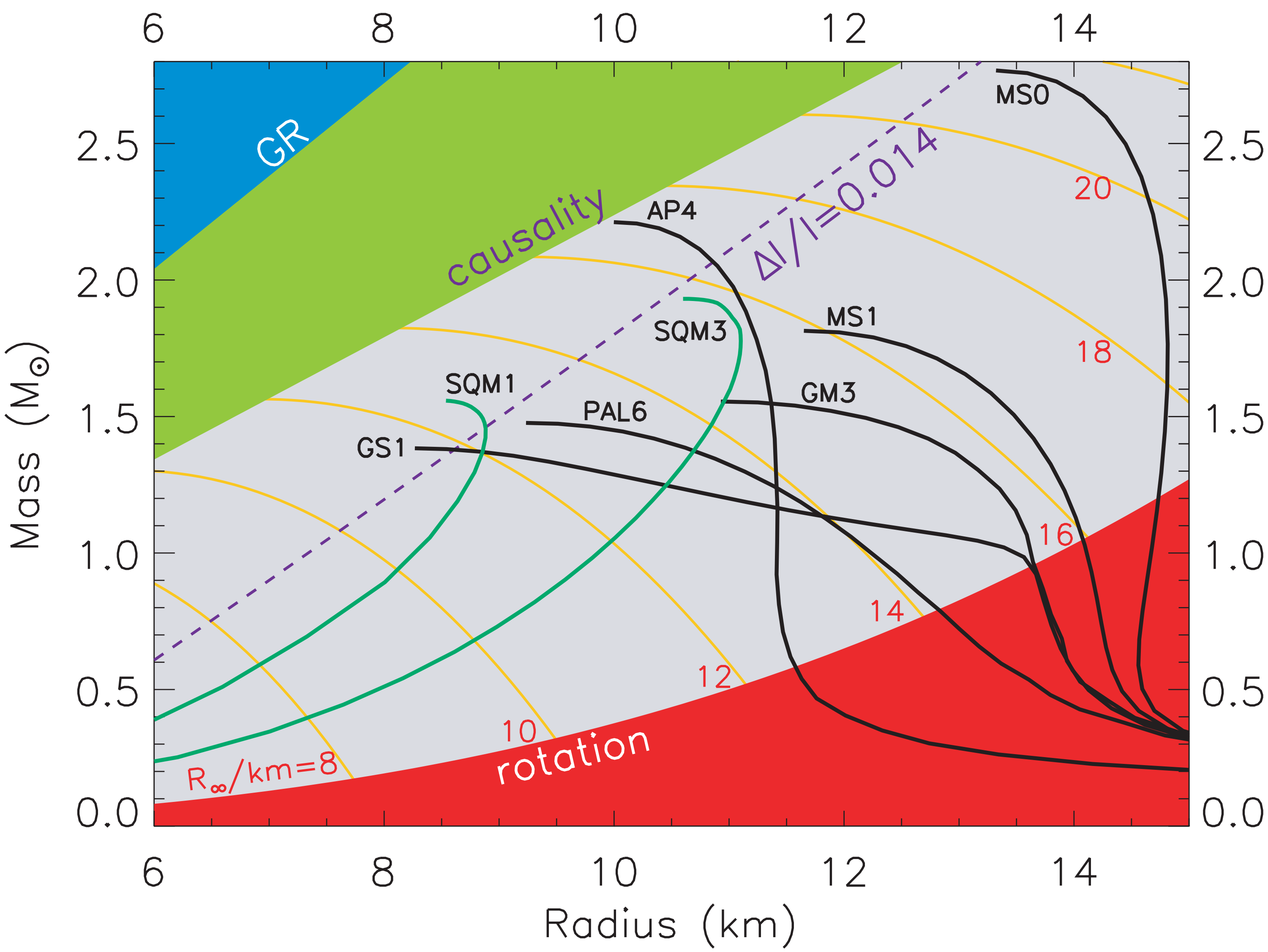
  ↓

  two relations between

  $$P(r),\ \rho(r),\ m(r)$$

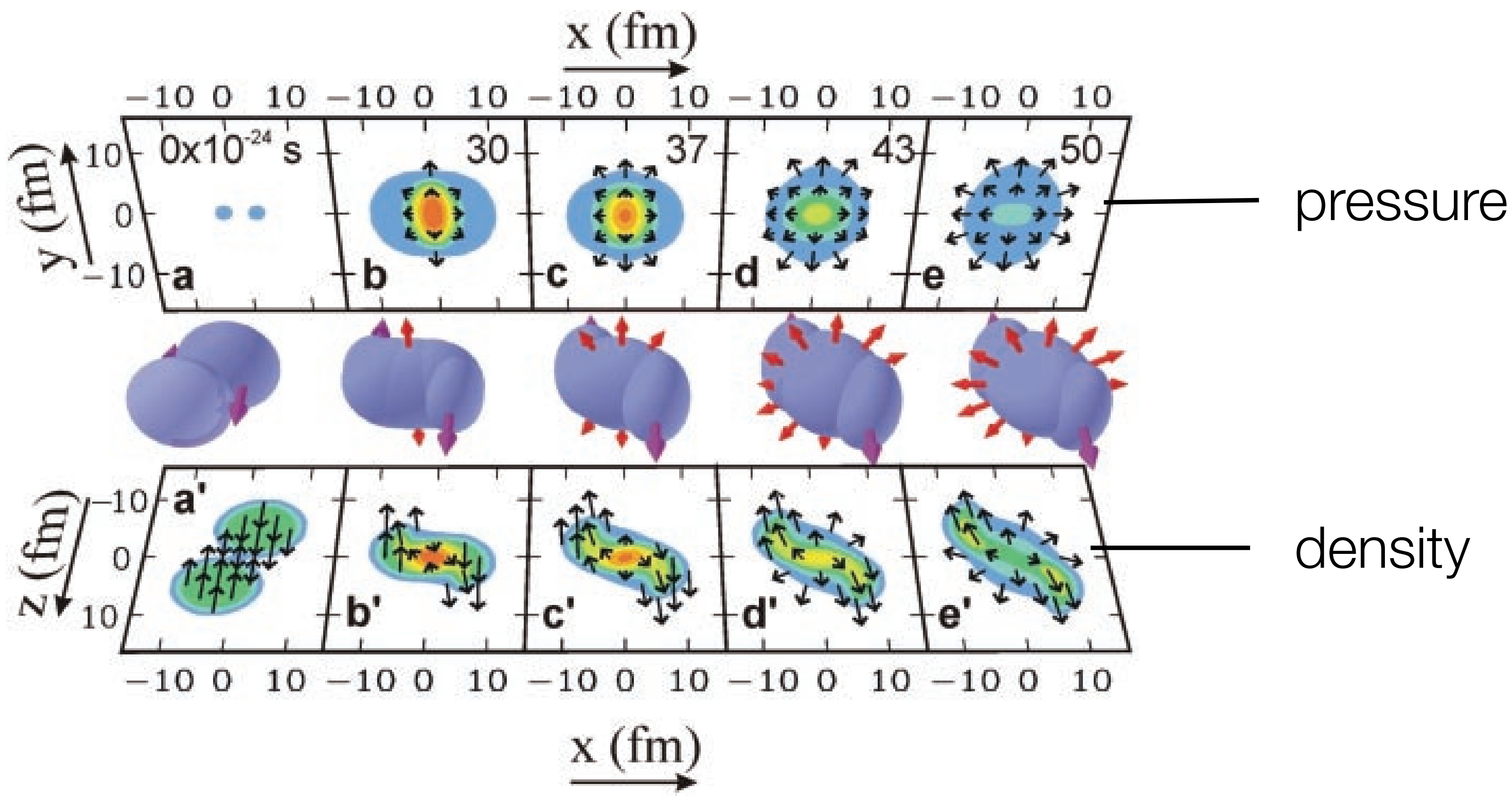
⇢ equation of state needed $P(\rho)$

⇢ transition to quark matter ?



Applications and constraints: heavy ions

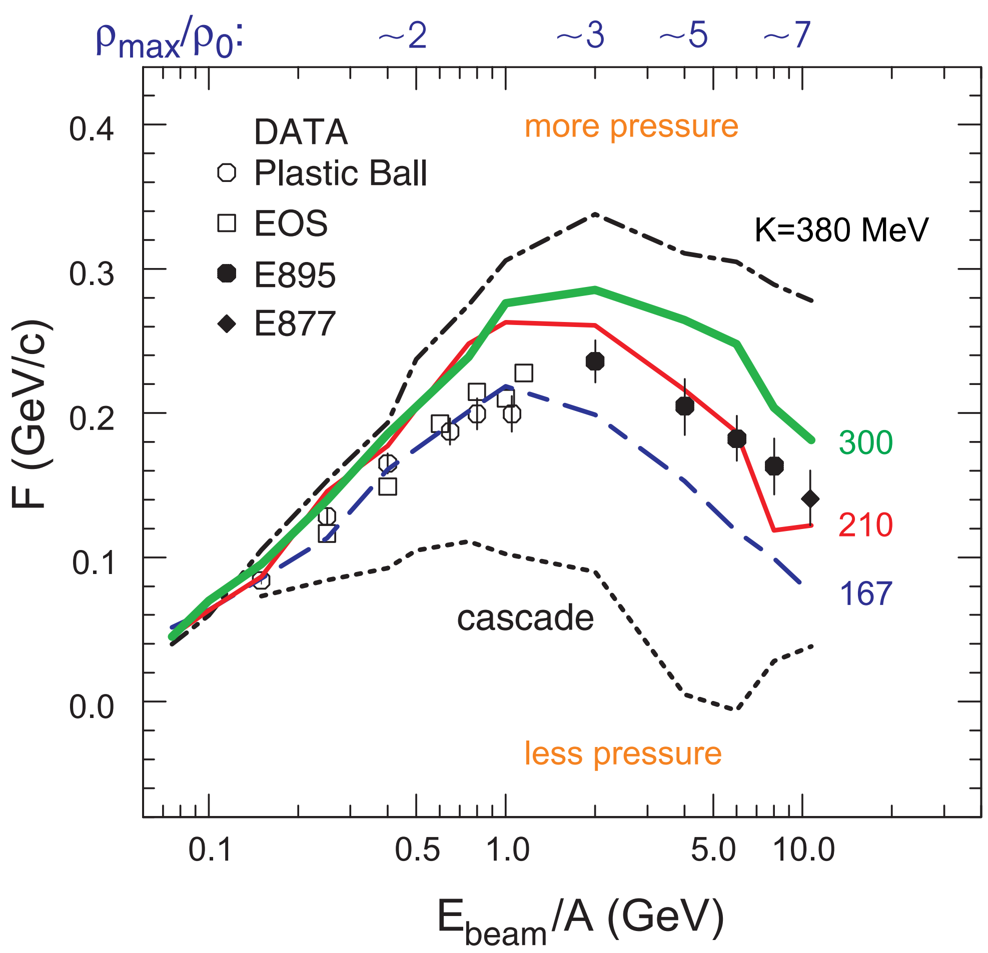
- Au-Au collisions at E_{beam}/A = [0.15 - 10] GeV, semi-peripheral
- information on the EoS from two kinds of flow: transverse and elliptic
- densities up to 2-5 times the nuclear saturation density



[Danielewicz et al., Science 298 (2002)]

Heavy ions: transverse and elliptic flow

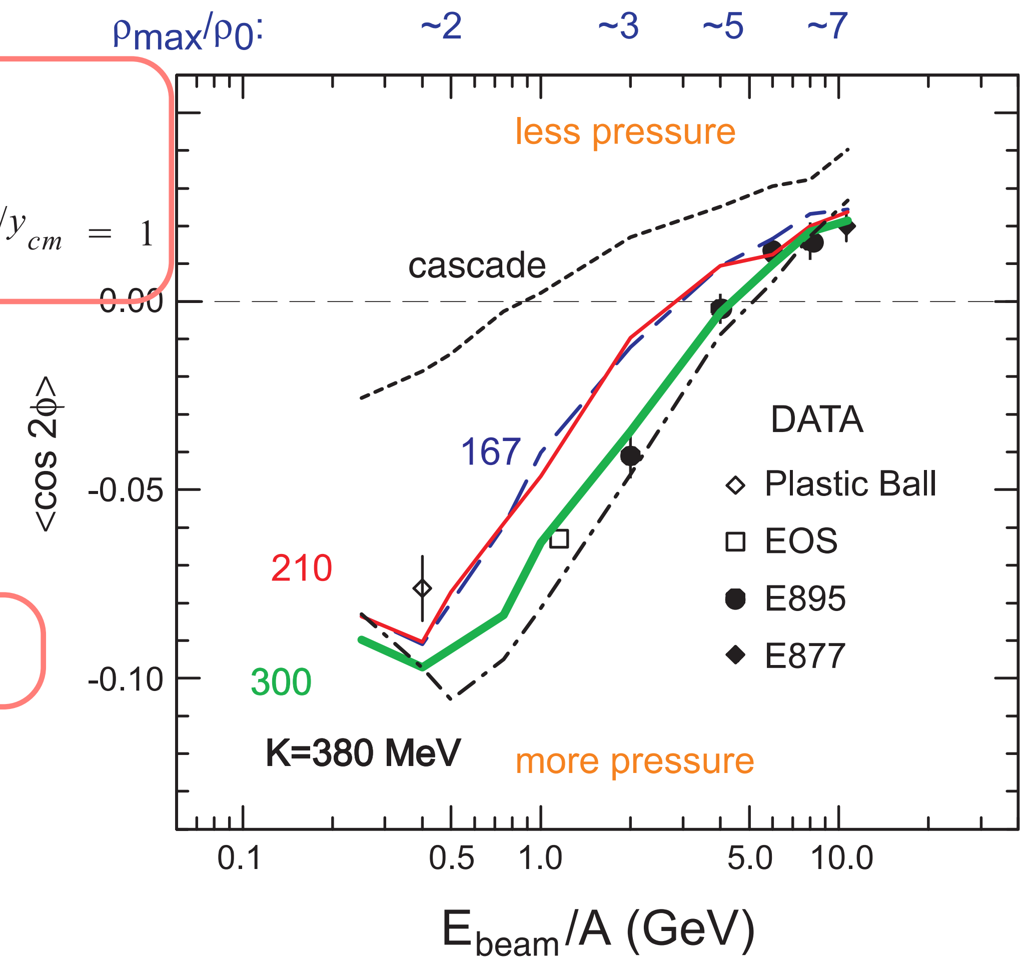
transverse flow



$$F = \left. \frac{d\langle p_x/A\rangle}{d(y/y_{\rm cm})} \right|_{y/y_{cm}\ =\ 1}$$

$$\tan\varphi = p_y/p_x$$

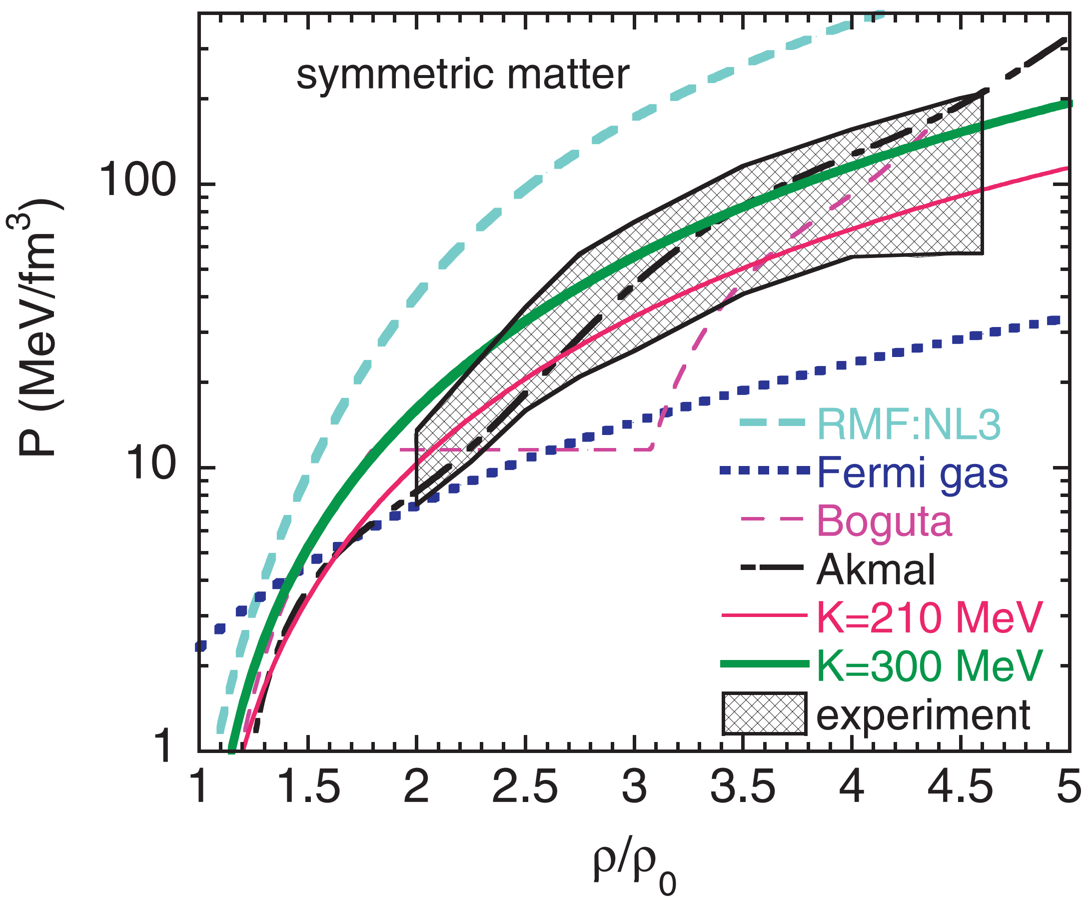
elliptic flow

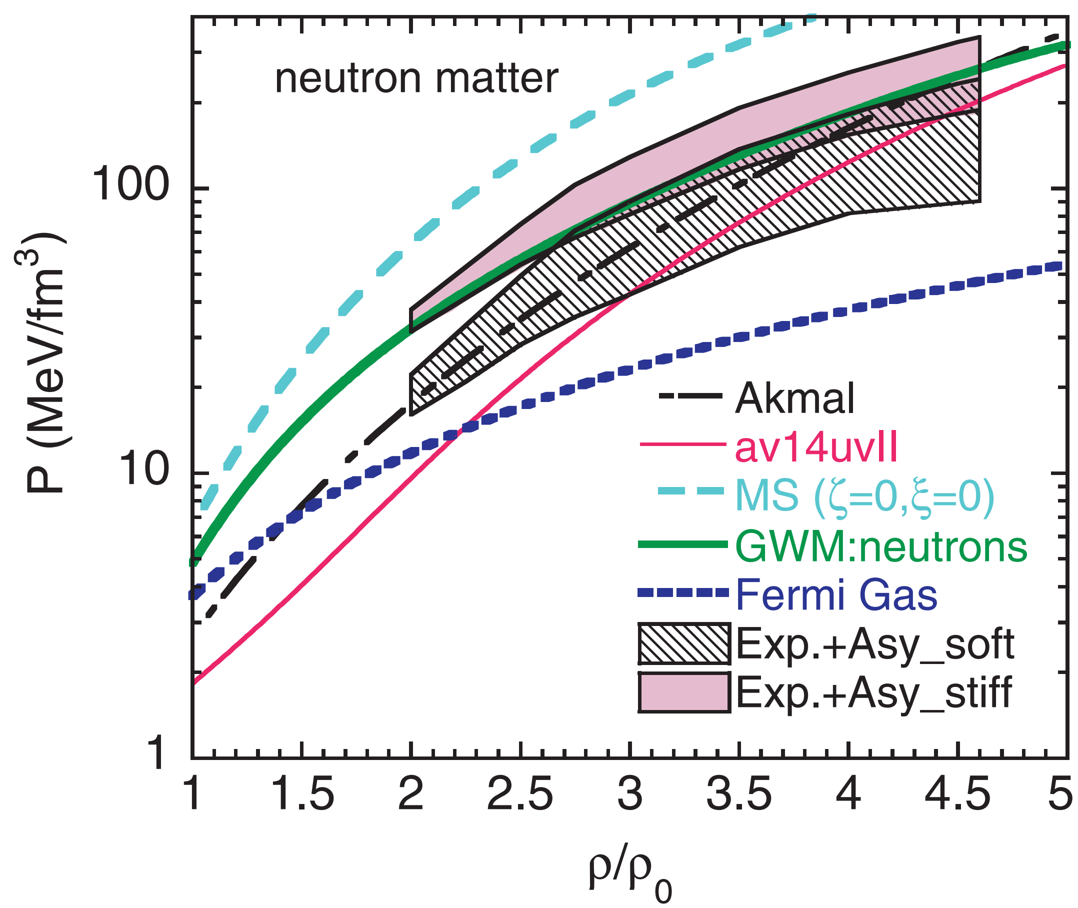


model:

- address the time-evolution of Wigner f(p,r,t) for stable/excited nucleons and pions
- model for the interaction energy ⇢ $U = (a\rho + b\rho^{\nu})/[1+(0.4\rho/\rho_0)^{\nu-1}] + \delta U_p$

Global constraints from flow observables





- not excluded a phase transition above $4\rho_0$

- extrapolation to neutron matter

↓

model for the symmetry energy

⍟ Summary of EoS applications and constraints ⍟

investigating nuclear matter properties:

neutron stars

- mass-radius: sensitivity on the global EoS
- cooling: sensitivity on the global EoS *and* on the single-particle properties

heavy-ion reactions

- flow global contraints: sensitivity on the global EoS

just examples: superfluid neutrons, symmetry energy, ...

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The nuclear many-body problem

- many-body system with two-body interaction

$$H = \sum_{i=1}^{N} T_i + \sum_{i<j}^{N} V_{ij}$$

in the nuclear case, the strong repulsive core precludes an ordinary perturbation expansion in terms of the *bare* interaction

- we need suitable methods to take into account the short-range correlations induced in the medium

➡ ab-initio calculations

- alternative: employ an *effective* potential (Skyrme, Gogny)

➡ phenomenological (mean-field) calculations

predictive power?

Variational approach

- calculates an upper bound to the ground state energy $$E = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \geq E_0$$
- wave function constructed from the unperturbed ground state

$$\Psi = \left[S \prod_{i<j} F(r_{ij}) \right] \Phi$$

correlation functions determined through the minimization

BHF approach

- rearrangement of the Hamiltonian

$$\boxed{H = T + V = T + U + \underbrace{V - U}_{\delta V} = T + U + \delta V}$$

- calculation of the ground state energy from a *perturbative* expansion in terms of δV

$$\Downarrow$$

- definition of G matrix from ladder diagrams
- Hartree-Fock calculation with the G-matrix interaction

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Green's functions basics

- mathematical definition $$\left[z-\mathfrak{L}(x,x')\right]G(x,x';z)=\delta(x-x')$$
- example: free particle $$\left[E+\frac{\nabla^2}{2m}\right]G(x,x';E)=\delta(x-x')$$

$\downarrow$

many-body interacting system

$$\left[i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}\right]G(xt,x't')=\delta(x-x')\,\delta(t-t')-i\int d^3y\,V(x-y)\,G_2(xt,x't';yt,y't^+)$$

... but then ... $G_N(x_1,t_1;....x_N,t_N)$

$$\boxed{\Psi(x_1,...,x_N;t)\longrightarrow G_N(x_1,....,x_N;t)\longrightarrow G(x_1,x_2;t)}$$

Self-consistent procedure

① start with $V(p,p')$ and $G(p,\omega)$ (ansatz)

② compute $G_2(P,k,k',\Omega)$

$\downarrow$

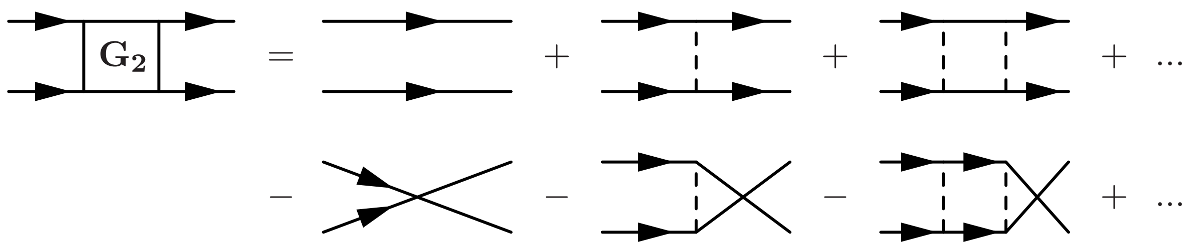
approximation $\longrightarrow$ self-energy $\Sigma(p,\omega)$

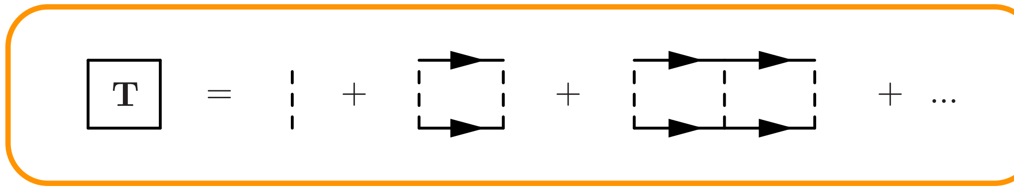
③ compute $G(p,\omega)$ by means of the Dyson eq. $G^{-1} = G_0^{-1} - \Sigma$

④ repeat ② and ③ until convergence is achieved

In-medium T-matrix

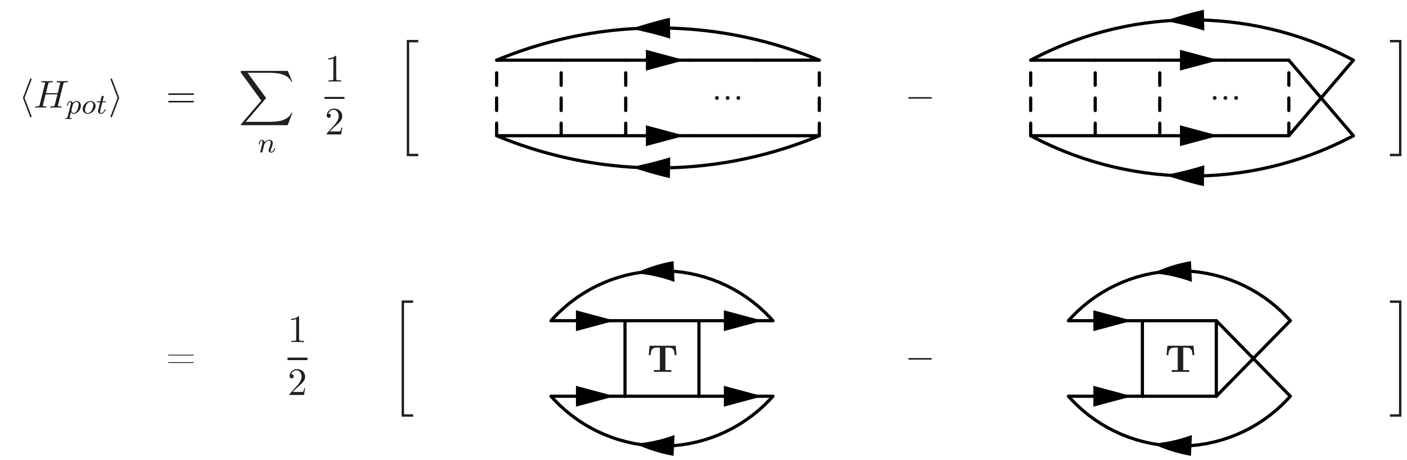
- T-matrix approximation for the two-particle propagator





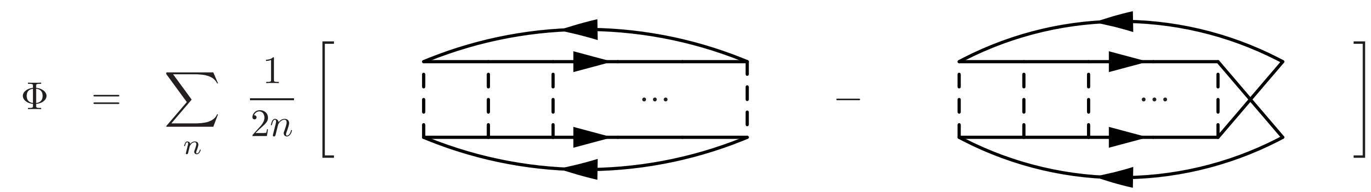
- energy per particle

$$\frac{E}{N} = \frac{1}{\rho}\left[\frac{\langle H_{tot}\rangle}{\mathcal{V}}\right] = \frac{1}{\rho}\left[\frac{\langle H_{kin}\rangle}{\mathcal{V}} + \frac{\langle H_{pot}\rangle}{\mathcal{V}}\right]$$



- grand-canonical potential

$$\Omega[G, \Sigma, \Phi] = -P\,\mathcal{V}$$



Spectral representation

- spectral GFs

$$i\,G^{>}(\mathbf{p},\omega) = [1 - f(\omega)]\,A(\mathbf{p},\omega)$$

particles

$$-i\,G^{<}(\mathbf{p},\omega) = f(\omega)\,A(\mathbf{p},\omega)$$

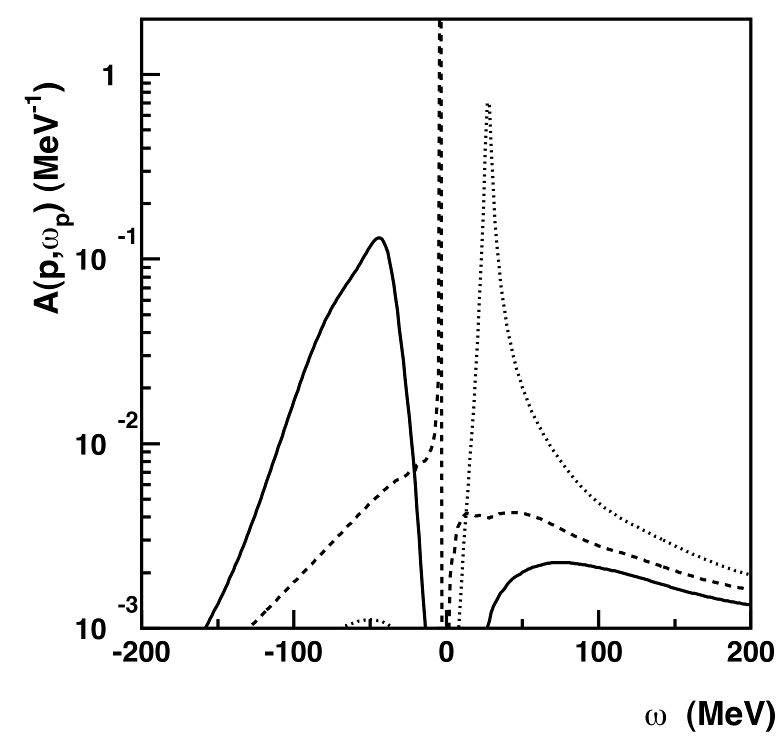
holes

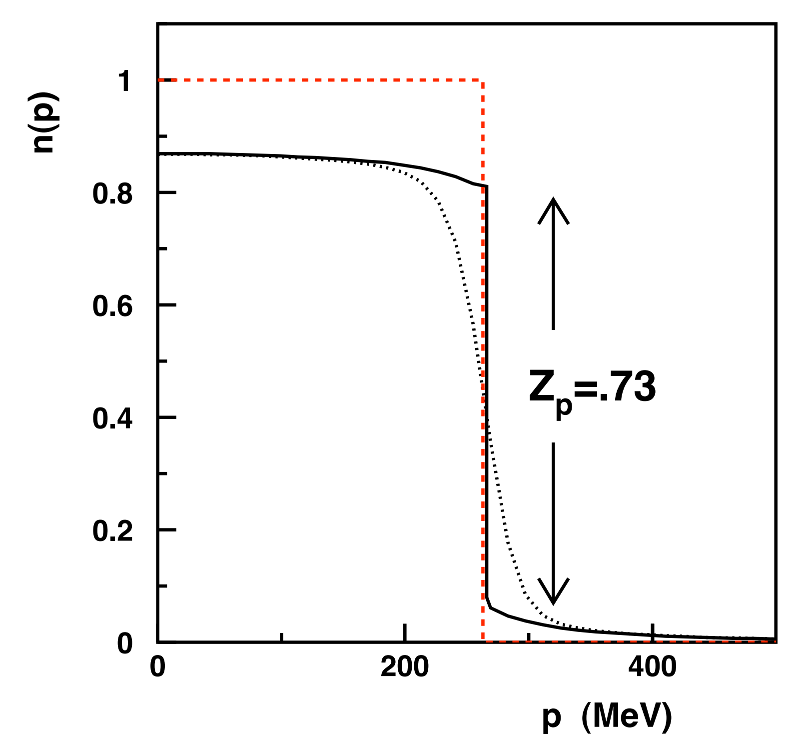
where $$f(\omega) = \frac{1}{e^{\beta\,(\omega-\mu)}+1}$$

and $$\int \frac{d\omega}{2\pi} A(\mathbf{p},\omega) = 1$$

⇢ example: free particle $A_0(\mathbf{p},\omega) = 2\pi\,\delta(\omega - p^2/2m)$

- density $-i\,G^{<}(\mathbf{p},\omega) = \langle n(\mathbf{p},\omega)\rangle$
- momentum distribution $n(\mathbf{p}) = \int \frac{d\omega}{2\pi} A(\mathbf{p},\omega)\,f(\omega)$

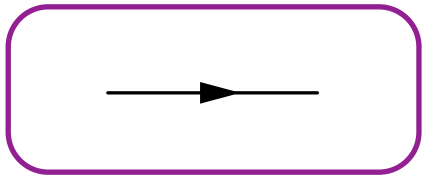




Finite temperature Green's functions

- single-particle propagator on the time-contour

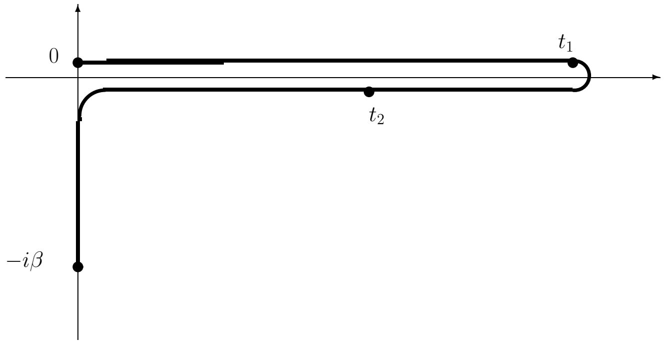
$$i\,\mathbf{G}_{A\alpha B\beta}(\mathbf{r},t;\mathbf{r}',t') = \left\langle \mathcal{T}\,\psi_{A\alpha}(\mathbf{r},t)\,\psi^\dagger_{B\beta}(\mathbf{r}',t')\right\rangle$$



weird, but...

carry statistical mechanics information of the system

$$\langle\hat{O}\rangle = \mathrm{tr}\left[\hat{\rho}\,\hat{O}\right] = \frac{\mathrm{tr}\left[e^{-\beta(H-\mu N)}\hat{O}\right]}{\mathrm{tr}\left[e^{-\beta(H-\mu N)}\right]}$$



➡ consistency between macroscopic and microscopic observables

other ab-initio approaches:

- *Bloch-De Dominicis (⇒ BBG)* → "frozen correlations" approximation
- *variational* → work in progress

Technical aspects

- numerical solution of coupled integro-differential equations

  → iterative scheme

- code in **`Fortran 77`**
- use Fast Fourier Transform (FFT) and convolution theorem

$$\int d\omega' F_1(\omega' - \omega)\, F_2(\omega') = \left[F_1^T(t) F_2^T(t)\right]^T$$

- discretization on a fixed-spacing grid
- cut-off dependence under control
- each point (T, ρ, δ, V) ~ 100 hours

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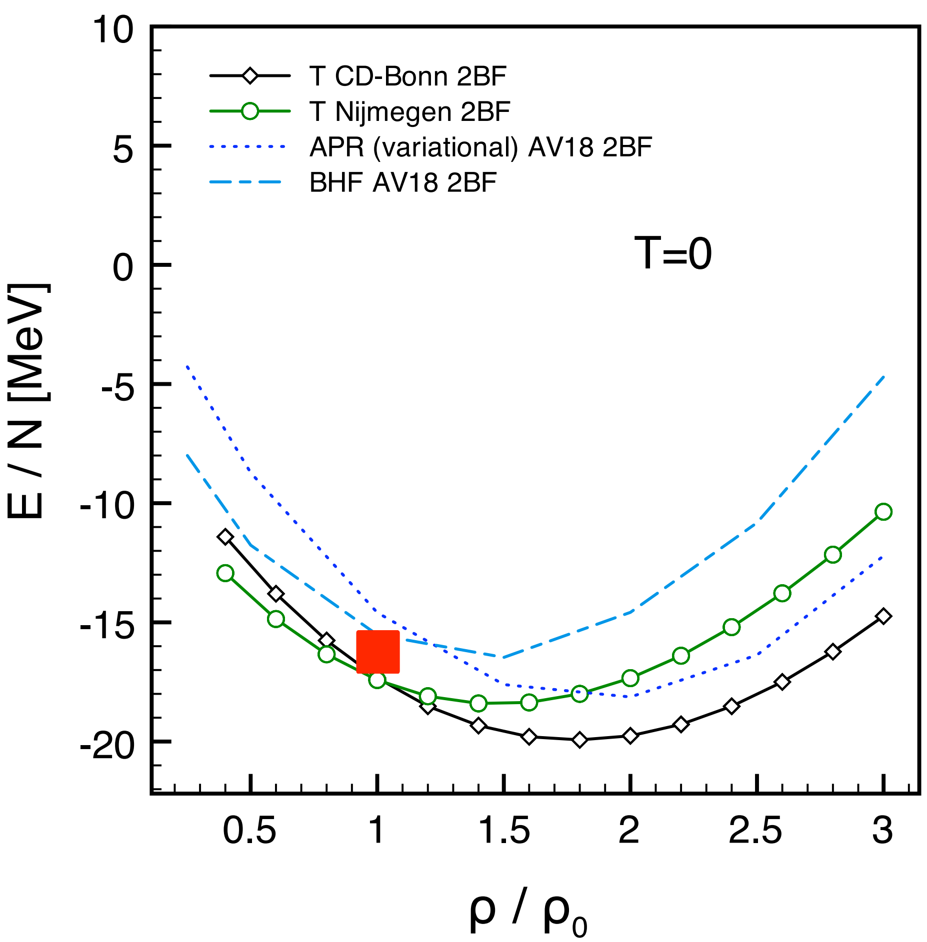
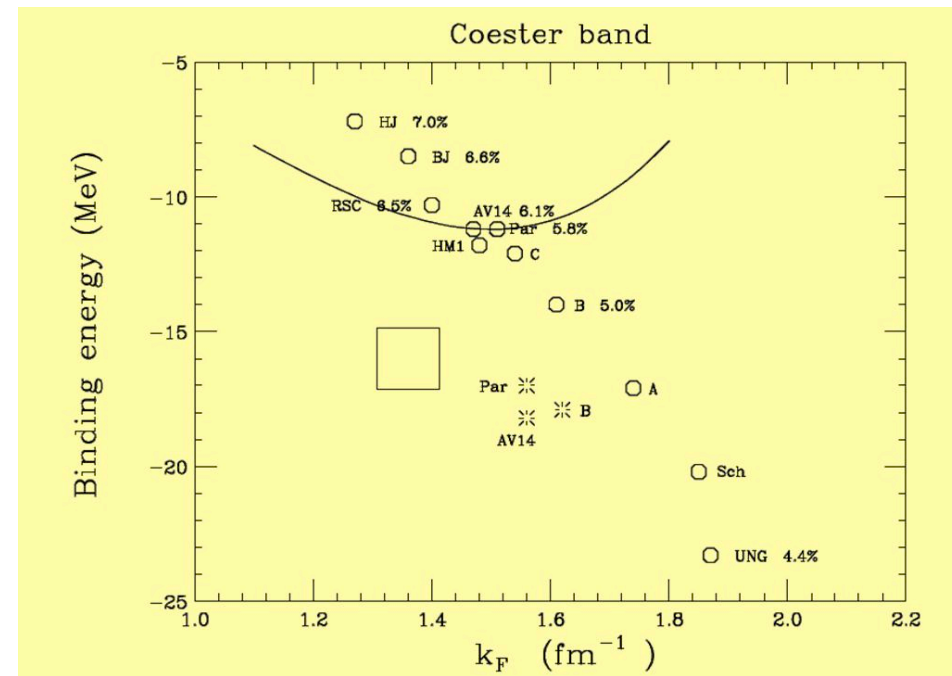
The need for three-body forces

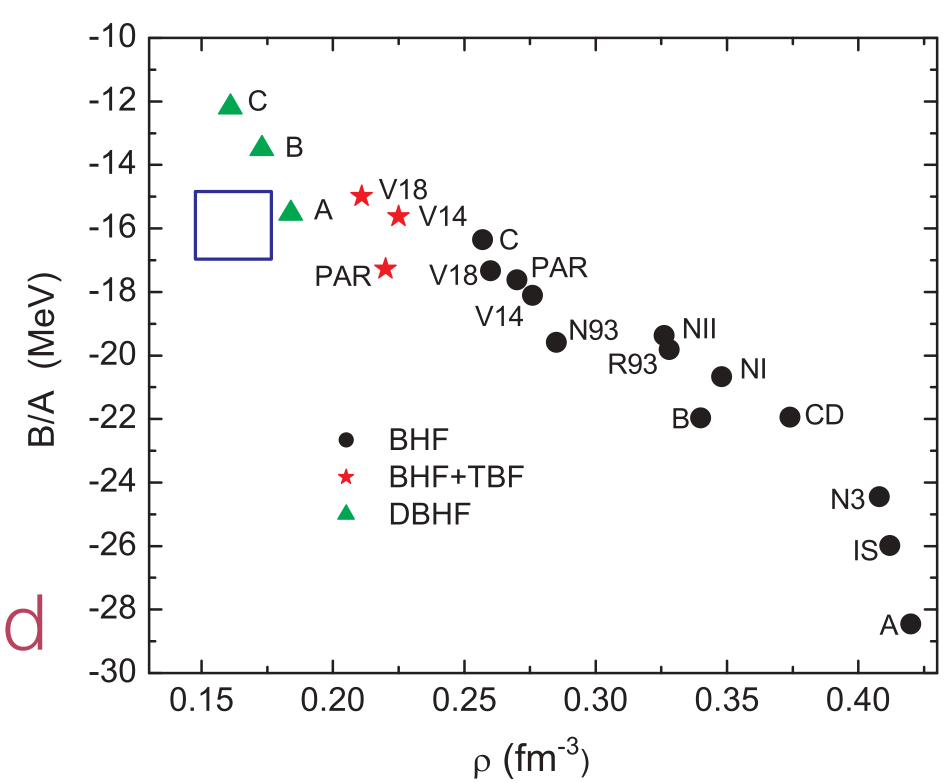
- empirical values for saturation

$$\rho_{\text{sat}} \equiv \quad \rho_0 \quad = 0.16 \pm 0.01\ \text{fm}^{-3}$$
$$E_{\text{sat}}/N \equiv \quad B \quad = 16 \pm 1\ \text{MeV}$$



Coester band





New Coester band

[Akmal et al., Phys. Rev. C 58 (1998)]

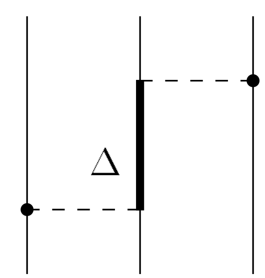
[Baldo and Maieron, J. Phys. G 34 (2007)]

Urbana three-body forces

$$V_{ijk}^{Urbana} = V_{ijk}^{2\pi} + V_{ijk}^{R}$$

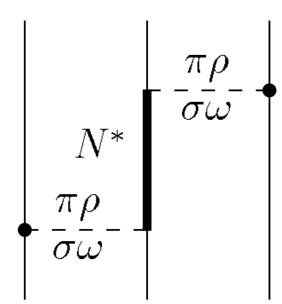
- modification of the internal structure of hadrons

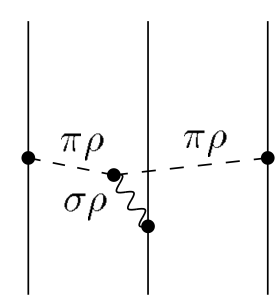
⇢ Δ-excitation

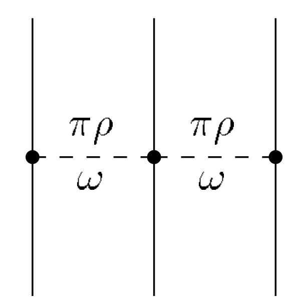


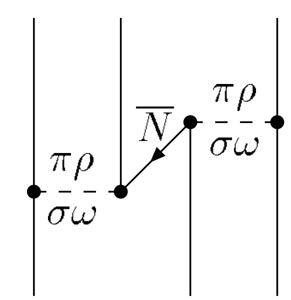
↔ 2π exchange

⇢ and others:









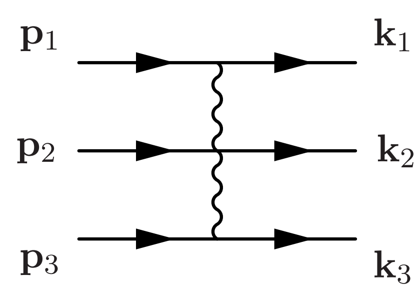
Derivation of the effective potential

- need to derive an effective two-body potential

$$V_3^{eff}(\mathbf{q},\mathbf{q}') = \sum_{\sigma\,\tau} \int \frac{d\mathbf{k}}{(2\pi)^3}\, n(\mathbf{k})\, V_3^{FT}(\mathbf{k},\mathbf{q},\mathbf{q}')$$

to be inserted in the T-matrix $\qquad V \to V' = V + V_3^{eff}$

Fourier transform



spin-isospin average

$$\begin{aligned} V_3^{eff}(\mathbf{q},\mathbf{q}') &= V_s^R(\mathbf{q},\mathbf{q}') + V_s^{2\pi}(\mathbf{q},\mathbf{q}') \\ &+ V_{\sigma\tau}^{2\pi}(\mathbf{q},\mathbf{q}')\, \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'\, \boldsymbol{\tau}\cdot\boldsymbol{\tau}' + V_{S\tau}^{2\pi}(\mathbf{q},\mathbf{q}')\, S(\mathbf{q},\mathbf{q}')\, \boldsymbol{\tau}\cdot\boldsymbol{\tau}' \end{aligned}$$

projection into partial waves

$$\langle q|V_J^{S=0}(P)|q'\rangle = \frac{1}{4\pi^2}\int d\Omega_{qq'}\, \mathrm{P}_J(\Omega_{qq'}) \times \begin{cases} V_s^R + V_s^{2\pi} - 3V_{\sigma\tau}^{2\pi} & \text{for } J \text{ even} \\ V_s^R + V_s^{2\pi} + 9V_{\sigma\tau}^{2\pi} & \text{for } J \text{ odd} \end{cases}$$

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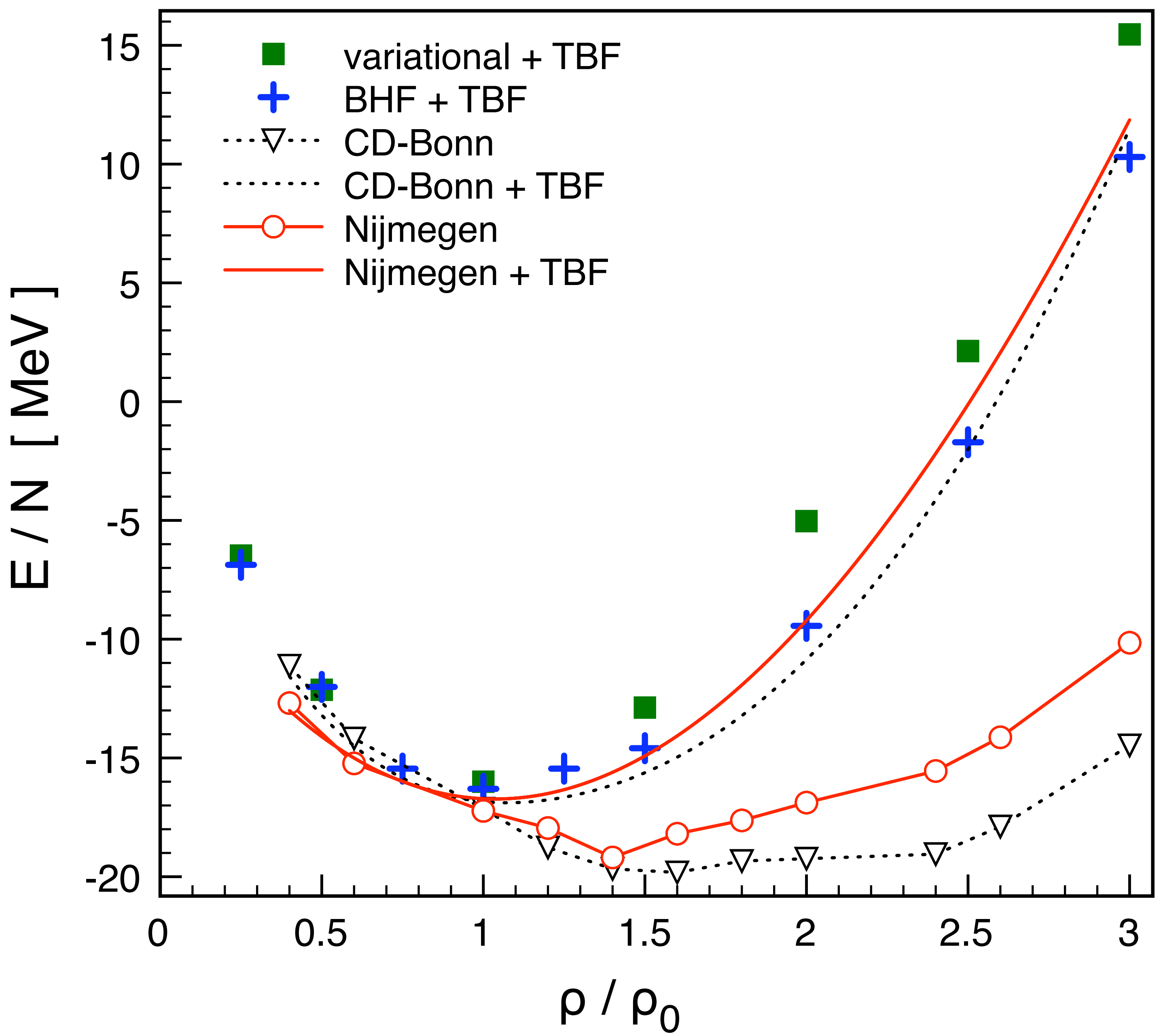
Energy per particle in symmetric matter

CD-Bonn

$$\begin{aligned} E/N &= -16.3\,\mathrm{MeV} \\ \rho &= 0.171\,\mathrm{fm}^{-3} \end{aligned}$$

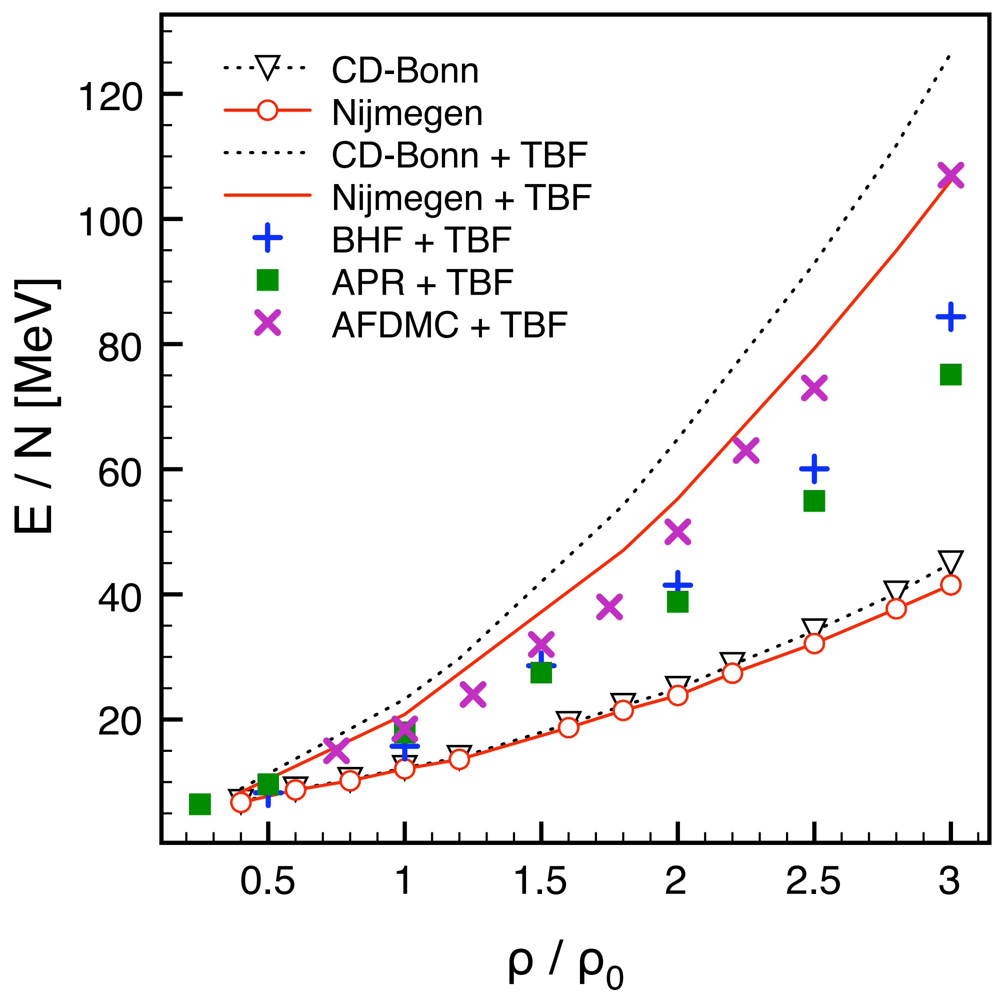
Nijmegen

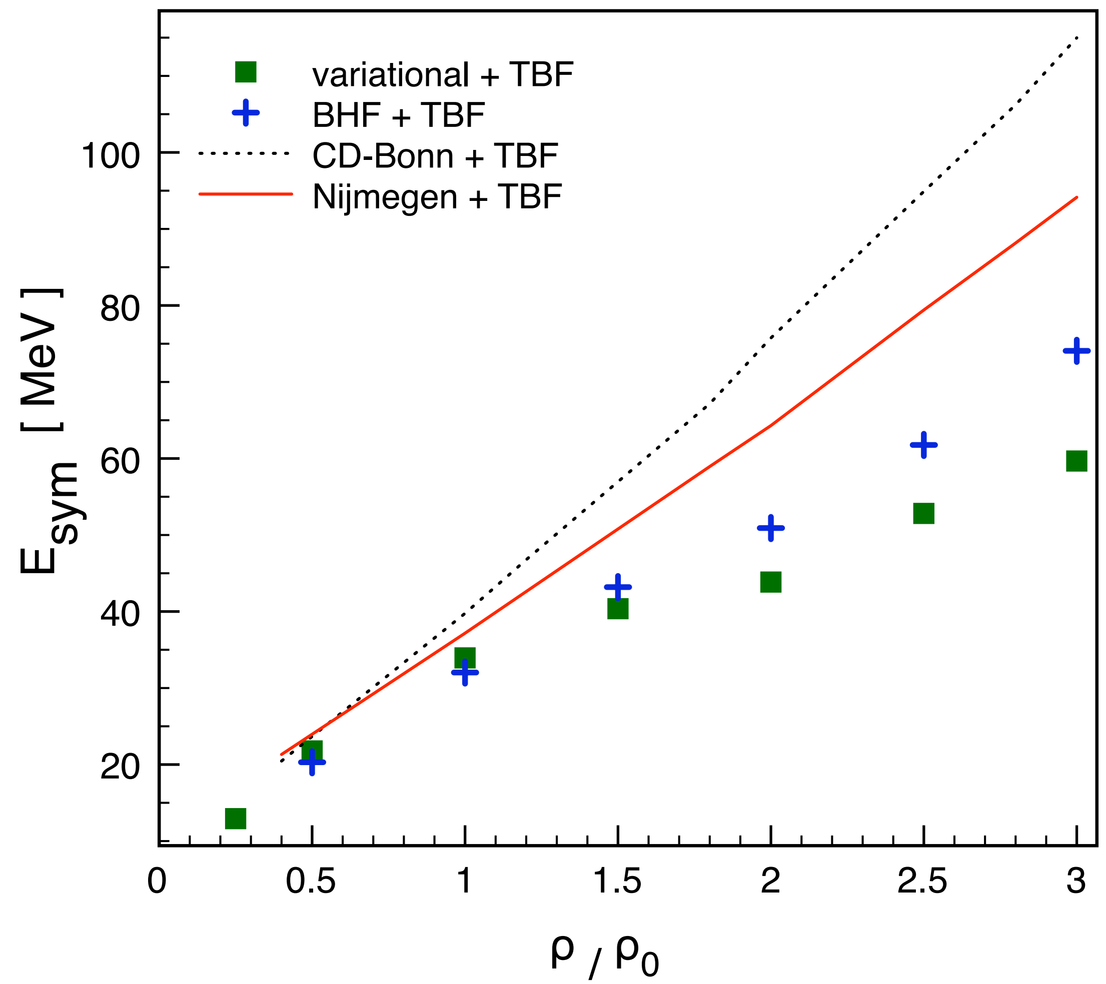
$$\begin{aligned} E/N &= -16.4\,\mathrm{MeV} \\ \rho &= 0.164\,\mathrm{fm}^{-3} \end{aligned}$$



[Somà and Bożek, Phys. Rev. C 78 (2008)]

Energy in neutron matter & symmetry energy





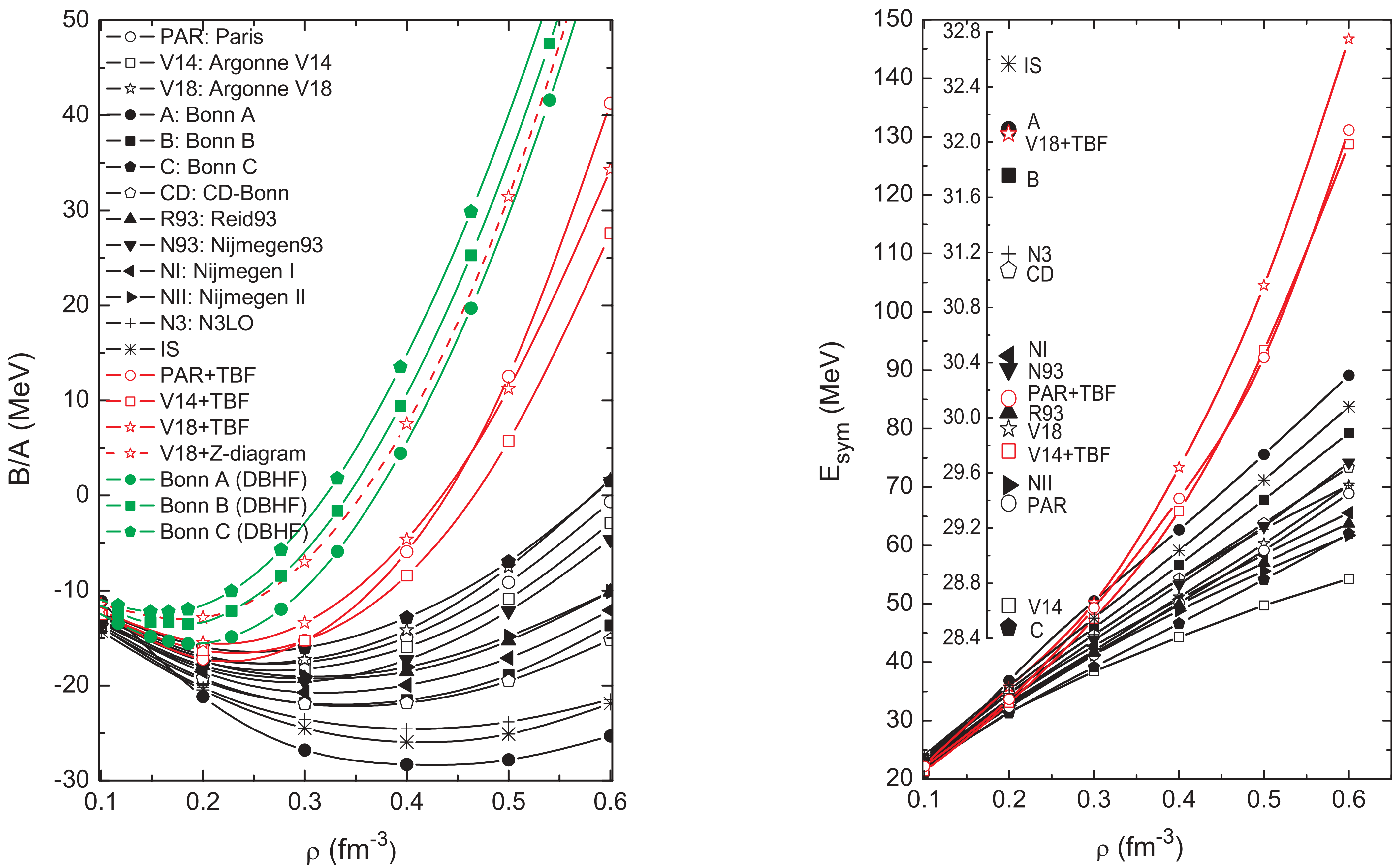
- parabolic approximation

$$\frac{E}{N}(\rho,\delta) = \frac{E}{N}(\rho,\delta=0) + \delta^2\, E_{sym}(\rho) + \mathcal{O}(\delta^4)$$

$$\delta \equiv \frac{\rho_n - \rho_p}{\rho_{tot}}$$

Density dependence

- large uncertainties in the density dependence of the energy and symmetry energy



[Li et al., Phys. Rev. C 74 (2006)]

Entropy and pressure

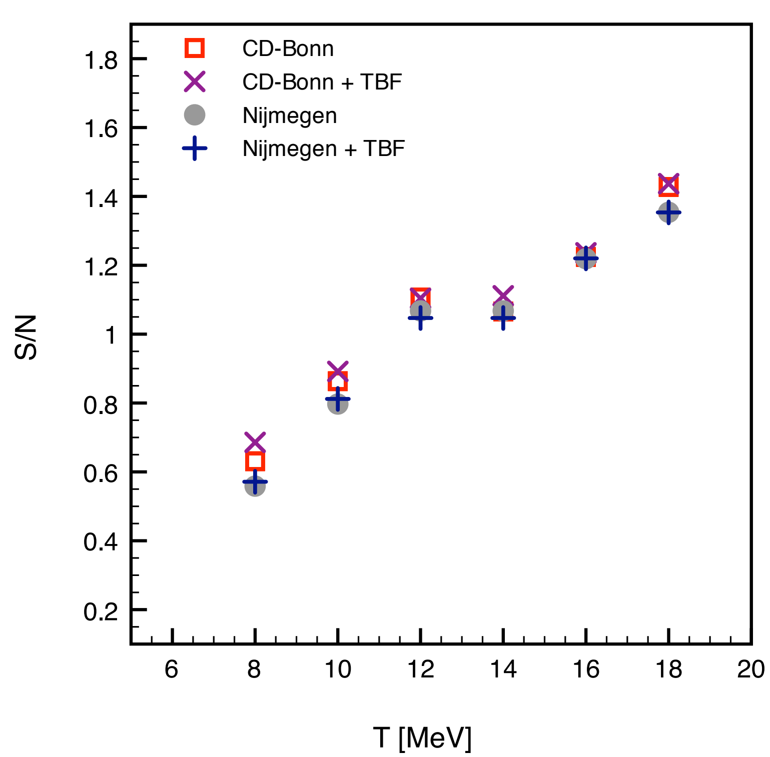
- direct (diagrammatic) calculation of P

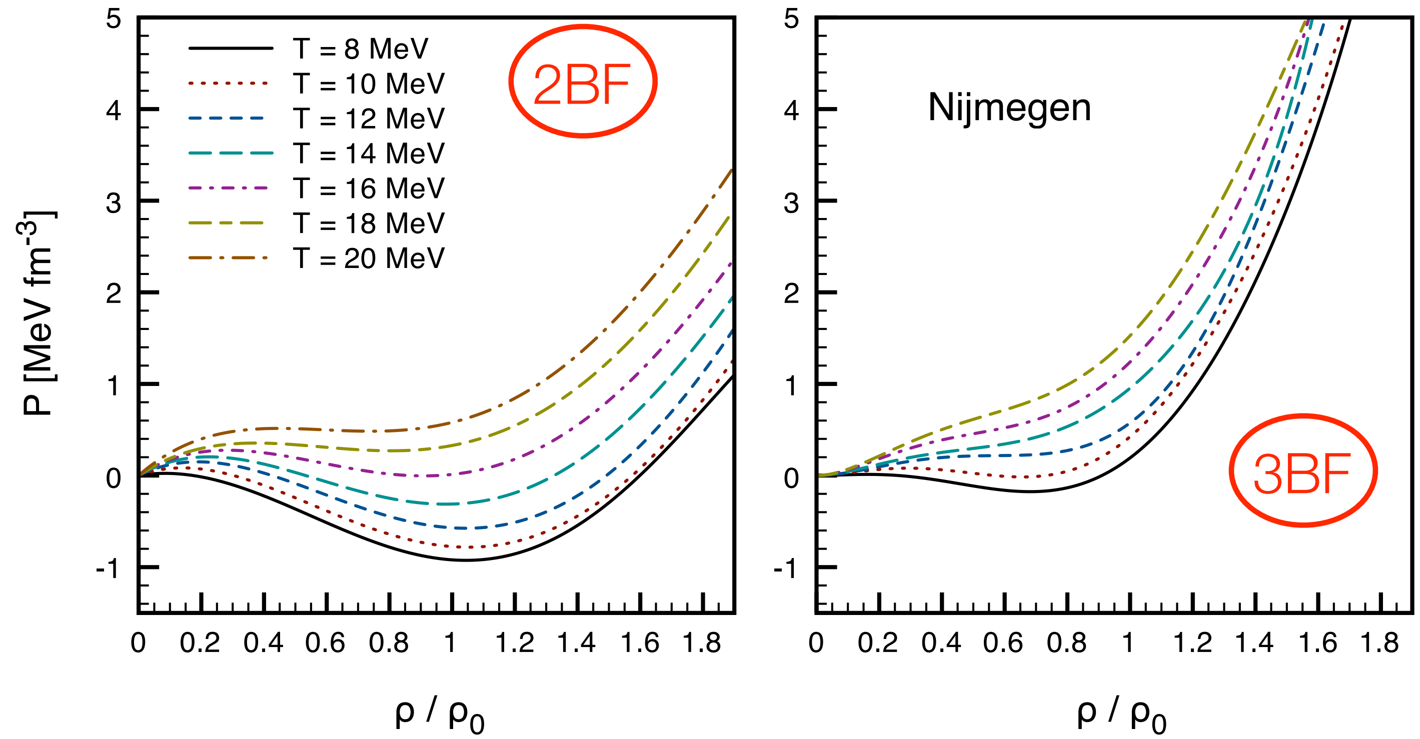
$$\Omega[G,\Sigma,\Phi] = -P\,\mathcal{V}$$

$$\Phi = \int_0^1 \frac{d\lambda}{\lambda} H_{pot}(\lambda V, G_{\lambda=1})$$

- entropy from

$$\frac{S}{N} = \frac{1}{T}\left(\frac{E}{N} - \mu + \frac{P}{\rho}\right)$$





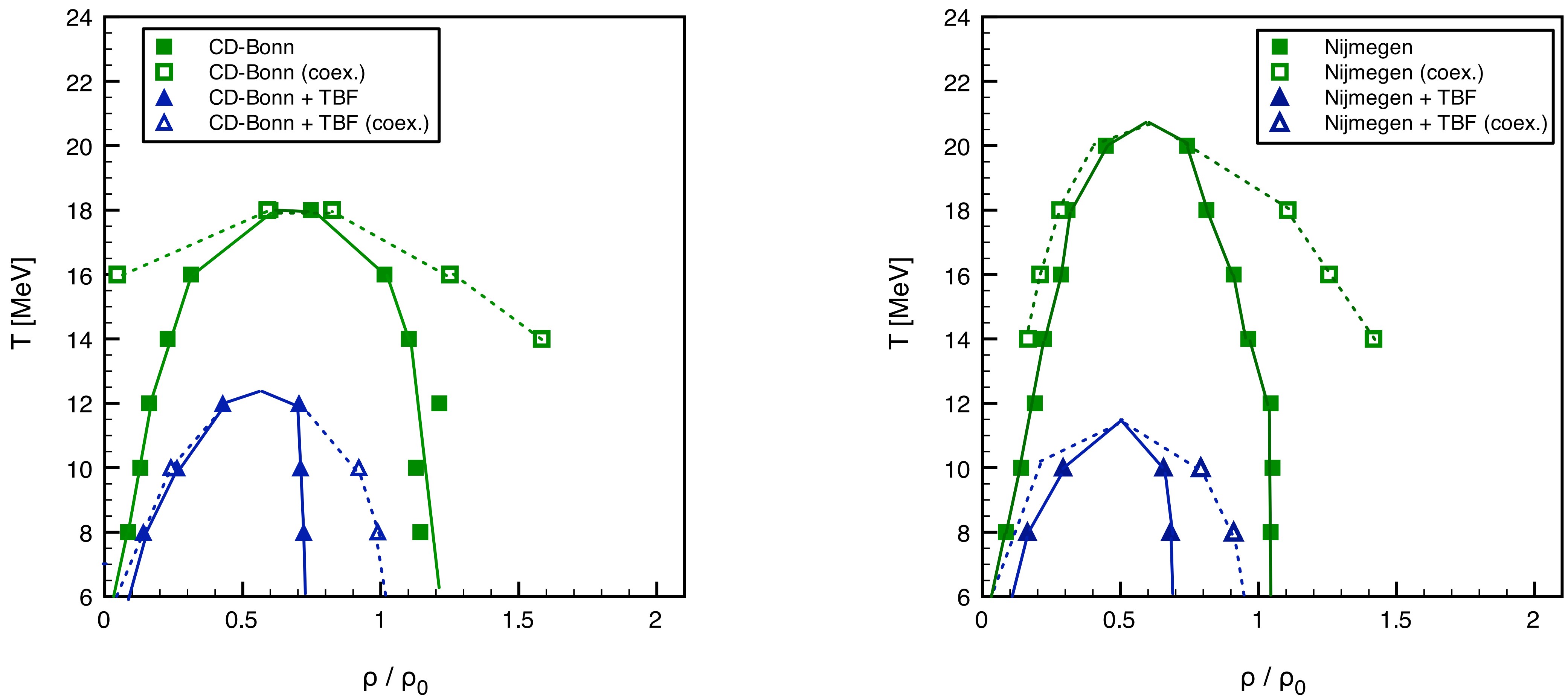
spinodal region

$$\left.\frac{\partial P}{\partial \rho}\right|_T < 0 \qquad \left.\frac{\partial \mu}{\partial \rho}\right|_T < 0$$

coexistence region

$$\mu(\rho_g) = \mu(\rho_l)$$
$$P(\rho_g) = P(\rho_l)$$

Spinodal region



potential	T_c (MeV)	ρ_c (fm^{-3})	P_c ($\mathrm{MeV\,fm}^{-3}$)	$\dfrac{P_c}{\rho_c T_c}$
CD-Bonn	18	0.107	0.43	0.22
CD-Bonn + TBF	12.5	0.096	0.14	0.12
Nijmegen	20.5	0.094	0.50	0.26
Nijmegen + TBF	11.5	0.088	0.15	0.14

[Somà and Bożek, Phys. Rev. C 80 (2009)]

Critical temperature in nuclear matter

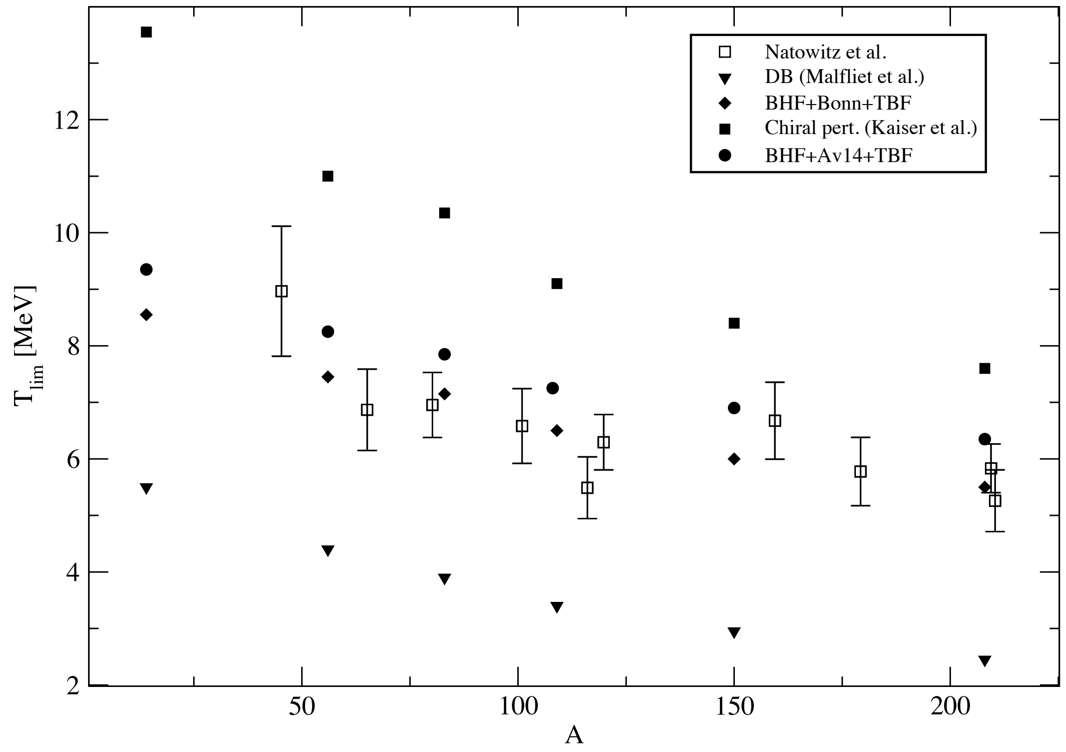
- Bloch-De Dominicis → 9 - 20 MeV [Das et al. ; Baldo et al.]
- Dirac-Brueckner-Hartree-Fock → 10 - 13 MeV [Ter Haar et al ; Huber et al.]
- Green's functions (NN only) → 16 - 19 MeV [Rios et al. ; VS, Bożek]
- Green's functions (NN+NNN) → ~12 MeV [VS, Bożek]

"Limiting temperature" in finite nuclei

- Coulomb and surface effects

the nucleus undergoes a mechanical instability before reaching T_c

$$\delta P = P_C + P_S(T)$$



[Baldo et al., Phys. Rev. C 69 (2004)]

Outline

- Introduction
  - ⇢ *what* is nuclear matter

    definitions, limits of validity, EOS
  - ⇢ *why* we study it

    applications and constraints
  - ⇢ *how* we study it

    phenomenological vs. ab-initio approaches
- Self-consistent Green's functions at finite temperature
- The need for three-body forces
- Results I : equation of state
- Results II : in-medium single-particle properties
- Conclusions and current plans

Single-particle properties

- spectral representation

$$-i\, G^{<}(\mathbf{p},\omega) = f(\omega)\, A(\mathbf{p},\omega)$$
$$i\, G^{>}(\mathbf{p},\omega) = [1 - f(\omega)]\, A(\mathbf{p},\omega)$$

recall that the free spectral function is $A_0(\mathbf{p},\omega) = 2\pi\,\delta(\omega - p^2/2m)$

⇢ *quasiparticle approximation* $A_{qp}(\mathbf{p},\omega) = 2\pi\,\delta(\omega - p^2/2m - \mathrm{Re}\,\Sigma(\mathbf{p},\omega))$

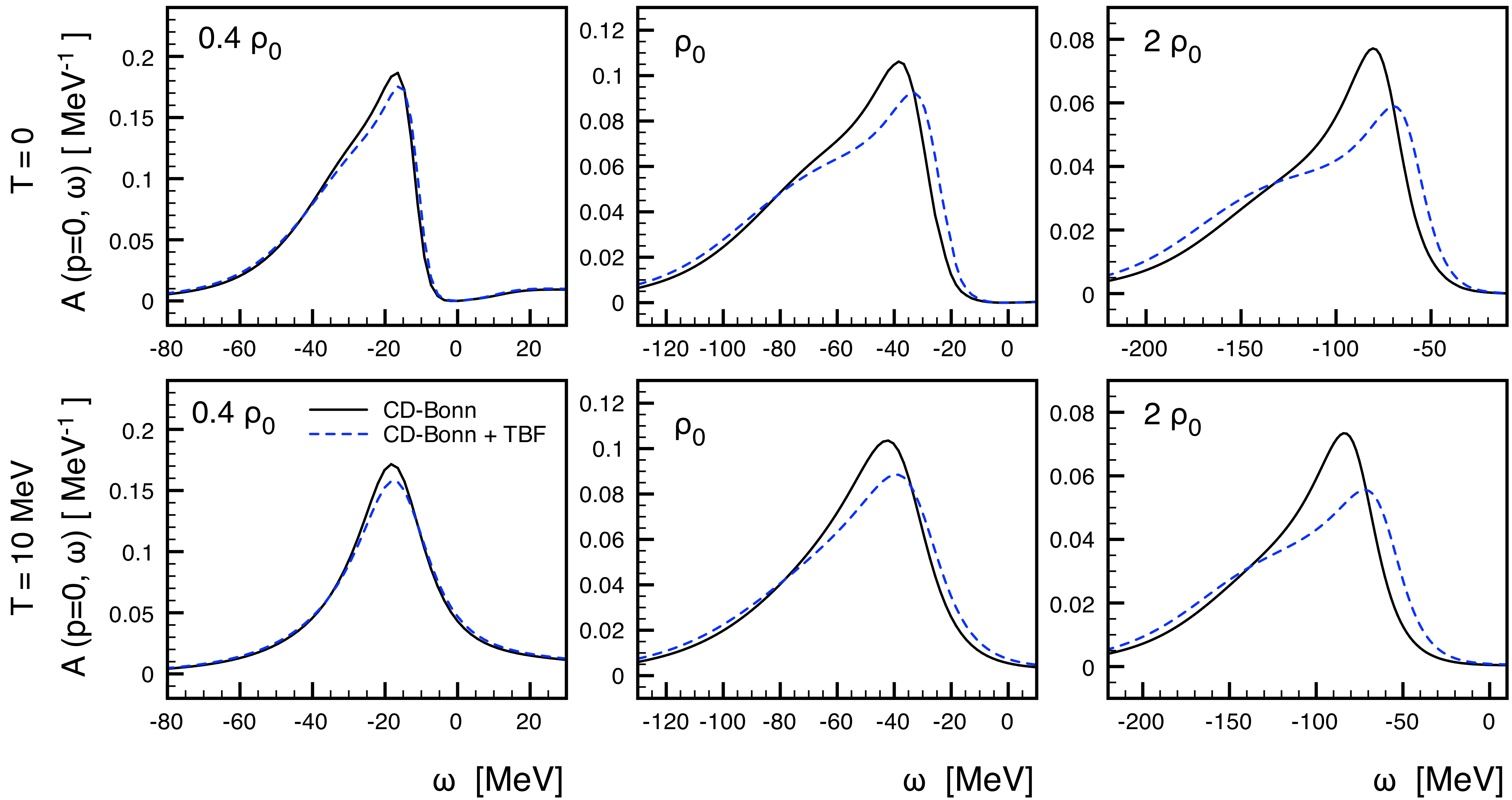
- effective mass

$$\left.\frac{\partial\,\omega_p}{\partial\,p^2}\right|_{p=p_F} = \frac{1}{2m^\star}$$

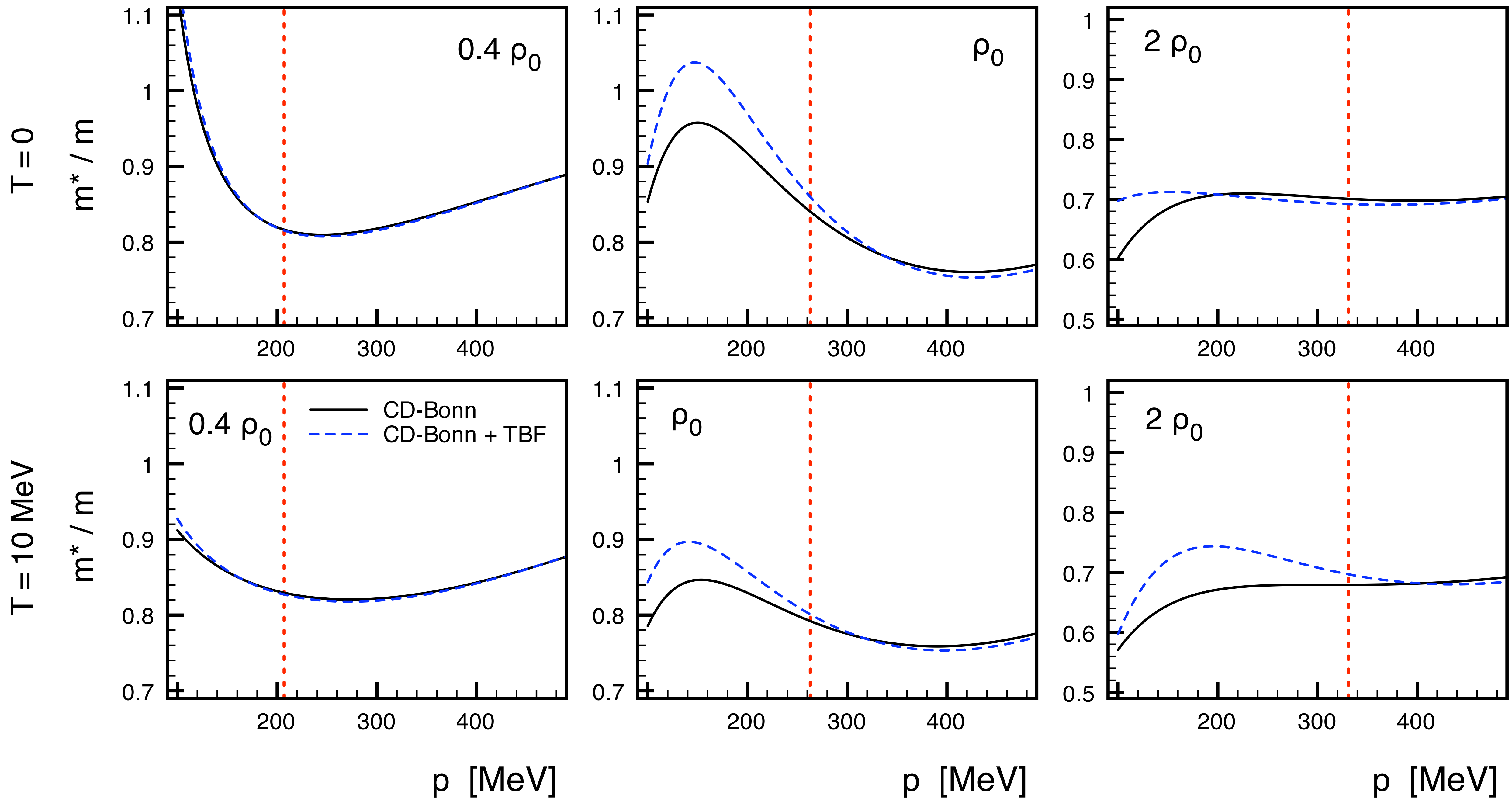
where

$$\omega_p = \frac{p^2}{2m} + \mathrm{Re}\,\Sigma(p,\omega_p)$$

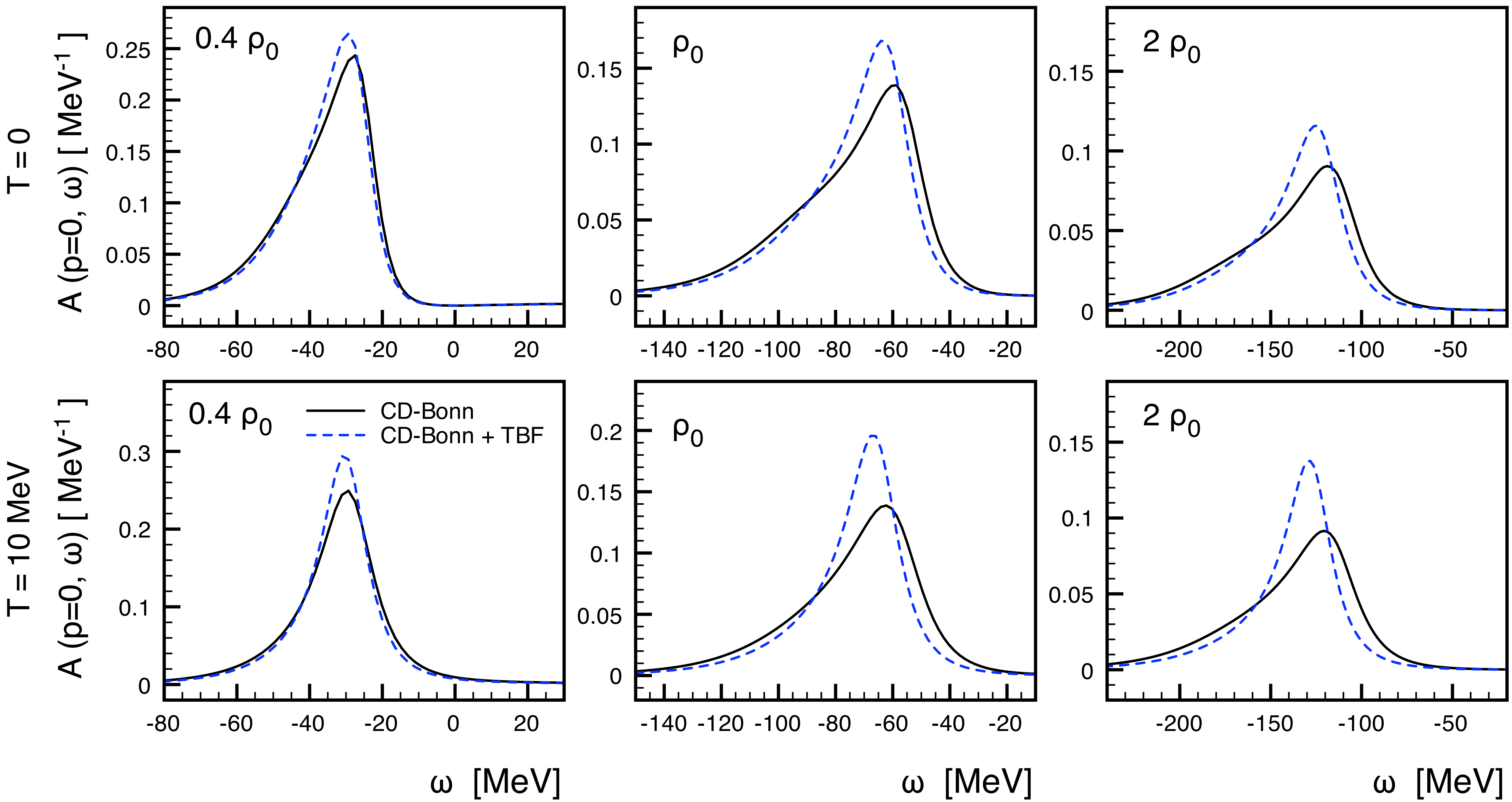
Spectral function - symmetric matter



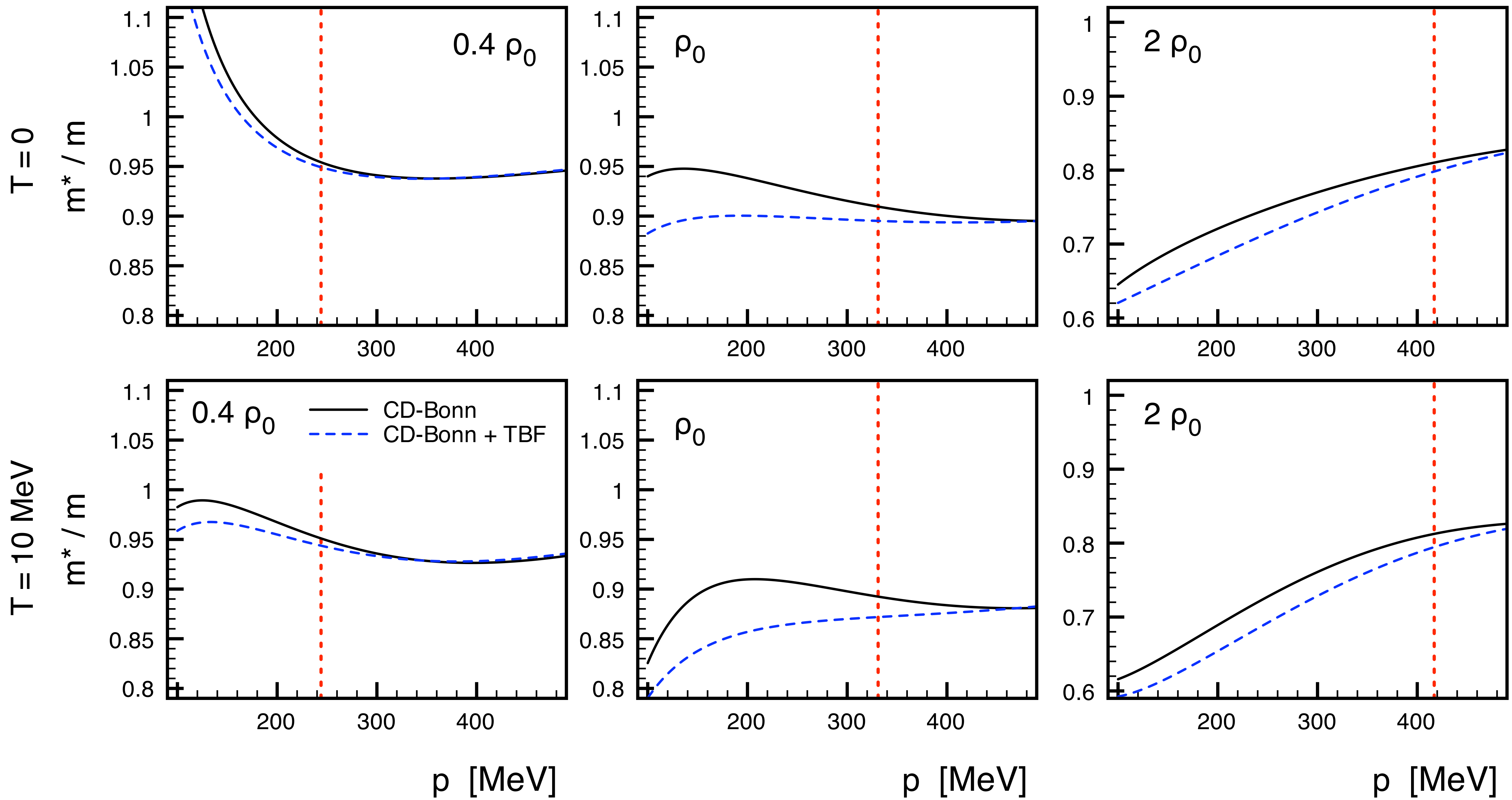
Effective mass - symmetric matter



Spectral function - neutron matter



Effective mass - neutron matter



⍟ Summary of the results ⍟

- first spectral calculations of the nuclear matter EOS with TBF
- correct saturation properties
- entropy not affected by nucleon correlations
- study of the liquid-gas phase transition $\longrightarrow$ $T_c \simeq 12\,\mathrm{MeV}$

- single-particle properties ⇢ TBF effects above saturation density
- spectral function ⇢ opposite effect in symmetric and in pure neutron matter

Extensions of the technique:

- asymmetric nuclear matter
- explicit inclusion of superfluidity
- application to nuclei

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Plans @ ESNT

The nuclear many-body problem

- many-body system with two-body interaction

$$H = \sum_{i=1}^{N} T_i + \sum_{i<j}^{N} V_{ij}$$

in the nuclear case, the strong repulsive core precludes an ordinary perturbation expansion in terms of the *bare* interaction

- we need suitable methods to take into account the short-range correlations induced in the medium

➡ ab-initio calculations

- alternative: employ an *effective* potential (Skyrme, Gogny)

➡ phenomenological (mean-field) calculations

predictive power?

The nuclear many-body problem

- many-body system with two-body interaction

$$H = \sum_{i=1}^{N} T_i + \sum_{i<j}^{N} V_{ij}$$

- we need suitable methods to take into account the short-range correlations induced in the medium

  ➡ ab-initio calculations

- take into account part of the short-range correlations already in the potential

  ➡ low-momentum interactions

- alternative: employ an *effective* potential (Skyrme, Gogny)

  ➡ phenomenological (mean-field) calculations

Bridging SCGF and EDFs

- in finite nuclei ab-initio calculations are limited
- energy density functionals ⇢ lack of predictive power

many-body theory $G(\mathbf{r}, t)$

DFT $n(\mathbf{r})$

EDF $E[n]$

many-body perturbation theory

low-momentum interactions
very promising...