THE CONTROVERSY CONCERNING THE DEFINITION OF QUARK AND GLUON ANGULAR MOMENTUM:

WHAT'S IT ALL ABOUT AND DOES IT MATTER?

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Meant to resolve controversy. Has opened up Pandora's box!

Important question: how are the momentum and angular momentum of a nucleon built up from the momenta and angular momenta of its constituents?

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• New controversy about angular momentum sum rules

OUTLINE

- Pedagogical: Canonical and Belinfante angular momentum

- The new schemes and the claim that all textbooks on QED have been wrong for past 50 years

- Physical content of the new schemes

- Interpretation of $\Delta G(x)$

Actually two kinds of problem:

- Any interacting particles

- Specific to gauge theories

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Since controversy arose in QCD, will first discuss gauge aspect
Since problem already arises in QED, will illustrate via QED

Theory invariant under space-time and Lorentz transformations

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Conserved energy-momentum tensor $t^{\mu\nu}$ $\qquad \partial_\mu t^{\mu\nu} = 0$

Conserved angular momentum tensor $\mathcal{M}^{\mu\nu\lambda}$ $\qquad \partial_\mu \mathcal{M}^{\mu\nu\lambda} = 0$

Theory invariant under space-time and Lorentz transformations

From Lagrangian via Noether's theorem, derive:

Conserved energy-momentum density

$$t^{\mu\nu} \qquad \partial_\mu t^{\mu\nu} = 0$$

Conserved angular momentum density

$$\mathcal{M}^{\mu\nu\lambda} \qquad \partial_\mu \mathcal{M}^{\mu\nu\lambda} = 0$$

Call these CANONICAL: $t^{\mu\nu}_{can}$ and $\mathcal{M}^{\mu\nu\lambda}_{can}$

Total 4-momentum: $P^{\mu}_{can} = \int d^3x \; t^{0\mu}_{can}(x)$

Total angular momentum: $M^{ij}_{can} \equiv \int d^3x \mathcal{M}^{0ij}_{can}(x)$

$$J^k_{can} = \frac{1}{2}\epsilon_{kij} \, M^{ij}_{can}$$

THESE ARE GENERATORS OF SPACE-TIME AND LORENTZ TRANSFORMATIONS

For any set of fields $\phi_r(x)$:

$$i\,[P^{\mu}_{can},\, \phi_r(x)] = \partial^{\mu}\, \phi_r(x)$$

$$i[M^{ij}_{can}\,, \phi_r(x)] = (x^i\partial^j - x^j\partial^i)\phi_r(x) + (\Sigma^{ij})_r{}^s\phi_s(x)$$

Pros and Cons of

$$J_{can}$$

Similar issues in QED and QCD: mainly discuss QED for simplicity.

$$
\begin{aligned}
\boldsymbol{J}_{can} &= \int d^3x\,\psi^\dagger \boldsymbol{\gamma}\gamma_5\psi + \int d^3x\,\psi^\dagger[\boldsymbol{x}\times(-i\boldsymbol{\nabla})]\psi \\
&+ \int d^3x\,(\boldsymbol{E}\times\boldsymbol{A}) + \int d^3x\,E^i[\boldsymbol{x}\times\boldsymbol{\nabla}A^i] \\
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\end{aligned}$$

Pros:

- $\boldsymbol{J}$ Looks like sum of free electron plus free photon terms

- Total energy looks like electron energy plus photon energy plus H_{int}

- Photon angular momentum is split into SPIN and ORBITAL parts: similar in QCD.....gluon spin!

Cons:

- ONLY electron spin term is Gauge Invariant

- All Textbooks on QED say:

  The angular momentum of the photon cannot be split in a gauge invariant way into a spin part and an orbital part

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Ji: Yes: If experimentally measurable, the operators should be gauge invariant.

E.L: No: What you measure are matrix elements. The physical matrix elements must be gauge invariant.

An example: Non-gauge invariance of the QED momentum operator

Theorem : Theory invariant under local c-number gauge transformations.

$$A^\mu(x) \to A^\mu(x) + \partial^\mu \Lambda(x)$$

where $\Lambda(x)$ is a c-number field satisfying $\Box\Lambda(x) = 0$ and vanishing at infinity.

P^μ the *total* momentum operator, defined as the generator of space-time translations.

Then P^μ cannot be a gauge invariant operator.

Proof: Let F be the generator of gauge transformations, so that

$$i[F, A^\mu(x)] = \partial^\mu \Lambda(x)$$

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$$[F, [P^{\mu}, A^{\nu}]] + [A^{\nu}, [F, P^{\mu}]] + [P^{\mu}, [A^{\nu}, F]] = 0$$

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Since $[A^\nu, F]$ is a c-number, one finds

$$[[F, P^\mu], A^\nu] \neq 0$$

so that P^μ is not gauge invariant.

However, lack of gauge invariance of no physical significance.

Example, covariantly quantized QED: generator of translations is P_{can}: show that the matrix element of P^j_{can} between any normalizable physical states, unaffected by gauge changes in the operator.

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Lautrup-Nakanishi Lagrangian density: combination of the Classical Lagrangian ($Clas$) and a Gauge Fixing part (Gf) $\quad \mathcal{L} = \mathcal{L}_{Clas} + \mathcal{L}_{Gf}$

$$\mathcal{L}_{Clas} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} [\bar{\psi}(i \not{\partial} - m + e \not{A})\psi + \text{h.c.}]$$

$$\mathcal{L}_{Gf} = B(x)\, \partial_\mu A^\mu(x) + \frac{\mathsf{a}}{2} B^2(x)$$

$B(x)$: gauge–fixing field.

$$\text{Generator}\, F = \int d^3x\, [(\partial_0 B)\Lambda - B\partial_0\Lambda + \partial_j(F^{0j}\Lambda)].$$

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For arbitrary physical states

$$\langle\Psi'|B(x)|\Psi\rangle = 0$$

Theorem: Physical matrix elements of P^j are invariant under gauge transformations.

Proof: Consider the general physical matrix element

$$\langle\Psi'|P^j|\Psi\rangle = \int d^3\boldsymbol{p}\, d^3\boldsymbol{p}'\, \phi(\boldsymbol{p})\, \phi'(\boldsymbol{p}')\, \langle\boldsymbol{p}'|P^j|\boldsymbol{p}\rangle$$

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Change induced in $\langle\boldsymbol{p}'|P^j|\boldsymbol{p}\rangle$ is $\langle\boldsymbol{p}'|i[F,P^j]|\boldsymbol{p}\rangle$.

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Using expression for F can show $\langle \Psi' | P^j | \Psi \rangle$ is indeed invariant under gauge transformations.

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We don't know how to do operator gauge transformations.

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Maybe Path Integral is only way.

The Belinfante energy-momentum and angular momentum densities

Suppose we add a spatial divergence, say $\frac{\partial}{\partial x^j} H^{j\mu}(x)$ to $t^{0\mu}_{can}(x)$ where

$$\lim_{|\boldsymbol{x}| \to \infty} H^{j\mu}(x) = 0.$$

Then

$$\int_{all\ space} d^3x \, \frac{\partial}{\partial x^j} H^{j\mu}(x) = H^{j\mu}(t, |\boldsymbol{x}| = \infty) = 0$$

Can define the Belinfante energy-momentum density which :

- is symmetric: $t^{\nu\mu}_{bel}(x) = t^{\mu\nu}_{bel}(x)$
- is gauge invariant
- $t^{0\mu}_{bel}(x) = t^{0\mu}_{can}(x) +$ spatial divergence

It follows that

$$P^{\mu}_{bel} \equiv \int d^3x\, t^{0\mu}_{bel}(x) = P^{\mu}_{can}$$

IF the fields vanish at infinity.

Similarly for Belinfante angular momentum density;

$$\mathcal{M}^{0ij}_{bel}(x) = \mathcal{M}^{0ij}_{can}(x) + \text{spatial divergence}$$

so that

$$\boldsymbol{J}_{bel} = \boldsymbol{J}_{can}$$

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Come back to this presently.

What does $\boldsymbol{J}_{bel}$ look like ?

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&+ \int d^3x\, \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B}) \\
&= \boldsymbol{S}_{bel}(el) + \boldsymbol{L}_{bel}(el) + \boldsymbol{J}_{bel}(\gamma)
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where

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Pros: Each term is gauge invariant

Cons: $\boldsymbol{J}_{bel}(\gamma)$ NOT split into spin and orbital parts.

There are several delicate questions involved in the above, even at classical level.

1) Classical: a circularly polarized light beam

Applying the above to a free classical electromagnetic field, one gets

$$\boldsymbol{J}_{can} = \underbrace{\int d^3x\,(\boldsymbol{E}\times\boldsymbol{A})}_{\text{spin term}} + \underbrace{\int d^3x\,E^i(\boldsymbol{x}\times\boldsymbol{\nabla}A^i)}_{\text{orbital term}}$$

and

$$\boldsymbol{J}_{bel} = \int d^3x\,[\boldsymbol{x}\times(\boldsymbol{E}\times\boldsymbol{B})]$$

Consider a left-circularly polarized (= positive helicity) beam propagating along OZ i.e. along $\boldsymbol{e}_{(z)}$:

$$A^\mu = \left(0,\, \frac{E_0}{\omega} \cos(kz - \omega t),\, \frac{E_0}{\omega} \sin(kz - \omega t),\, 0\right)$$

gives correct $\boldsymbol{E}$ and $\boldsymbol{B}$.

$\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{A}$ all rotate in the XY plane.

Now consider the component of $\boldsymbol{J}$ along OZ.
Note that

$$\boldsymbol{\nabla} A_{x,y} \propto \boldsymbol{e}_{(z)} \quad \text{so that} \quad (\boldsymbol{x} \times \boldsymbol{\nabla} A_{x,y})_z = 0$$

so only the spin term contributes to $J_{can,\,z}$.

Find

$$J_{can,\,z} \text{ per unit volume} = \frac{E_0^2}{\omega}$$

For one photon per unit volume $E_0^2 = \hbar\omega$ so that

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For Belinfante case

$$(\boldsymbol{E} \times \boldsymbol{B}) \propto \boldsymbol{e}_z$$

so that

$$J_{bel,\,z} = \int d^3x\, [\boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})]_z = 0 \qquad \times$$

2) **Quantum: what does it mean to say an operator vanishes at infinity?**

Usually we are interested in expectation values of these operators i.e their forward matrix elements. For these it may be possible to justify neglecting the contribution at infinity.

2.1) **Spatial divergence of a local operator**

A local operator $O(x)$ is defined at one space-time point and must satisfy the law of translation i.e.

$$e^{ia\cdot P}O(x)e^{-ia\cdot P} = O(x+a)$$

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For the spatial divergence of a local operator we have

$$\begin{aligned}
\langle \boldsymbol{p}' \,|\, \partial_j O(x) \,|\boldsymbol{p}\,\rangle \;&=\; \frac{\partial}{\partial x^j}\,\langle \boldsymbol{p}' \,|\, O(x) \,|\boldsymbol{p}\,\rangle \\
&= \left[\frac{\partial}{\partial x^j}\, e^{-i\boldsymbol{x}\cdot(\boldsymbol{p}-\boldsymbol{p}')}\right]\langle \boldsymbol{p}'|O(0)|\boldsymbol{p}\,\rangle \\
&= i(p'^j - p^j)\langle \boldsymbol{p}' \,|\, O(0) \,|\boldsymbol{p}\,\rangle\, e^{-i\boldsymbol{x}\cdot(\boldsymbol{p}-\boldsymbol{p}')}
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\end{aligned}$$

Therefore as $\boldsymbol{p}' \to \boldsymbol{p}$

$\langle \boldsymbol{p}\,|\,\partial_j O(x)\,|\boldsymbol{p}\,\rangle = 0$ if $\langle \boldsymbol{p}\,|\,O(0)\,|\boldsymbol{p}\,\rangle$ is non-singular

2.2) **Spatial divergence of a compound operator**

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To see this suppose that $Q(x) = x\,O(x)$ is a local operator. Then

$$
\begin{aligned}
Q(x) &= e^{-ix\cdot P} Q(0)\, e^{ix\cdot P} \\
&= 0 \quad \text{for all } x\text{, since} \quad Q(0) = 0
\end{aligned}
$$

It is then much more difficult to show that one can neglect the expectation value of the spatial divergence of a compound operator. It can be done, but requires use of localised wave packets, as demonstrated by Shore and White .

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Conclusion

For momentum and angular momentum it is safe to neglect spatial divergence terms.

The problem of defining separate quark and gluon momenta

Two separate issues:

(1) general problem of how to define the separate momenta for a system of interacting particles,

(2) more specific to gauge theories and includes the issue of splitting the angular momentum of a gauge particle into a spin and orbital part.

(1) The general problem: System of interacting particles E and F. Split the total momentum into two pieces

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Note that this expression is totally misleading, and should be written

$$P^j = P_E^j(t) + P_F^j(t)$$

to reflect the fact that the particles exchange momentum as a result of their interaction.

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and similarly for F

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and similarly for F

But there is no way we can check this, since $P_E^j(t)$ depends on t and, without solving the entire theory, we are only able to compute equal time commutators .

We suggest, therefore, that *the minimal requirement for identifying an operator* P_E^j *with the momentum carried by* E , is to demand that *at equal times*

$$i[P_E^j(t)\,,\phi^E(t,\boldsymbol{x})] = \partial^j \phi^E(t,\boldsymbol{x}).$$

Analogously, *for an angular momentum operator* M_E^{ij} *(* $J^i = \epsilon^{ijk} M^{jk}$*) we suggest* that at equal times

$$i[M_E^{ij}(t)\,,\phi_r^E(t,\boldsymbol{x})] = (x^i\partial^j - x^j\partial^i)\phi_r^E(t,\boldsymbol{x}) + (\Sigma^{ij})_r{}^s\phi_s^E(t,\boldsymbol{x})$$

where r and s are spinor or Lorentz labels and $(\Sigma^{ij})_r{}^s$ is the relevant spin operator.

Implications

For the **total** momentum there is no essential difference between P_{can} and P_{bel} since their integrands differ by the spatial divergence of a local operator.

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But, if we split P_{can} into $P_{can,\,E} + P_{can,\,F}$ and P_{bel} into $P_{bel,\,E} + P_{bel,\,F}$, then the integrands of $P_{can,\,E}$ and $P_{bel,\,E}$ do *not* differ by a spatial divergence.

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But, if we split P_{can} into $P_{can,\,E} + P_{can,\,F}$ and P_{bel} into $P_{bel,\,E} + P_{bel,\,F}$, then the integrands of $P_{can,\,E}$ and $P_{bel,\,E}$ do *not* differ by a spatial divergence.

Hence $P_{can,\,E}$ and $P_{bel,\,E}$ do **not** generate the same transformation on $\phi^E(x)$, and similarly for F.

Since, by construction, $P_{can,\,E}$ and $P_{can,\,F}$ do generate the correct transformations on $\phi_E(x)$ and $\phi_F(x)$ respectively, we conclude that with the above minimal requirement we are forced to associate the momentum and angular momentum of E and F with the canonical version of the relevant operators.

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However you

CAN NO LONGER TALK ABOUT $\boldsymbol{J}$(quark) and $\boldsymbol{J}$(gluon)

MUST SPECIFY WHICH SCHEME FOR $\boldsymbol{J}$

YOU ARE USING

and similarly for momentum $\boldsymbol{P}$.

No worse than realization that for PDFs must specify factorization scheme:

$$q(x)_{MS} \qquad q(x)_{\overline{MS}} \qquad q(x)_{DIS}$$

THE CONTROVERSY

Chen, Lu, Sun, Wang and Goldman (Chen *et al*):

They insist on gauge invariant operators, but then using $\boldsymbol{J}_{bel}$ in analogous QCD case, what do you mean by the gluon spin???????

IT IS POSSIBLE to split photon or gluon angular momentum into a spin part and an orbital part in a GAUGE INVARIANT way !!!

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IT IS POSSIBLE to split photon angular momentum into a spin part and an orbital part in a GAUGE INVARIANT way !!!

Put $\boldsymbol{A} = \boldsymbol{A}_{phys} + \boldsymbol{A}_{pure}$ with

$$\boldsymbol{\nabla}.\boldsymbol{A}_{phys} = 0 \qquad \boldsymbol{\nabla} \times \boldsymbol{A}_{pure} = 0$$

Corresponds exactly to what is usually called the transverse $\boldsymbol{A}_{\perp}$ and longitudinal $\boldsymbol{A}_{\|}$ parts respectively

Adding a spatial divergence to $\boldsymbol{J}_{can}$ they get

$$\begin{aligned}
\boldsymbol{J}_{chen} &= \int d^3x\, \psi^\dagger \boldsymbol{\gamma}\gamma_5 \psi + \int d^3x\, \psi^\dagger [\boldsymbol{x} \times (-i\boldsymbol{D}_{pure})]\psi \\
&+ \int d^3x\, (\boldsymbol{E} \times \boldsymbol{A}_{phys}) + \int d^3x\, E^i [\boldsymbol{x} \times \boldsymbol{\nabla} A^i_{phys}] \\
&= \boldsymbol{S}_{ch}(el) + \boldsymbol{L}_{ch}(el) + \boldsymbol{S}_{ch}(\gamma) + \boldsymbol{L}_{ch}(\gamma)
\end{aligned}$$

where

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where

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Under gauge transformation:

$$\boldsymbol{A}_{pure} \to \boldsymbol{A}_{pure} + \boldsymbol{\nabla}\Lambda(x) \qquad \boldsymbol{A}_{phys} \to \boldsymbol{A}_{phys}$$

so each term in $\boldsymbol{J}_{chen}$ is indeed gauge invariant.

Does this imply that all textbooks of past 50 years are wrong?

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NO!

$\boldsymbol{A}_{phys}$ is not a local field:

$$\boldsymbol{A}_{phys} = \boldsymbol{A} - \frac{1}{\nabla^2}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{A})$$

Recall

$$\frac{1}{\nabla^2}f(x) \equiv \frac{1}{4\pi}\int d^3x'\,\frac{f(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

What does this actually mean physically?

Since Chen *et al.* is gauge invariant, can choose gauge $\boldsymbol{A}_{pure} = 0$ i.e. $\boldsymbol{A}_{phys} = \boldsymbol{A}$ which implies

$$\boldsymbol{\nabla}.\boldsymbol{A} = 0$$

which is the Coulomb gauge !

Thus

$$\boldsymbol{J}_{chen} \equiv \boldsymbol{J}_{can}\big|_{\text{Coulomb Gauge}}$$

Summary of Chen, Lu, Sun, Wang and Goldman

- it is a Gauge Invariant Extension of the Canonical case in the Coulomb gauge

- it involves non-local fields

- A^μ does not transform as a 4-vector under Lorentz transformations

- the physical content is exactly the same as in the canonical case in the Coulomb gauge

FURTHER DEVELOPMENTS

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Split

$$A^\mu = A^\mu_{phys} + A^\mu_{pure}$$

$$F^{\mu\nu}_{pure} = 0$$

NB Wakamatsu does NOT give a specific formula for A^μ_{pure}

Lorcé: Exists *Stueckelberg* transformation

$$A^{\mu}_{pure} \to A^{\mu}_{pure} + \partial^{\mu} C(x) \quad A^{\mu}_{phys} \to A^{\mu}_{phys} - \partial^{\mu} C(x)$$

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So there are an INFINITE number of possible A^μ_{pure}.

QED: $A^\mu_{pure} = \partial^\mu \Lambda(x)$ any Λ

QCD: $A^\mu_{pure} = U^{-1}\partial^\mu U$; U any $SU(3)$ matrix

Physical content of Wakamatsu II

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Suppose we uniquely specify the scheme $\boldsymbol{J}^{F}_{wakII}$ by fixing A^{μ}_{pure} via

$$A^{\mu}_{pure}|_{F} = F(A^{\mu})$$

where F is some given function.

Since the scheme is gauge invariant, choose the gauge which makes $A^{\mu}_{pure}|_F = 0$. Call it Gauge F.

Then

$$A^{\mu}_{phys} = A^{\mu}|_{\text{Gauge F}}$$

and from the expression for $\boldsymbol{J}_{wakII}$ one sees that

$$\boldsymbol{J}^{F}_{wakII} = \boldsymbol{J}_{can}|_{\text{Gauge F}}$$

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Thus the family of schemes Wakamatsu II is identical to the canonical scheme in various choices of gauge.

Hatta

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It turns out that

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corresponds to the lightcone gauge $A^{+} = 0$.

$$\text{Thus } \boldsymbol{J}_{Hatta} = \boldsymbol{J}_{can}|_{\text{Gauge}\, A^{+}=0}$$

Several other papers———no time to discuss:

Bashinsky and Jaffe; Stoilov; Cho, Ge and Zhang; Zhang and Pak; Zhou and Huang; Xiang-Song Chen.

Lorcé: general underlying mathematical structure.

WHICH SCHEME SHOULD YOU LOVE AND TRUST??

Two fundamental schemes: Canonical and Belinfante

Belinfante favoured by Ji and collaborators:
Pros:

▲ Each term is gauge invariant

▲ Nucleon expectation values can be related to GPDs

.....but this is now controversial!

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▼Photon (gluon) angular momentum NOT split into spin and orbital parts

▼Operators do NOT generate rotations

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▲ At equal times operators are generators of rotations

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▼ Terms are not gauge invariant

SUMMARY

1) There are, I believe, only two fundamental schemes: Canonical and Belinfante

2) You may use whichever you prefer, but you must indicate which scheme you are using

3) Though not gauge invariant, I prefer Canonical because the operators generate rotations at least at equal times

4) All the new gauge invariant schemes involve non-local fields and correspond to the Canonical version viewed in a particular choice of gauge. Thus the new schemes, in my opinion, do not contain any new physics.

EXTRA SLIDES

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In fact, no contradiction in the special case of the *longitudinal* components of the momentum and angular momentum.

From the gauge invariant expression for the unpolarized quark number density $q(x)$ (including Wilson line operator) one finds

$$\int_0^1 dx\, x\, [\, q(x) + \bar{q}(x)\,] = \frac{i}{4(P^+)^2} \langle\, P \,|\, \bar{\psi}(0)\, \gamma^+ \overleftrightarrow{D}^+ \psi(0) \,|\, P \,\rangle$$

with

$$\overleftrightarrow{D}^+ = \overrightarrow{\partial}^+ - \overleftarrow{\partial}^+ - 2igA^+(0).$$

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with

$$\overleftrightarrow{D}^+ = \overrightarrow{\partial}^+ - \overleftarrow{\partial}^+ - 2igA^+(0). \qquad (2)$$

But the quark part of $t^{\mu\nu}_{bel}(qG)$ is given by

$$t^{\mu\nu}_{q,\,bel}(z) = \frac{i}{4}[\bar{\psi}(z)\gamma^\mu \overleftrightarrow{D}(z)^\nu \psi(z) + (\mu \leftrightarrow \nu)] - g^{\mu\nu}\mathcal{L}_q$$

where $\mathcal{L}_q$ is the quark part of $\mathcal{L}_{qG}$.

Since $g^{++} = 0$

$$t^{++}_{q,\,bel}(0) \;= \frac{i}{2}\{\bar{\psi}(0)\,\gamma^{+}\overleftrightarrow{D}^{+}\,\psi(0)\}$$

so that

$$\int_0^1 dx\,x\,[\,q(x) + \bar{q}(x)\,] = \frac{1}{2(P^{+})^2}\langle\, P\,|\,t^{++}_{q,\,bel}(0)\;|\,P\,\rangle.$$

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Consider the physical interpretation of the LHS in the parton model. The parton model is not synonymous with QCD. It is a picture of QCD in the gauge $A^{+} = 0$ and it is in this gauge, and in an infinite momentum frame that x can be interpreted as the momentum fraction carried by a quark in the nucleon.

But since $A^+ = 0$ we have

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$$t^{++}_{q,\,can}(0) = t^{++}_{q,\,bel}(0) \qquad (\text{gauge}\, A^+ = 0).$$

Hence the fraction of *longitudinal* momentum carried by the quarks in an infinite momentum frame is given equally well by either the canonical or Belinfante versions of the energy momentum tensor density.

A key issue: the polarized gluon density $\Delta G(x)$

$\Delta G(x)$ is measurable
$\Delta G(x)$ is gauge invariant

In what sense does it correspond to the spin of the gluon?

As stated: my view: The parton model is a PICTURE of QCD in the gauge $A^{+} = 0$.

All is well, since can show that

$$\Delta G(x) = \langle\, \hat{\boldsymbol{P}} \cdot \boldsymbol{S}_{can}(\text{gluon})\, \rangle|_{\text{Gauge}\, A^{+}=0}$$

where $\langle .. \rangle$ means expectation value in a longitudinally polarized nucleon.