Seminar
Vendredi 09/09/2016, 11h00-12h00

High density matter, neutron stars and finite nuclei

J. R. Stone
Tennessee/Oxford/Maryland/RIKEN

jirina.stone@physics.ox.ac.uk

Outline:

1. The whole picture

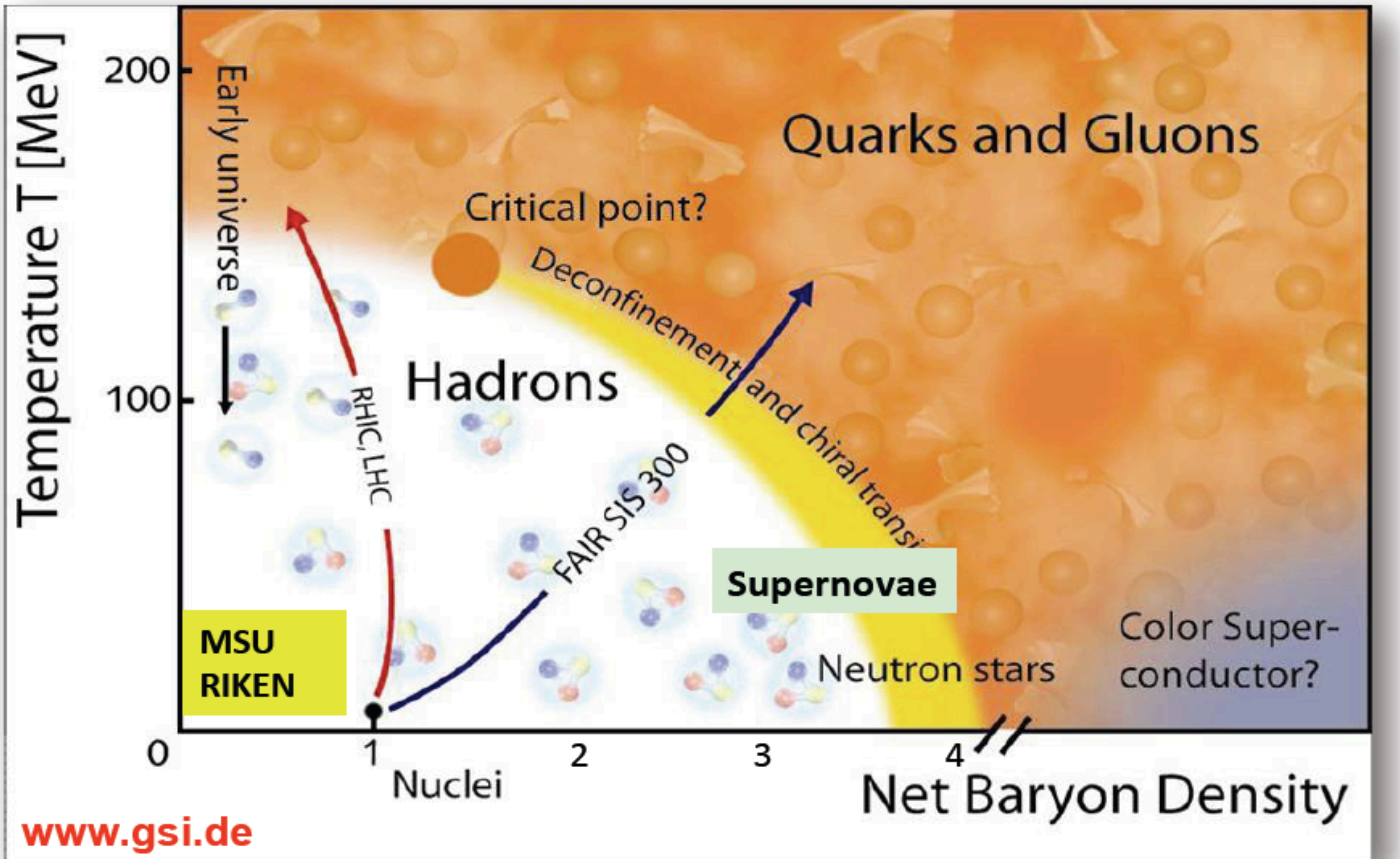
2. High density cold matter and neutron stars

3. Problems of current low energy nuclear theory
   Ab initio, Shell model, mean field models

4. Quark-meson coupling model
   Finite nuclei, nuclear matter and neutron stars

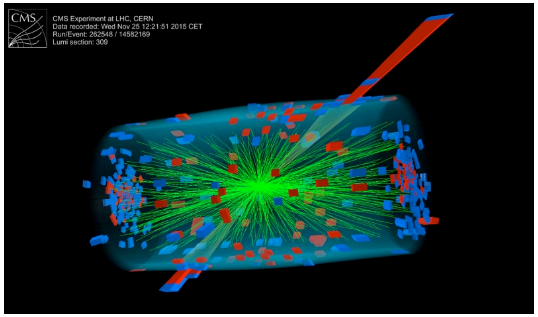
5. Summary remarks

QCD phase diagram



Nuclei comprise 99.9% of all baryonic matter in the Universe and are the fuel that burns in stars. The rather complex nature of the nuclear forces among protons and neutrons generates a broad range and diversity in the nuclear phenomena that can be observed. (SciDAC review)

THE SAME LAWS GOVERN TERRESTRIAL NUCLEI AND NUCLEI IN COSMOS.

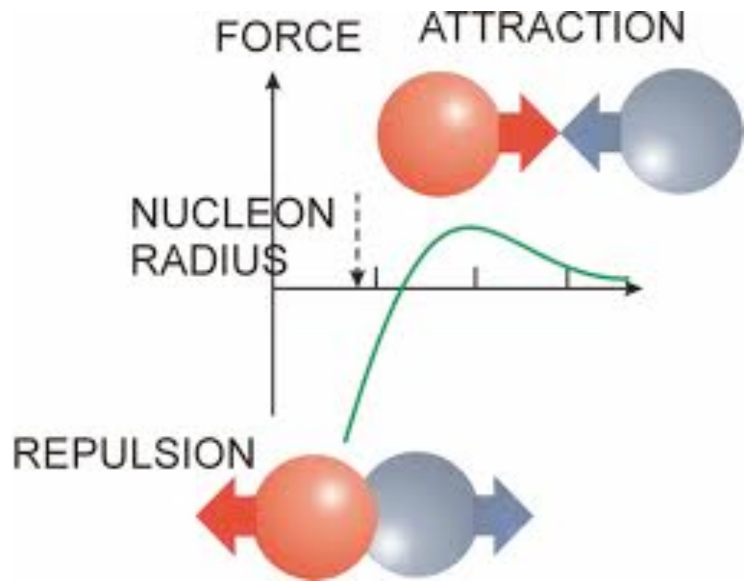


But there is fundamental problem:

We do not know the nature of the nuclear force from first principles and have to rely on models

Nuclear force:

TWO NUCLEONS IN VACUUM :
nucleon-nucleon scattering
tractable with many parameters
no unique model as yet

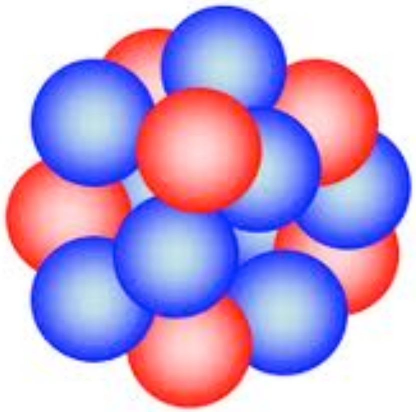


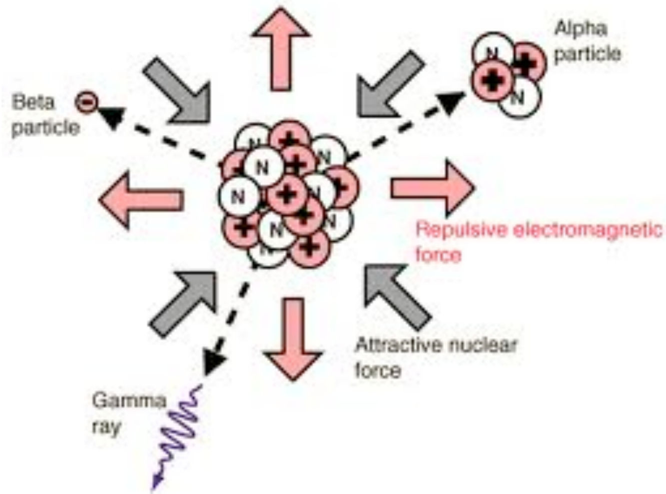
(Argonne, Bonn, Nijmegen, Paris etc)
'realistic' potentials

Empirical or One-Boson-Exchange

Chiral effective field theories – relation with Quantum Chromodynamics
(QDC non-perturbative at low energies)

NUCLEON-NUCLEON INTERACTION IN NUCLEAR MEDIUM:
force depends on density and momentum –
strong and electro-weak interactions play role – intractable?



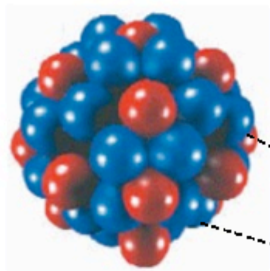


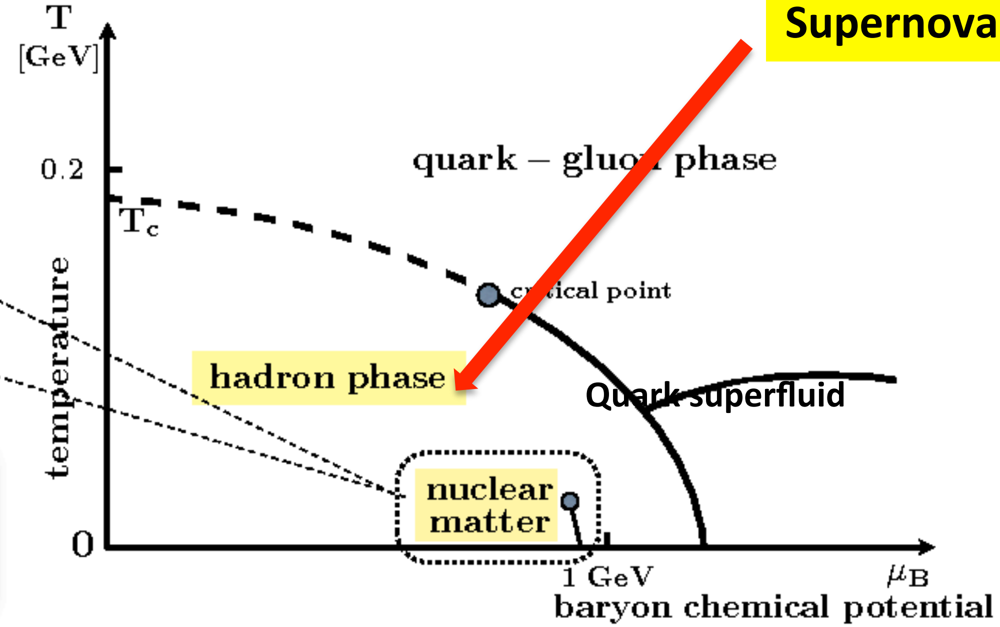
NUCLEAR MANY-BODY PROBLEM

Nuclear matter

Finite nuclei
Neutron stars
Core-Colapse
Supernovae

nuclei





Scales in **nuclear matter**:

- momentum scale:
  Fermi momentum $p_F \simeq 1.4 \text{ fm}^{-1} \sim 2m_\pi$
- NN distance: $d_{NN} \simeq 1.8 \text{ fm} \simeq 1.3\ m_\pi^{-1}$
- energy per nucleon: $E/A \simeq -16 \text{ MeV}$
- compression modulus: $K = (260 \pm 30) \text{ MeV} \sim 2m_\pi$

CONCEPT OF NUCLEAR MATTER

Infinite A in an infinite volume V but A/V finite

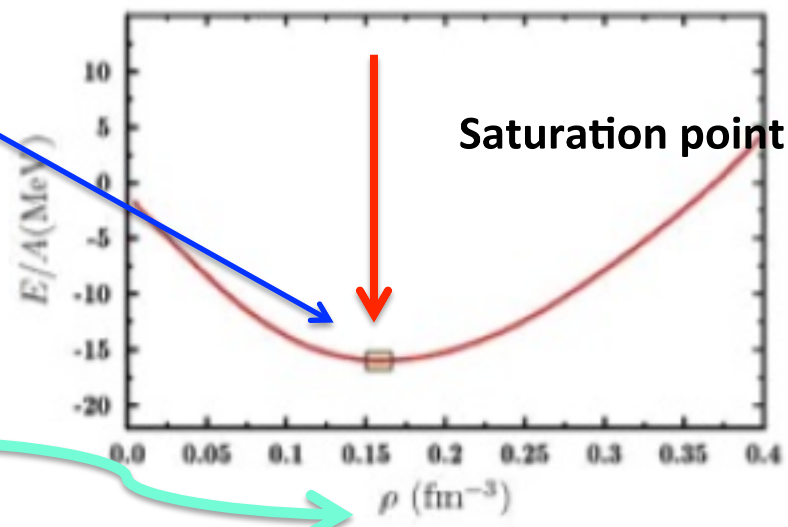
No Coulomb force , NO SURFACE EFFECTS

Uniform density distribution

$$\frac{E(N,Z)}{A} = a_V - a_S A^{-1/3} - a_C Z^2 A^{-4/3} - a_{Sym} \frac{(N-Z)^2}{A^2} - \lambda\, a_P A^{-7/4}$$

$$\frac{E(N,Z=0)}{A} - \frac{E(N=Z)}{A} = a_{Sym}$$

$$R = r_0 A^{1/3} \quad V = \frac{4}{3}\pi R^3 \quad \rho_0 = \frac{A}{V}$$



Two reasons for using nuclear matter:

1. **Simple bench-marks for testing nuclear models**

$$a_v \approx -16 \text{ MeV}$$

$$a_{Sym} \approx 30 \text{ MeV}$$

$$\rho_0 \approx 0.16 \text{ fm}^{-3}$$

2. **Nuclear matter exists on astrophysical objects (neutron stars or supernovae) as well as in the interior of heavy nuclei**

Realistic bare nucleon

20-60 adjustable parameters

Several thousands of data points:
Free nucleon-nucleon scattering and properties of deuteron

Used in nuclear matter calculations, shell model, ab-initio theories

NO DENSITY DEPENDENT FURTHER TREATMENT NEEDED TO USE IN NUCLEAR ENVIRONMENT

Phenomenological density dependence

10-15 adjustable parameters

Several tens of data points:
Symmetric nuclear matter at saturation, Ground state properties of finite nuclei

Used in mean-field models non-relativistic Hartree-Fock

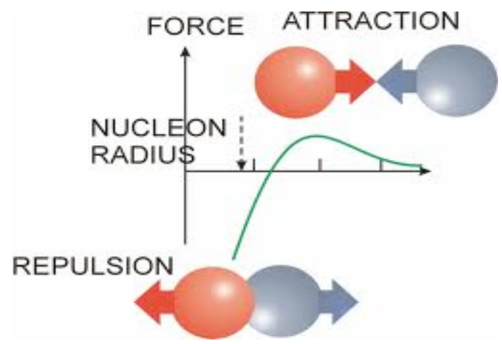
DENSITY DEPENDENCE INCLUDED IN AN EMPIRICAL WAY THROUGH PARAMETERS

EXAMPLE 1: REALISTIC POTENTIAL **A18+δv+UIX*** (Akmal et al, PRC58,1804 (1998))

EXAMPLE 1: REALISTIC BARE NUCLEON POTENTIAL – ARGONNE 18

A18 (two-body):
static, long range
one-pion exchange
+
medium/short range
18 two-body operators

UIX (three-body)
static, long-range
two-pion exchange
+
medium/short range
empirical repulsive term



+
relativistic boost
+
correction

$$\gamma_2 \rho^2 e^{\gamma_3 \rho}$$

Normal density in fm^{-3} and E/A in MeV in SNM

P_0	**A18**	**A18+δv**	**A18+UIX**	**A18+δv+UIX***	**corr**
0.12			-10.52	-10.54	- 15.04
0.16	-14.59	-12.54	-11.85	-12.16	-16.00
0.20			-11.28	-12.21	-15.09

Example of the structure of an Argonne potential

The strong interaction part of the potential is projected into an operator format with 18 terms: A charge-independent part that has 14 operator components (as in the older Argonne v_{14})

$$1,\ \ \sigma_i \cdot \sigma_j,\ \ S_{ij},\ L \cdot S,\ \ L^2,\ \ L^2\,\sigma_i \cdot \sigma_j,\ \ (L \cdot S)^2$$

$$\tau_i \cdot \tau_j,\ \ (\sigma_i \cdot \sigma_j)\,(\tau_i \cdot \tau_j),\ \ S_{ij}\,(\tau_i \cdot \tau_j),\ L \cdot S\,(\tau_i \cdot \tau_j),\ \ L^2(\tau_i \cdot \tau_j),\ \ L^2\,(\sigma_i \cdot \sigma_j)\,(\tau_i \cdot \tau_j),\ \ (L \cdot S)^2\,(\tau_i \cdot \tau_j)$$

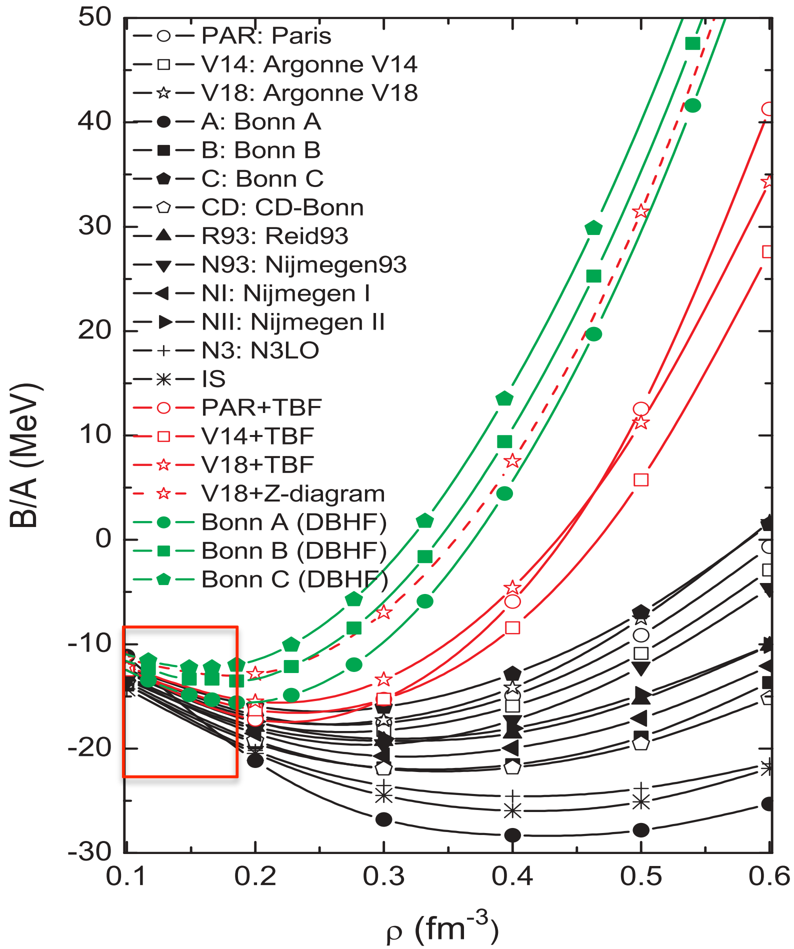
And a charge-independence breaking part that has three charge-dependent operators

$$T_{ij},\ \ (\sigma_i \cdot \sigma_j)\,T_{ij},\ \ S_{ij}\,T_{ij}$$

where $T_{ij} = 3\tau_{zi}\tau_{zj} - \tau_i \cdot \tau_j$ is the isotensor operator, defined analogous to the S_{ij} operator; and one charge-asymmetric operator

$$\tau_{zi} + \tau_{zj}$$

The potential includes also a complete electromagnetic potential, containing Coulomb, Darwin-Foldy, vacuum polarization, and magnetic moment terms with finite-size effects.



The minimum of all curves should be P = 0.16 fm^{-3} and the corresponding B/A should be -16 MeV

Additional adjustment such as an addition of 3 body forces Is needed

Binding energy per particle In symmetric nuclear matter As calculated using various "realistic" models. Empirical data to match are B/A = -16 MeV at ρ = 0.16 fm^{-3}

Li et al., PRC74, 047304 (2006)

EXAMPLE 2: PHENOMENOLOGICAL DENSITY DEPENDENT POTENTIAL: THE SKYRME INTERACTION

$$\mathcal{E}_{\rm Sk} = \mathcal{E}_{\rm Sk,even} + \mathcal{E}_{\rm Sk,odd}\ , \tag{6a}$$

$$\begin{aligned}
\mathcal{E}_{\rm Sk,even} = {} & C_0^{\rho}\,\rho_0^2 && + C_1^{\rho}\,\rho_1^2 \\
& + C_0^{\rho,\alpha}\,\rho_0^{2+\alpha} && + C_1^{\rho,\alpha}\,\rho_1^2\rho_0^{\alpha} \\
& + C_0^{\Delta\rho}\,\rho_0\Delta\rho_0 && + C_1^{\Delta\rho}\,\rho_1\Delta\rho_1 \\
& + C_0^{\nabla J}\,\rho_0\nabla\cdot\boldsymbol{J}_0 && + C_1^{\nabla J}\,\rho_1\nabla\cdot\boldsymbol{J}_1 \\
& + C_0^{\tau}\,\rho_0\tau_0 && + C_1^{\tau}\,\rho_1\tau_1 \\
& + C_0^{J}\,\boldsymbol{J}_0^2 && + C_1^{J}\,\boldsymbol{J}_1^2
\end{aligned} \tag{6b}$$

$$\begin{aligned}
\mathcal{E}_{\rm Sk,odd} = {} & C_0^{\sigma}\,\boldsymbol{\sigma}_0^2 && + C_1^{\sigma}\,\boldsymbol{\sigma}_1^2 \\
& + C_0^{\sigma,\alpha}\,\boldsymbol{\sigma}_0^2\rho_0^{\alpha} && + C_1^{\sigma,\alpha}\,\boldsymbol{\sigma}_1^2\rho_0^{\alpha} \\
& + C_0^{\Delta\sigma}\,\boldsymbol{\sigma}_0\Delta\boldsymbol{\sigma}_0 && + C_1^{\Delta\sigma}\,\boldsymbol{\sigma}_1\Delta\boldsymbol{\sigma}_1 \\
& + C_0^{\nabla J}\,\boldsymbol{\sigma}_0\cdot\nabla\times\boldsymbol{j}_0 && + C_1^{\nabla J}\,\boldsymbol{\sigma}_1\cdot\nabla\times\boldsymbol{j}_1 \\
& - C_0^{\tau}\,\boldsymbol{j}_0^2 && - C_1^{\tau}\,\boldsymbol{j}_1^2 \\
& - \tfrac{1}{2}C_0^{J}\boldsymbol{\sigma}_0\cdot\boldsymbol{\tau}_0 && - \tfrac{1}{2}C_1^{J}\boldsymbol{\sigma}_1\cdot\boldsymbol{\tau}_1
\end{aligned} \tag{6c}$$

P.-G. Reinhard, Phys. Scr. 91, 023002 (2016).

Tony Hilton Royle Skyrme
1922 - 1987

Parameters adjustable to experiment:
t0, t1, t2, t3, t4, t5, x0, x1, x2, x3, x4, x5, α, β, γ

$$C_0^\rho = \tfrac{3}{8}t_0 + \tfrac{3}{48}t_3\rho^\alpha, \tag{A30a}$$

$$C_1^\rho = -\tfrac{1}{4}t_0\left(\tfrac{1}{2} + x_0\right) - \tfrac{1}{24}t_3(1 + x_3)\rho^\alpha, \tag{A30b}$$

$$C_0^s = -\tfrac{1}{4}t_0\left(\tfrac{1}{2} - x_0\right) - \tfrac{1}{24}t_3\left(\tfrac{1}{2} - x_3\right)\rho^\alpha, \tag{A30c}$$

$$C_1^s = -\tfrac{1}{8}t_0 - \tfrac{1}{48}t_3\rho^\alpha, \tag{A30d}$$

$$C_0^\tau = \tfrac{3}{16}t_1 + \tfrac{1}{4}t_2\left(\tfrac{5}{4} + x_2\right) + \tfrac{3}{16}t_4\rho^\beta + \tfrac{1}{4}t_5\left(\tfrac{5}{4} + x_5\right)\rho^\gamma, \tag{A30e}$$

$$C_1^\tau = -\tfrac{1}{8}t_1\left(\tfrac{1}{2} + x_1\right) + \tfrac{1}{8}t_2\left(\tfrac{1}{2} + x_2\right) - \tfrac{1}{8}t_4\rho^\beta\left(\tfrac{1}{2} + x_4\right) + \tfrac{1}{8}t_5\rho^\gamma\left(\tfrac{1}{2} + x_5\right), \tag{A30f}$$

$$C_0^T = -\tfrac{1}{8}\Big[t_1\left(\tfrac{1}{2} - x_1\right) - t_2\left(\tfrac{1}{2} + x_2\right) + t_4\rho^\beta\left(\tfrac{1}{2} - x_4\right) - t_5\rho^\gamma\left(\tfrac{1}{2} + x_5\right)\Big], \tag{A30g}$$

NUCLEAR MATTER PROPERTIES FROM MEAN FIELD MODELS WITH DENSITY DEPENDENT SKYRME EFFECTIVE INTERACTION:

1. **240 non-relativistic models based on the Skyrme interaction - density dependent effective nucleon-nucleon force dependent on up to 15 adjustable parameters were recently tested against the most up-to-date constraints on properties of nuclear matter:**

5 satisfied all the constraints M. Dutra et al., PRC 85, 035201 (2012)
BUT ONLY 2 OF THEM (SQMC(700) and KDEv1)
ALSO WORK SATISFACTORILY) IN FINITE NUCLEI!!!
P. Stevenson et al., arXiv:1210.1592 (2012)

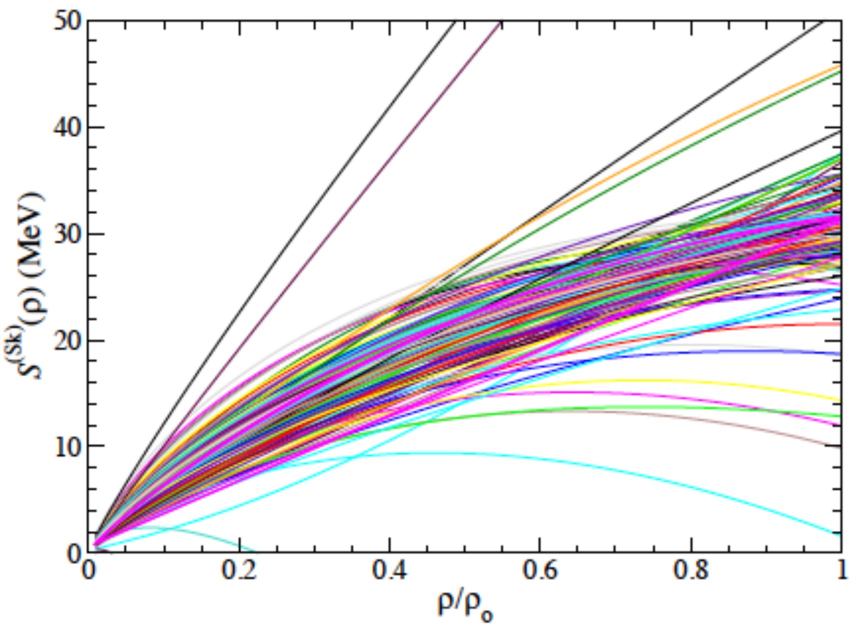
TABLE VI. Parameters of the Skyrme interactions consistent with the macroscopic constraints. and t_{3i} is in MeV fm$^{3+3\sigma_i}$. x_0, x_1, x_2, x_{3i}, and σ_i are dimensionless. For all parametrizations $t_4 = x_4$

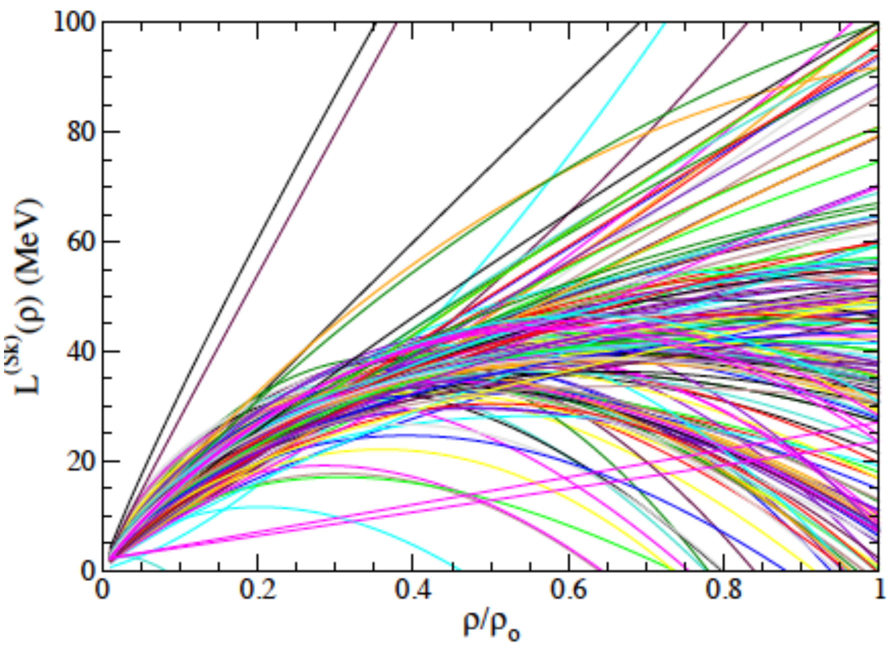
Skyrme	t_0	t_1	t_2	t_{31}	t_{32}	t_{33}	x_0	x_1	x_2	
GSkI	−1855.5	397.2	264.6	13858.0	−2694.1	−319.9	0.12	−1.76	−1.81	
GSkII	−1856.0	393.1	266.1	13842.9	−2689.7	–	0.09	−0.72	−1.84	-
KDE0v1	−2553.1	411.7	−419.9	14603.6	–	–	0.65	−0.35	−0.93	
LNS	−2485.0	266.7	−337.1	14588.2	–	–	0.06	0.66	−0.95	-
MSL0	−2118.1	395.2	−64.0	12875.7	–	–	−0.07	−0.33	1.36	-
NRAPR	−2719.7	417.6	−66.7	15042.0	–	–	0.16	−0.05	0.03	
Ska25s20	−2180.5	281.5	−160.4	14577.8	–	–	0.14	−0.80	0.00	
Ska35s20	−1768.8	263.9	−158.3	12904.8	–	–	0.13	−0.80	0.00	
SKRA	−2895.4	405.5	−89.1	16660.0	–	–	0.08	0.00	0.20	
SkT1	−1794.0	298.0	−298.0	12812.0	–	–	0.15	−0.50	−0.50	
SkT2	−1791.6	300.0	−300.0	12792.0	–	–	0.15	−0.50	−0.50	
SkT3	−1791.8	298.5	−99.5	12794.0	–	–	0.14	−1.00	1.00	
Skxs20	−2885.2	302.7	−323.4	18237.5	–	–	0.14	−0.26	−0.61	
SQMC650	−2462.7	436.1	−151.9	14154.5	–	–	0.13	0.00	0.00	
SQMC700	−2429.1	371.0	−96.7	13773.6	–	–	0.10	0.00	0.00	
SV-sym32	−1883.3	319.2	197.3	12559.5	–	–	0.01	−0.59	−2.17	-

EXAMPLES OF RESULTS OF THE SYMMETRY ENERGY OF NUCLEAR MATTER USING DIFFERENT SKYRME MODELS

Energy per particle $E(\rho, \delta) = E_0(\rho, \delta = 0) + S(\rho)\delta^2, \quad \delta = (\rho_n - \rho_p)/\rho$

Symmetric nuclear matter: $S(\rho) = S(\rho_0) - L\frac{(\rho_0 - \rho)}{3\rho_0}$ $\qquad S(\rho_0) \approx a_{Sym} \approx 30 \text{ MeV}$



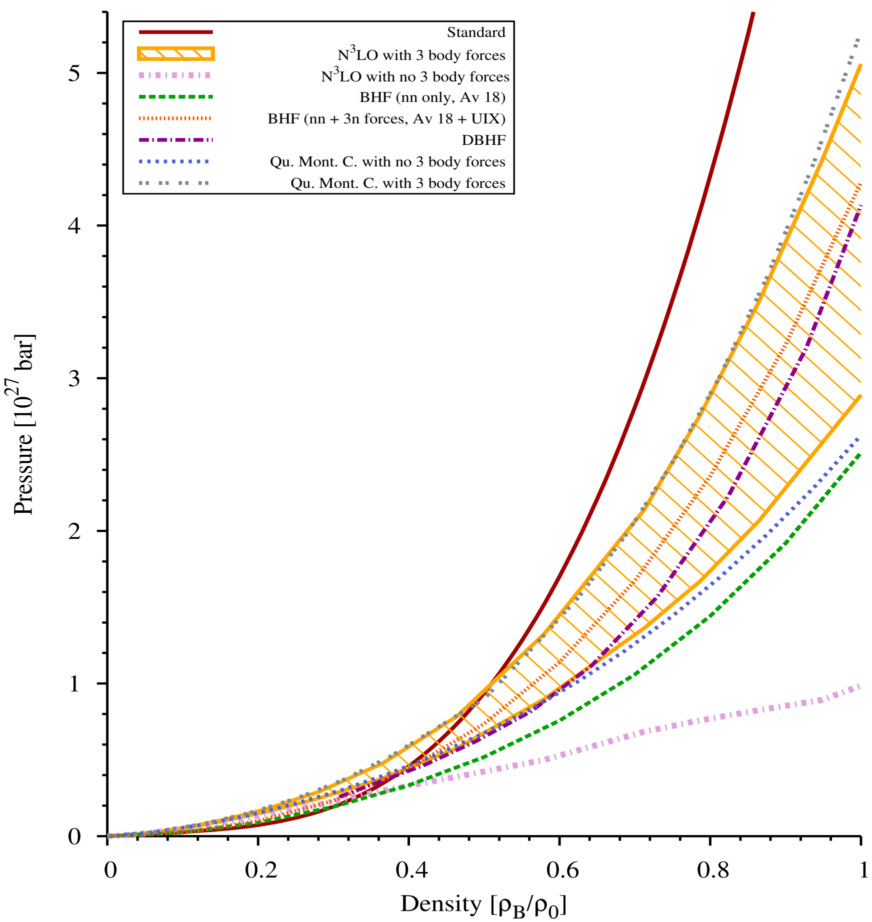


Santos et al., arXiv:1507.05856v1 (2015)

EXAMPLE 3.

Other techniques: Quantum Monte Carlo, Chiral Effective Field Theory

Theoretical calculations of properties of PNM at sub-saturation density:



N^{3}LO, BHF, QuMoCa

With and without
3-body forces

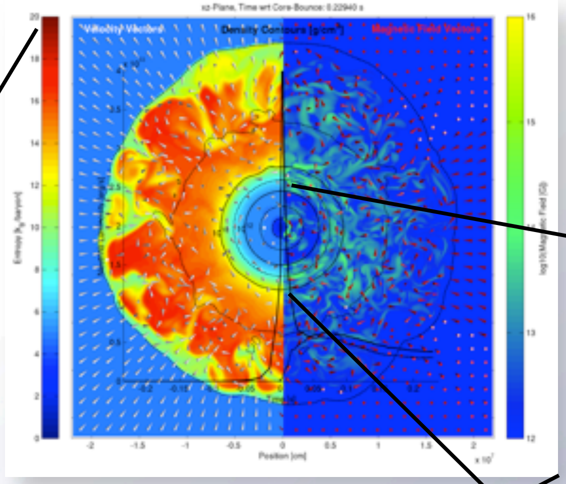
Pressure is too low
In all models without
3-body force, but this force
Is unknown and empirical
expressions introduce
additional parameters

Solid line: QMC model

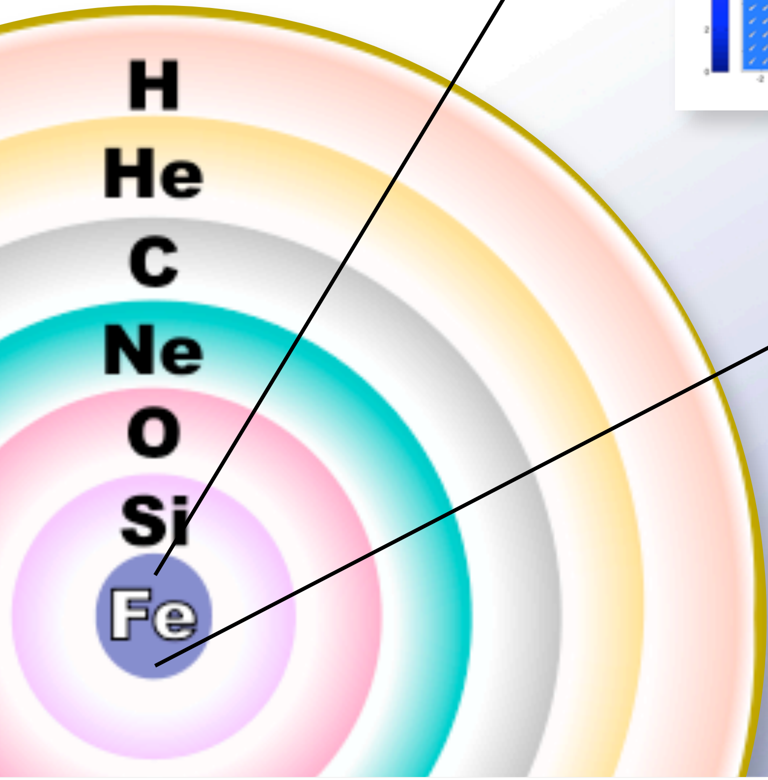
Whittenbury et al.2014

From progenitor stars via CCSNe to neutron stars

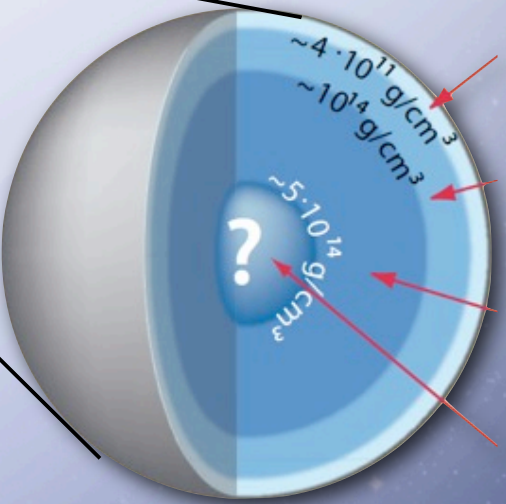
core-collapse
supernova explosion



progenitor star at
onset of collapse



cold neutron star



- what is the state of matter during all these stages?

Slide from M.Hempel, Russbach, 2014

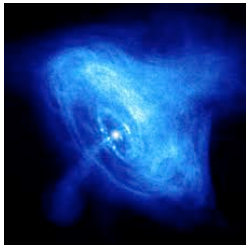
The Equation of State (EoS)

Relation between pressure P, energy density ε, particle number density ρ at temperature T

$$P = \rho^2 \left(\frac{\partial(\varepsilon / \rho)}{\partial \rho} \right)_{s/\rho} \qquad \varepsilon(\rho,T) = \sum_f \varepsilon_f(\rho,T)$$

Summation over f includes all hadronic (baryons, mesons), leptonic and quark (if applicable) components present in the system at density ρ and temperature T

INPUT TO MODEL CALCULATION OF NEUTRON STARS:



↓

Gravitational mass a related radius of a cold neutron star

Basic model of (non-rotating) neutron star properties:

Tolman-Oppenheimer-Volkoff (TOV) equations for hydrostatic equilibrium of a spherical object with isotropic mass distribution in general relativity:

- **Input:** **The Equation of State P(ε) – pressure as a function of energy density**
- **Output: Mass as a function of Radius M(R)**

$$\frac{dP}{dr} = -\frac{GM(r)\varepsilon}{r^2}\frac{(1+P/\varepsilon c^2)(1+4\pi r^3 P/M(r)c^2)}{1-2GM(r)/rc^2}$$

$$M(r) = \int_0^r 4\pi r'^2 \varepsilon(r')dr'$$

I. **Precise determination of a neutron star mass alone is not sufficient to compare models with observation.**

II. **Strong dependence on the equation of state –**
NUCLEAR AND PARTICLE PHYSICS

Mass-Radius Diagram and Theoretical Constraints

GR:

$R > 2GM/c^2$

$P < \infty$:

$R > (9/4)GM/c^2$

causality:

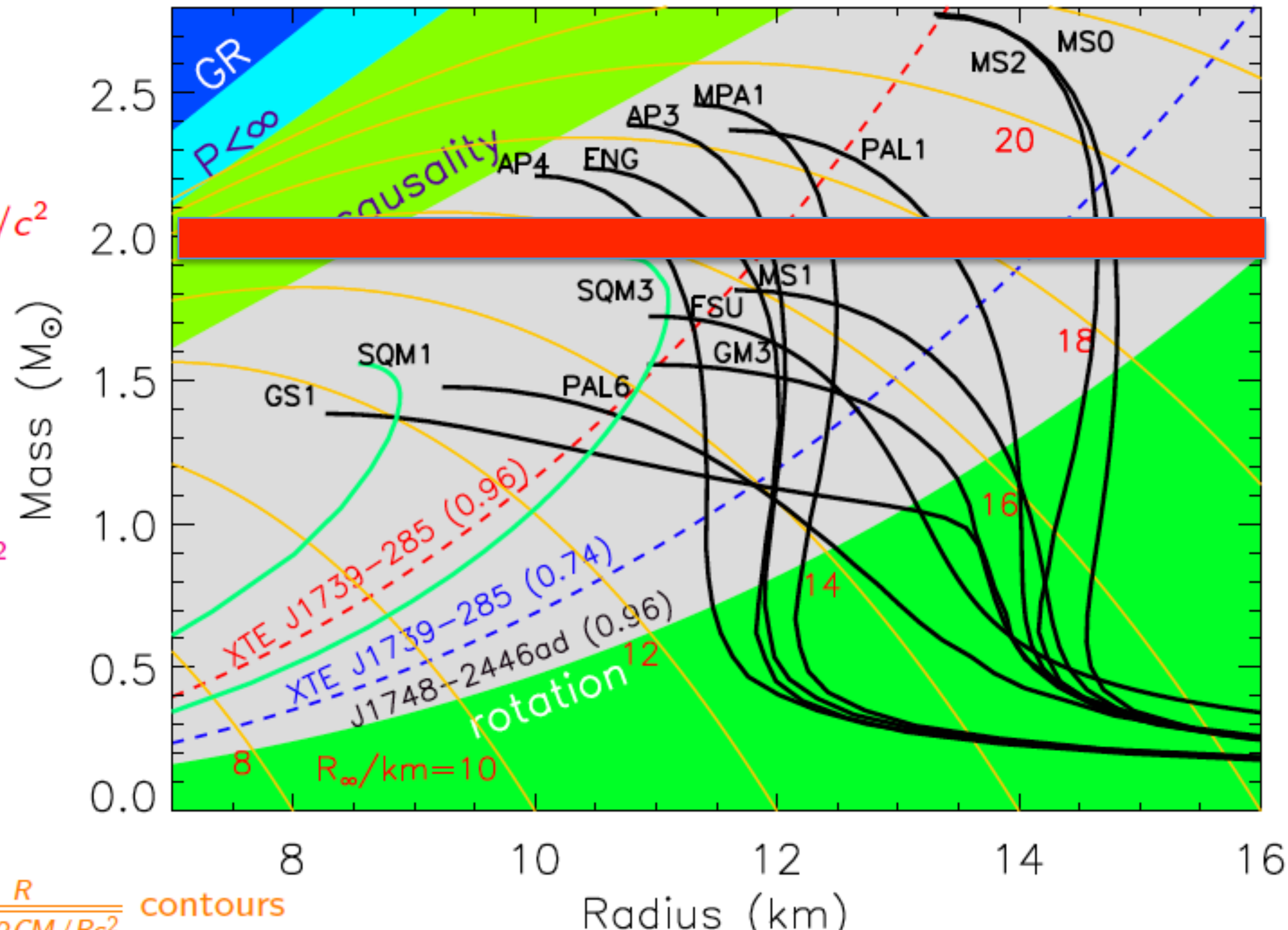
$R \gtrsim 2.9GM/c^2$

— normal NS

— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours

From J.M.Lattimer



SUMMARY I:

1. Nuclear matter constraints on the nuclear force are not adequate:

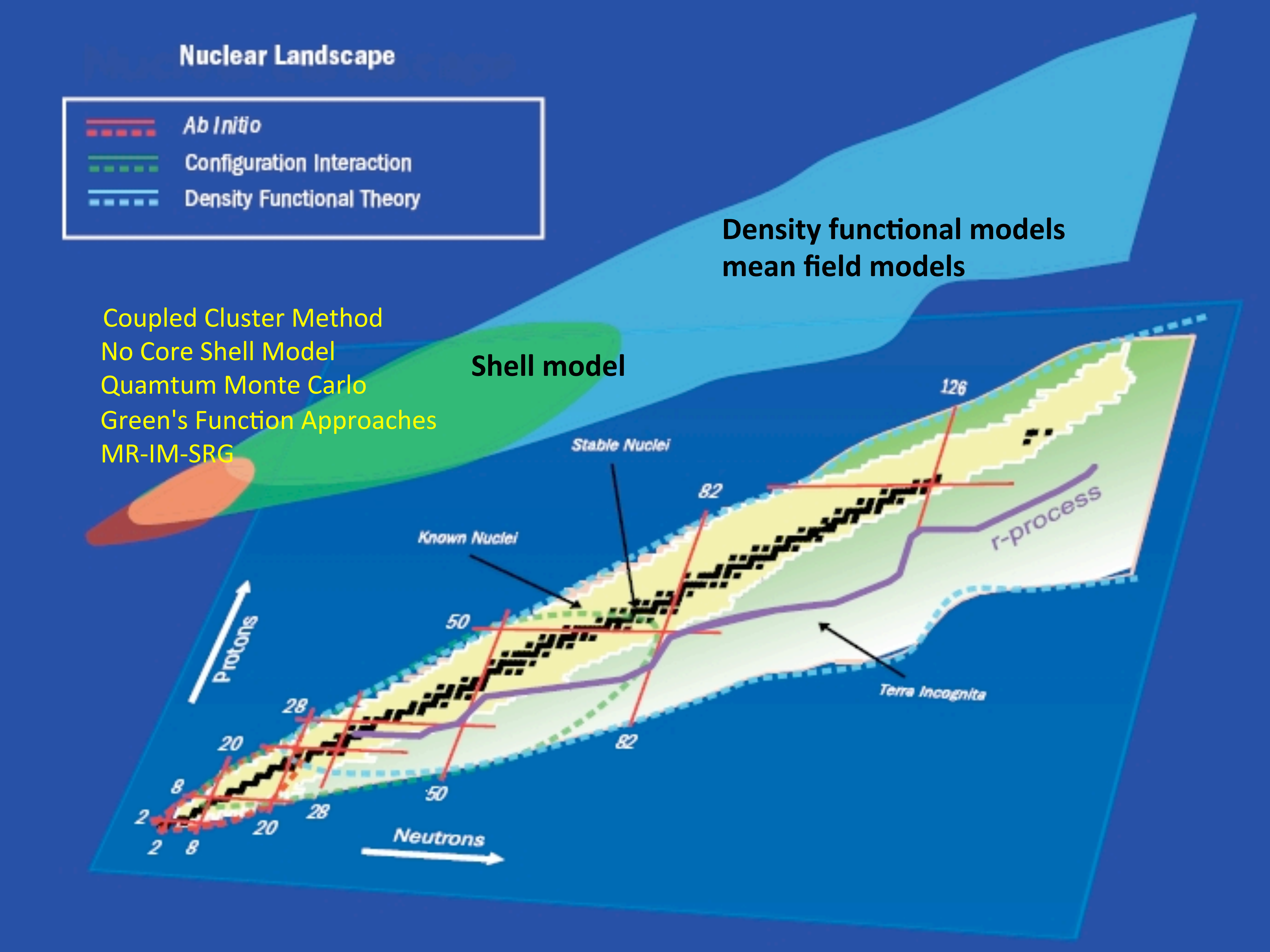
2. The models have too many correlated parameters for much fewer reliable experimental data which are also correlated.
   Their sensitivity to the parameters is variable to say the least.

3. The only firm parameter of neutron stars is their mass which is however dependent on the radius.

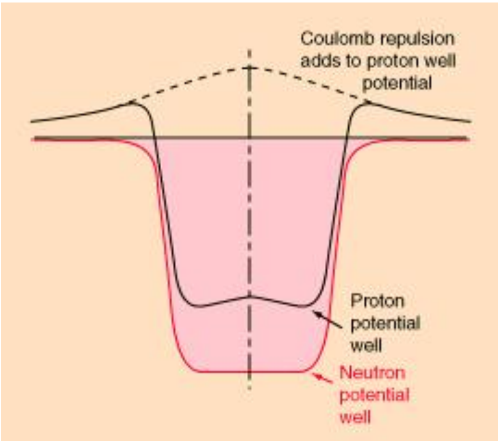
4. Radii are very hard to measure and it is very rare to have both mass and radius known with enough accuracy for the same star.

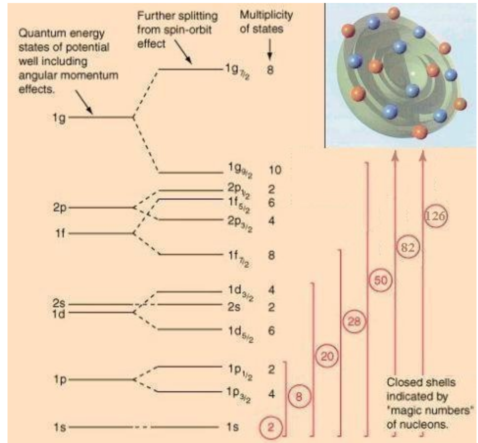
See review with details: JRS, Eur. Phys. J. A (2016) 52: 66

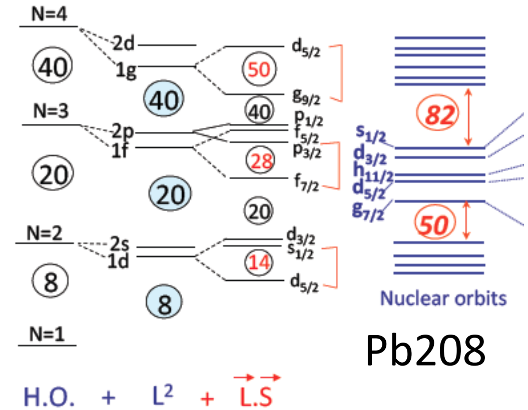
Finite Nuclei



Nuclear Landscape
Ab Initio
Configuration Interaction
Density Functional Theory
Density functional models
mean field models
Coupled Cluster Method
No Core Shell Model
Quamtum Monte Carlo
Green's Function Approaches
MR-IM-SRG
Shell model
126
Stable Nuclei
82
Known Nuclei
r-process
50
Protons
28
20
8
2
Terra Incognita
82
50
28
20
2
8
Neutrons







Courtesy of O.Sorlin

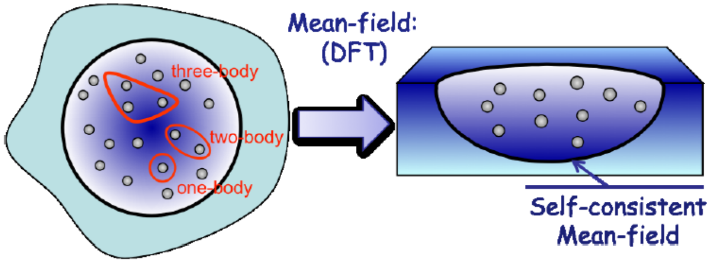
Advantages: Calculates excited states, transition probabilities, decays, electromagnetic moments, etc for spherical and deformed nuclei

Disadvantages: Limited valence space when going away from major shells, need to use different effective interaction in different shells, no calculation for ground states, no nuclear matter

Caurier, E. et al., Rev. Mod.Phys. 77, 427 (2005)
Brown, B. A., Prog. Part. Nucl. Phys. 47, 517(2001).
Otsuka, T., M. Honma, et al. Prog. Part. Nucl. Phys. 47, 319 (2001).

Density functional theory -> Energy density functional

One N – body problem



N one-body problems

Hartree-Fock method

Needs input of a phenomenological density dependent nucleon-nucleon potential Skyrme, Gogny etc. introduces uncertainty as these potentials have many parameters

Advantages: calculation of ground state properties, binding energies, radii, HF single-particle energies, spin-orbit splitting, deformations, giant resonances

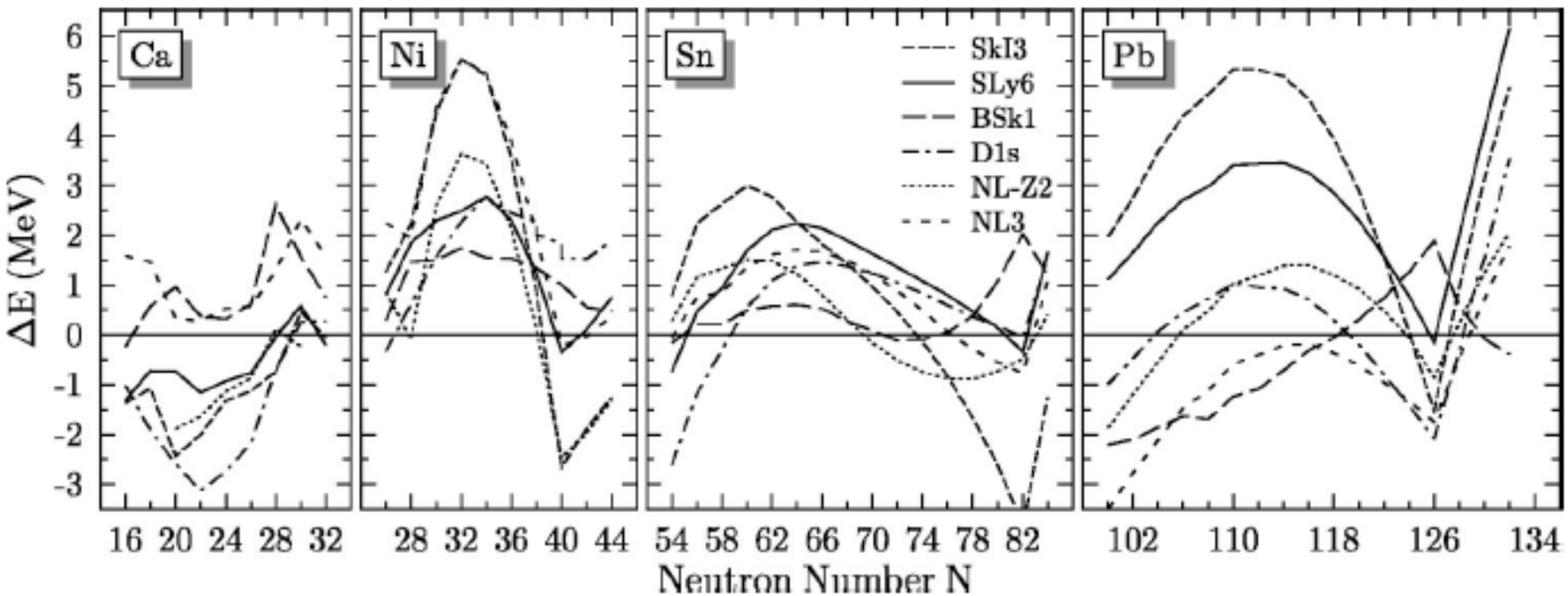
Disadvantages: calculation of realistic excitation states difficult (RPA), no transitions probabilities, beta-decay, electromagnetic moments, pairing has to be added

http://ejc2011.sciencesconf.org/conference/ejc2011/EJC2011_lacroix.pdf

EXAMPLES OF RESULTS OF VARIOUS SKYRME HARTREE-FOCK MODELS I

Deviations in calculated and experimental binding energies for various forces

Differences between theory and experiment for ground state binding energy as calculated using different Skyrme models

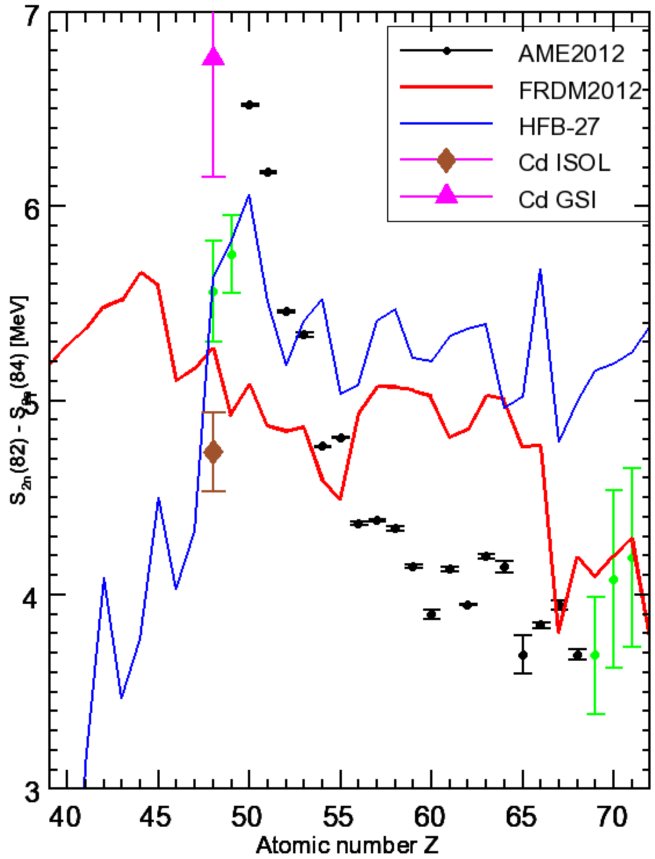


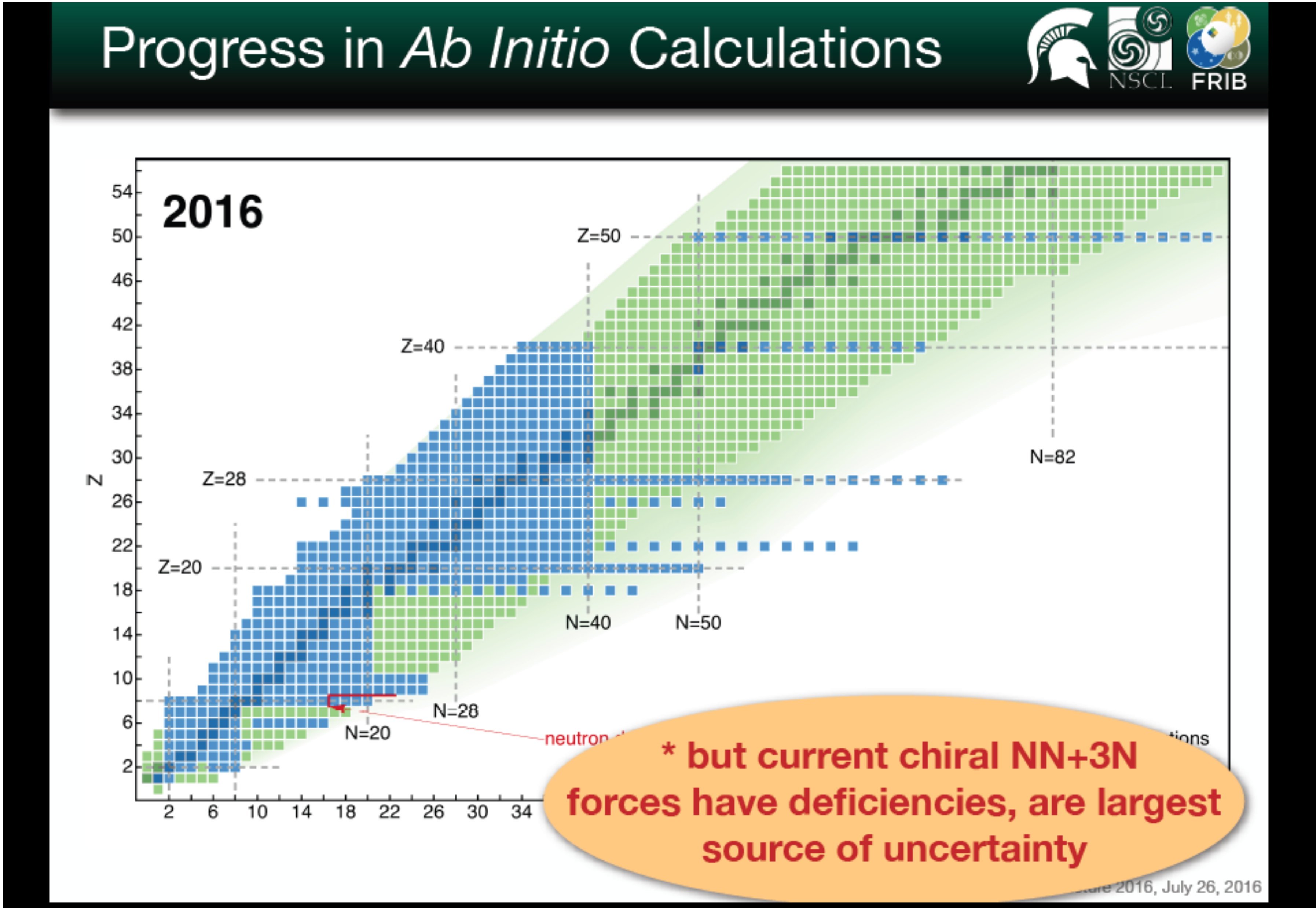
Rev. Mod. Phys., Vol. 75, No. 1, January 2003

Current status od the two-neutron separation energies in Cd region.

Data essential for modeling of the r-process nucleosynthesis

Takem from K.-L. Kratz





Courtesy of Heiko Hergert, NS 2016, Knoxville TN

AB-INITIO: (eg. MR-iM-SRG, Coupled clusters, No-core shell model, Green's function models, Quantum Monte Carlo etc.

Techniques based on Effective field theory – include only nucleon-pion interaction.

Advantage: Provide ground-state and excited states properties (include correlations in the mean field) calculate interaction + operators, estimate errors

Disadvantages: Dependence on cut-off parameters, uncertainty in 3-body forces, computationally expensive to include higher orders in theories and to apply to heavier nuclei.
ALREADY MULTIPLE MODELS EXIST

S. Bogner, R. Furnstahl, and A. Schwenk, Prog. Part. Nucl. Phys. 65 (2010) 94
R. Machleidt and D. R. Entem, Phys. Rept. 503, (2011) 1
H. Hergert et al., Phys. Rept. 621 (2016)165
G. R. Jansen et al., Phys. Rev. Lett. 113 (2014) 142502

SUMMARY II:

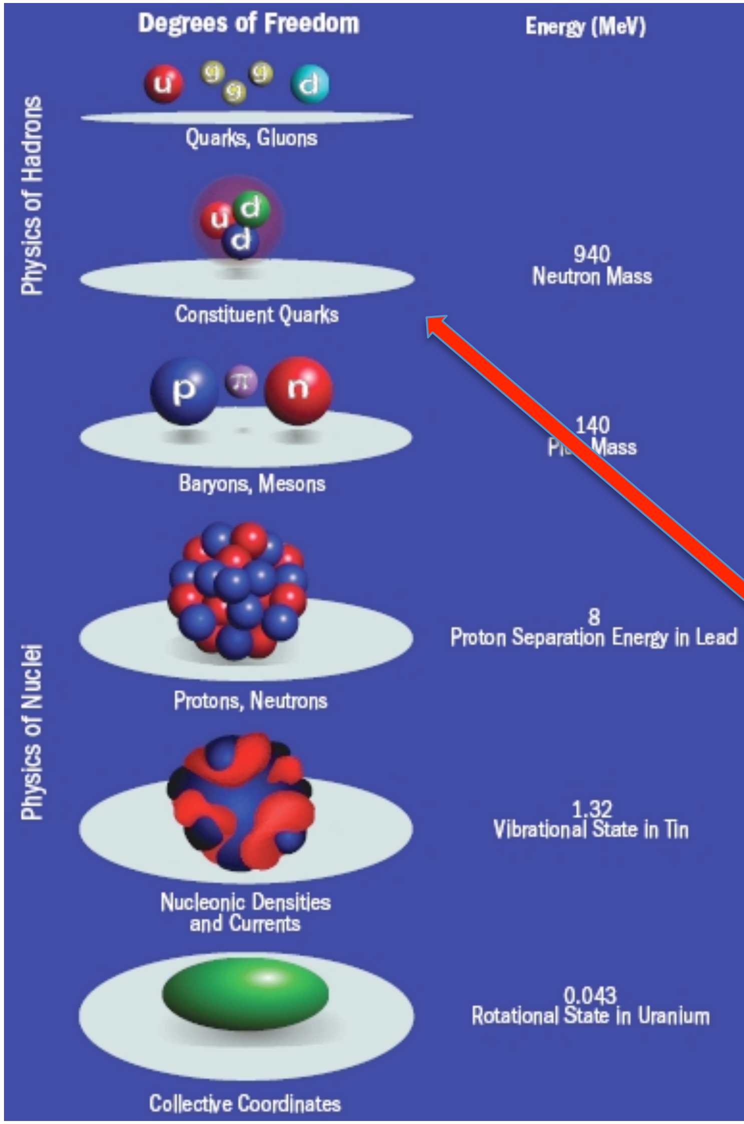
Current models have limited predictive power – they have too many parameters and it is impossible to constrain them unambiguously

Models are often adjusted to fit only a selected class of data well, but they failure elsewhere is neglected . Such models cannot be right. Even "minimal" models are of a limited use in a broader context.

Suggested path towards a solution ?

Look for new physics!

Energy scales in hadron and Nuclei



FUNDAMENTAL QUESTIONS:

1. Is the nucleon immutable?

2. When immersed to a nuclear medium with applied scalar field with strength of order of half of its mass is it really unchangeable?

3. Is this effect relevant to nuclear structure?

Replace interaction between nucleons

↓

By interaction between valence quarks in individual non-overlapping nucleons

Look for the modification of the quark dynamics in a nucleon due to presence of other nucleons

ACCOUNT FOR THE MEDIUM EFFECT

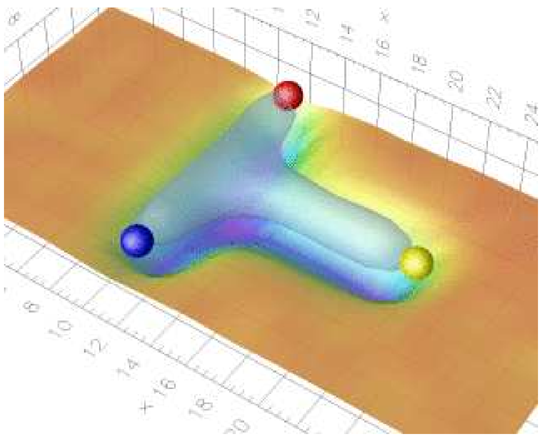
QUARK-MESON-COUPLING MODEL

History:

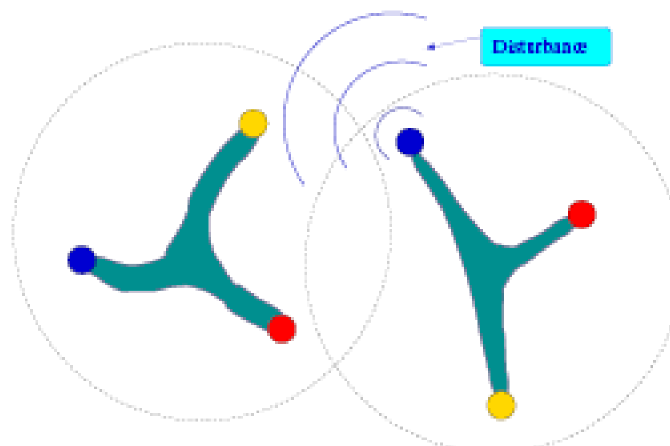
Original: Pierre Guichon (Saclay), Tony Thomas (Adelaide) 1980'

Several variants developed in Japan, Europe, Brazil, Korea, China

Latest: JRS, Guichon, P.-G.Reinhard and Thomas, PRL 116. 092501 (2016)



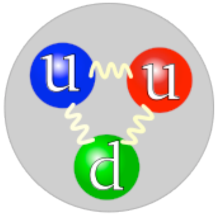
Lattice QCD simulations



Schematic *(Guichon)*

WHAT WE DO:

1. Start with a baryon as an MIT (Massachusetts Institute of Technology) bag (with one qluon exchange) immersed in a mean scalar field created by the other nucleons



2. Solve the bag equations in the density dependent scalar field to obtain a dynamical nucleon effective mass

$$M_N^* = M_N - g_{\sigma N}\bar{\sigma} + \frac{d}{2}\left(g_{\sigma N}\bar{\sigma}\right)^2$$

The last term represents the response of the nucleon to the scalar field with d being the scalar polarizability – the ORIGIN OF MANY- BODY FORCES in QMC.

$d = 0.0044 + 0.211R_B - 0.0357R_B^2$,

where R_B is the bag radius and the coupling constant $g_{\sigma N}$ of the composite nucleon to the σ field at zero density is a parameter to be fitted to data.

Application to nuclear matter:

Obtain Lagrangian density on a hadronic level (baryons, mesons) +leptons, using the effective baryon mass M^*_N, solve the field equations in a mean field approximation (Hartree-Fock), and proceed to calculate standard observables.

$$\mathcal{L} = \sum_B \mathcal{L}_B + \sum_m \mathcal{L}_m + \sum_\ell \mathcal{L}_\ell,$$

for the octet of baryons $B \in \{N, \Lambda, \Sigma, \Xi\}$, selected mesons $m \in \{\sigma, \omega, \rho, \pi\}$, and leptons $\ell \in \{e^-, \mu^-\}$ with the individual Lagrangian densities,

For technical details see Guichon et al NPA772,1 (2006), Stone et al NPA792, 341 (2007) , Whittenbury et al. PRC 89, 065801(2014)

Parameters (very little maneuvering space) :

I. 3 nucleon-meson coupling constants in vacuum $g_{\sigma N}, g_{\omega N}, g_{\rho N}$

$$g_{\sigma N} = 3 g_\sigma^q \int_{Bag} d\vec{r}\, \overline{q} q(\vec{r}) \qquad g_{\omega N} = 3 g_\omega^q \qquad g_{\rho N} = g_\rho^q$$

$$G_{\sigma N} = g_{\sigma N}^2 / m_\sigma^2 \qquad G_{\omega N} = g_{\omega N}^2 / m_\omega^2 \qquad G_{\rho N} = g_{\rho N}^2 / m_\rho^2$$

Constrained by saturation properties of symmetric nuclear matter (saturation density and energy) and the symmetry energy (difference between the energy per particle in SNM and PNM)

II. Meson masses: ω, ρ, π keep their physical values

$650 \text{ MeV} < M_\sigma < 700 \text{ MeV}$

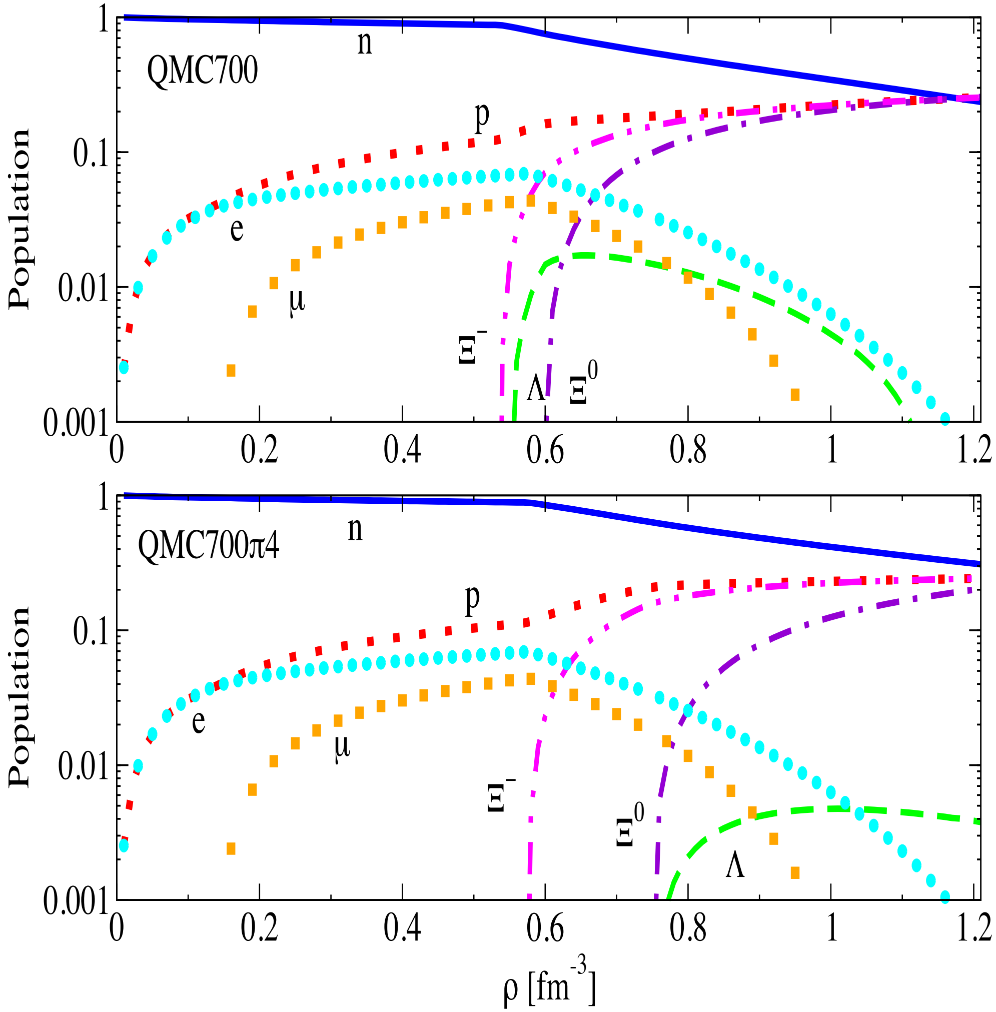
III. Bag radius (free nucleon radius):

1 fm (limited sensitivity within change +/- 20%)

All other parameters either calculated within the model or fixed by symmetry.

Results of QMC to dense nuclear matter and neutron stars

Composition of matter in a neutron star core as calculated in the QMC model (nucleon-hyperon interaction calculated in a mean field approximation)



Existence of Λ - hypernuclei

Non-Existence of bound Σ hypernuclei

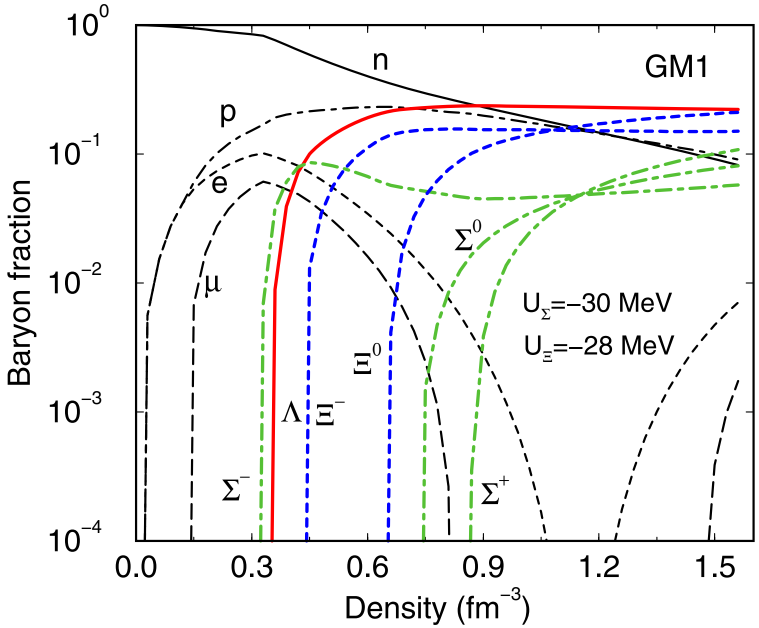
Existence of cascade-hypernucleus

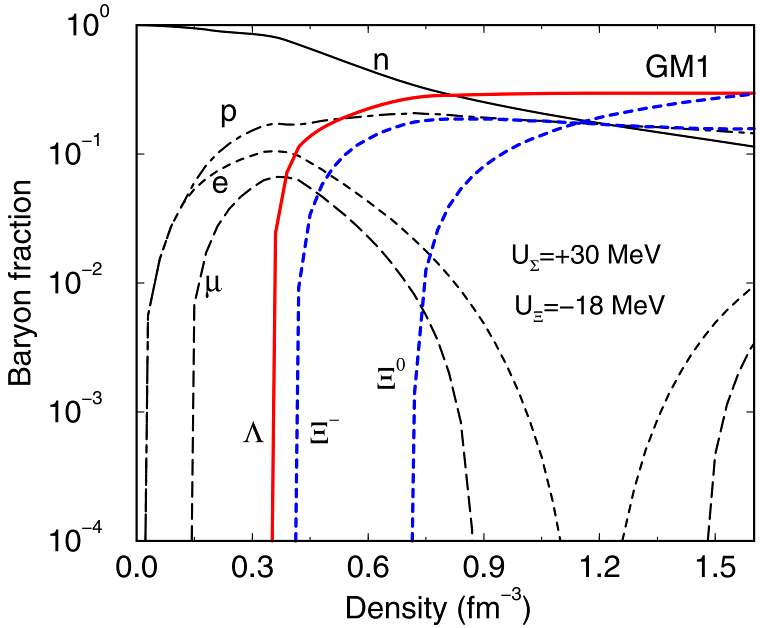
The first evidence of a deeply bound state of Ξ--14N system"K.Nakazawa et al.,Prog. Theor. Exp. Phys. (2015),

New results Emiko Hyama – private communication

JRS, Guichon, Matevosyan, Thomas, NPA 792, 341 (2007)

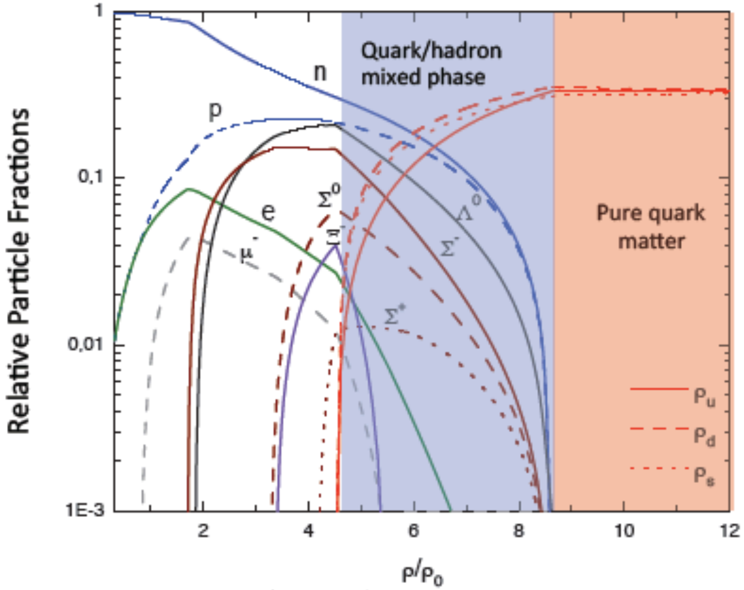
Relativistic mean fields with GM1 interaction
Empirical hyperon-N potentials fitted self-consistently to data.
(J. Schaeffner-Bielich)



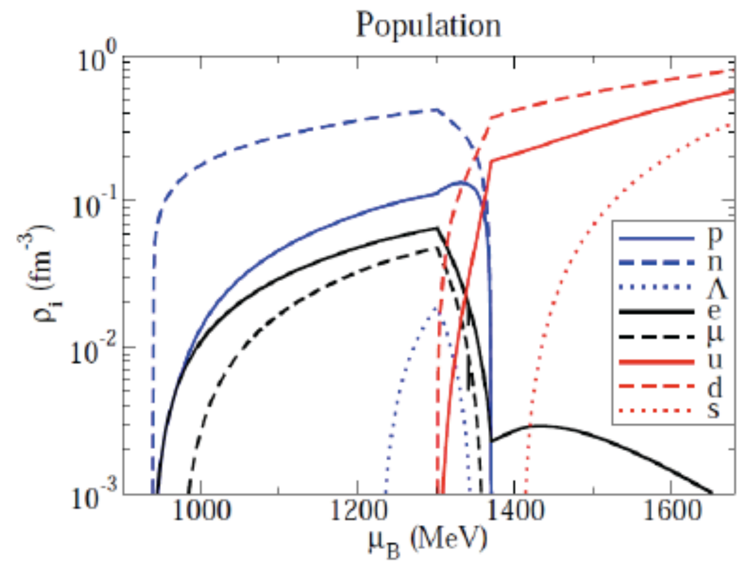


In these models the hyperon-nucleon interaction have to be put in by hand (or fitted).
QMC calculates it within the model.

Model Neutron Star Matter Composition
Non-local SU(3) NJL with vector coupling



Milva Orsaria & Hilario Rodrigues (June 2012)

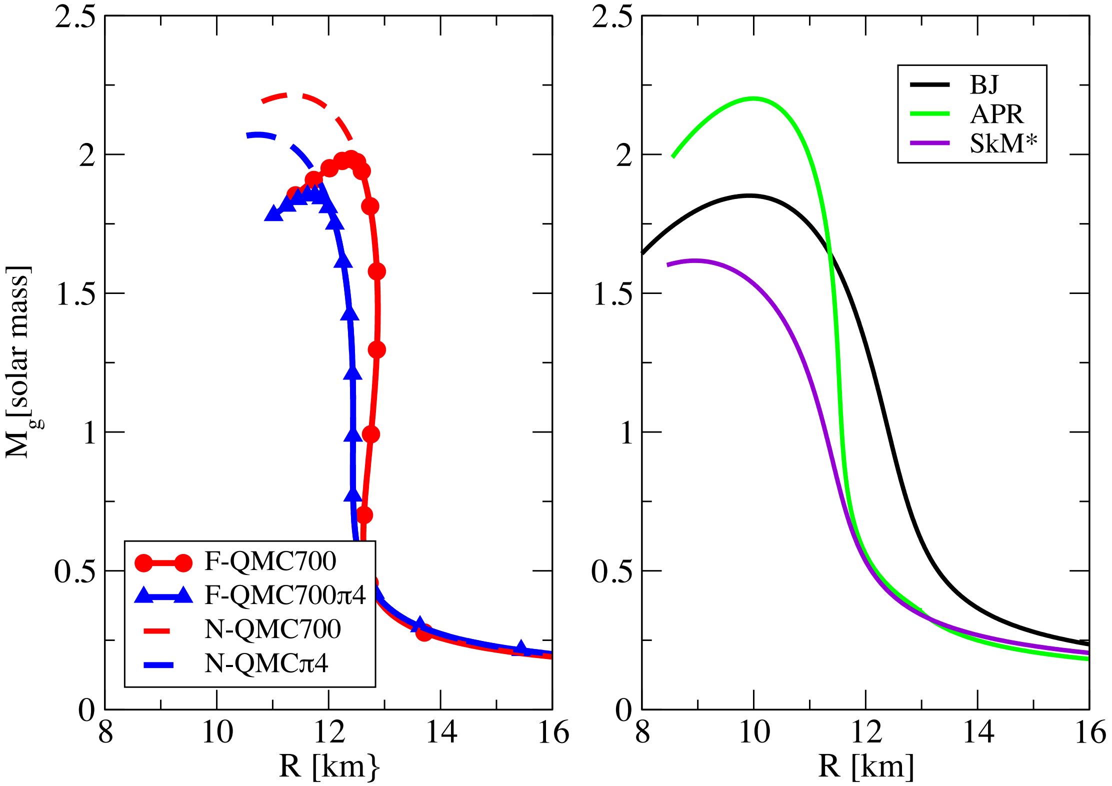


Dexheimer and Schramm, PRC81 045201 (2010)

Physical conditions for appearance: hyperons,
π and K meson condensates
u d s matter +

THRESHOLD DENSITIES UNKNOWN - STRONGLY MODEL DEPENDENT

Mass-radius of a neutron star prediction by the QMC model (relativistic version)



2 M_{solar} mass neutron star with full hyperon octet predicted 3 years before its observation – no hyperon puzzle

Parameters of the QMC models for nuclear matter as derived in JRS, Guichon, Matevosyan, Thomas, NPA 792, 341 (2007)

The couplings for different versions of the model. The column π is the number by which the pion contribution has been multiplied. $\mathcal{E}$ is the binding energy of symmetric nuclear matter and K_∞ its incompressibility modulus

Model	m_σ (MeV)	π	$\mathcal{E}$ (MeV)	G_σ (fm^2)	G_ω (fm^2)	G_ρ (fm^2)	K_∞ (MeV)
QMC600	600	0	−15.86	11.23	7.31	4.81	344
QMC700	700	0	−15.86	11.33	7.27	4.56	340
QMCπ1	700	1	−15.86	10.64	7.11	3.96	322
QMCπ2	700	1	−14.5	10.22	6.91	3.90	301
QMCπ3	700	1.5	−14	9.69	6.73	3.57	283
QMCπ4	700	2	−13	8.97	6.43	3.22	256

Application to finite nuclei

Application to finite nuclei(non-relativistic approximation):

Derive local QMC energy functional

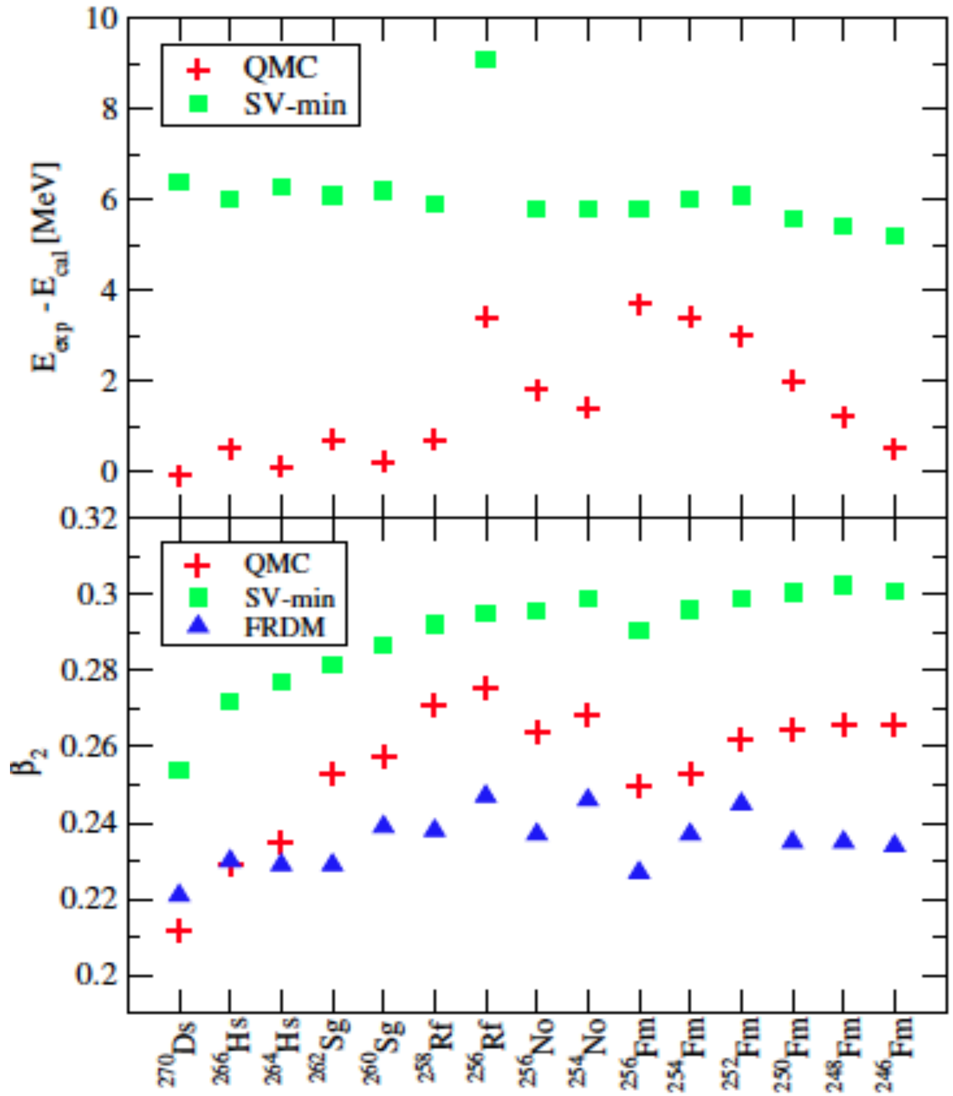
$$\langle H(\vec{r})\rangle = \rho M + \frac{\tau}{2M} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\rm eff} + \mathcal{H}_{\rm fin} + \mathcal{H}_{\rm so}$$

ELAIDE

implemented into a 2D Hartree-Fock + BCS model (spherical and quadrupole and octupole deformation)

Stone, Guichon, Reinhard, Thomas PRL 116, 092501 (2016)

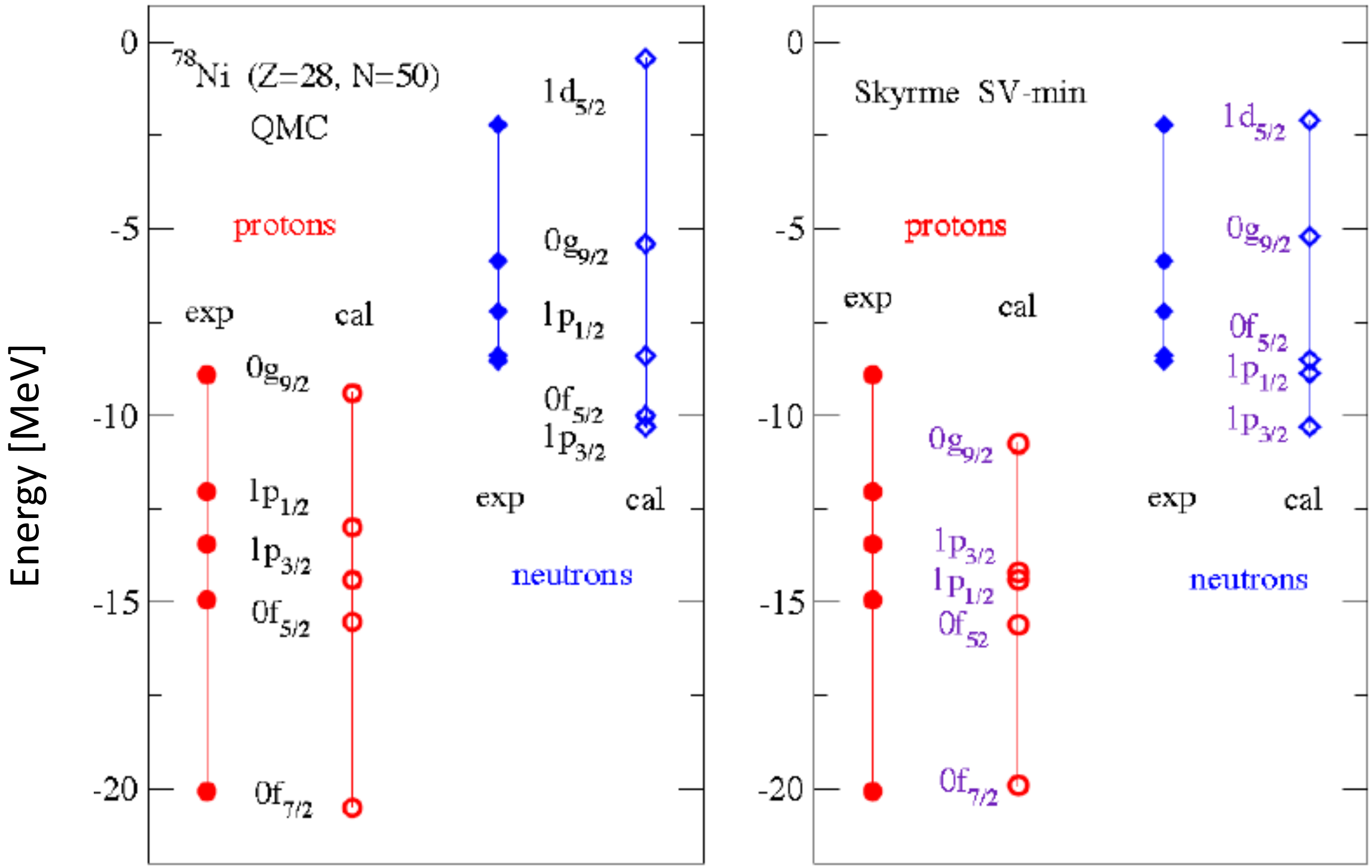
Quadrupole deformation and ground state binding energy for selected SHE nuclei



Skyrme SV-min
P. Klupfel, P.-G. Reinhard,
T. J. Burvenich, and J. A. Maruhn,
Phys. Rev. C **79**, 034310 (2009)

Finite Range Droplet Model (FRDM)
P. Moller et al., ADNDT, 59, 185
(1995)

Spectrum of single-particle energies in the ground state of ^{78}Ni



Data taken from Grawe et al., Rep.Prog.Phys. 70, 1525 (2007)

Final parameters:

$$G_\sigma = 11.847 \pm 0.020 \text{ fm}^2$$

$$G_\omega = 8.268 \pm 0.020 \text{ fm}^2$$

$$G_\rho = 7.682 \pm 0.025 \text{ fm}^2$$

$$M_\sigma = 3.66 \pm 0.01 \text{ fm}^2$$

Consistent with SNM properties:

E_0 =-16.03 MeV, ρ_0 = 0.153 fm^{-3} ,

K_0 = 340 MeV, S_0 = 29.99 MeV,

L = 23.35 MeV

m*/m = 0.77

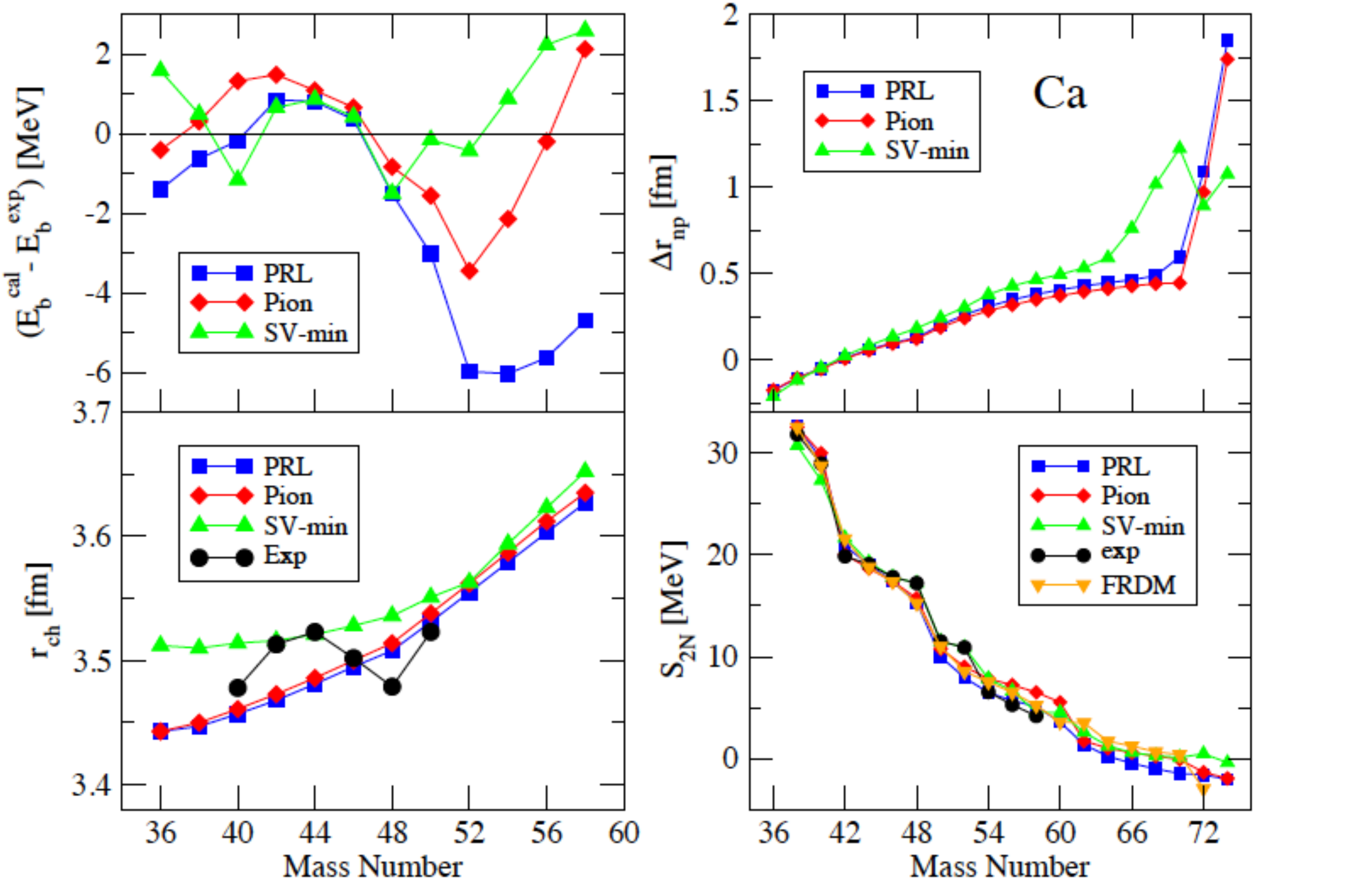
Note that in QMC we determine error of the parameters.

The set is unique within these errors for the current Hamiltonian

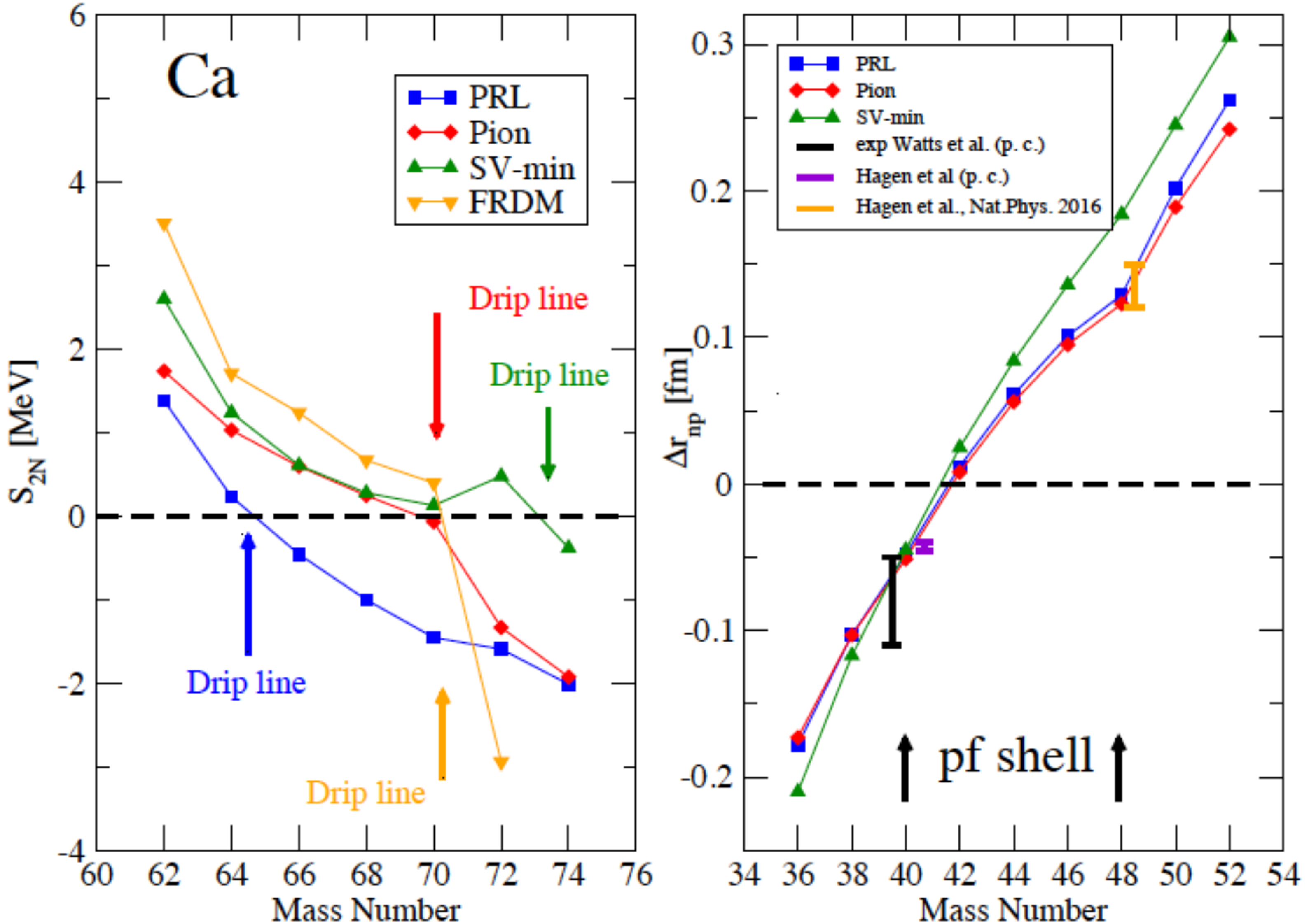
(Practically impossible to identify a unique set for other mean-field models with many more parameters – see eg. Klupfel et al., PRC 79, 034310 (2009))

Addition of the pion exchange

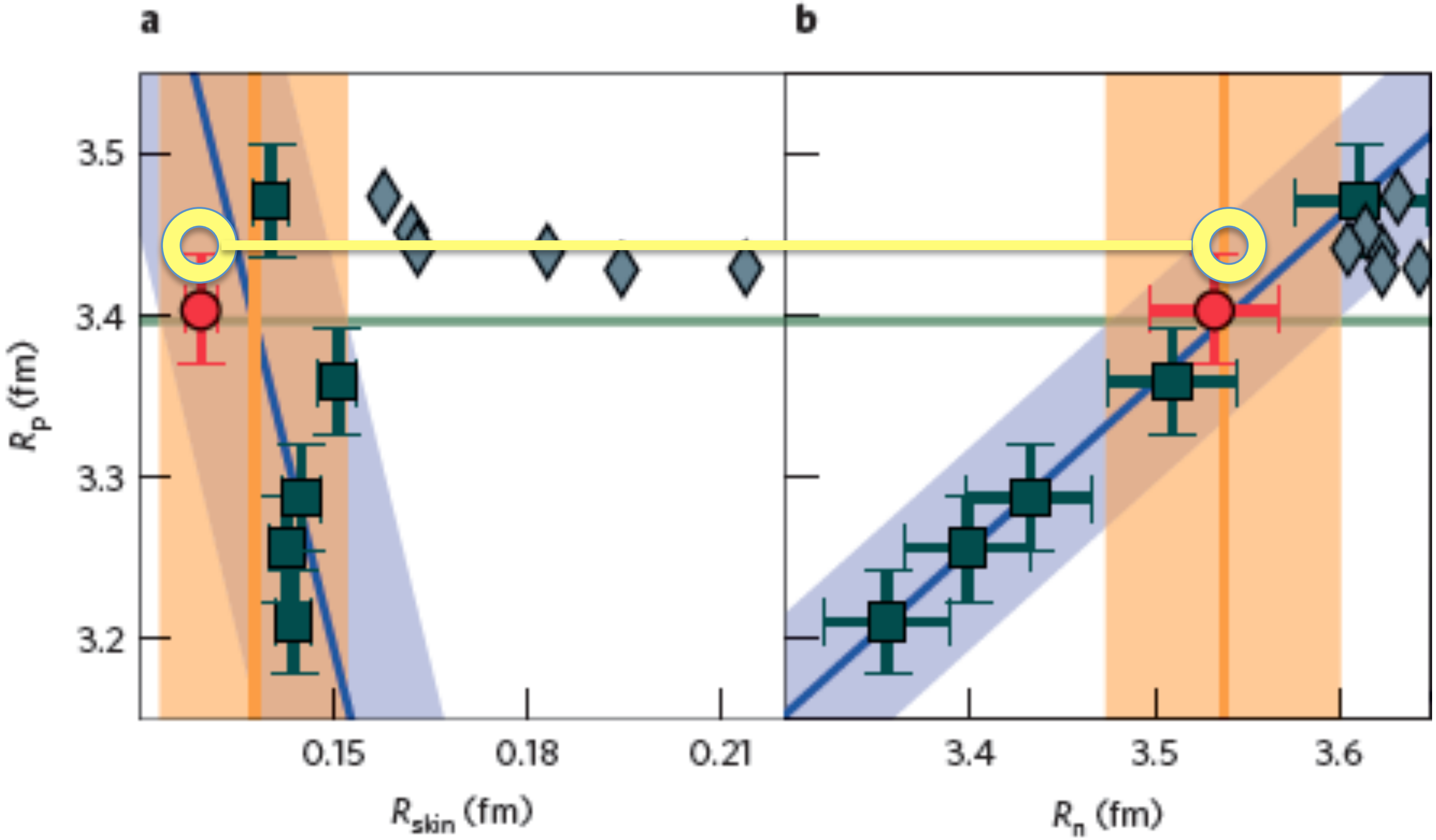
The effect of the pion – Ca region I



The effect of the pion-II



Correlation of neutron skin, point proton and neutron radii in ^{48}Ca



From Hagen et al., Nat.Phys. 12, 186 (2016)

Red circles: **NNLO$_{sat}$ PRC 91, 051301(R) (2015)**

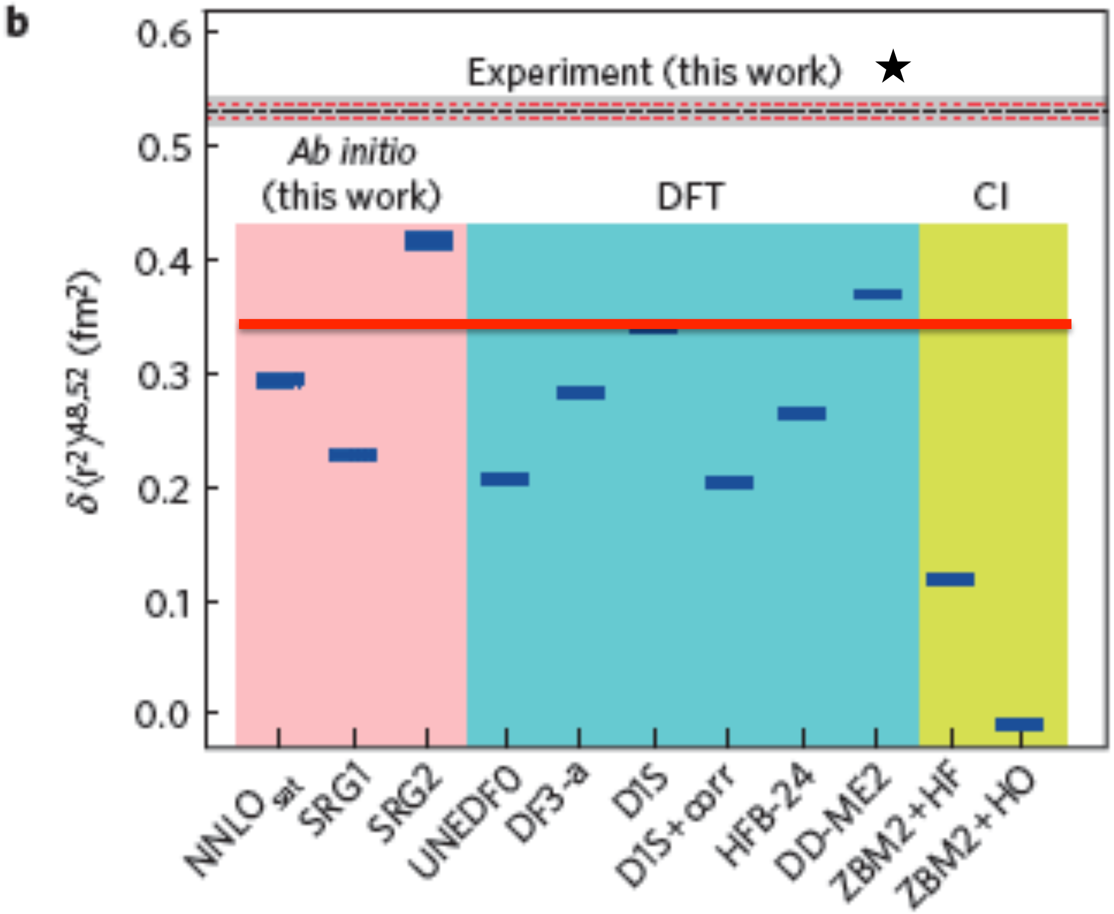
Blue squares: **Ch-Int. PRC 83, 031301 (2011)**

Grey diamonds: **DFT PRC 85, 041302 (2012)**

Yellow symbols: QMC(π)

Sudden increase in nuclear size above ^{48}Ca

Isotope shift



$$\delta < r_{ch}^2 >^{48,52}$$

0.530(5) fm² exp

0.340 fm² QMC(π)

0.192 fm² SV-min

$$\delta < r_{ch}^2 >^{48,50}$$

0.293(37) fm² exp

0.169 fm² QMC(π)

0.107 fm² SV-min

★ Garcia Ruiz et al., Nat. Phys. 12, 594 (2016)

Ab initio predictions of observables in ^{48}Ca for models with varying two-body cut-off parameters and EM and PWA two-body potentials (Basis for estimation of uncertainties)

Supplementary material, Hagen et al., Nat.Phys. 12, 186 (2016)

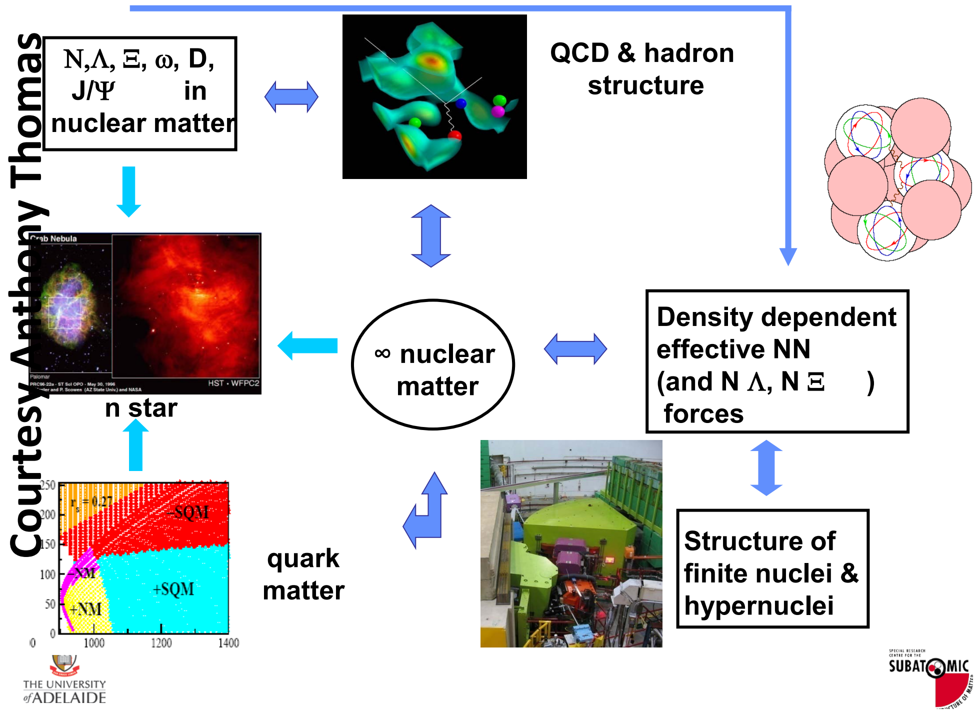
Interaction	BE	S_n	Δ	R_{ch}
$NNLO_{sat}$	404(3)	9.5	2.69	3.48
1.8/2.0 (EM)	420(1)	10.1	2.69	3.30
2.0/2.0 (EM)	396(2)	9.3	2.66	3.34
2.2/2.0 (EM)	379(2)	8.8	2.61	3.37
2.8/2.0 (EM)	351(3)	8.0	2.41	3.44
2.0/2.0 (PWA)	346(4)	7.8	2.82	3.55
Experiment	415.99	9.995	2.399	3.477

See also Hebeler et al., PRC 83, 031301 (2011) more explanation

BE, S_N and $\Delta = (Sn(^{48}Ca) - Sn(^{49}Ca))/2$ in MeV, R_{ch} in fm.

SUMMARY III:

If a model works it has to work everywhere



IF IT FAILS, PLAYING WITH PARAMETERS OF ADDITION OF TERMS WITHOUT
A CLEAR PHYSICAL MEANING IS NOT THE WAY FORWARD

IT IS THE PHYSICS WHICH HAS TO BE LOOKED INTO.

It is just the beginning…….

Back-up slides

	QMC force	Skyrme force
Parameters:	4 (unique)	10+ (infinite)
Physics base:	more fundamental	more empirical
Nuclear matter:	valid	sometimes valid
Finite nuclei:	1 – 2 % level	less than 1 – 2 % level
Neutron stars: (rel)	valid up to ~6-7 ρ_0 hyperons	not valid above ~ 3 ρ_0 nucleon only
Future:	development (pions +)	?

$$\mathcal{H}_0 + \mathcal{H}_3 = \rho^2 \left[\frac{-3G_\rho}{32} + \frac{G_\sigma}{8(1+d\rho G_\sigma)^3} - \frac{G_\sigma}{2(1+d\rho G_\sigma)} + \frac{3G_\omega}{8} \right] + (\rho_n - \rho_p)^2 \left[\frac{5G_\rho}{32} + \frac{G_\sigma}{8(1+d\rho G_\sigma)^3} - \frac{G_\omega}{8} \right],$$

$$\mathcal{H}_{\rm eff} = \left[\left(\frac{G_\rho}{8m_\rho{}^2} - \frac{G_\sigma}{2m_\sigma{}^2} + \frac{G_\omega}{2m_\omega{}^2} + \frac{G_\sigma}{4M_N{}^2} \right) \rho_n + \left(\frac{G_\rho}{4m_\rho^2} + \frac{G_\sigma}{2M_N^2} \right) \rho_p \right] \tau_n + p \leftrightarrow n,$$

$$\mathcal{H}_{\rm fin} = \left[\left(\frac{3G_\rho}{32m_\rho{}^2} - \frac{3G_\sigma}{8m_\sigma{}^2} + \frac{3G_\omega}{8m_\omega{}^2} - \frac{G_\sigma}{8M_N{}^2} \right) \rho_n + \left(\frac{-3G_\rho}{16m_\rho{}^2} - \frac{G_\sigma}{2m_\sigma{}^2} + \frac{G_\omega}{2m_\omega{}^2} - \frac{G_\sigma}{4M_N{}^2} \right) \rho_p \right] \nabla^2(\rho_n) + p \leftrightarrow n,$$

$$\mathcal{H}_{\rm so} = \nabla \cdot J_n \left[\left(\frac{-3G_\sigma}{8M_N{}^2} - \frac{3G_\omega(-1+2\mu_s)}{8M_N{}^2} - \frac{3G_\rho(-1+2\mu_v)}{32M_N{}^2} \right) \rho_n + \left(\frac{-G_\sigma}{4M_N{}^2} + \frac{G_\omega(1-2\mu_s)}{4M_N{}^2} \right) \rho_p \right] + p \leftrightarrow n.$$

Overview of the results (compared to a Skyrme interaction SV-min *)

	rms deviations			
	[%]		[absolute]	
Data	QMC	SV-min	QMC	SV-min
Fit nuclei:				
Binding energies	0.36	0.24	2.85 MeV	0.62 MeV
Diffraction radii	1.62	0.91	0.064 fm	0.029 fm
Surface thickness	10.9	2.9	0.080 fm	0.022 fm
rms radii	0.71	0.52	0.025 fm	0.014 fm
Pairing gap (n)	57.6	17.6	0.49 MeV	0.14 MeV
Pairing gap (p)	25.3	15.5	0.052 MeV	0.11 MeV
Spin-orbit splitting (p)	15.8	18.5	0.16 MeV	0.18 MeV
Spin-orbit splitting (n)	20.3	16.3	0.30 MeV	0.20 MeV
Nuclei not included in the fit:				
Superheavy nuclei	0.10	0.32	1.97 MeV	6.17 MeV
$N = Z$ nuclei	2.54	1.44	5.89 MeV	3.47 MeV
Mirror nuclei	3.16	2.83	5.27 MeV	3.37 MeV
Other	0.51	0.30	4.27 MeV	3.19 MeV

*) P. Klupfel et al., Phys. Rev. C **79**, 034310 (2009)

Phase transitions:

Gas –liquid crust – core transition

Uniform beta equilibrium hadronic matter:
nucleons, hyperons, boson condensates

Supernova matter (no equilibrium)

hadronic – quark matter

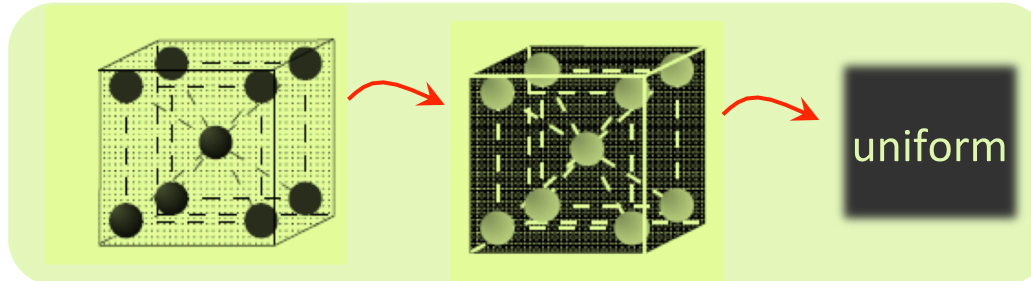
quark matter (superfluid phases)

Nuclear "pasta" structures

Courtesy Toshi Maruyama

- Baym, Bethe, Pethick, 1971

  "Nuclei inside-out"



- Ravenhall et al 1983 & Hashimoto et al 1984

  Concept of "pasta" structures. Minimizing free-energy of the inhomogeneous structure, i.e., achieving the balance between **surface tension** and the **Coulomb repulsion**
  → nuclear **pasta**

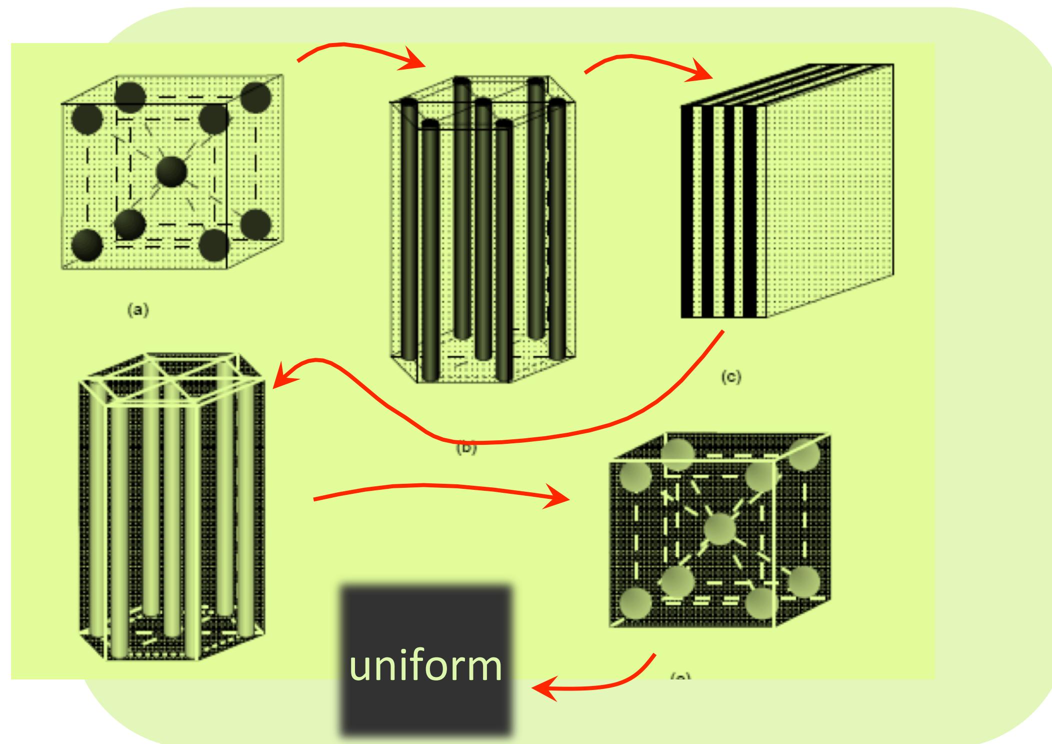


Figure from K. Oyamatsu, NP**A561**, 431 (1993)

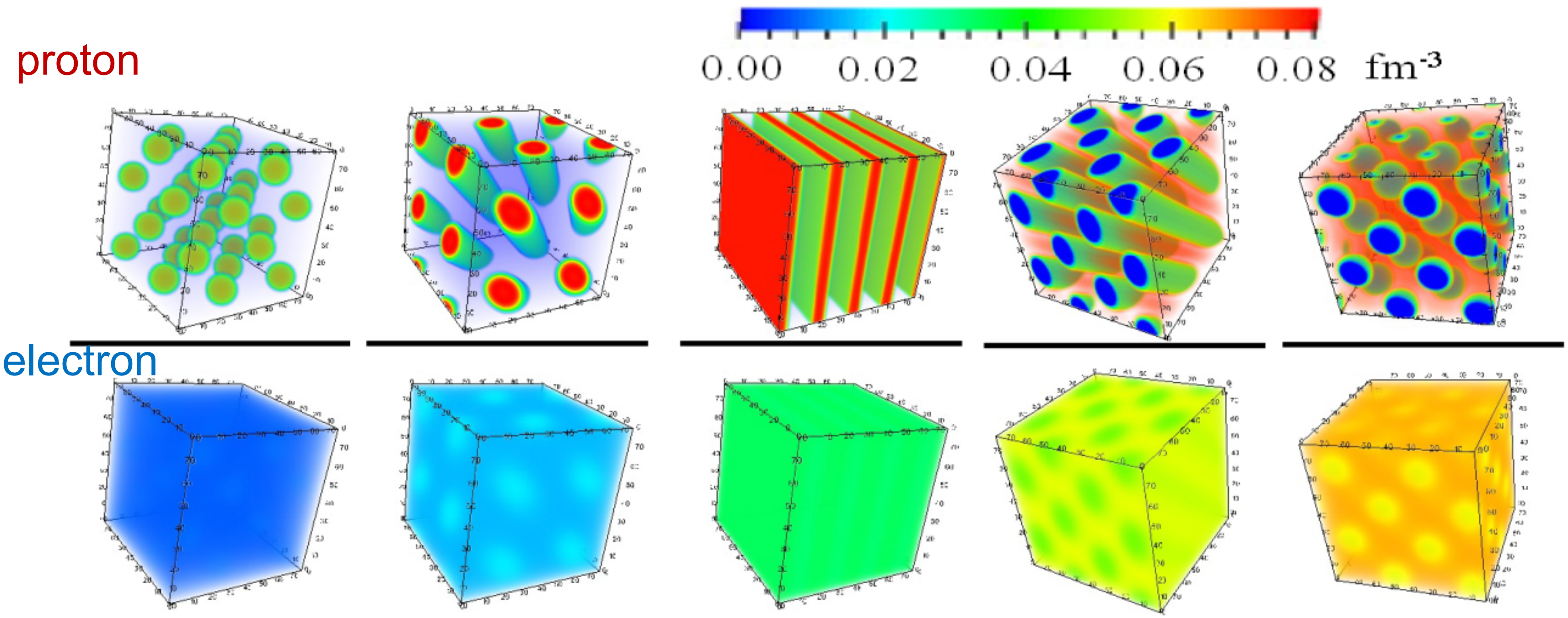
Courtesy Toshi Maruyama

Courtesy Toshi Maruyama

Fully 3D RMF calculations

[**Phys.Lett. B713 (2012) 284**]

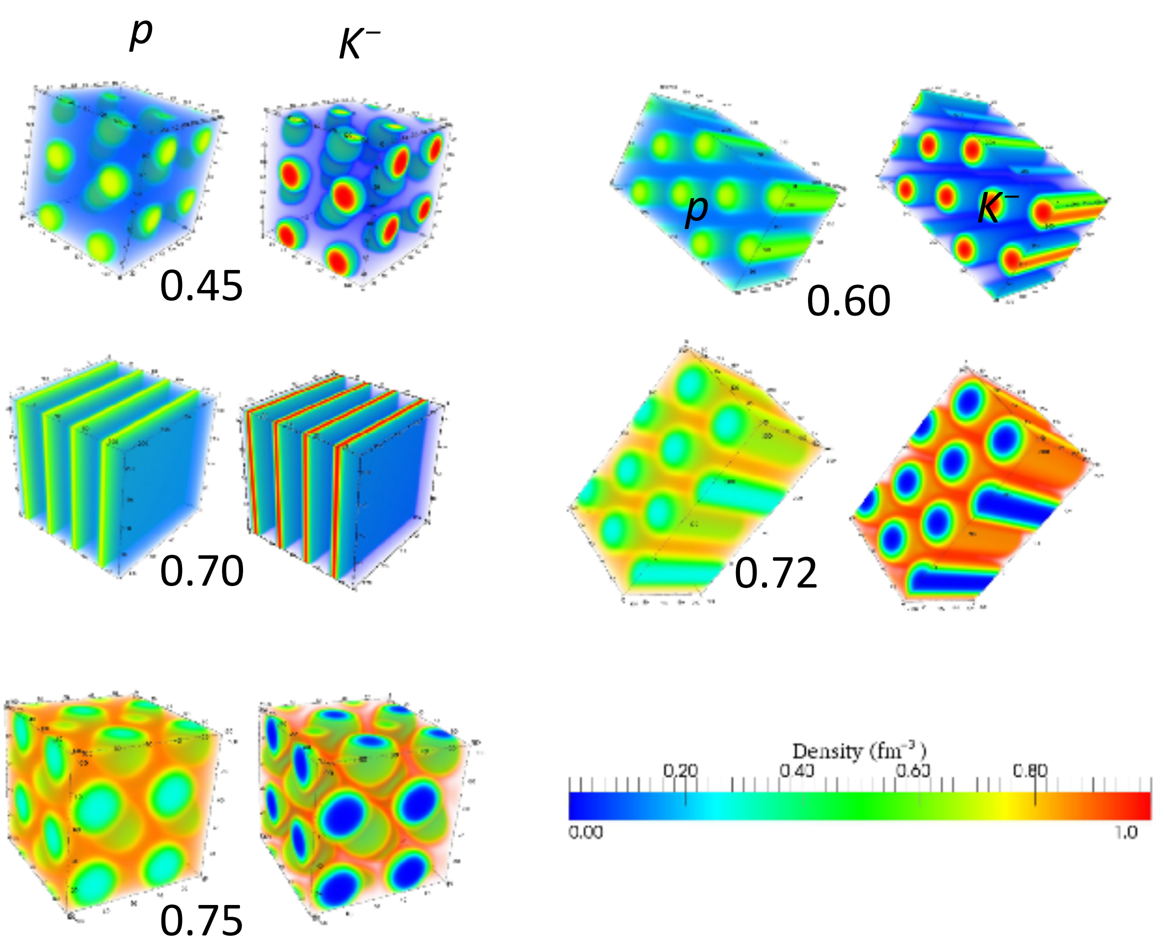
$Y_p = Z/A = 0.5$

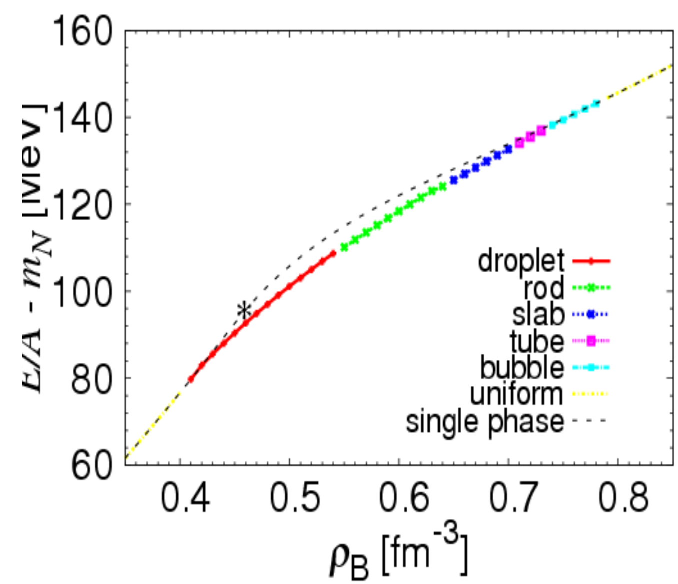


"droplet" [fcc]	"rod" [honeycomb]	"slab"	"tube" [honeycomb]	"bubble" [fcc]
ρ_B = 0.012 fm^{-3}	0.024 fm^{-3}	0.05 fm^{-3}	0.08 fm^{-3}	0.094 fm^{-3}

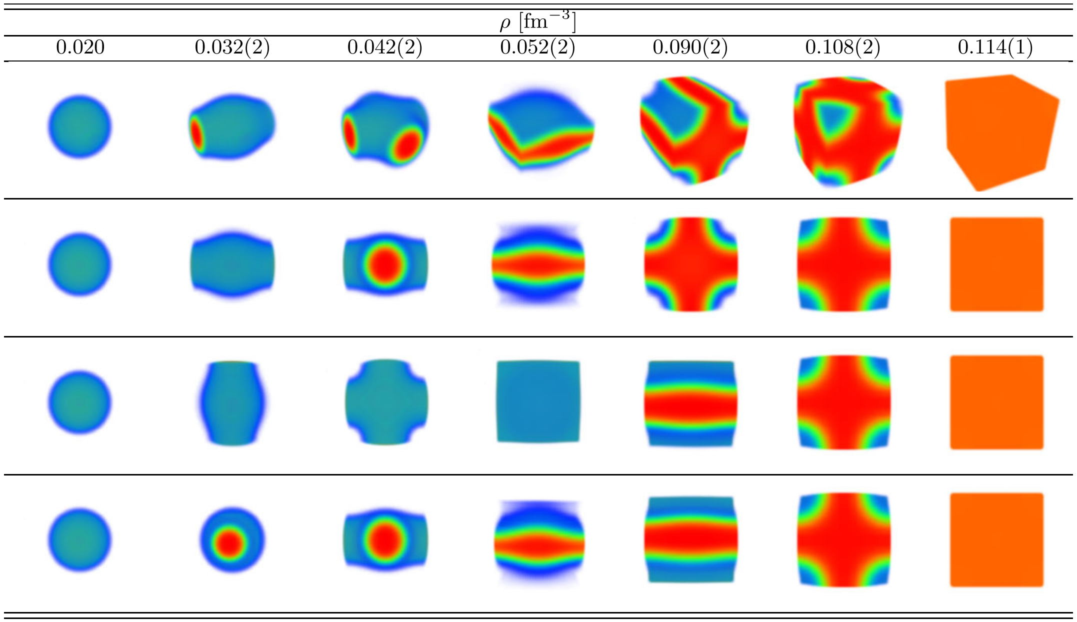
[M. Okamoto, PhD thesis, Univ. Tsukuba]

Fully 3D calculation – pasta in kaon condensate



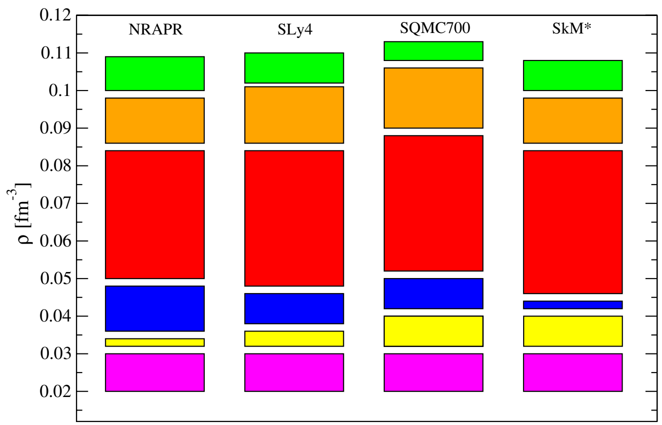


Courtesy Toshi Maruyama



Pais and Stone: PRL 109, 2012

First row: Pasta phases in neutron matter calculated using the SQMC700 Skyrme interaction, T = 2 MeV and yp = 0.3 Rows 2, 3, 4: 2D projection of the pasta phases on the (y, x), (x, z), and (y, z) planes, respectively.

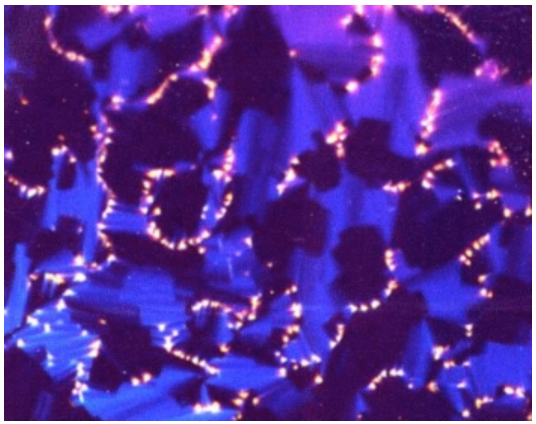


Comparison of phase diagrams at T= 2 MeV and yp =3 as calculated for the four Skyrme interactions used in the the fully selfconsistent 3D-SHF model.

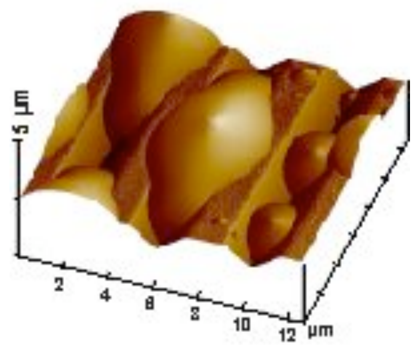
Max Plank Institute for Dynamics and Self – Organisation

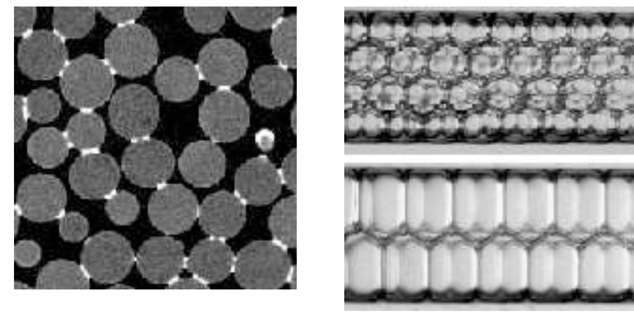
Soft solids: emulsions, foams, colloids, polymers, gels , liquid crystals, cytoplasma

Flexible internal structure, weak interactions, easily influenced by external conditions



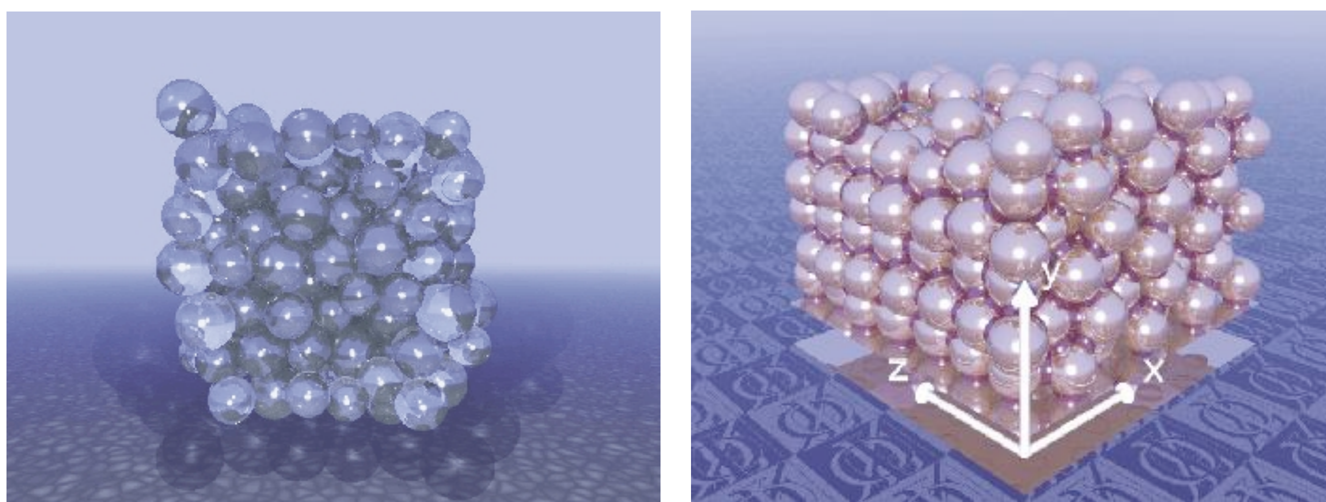
Liquid crystal



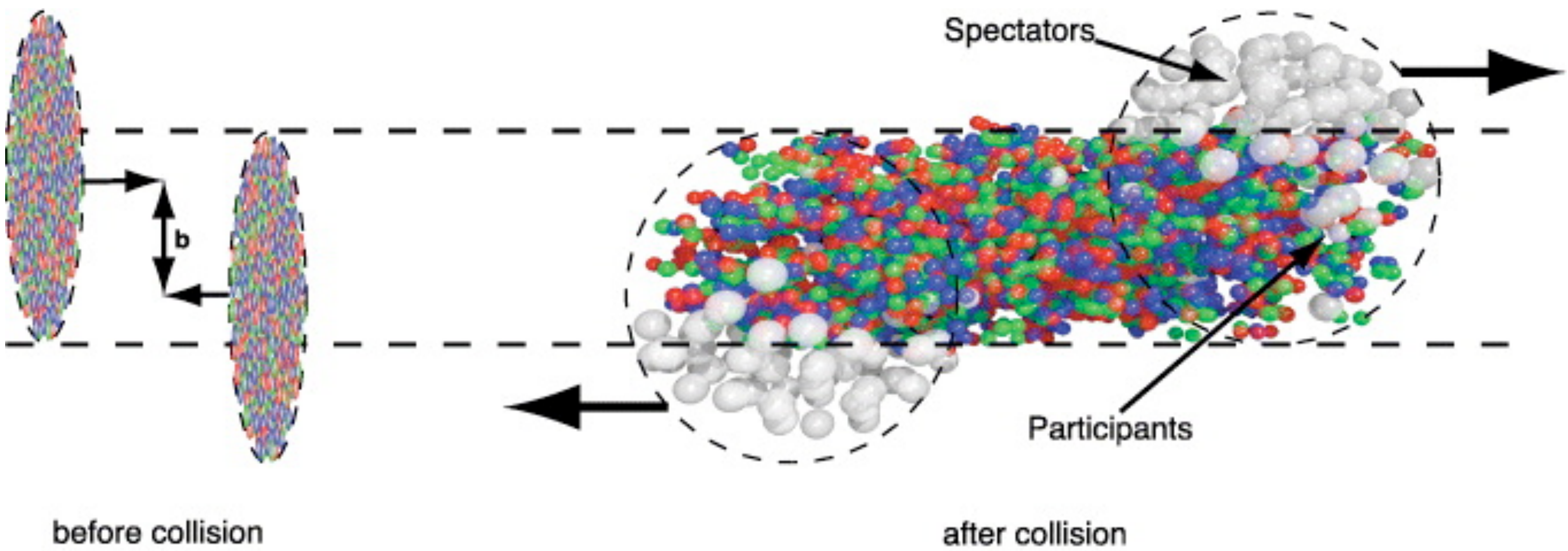


Geometry of fluid interfaces

Granular matter
under stress



Heavy Ion Collisions



Heavy Ion collisions:

GSI, MSU, Texas A&M, RHIC, LHC — existing
FAIR (GSI), NICA (Dubna, Russia) — planned

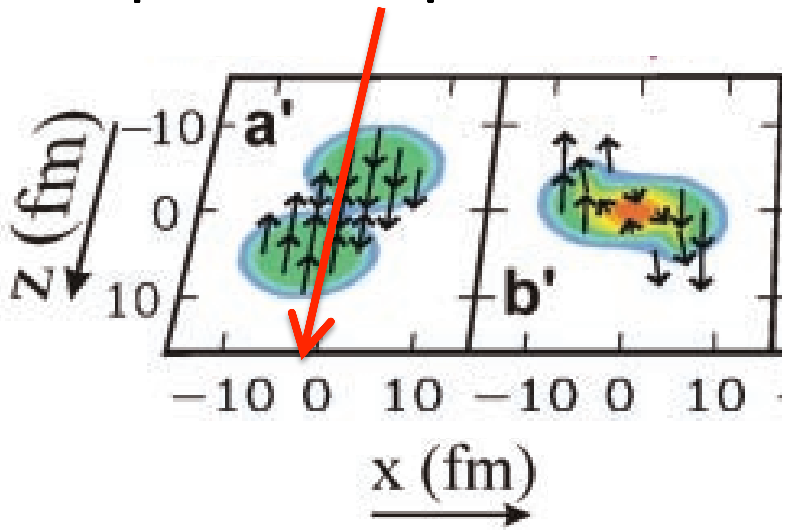
Measurement: Beam energy 35 A MeV – 5.5 A TeV
Collisions (Au,Au), (Sn,Sn) , (Cu,Cu)
but also (p,p) for a comparison
Transverse and Elliptical particle flow

Calculation: Transport models -- empirical mean field potentials
Fit to data → energy density → P (ε) → the EoS
(extrapolation to equilibrium, zero temperature, infinite matter) (e.g Danielewicz et al., Science 298, 2002, Bao-An Li et al., Phys.Rep. 464, 2008)

Quantum Molecular Dynamics
(e.g. Yingxun Zhang, Zhuxia Li, Akira Ono)

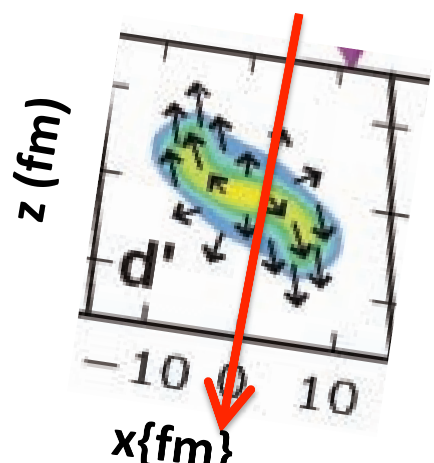
Two EoS sensitive observables:

Elliptic flow: Comparison of in-plane to out-of- plane emission rates



Red line – beam
Reaction plane x z
Spectator nucleon blocking

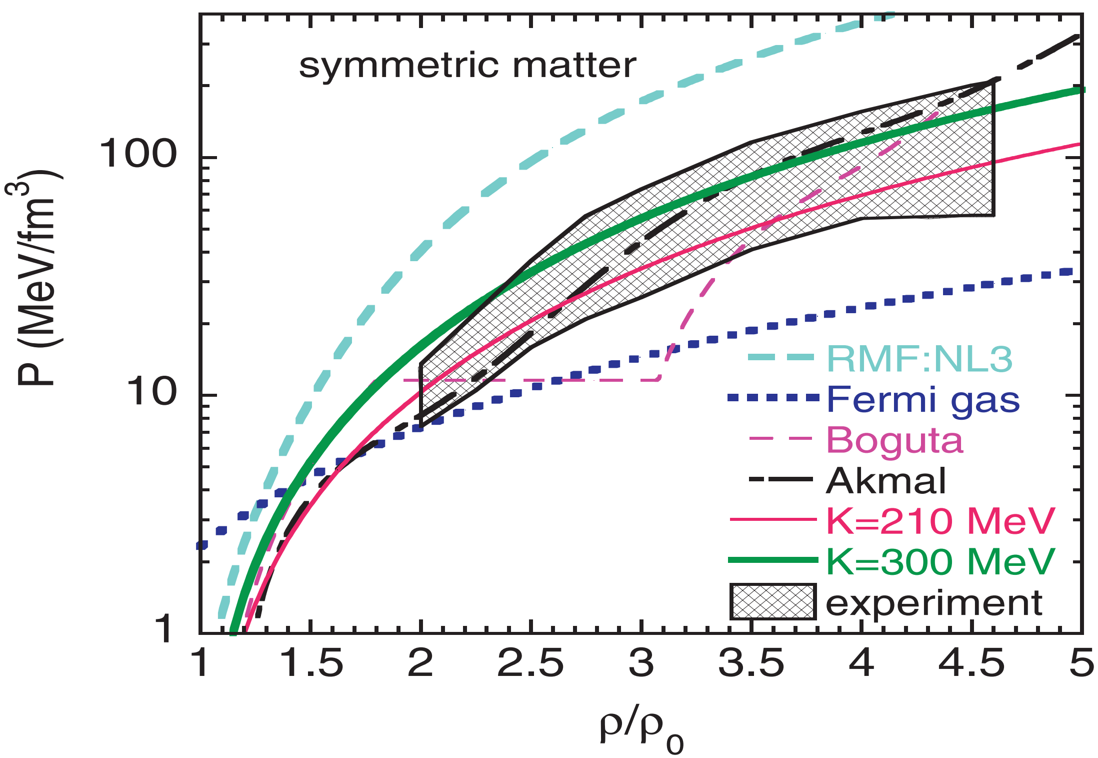
Transverse flow: Sideways deflection of spectator nucleons within the reaction plane, due to the pressure of the compressed region



Spectator nucleons - peripheral nucleons not participating in the collision

Theoretical predictions for the EoS for symmetrical matter inferred from dynamical calculations by Danielewicz et al, 2002.

The mean field potential was fitted to transverse and elliptical flow.



Limits from experiment/simulation shown by solid black lines

ρ = particle number density, $\rho_0 = 0.16\ fm^{-3}$ saturation density of symmetric nuclear matter

Matter in HIC and compact objects have different EoS:

Central A-A collision:

Strongly beam energy dependent
Beam energy < 1GeV/ A:

Temperature: < 50 MeV
Energy density: ~ 1 -2 GeV/fm^3
Baryon density < ρ_0
Time scale to cool-down: 10^{-22-24} s
No neutrinos

Strong Interaction: (S, B and L conserved)
Time scale 10^{-24} s

NEARLY SYMMETRIC MATTER

Inelastic NN scatterings,
N,N*, Δ's
LOTS of PIONS
strangeness
less important (kaons)

? (Local)EQUILIBRIUM?

Proto-neutron star:

(progenitor mass dependent)
~ 8 – 20 solar mass

Temperature: < 50 MeV
Energy density: ~ 1 GeV/fm3
Baryon density ~ 2-3 ρ_s
Time scale to cool-down: 1 -10 s
Neutrino rich matter

Strong +Weak Interaction: (B and L con)
Time scale 10^{-10} s (ρ and T dependent)

HIGHLY ASYMMETRIC MATTER

Higher T: strangeness produced in
in weak processes
Lower T: freeze-out
N, strange baryons and mesons,
NO PIONS, leptons

EQUILIBRIUM