Recherches inspirées par la thèse de Vincent Brindejonc

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Deux approches de gravite forte en chromodynamique quantique :
Théorie effective et Reformulation.
Vincent Brindejonc

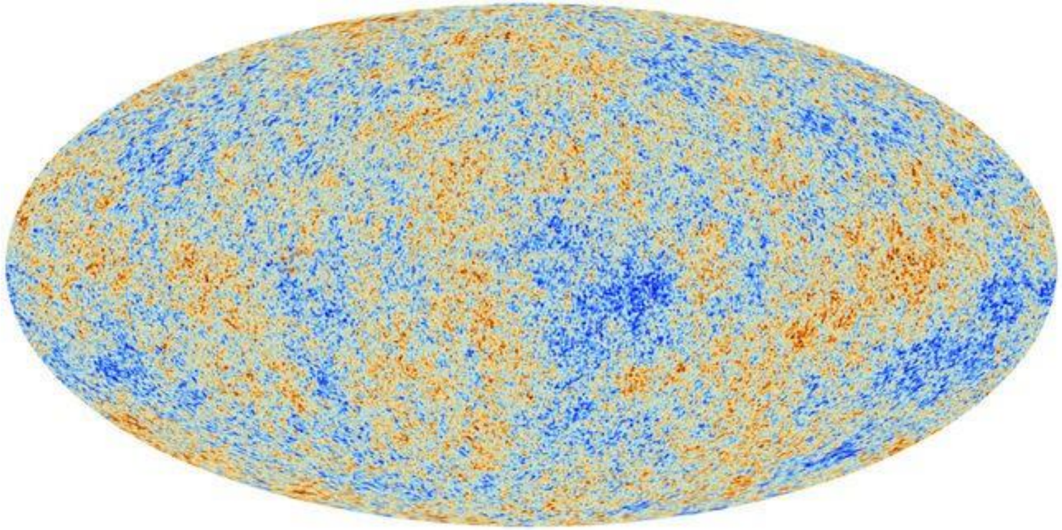
Résumé

Le but de cette thèse est d'étudier les effets d'une gravitation propre à l'interaction forte sur le confinement. L'idée d'une telle gravitation est apparue dans les années 70 sous le nom de gravité forte et est réapparue récemment dans le cadre de la chromodynamique quantique.

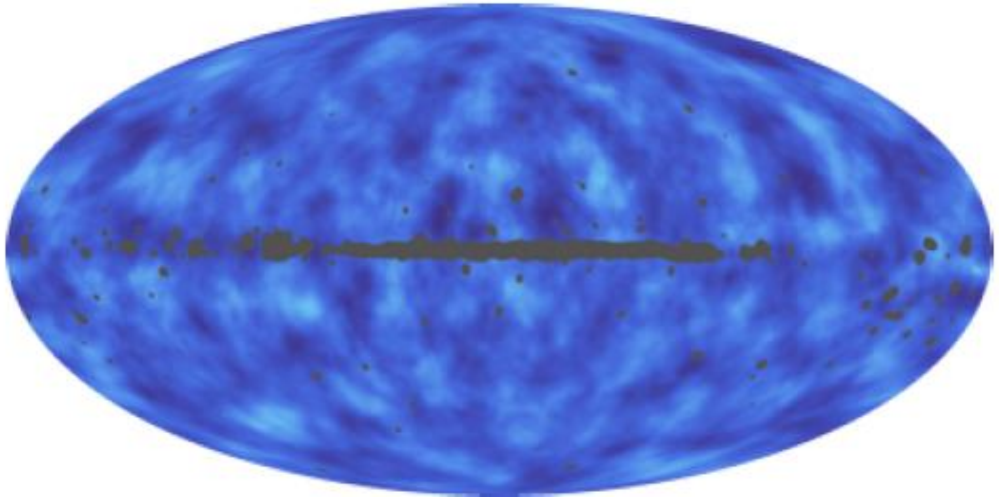
Pour mener à bien cette étude, deux approches ont été développées. D'une part, nous avons bâtit un modèle dans lequel la chromodynamique est quantifiée sur un espace-temps courbe. Dans ce modèle, les seuls degrés de liberté à haute énergie sont les quarks et les gluons mais une métrique forte, dynamique, apparait quand une théorie effective de basse énergie en est extraite. L'action effective est similaire à celle d'Einstein-Hilbert avec constante cosmologique. Son exploitation classique montre qu'étant donné son signe, cette constante confine les sources de couleur dans une sorte de sac cosmologique.

La seconde approche suivie, plus formelle, consiste en la reformulation d'une théorie de Yang Mills comme une gravitation. Une telle reformulation est possible à trois dimensions. Nous en avons cerné les limites dues en particulier à des singularités, nous l'avons étendue à un espace de fond courbe, à un terme de Chern-Simons et à la présence de sources de couleur. Cette re-formulation est pour l'instant classique et le confinement n'y est pas encore explicite, néanmoins, la pseudo métrique pourrait fournir un état asymptotique non coloré. Nous avons développé une application inattendue de la reformulation aux équations d'Einstein-Yang-Mills à trois dimensions. Cette méthode permet de générer des solutions de ces équations dès qu'une solution des équations d'Einstein dans le vide est connue.

The CMB and the results of the Planck experiment shown in two full sky maps

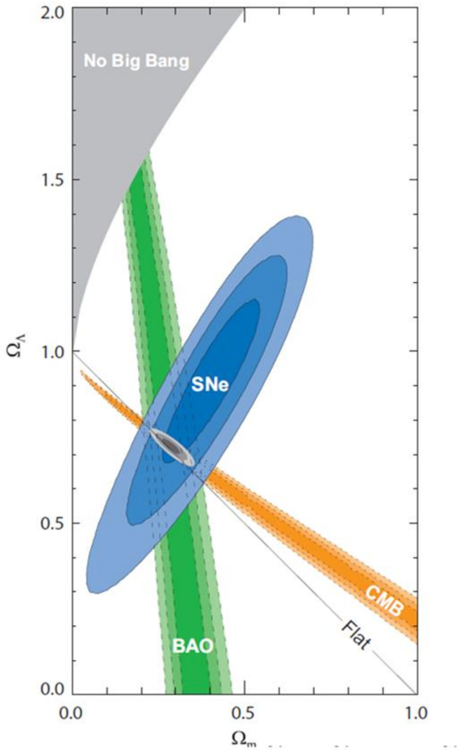


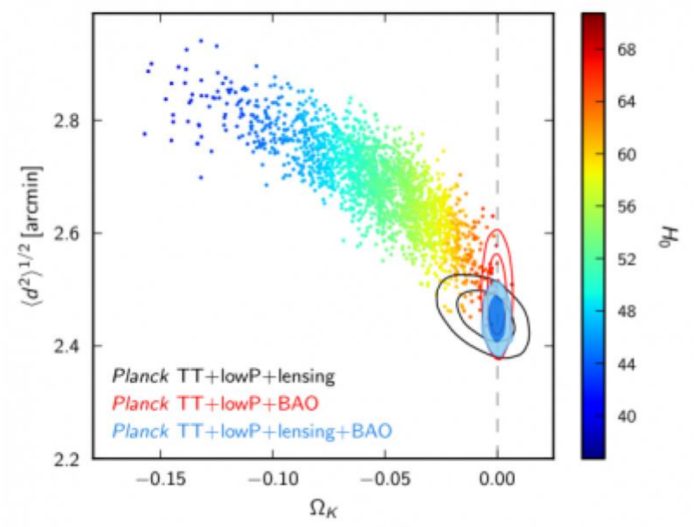
The full sky map of the CMB

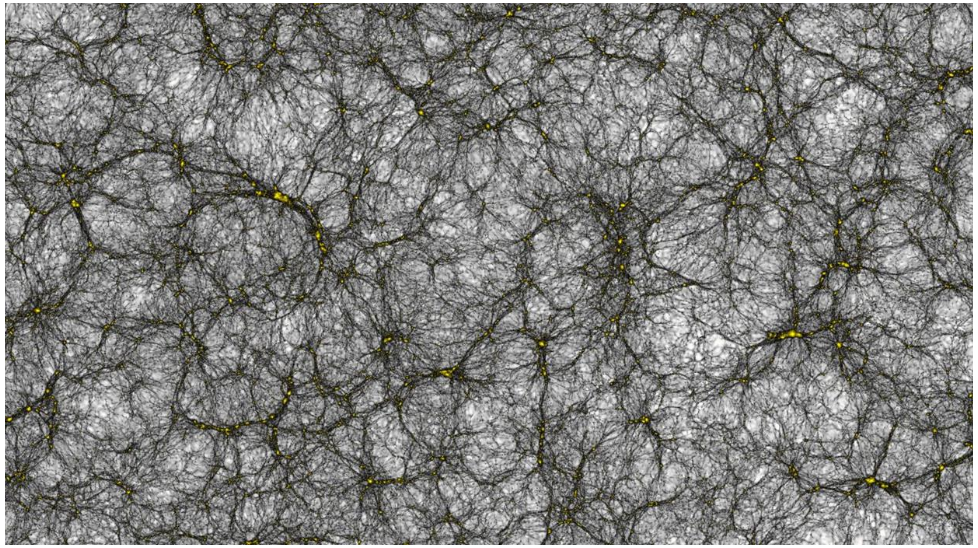


The full sky map of the lensing potential

ΛCDM is a gedanken experiment that consists in bringing back into the tableau the visible matter that has been removed as a foreground possibly interfering with the CMB. This is what the reconciliation of all observations (CMB, BAO, SNe) and simulations can do.

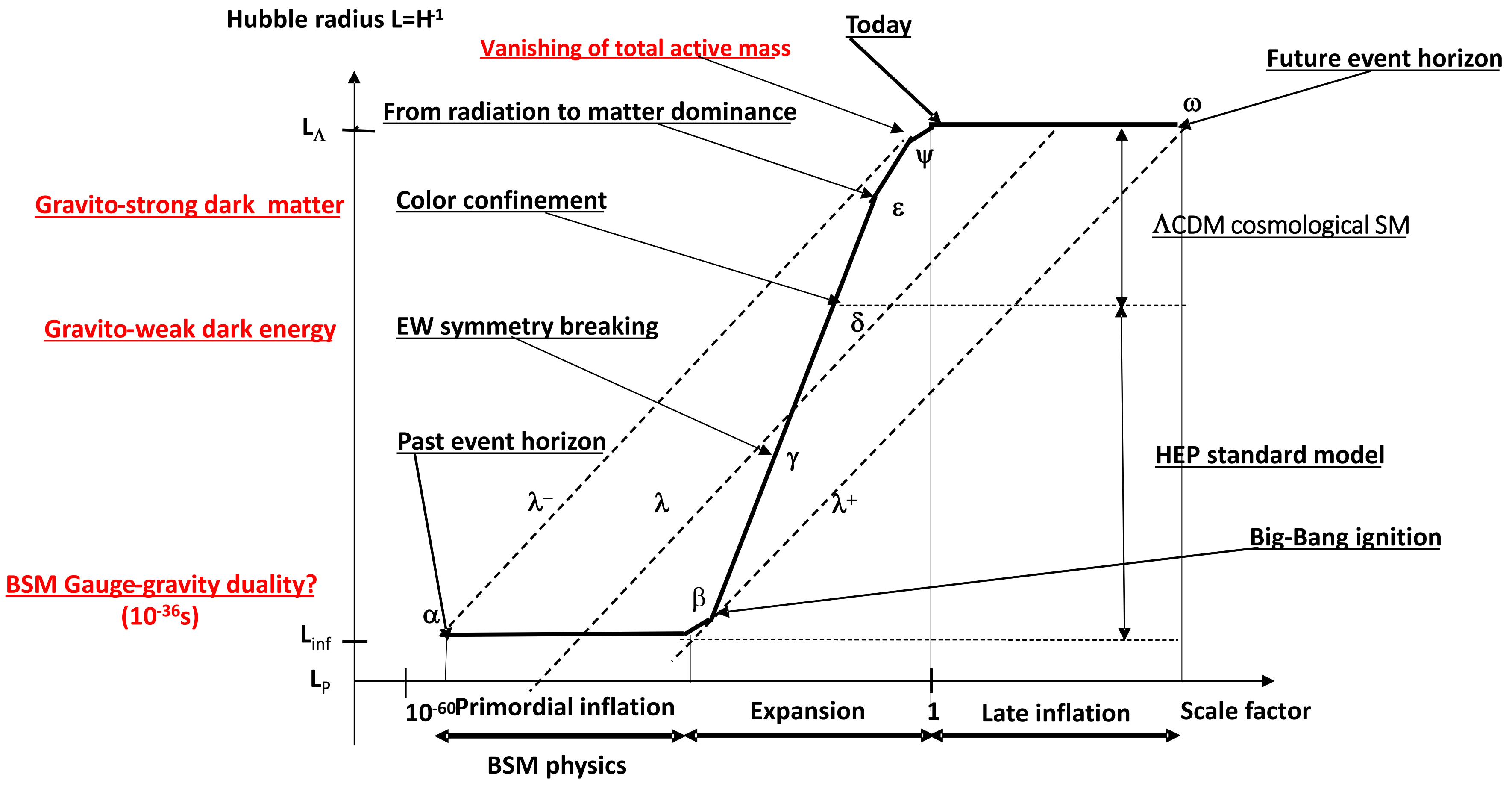




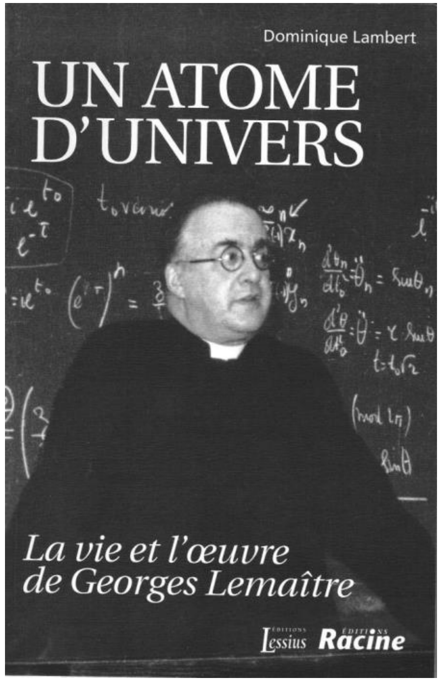


The methodology of effective theories and emergent gravity

All densities and pressures, including the cosmological constant, are replaced by ***effective, comoving quantities, i.e. thermal time dependent***

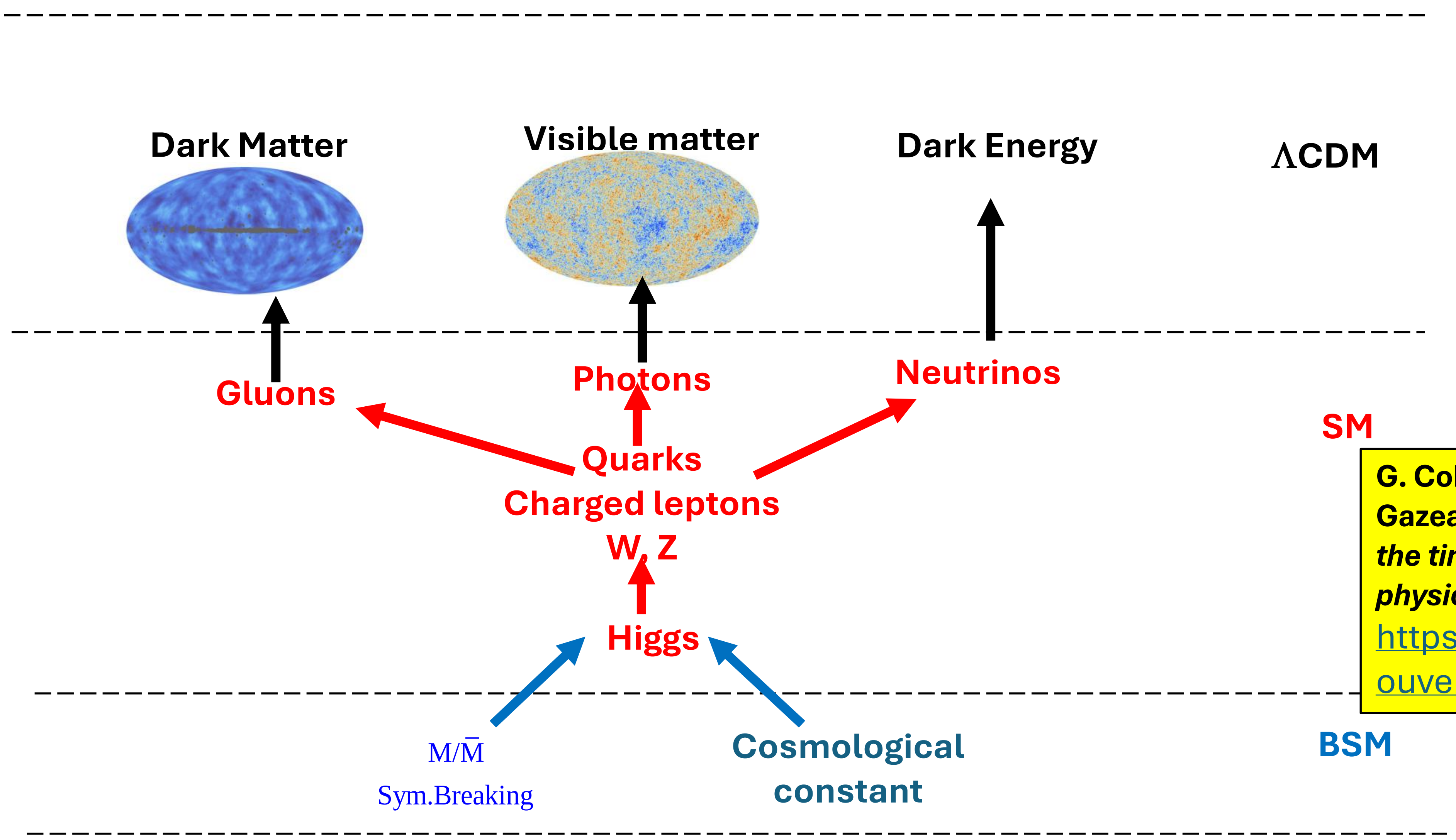


What is scientific cosmogony?

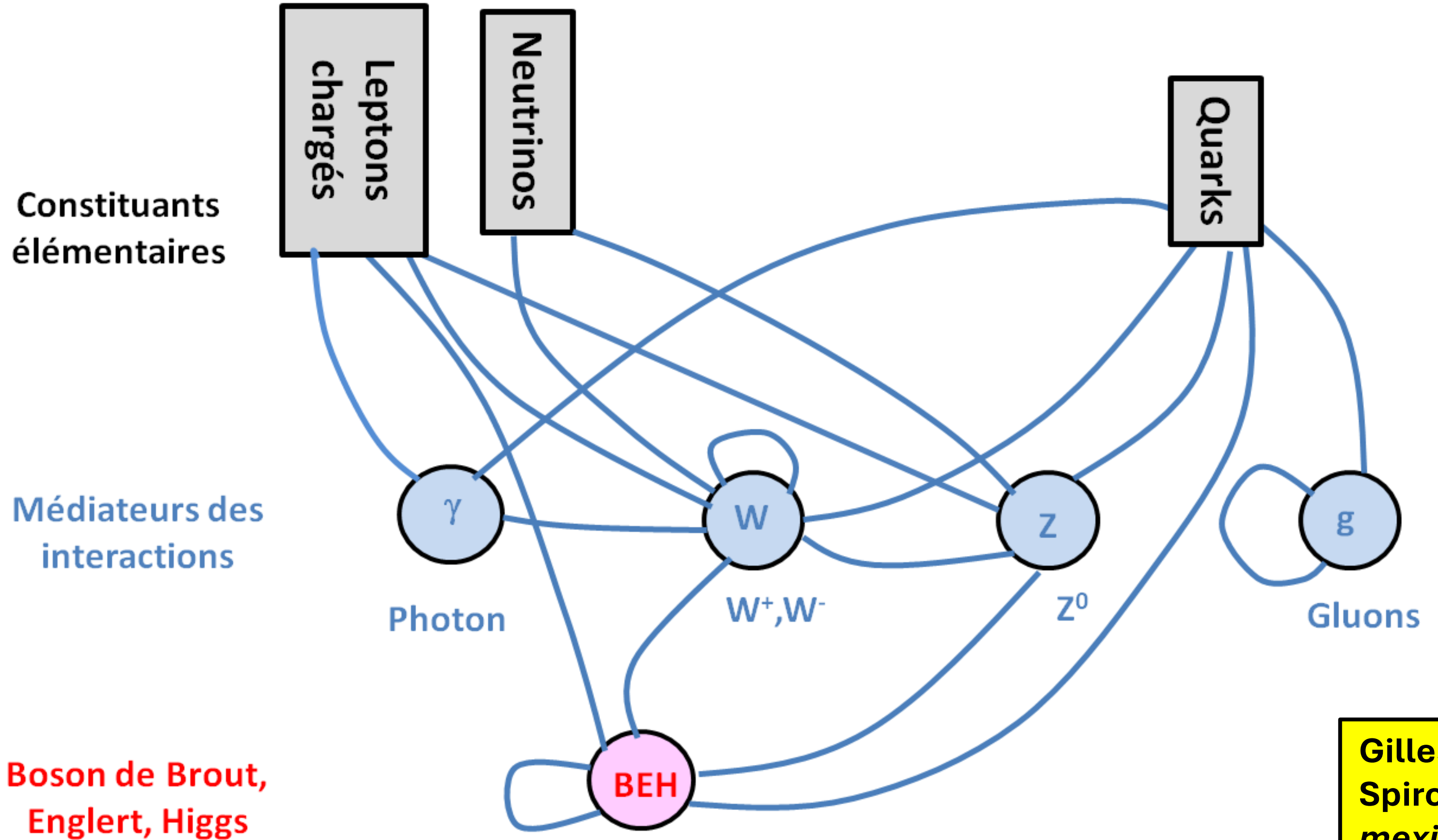


« L'objet d'une théorie cosmogonique est de rechercher des conditions initiales idéalement simples d'où a pu résulter, par le jeu des forces physiques connues, le monde actuel dans toute sa complexité » ***Georges Lemaître, l'hypothèse de l'atome primitif – Essai de cosmogonie -***

"The object of a cosmogonic theory is to search for ideally simple initial conditions from which, through the play of known physical forces, the current world in its full complexity could have resulted"
Georges Lemaître, the **primitive atom hypothesis – Essay of Cosmogony -**



G. Cohen-Tannoudji and J.P. Gazeau, *Scientific cosmogony, the time in quantum relativistic physics*
https://hal.archives-ouvertes.fr/hal-03538740



Gilles Cohen-Tannoudji and M. Spiro, *Le boson et le chapeau mexicain,* Gallimard , 2013

Le boson de Higgs, avatar moderne de l'atome primitif de Lemaître?

Einstein's equation

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu}$$

World Matter energy density

$$T_{\mu\nu} = -P g_{\mu\nu} + (P+\rho) u_\mu u_\nu$$

$$T^{\Lambda}_{\mu\nu} = -\Lambda P g_{\mu\nu}$$

$\Lambda>0$: de Sitter WM

$\Lambda<0$: anti-de Sitter WM

The cosmological constant in the r.h.s, constant of integration

Friedman-Lemaître equation

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3}(\rho + 3P)$$

$$\dot{\rho} = -3H(\rho + P)$$

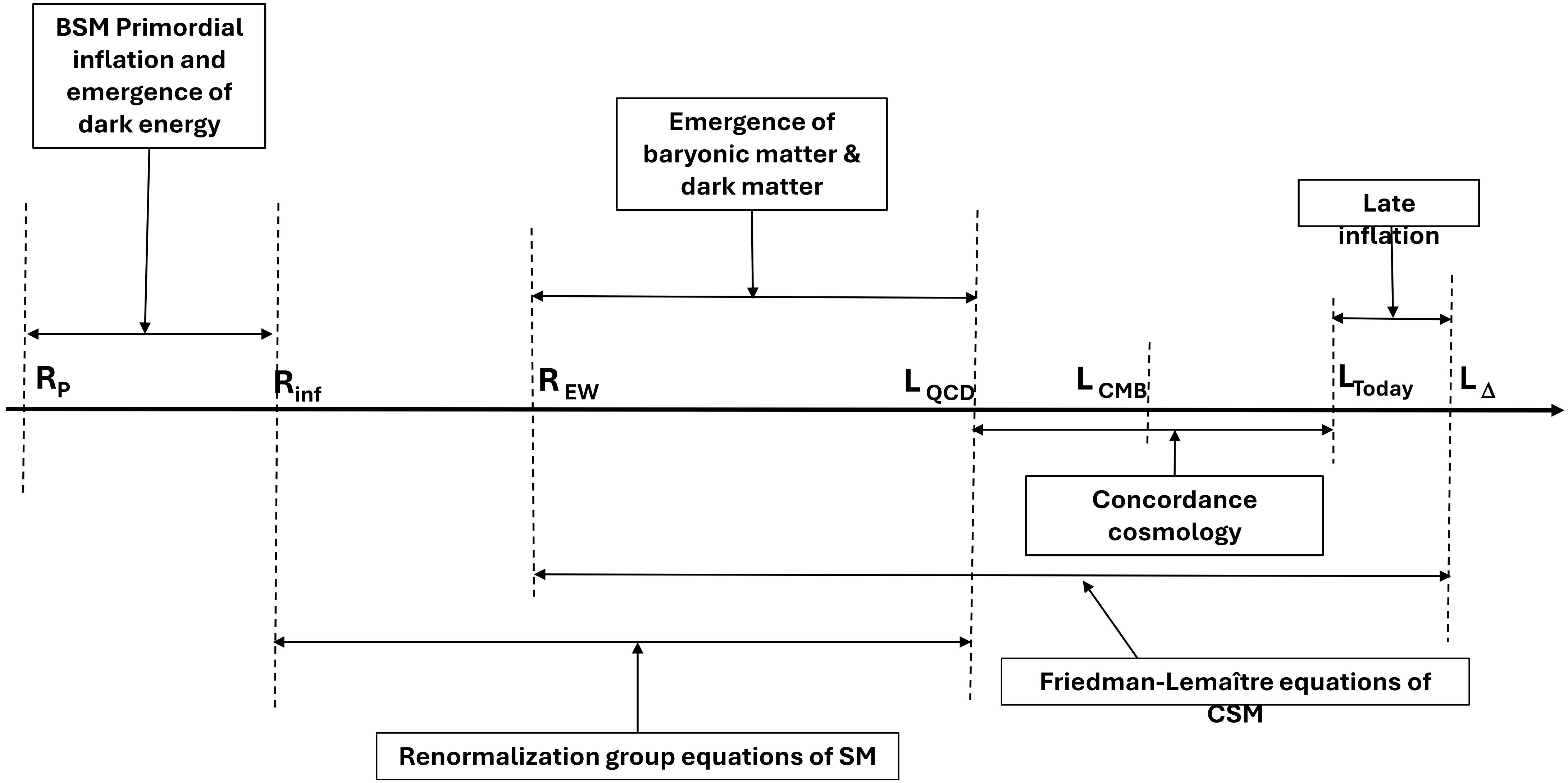
Flatness sum rule

$$\rho_c \equiv \frac{3H^2}{8\pi G_N}$$

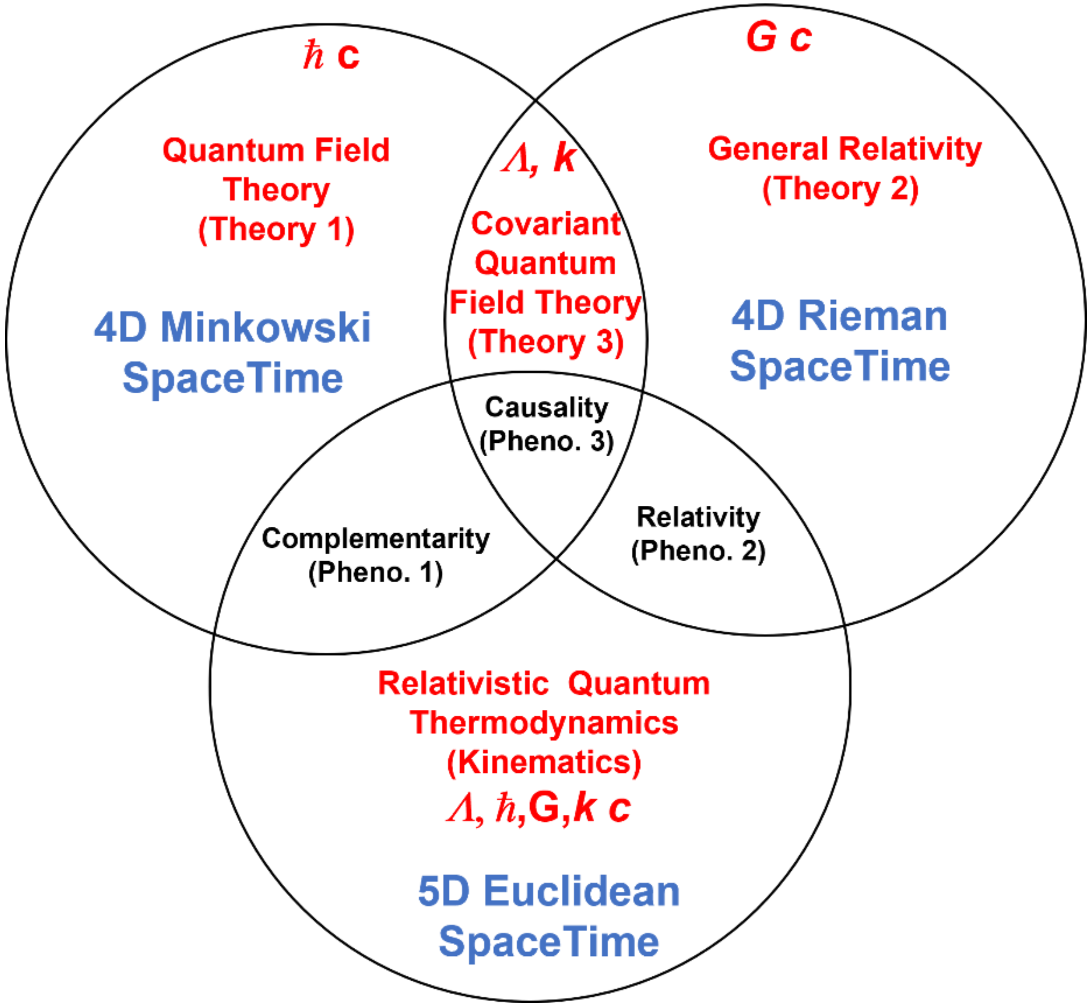
$$\rho_{\mathrm{b}} + \rho_{\mathrm{R}} + \rho_{\mathrm{DM}} + \rho_{\mathrm{DE}} - \rho_{\mathrm{c}} = 0$$

with $\rho_{\mathrm{DE}} = \Lambda / 8\pi G_N$

Overlap of renormalization group equations of SM and Friedman-Lemaître equations of CSM



The landscape of scientific cosmogony implied by its five universal constants



G. Cohen-Tannoudji and J.P. Gazeau, ***Scientific cosmogony, the time in quantum relativistic physics***
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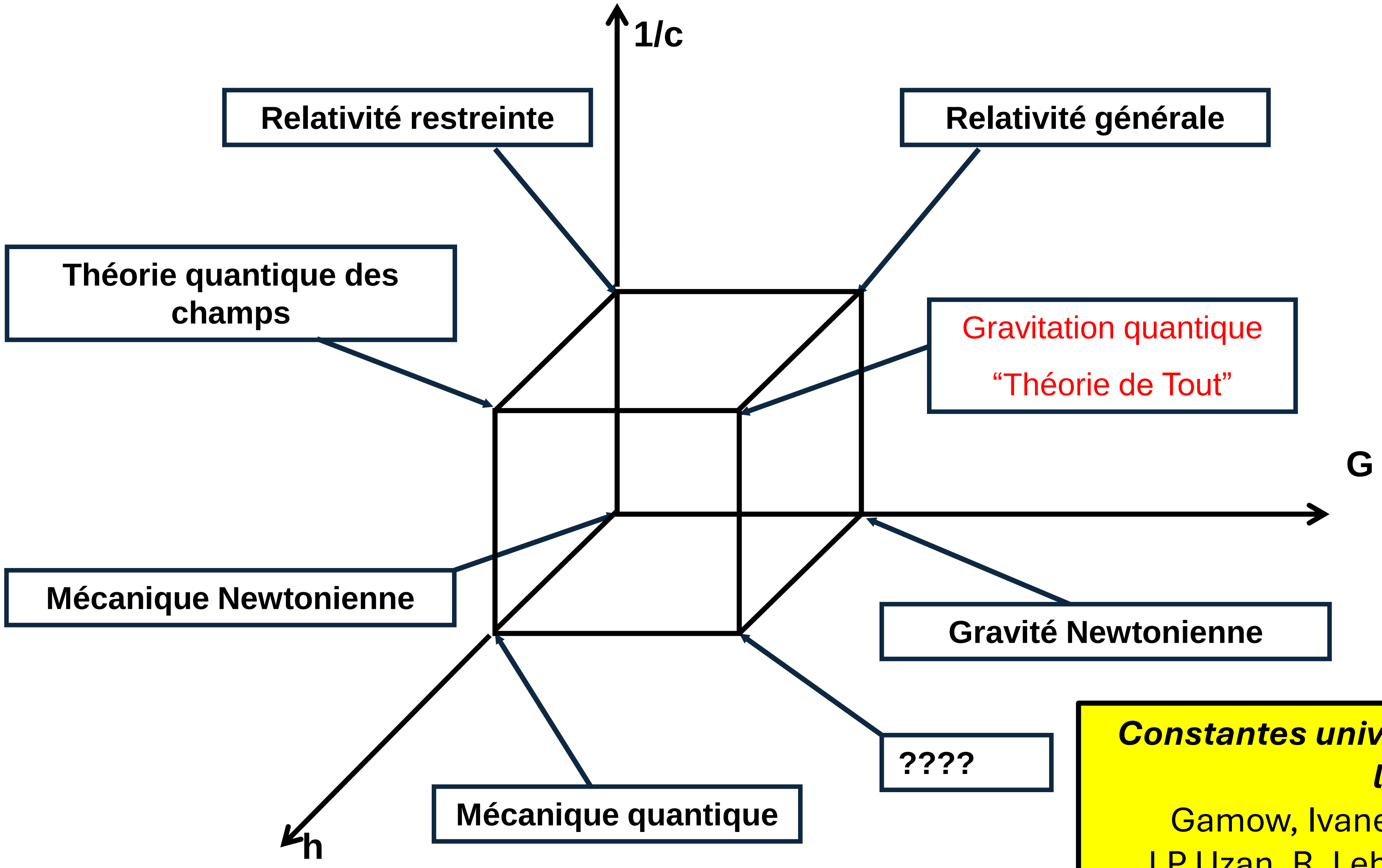
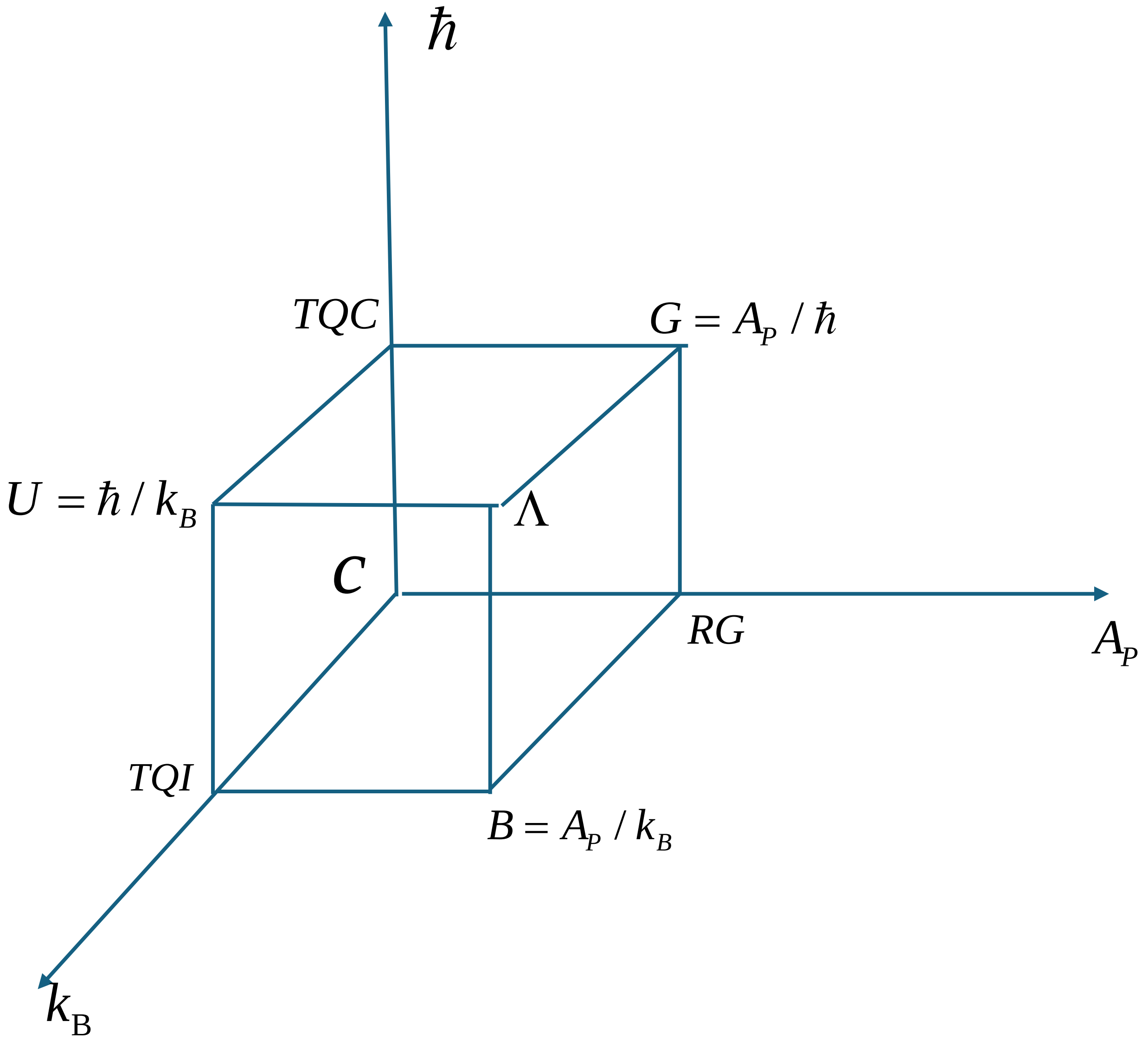
L’ancien cube des théories



Constantes universelles et transitions limites

Gamow, Ivanenko, Landau (1928)

J.P Uzan, R. Lehoucq *Les constantes fondamentales* p. 133 Belin 2005



Le nouveau cube des théories

Expansion of the universe and the need of an extra dimension of time

The extra-mundane time of de Sitter

The three-dimensional world must, in order to be able to perform "motions", i.e. in order that its position can be a variable function of the time, be thought movable in an "absolute" space of three or more dimensions (*not* the time-space x, y, z, ct). The four-dimensional world requires for its "motion" a four- (or more-) dimensional absolute space, and moreover an *extra-mundane "time"* which serves as independent variable for this motion.

W. de Sitter, *On the relativity of inertia. Remarks concerning Einstein's latest hypothesis, in:* KNAW, Proceedings, 19 II, 1917, Amsterdam, 1917, pp. 1217-1225

The thermal time of Alain Connes and Carlo Rovelli

In this case we have two independent and compatible definitions of time flow in this system: the thermal time flow αt and the flow βτ determined by the proper time.

A. Connes and C.Rovelli Von Neumann algebra automorphisms and time-thermodynamics relation in general covariant quantum theories http://arxiv.org/abs/gr-qc/9406019v1

De Sitter/Anti-de Sitter Space-time a possible kinematics

Gilles Cohen-Tannoudji and Jean-Pierre Gazeau, *Dark matter as a QCD effect in an anti de Sitter geometry: Cosmogonic implications of de Sitter, anti de Sitter and Poincaré symmetries*
SciPost Phys. Proc. 14, 004 (2023)

Bacry, H. and Lévy-Leblond, J.-M. ***Possible Kinematics*****, J. Math. Phys. 1968, 9, 1605.**

Gazeau, J.-P. Mass in ***de Sitter and Anti-de Sitter Universes with Regard to Dark Matter,*** **Universe 2020, 6 (5), 66; (https://www.mdpi.com/2218-1997/6/5/66)**

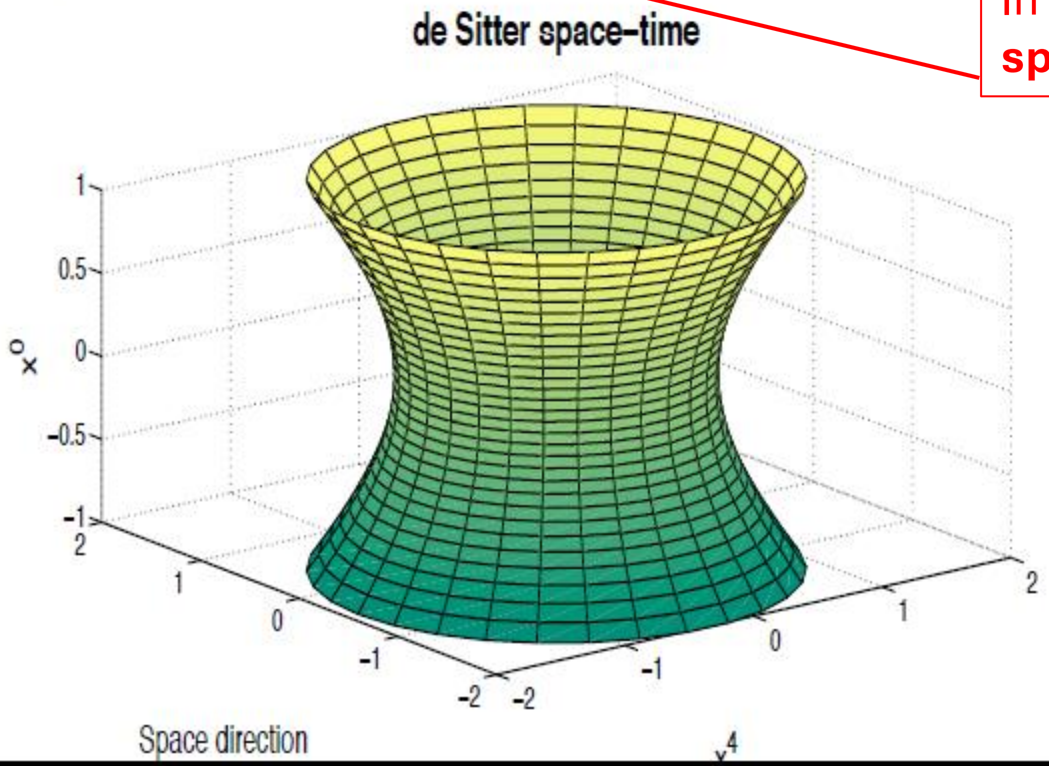
de Sitter geometry

- de Sitter space can be viewed as a one-sheeted hyperboloid embedded in a five-dimensional Minkowski space (but keep in mind that all points are physically equivalent)

$$M_{dS} \equiv \{x \in \mathbb{R}^5;\; x^2 = \eta_{\alpha\beta}\, x^\alpha x^\beta = -\frac{\Lambda_{dS}}{3}\}, \qquad \alpha, \beta = 0, 1, 2, 3, 4,$$

where $\eta_{\alpha\beta}$ =diag$(1, -1, -1, -1, -1)$

In de Sitter, the fifth dimension is **space like**



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Jean Pierre Gazeau

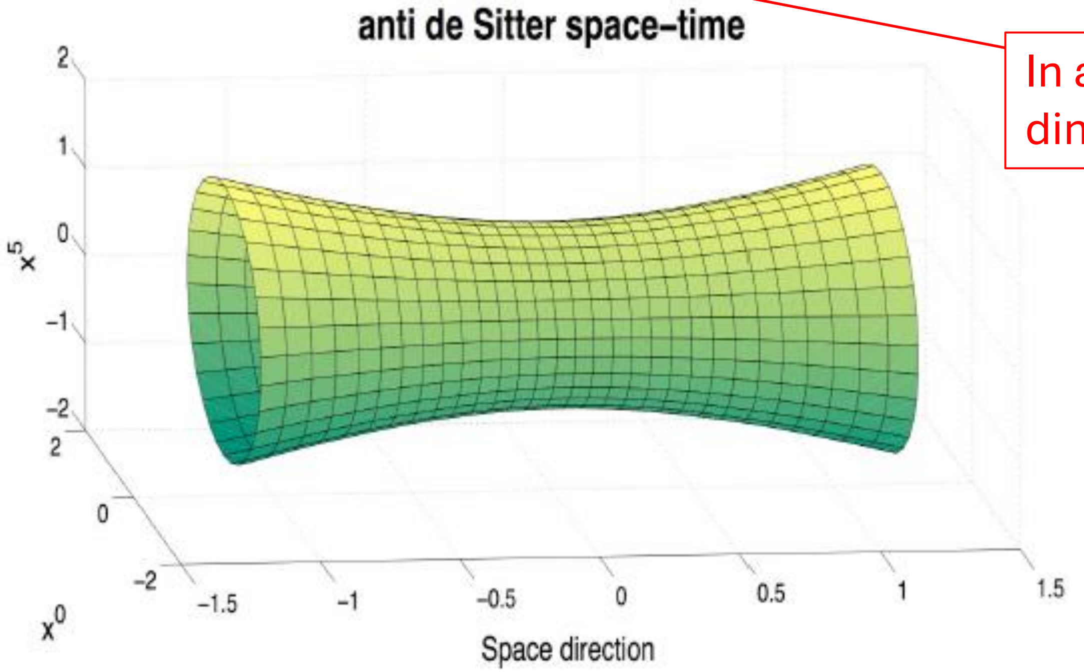
Anti de Sitter geometry

- Anti de Sitter space can be viewed as a one-sheeted hyperboloid embedded in another five-dimensional space with different metric (here too all points are physically equivalent) :

$$M_{AdS} \equiv \{x \in \mathbb{R}^5;\ x^2 = \eta_{\alpha\beta}\, x^\alpha x^\beta = \frac{|\Lambda_{\rm AdS}|}{3}\}, \quad \alpha, \beta = 0, 1, 2, 3, 5,$$

where $\eta_{\alpha\beta}$ =diag$(1, -1, -1, -1, 1)$.

In anti de Sitter, the fifth dimension is **time like**



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Jean Pierre Gazeau

- Each Anti-deSitterian quantum elementary system (in the Wigner sense) has a rest energy

$$E^{\mathrm{rest}}_{\mathrm{AdS}} = \left[m^2 c^4 + \hbar^2 \omega^2_{\mathrm{AdS}} \left(s - \frac{1}{2} \right)^2 \right]^{1/2} + \frac{3}{2} \hbar \omega_{\mathrm{AdS}} \, , \qquad (1)$$

with frequency $\omega_{\mathrm{AdS}} := \sqrt{\frac{|\Lambda_{\mathrm{AdS}}|}{3}}\, c$.

- Hence, to the order of $\hbar$, an AdS elementary system in the Wigner sense is a deformation of both a relativistic free particle with rest energy mc^2 and a 3d isotropic quantum harmonic oscillator with ground state energy $\frac{3}{2}\hbar\omega_{\mathrm{AdS}}$.
- In contrast to AdS, for Poincaré and DS symmetries the energy spectrum is continuous $\geq mc^2$. For dS :

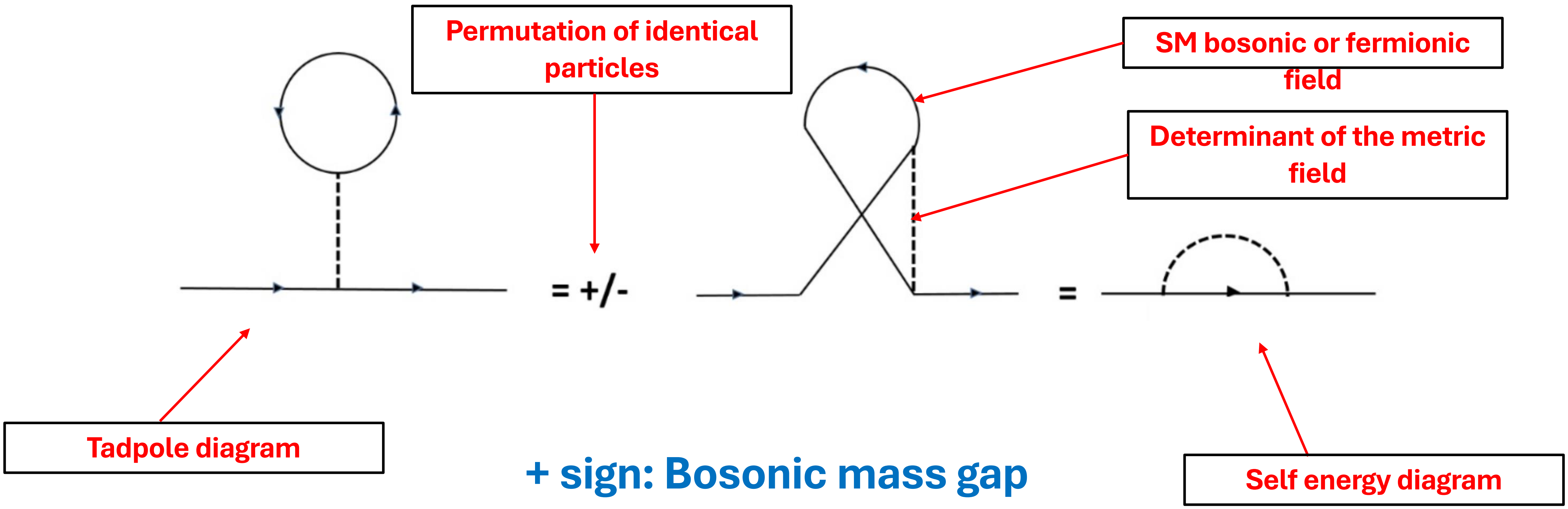
$$E^{\mathrm{rest}}_{\mathrm{dS}} = \pm \left[m^2 c^4 - \hbar^2 c^2 \frac{\Lambda_{\mathrm{dS}}}{3} \left(s - \frac{1}{2} \right)^2 \right]^{1/2} . \qquad (2)$$

- Note the noticeable simplification in both AdS and dS for spin $s = 1/2$:

$$\text{for dS :} \quad E^{\mathrm{rest}}_{\mathrm{dS}} = \pm mc^2 \, , \qquad (3)$$

$$\text{for AdS :} \quad E^{\mathrm{rest}}_{\mathrm{AdS}} = mc^2 + \frac{3}{2}\hbar\omega_{\mathrm{AdS}} \, . \qquad (4)$$

Dark matter-energy in AdS spacetime



+ sign: Bosonic mass gap

- sign: Fermionic energy gap

Notice the simplification in AdS spacetime for particles with spin=1/2

QCD Trace anomaly

$$\left\langle T^{\mu}_{\mu tes} \right\rangle = -\frac{1}{8}\left[11N_{\mathrm{c}} - 2N_{\mathrm{f}}\right]\left\langle \frac{\alpha_{\mathrm{S}}}{\pi}\left(F^{a}_{\mu\nu}F^{a\mu\nu}\right)^{r} \right\rangle_{0}$$

The prefactor is present in the contributions of all QCD condensates

The term in N_{c} represents the contribution of gluon loops to the dark matter

The term in N_{f} represents the contribition of flavored quark loops to the baryonic matter

Stephan Adler, *Einstein gravity as a symmetry-breaking effect in quantum field theory* *Reviews of Modern Physics, Vol. 54, No. 3, July 1982*

Holographic equipartition and spacetime thermodynamics

Holographic equipartition

$$N_{\mathrm{sur}} = \frac{4\pi}{L_{\mathrm{P}}{}^{2} H^{2}}; \text{ one degree of freedom (one bit) per Planck area}$$

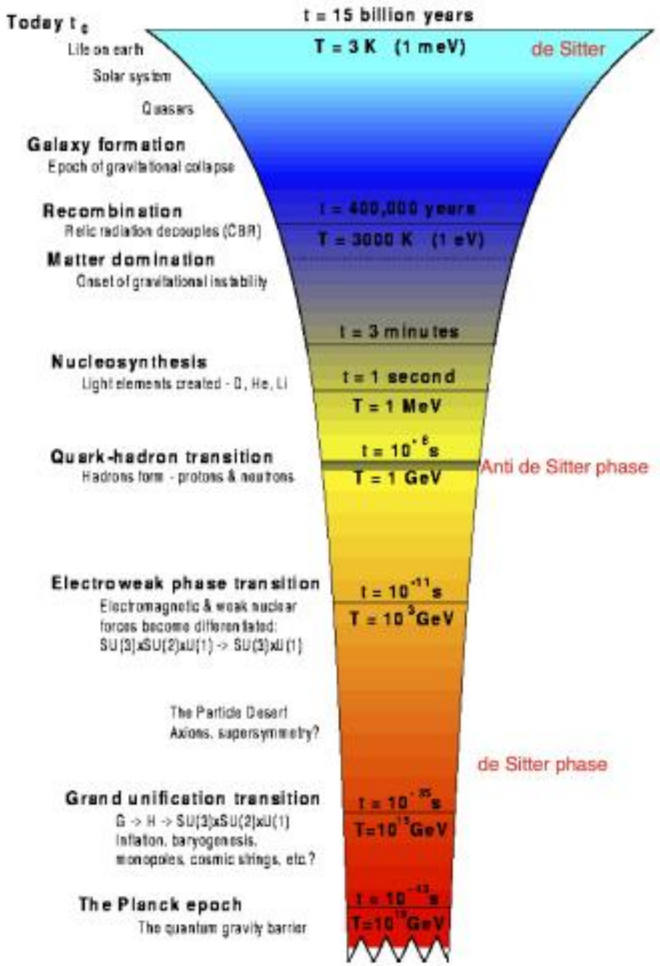
$$N_{\mathrm{bulk}} = \frac{|E|}{1/2 k_{\mathrm{B}} T}; \; k_{\mathrm{B}} T = \frac{H}{2\pi}; \; |E| = |\rho + 3P| V; \; V = \frac{4\pi}{3H^{3}}$$

$$\Rightarrow N_{\mathrm{bulk}} = -\frac{E}{1/2 k_{\mathrm{B}} T} = -\frac{2(\rho + 3P)V}{k_{\mathrm{B}} T} \text{ (for positive } \rho)$$

$$P = -\rho \text{ (vacuum)} \Rightarrow H^{2} = 8\pi L_{\mathrm{P}}{}^{2} \rho / 3 \Leftrightarrow N_{\mathrm{bulk}} = N_{\mathrm{sur}}$$

Chirco et Al , *Spacetime thermodynamics without hidden degrees of freedom* ArXiv:1401,5262v1 [gr-qc]

Cosmology chronology : de Sitter and Anti de Sitter phases



Gazeau, J.-P. Mass in de Sitter and Anti-de Sitter Universes with Regard to Dark Matter. Universe 2020, 6, 66. Available online: https://www.mdpi.com/2218-1997/6/5/66

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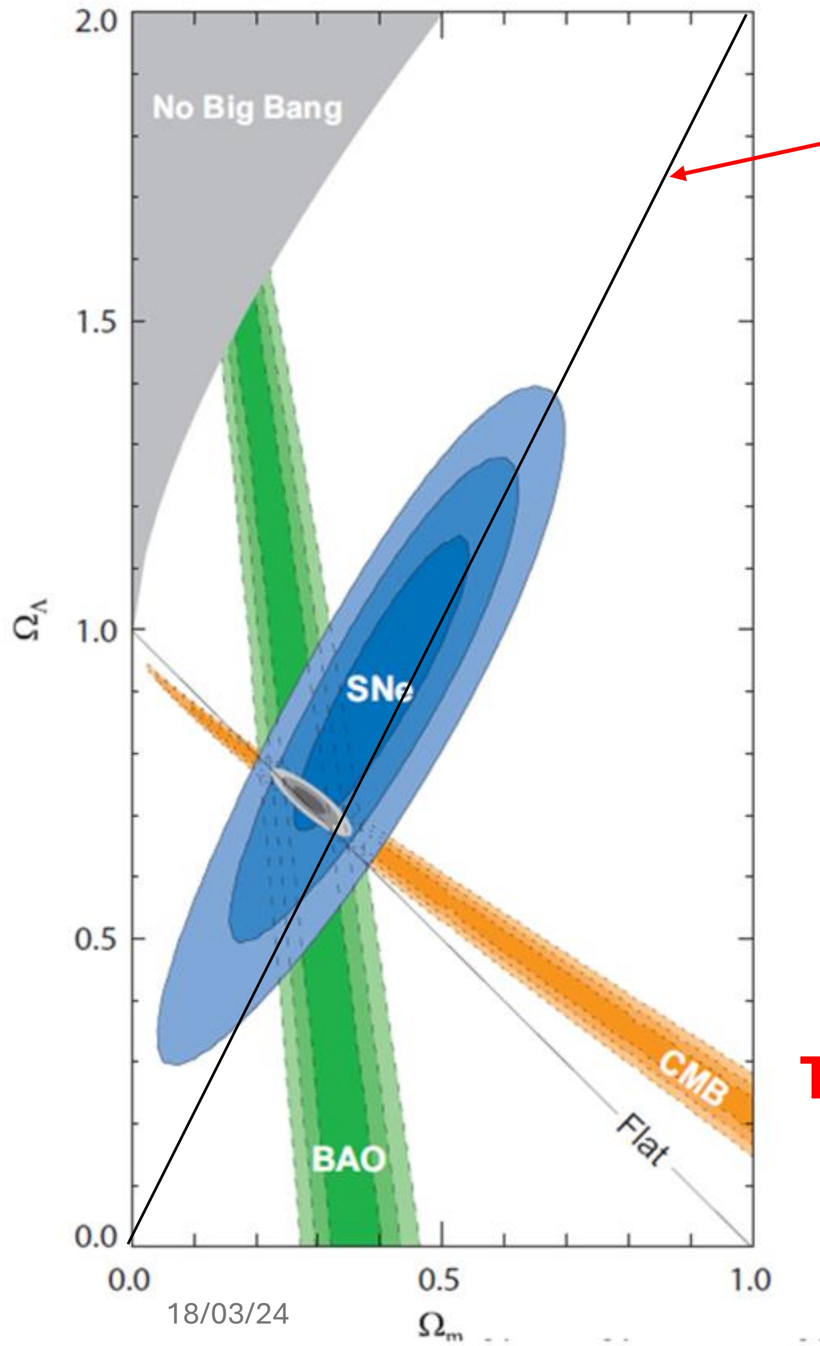

Ω_{DM}=1/2Ω_Λ vanishing of total active mass

The effective quantum vacuum (visible matter set to zero)

Flatness | Darkness | Emptyness

$$\rho_{DM} + \rho_{DE} = \rho_{Vac} = \rho_c \qquad \rho_{Vac} + P_{Vac} = 0 \qquad \rho_{Vac} + 3P_{Vac} = 0$$

$$\rho_{DE} = \rho_\Lambda \Rightarrow \rho_c = \rho_{Vac} = \frac{3}{2}\rho_\Lambda$$

$$\Omega_\Lambda^{Vac} = \frac{2}{3}; \Omega_M^{Vac} = \frac{1}{3}$$

$\rho_{Vis}^{as} = 0$: emergence of matter in the remote past and disappearance in the far future

The whole history of the visible universe from point α to point ω is the one of a gigantic fluctuation of the quantum vacuum

ΛCDM = Reconciliation of Einstein' and de Sitter' models !

$$\text{Flatness sum rule } \rho_{\text{vis}} + \rho_{\text{DM}} + \rho_{\text{DE}} = \rho_{\text{c}} = \rho_{\phi} = \frac{3}{2}\rho_{\Lambda}$$

Equation of state of dilaton field (determinant of the metric) ϕ

$$\mathrm{W}_{\phi} = \frac{P_{\phi}}{\rho_{\phi}} = -1/3$$

$$\rho_{\text{vis}} + \rho_{\text{DM}} \equiv \rho_{\text{M}} = -P_{\phi}; \rho_{\text{DE}} = -2P_{\phi}$$

$$\rho_{M} = \frac{1}{2}\rho_{\Lambda} = -\rho_{\bar{\text{M}}} = -P_{M}$$

$$\Rightarrow \rho_{\text{M}} + P_{M} = 0$$ (Einstein model, static universe)

$$\Rightarrow \rho_{\text{M}} + \rho_{\bar{\text{M}}} = 0$$ (de Sitter model, pure cosmological constant)

Confronting our model with data and Planck's results

When adding ρ_{Vis} to $\rho_{\Lambda}^{\text{Vac}}$ to obtain ρ_{Λ} note that,

only half of $\rho_{_{\text{Vis}}} = \rho_{\bar{F}}$ (anti-fermion) has to be taken in account

since the fermion contribution remains in ρ_{Vis}

So, equating $\rho_{\text{M}}^{\text{Vac}} = \frac{1}{2}\rho_{\Lambda}$ with $-\rho_{\bar{M}}$ we have

$$\rho_{\Lambda} = \rho_{\Lambda}^{\text{Vac}} + \frac{1}{2}\rho_{Bar}\,;\ \rho_{M} = \rho_{\text{M}}^{\text{Vac}} - \frac{1}{2}\rho_{\text{Bar}} \text{ in terms of "omegas"}$$

$$\Omega_{\Lambda} = \frac{2}{3} + \frac{1}{2}\Omega_{Bar}$$

$$\Omega_{\text{M}} = \frac{1}{3} - \frac{1}{2}\Omega_{Bar}$$

Then, since $\Omega_{\text{DM}} = \Omega_{\text{M}} - \Omega_{Bar}$,

$$\Omega_{\text{DM}} = \frac{1}{3} - \frac{3}{2}\Omega_{\text{Bar}}$$

Planck 2018 cosmological parameters

Table 2. Parameter 68 % intervals for the base-ΛCDM model from *Planck* CMB power spectra, in combination with CMB lensing reconstruction and BAO. The top group of six rows are the base parameters, which are sampled in the MCMC analysis with flat priors. The middle group lists derived parameters. The bottom three rows show the temperature foreground amplitudes $f^{TT}_{\ell=2000}$ for the corresponding frequency spectra (expressed as the contribution to $D^{TT}_{\ell=2000}$ in units of $(\mu\mathrm{K})^2$). In all cases the helium mass fraction used is predicted by BBN (posterior mean $Y_P \approx 0.2454$, with theoretical uncertainties in the BBN predictions dominating over the *Planck* error on $\Omega_b h^2$). The reionization redshift mid-point z_{re} and optical depth τ here assumes a simple tanh model (as discussed in the text) for the reionization of hydrogen and simultaneous first reionization of helium. Our baseline results are based on *Planck* TT,TE,EE+lowE+lensing (as also given in Table 1).

Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0 [km s^{-1} Mpc^{-1}]	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.679 ± 0.013	0.699 ± 0.012	$0.711^{+0.033}_{-0.026}$	0.6834 ± 0.0084	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.321 ± 0.013	0.301 ± 0.012	$0.289^{+0.026}_{-0.033}$	0.3166 ± 0.0084	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1434 ± 0.0020	0.1408 ± 0.0019	$0.1404^{+0.0034}_{-0.0039}$	0.1432 ± 0.0013	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09589 ± 0.00046	0.09635 ± 0.00051	$0.0981^{+0.0016}_{-0.0018}$	0.09633 ± 0.00029	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8118 ± 0.0089	0.793 ± 0.011	0.796 ± 0.018	0.8120 ± 0.0073	0.8111 ± 0.0060	0.8102 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.840 ± 0.024	0.794 ± 0.024	$0.781^{+0.052}_{-0.060}$	0.834 ± 0.016	0.832 ± 0.013	0.825 ± 0.011
$\sigma_8 \Omega_m^{0.25}$	0.611 ± 0.012	0.587 ± 0.012	0.583 ± 0.027	0.6090 ± 0.0081	0.6078 ± 0.0064	0.6051 ± 0.0058
z_{re}	7.50 ± 0.82	$7.11^{+0.91}_{-0.75}$	$7.10^{+0.87}_{-0.73}$	7.68 ± 0.79	7.67 ± 0.73	7.82 ± 0.71
$10^9 A_s$	2.092 ± 0.034	2.045 ± 0.041	2.116 ± 0.047	$2.101^{+0.031}_{-0.034}$	2.100 ± 0.030	2.105 ± 0.030
$10^9 A_s e^{-2\tau}$	1.884 ± 0.014	1.851 ± 0.018	1.904 ± 0.024	1.884 ± 0.012	1.883 ± 0.011	1.881 ± 0.010
Age [Gyr]	13.830 ± 0.037	13.761 ± 0.038	$13.64^{+0.16}_{-0.14}$	13.800 ± 0.024	13.797 ± 0.023	13.787 ± 0.020
z_*	1090.30 ± 0.41	1089.57 ± 0.42	$1087.8^{+1.6}_{-1.7}$	1089.95 ± 0.27	1089.92 ± 0.25	1089.80 ± 0.21
r_* [Mpc]	144.46 ± 0.48	144.95 ± 0.48	144.29 ± 0.64	144.39 ± 0.30	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04097 ± 0.00046	1.04156 ± 0.00049	1.04001 ± 0.00086	1.04109 ± 0.00030	1.04110 ± 0.00031	1.04119 ± 0.00029
z_{drag}	1059.39 ± 0.46	1060.03 ± 0.54	1063.2 ± 2.4	1059.93 ± 0.30	1059.94 ± 0.30	1060.01 ± 0.29
r_{drag} [Mpc]	147.21 ± 0.48	147.59 ± 0.49	146.46 ± 0.70	147.05 ± 0.30	147.09 ± 0.26	147.21 ± 0.23
k_D [Mpc^{-1}]	0.14054 ± 0.00052	0.14043 ± 0.00057	0.1426 ± 0.0012	0.14090 ± 0.00032	0.14087 ± 0.00030	0.14078 ± 0.00028
z_{eq}	3411 ± 48	3349 ± 46	3340^{+81}_{-92}	3407 ± 31	3402 ± 26	3387 ± 21
k_{eq} [Mpc^{-1}]	0.01041 ± 0.00014	0.01022 ± 0.00014	$0.01019^{+0.00025}_{-0.00028}$	0.010398 ± 0.000094	0.010384 ± 0.000081	0.010339 ± 0.000063
$100\theta_{s,eq}$	0.4483 ± 0.0046	0.4547 ± 0.0045	0.4562 ± 0.0092	0.4490 ± 0.0030	0.4494 ± 0.0026	0.4509 ± 0.0020
f^{143}_{2000}	31.2 ± 3.0			29.5 ± 2.7	29.6 ± 2.8	29.4 ± 2.7
$f^{143\times217}_{2000}$	33.6 ± 2.0			32.2 ± 1.9	32.3 ± 1.9	32.1 ± 1.9
f^{217}_{2000}	108.2 ± 1.9			107.0 ± 1.8	107.1 ± 1.8	106.9 ± 1.8

Agreement within the error bars

	Our model	Planck' results
$\Omega_\Lambda = \frac{2}{3} + \frac{1}{2}\Omega_{\text{Bar}}$	0,666+0,049/2=0,690	0,6889
$\Omega_{\text{M}} = \frac{1}{3} - \frac{1}{2}\Omega_{Bar}$	0,333-0,049/2=0,309	0,311
$\Omega_{\text{DM}} = \frac{1}{3} - \frac{3}{2}\Omega_{Bar}$	0,333-1,5X0,049/2=0,260	0,261

Prediction for the ratio Dark to visible matter

With N_f=3, the ratio is predicted to 5,5 to be compared to 0,261/0,049=5,326. It is reasonable to consider N_f as a parameter allowing to fit the ratio

Cold dark matter : Bose-Einstein condensation of gluons in Anti-de Sitter space time

Gilles Cohen-Tannoudji and J-P Gazeau, Universe 2021, 7, 402.
https ://doi.org/10.3390/universe7110402

- A parallel between dark matter and CMB :
  - CMB $\rightarrow$ photon decoupling, i.e. photons started to travel freely through space rather than constantly being scattered by electrons and protons in plasma (QED effect).
  - Dark matter $\rightarrow$ gluonic component of the quark epoch (quark-gluon plasma) which freely subsists after hadronization within an effective AdS environment (QCD effect)
- As an assembly of N_G non-interacting entities with individual energies $E_n = (n+2)\hbar\omega_{\mathrm{AdS}}$ and degeneracy $g_n = (n+1)(n+3)$, those remnant gluons are assumed to form a grand canonical Bose-Einstein ensemble whose the chemical potential μ is, at temperature T, fixed by

$$N_G = \sum_{n=0}^{\infty} \frac{g_n}{\exp\left[\frac{\hbar\omega_{\mathrm{AdS}}}{k_B T}(n+2-\mu)\right]-1}\,. \tag{8}$$

- Since this number is very large this gas condensates at temperature

$$T_c \approx \frac{\hbar\omega_{\mathrm{AdS}}}{k_B}\left(\frac{N_G}{\zeta(3)}\right)^{1/3} \tag{9}$$

to become the currently observed dark matter.

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Jean Pierre Gazeau, Sonia Gazeau

Gazeau, J-P., Habonimana, C., Signal analysis and quantum formalism: Quantizations with no Planck constant, in: Landscapes of Time-Frequency Analysis, Vol. 2, Applied in Numerical and Harmonic Analysis series, New York: Springer International Publishing, 2020. : arXiv:2001.04916 [quant-ph]

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Conclusion

"We have tentatively explained dark matter by actually asking a simple question (!): what becomes the huge amount of gluons after the transition from QGP period to hadronization? Similarly to the emergence of the two validated CMB (QED effect) and CNB (electroweak effect), we propose to consider Dark Matter, observed through its gravitational effects, as a pure QCD effect. From our viewpoint it would be legitimate to replace the puzzling expression "Dark Matter" with the realistic "Cosmic Gluonic Background".

Gilles Cohen-Tannoudji and Jean-Pierre Gazeau, *Dark matter as a QCD effect in an anti de Sitter geometry: Cosmogonic implications of de Sitter, anti de Sitter and Poincaré symmetries*
SciPost Phys. Proc. 14, 004 (2023)