Time Reversed Waves and Super-Resolution: From Acoustics to Electromagnetism

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Time-reversed acoustics in a non dissipative medium

$\boldsymbol{p}(\vec{\boldsymbol{r}},\boldsymbol{t})$ acoustic pressure field (scalar)

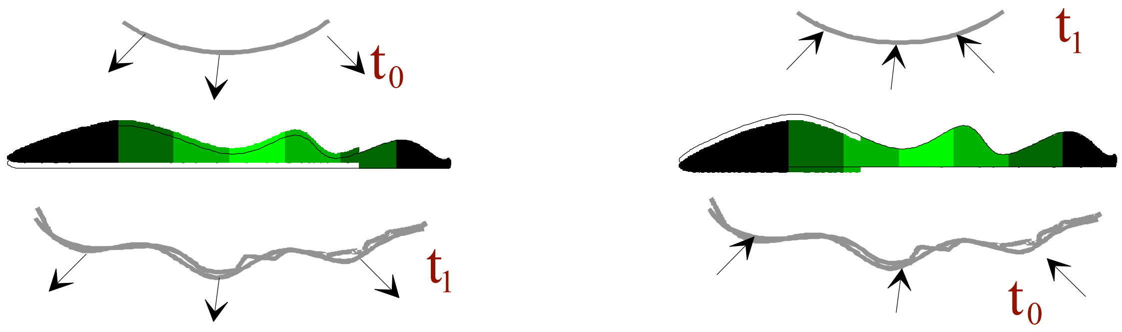
$\rho(\vec{\boldsymbol{r}})$ is the density and $\boldsymbol{c}(\vec{\boldsymbol{r}})$ is the sound velocity in an heterogeneous medium

The wave equation in a domain **without source**

$$div\left\{\frac{grad(p(\vec{r},t))}{\rho(\vec{r})}\right\}-\frac{1}{\rho(\vec{r})c^2(\vec{r})}\frac{\partial^2 p(\vec{r},t)}{\partial t^2}=0$$

Spatial Reciprocity   Time Reversal Invariance

Dual solutions



$p(\vec{r},t)$ $p(\vec{r},-t)$

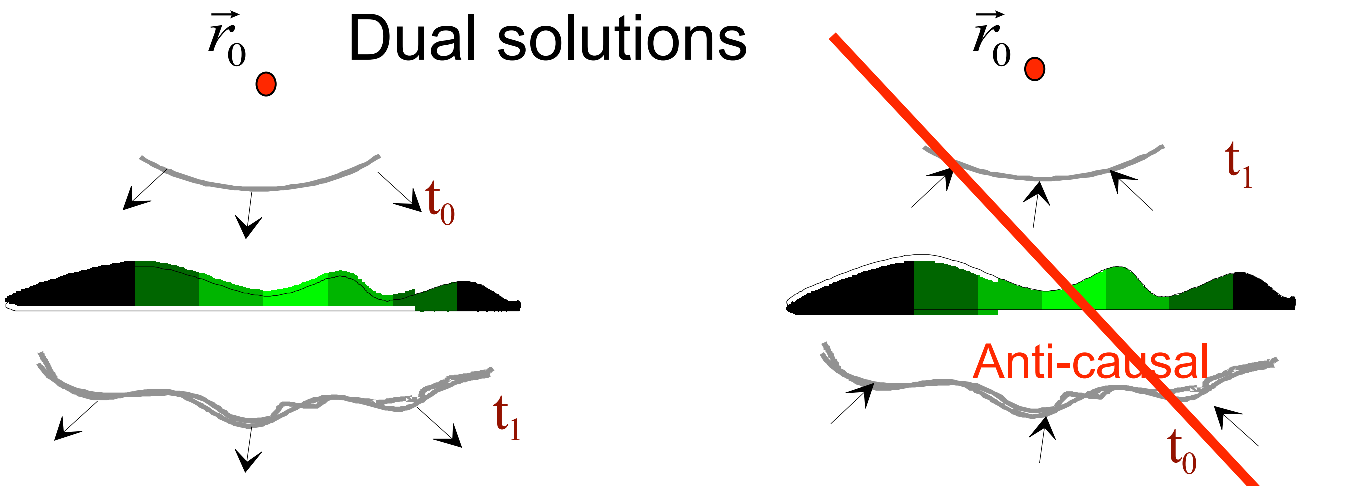
Time-Reversed Acoustics and Causality

The wave equation in a non dissipative domain **with a ponctual source**

$$\left\{ div\left[\frac{grad}{\rho(\vec r)}\right] - \frac{1}{\rho(\vec r)c^2(\vec r)}\frac{\partial^2}{\partial t^2}\right\} G(\vec r,\vec r_0;t) = -\delta\left(\vec r-\vec r_0\right)\delta\left(t\right)$$

Green's function

Dual solutions



$G_{ret}(\vec r,\vec r_0;t)$

$G_{ret}(\vec r,\vec r_0;-t) = G_{adv}(\vec r,\vec r_0;t)$

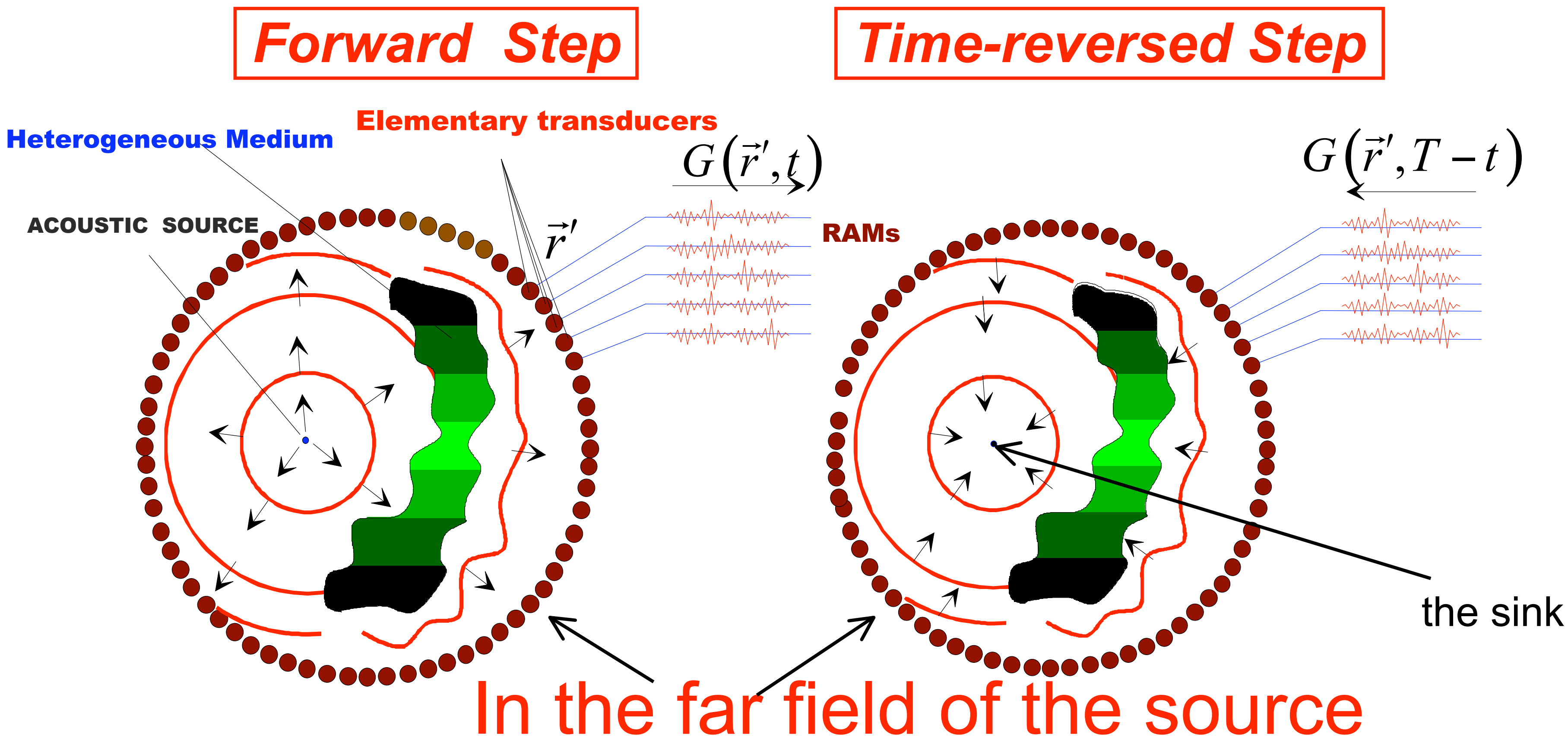
Wheeler and Feynman : the absorbing universe

$$\left\{G_{ret}(\vec r,\vec r_0;t) + G_{adv}(\vec r,\vec r_0;t)\right\}\Big/2$$

To build the ... d the causal field ... ne reversal cav...

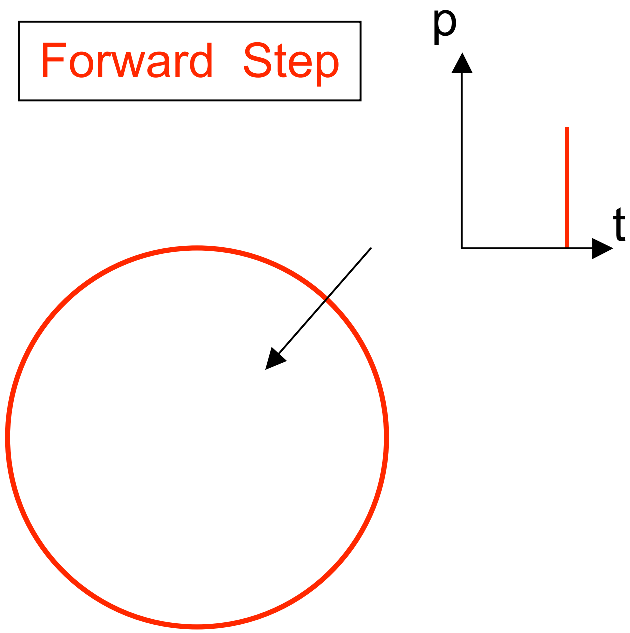
TR on the boundary : the TR Cavity

- record on the boundary $G(\vec{r}\,',\vec{r}_0;t);\partial_n G(\vec{r}\,',\vec{r}_0;t)$
- transmit from the boundary $G(\vec{r}\,',\vec{r}_0;T-t);\partial_n G(\vec{r}\,',\vec{r}_0;T-t)$

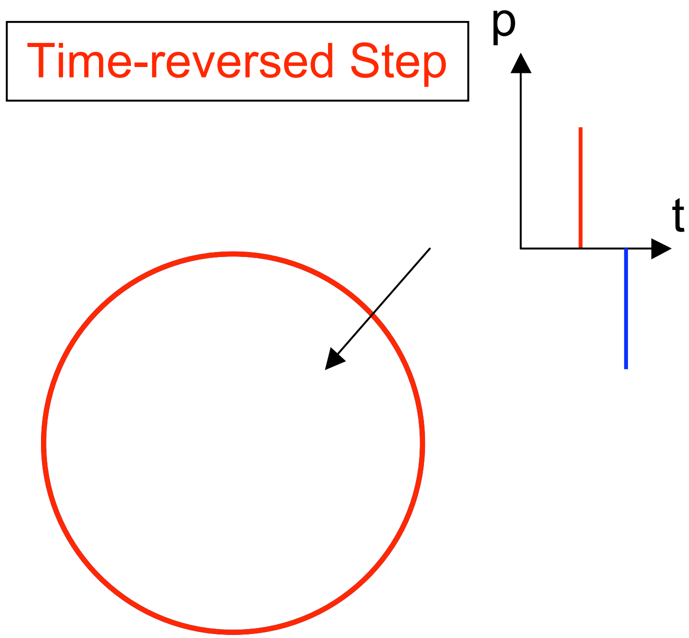


Origin of Diffraction Limits in Wave Physics

Pulsed mode – the homogeneous medium



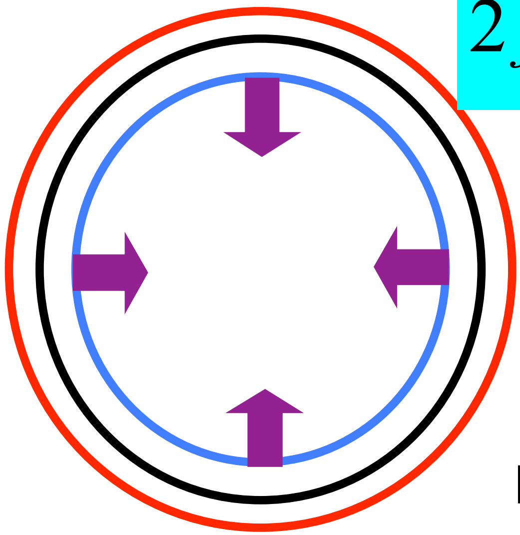
$$G^0_{ret}(R,t)$$



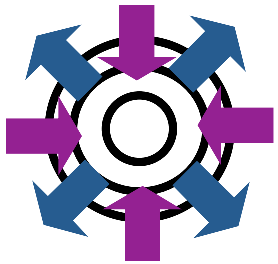
$$G^0_{adv}(R,t) - G^0_{ret}(R,t)$$

The time-reversed step for monochromatic waves : origin of the diffraction limits

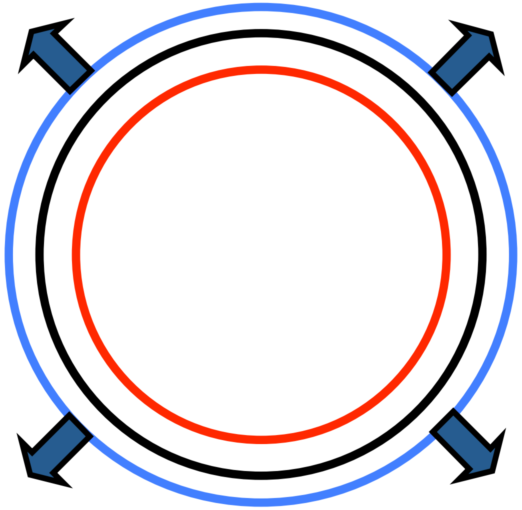
$$2\,j\,\mathrm{Im}\left\{\hat{G}^{0}_{ret}(R,\omega)\right\}$$



Converging only



Both converging and diverging waves interfere



Diverging only

$$\hat{G}^{0}_{adv}(R,\omega)=\frac{\exp\{j(-kR-\omega t)\}}{R}$$

with a singularity

$$\frac{\sin\{kR\}}{R}\exp(-j\omega t)$$

without singularity

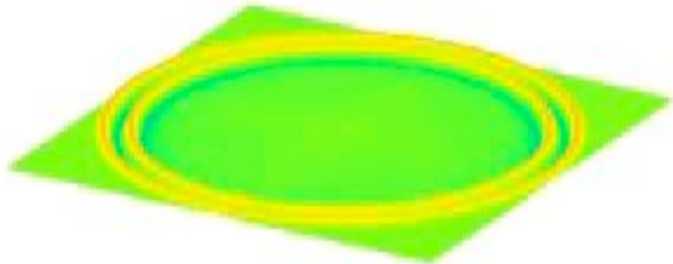
Diffraction limit (λ/2)

$$-\hat{G}^{0}_{ret}=-\frac{\exp\{j(kR-\omega t)\}}{R}$$

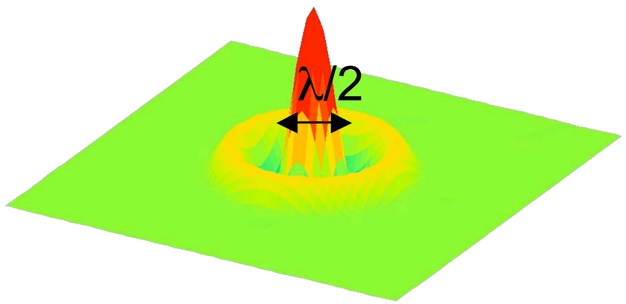
with a singularity

J. de Rosny, M. Fink

Broadband Focusing

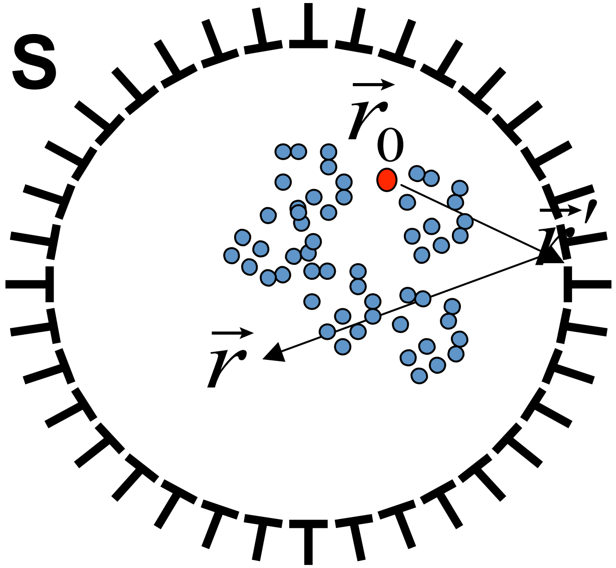


Instantaneous focal spot at the focal time (collapse time)



Theoretical description of an ideal time TR Cavity.

For any heterogeneous medium



For a Dirac excitation

$$\varphi_{tr}\left(\vec{r},t\right)=G_{ret}\left(\vec{r},\vec{r}_0;-t\right)-G_{ret}\left(\vec{r},\vec{r}_0;t\right)$$

For a monochromatic signal

$$\widehat{\Phi}_{tr}\left(\vec{r};\omega\right)=-2j\,\mathrm{Im}\,\widehat{G}\left(\vec{r},\vec{r}_0;\omega\right)$$

For a limited bandwidth signal

$$\varphi_{tr}\left(\vec{r},t=0\right)=-2j\int_{\Delta\omega}\mathrm{Im}\,\widehat{G}\left(\vec{r},\vec{r}_0;\omega\right)d\omega$$

Field at the focal time.
At $\vec{r}=\vec{r}_0$
related to LDOS

D. Cassereau, M. Fink, Sept 1992, IEEE UFFC

The TR formula in Electromagnetism

For a monochromatic signal

Dipole source

$$\mathbf{E}(\vec{r},\omega) = \mu_0 \omega^2 \overleftrightarrow{\mathbf{G}}(\vec{r},\vec{r}_0,\omega)\mathbf{p}$$

with $\nabla'_r \times \nabla'_r \times \overleftrightarrow{\mathbf{G}}(\vec{r},\vec{r}_0,\omega) - \frac{\omega^2}{c^2}\varepsilon(\vec{r},\omega) = -\delta(\vec{r}-\vec{r}_0)\overleftrightarrow{\mathbf{I}}$

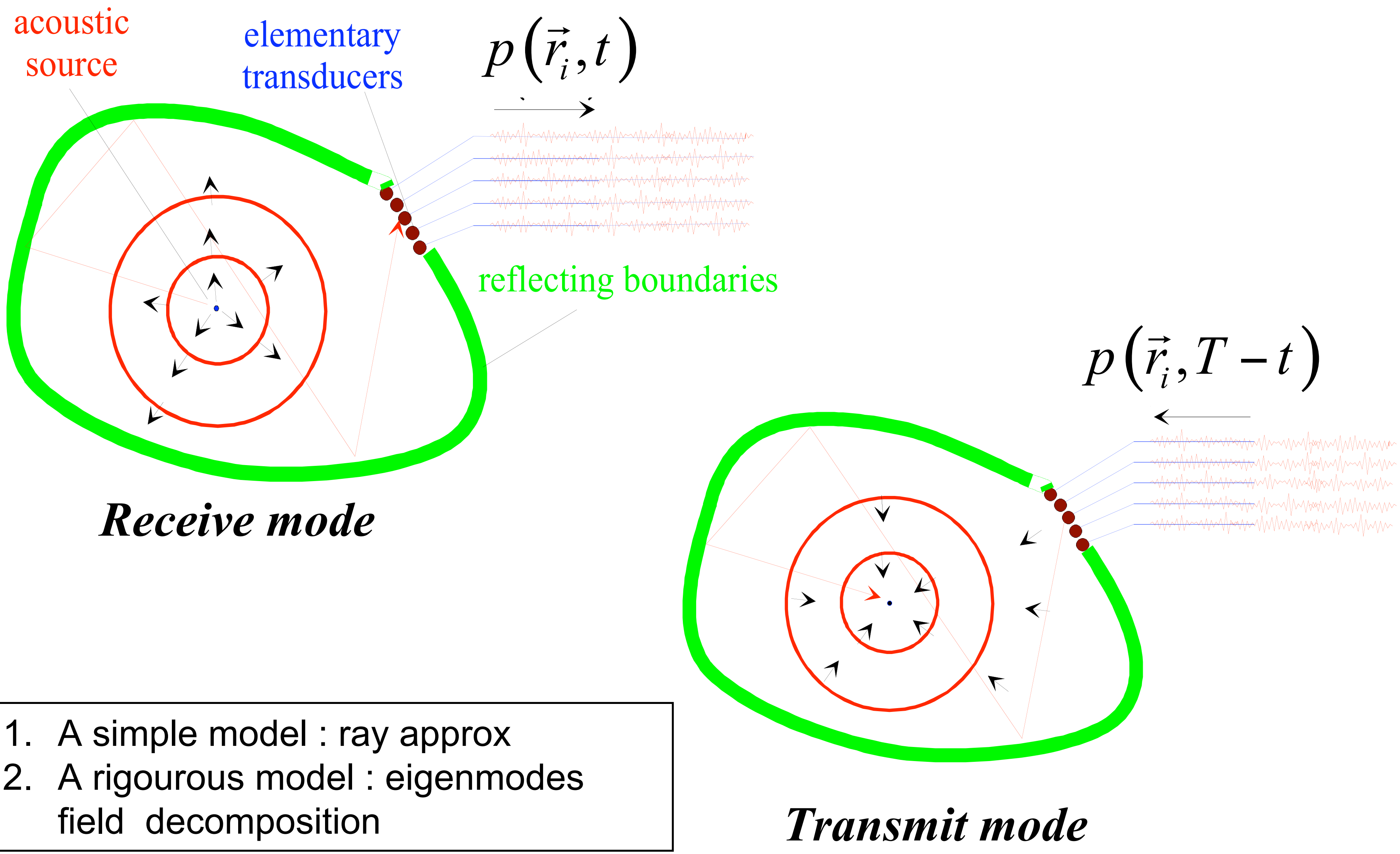
Dyadic Green's Function

$$\mathbf{E}_{tr}(\vec{r},\omega) = -2i\,\mu_0\,\omega^2\,\mathrm{Im}\left[\overleftrightarrow{\mathbf{G}}(\vec{r},\vec{r}_0,\omega)\right]\mathbf{p}^*$$

$$\mathbf{E}_{tr}(\vec{r}=\vec{r}_0,\omega) = -2i\,\mu_0\,\omega^2\,\mathrm{Im}\left[\overleftrightarrow{\mathbf{G}}(\vec{r}_0,\vec{r}_0,\omega)\right]\mathbf{p}^* \prec LDOS$$

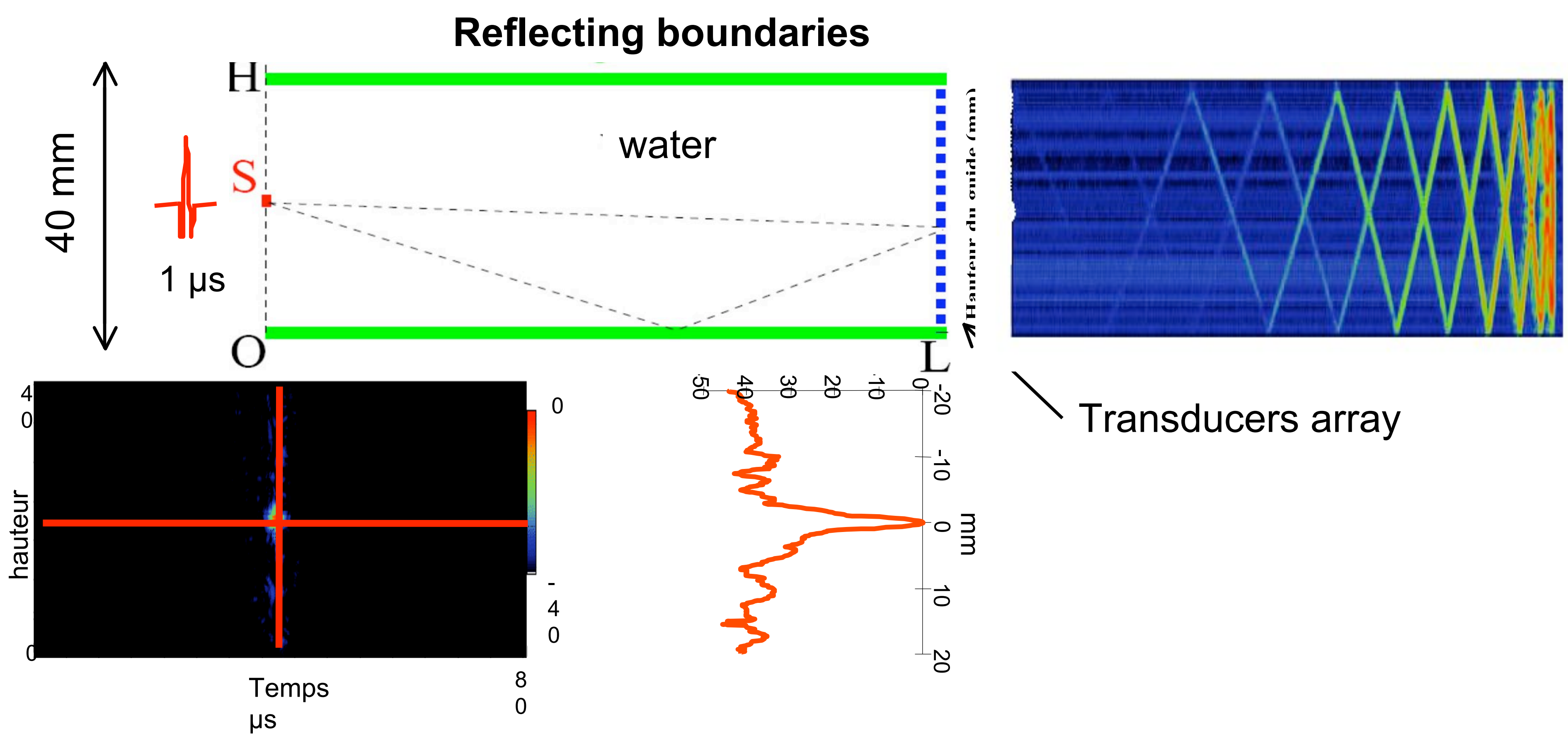
R. Carminati, M. Fink, R. Pierrat, J de Rosny

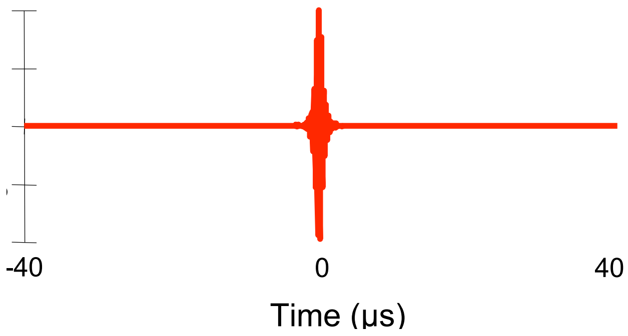
The effect of boundaries on Time-reversal Mirror



1. A simple model : ray approx
2. A rigourous model : eigenmodes field decomposition

Time Reversal in an ultrasonic waveguide





P. Roux, M. Fink

A 78 m Long Time Reversal Mirror

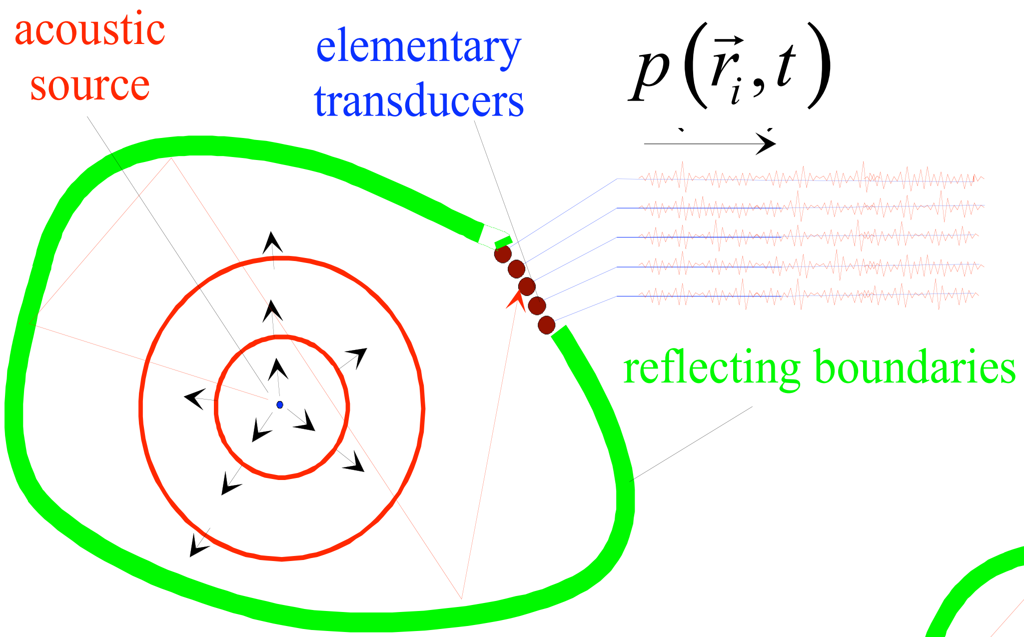
L = 78 m

N = 29

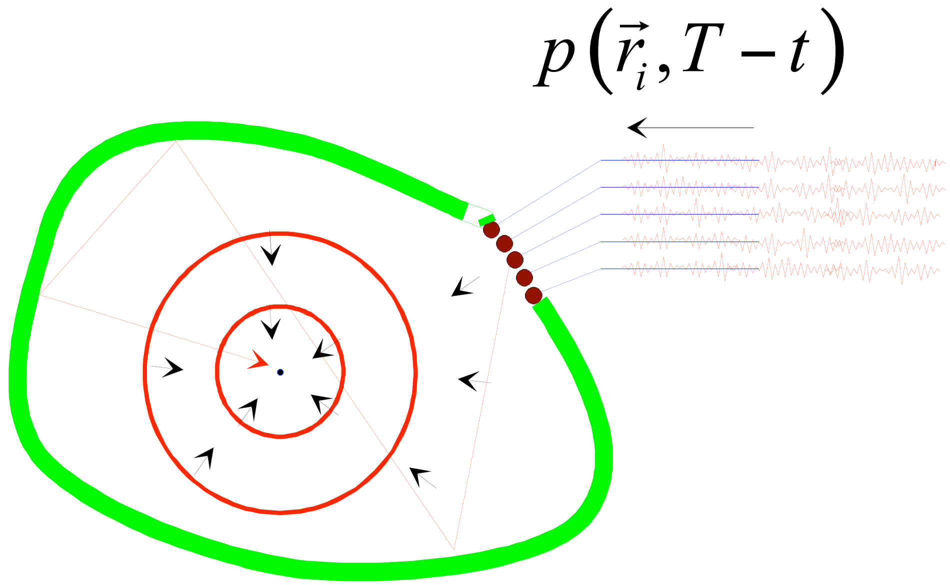
3.5 kHz , λ = 50 cm transducers

Underwater acoustics , B. Kuperman Group, Scripps, San Diego

The effect of boundaries on Time-reversal Mirror



Receive mode



Transmit mode

1. A simple model : ray approx
2. A rigourous model : eigenmodes field decomposition

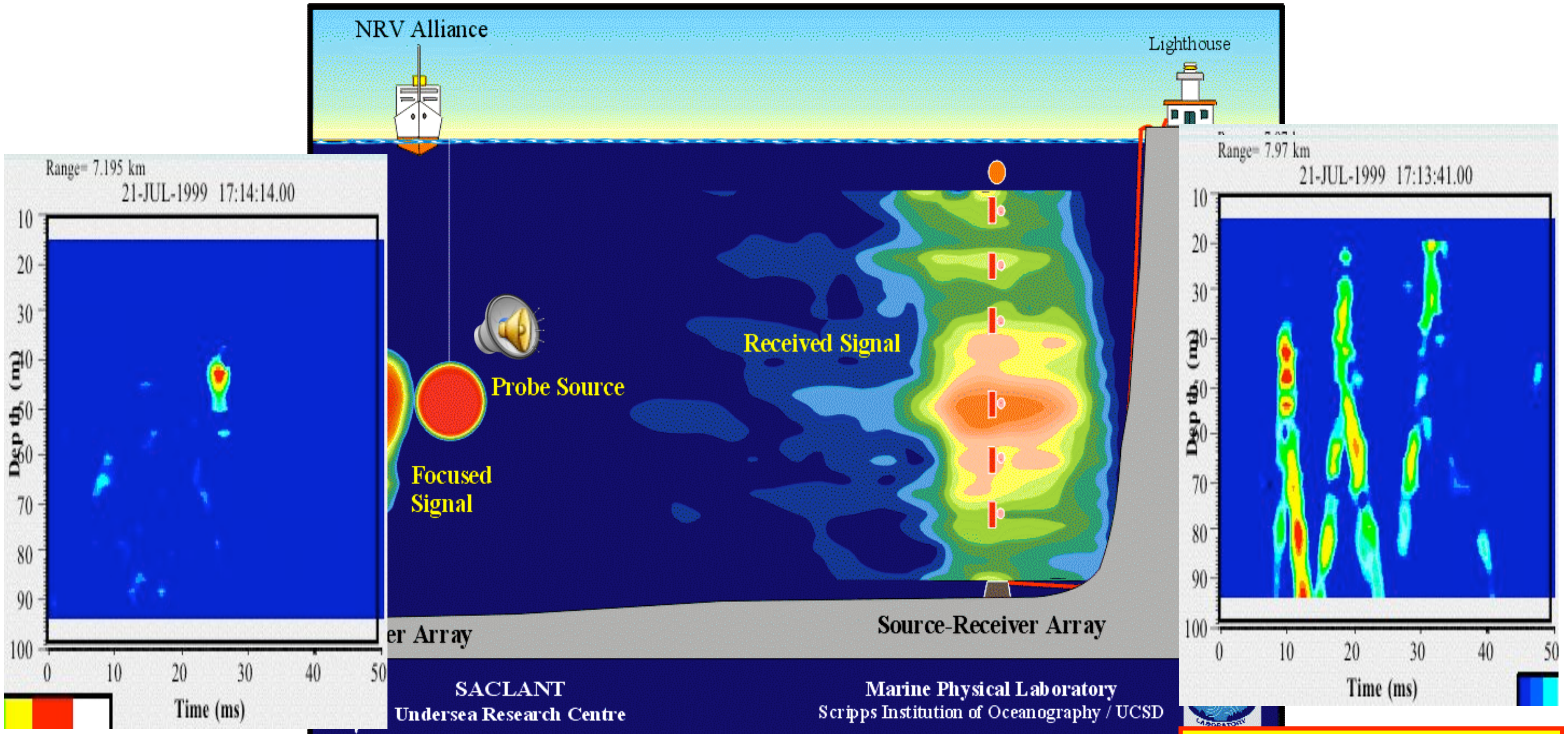
A 78 m Long Time Reversal Mirror

L = 78 m
N = 29

3.5 kHz , λ = 50 cm
transducers

Underwater acoustics , B. Kuperman Group, Scripps, San Diego

Elba Island Experiment

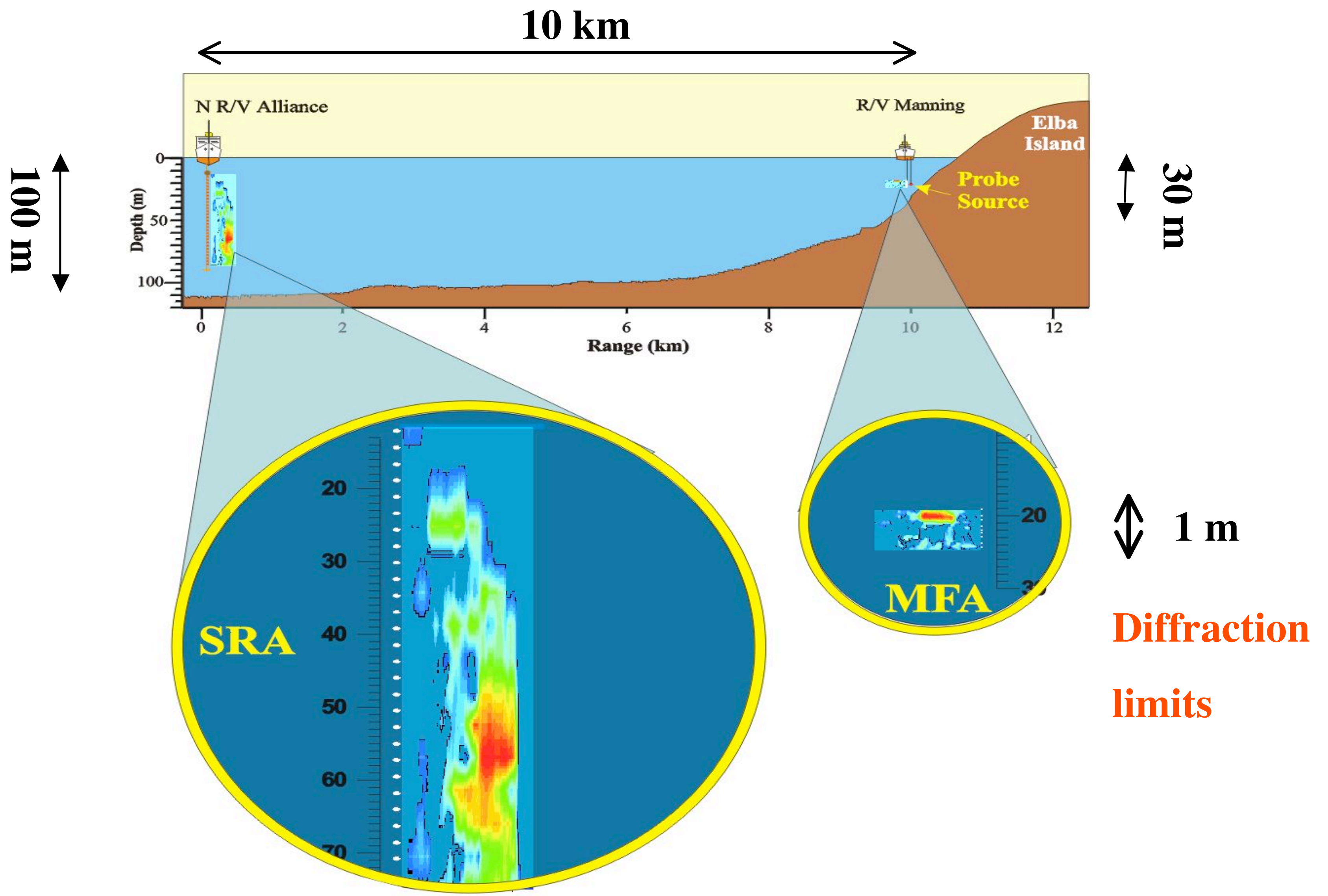


Focused pulse back at source

TR = last in first out

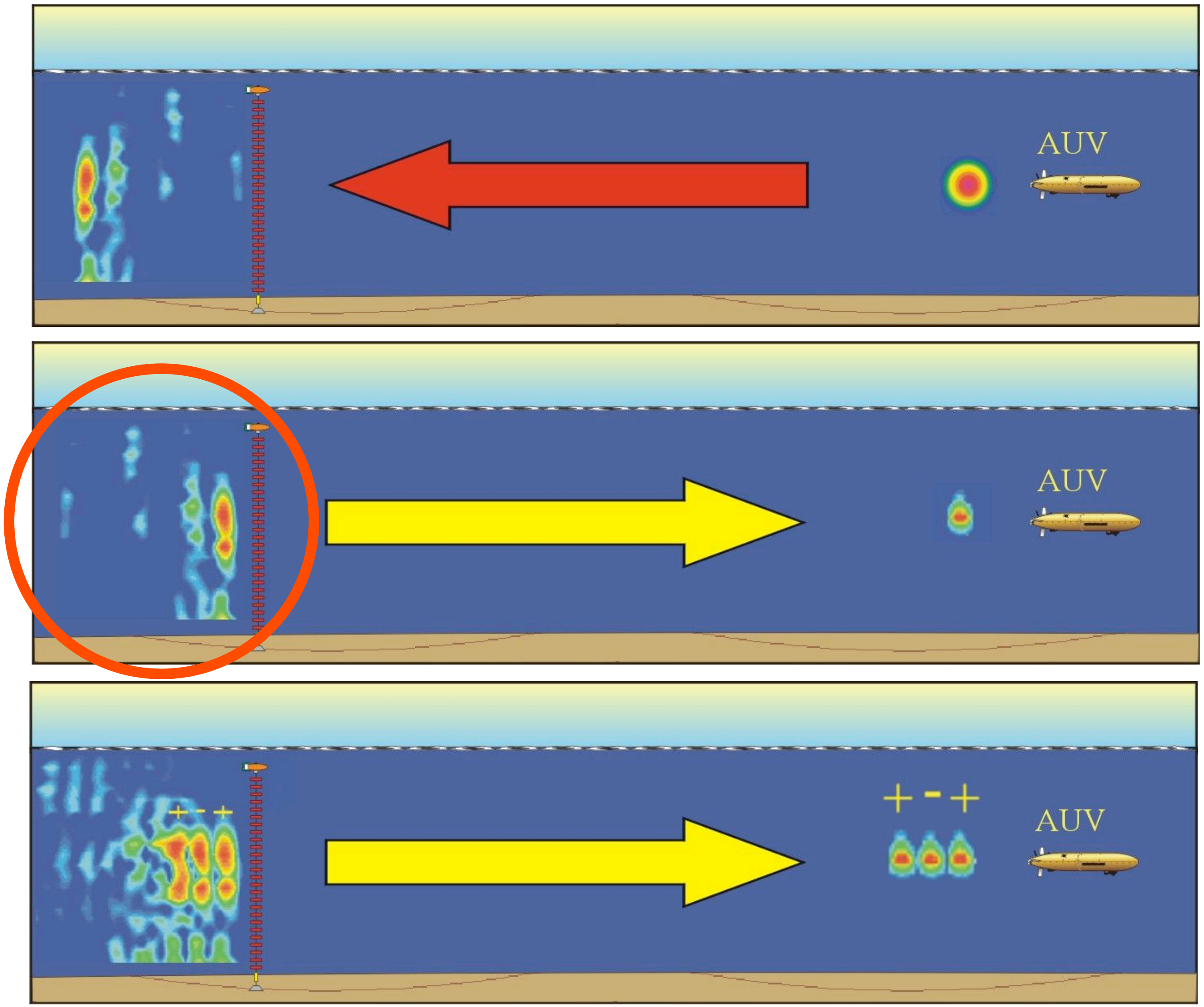
P. Roux, B Kuperman

Elba Island Experiments



Diffraction limits

Underwater communications

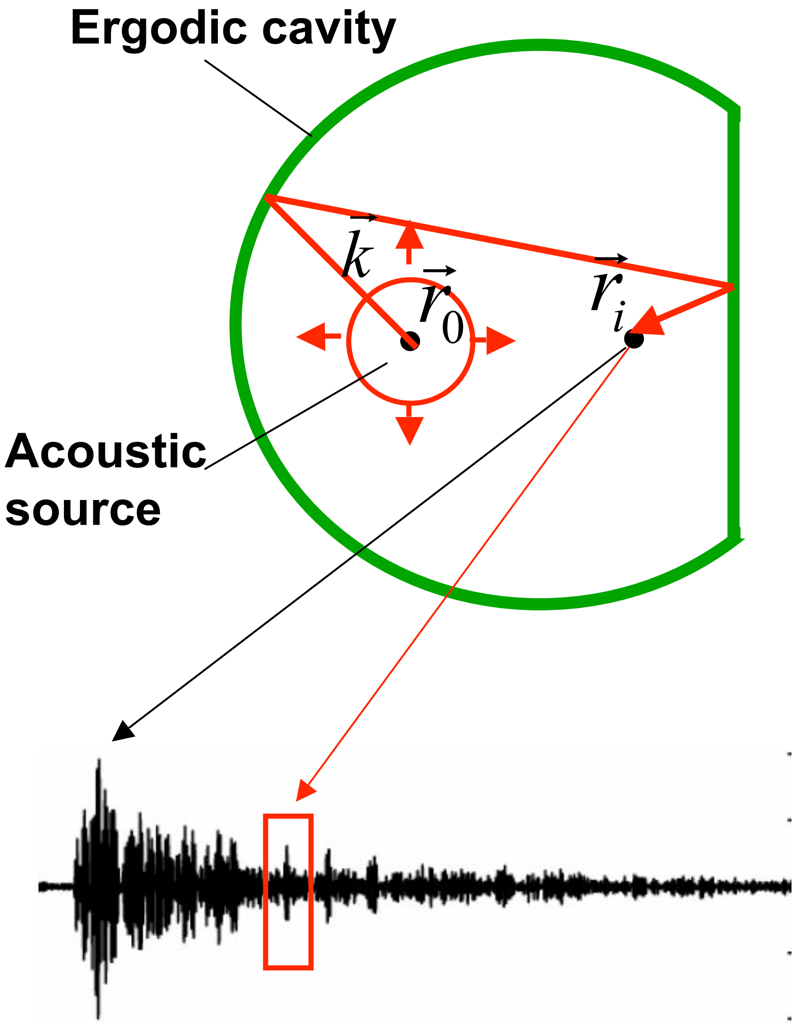


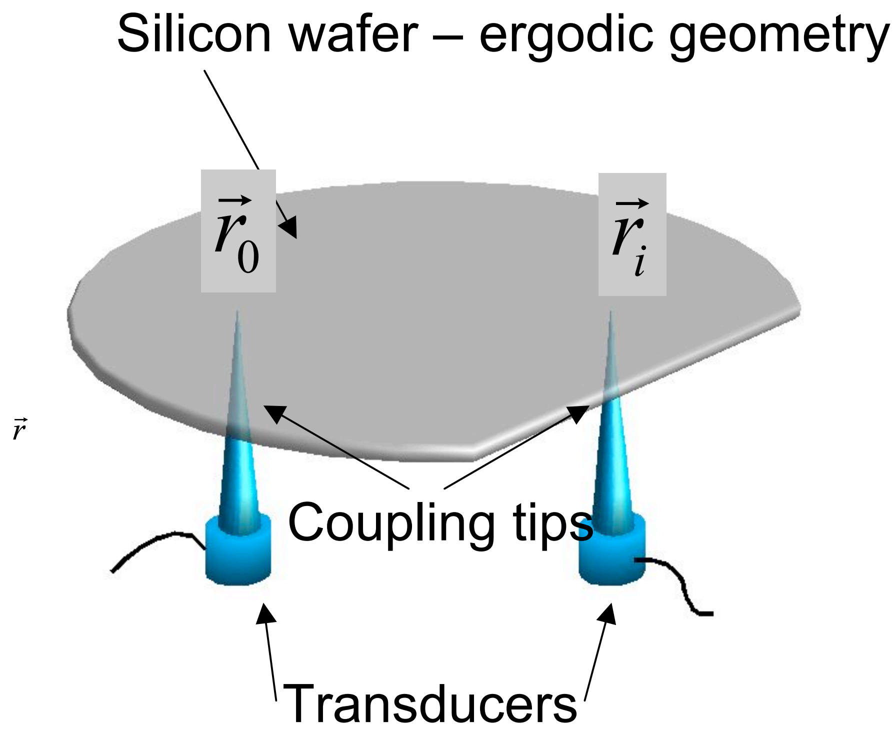
Reference pulse

Time-reversal

Time-reversal Communications

Time-Reversal in a chaotic billiard with a one channel TRM



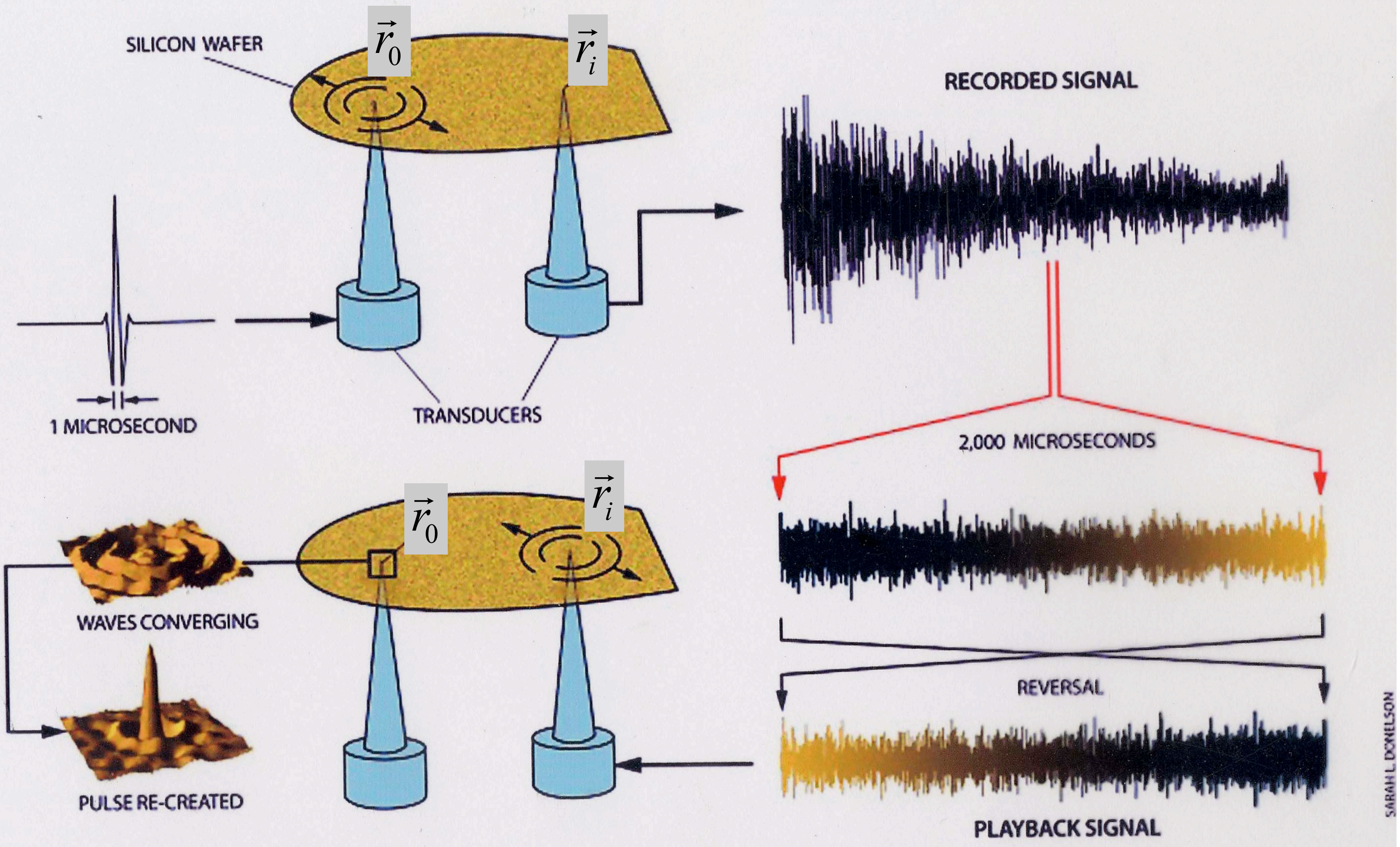


Space – time coupling

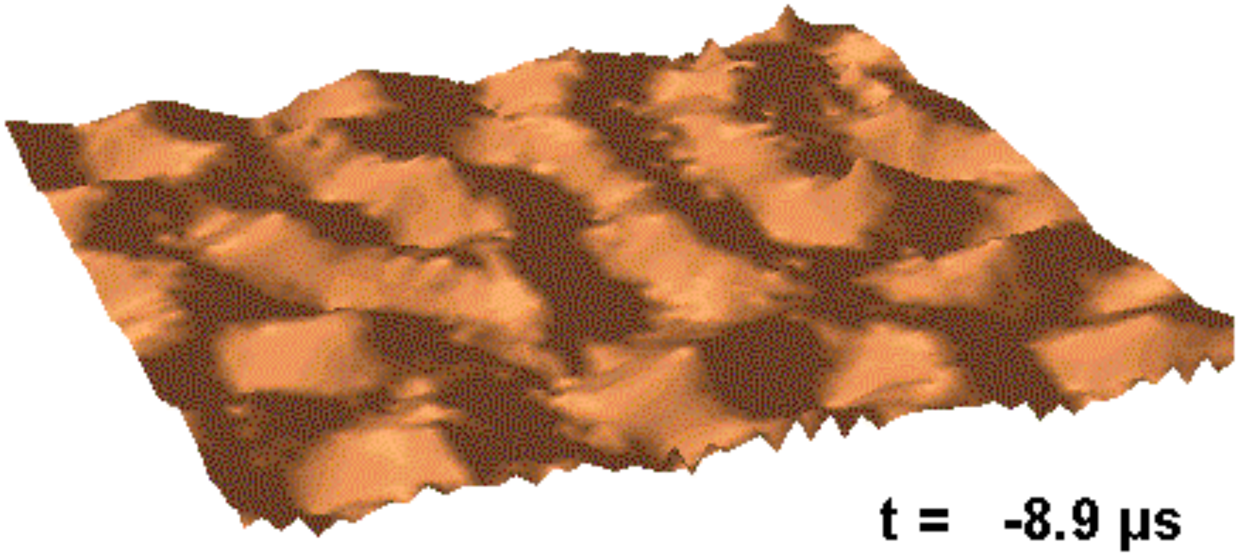
each $\vec{k}$ **is coded in a time t**

$$\Delta k_i . \Delta r_i \Rightarrow \Delta t . \Delta \omega$$

C Draeger, J de Rosny, M.

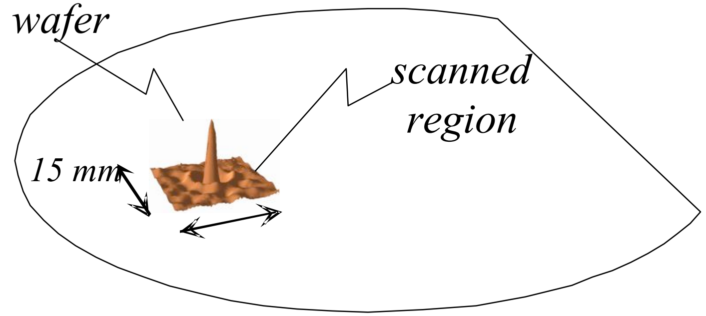


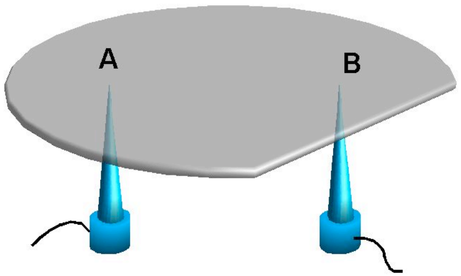
SINGLE TRANSDUCER can time-reverse a wave in an enclosed "cavity." A source transducer emits a pulse at location A on a small silicon wafer (*top*). A transducer at location B records chaotic reverberations of the pulse reflected off the wafer edges hundreds of times. The transducer at B plays back a short segment of that signal in reverse (*bottom*). After many reflections, these recombine to re-create the short pulse focused again at location A, as was revealed by imaging the waves on the wafer near A (*bottom left*).

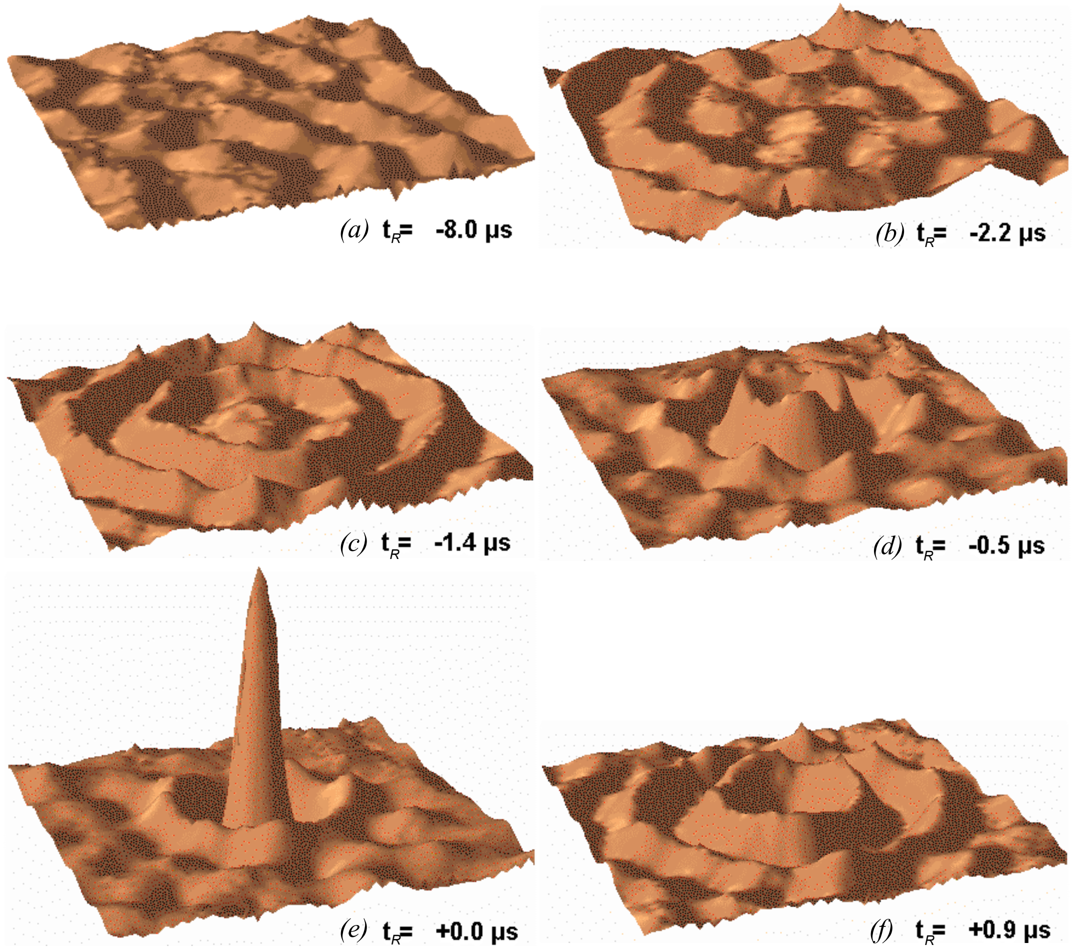


A 2ms window corresponds to the Heisenberg time of the cavity : $\tau_{Heis} = \frac{1}{\delta\omega}$

with $\delta\omega$ being the mean distance between modes



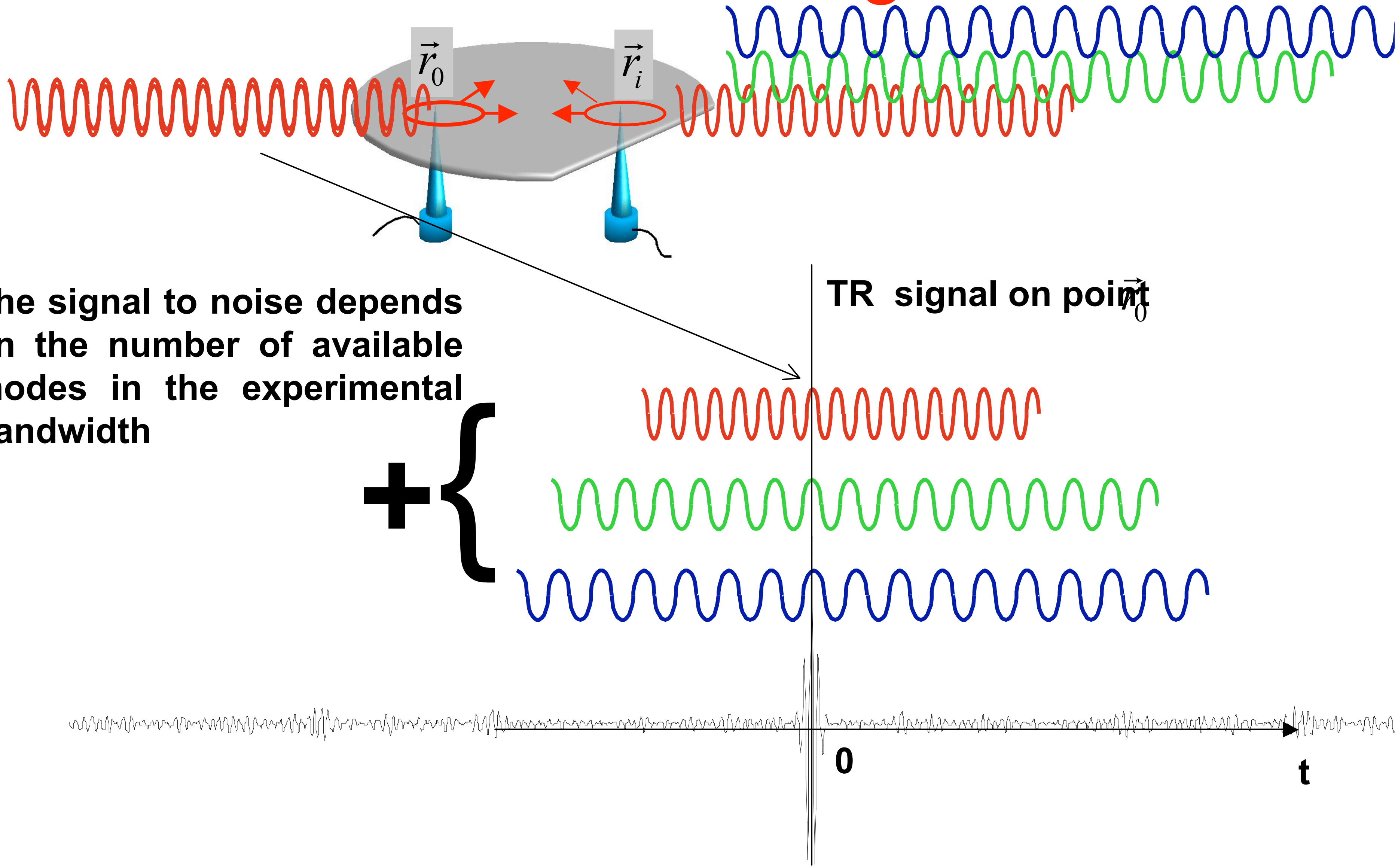




The signal to noise
Is proportional to the
bandwidth square: $\sqrt{\dfrac{\Delta\omega}{\delta\omega}}$

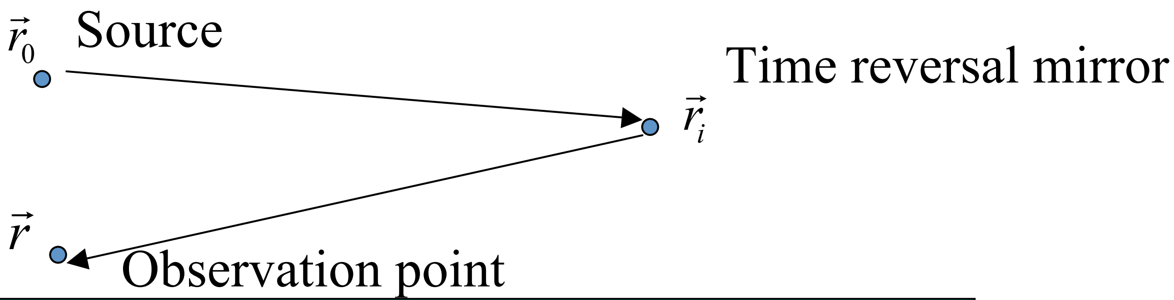
Indeed, a monochromatic source located at B will never focused at A

The one-channel TRM works only for broadband signal



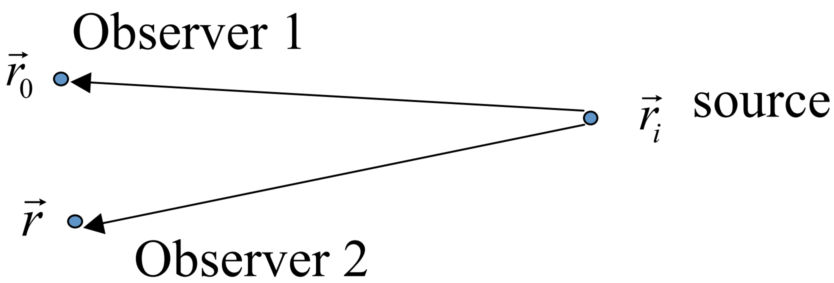
The signal to noise depends on the number of available modes in the experimental bandwidth

One channel TRM as a Spatial Correlator



$$\varphi_{tr}\left(\vec{r},t\right) \propto \frac{\partial}{\partial t}\left\{ G(\vec{r}_i,\vec{r}_0,-t) \overset{t}{\otimes} G(\vec{r}_i,\vec{r},t) \right\}$$

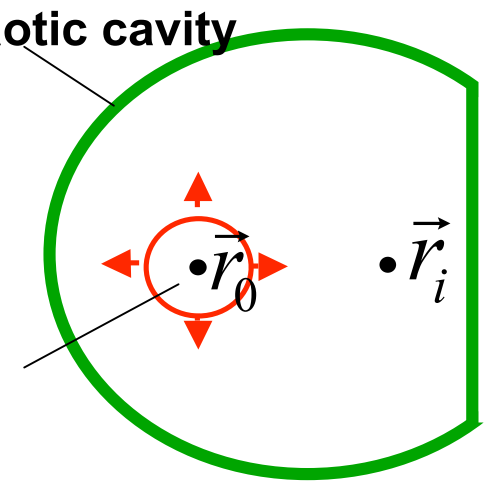
Time-reversed field observed at point $\vec{r}$ from a source at $\vec{r}_0$



$$C\left(\vec{r}_0,\vec{r},t\right) = G\left(\vec{r}_0,\vec{r}_i,-t\right) \overset{t}{\otimes} G\left(\vec{r},\vec{r}_i,t\right)$$

The time-reversed field is an estimate of the derivative of the spatial correlation of the field radiated by point $\vec{r}_i$

An important formula



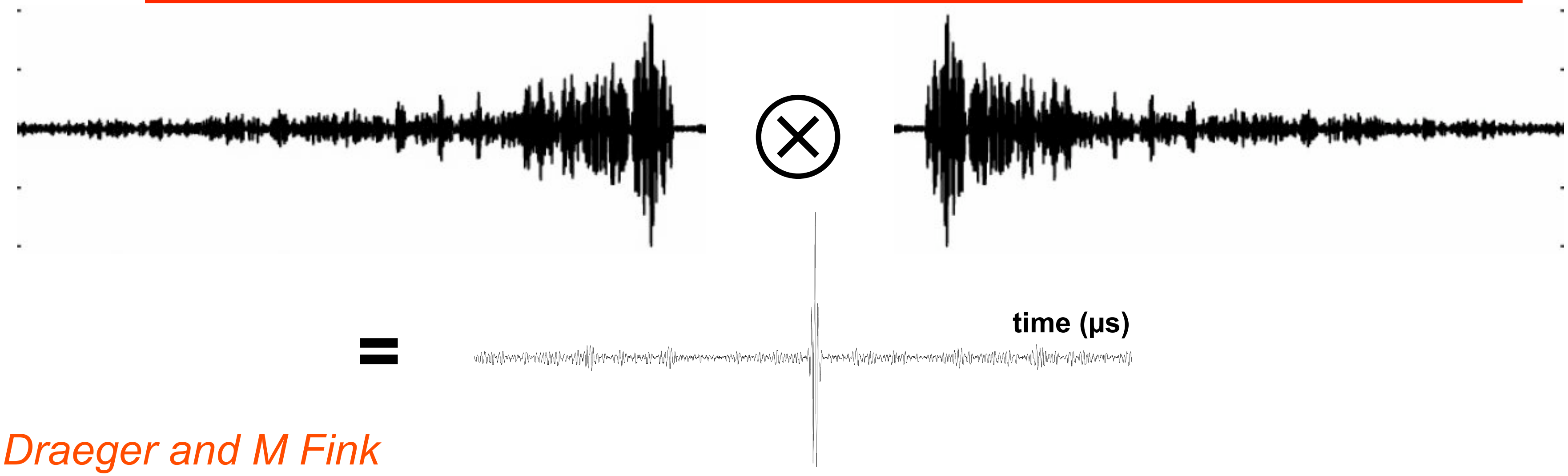
In terms of the cavity modes, $\vec{r}_0$ and $\vec{r}_i$ cannot exchange all informations, because $\vec{r}_0$ and $\vec{r}_i$ are always at the **nodes** of some modes.

$$G(\vec{r}_i,\vec{r}_0;t)=\sum_n u_n(\vec{r}_0)u_n(\vec{r}_i)\frac{\sin(\omega_n t)}{\omega_n}$$

eigenmodes u_n $\quad \Delta u_n = -\frac{\omega_n^2}{c^2}u_n$

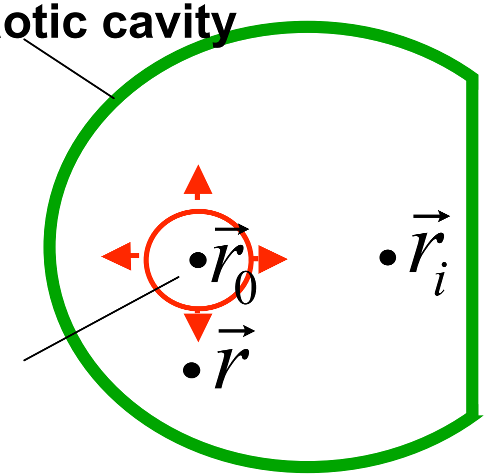
Mode repulsion

$$G(\vec{r}_i,\vec{r}_0;-t)\otimes G(\vec{r}_i,\vec{r}_0;t)=G(\vec{r}_0,\vec{r}_0;-t)\otimes G(\vec{r}_i,\vec{r}_i;t)$$



C Draeger and M Fink

The field at any $\vec{r}$



$$G\left(\vec{r}_i,\vec{r}_0;t\right) = \sum_n u_n(\vec{r}_0)u_n(\vec{r}_i)\frac{\sin(\omega_n t)}{\omega_n}$$

eigenmodes u_n

$$\Delta u_n = -\frac{\omega_n^2}{c^2}u_n$$

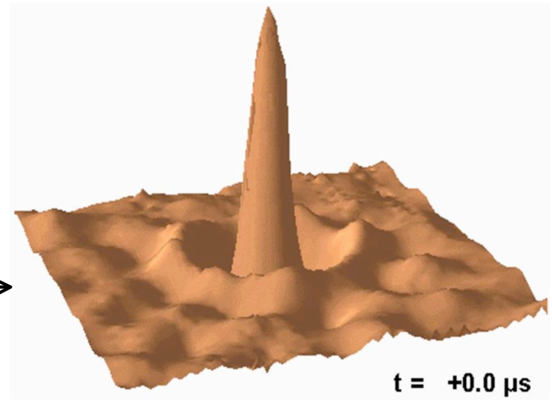
$$G\left(\vec{r}_i,\vec{r}_0;-t\right)\otimes G\left(\vec{r}_i,\vec{r};t\right) = G\left(\vec{r}_0,\vec{r};-t\right)\otimes G\left(\vec{r}_i,\vec{r}_i;t\right)$$

C Draeger and M Fink

The focal spot

$$\varphi_{tr}\left(\vec{r},t\right) \propto \frac{\partial}{\partial t}\left\{G(\vec{r}_i,\vec{r}_0,-t)\overset{t}{\otimes} G(\vec{r}_i,\vec{r},t)\right\}$$

$$\varphi_{tr}(\vec{r},t=0) \propto \frac{\partial}{\partial t}\sum_n \frac{1}{\omega_n^2} u_n(\vec{r}_0)u_n(\vec{r})u_n^2(\vec{r}_i)$$



Frequency Average

$$\left\langle u_n(\vec{r}_0)u_n(\vec{r})u_n^2(\vec{r}_i)\right\rangle = \left\langle u_n(\vec{r}_0)u_n(\vec{r})\right\rangle\left\langle u_n^2(\vec{r}_i)\right\rangle$$

$$J_0(2\pi\left|\vec{r}-\vec{r}_0\right|/\lambda_n)$$

If chaotic rays support irregular modes, Berry Conjecture

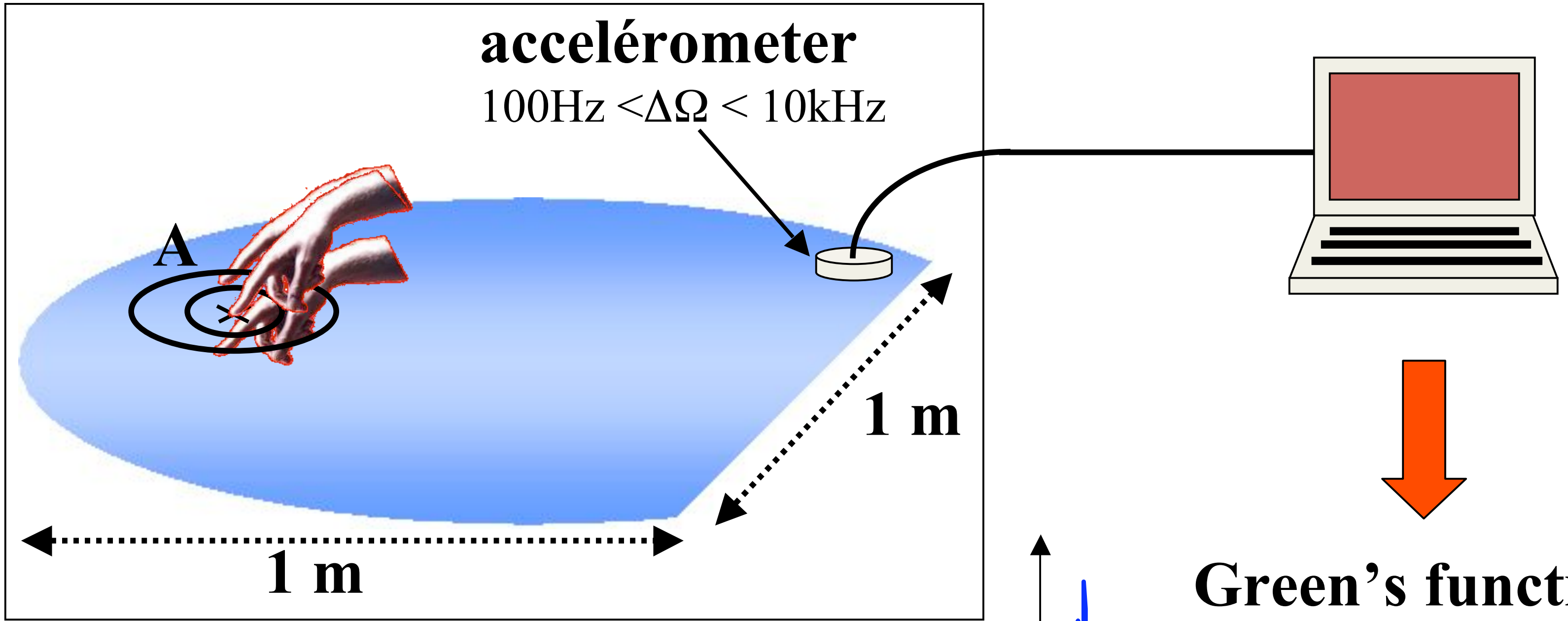
$$\varphi_{tr}\left(\vec{r},t=0\right)\approx -2j\int_{\Delta\omega} \mathrm{Im}\,\widehat{G}\left(\vec{r},\vec{r}_0;\omega\right)d\omega$$

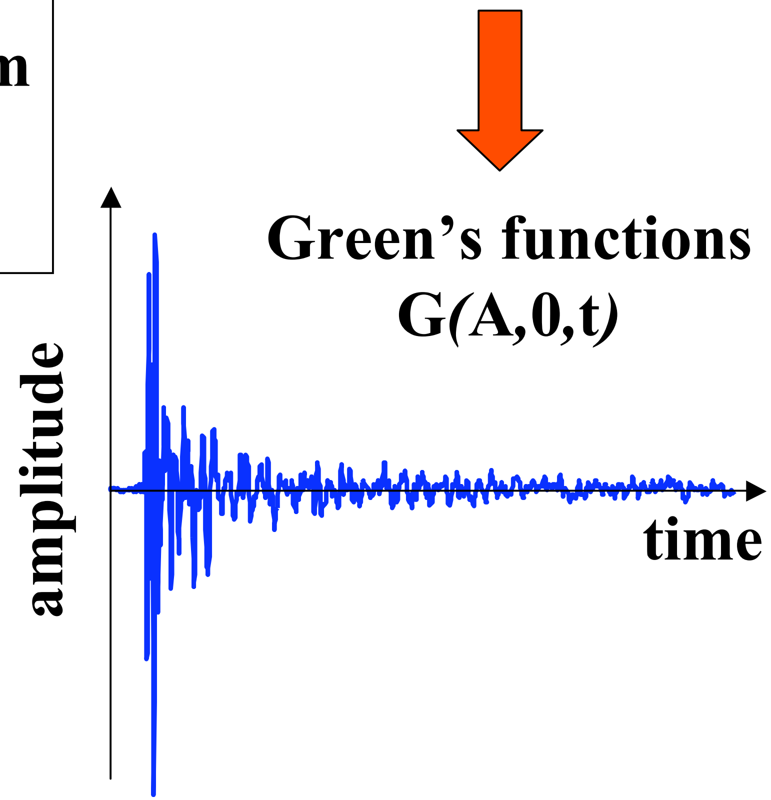
Time Reversal is Self -averaging

many uncorellated eigenmodes =400

A nice application: Tactile objects

How to transform a solid object in a tactile screen ?

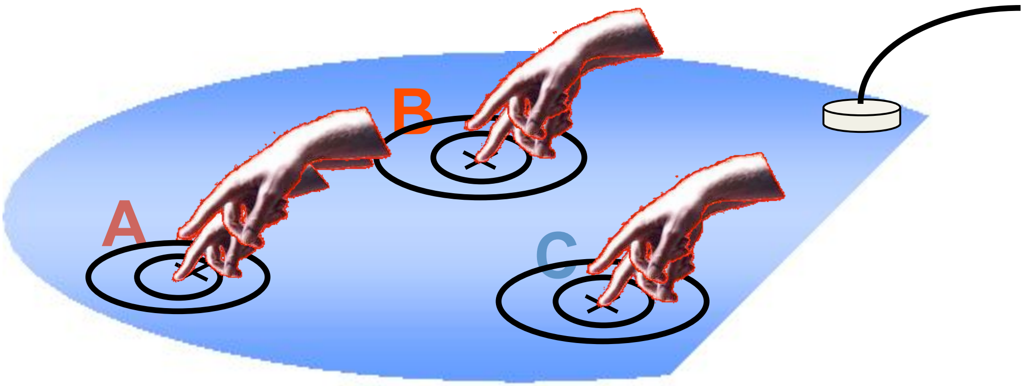


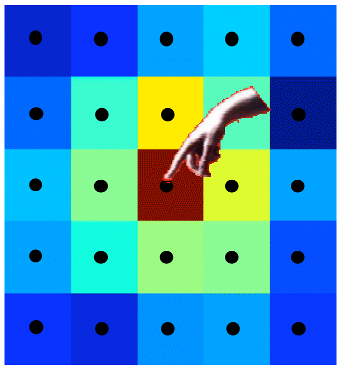


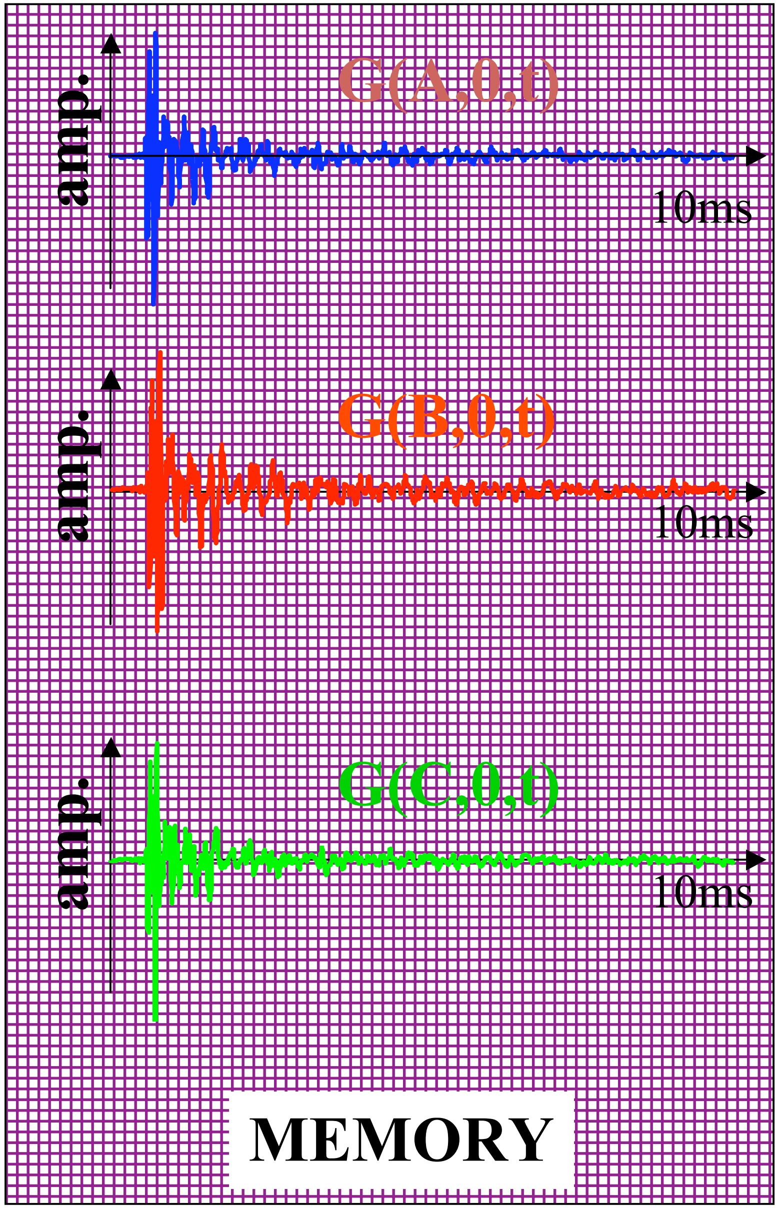
Sensitive Object

R.K Ing, N. Quieffin, S. Catheline, M. Fink

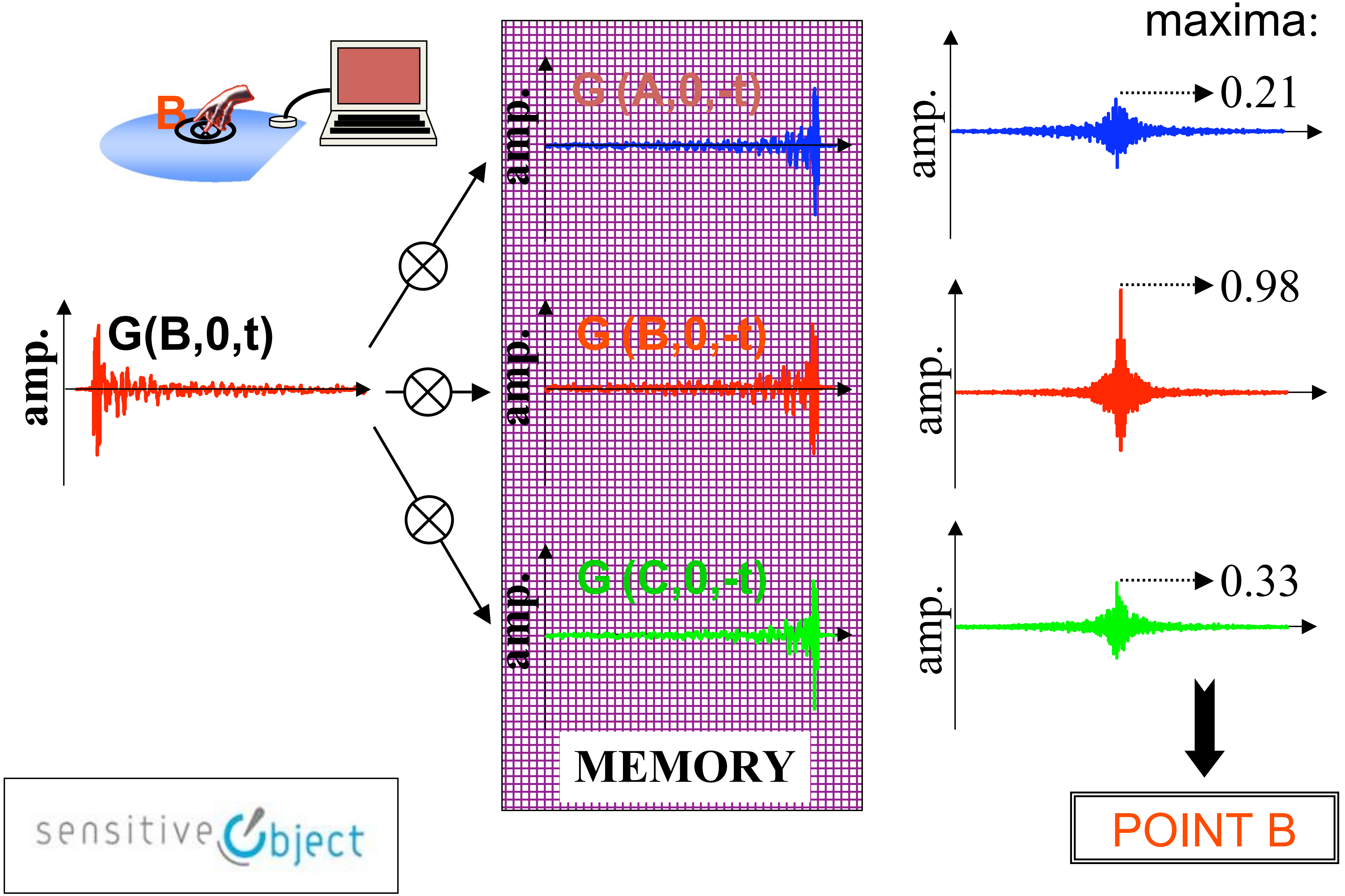
Teaching the Green's functions



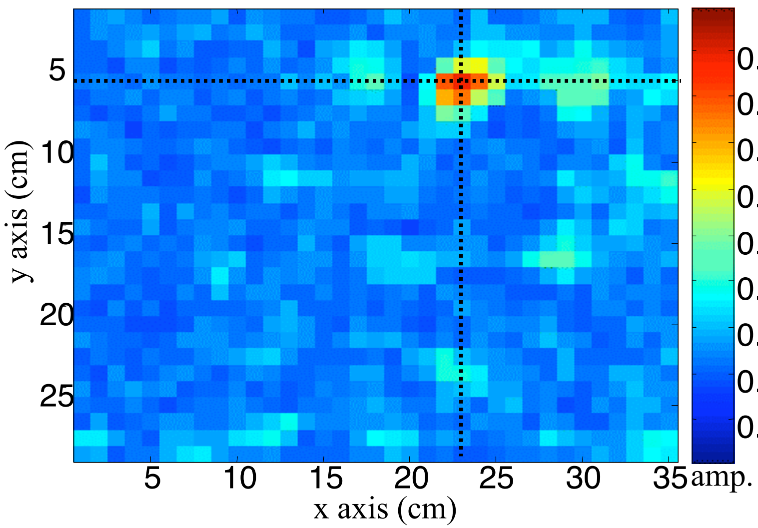




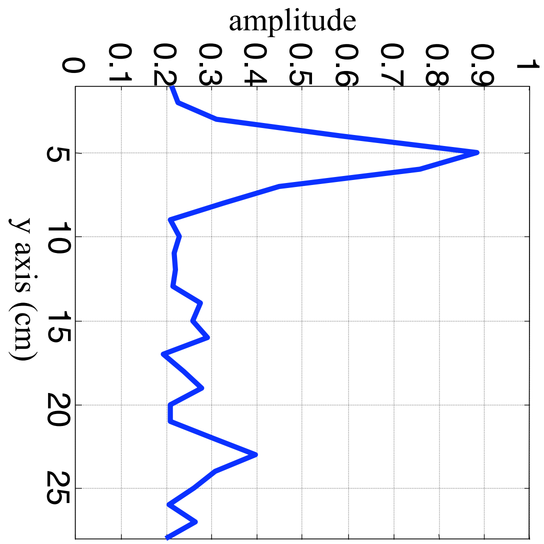
Source localisation by cross-corelation : numerical simulation of TR



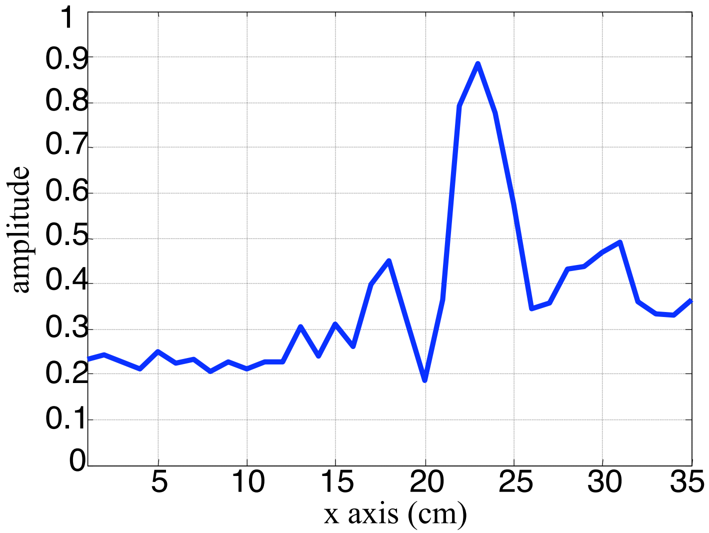
Focal spot



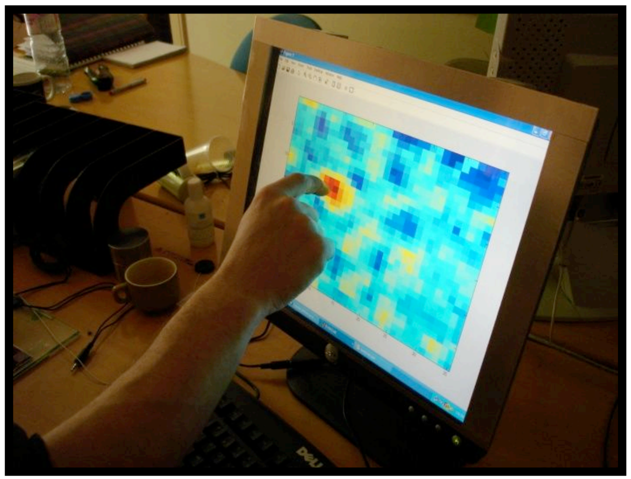
y axis (cm)
5
10
15
20
25
5
10
15
20
25
30
35
x axis (cm)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
amp.



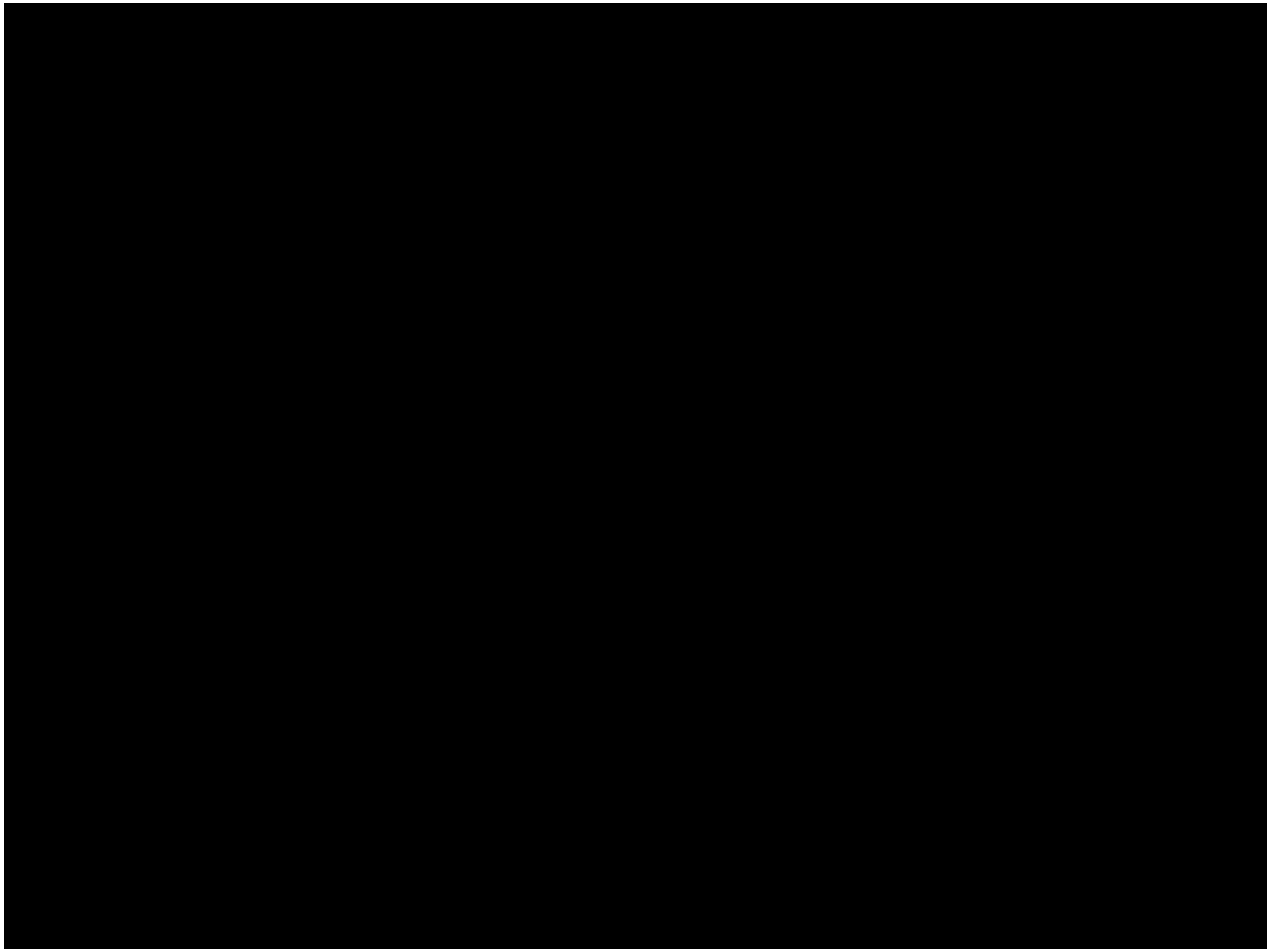
amplitude
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
y axis (cm)
5
10
15
20
25



amplitude
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20
25
30
35
x axis (cm)

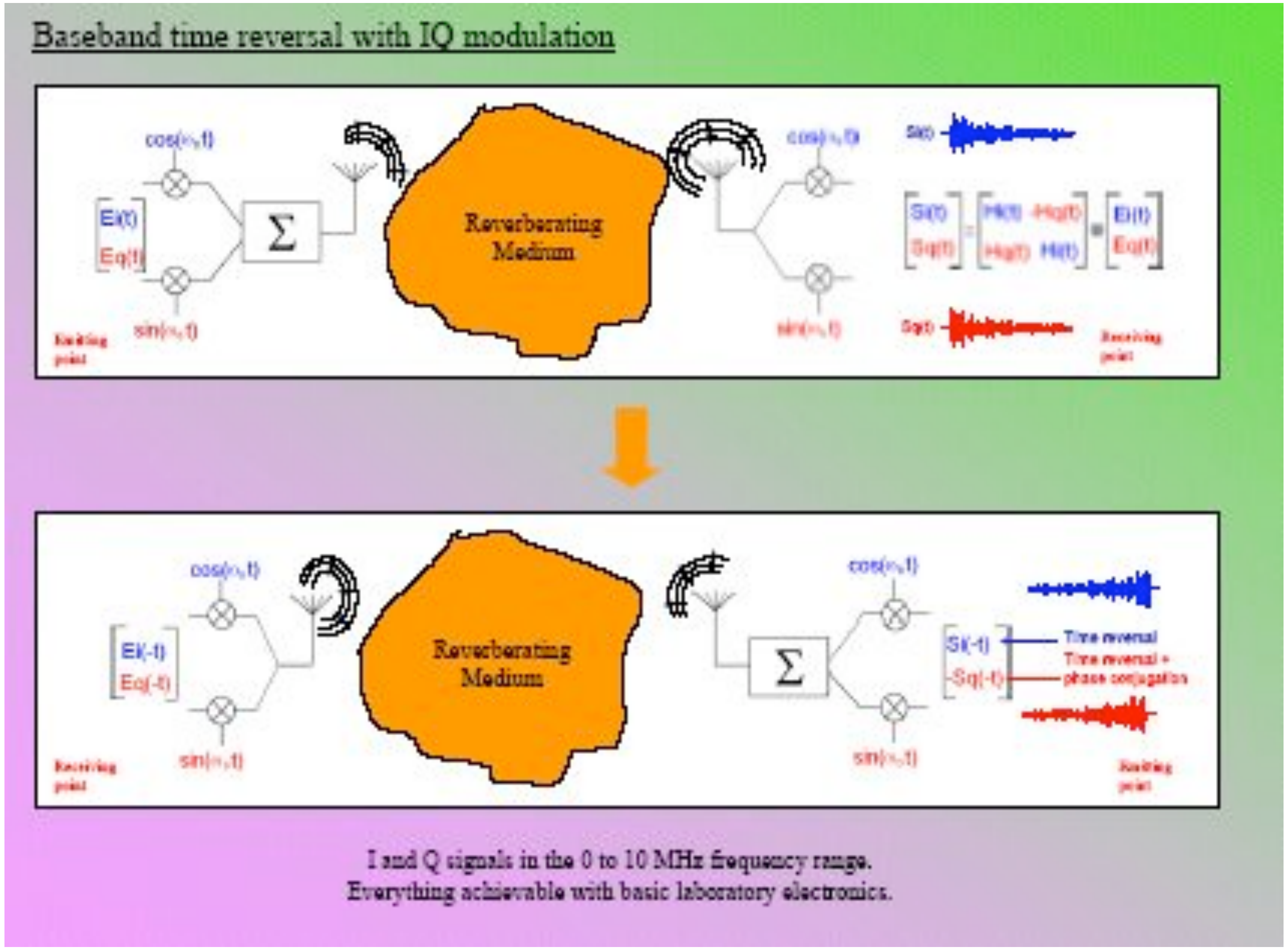


Products



sensitive object

Time Reversal with Electromagnetic Waves (2.4 GHz)

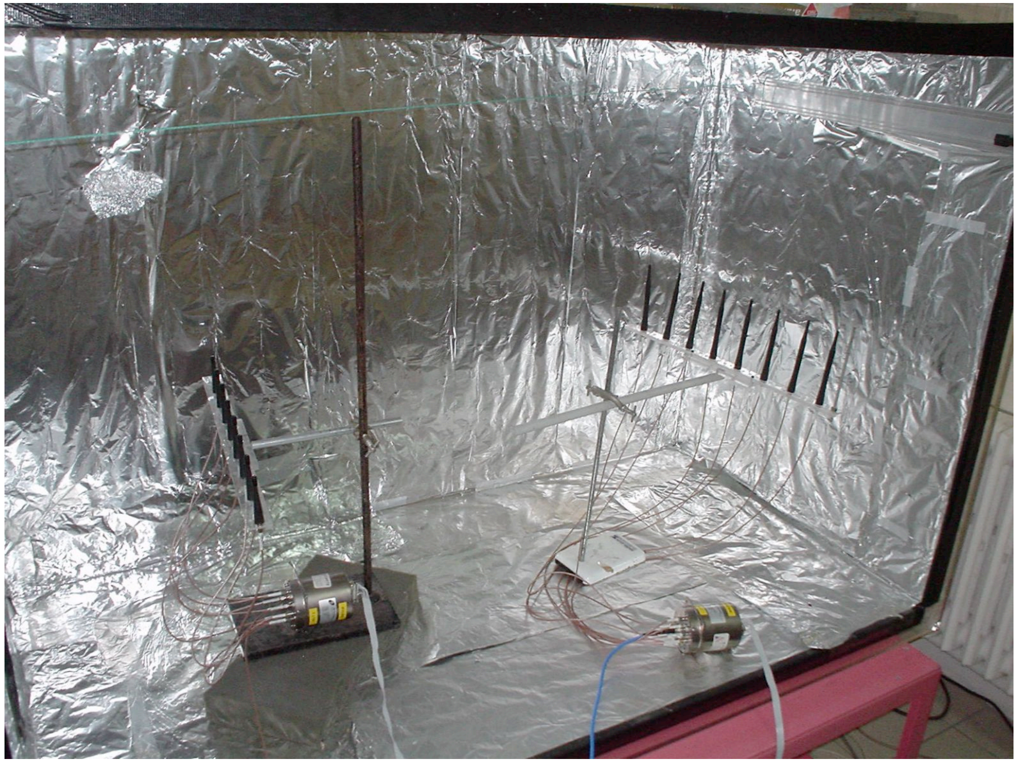


G.Lerosey , J de Rosny, A Tourin, A Derode, G Montaldo M Fink,

Electromagnetic TRM

• 2 arrays of 8 antennas separated of approx 6.15 cm, i.e. half a wavelength (12.3cm @ 2.44 GHz)

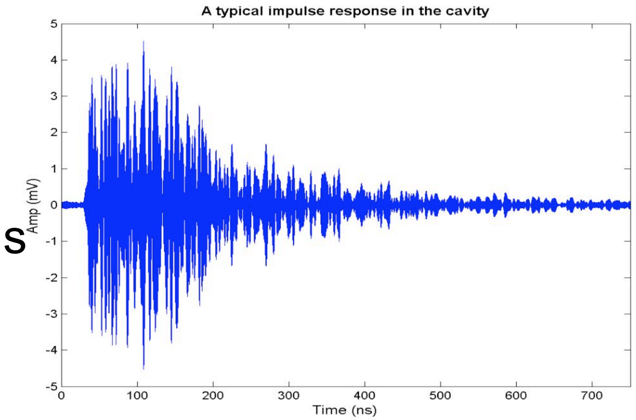
•

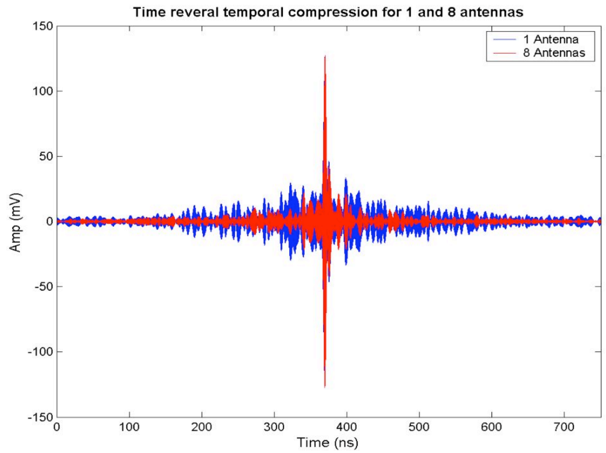


G Lerosey, J. de Rosny, A Tourin, M Fink

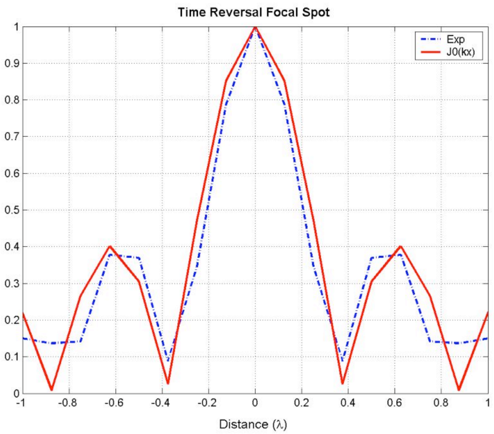
Focalisation spatio-temporel pour la communication à très haut débit

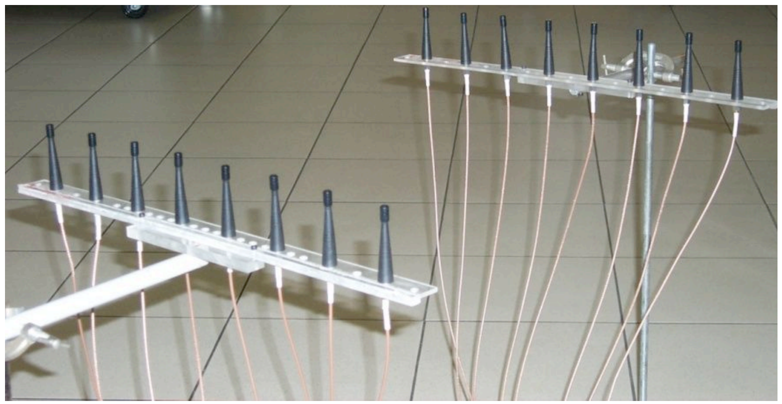
Impulsion initiale 10 ns





Compression temporelle



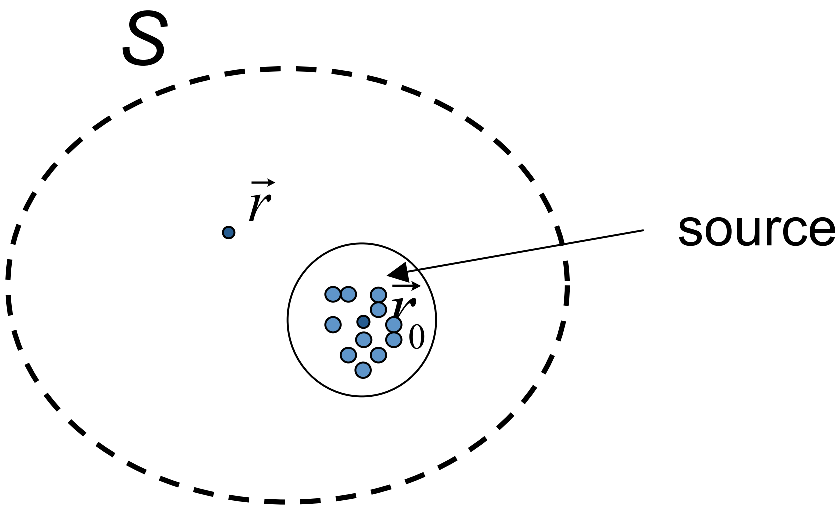


Focalisation spatiale $\lambda/2$

Super Resolution

2- Media with sophisticated Green's functions

Sophisticated Green's functions



- select a medium with $\mathrm{Im}\left\{\hat{G}_{ret}\left(\vec{r},\vec{r}_0,\omega\right)\right\}$ that oscillate faster than the wavelength

An homogenous medium is not interesting

$$\hat{G}^0_{ret}\left(\vec{r},\vec{r}_0,\omega\right) = \frac{\exp\left(jk\left|\vec{r}-\vec{r}_0\right|\right)}{k\left|\vec{r}-\vec{r}_0\right|}$$

$$\mathrm{Im}\left\{\hat{G}^0_{ret}\left(\vec{r},\vec{r}_0,\omega\right)\right\} \prec \frac{\sin\left(k\left|\vec{r}-\vec{r}_0\right|\right)}{k\left|\vec{r}-\vec{r}_0\right|}$$

Build media with complex pattern in the near field of the source : obstacles or antenna

$$\varphi_{tr}\left(\vec{r},t=0\right) = -2j\int_{\Delta\omega} \mathrm{Im}\,\widehat{G}\left(\vec{r},\vec{r}_0;\omega\right)d\omega$$

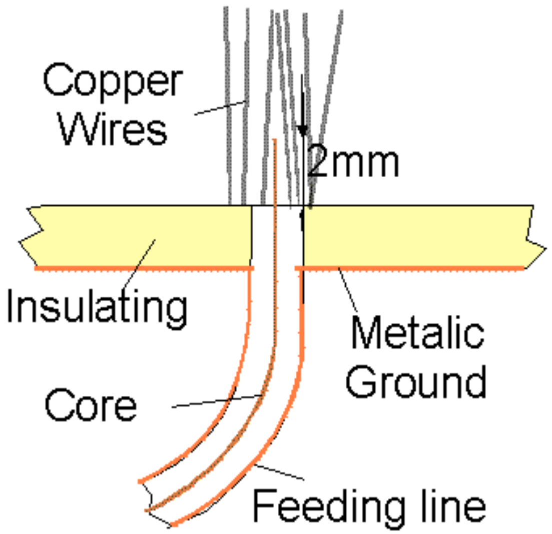
$$\varphi_{tr}\left(\vec{r}=\vec{r}_0,t=0\right) = -2j\int_{\Delta\omega} \mathrm{Im}\,\widehat{G}\left(\vec{r}_0,\vec{r}_0;\omega\right)d\omega$$

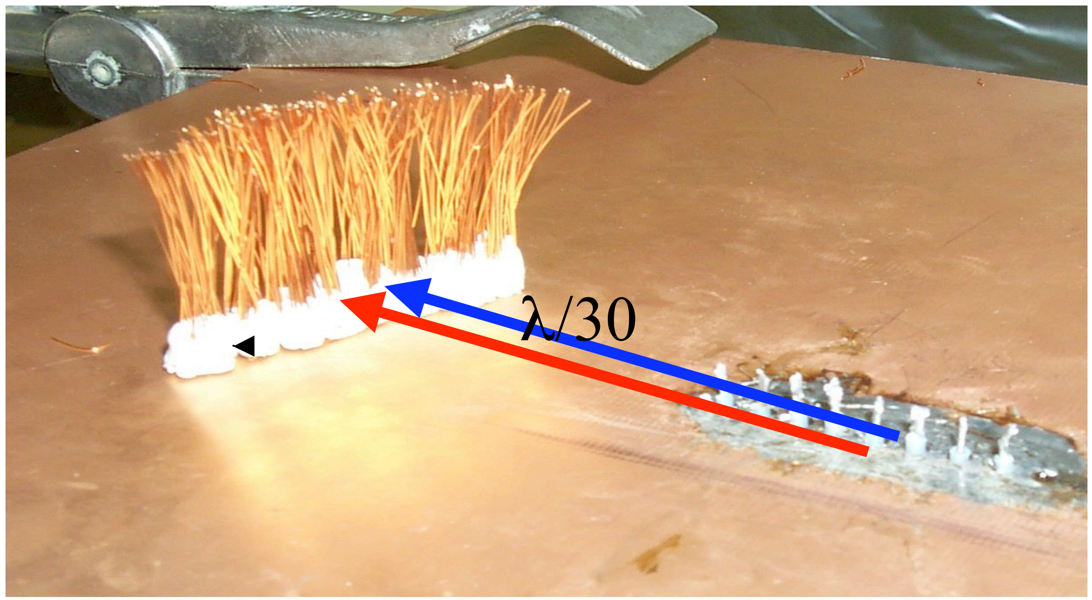
Number of modes

How to create a fast oscillating Im{G} around the source ?

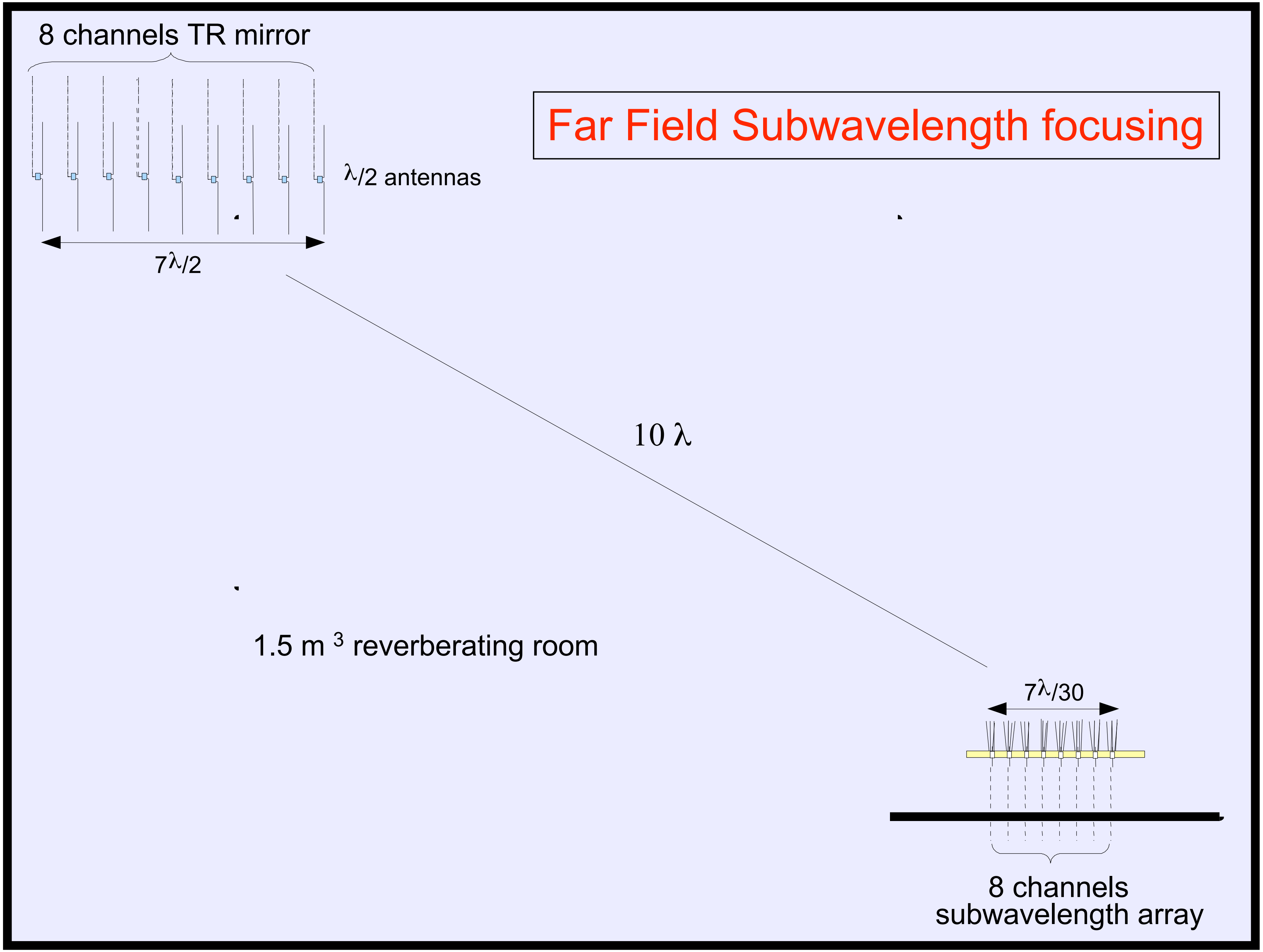
G Lerosey, J de Rosny, A Tourin,M Fink

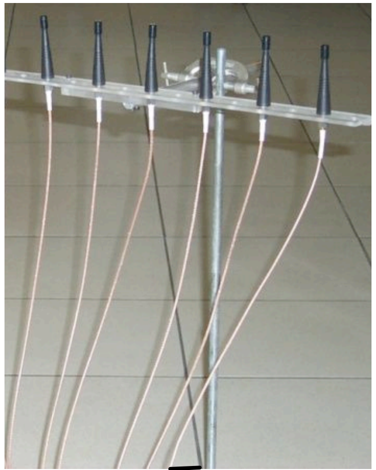
An Electromagnetic Example

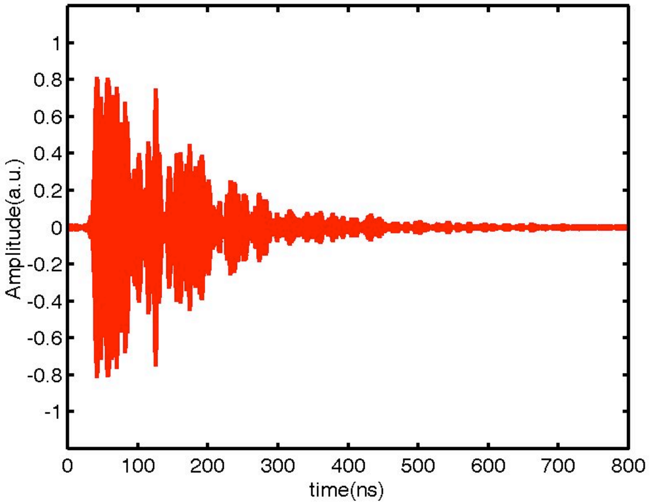


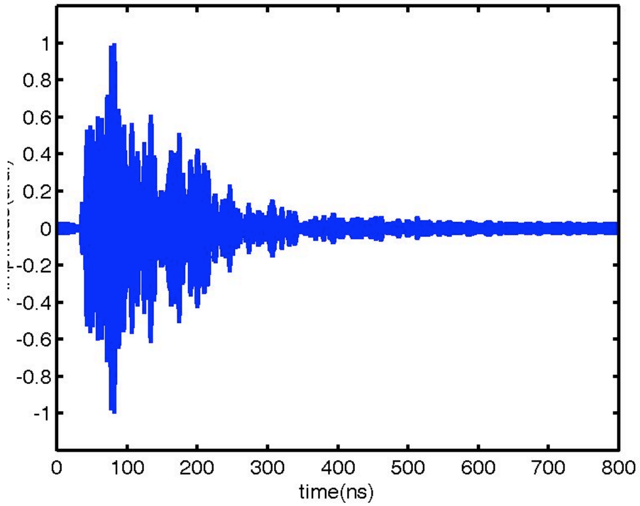


Far Field Subwavelength focusing



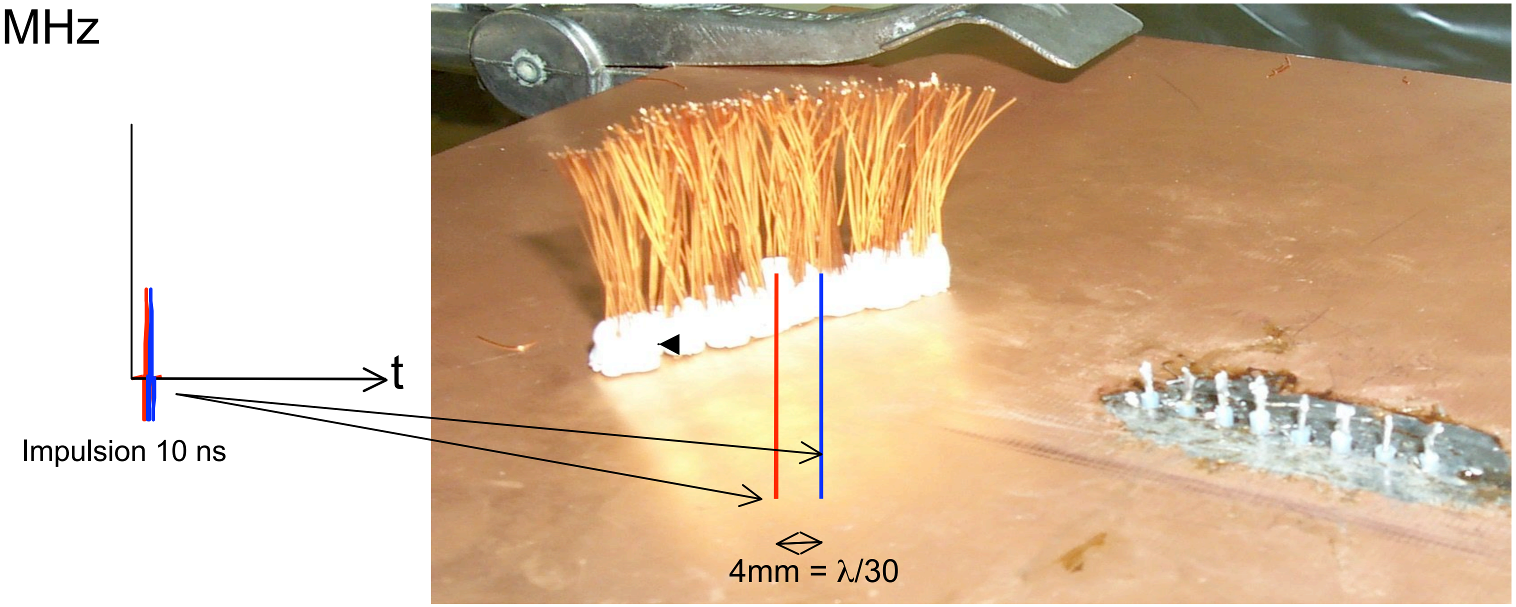






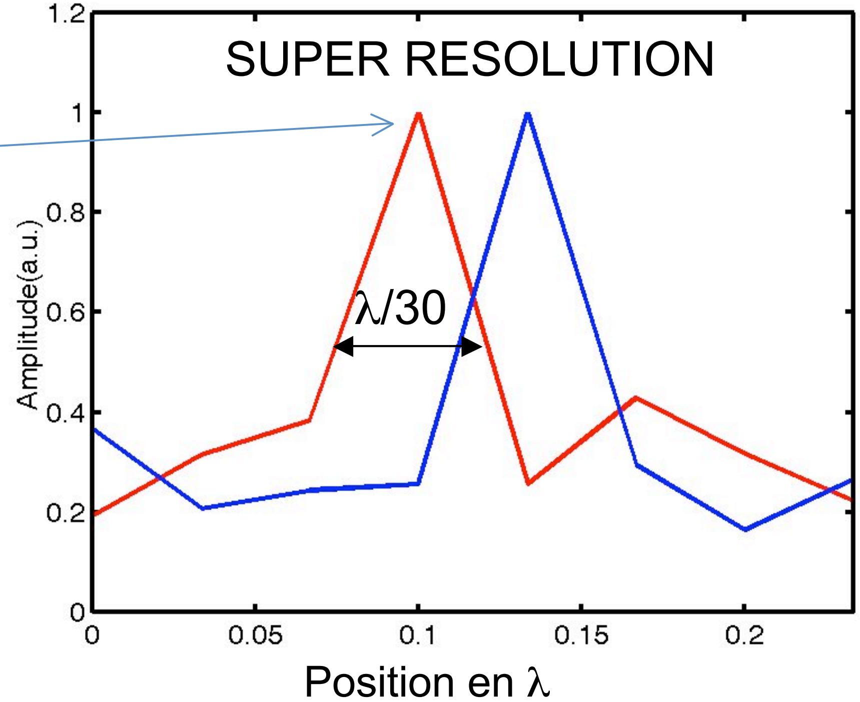
Signal recorded in far field
by
one antenna of the TRM

Bandwidth 100 MHz

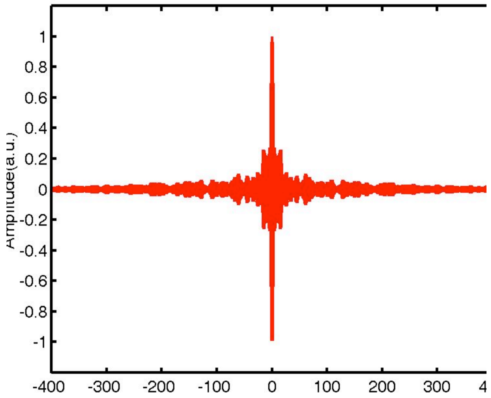


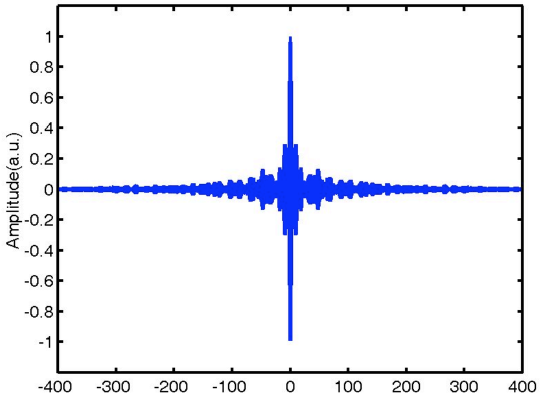
Sub-wavelength resolution with far field time reversal

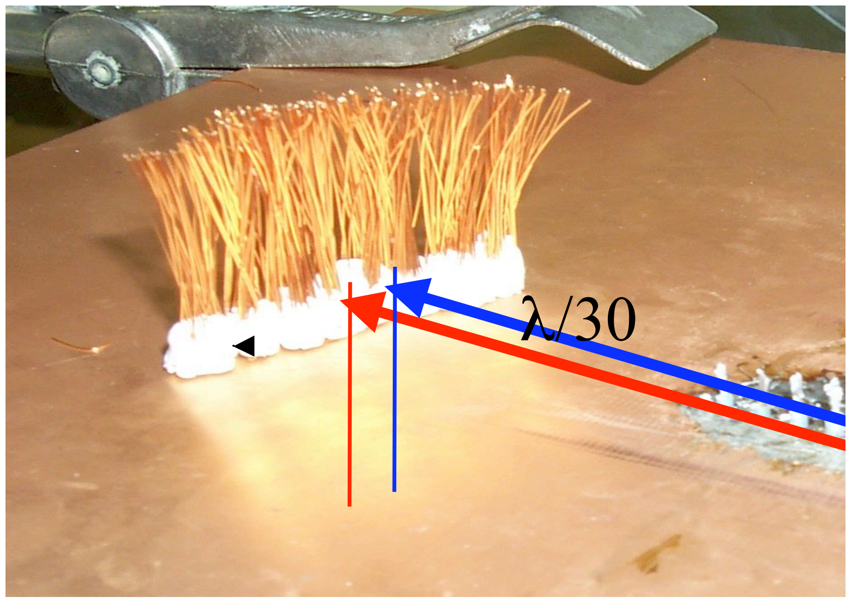
Number of modes



Signals observed on wire red or l
from the time-reversal signals





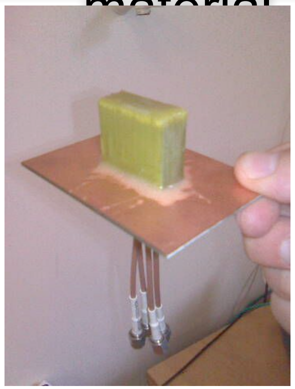


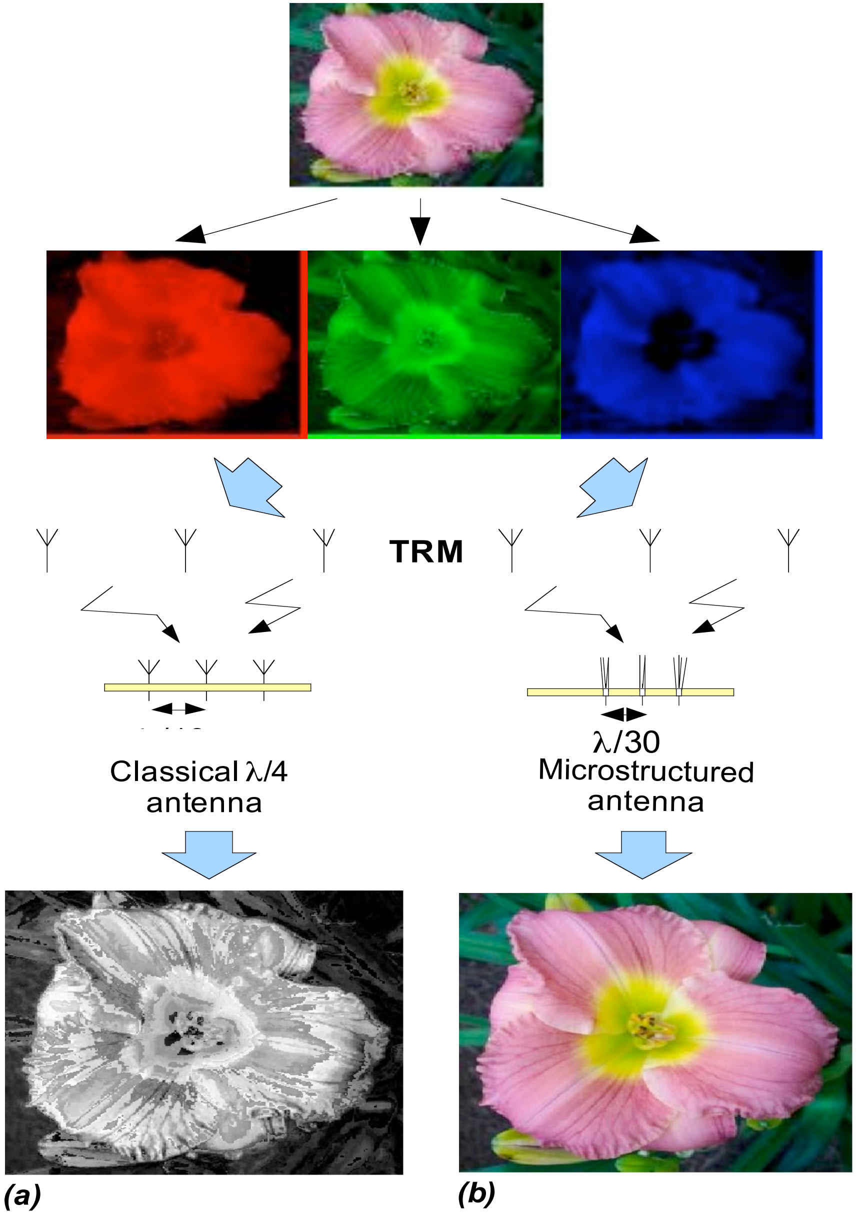
Telecommunications

3 bitstreams (RGB) with a data rate of 50 Mbits/s each.
The intended global data rate is thus 150 Mbits/s.

The TRM is made of 3 antenna
2.45 GHz central frequency
180 MHz bandwidth

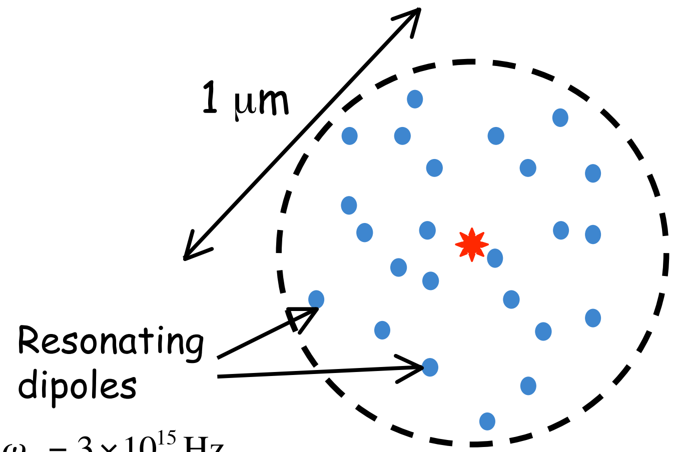
New prototype in PCB





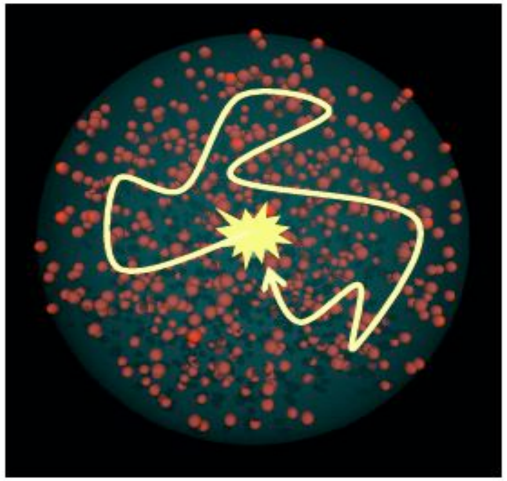
SA Time Reversal

A numerical simulation of a random distribution of resonating dipc



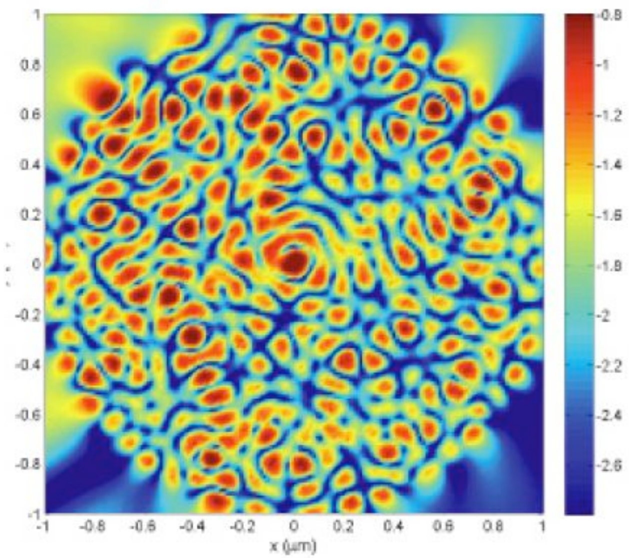
$\omega_0 = 3\times 10^{15}$ Hz

$\Gamma = 0.2\times 10^{15}$ Hz

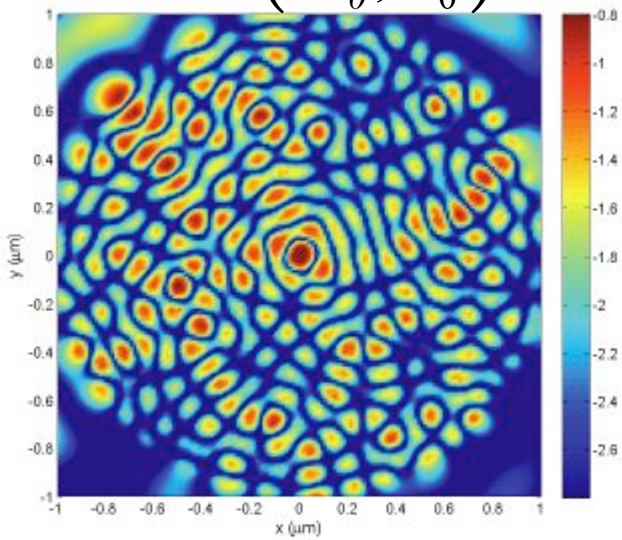


$\vec{r}_0$ can be at a zero LDOS point for ω_0

$\left|\hat{G}\left(\vec{r},\vec{r}_0;\omega_0\right)\right|$



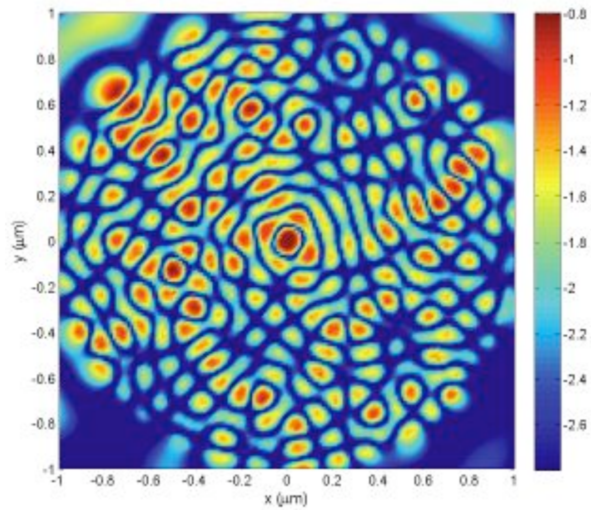
$\mathrm{Im}\,\hat{G}\left(\vec{r},\vec{r}_0;\omega_0\right)$ at one frequency



C. Vandenbem, R. Carminati ,

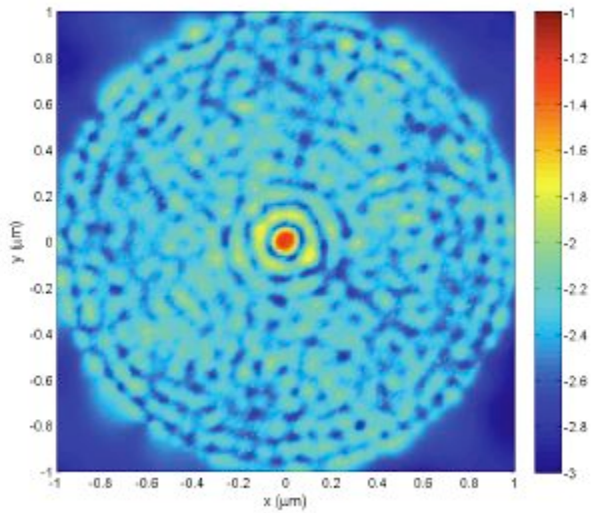
TR field at ω_0

Monochromatic



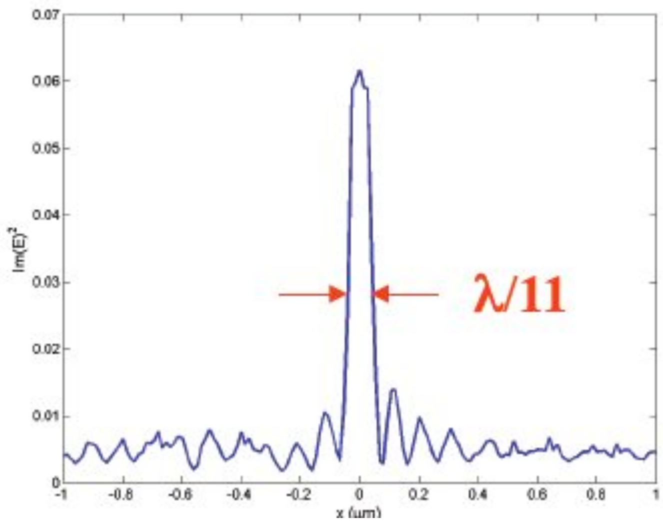
$$\operatorname{Im} G\left(\vec{r},\vec{r}_0;\omega_0\right)$$

$$\Delta\omega = 0.25\times 10^{15}\ \mathrm{Hz}$$



$$\int_{\Delta\omega} \operatorname{Im} G\left(\vec{r},\vec{r}_0;\omega\right)d\omega$$

Broadband time-reversed field at the focal time



Sub-wavelength control of nano-optical fields

Xiangting Li[1,2,*] and Mark I. Stockman[1,†]
PHYSICAL REVIEW B 77, 195109 (2008)

Surface Plasmon modes

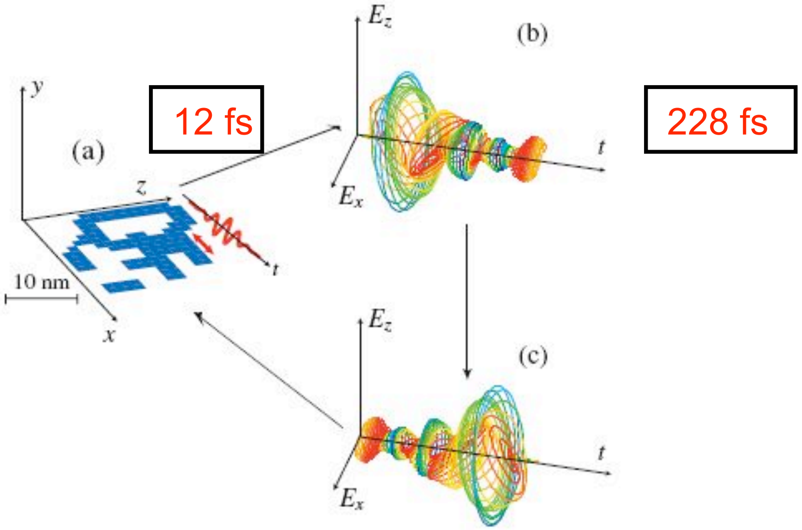
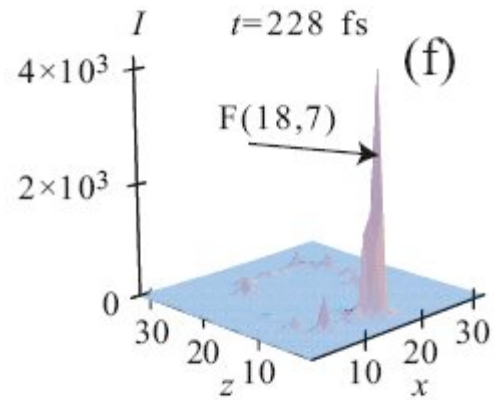
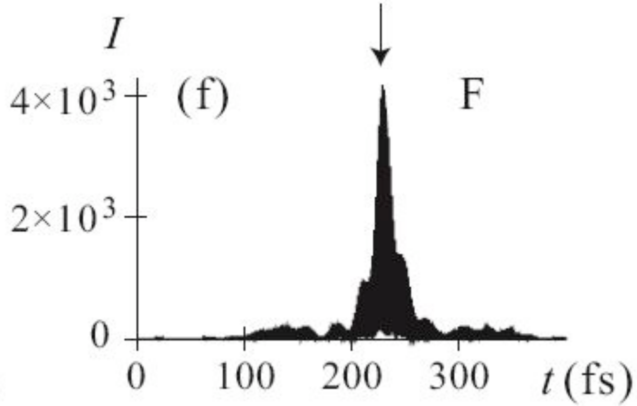


Figure 3. (a) Geometry of nanosystem, initial excitation dipole and its oscillation waveform. The nanosystem as a thin nanostructured silver film is depicted in blue. A position of the oscillating dipole that initially excites the system is indicated by a double red arrow, and its oscillation in time is shown by a bold red waveform. (b) Field in the far-field zone that is generated by the system following the excitation by the local oscillating dipole: vector $\{E_x(t), E_z(t)\}$ is shown as a function of the observation time t. The color corresponds to the instantaneous ellipticity as explained in the text in connection with (c), the same as in (b) but for a time-reversed pulse in the far zone that is used as an excitation pulse to drive the optical energy nanolocalization at the position of the initial dipole.

TR refocusing

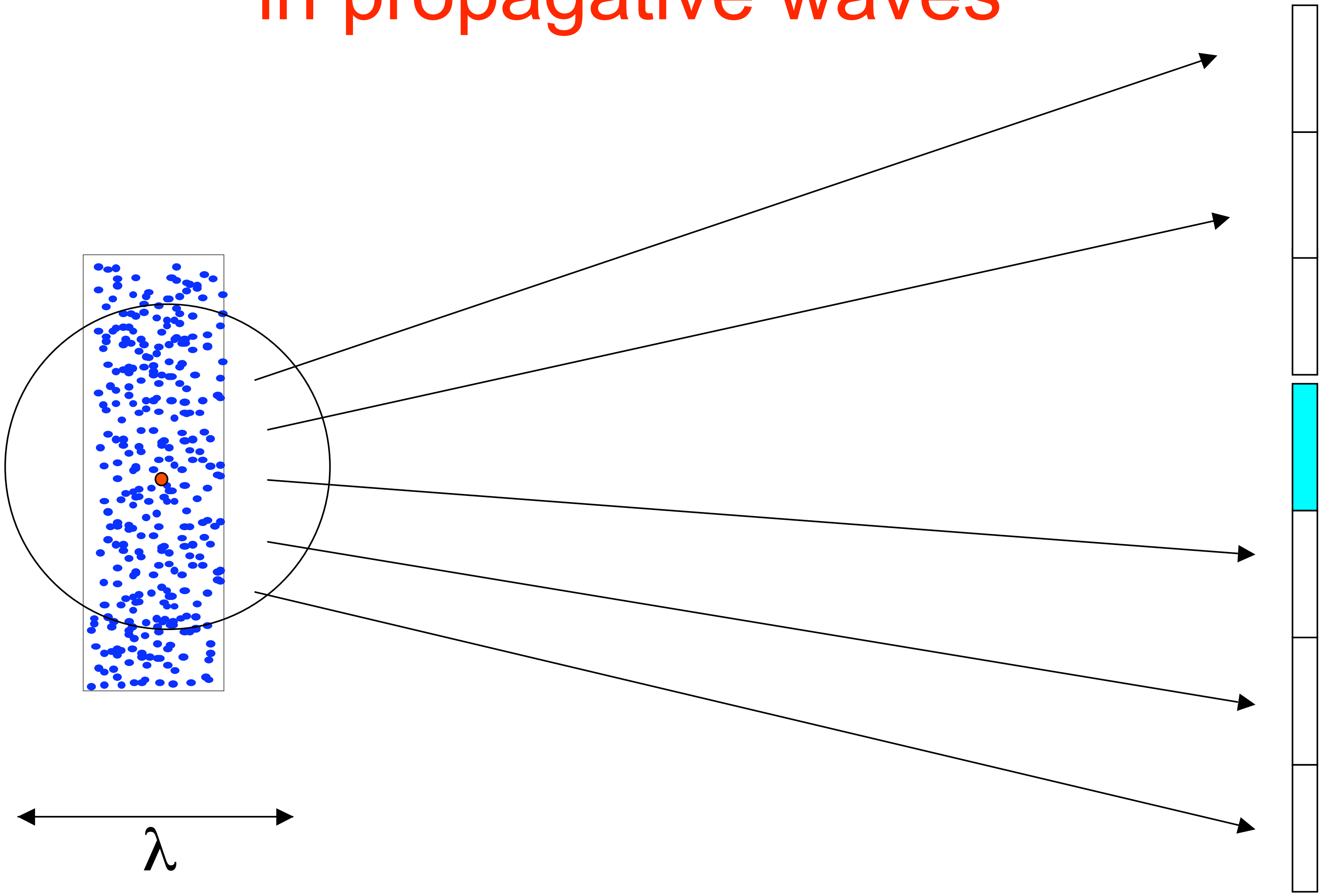


Time compression

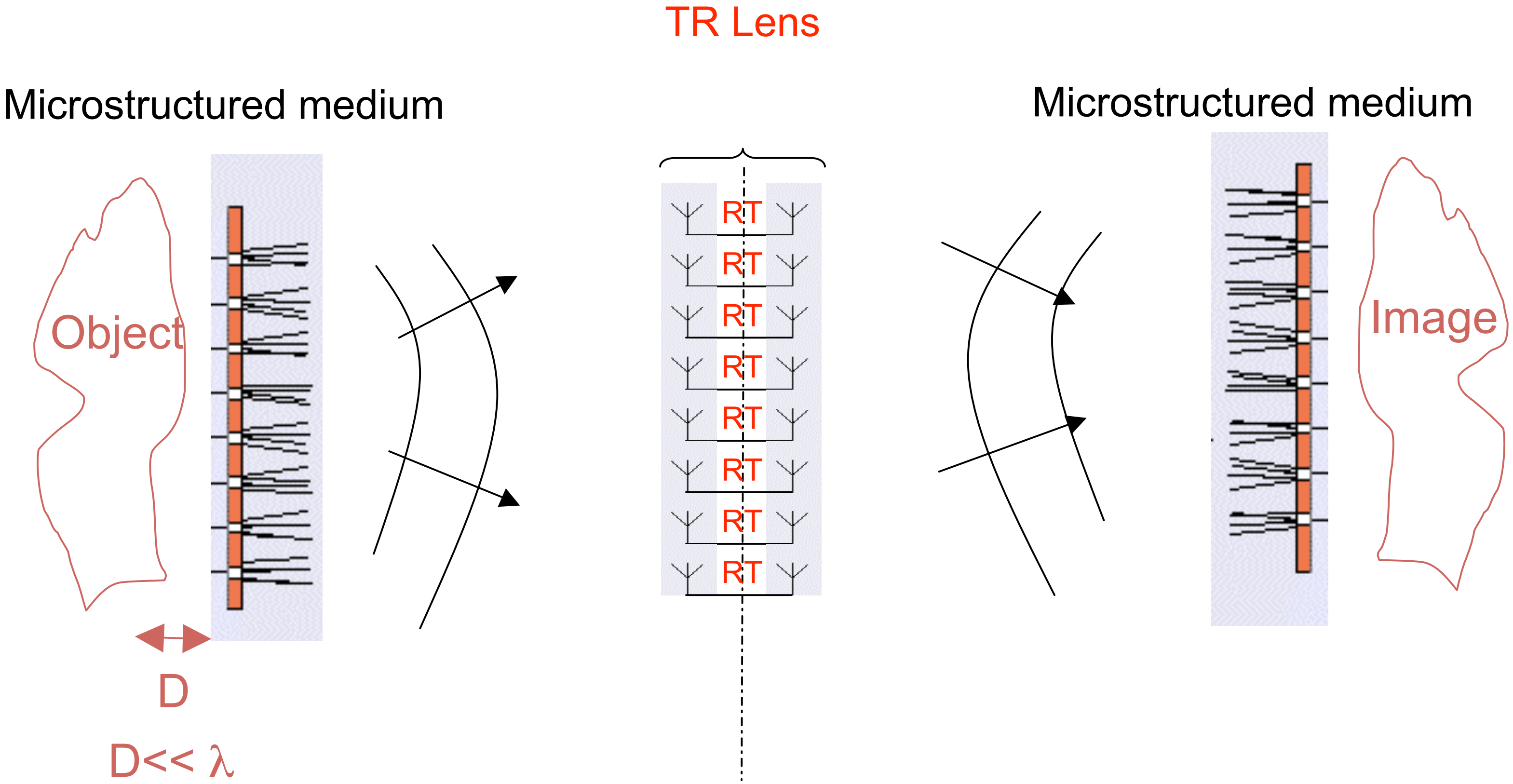


The nanosystem as a thin nanostructured silver film is depicted in blue. A position of the oscillating dipole that initially excites the system is indicated by a double red arrow,

The evanescent field is converted in propagative waves

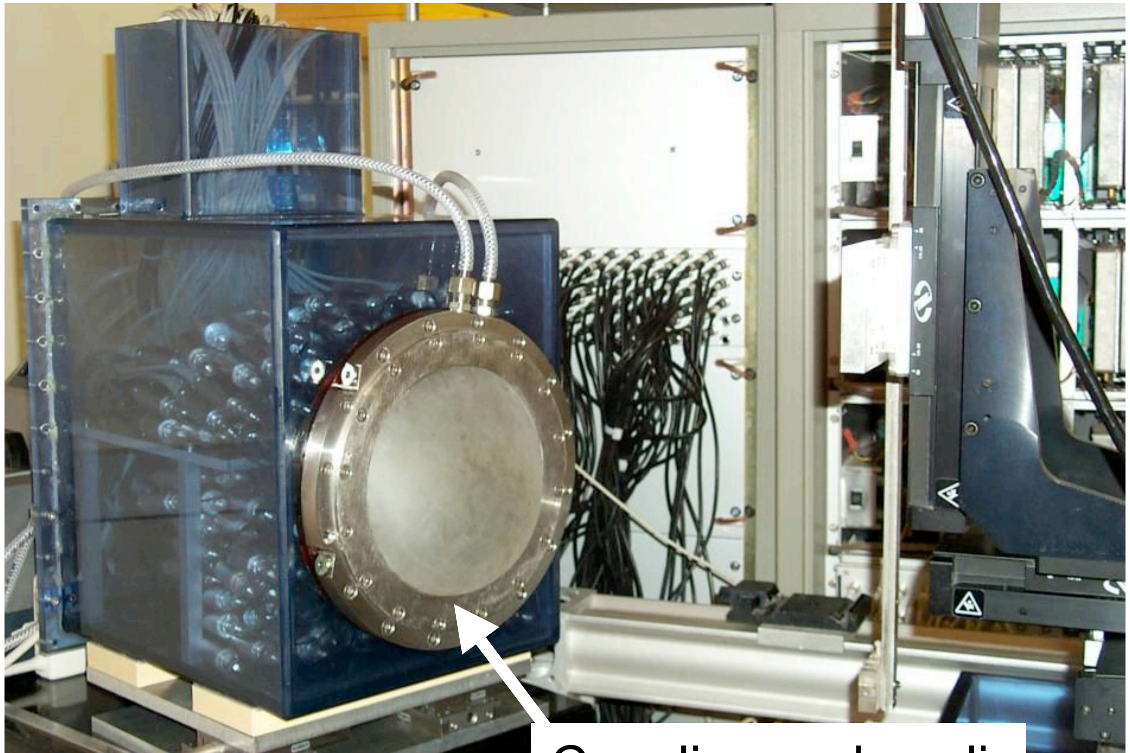


A time-reversal Hyperlens!!!

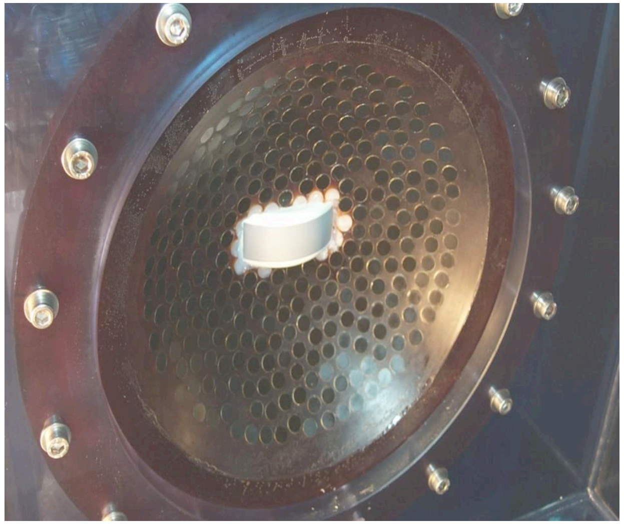


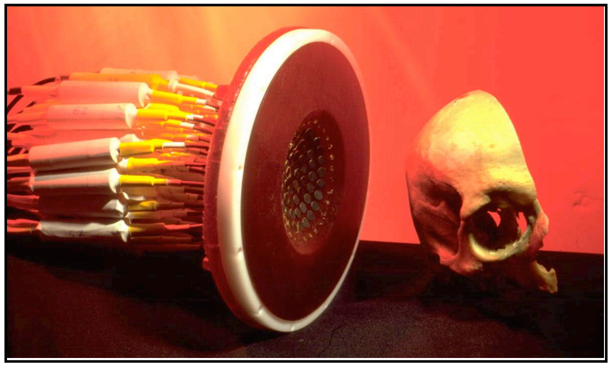
High power time-reversal mirror for ultrasound therapy

With high intensity focused ultrasound (HIFU) in sinusoidal r
(70 bars), a beam transmitted during several seconds induce
temperature increase sufficient to necrose tissue (proteins
coagulation temperature)

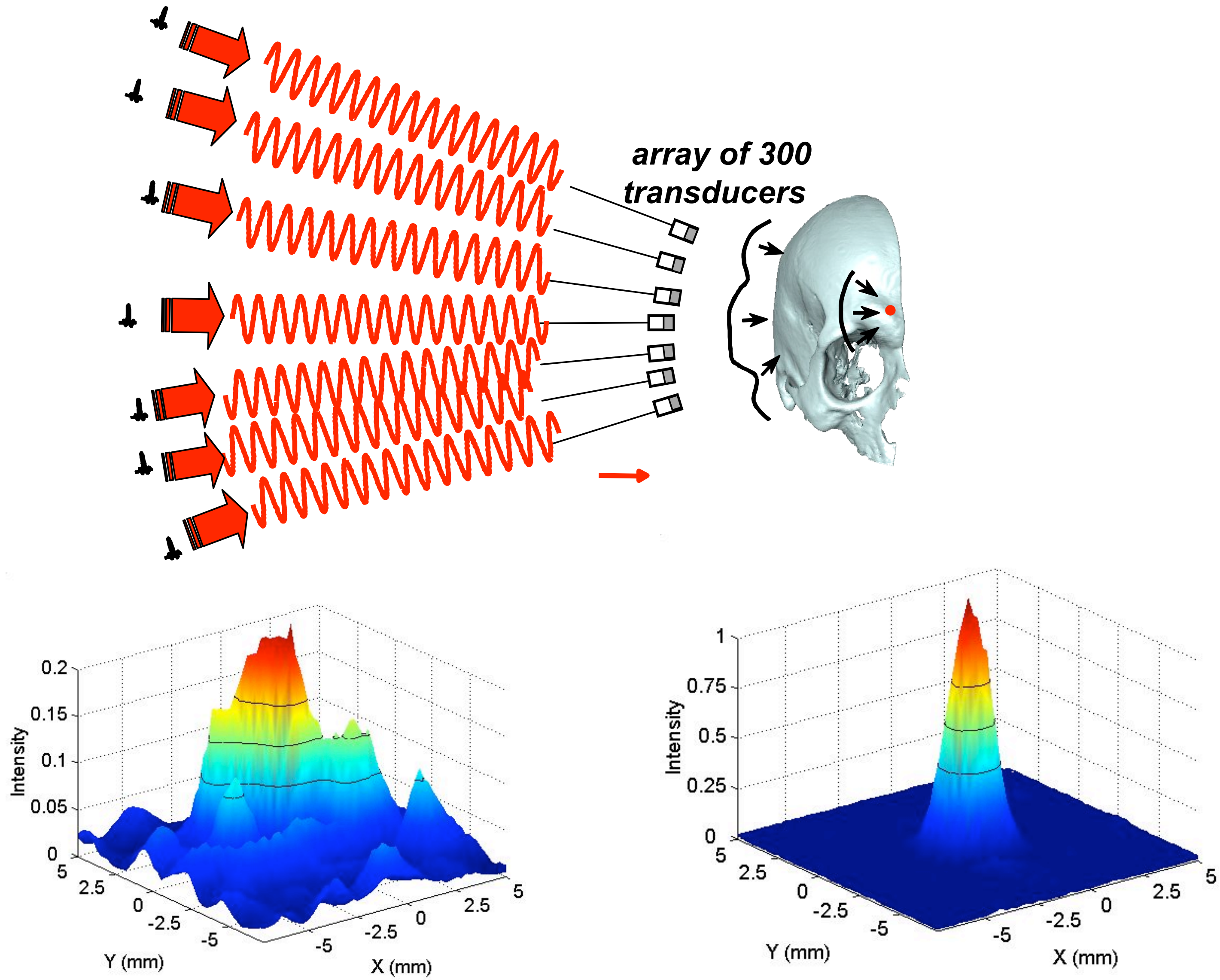


Coupling and cooling system



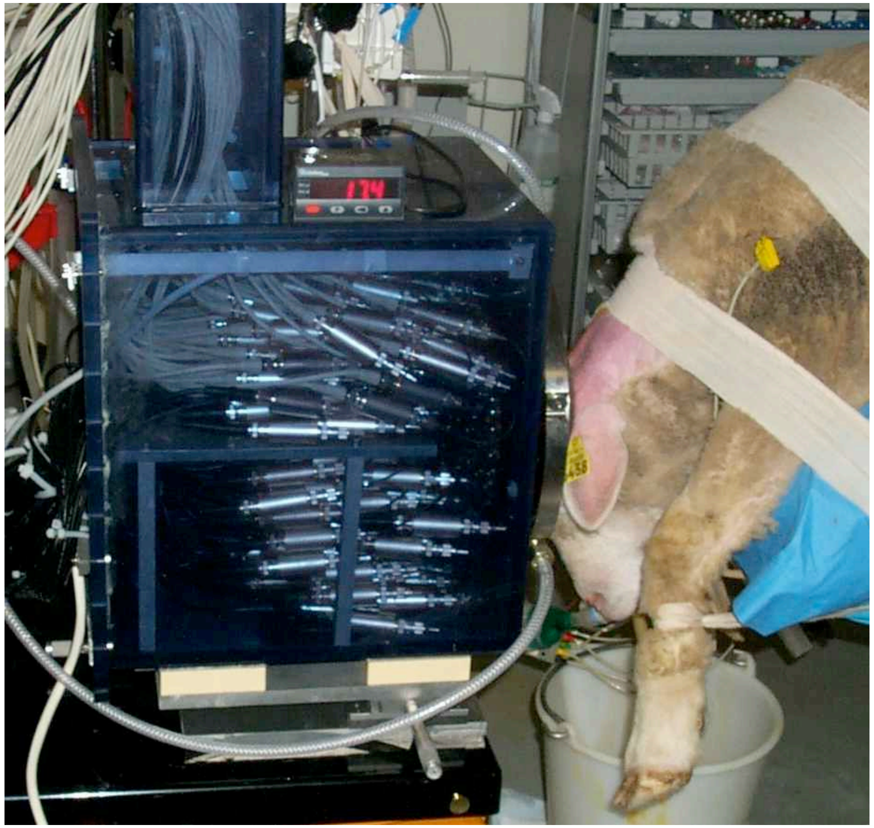


Mickael Tanter, Jean-François Aubry
Mathieu Pernot, Mathias Fink



Acoustic pressure at focal point:
- 70 Bars, 1600 W.cm^{-2} (with TR)
- 15 Bars, 80 W.cm^{-2} (without

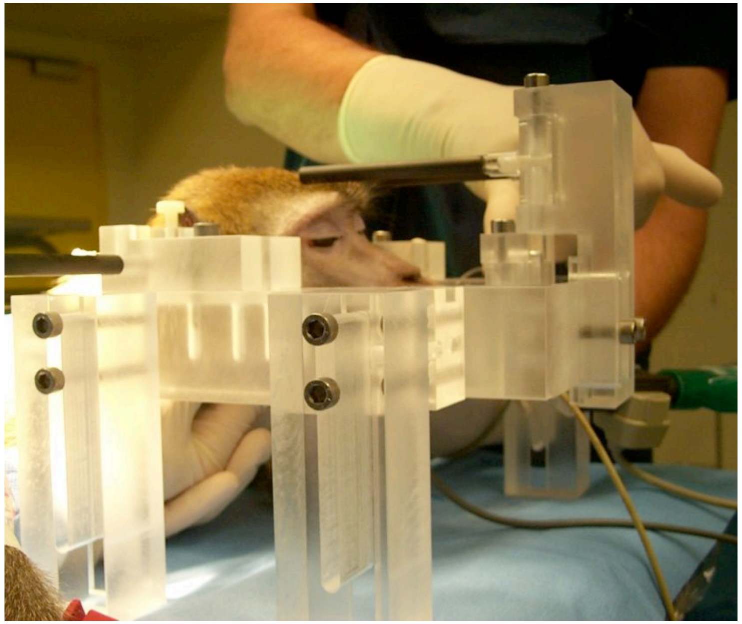
Experiment on sheep, Institut Montsouris



Mickael Tanter, Jean-François Aubry
Anne-Laure Boch, Pitié Salpetrière
Mathieu Pernot, Mathias Fink
Fondation de l'Avenir

Experiment on monkeys, Institut Montsouris

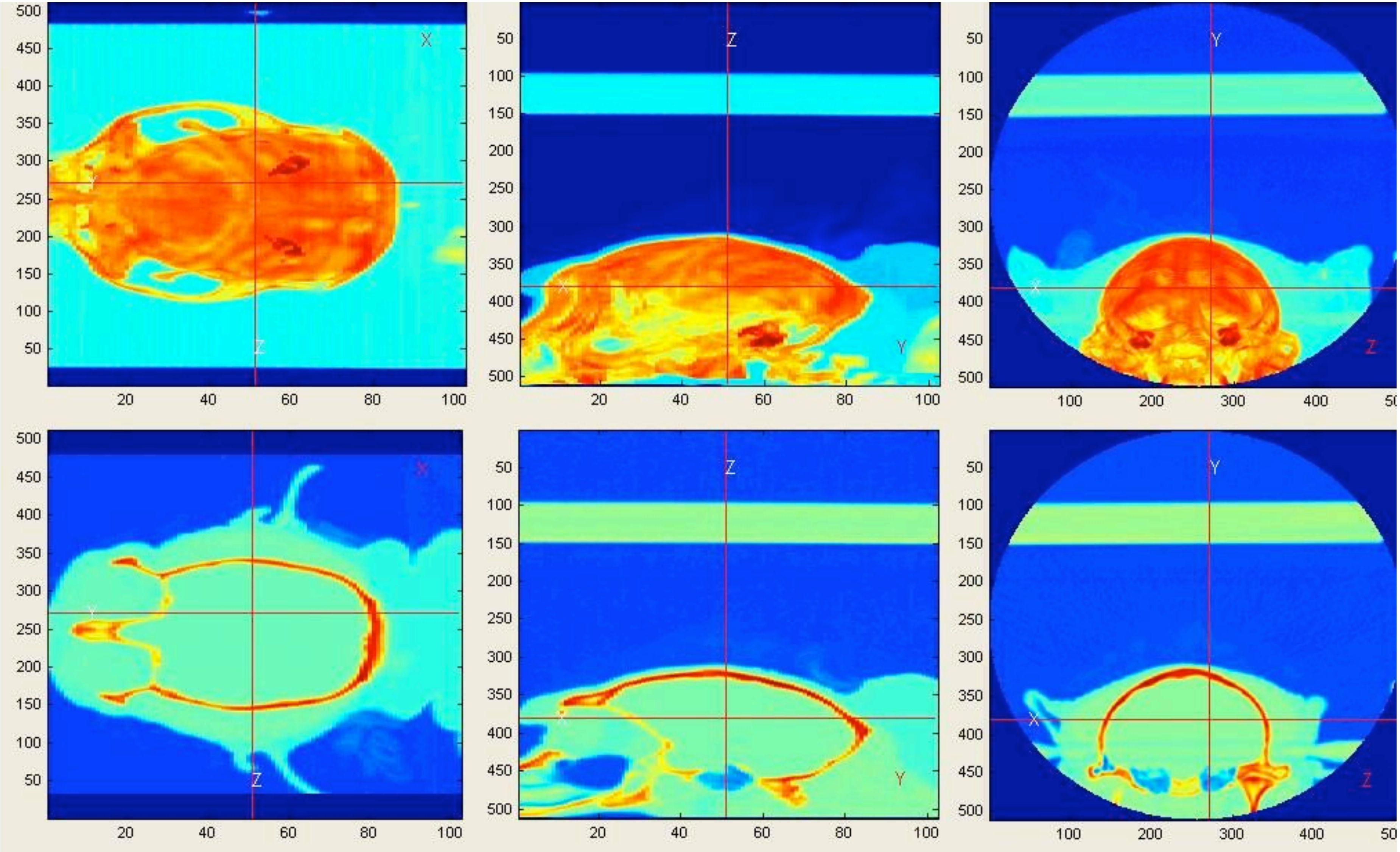
One use a 3D image of the skull obtained with X ray CT to deduce a 3D model of ultrasonic propagation. A numerical simulation of the time reversal from virtual sources is made.



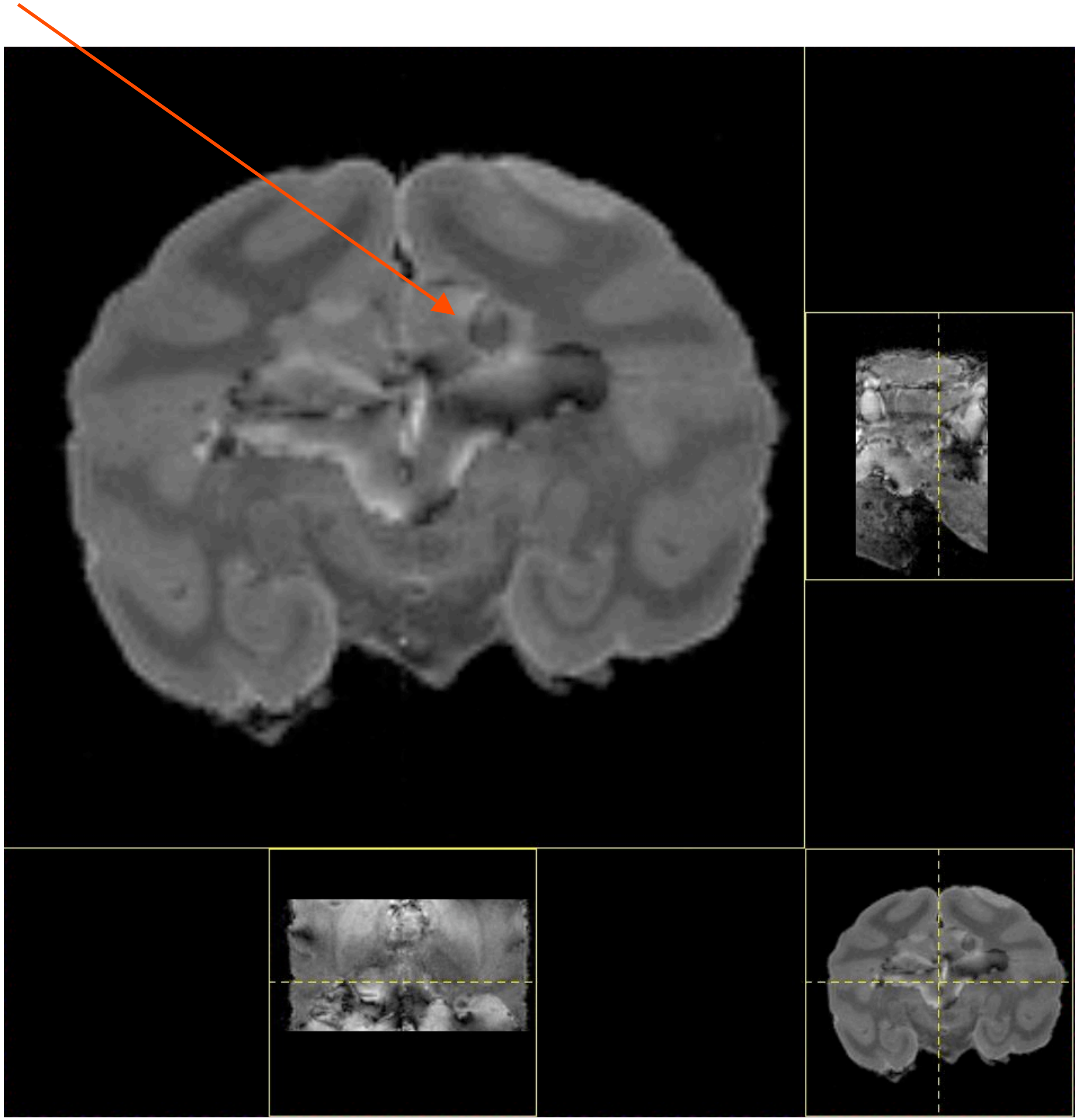
Stereotaxic frame

Monkey located in a CT scan

CT scan image from skull



Thermal necrosis obtained in vivo on monkey



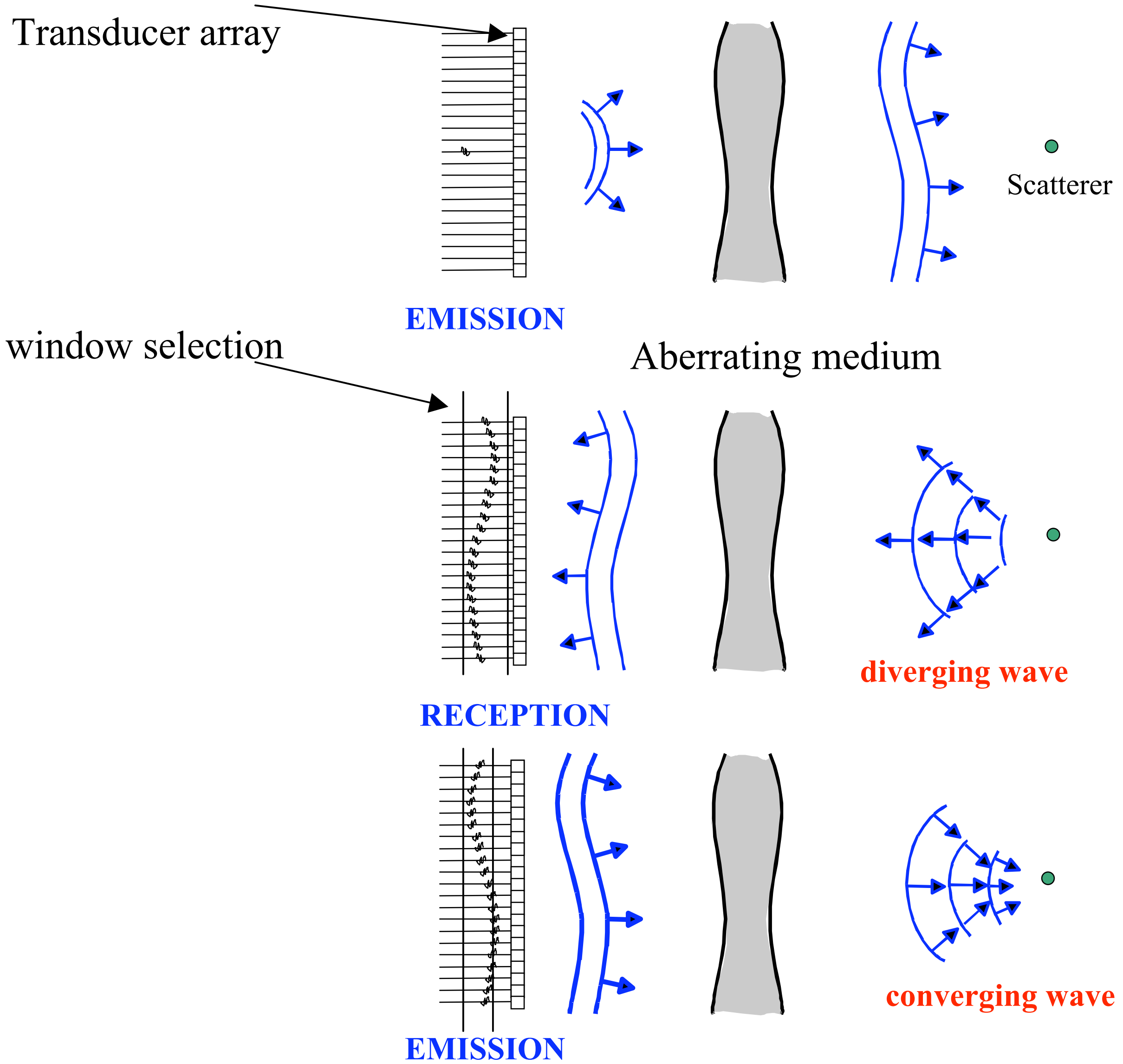
New TRM, MR compatible, Supersonic Imagine

Location at CIERM (Kremlin Bicètre) in April 2009

TRM, 512 éléments

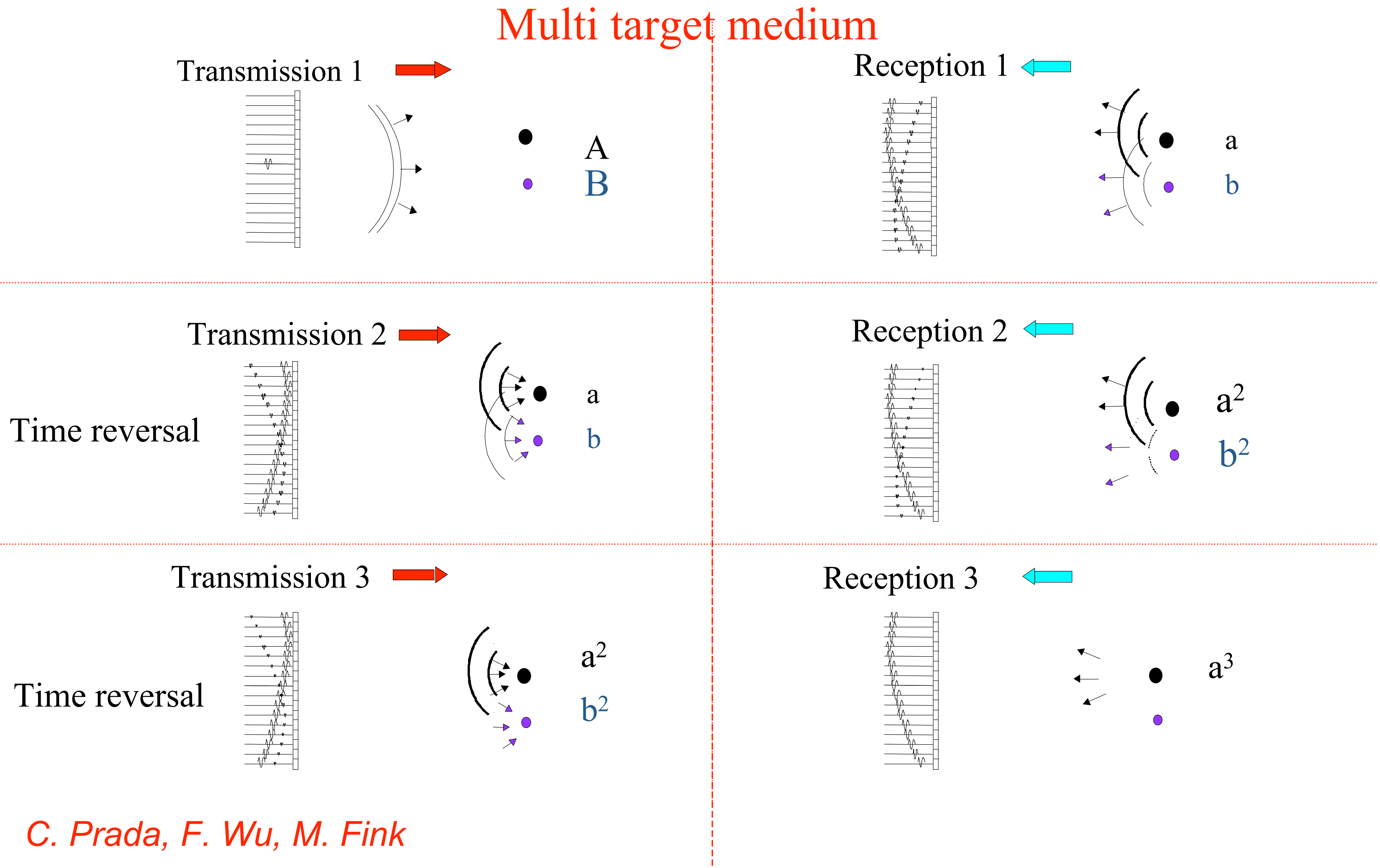
New TRM, MR compatible, Supersonic Imagine

Radar et Sonar à retournement temporel : 1 cible



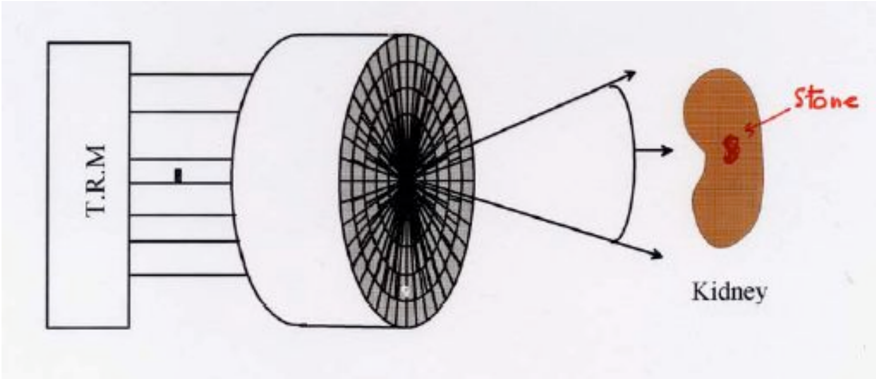
C. Prada, M. Fink, 1988

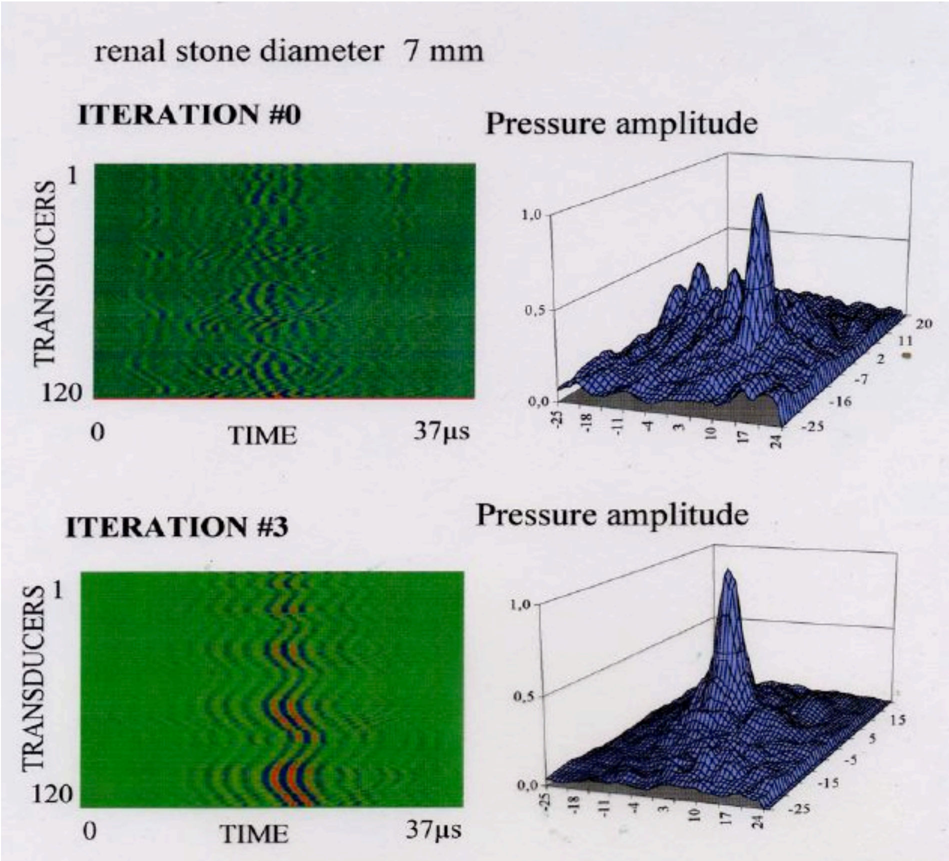
Retournement temporel iteratif : multi-cible



C. Prada, F. Wu, M. Fink

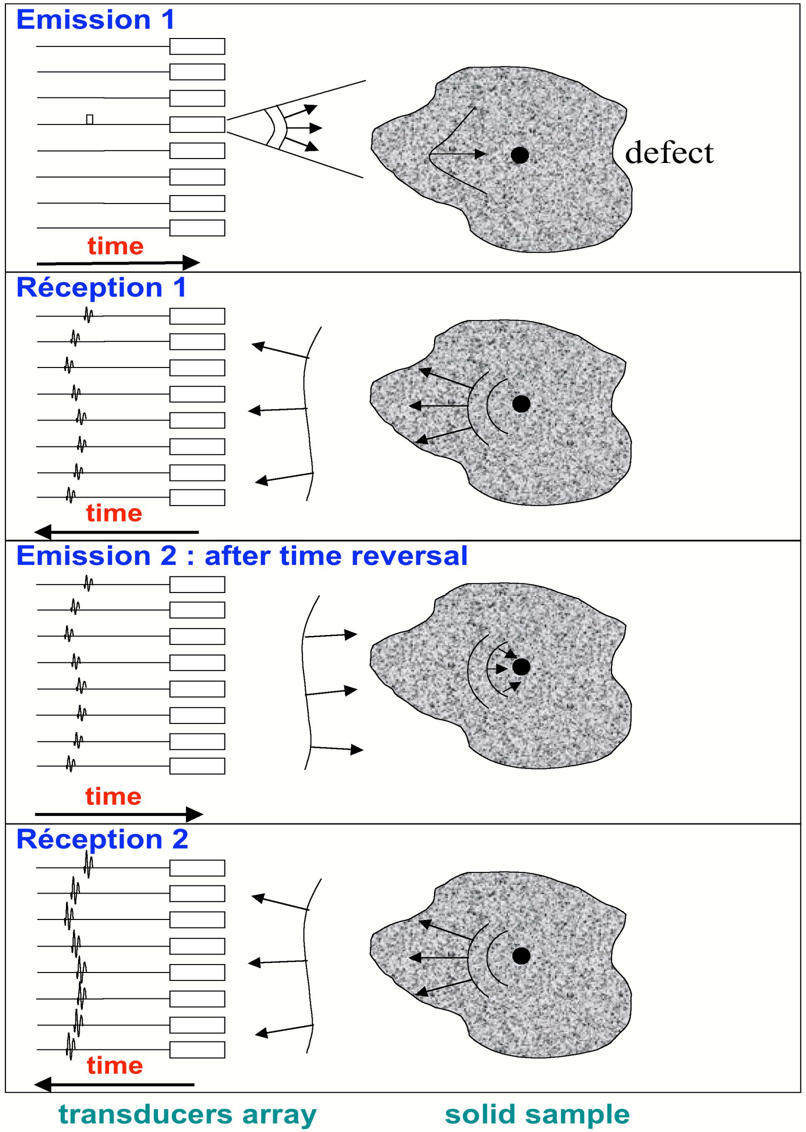
Application of TRM to Lithotripsy



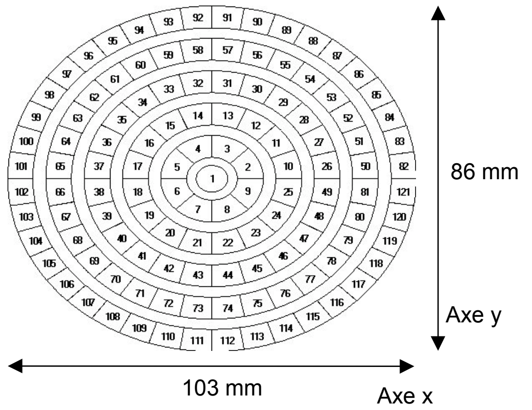


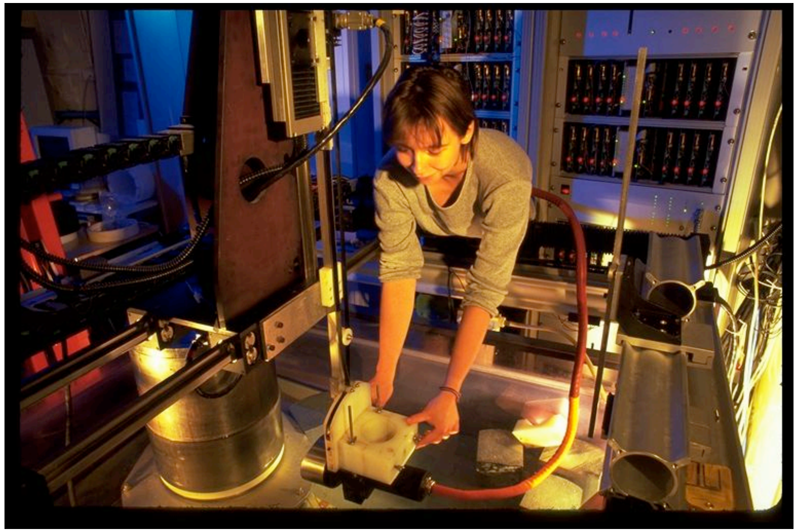
J.L. Thomas, F. Wu, M. Fink

Time Reversal Mirror in non-destructive testing



Applications to defect detection in titanium alloy (SNECMA)

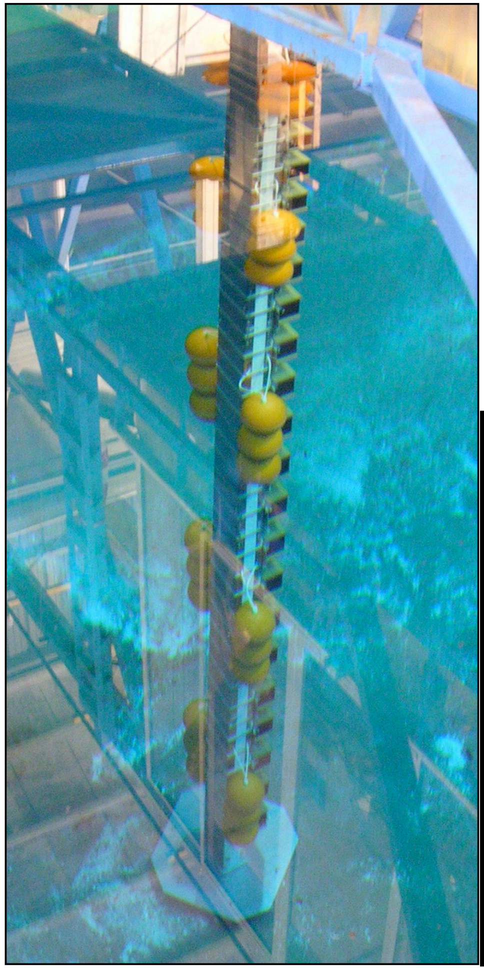




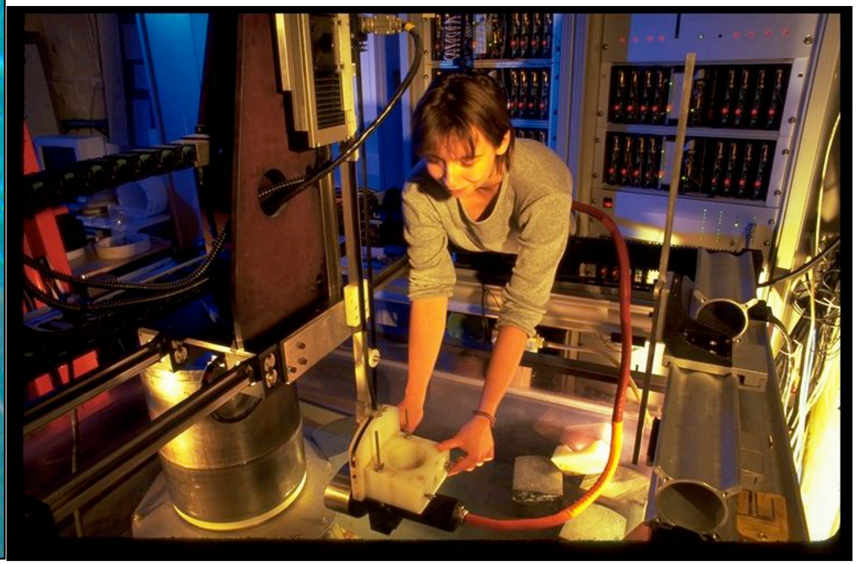
F. Wu, D. Cassereau, N. Chakroun, V. Miette, M. Fink

Sonars à Retournement Temporel

Protection des ports
(DGA, Atlantide)

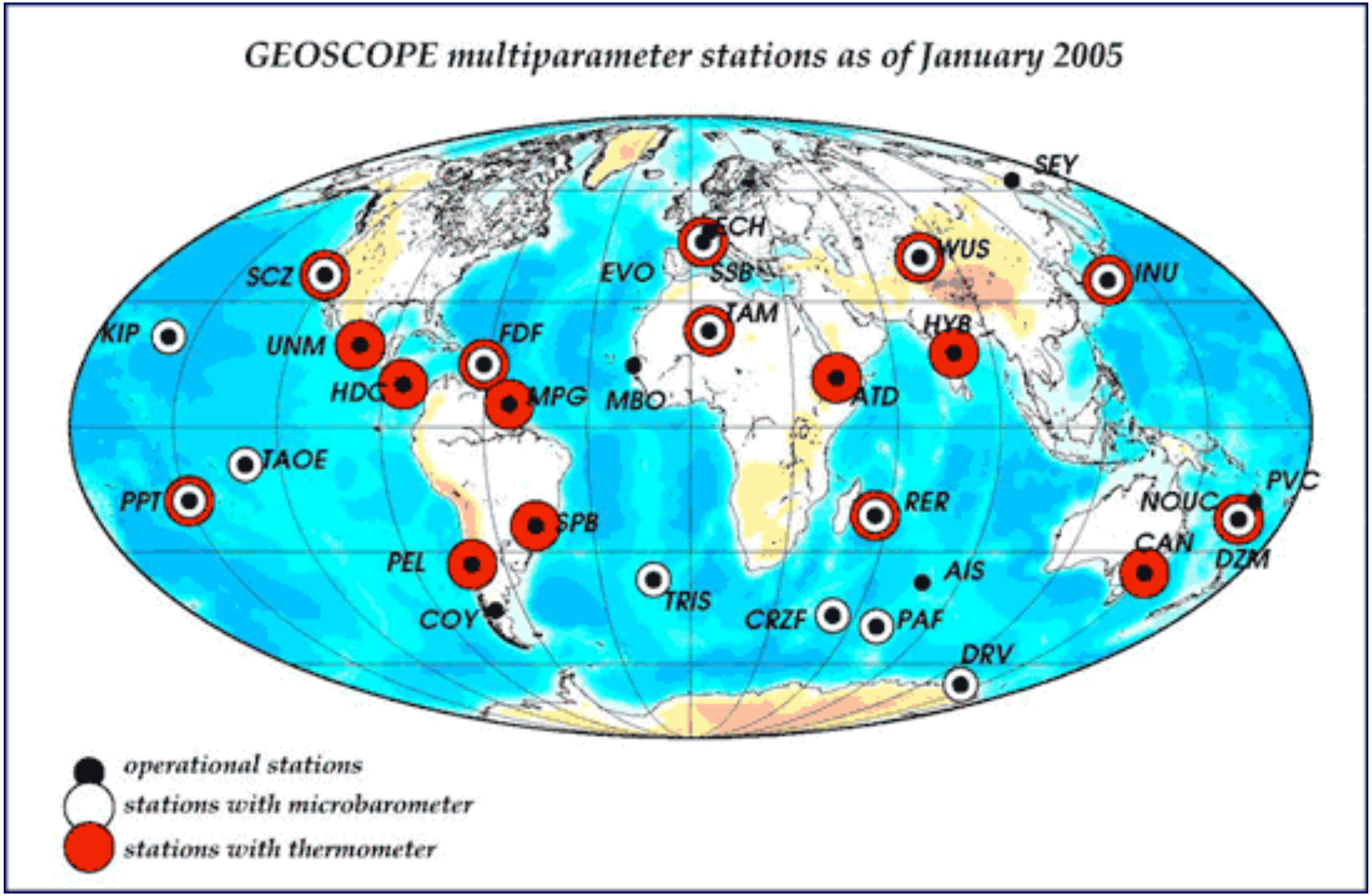


Détection de défauts dans les alliages
de Titane (SNECMA, SAFRAN)



Tracking et
destruction
de calculs rénaux
(TECHNOMED)

TR en Sismologie



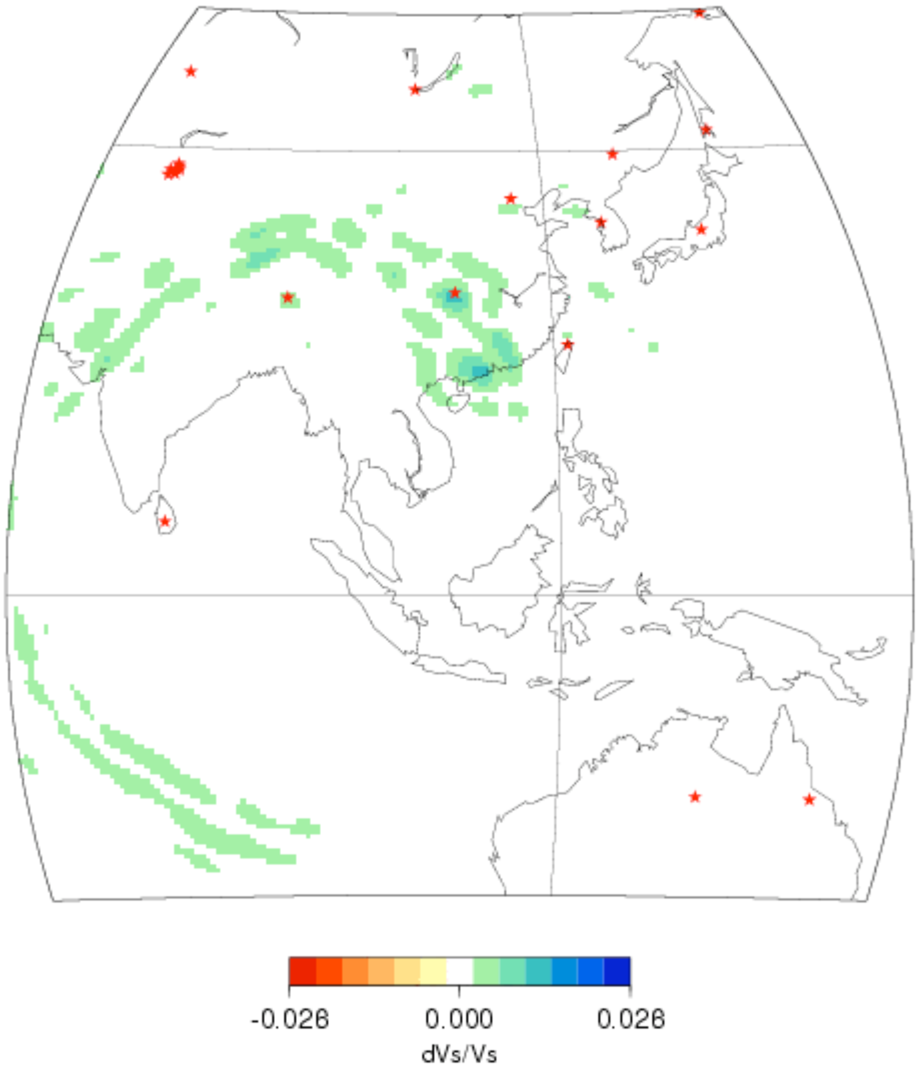
Jean-Paul Montagner, Carene Larmat, IPG, Arnaud Tourin, Mathias Fink

TIME REVERSAL IN SEISMOLOGY

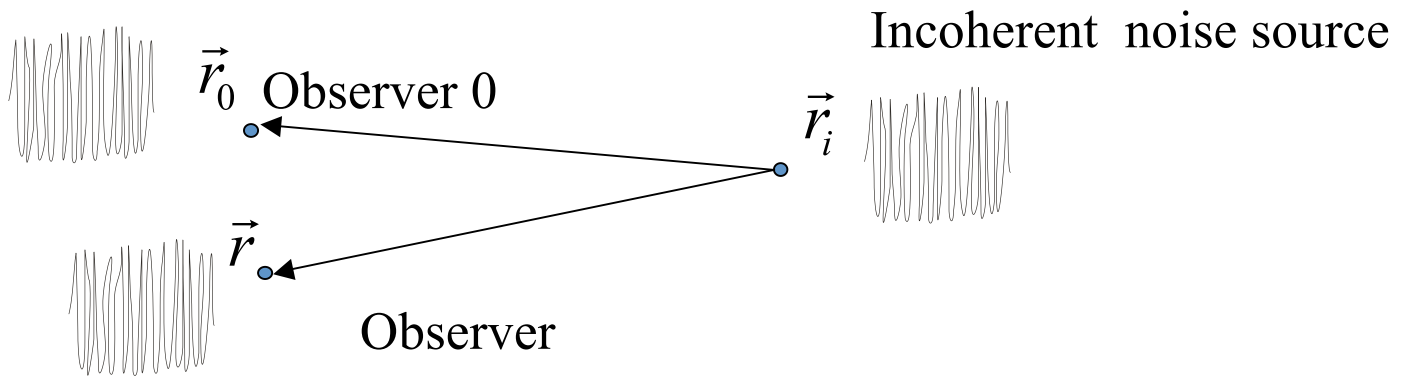
- Application to real seismograms with broadband FDSN stations (165)
- Spatio-temporal Imaging of seismic source
- Detection of unknown seismic sources (“quiet” earthquakes, Seismic “Hum” of the Earth)
- Applications to seismic Tomography- Detection of mantle plumes

Jean-Paul Montagner, Carene Larmat, Arnaud Tourin, Mathias Fink

Time reversal of Sumatra traces



Spatial Correlation of Noise from a Point Source



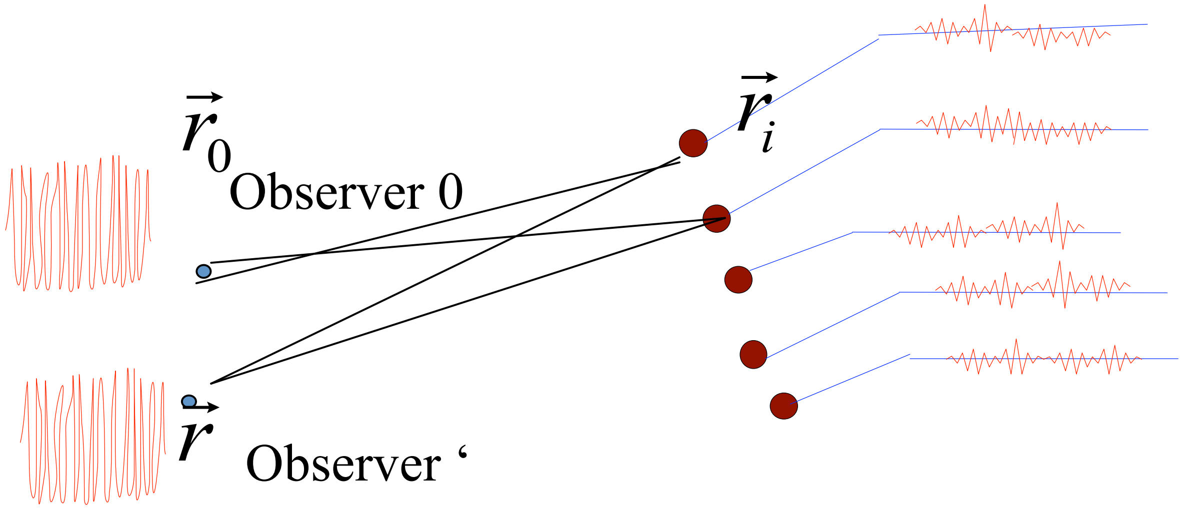
$$Corr\left(\vec{r}_0,\vec{r},t\right) \prec \left\{G\left(\vec{r},\vec{r}_i;t\right)\otimes n\left(t\right)\right\}\otimes\left\{G\left(\vec{r}_0,\vec{r}_i;-t\right)\otimes n\left(-t\right)\right\}$$

If $n\left(t\right)\otimes n\left(-t\right)=\delta\left(t\right)$

The noise correlation recorded by two observers gives, within a time derivative, the same result than a time-reversal experiment conducted with a one channel TRM

Spatially Distributed Source of Noise

with spatial correlation $\langle n(\vec{r},t)\,n(-\vec{r},-t)\rangle = \delta(\vec{r},t)$



$$Corr(\vec{r}_0,\vec{r},t) \prec \iint \left\{G(\vec{r}_0,\vec{r}_i;t)\otimes n(\vec{r}_i,t)\right\}\otimes\left\{G(\vec{r},\vec{r}_i;-t)\otimes n(-\vec{r}_i,-t)\right\} d^2\vec{r}_i$$

$$\frac{\partial}{\partial t} Corr(\vec{r}_0,\vec{r},t) \prec p_{tr}(\vec{r},t) = G(\vec{r},\vec{r}_0;T-t) - G(\vec{r},\vec{r}_0;t-T)$$

"By cross-correlating noise traces recorded at two locations, we can construct the wavefield that would be recorded at one locations if there was a source at the other"

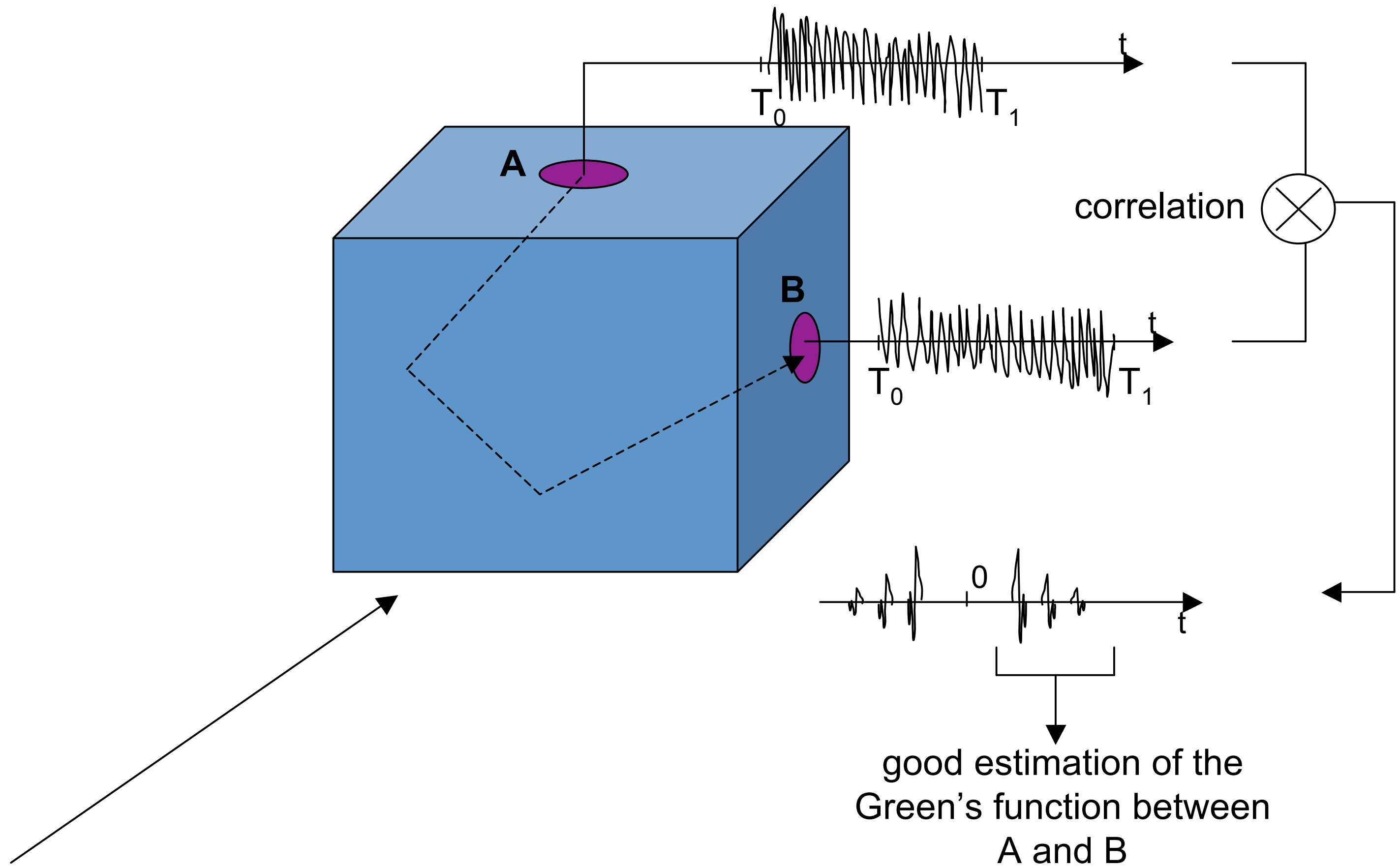
Claerbout 's conjecture

- Helioseismology: (Solar impulse response) (<0.01Hz). *J.Claerbout & J. Rickett, Leading Edge,1999.*
- Geophysics: using coda arrivals or ambient seismic noise (0.1 – 0.3 Hz) . *Campillo & Paul, Science, 2003; Shapiro & Campillo, Geophys. Res. Lett. 2004.*
- Underwater Acoustics (70-130 Hz) *P. Roux and W. Kuperman, ASA 2003*.
- Ultrasonics: with diffuse and thermal noise in cavities (0.1 – 0.9 MHz) *R. Weaver & O. Lobkiss JASA 2001 & 2003.*
- Ultrasonics: in scattering medium with several sources (1MHz). *A. Derode, E. Larose, M. Campillo, M. Fink JASA 2003*,

Coherent signals from noise data

Experimental demonstration in ultrasonics (0.1 – 0.9 MHz)

R.L. Weaver & O.I. Lobkis, Phys. Rev. Lett., 2001



Bruit thermique : phonons