## Quantum Optics with Levitating Diamonds

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## Outline

- Quantum optics with levitating macroscopic particles
- Coupling a single electron spin to the motion of levitating particle
- NV centers in Diamonds
- Towards quantum optical experiments with levitating diamonds


## Quantum optics with macroscopic oscillators



Millions of atoms

## Quantum optics with macroscopic oscillators



$$
\hat{H}=\frac{{\hat{p_{x}}}^{2}}{2 m}+\frac{1}{2} m \omega^{2} \hat{x}^{2}
$$

1D quantum harmonic oscillator

## Quantum optics with macroscopic oscillators


Millions of atoms
Number of phonons
$\hat{H}=\frac{{\hat{p_{x}}}^{2}}{2 m}+\frac{1}{2} m \omega^{2} \hat{x}^{2}$


1D quantum harmonic oscillator

$$
\left\{\begin{array}{l}
\hat{x}=\sqrt{\frac{\hbar}{2 m \omega}}\left(\hat{a}+\hat{a}^{\dagger}\right) \\
\hat{p_{x}}=-i \sqrt{\frac{\hbar}{2 m \omega}}\left(\hat{a}-\hat{a}^{\dagger}\right)
\end{array} \quad\left[\hat{a}, \hat{a}^{\dagger}\right]=1\right.
$$

Introducing ladder operators satisfying bosonic commutation relations

## Quantum optics with macroscopic oscillators



Millions of atoms
Energy in the centre of mass (COM) mode

$\hbar \omega$
$\hat{H}=\frac{{\hat{p_{x}}}^{2}}{2 m}+\frac{1}{2} m \omega^{2} \hat{x}^{2}$
$\sqrt{\left\langle\hat{x}^{2}\right\rangle}=\sqrt{\frac{\hbar}{2 m \omega}}$

## Quantum optics with macroscopic oscillators



Energy in the centre of mass (COM) mode


Ground state cooling of a mechanical oscillator A. D. O'Connell et al. Nature 464, 697-703 (2010)...

## Quantum optics with macroscopic oscillators



Ground state cooling of a mechanical oscillator A. D. O'Connell et al. Nature 464, 697-703 (2010)...

Offers the prospect of creating macroscopic quantum superpositions of the form :

$$
\rightarrow|\psi\rangle=\frac{|n=0\rangle+|n=1\rangle}{\sqrt{2}}
$$

$$
\text { if } k T_{c . m .} \ll \hbar \omega
$$

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Offers the prospect of creating macroscopic quantum superpositions of the form :


## Ultra cold levitating macroscopic objects

Energy in the center of mass (COM) mode


Confining
Potential


## Ultra cold levitating macroscopic objects



- No need to cool the particles themselves (mandatory with clamped oscillators).
- Ground state extension of the COM ~ picometer.

Ashkin A, APL (1976)
Chang D. et al. PNAS (2010)

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State of the art method : Optical trapping The trapped object seeks high intensities (typically 300 mW of laser power with a 1 micron beam waist).


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State of the art method : Optical trapping The trapped object seeks high intensities (typically 300 mW of laser power with a 1 micron beam waist).


Problem: The laser light can hit up the particle and/or make it unstable
L. P. Neukirch, Nat. Phot. (2016)
A. Rahman, Scientific Reports (2016)

## Ultra cold levitating macroscopic objects



Thermalisation :
$\mathrm{T}_{\mathrm{gaz}}=\mathrm{T}_{\text {particle }}$

Atmospheric pressure


Out of equilibrium :
The particle can warm up significantly

High vacuum

## Scattering free trapping

## The Paul trap



Charged trapped particle in an electrodynamical potential

- No laser light
- Large potential depth $\rightarrow$ stays in the trap for days
- Single ion experiments showed control of the motion at the quantum level
A. Kulicke et al. APL (2014)
J. Millen, et al. PRL (2015)


## How to trap a charged particle ?

Static Coulomb force :


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Static Coulomb force :


Restoring force in the $\mathbf{3}$ directions of space :

Confinement along the ring plane.

But no confinement along the $z$ direction anymore :(


## How to trap a charged particle ?

## Other possibility:


Electric
Potential $\mathbf{U}$

In electrostatics, whatever the geometry, at least one direction will not be confining!

Consequence of the conservation of the electric flux on a closed surface


## How to trap a charged particle ?

In electro-statics, one cannot confine a charged particle. Idea : make the electric field oscillate.


One can feel that the electric field has to oscillate more rapidly than the period $T_{s}=2 \pi / \omega_{0}$ in statics.

However, a priori, one cannot see why the force that brings the particle towards the center would compensate the force that pushes it away from it

## How to trap a charged particle ?

$F=-k x \cos (\omega t)$ where $k$ depends on the tension applied to the electrode.
During one cycle of oscillation with a period $\quad T=\frac{2 \pi}{\omega} \ll T_{S}=\sqrt{\frac{m}{k}}$



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Little by little, the particle gets closer to the center...

## The Paul trap



Stable confinement if the trapping frequency is larger than the static curvature

## Linear Paul trap with Lycopodium particles


www.newtonianslabs.com

Loading Coulomb crystals with macroscopic particle

## Experimental set-up

Phase contrast imaging


Injection : Approaching a diamond coated metallic tip to the trap center.


Trap frequency : $\omega_{z} / 2 \pi=1 \mathrm{kHz}$
$\rightarrow$ Charge surface : $5000 e^{-}$

## Experimental set-up

Catching the levitating diamonds with a nano-fiber...

## Quantum control of the motion using a single electron



## Quantum optics with levitating systems

## Cooling

$$
\hat{H}=\hbar \omega_{k}\left(\hat{a}^{\dagger} \hat{a}+\frac{1}{2}\right)
$$

Schrödinger cat states
$|\psi\rangle=\frac{|\mathrm{n}=0\rangle+|\mathrm{n}=1\rangle}{\sqrt{2}}$

Energy in the centre
of mass (COM) mode

$\rightarrow$ Use a single embedded atom to cool the collective motion of billions of atoms and prepare Schrödinger cat states

## Single electron coupling to the motion

## Read-out and coupling to the center of mass motion



A single embedded spin moving in a magnetic field gradient
$\rightarrow$ Single atom resolution of the motion



Frequency shift proportional to the displacement

## Single electron coupling to the motion

## Read-out and coupling to the rotational mode



Read-out of the angular motion

## Single electron coupling to the motion

## Read-out and coupling to the rotational mode




Magnetic compas

## Single electron coupling to the motion

## Read-out and coupling to the rotational mode




Single atom magnetic compas

## Coupling to the center of mass mode

## COM mode


$H_{\text {int }}=\vec{\mu} \cdot \vec{B}=\lambda_{\text {com }} S_{z}\left(a+a^{\dagger}\right)$
Coupling rate to the the COM mode :

$$
\lambda_{\text {com }}=g_{s} \mu_{B} G_{m} a_{0} \rightarrow a_{0}=\sqrt{\hbar / 2 m \omega_{\text {com }}}
$$

Energy in the centre
of mass (COM) mode


- a= Creation and annihilation operators of the center of mass mode.
- $\quad \mathbf{S z}=$ Pauli operator for the NV electronic spin


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Strong coupling requires magnetic field gradients Gm in the $10^{5}$ to $10^{7} \mathrm{~T} / \mathrm{m}$ range!

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of mass (COM) mode


- a= Creation and annihilation operators of the center of mass mode.
- $\quad$ Sz = Pauli operator for the NV electronic spin
P. Rabl et al. PRB (2009)


## Coupling to the rotational mode

## COM mode


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## Rotational mode



$$
\lambda_{\phi}=g_{s} \mu_{B} B \phi_{0}
$$

where $\phi_{0}=\sqrt{\hbar /\left(2 I_{y} \omega_{\phi}\right)}$ magnetic fields in the mT range
T. Delord et al. ArXiv (2017)

## Towards quantum optical experiments

Ground state cooling


Use a single « atom » to cool the motion of thousands of atoms. $\rightarrow$ Picometer precision of a nanometer sized object.

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Ground state cooling


Use a single « atom » to cool the motion of thousands of atoms.
$\rightarrow$ Picometer precision of a nanometer sized object.
Two requirements :

- Angular stability of the diamond in the trap
- Low vacuum


## Summary

## Trapping



## Paul trap

- No laser light
- Large potential depth
$\rightarrow$ stays in the trap for days
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## Read-out

A single embedded spin moving in a magnetic field gradient
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Frequency shift proportional to the displacement

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Frequency shift proportional to the displacement
$\rightarrow$ NV centres in diamond
P. Rabl et al. PRB (2009)
O. Arcizet et al. Nat. Phys.(2011)

## NV centers in diamond



## NV centers in diamond

Cristalline defect formed by one nitrogen atom ( N ) and one vacancy ( V ) on two adjacent sites of the diamond matrix

$\longrightarrow$ « artificial atom» in diamond
$\longrightarrow$ photoluminescence (PL) perfectly stable at room temperature
$\Rightarrow$ Single NV isolation Gruber et al., Science 276 (1997)
Detection via confocal microscopy


## Spin properties of the NV center

- $S=1$, the fondamental level is a spin triplet



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| Optical |
| :---: |
| pumping in the |
| state $\left\|0_{e}\right\rangle$ |
| Spin |
| initialisation |


$\Rightarrow$| The PL level depends |
| :---: |
| upon spin state |
| $\left\|0_{e}\right\rangle \longrightarrow$ «bright» state |
| $\left\| \pm 1_{e}\right\rangle \rightarrow$ dark state |
| $\Longrightarrow$ Optical read-out of |
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Optical
pumping in the
state $\left|0_{e}\right\rangle$
$\Rightarrow$ Spin
initialisation

The PL level depends upon spin state
$\left|0_{e}\right\rangle \longrightarrow$ «bright» state
$\left| \pm 1_{e}\right\rangle \rightarrow$ dark state
$\Longrightarrow$ Optical read-out of spin state

Optical detection of spin magnetic resonance


## Applications of the NV center

Room temperature read out of a single electronic spin in diamond


At the core of many applications in quantum physics

## Applications of the NV center

Sensing magnetic fields with $n m$ resolution


Single nuclear spins


Ferromagnetic vorteces

## Applications of the NV center

Sensing magnetic fields with $n m$ resolution


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## Applications of the NV center

Sensing magnetic fields with nm resolution


Single nuclear spins


Ferromagnetic vorteces


Quantum information


## Applications of the NV center

Sensing magnetic fields with nm resolution


Single nuclear spins
$D(T)$
Nanoscale thermometer


Ferromagnetic vorteces

Quantum information


And for studying levitating quantum systems...

## Experimental set-up



## ESR with NVs in levitating diamonds

Resonance
No magnetic field
ESR contrast comparable to the ESR with deposited diamonds

Electron

## ESR with NVs in levitating diamonds

Resonance
No magnetic field
ESR contrast comparable to the ESR with deposited diamonds

With a magnetic field
4 possible orientations


Electron Spin

## ESR with NVs in levitating diamonds

Resonance
No magnetic field
ESR contrast comparable to the ESR with deposited diamonds

With a magnetic field
4 possible orientations




Angular stability !

## Deterministic angle change

ESR with $\mathbf{1 0}$ microns diameters diamonds



## Deterministic angle change

ESR with $\mathbf{1 0}$ microns diameters diamonds




## Deterministic angle change

An asymetrical particle enables locking the rotation along the most confining trap axis

## Angular stability

MEB image of microdiamonds on the tungsten tip


Highly asymmetric particles

## Angular stability

Newton's law for an ellipsoidal particule :

$$
\ddot{\phi}-\sqrt{2} \omega_{\phi} \Omega \cos (\Omega t) \frac{\sin (2 \phi)}{2}=0
$$

In the small angle limit $\rightarrow$ Matthieu equation.

Angular stability with harmonic confinement at the frequency :

$$
\begin{aligned}
& \omega_{\phi}=\omega_{z} m S_{I} / I_{y y} \longleftarrow \begin{array}{c}
\text { Inertia } \\
\text { momentum } \\
\text { Along } y
\end{array} \\
& \mathrm{~S}_{\mathrm{I}}=\frac{3}{\mathrm{~S}} \iint\left(\mathrm{z}^{2}-\mathrm{x}^{2}\right) \mathrm{dS}
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$$

Energy in the rotational mode


## Towards quantum optical experiments with macroscopic oscillators

Quantum jump of a single photon!


Watching the life and death of a photon Gleize et al. Nature (2007) - S. Haroche's group

## Towards quantum optical experiments

Ground state cooling


Use a single « atom » to cool the motion of thousands of atoms.
$\rightarrow$ Picometer precision of a nanometer sized object.

Final phonon number limited by the collision rate $1 / \tau_{\text {coll }}$ with surrounding gaz particles. One needs $\lambda_{\phi} \gg 1 / \tau_{\text {coll }}$

## Levitating diamonds under vacuum



## Diamonds levitating under vacuum

$$
\mathrm{P}=1 \text { bar }
$$



## Diamonds levitating under vacuum

$$
P=0.1 \text { mbar }
$$



## Diamonds levitating under vacuum

$$
P=0.1 \mathrm{mbar}
$$



The diamond heats up !

## NV thermometry



Thermalisation :
$\mathrm{T}_{\mathrm{gaz}}=\mathrm{T}_{\text {particle }}$


Out of equilibrium :
The particle can warm up significantly

## NV thermometry



- Significant diamond heating at 0.1 mbar
- Depends linearly on the green laser power


## NV thermometry



- Heating depends on the gaz pressure
- At 0.01 mbars, the diamond escapes from the trap...


## NV thermometry



Solution : use
ultra-pur diamonds
(CVD grown)


## Conclusions / Perspectives

## Conclusion :

- We observe efficient driving of NV centers in a diamond levitating in a Paul trap.
- The spin properties of deposited diamond particles are retained.
- We observed angle stability of single trapped monocrystals $\rightarrow$ Necessary step towards spin-controlled levitating macroscopic objects.
- NV spin enables reading locally the temperature of levitating objects


## Perspectives :

- Increase the frequency $\rightarrow$ UV light, electron gun to increase the charge surface
- Ground state cooling of a massive object using a single electron
- Quantum non-demolition read-out of the collective modes


## Conclusion

## Collaborations :

J.-F. Roch, François Treussard, Loic Rondin (LAC, Paris)
V. Jacques (L2C, Montpellier)
L. Guidoni
A. Tallaire (LSPM- Villetaneuse)
P. Maletinsky (Basel)
C. Becher(Saarbrücken)

Optics team at LPA :


Team : Baptiste Vindolet, Tom Delord, Lucien Schwab, Martina Bodini, Louis Nicolas


Laboratoire Pierre Aigrain

## Aim 3: Entangle the motion of distant macroscopic objects



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Methodology :
$\rightarrow$ Entangle the spins from distant NV centres in diamonds using single photon scattering
$\rightarrow$ Transfer spin entanglement to motional entanglement


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$\rightarrow$ Entangle the spins from distant NV centres in diamonds using single photon scattering
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- Long lived entangled state $\rightarrow$ quantum memory
- Quantum information
- Sensitive detection of gravitational effects


## Optical spectra



NV spectra from deposited nanodiamonds on a quartz coverslip

## Optical spectra



NV spectra from deposited nanodiamonds on a quartz coverslip


NV spectra from levitating nanodiamonds

No apparent change in the photophysical properties

## 2D trap



## The electrodynamical trap



General law of electrostatics
Whatever the geometry, at least one direction will not be confining


## The electrodynamical trap

Idea (W. Paul 1967), use oscillating electric field


But why should the force that pushes the particule towards the center win over the one that pushes it away ?
One could expect no net force.

## One dimensional case :

$$
\mathrm{t}=0
$$

On average over one cycle, there is a restoring

$$
\mathrm{t}=\mathrm{T} / 2
$$ force due to the fast oscillating field.



Dynamical stabilisation
The micromotion induced by the fast oscillatin


Torsional quantum levels

FIG. 2: Principle of hybrid torsional cooling.

## The Paul trap



- High optical access
- Tunable trap parameters after injection
- Ambient conditions


Whatever the geometry, at least one direction will not be confining $\rightarrow$ General law of electrostatics

W. Paul (1967) proposed to use oscillating electric fields. :


## Radiation pressure force :

$$
\begin{aligned}
F_{r a d} & =\int_{-\theta_{m}}^{\theta_{m}} \frac{h}{\lambda} 2 R_{n} \cos \theta \frac{P \lambda}{h c} \frac{d \theta}{2 \theta_{m}} \\
& =\frac{2 R_{n} P}{c} \operatorname{sinc}\left(\theta_{m}\right) .
\end{aligned}
$$

Displacement due to the laser

$$
\Delta x=\frac{F_{r a d}}{m \omega_{x}^{2}}
$$

$\Delta x / P \sim 350 \mathrm{~nm} / \mathrm{mW} \quad$ For a micron size particule

$$
\Delta x / P \sim 11 \mu \mathrm{~m} / \mathrm{mW} \quad \text { For a particule size of } 100 \mathrm{~nm}
$$

## Coupling to the center of mass via the NV spin



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Non-adiabatic regime : $\omega_{\text {com }} \gg \gamma$



The Hamiltonian :
$H_{\mathrm{int}}=\vec{\mu} \cdot \vec{B}=\lambda_{\mathrm{com}} S_{z}\left(a+a^{\dagger}\right)$
Coupling rate to the the COM mode :
$\lambda_{\text {com }}=g_{s} \mu_{B} G_{m} a_{0} \longrightarrow a_{0}=\sqrt{\hbar / 2 m \omega_{\text {com }}}$
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Consequence of the conservation of the electric flux on a closed surface

$\oiint \vec{E} \cdot \overrightarrow{d S}=0$


## Solution?

In statics, one cannot confine a charged particle. Idea: make the electric field oscillate.


One can feel that the electric field has to oscillate more rapidly than the period $T_{s}=2 \pi / \omega_{0}$ in statics.

However, a priori, one cannot see why the force that brings the particle towards the center would compensate the force that pushes it away from it

## One dimensional case

$F=-k x \cos (\omega t)$ where $\mathbf{k}$ depends on the tension applied to the electrode. During one cycle of oscillation with a period $T=\frac{2 \pi}{\omega} \ll T_{s}=\sqrt{\frac{m}{k}}$


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$x_{\uparrow}$
$\dagger$


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