Why the standard model

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(based on previous work with M. Marcolli)

Space-Time

Flat space (Poincaré, Einstein, Minkowski)

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Curved space, gravitational potential $g_{\mu\nu}$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Action principle

$$S_E[g_{\mu\nu}] = \frac{1}{16\pi G} \int_M R\sqrt{g}\; d^4x$$

$$S = S_E + S_{SM}$$

Classical $\rightarrow$ Quantum (Feynman)

$$e^{i\frac{S}{\hbar}}$$

Standard Model

$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu$$

$$-\partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu$$

$$-igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu)$$

$$+Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)$$

$$-A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu))$$

$$-\frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu$$

$$+g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu)$$

$$+g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2}m_h^2 H^2$$

$$-\partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M^2 \phi^0 \phi^0$$

$$-\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-) \right) + \frac{2M^4}{g^2}\alpha_h$$

$$-g\alpha_h M\left(H^3+H\phi^0\phi^0+2H\phi^+\phi^-\right)$$

$$-\frac{1}{8}g^2\alpha_h\left(H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2\right)$$

$$-gMW_\mu^+W_\mu^-H-\frac{1}{2}g\frac{M}{c_w^2}Z_\mu^0Z_\mu^0H$$

$$-\frac{1}{2}ig\left(W_\mu^+(\phi^0\partial_\mu\phi^--\phi^-\partial_\mu\phi^0)-W_\mu^-(\phi^0\partial_\mu\phi^+-\phi^+\partial_\mu\phi^0)\right)$$

$$+\frac{1}{2}g\left(W_\mu^+(H\partial_\mu\phi^--\phi^-\partial_\mu H)+W_\mu^-(H\partial_\mu\phi^+-\phi^+\partial_\mu H)\right)$$

$$+\frac{1}{2}g\frac{1}{c_w}(Z_\mu^0(H\partial_\mu\phi^0-\phi^0\partial_\mu H)-ig\frac{s_w^2}{c_w}MZ_\mu^0(W_\mu^+\phi^--W_\mu^-\phi^+)$$

$$+igs_wMA_\mu(W_\mu^+\phi^--W_\mu^-\phi^+)-ig\frac{1-2c_w^2}{2c_w}Z_\mu^0(\phi^+\partial_\mu\phi^--\phi^-\partial_\mu\phi^+)$$

$$+igs_wA_\mu(\phi^+\partial_\mu\phi^--\phi^-\partial_\mu\phi^+)-\frac{1}{4}g^2W_\mu^+W_\mu^-\left(H^2+(\phi^0)^2+2\phi^+\phi^-\right)$$

$$-\frac{1}{8}g^2\frac{1}{c_w^2}Z_\mu^0Z_\mu^0\left(H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-\right)$$

$$-\frac{1}{2}g^2\frac{s_w^2}{c_w}Z_\mu^0\phi^0(W_\mu^+\phi^-+W_\mu^-\phi^+)-\frac{1}{2}ig^2\frac{s_w^2}{c_w}Z_\mu^0H(W_\mu^+\phi^--W_\mu^-\phi^+)$$

$$+\frac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^+\phi^-+W_\mu^-\phi^+)+\frac{1}{2}ig^2s_wA_\mu H(W_\mu^+\phi^--W_\mu^-\phi^+)$$

$$-g^2\frac{s_w}{c_w}(2c_w^2-1)Z_\mu^0A_\mu\phi^+\phi^--g^2s_w^2A_\mu A_\mu\phi^+\phi^-$$

$$+\frac{1}{2}ig_s\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j$$

$$-\bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_wA_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right)$$

$$+\frac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s^2_w - 1 - \gamma^5)e^\lambda)$$

$$+(\bar{d}^\lambda_j\gamma^\mu(\frac{4}{3}s^2_w - 1 - \gamma^5)d^\lambda_j) + (\bar{u}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s^2_w + \gamma^5)u^\lambda_j)\}$$

$$+\frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right)$$

$$+\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_jC^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right)$$

$$+\frac{ig}{2\sqrt{2}}\frac{m^\lambda_e}{M}\left(-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)\right)$$

$$-\frac{g}{2}\frac{m^\lambda_e}{M}\left(H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5e^\lambda)\right)$$

$$+\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_jC_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_jC_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right)$$

$$+\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right)$$

$$-\frac{g}{2}\frac{m^\lambda_u}{M}H(\bar{u}^\lambda_ju^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar{d}^\lambda_jd^\lambda_j) + \frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5d^\lambda_j)$$

$$+\bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^-$$

$$+\bar{X}^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0)$$

$$+igs_w W^+_\mu(\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+)$$

$$+igs_w W^-_\mu(\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z^0_\mu(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-)$$

$$+igs_w A_\mu(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-)$$

$$-\frac{1}{2}gM\left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H\right)$$

$$+\frac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-\right) + \frac{1}{2c_w}igM\left(\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-\right)$$

$$+igMs_w\left(\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-\right) + \frac{1}{2}igM\left(\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0\right) .$$

Notations

- Gauge bosons : $A_\mu, W_\mu^\pm, Z_\mu^0, g_\mu^a$
- Quarks : u_j^κ, d_j^κ, collective : q_j^σ
- Leptons : e^λ, ν^λ
- Higgs fields : $H, \phi^0, \phi^+, \phi^-$
- Ghosts : G^a, X^0, X^+, X^-, Y,
- Masses : $m_d^\lambda, m_u^\lambda, m_e^\lambda, m_h, M$ (the latter is the mass of the W)
- Tadpole Constant β_h
- Cosine and sine of the weak mixing angle c_w, s_w
- Coupling constants $s_w g = \sqrt{4\pi\alpha}$ (fine structure), g_s = strong, $\alpha_h = \frac{m_h^2}{4M^2}$
- Cabibbo–Kobayashi–Maskawa mixing matrix : $C_{\lambda\kappa}$
- Structure constants of SU(3) : f^{abc}
- The Gauge is the Feynman–t'Hooft gauge.

Symmetries of $S = S_E + S_{SM}$

$$G = U(1) \times SU(2) \times SU(3)$$

$$\mathcal{G} = \mathrm{Map}(M, G) \rtimes \mathrm{Diff}(M)$$

Question :

Is there a space X whose group of diffeomorphisms is directly of that form ?

Answer :

No : for ordinary "commutative spaces"

Yes : for noncommutative spaces

$$\mathcal{A} = C^\infty(M, M_n(\mathbb{C})) = C^\infty(M) \otimes M_n(\mathbb{C})$$

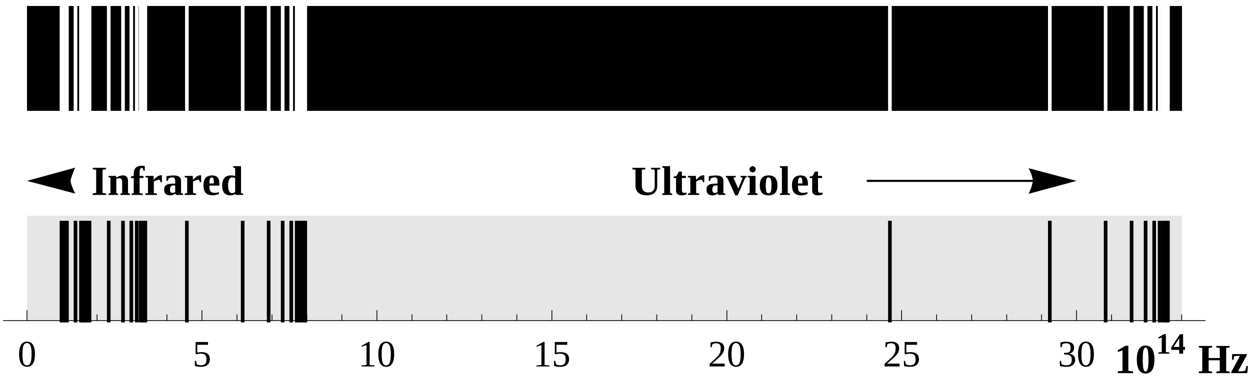
The group $\mathrm{Inn}(\mathcal{A})$ is locally isomorphic to the group $\mathcal{G}$ of smooth maps from M to the small gauge group $G = PSU(n)$ (quotient of $SU(n)$ by its center)

$$1 \to \mathrm{Inn}(\mathcal{A}) \to \mathrm{Aut}(\mathcal{A}) \to \mathrm{Out}(\mathcal{A}) \to 1$$

$$1 \to \mathrm{Map}(M, G) \to \mathcal{G} \to \mathrm{Diff}(M) \to 1.$$

What is a metric in spectral geometry

- It contains the Riemannian paradigm $(M, g_{\mu\nu})$ as a special case.
- It does not require the commutativity of coordinates.
- It contains spaces X_z of complex dimension z suitable for the Dim-Reg procedure.
- It provides a way of expressing the full standard model coupled to Einstein gravity as pure gravity on a modified space-time geometry.
- It allows for quantum corrections to the geometry.



Meter → Wave length (Krypton (1967) spectrum of 86Kr then Caesium (1984) hyperfine levels of C133)

In fact, the actual definition of the unit of length m in the metric system is as a specific fraction $\frac{9192631770}{299792458} \sim 30.6633...$ of the wave length of the radiation coming from the transition between two hyperfine levels of the Cesium 133 atom. Indeed the speed of light is fixed once and for all at the value of

$$c = 299792458\, m/s$$

and the second s, which is the unit of time, is defined as the time taken by 9192631770 periods of the above radiation.

Spectral Triples

A spectral triple $(\mathcal{A}, \mathcal{H}, D)$ is given by an involutive unital algebra $\mathcal{A}$ represented as operators in a Hilbert space $\mathcal{H}$ and a self-adjoint operator D with compact resolvent such that all commutators $[D, a]$ are bounded for $a \in \mathcal{A}$.

A spectral triple is *even* if the Hilbert space $\mathcal{H}$ is endowed with a $\mathbb{Z}/2$- grading γ which commutes with any $a \in \mathcal{A}$ and anticommutes with D.

Real Structure

A real structure of KO-dimension $n \in \mathbb{Z}/8$ on a spectral triple $(\mathcal{A}, \mathcal{H}, D)$ is an antilinear isometry $J : \mathcal{H} \to \mathcal{H}$, with the property that

$$J^2 = \varepsilon, \qquad JD = \varepsilon' DJ, \quad \text{and} \quad J\gamma = \varepsilon'' \gamma J \qquad (1)$$

The numbers $\varepsilon, \varepsilon', \varepsilon'' \in \{-1, 1\}$ are a function of n mod 8 given by

n	0	1	2	3	4	5	6	7
ε	1	1	-1	-1	-1	-1	1	1
ε'	1	-1	1	1	1	-1	1	1
ε''	1		-1		1		-1	

$$[a, b^0] = 0 \quad \forall\, a, b \in \mathcal{A}, \;\; b^0 = Jb^* J^{-1} \qquad (2)$$

$$[[D, a], b^0] = 0 \qquad \forall\, a, b \in \mathcal{A}\,. \qquad (3)$$

Our road to F is through the following steps

1. We classify the irreducible triplets $(\mathcal{A}, \mathcal{H}, J)$.
2. We study the $\mathbb{Z}/2$-gradings γ on $\mathcal{H}$.
3. We classify the subalgebras $\mathcal{A}_F \subset \mathcal{A}$ which allow for an operator D that does not commute with the center of $\mathcal{A}$ but fulfills the "order one" condition (3)

$$[[D, a], b^0] = 0 \qquad \forall\, a, b \in \mathcal{A}_F\,.$$

Assume irreducibility, then one of the following cases holds

- The center $Z(\mathcal{A}_{\mathbb{C}})$ is reduced to $\mathbb{C}$.
- One has $Z(\mathcal{A}_{\mathbb{C}}) = \mathbb{C} \oplus \mathbb{C}$ and $Je_1J^{-1} = e_2$ where $e_j \in Z(\mathcal{A}_{\mathbb{C}})$ are the minimal projections of $Z(\mathcal{A}_{\mathbb{C}})$.

The case $Z(\mathcal{A}_{\mathbb{C}}) = \mathbb{C}$

Let $\mathcal{H}$ be a Hilbert space of dimension n. Then an irreducible solution with $Z(\mathcal{A}_{\mathbb{C}}) = \mathbb{C}$ exists iff $n = k^2$ is a square. It is given by $\mathcal{A}_{\mathbb{C}} = M_k(\mathbb{C})$ acting by left multiplication on itself and antilinear involution

$$J(x) = x^*\,, \quad \forall x \in M_k(\mathbb{C})\,.$$

Three possibilities
– $\mathcal{A} = M_k(\mathbb{C})$ (unitary case)
– $\mathcal{A} = M_k(\mathbb{R})$ (orthogonal case)
– $\mathcal{A} = M_a(\mathbb{H})$, for even $k = 2a$, (symplectic case)

The case $Z(\mathcal{A}_{\mathbb{C}}) = \mathbb{C} \oplus \mathbb{C}$

Let $\mathcal{H}$ be a Hilbert space of dimension n. Then an irreducible solution with $Z(\mathcal{A}_{\mathbb{C}}) = \mathbb{C} \oplus \mathbb{C}$ exists iff $n = 2k^2$ is twice a square. It is given by $\mathcal{A}_{\mathbb{C}} = M_k(\mathbb{C}) \oplus M_k(\mathbb{C})$ acting by left multiplication on itself and antilinear involution

$$J(x, y) = (y^*, x^*)\,, \quad \forall x, y \in M_k(\mathbb{C})\,.$$

Intrinsic description

$$\mathcal{A}_{\mathbb{C}} = \mathsf{End}_{\mathbb{C}}(W) \oplus \mathsf{End}_{\mathbb{C}}(V)\,.$$

$$\mathcal{E} = \mathsf{Hom}_{\mathbb{C}}(V, W)\,, \quad \mathcal{E}^* = \mathsf{Hom}_{\mathbb{C}}(W, V)$$

$$\mathcal{H} = \mathcal{E} \,\oplus\, \mathcal{E}^*\,, \quad J(\xi, \eta) = (\eta^*, \xi^*)$$

F **of** KO**-dimension** 6 **(mod** 8**)**

In the case $Z(\mathcal{A}_{\mathbb{C}}) = \mathbb{C}$, let γ be a $\mathbb{Z}/2$-grading of $\mathcal{H}$ such that $\gamma \mathcal{A} \gamma^{-1} = \mathcal{A}$ and $J\gamma = \epsilon'' \gamma J$ for $\epsilon'' = \pm 1$. Then $\epsilon'' = 1$.

$$\Downarrow$$

case $Z(\mathcal{A}_{\mathbb{C}}) = \mathbb{C}$ is excluded

$\mathbb{Z}/2$-**grading**

We make the *hypothesis* that both the grading and the real form come by assuming that the vector space W is a right vector space over $\mathbb{H}$ and is non-trivially $\mathbb{Z}/2$-graded.

$$\Downarrow$$

$$\text{Simplest case : } W = \mathbb{H}^2,\ V = \mathbb{C}^4$$

(The space $\mathcal{E} = \mathrm{Hom}_{\mathbb{C}}(V, W)$ is related to the classification of instantons)

Up to an automorphism of $\mathcal{A}^{\text{ev}}$, there exists a unique involutive subalgebra $\mathcal{A}_F \subset \mathcal{A}^{\text{ev}}$ of maximal dimension admitting off-diagonal Dirac operators. It is given by

$$\mathcal{A}_F = \{(\lambda \oplus q, \lambda \oplus m) \mid \lambda \in \mathbb{C}\,,\ q \in \mathbb{H}\,,\ m \in M_3(\mathbb{C})\}$$

$$\subset \mathbb{H} \oplus \mathbb{H} \oplus M_4(\mathbb{C})\,,$$

using a field morphism $\mathbb{C} \to \mathbb{H}$. The involutive algebra $\mathcal{A}_F$ is isomorphic to $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ and together with its representation in $(\mathcal{H}, J, \gamma)$ it gives the noncommutative geometry F.

The product geometry

$$\mathcal{A} = \mathcal{A}_1 \otimes \mathcal{A}_2 \ , \ \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \ ,$$

$$D = D_1 \otimes 1 + \gamma_1 \otimes D_2 \, , \ \gamma = \gamma_1 \otimes \gamma_2 \, , \ J = J_1 \otimes J_2$$

$$\mathcal{A} = C^\infty(M) \otimes \mathcal{A}_F = C^\infty(M, \mathcal{A}_F)$$

$$\mathcal{H} = L^2(M, S) \otimes \mathcal{H}_F = L^2(M, S \otimes \mathcal{H}_F)$$

$$D = \partial\!\!\!/_M \otimes 1 + \gamma_5 \otimes D_F$$

where $\partial\!\!\!/_M$ is the Dirac operator on M.

Spectral Model

Let M be a Riemannian spin 4-manifold and F the finite noncommutative geometry of KO-dimension 6 described above. Let $M \times F$ be endowed with the product metric.

1. The unimodular subgroup of the unitary group acting by the adjoint representation Ad(u) in $\mathcal{H}$ is the group of gauge transformations of SM.
2. The unimodular inner fluctuations of the metric give the gauge bosons of SM.
3. The full standard model (with neutrino mixing and seesaw mechanism) minimally coupled to Einstein gravity is given in Euclidean form by the action functional

   $$S = \mathrm{Tr}(f(D_A/\Lambda)) + \frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle , \quad \tilde{\xi} \in \mathcal{H}^+_{cl},$$

   where D_A is the Dirac operator with the unimodular inner fluctuations.

Predictions

– Unification of couplings

$$\frac{g_3^2\, f_0}{2\pi^2} = \frac{1}{4}, \qquad g_3^2 = g_2^2 = \frac{5}{3}\, g_1^2 \, .$$

– See-saw mechanism for neutrino masses with large $M_R \sim \Lambda$.

– The mass matrices satisfy the constraint at unification

$$\sum_\sigma (m_\nu^\sigma)^2 + (m_e^\sigma)^2 + 3\,(m_u^\sigma)^2 + 3\,(m_d^\sigma)^2 = \, 8\, M^2$$

$$Y_2 = \sum_\sigma (y_\nu^\sigma)^2 + (y_e^\sigma)^2 + 3\,(y_u^\sigma)^2 + 3\,(y_d^\sigma)^2$$

$$Y_2(S) = 4\, g^2.$$

This yields a value of the top mass which is 1.04 times the observed value when neglecting the yukawa couplings of the bottom quarks etc...and is hence compatible with experiment.

– The Higgs scattering parameter

$$\tilde{\lambda}(\Lambda) = g_3^2 \frac{b}{a^2}.$$

The numerical solution to the RG equations with the boundary value $\lambda_0 = 0.356$ at $\Lambda = 10^{17}$ GeV gives $\lambda(M_Z) \sim 0.241$ and a Higgs mass of the order of 170 GeV.

– Newton constant ($f_2 \sim 5 f_0$)

– No proton decay

$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \beta_h\left(\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h - g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \frac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \frac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + M\left(\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \frac{1}{8}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \frac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \frac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + (\bar{u}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa)\right) + \frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa)\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \frac{g}{2}\frac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \frac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa - \frac{1}{4}\,\overline{\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa} + \frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j)\right) + \frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j)\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) - \frac{1}{2}gM\left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H\right) + \frac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-\right) + \frac{1}{2c_w}igM\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + igMs_w\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + \frac{1}{2}igM\left(\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0\right)\,.$$

Inner fluctuations of metric

One defines a right $\mathcal{A}$-module structure on $\mathcal{H}$ by

$$\xi\, b = b^0\, \xi\,, \quad \forall\, \xi \in \mathcal{H}\,, \quad b \in \mathcal{A}$$

The unitary group of the algebra $\mathcal{A}$ then acts by the "adjoint representation" in $\mathcal{H}$ in the form

$$\xi \in \mathcal{H} \to \mathrm{Ad}(u)\, \xi = u\, \xi\, u^*\,, \quad \forall\, \xi \in \mathcal{H}$$

The inner fluctuations of the metric are given by

$$D \to D_A = D + A + \varepsilon'\, J\, A\, J^{-1}$$

where A is a self-adjoint operator of the form

$$A = \sum a_j [D, b_j]\,, \quad a_j, b_j \in \mathcal{A}.$$

The bosonic part of the spectral action is

$$
\begin{aligned}
S &= \frac{1}{\pi^2}(48\, f_4\, \Lambda^4 - f_2\, \Lambda^2\, c + \frac{f_0}{4}\, d) \int \sqrt{g}\, d^4x(1) \\
&+ \frac{96\, f_2\, \Lambda^2 - f_0\, c}{24\pi^2} \int R\, \sqrt{g}\, d^4x \\
&+ \frac{f_0}{10\, \pi^2} \int (\frac{11}{6}\, R^* R^* - 3\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma})\, \sqrt{g}\, d^4x \\
&+ \frac{(-2\, a\, f_2\, \Lambda^2 + e\, f_0)}{\pi^2} \int |\varphi|^2\, \sqrt{g}\, d^4x \\
&+ \frac{f_0}{2\, \pi^2} \int a\, |D_\mu \varphi|^2\, \sqrt{g}\, d^4x \\
&- \frac{f_0}{12\, \pi^2} \int a\, R\, |\varphi|^2\, \sqrt{g}\, d^4x \\
&+ \frac{f_0}{2\, \pi^2} \int (g_3^2\, G_{\mu\nu}^i\, G^{\mu\nu i} + g_2^2\, F_{\mu\nu}^\alpha\, F^{\mu\nu\alpha} \\
&+ \frac{5}{3}\, g_1^2\, B_{\mu\nu}\, B^{\mu\nu})\, \sqrt{g}\, d^4x \\
&+ \frac{f_0}{2\, \pi^2} \int b\, |\varphi|^4\, \sqrt{g}\, d^4x
\end{aligned}
$$

where

$$R^* R^* = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} R_{\mu\nu}^{\alpha\beta} R_{\rho\sigma}^{\gamma\delta}$$

is the Euler characteristic.

We first perform a trivial rescaling of the Higgs field φ so that kinetic terms are normalized. To normalize the Higgs fields kinetic energy we have to rescale φ to :

$$\mathbf{H} = \frac{\sqrt{a\, f_0}}{\pi}\, \varphi \,,$$

so that the kinetic term becomes

$$\int \frac{1}{2} |D_\mu \mathbf{H}|^2 \sqrt{g}\, d^4x$$

The normalization of the kinetic terms imposes a relation between the coupling constants g_1, g_2, g_3 and the coefficient f_0, of the form

$$\frac{g_3^2\, f_0}{2\pi^2} = \frac{1}{4}, \qquad g_3^2 = g_2^2 = \frac{5}{3}\, g_1^2 \,.$$

The bosonic action then takes the form

$$
\begin{aligned}
S = & \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 \, C_{\mu\nu\rho\sigma} \, C^{\mu\nu\rho\sigma} + \quad \gamma_0 + \tau_0 \, R^* R^* \\
+ & \ \frac{1}{4} G^i_{\mu\nu} \, G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} \, F^{\mu\nu\alpha} + \ \frac{1}{4} B_{\mu\nu} \, B^{\mu\nu} \\
+ & \ \frac{1}{2} |D_\mu \, \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 \, R \, |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g} \, d^4 x
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{1}{\kappa_0^2} = & \quad \frac{96 \, f_2 \, \Lambda^2 - f_0 \, c}{12 \, \pi^2} \\
\mu_0^2 = & \quad 2 \, \frac{f_2 \, \Lambda^2}{f_0} - \ \frac{e}{a} \\
\alpha_0 = & \ -\frac{3 \, f_0}{10 \, \pi^2} \\
\tau_0 = & \ \frac{11 \, f_0}{60 \, \pi^2} \\
\gamma_0 = & \quad \frac{1}{\pi^2} (48 \, f_4 \, \Lambda^4 - f_2 \, \Lambda^2 \, c + \frac{f_0}{4} \, d) \\
\lambda_0 = & \ \frac{\pi^2}{2 \, f_0} \frac{b}{a^2} \\
\xi_0 = & \ \frac{1}{12}
\end{aligned}
$$

Standard Model	**notation**	**notation**	**Spectral Action**
Higgs Boson	$\varphi = (\frac{2M}{g} + H - i\phi^0, -i\sqrt{2}\phi^+)$	$\mathbf{H} = \frac{1}{\sqrt{2}} \frac{\sqrt{a}}{g}(1+\psi)$	**Inner metric**$^{(0,1)}$
Gauge bosons	$A_\mu, Z^0_\mu, W^\pm_\mu, g^a_\mu$	(B, W, V)	**Inner metric**$^{(1,0)}$
Fermion masses u, ν	m_u, m_ν	$Y_{(\uparrow 3)} = \delta_{(\uparrow 3)}, Y_{(\uparrow 1)} = \delta_{(\uparrow 1)}$	**Dirac**$^{(0,1)}$ **in** $\uparrow$
CKM matrix **Masses down**	C^κ_λ, m_d	$Y_{(\downarrow 3)} = C\,\delta_{3,\downarrow}\,C^\dagger$	**Dirac**$^{(0,1)}$ **in** $(\downarrow 3)$
Lepton mixing **Masses leptons** e	$U^{lep}{}_{\lambda\kappa}, m_e$	$Y_{(\downarrow 1)} = U^{lep}\,\delta_{(\downarrow 1)}\,U^{lep\dagger}$	**Dirac**$^{(0,1)}$ **in** $(\downarrow 1)$
Majorana **mass matrix**	M_R	Y_R	**Dirac**$^{(0,1)}$ **on** $E_R \oplus J_F E_R$
Gauge couplings	$g_1 = g\tan(\theta_w), g_2 = g, g_3 = g_s$	$g_3^2 = g_2^2 = \frac{5}{3}\,g_1^2$	**Fixed at unification**
Higgs scattering parameter	$\frac{1}{8}\,g^2\,\alpha_h, \alpha_h = \frac{m_h^2}{4M^2}$	$\lambda_0 = g^2\,\frac{b}{a^2}$	**Fixed at unification**
Tadpole constant	$\beta_h, (-\alpha_h\,M^2\,+\frac{\beta_h}{2})\,\|\varphi\|^2$	$\mu_0^2 = 2\,\frac{f_2\Lambda^2}{f_0} - \frac{e}{a}$	$-\mu_0^2\,\|\mathbf{H}\|^2$
Graviton	$g_{\mu\nu}$	$\partial\!\!\!/_M$	**Dirac**$^{(1,0)}$

TABLE 1. Conversion from Spectral Action to Standard Model

The mass relation

The relation between the mass matrices comes from the equality of the Yukawa coupling terms $\mathcal{L}_{Hf}$.

After Wick rotation to Euclidean and the chiral transformation $U = e^{i\frac{\pi}{4}\gamma_5} \otimes 1$ they are the same provided the following equalities hold

$$
\begin{aligned}
(k_{(\uparrow 3)})_{\sigma\kappa} &= \frac{g}{2M}\, m_u^\sigma\, \delta_\sigma^\kappa \\
(k_{(\downarrow 3)})_{\sigma\kappa} &= \frac{g}{2M}\, m_d^\mu\, C_{\sigma\mu}\delta_\mu^\rho C^\dagger_{\rho\kappa} \\
(k_{(\uparrow 1)})_{\sigma\kappa} &= \frac{g}{2M}\, m_\nu^\sigma\, \delta_\sigma^\kappa \\
(k_{(\downarrow 1)})_{\sigma\kappa} &= \frac{g}{2M}\, m_e^\mu\, {U^{lep}}_{\sigma\mu}\delta_\mu^\rho {U^{lep}}^\dagger_{\rho\kappa}
\end{aligned}
$$

Here the symbol δ_i^j is the Kronecker delta (not to be confused with the previous notation $\delta_{\uparrow\downarrow}$).

It might seem at first sight that one can simply use the above equation to define the matrices k_x but this overlooks the fact that one has the constraint

$$\mathrm{Tr}(k^*_{(\uparrow 1)} k_{(\uparrow 1)} + k^*_{(\downarrow 1)} k_{(\downarrow 1)}$$

$$+3(k^*_{(\uparrow 3)} k_{(\uparrow 3)} + k^*_{(\downarrow 3)} k_{(\downarrow 3)})) = 2\, g^2$$

The mass matrices thus satisfy the constraint

$$\sum_\sigma (m^\sigma_\nu)^2 + (m^\sigma_e)^2 + 3\,(m^\sigma_u)^2 + 3\,(m^\sigma_d)^2 = 8\, M^2$$

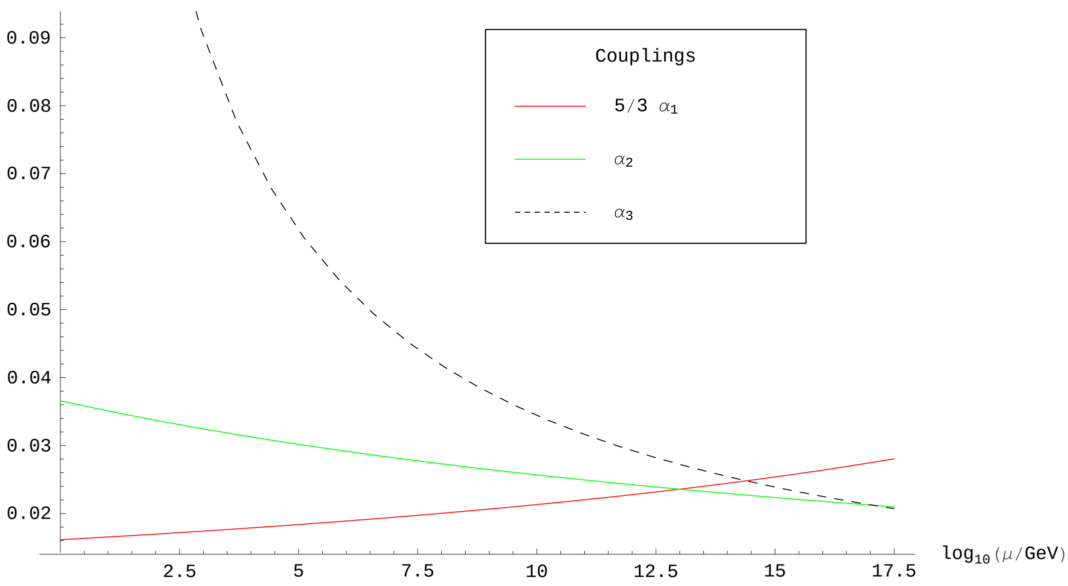
Running of coupling constants

At one loop :

$$\beta_{g_i} = (4\pi)^{-2}\, b_i\, g_i^3, \qquad \text{with} \quad b = (\frac{41}{6}, -\frac{19}{6}, -7),$$

$$\begin{aligned}
\alpha_1^{-1}(\Lambda) &= \alpha_1^{-1}(M_Z) - \frac{41}{12\pi}\,\log\,\frac{\Lambda}{M_Z} \\
\alpha_2^{-1}(\Lambda) &= \alpha_2^{-1}(M_Z) + \frac{19}{12\pi}\,\log\,\frac{\Lambda}{M_Z} \\
\alpha_3^{-1}(\Lambda) &= \alpha_3^{-1}(M_Z) + \frac{42}{12\pi}\,\log\,\frac{\Lambda}{M_Z}
\end{aligned}$$

where M_Z is the mass of the Z^0 vector boson.


0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
Couplings
5/3 α_1
α_2
α_3
2.5
5
7.5
10
12.5
15
17.5
$\log_{10}(\mu/\mathrm{GeV})$

Constraint on Higgs scattering parameter

$$\frac{f_0}{2\,\pi^2}\int b\,|\varphi|^4\,\sqrt{g}\,d^4x = \frac{\pi^2}{2\,f_0}\frac{b}{a^2}\int |\mathbf{H}|^4\,\sqrt{g}\,d^4x$$

gives a further relation in our theory between the $\tilde{\lambda}|\mathbf{H}|^4$ coupling and the gauge couplings to be imposed at the scale Λ. This is of the form

$$\tilde{\lambda}(\Lambda) = g_3^2\,\frac{b}{a^2}.$$

Running of Higgs scattering parameter

$$\frac{d\lambda}{dt} = \lambda\gamma + \frac{1}{8\pi^2}(12\lambda^2 + B)$$

where

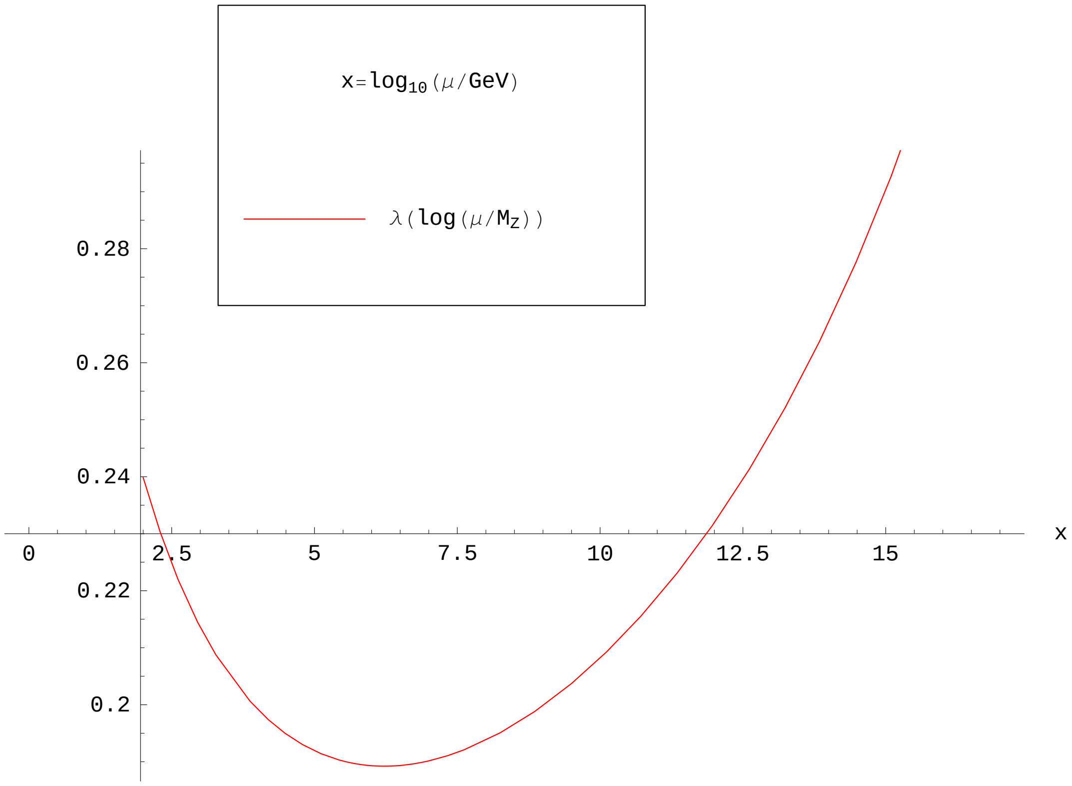
$$\begin{aligned}
\gamma &= \frac{1}{16\pi^2}(12\, y_t^2 - 9\, g_2^2 - 3\, g_1^2) \\
B &= \frac{3}{16}(3\, g_2^4 + 2 g_1^2\, g_2^2 + g_1^4) - 3\, y_t^4\,.
\end{aligned}$$

Higgs mass

The Higgs mass is then given by

$$m_H^2 = 8\lambda \frac{M^2}{g^2}, \quad m_H = \sqrt{2\lambda}\,\frac{2M}{g}$$

The numerical solution to these equations with the boundary value $\lambda_0 = 0.356$ at $\Lambda = 10^{17}$ GeV gives $\lambda(M_Z) \sim 0.241$ and a Higgs mass of the order of 170 GeV.



x=log10(μ/GeV)
λ(log(μ/MZ))
0.28
0.26
0.24
0.22
0.2
0
2.5
5
7.5
10
12.5
15
x

The fermion–boson mass relation

In terms of the Yukawa couplings ($y_{\cdot}^{\sigma}$) the mass constraint reads as

$$\frac{v^2}{2}\sum_{\sigma}(y_{\nu}^{\sigma})^2+(y_{e}^{\sigma})^2+3\,(y_{u}^{\sigma})^2+3\,(y_{d}^{\sigma})^2 = 2\,g^2\,v^2,$$

with $v = \frac{2M}{g}$ the vacuum expectation value of the Higgs, as above.

In the traditional notation for the Standard Model the combination

$$Y_2 = \sum_{\sigma}(y_{\nu}^{\sigma})^2+(y_{e}^{\sigma})^2+3\,(y_{u}^{\sigma})^2+3\,(y_{d}^{\sigma})^2$$

is denoted by $Y_2 = Y_2(S)$. Thus, the mass constraint is of the form

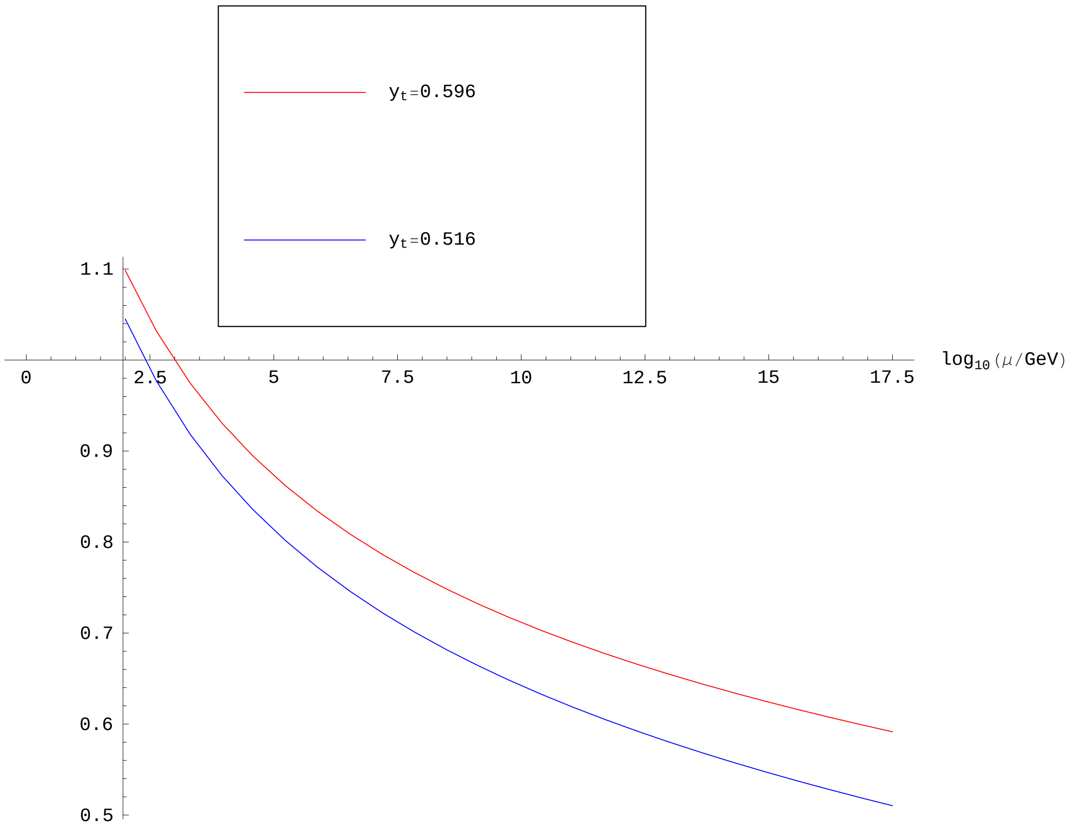
$$Y_2(S) = 4\,g^2.$$

Running of top Yukawa coupling

$$\frac{v}{\sqrt{2}}(y^{\sigma}_{\cdot}) = (m^{\sigma}_{\cdot}),$$

$$\frac{dy_t}{dt} = \frac{1}{16\pi^2}\left[\frac{9}{2}y_t^3 - \left(a\,g_1^2 + b\,g_2^2 + c\,g_3^2\right)y_t\right],$$

$$(a, b, c) = (\frac{17}{12}, \frac{9}{4}, 8)$$



y_t=0.596
y_t=0.516
1.1
0.9
0.8
0.7
0.6
0.5
0
2.5
5
7.5
10
12.5
15
17.5
log_10(μ/GeV)

Gravitational terms

$$\int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 \, C_{\mu\nu\rho\sigma} \, C^{\mu\nu\rho\sigma} + \gamma_0 \right.$$

$$\left. + \tau_0 \, R^* R^* - \xi_0 \, R \, |\mathbf{H}|^2 \right) \sqrt{g} \, d^4 x,$$

Curvature square terms :

$$\int \left(\frac{1}{2\eta} C_{\mu\nu\rho\sigma} \, C^{\mu\nu\rho\sigma} - \frac{\omega}{3\eta} R^2 + \frac{\theta}{\eta} E \right) \sqrt{g} \, d^4 x$$

$$\chi(M) = \frac{1}{32\pi^2} \int E \sqrt{g} \, d^4 x = \frac{1}{32\pi^2} \int R^* R^* \sqrt{g} \, d^4 x$$

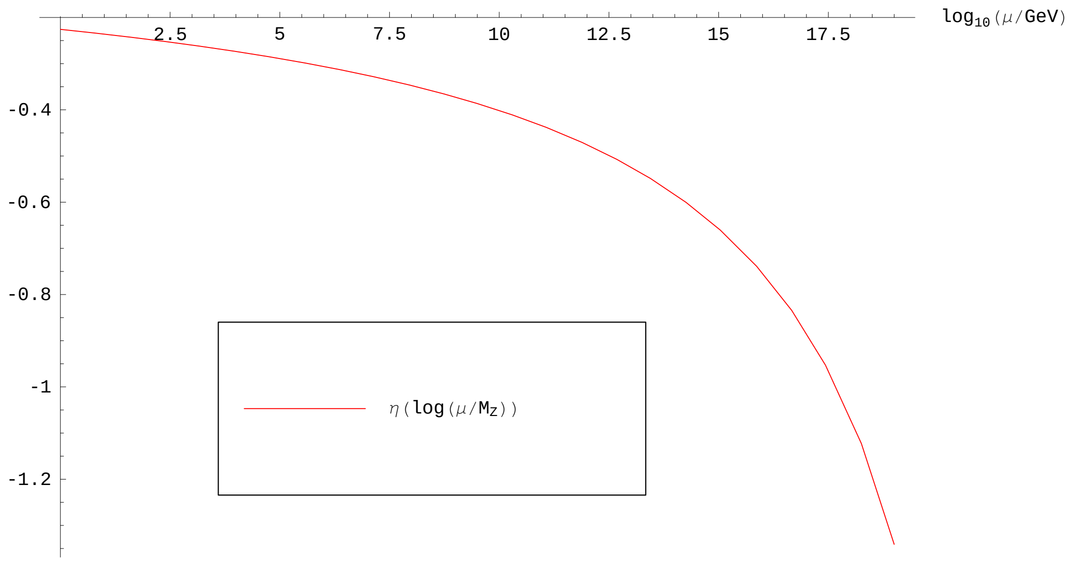
Running of gravitational terms

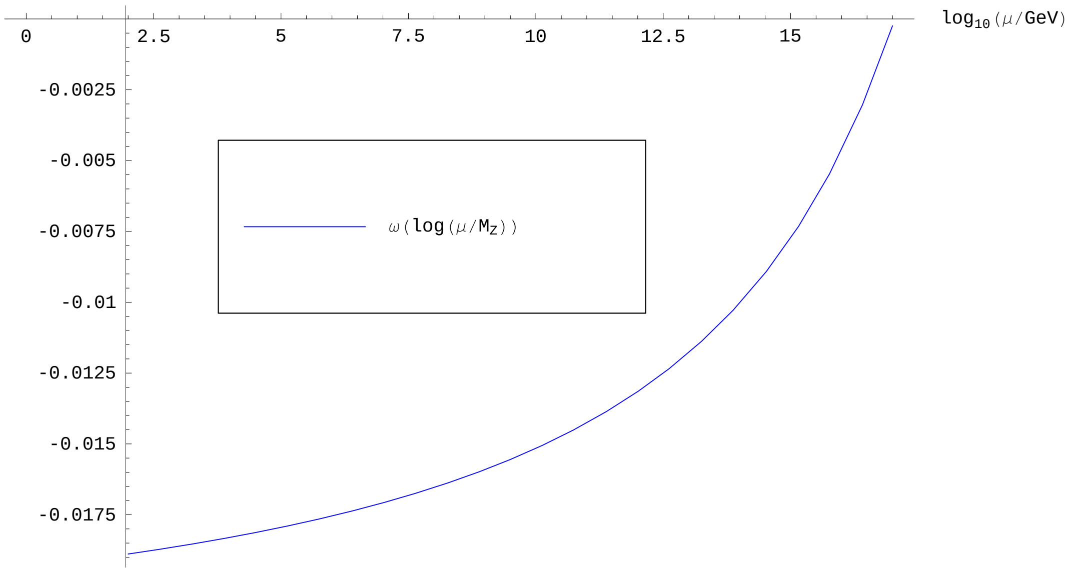
It is gauge independent and known (*cf.* Avramidi)

$$\beta_\eta = -\frac{1}{(4\pi)^2}\frac{133}{10}\eta^2$$

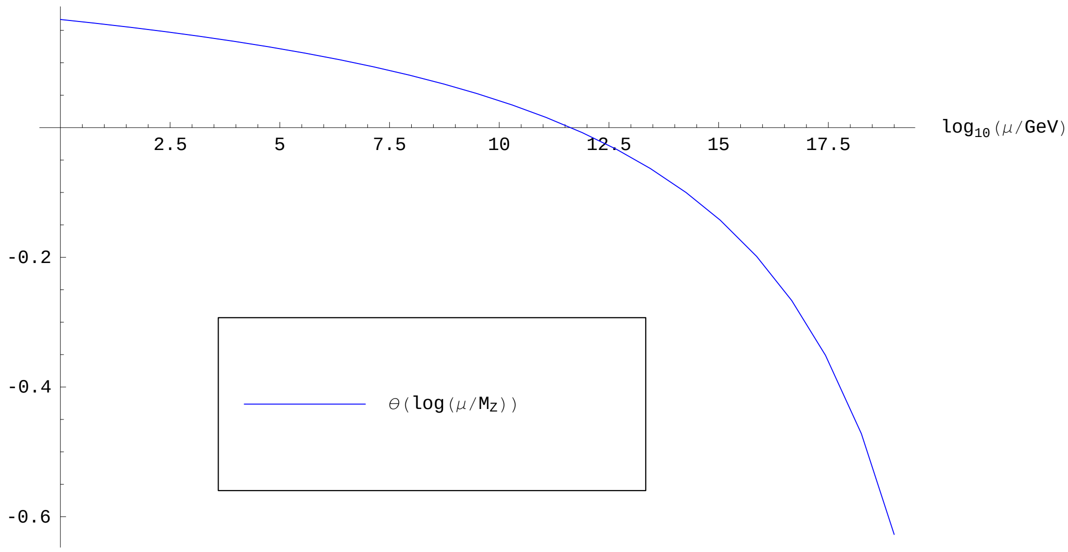
$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25 + 1098\,\omega + 200\,\omega^2}{60}\eta$$

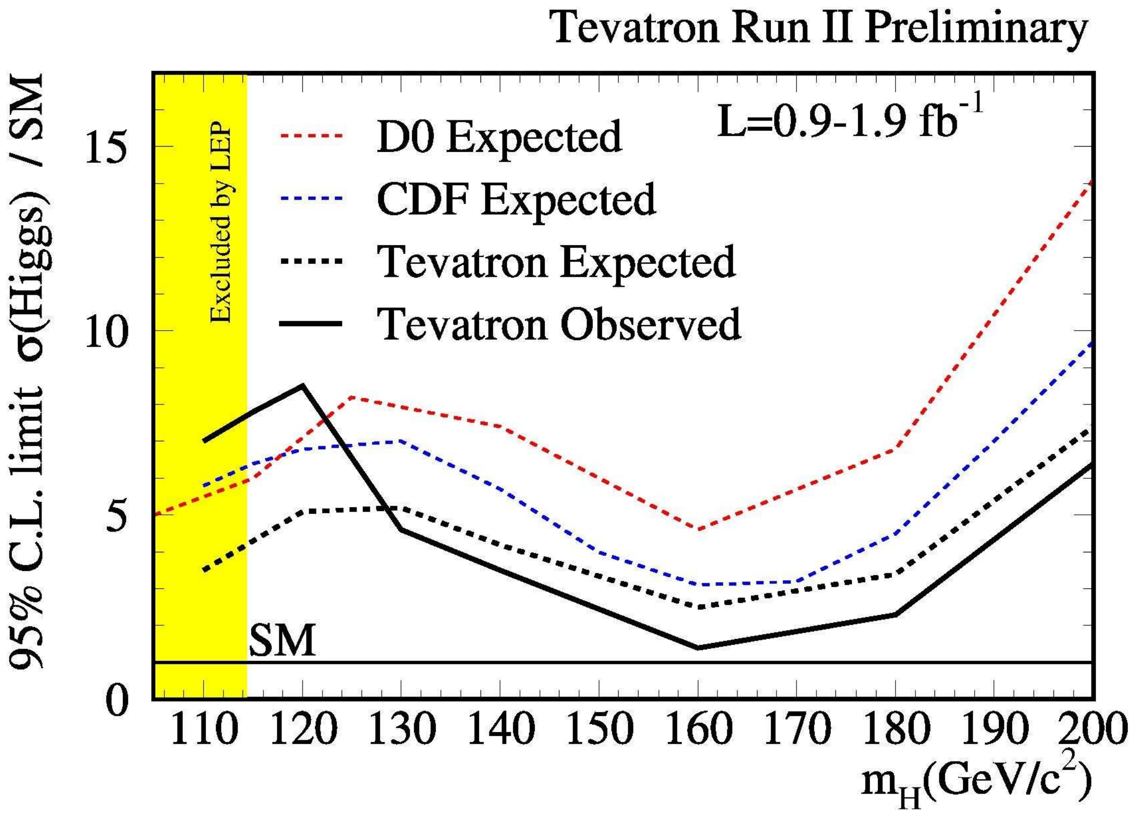
$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7\,(56 - 171\,\theta)}{90}\eta$$


log10(μ/GeV)
2.5
5
7.5
10
12.5
15
17.5
-0.4
-0.6
-0.8
-1
-1.2
η(log(μ/MZ))



0
2.5
5
7.5
10
12.5
15
log10(μ/GeV)
-0.0025
-0.005
-0.0075
-0.01
-0.0125
-0.015
-0.0175
ω(log(μ/MZ))


$\log_{10}(\mu/\mathrm{GeV})$
2.5
5
7.5
10
12.5
15
17.5
-0.2
-0.4
-0.6
$\Theta(\log(\mu/M_Z))$


Tevatron Run II Preliminary
95% C.L. limit σ(Higgs) / SM
Excluded by LEP
D0 Expected
CDF Expected
Tevatron Expected
Tevatron Observed
L=0.9-1.9 fb-1
SM
0
5
10
15
110
120
130
140
150
160
170
180
190
200
mH(GeV/c2)