Colloquium: Saclay 25 mai 2010

The (most) perfect fluid

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Quantum Chromodynamics

- The story starts with the "toughest" interaction in the Standard Model: Quantum Chromodynamics (alias: the strong interaction)
- Quantum Chromodynamics has been established as the correct theory of the strong interactions (in the past 35 years).
- It is described by a deceptively simple action:

$$S = \frac{1}{4g^2} Tr[F_{\mu\nu}F^{\mu\nu}] + \bar{q}_L(i\partial + A)q_L + \bar{q}_R(i\partial + A)q_R$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \in SU(3)_{\text{color}}$$

• Even in the absence of quarks the theory has defied analytical treatment so far.



• RG analysis indicates that the effective coupling constant becomes large in the IR while it becomes weak in the UV

$$\frac{1}{g_{eff}^2(E)} = \frac{1}{g_{eff}^2(\Lambda)} + b_0 \log \frac{E^2}{\Lambda^2} + \cdots$$

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Confinement

• The theory exhibits confinement of color and a mass gap (this is one of the seven millenium problems of the Clay Mathematics Institute. To-date no proof of confinement exists).

• The force is "short range", and color flux is confined into thin flux tubes.

$$V_{q\bar{q}}(r) = \sigma \ r + \frac{\beta}{r} + \cdots$$
, $\sigma \rightarrow$ string tension

- Quarks are permanently confined into colorless hadrons:
- A Mesons of the $\bar{q}q$ type (pions, Kaons etc.)
- A Baryons of qqq type (protons, neutrons etc) and their antiparticles.

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De-confinement

• It has been speculated since a long time that at high-temperature confinement will be lost and the quarks and gluons will be liberated.

Collins+Perry 1975

• The resulting state of matter was thought to be a (weakly coupled) plasma similar to that of EM plasmas. It was named Quark-Gluon-Plasma (QGP).

Shuryak, 1978

• A phase transition was expected to separate the confined from the deconfined phase in the pure gauge theory.

• It took twenty years of lattice simulations and many false paths to eventually reach a conclusion in the pure gauge theory: the transition is first order.



Figure 3: The trace anomaly, $\Theta^{\mu\mu}(T) \equiv \varepsilon - 3p$, in units of T^4 (left) and the entropy density, $s \equiv \varepsilon + p$, in units of T^3 calculated with the p4fat3 action [6] on lattices with temporal extent $N_{\tau} = 4$ and 6. For $\Theta^{\mu\mu}/T^4$ we also show results on $N_{\tau} = 8$ lattices (diamonds) obtained at high temperature. For $N_{\tau} = 6$ results from calculations with the asquad action are also shown [5]. In the right hand figure we also show the temperature scale Tr_0 (upper x-axis) which has been obtained from an analysis of static quark potentials at zero temperature [6]. The MeV-scale shown on the lower x-axis has been extracted from this using $r_0 = 0.469$ fm.

F. Karsch, 2002

• It looks highly plausible that it is a crossover when quarks are added.

• QCD seems to have a complex phase diagram, most regions of which are unexplored and speculative.

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A weakly coupled plasma?



- The pure gauge theory (first-order) critical temperature is $T_c \simeq 240 \pm 15$ MeV.
- It is interesting that the lightest bound state (glueball) in the pure gauge theory has a mass 1700 MeV so that $\frac{T_c}{M_{0++}} \simeq 0.14$
- The crossover with almost physical quarks is at $T_c \simeq 175 \pm 15$ MeV $\simeq 10^{12}$ 0 K. $\rightarrow 10^{-6}$ sec

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• The energy density corresponding to the deconfinement transition is $E_c \sim 1 \text{GeV/fm}^3$ (1 fm=10⁻¹⁵ m, radius of a proton $\simeq 0.8$ fm)

• The idea is to collide heavy-ion nuclei with the hope that for a short while they will create enough density and thermalize to probe the deconfined phase. A series of experimental efforts was devised:

• The first attempt: 1 Gev/nucleon at LBL's Bevalac. No signals.

• The second attempt was made in AGS (Brookhaven) by sending 10 GeV/nucleon Si and Au nuclei on a fixed target. That amounted to 5 GeV/nucleon in the collision rest frame that was not enough!

• The next attempt was made at SPS (CERN). S and Pb nuclei were accelerated and collided on fixed target with 17 Gev/nucleon in the collision rest frame. That was still not enough!

• The CERN experiments after 15 years of running (in 2000) saw some hints of collective behavior beyond the known hadronic interactions.

Relativistic Heavy Ion Collider (RHIC)



• The major breakthrough came at RHIC: two beams of Au or Cu nuclei colliding at 200 GeV/nucleon at the centerof-mass frame.

• Four experimental collaborations: BRAHMS, PHENIX, PHOBOS, STAR.

• For every almost central Au+ collision we get about 7000 particles (fragments, most of them mesons).

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RHIC head-on collision





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RHIC collision:animation



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- The crossing time for Au nuclei (with radius 8 fm) is $\sim 0.1 \text{fm/c} \simeq 3 \times 10^{-25}$ seconds.
- The particles with small v_L are produced after 1 fm/c $\simeq 3 \times 10^{-24}$ seconds. Those with higher v_L are produced later due to time dilation.

• Use the rapidity variable
$$y = \frac{1}{2} \log \left[\frac{1 + \frac{v_L}{c}}{1 - \frac{v_L}{c}} \right]$$
. Δy is Lorentz invariant.

• The "new matter" (free of fragments) is produced near $y \simeq 0$. This is what we are looking for.

• This can be tested by looking at how much "baryon" number is at mid-rapidity



• Distribution of zero baryon number and net baryon number particles as a function of rapidity (from BRAHMS)

• Each beam nucleon looses 73 ± 6 GeV on the average that goes into creating new particles. Therefore there is 26 TeV worth of energy available for particle production.

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Phases of a collision



The "initial" energy density is given by the Bjorken formula



PHENIX (triangles), STAR(stars), BRAHMS (circles) PHOBOS (crosses) particle ratios, at Au+Au (s=200 GeV) at mid-rapidity vs thermal ensemble predictions.

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- In an off-center collision, an initial elliptic pattern is produced.
- If the subsequent interactions are weak particles are free streaming and this elliptic pattern is wiped-out
- If the interactions are strong, this pattern persists and is visible in the detectors.

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Elliptic flow

$$\frac{1}{p_T} \frac{dN}{dp_T \, d\phi} = \frac{1}{p_T} \frac{dN}{dp_T} \, \left(1 + \frac{2 v_2(p_T)}{\cos(2\phi)} + \dots\right)$$



Elliptic flow is large X:Y $\sim 2.0:1$

• Such Elliptic flow has been observed recently in strongly coupled cold gases.

Elliptic flow in ultracold gases



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Elliptic flow data from STAR as a function of p_T (right) compared to relativistic hydrodynamics calculations with non-zero shear viscosity, from Luzum+Romanschke (2008). (see also PHOBOS data)

• Finite-temperature (equilibrium) relativistic hydrodynamics describes well the data with

$$rac{\eta}{s}\simeq$$
 (0.08 – 0.16) \hbar

- Perturbative (weak-coupling) QCD gives a large ratio: $\frac{\eta}{s} \simeq \frac{1}{g^4 \log(1/g)} \sim (5-10) \hbar$.
- Conclusion : The QGP produced is strongly coupled.

Lessons from RHIC data

- Very quickly after the collision a ball of QGP forms.
- It is in thermodynamic equilibrium and well into the QGP phase
- Its expansion is well described by relativistic fluid mechanics with $\frac{\eta}{s} \sim (0.08 0.16)\hbar$.
- After its temperature falls below $\sim 170~\text{MeV}$ the fluid hadronizes and the hadrons eventually reach the detectors.
- In the QGP phase the coupling is strong and there is jet-quenching.
- Heavy flavor production (charm and up) is suppressed.

Viscosity



 Viscosity: introduced by Claude Navier (1822) into what now is called the Navier-Stokes equation

 Defined in terms of the friction force between two plates (shear viscosity)
 F = η A dux/dy
 It is related to entropy production

$$\frac{\partial s}{\partial t} = \frac{\eta}{T} \left[\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial \cdot v \right]^2 + \frac{\zeta}{T} (\partial \cdot v)^2$$

- In the kinetic theory, viscosity is to due to collisions.
- \bullet Hydro is valid as an effective description when relevant length scales \gg mean-free-path

Maxwell : $\eta \sim \rho \ v \ \ell \sim \text{density} \times \text{velocity} \times \text{mean} - \text{free} - \text{path}$

• Viscosity becomes large when interactions become weak. (Paradox?)

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Viscosities of fluids

Table 8.4.1. Viscosities η for some common relations (10⁻² erg s/cm³).

Substance	Temperature	Viscosity (cp)
Air	18°C	0.018
Water	$0^{\circ}C$	1.8
Water	20°C	1
Water	$100^{\circ}C$	0.28
Glycerin	20°C	1500
Mercury	20°C	1.6
n-Fentane	20°C	0.23
Argon	85K	0.28
11e ⁴	4.2K	0.033
Superfluid He ⁴	$< 2.1 \mathrm{K}$	Q
Glass		$> 10^{15}$
692 - 39 (552) - 539(53)(59)		

Note that, by popular convention, the designation "glass" is applied to any disordered material once 'ts viscosity exceeds 10^{15} cp.

Viscosity spans a large range of magnitudes

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Liquid viscosity can be large

The pitch-drop experiment: University of Queensland





- 8 drops since 1930
- No one has witnessed a drop fall
- \checkmark Viscosity $\sim 10^{11}$ times of water
- 2005 Ig Nobel Prize in Physics

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Viscosity over entropy density

- $\frac{\eta}{s}$ has dimensions of action/angular momentum
- In the kinetic theory we can estimate

$$\eta\sim
ho~v~\ell~~,~~s\sim n\simrac{
ho}{m}$$

$$rac{\eta}{s} \sim rac{
ho \ v \ \ell}{rac{
ho}{m}} \sim mv \ \ell \sim \hbar \ rac{\ell}{rac{\hbar}{mv}} \sim \hbar \ rac{ extsf{mean-free-path}}{ extsf{de Broglie wavelength}}$$

 \bullet We expect that mean-free-path $\ \gtrsim \ {\rm de} \ {\rm Broglie} \ {\rm wavelength}$ so that

$$rac{\eta}{s}$$
 \gtrsim pure number \cdot \hbar

• Therefore we expect a lower bound on $\frac{\eta}{s}$.

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 η/s for real materials

- $\frac{\eta}{s}$ reaches a minimum near the critical point of a liquid-gas transition.
- This does not happen at the critical point as η diverges there (as (correlation length)^{$x\eta$}, with $x_{\eta} \sim 0.05 0.07$.)

Substance	$\left(\frac{\eta}{s}\right)_{\min}$	Substance	$\left(\frac{\eta}{s}\right)_{\min}$	Substance	$\left(\frac{\eta}{s}\right)_{\min}$
H, He	8.8				
Ne	17	H_2O	25	CO	35
Ar	37	H_2S	35	CO_2	32
Kr	57	N_2	23	SO_2	39
Xe	84	O_2	28		

- The values above are in units of $\hbar/(4\pi)$. The minimum on the table are for the most "quantum" liquids, H, He.
- For superfluids, the normal component has finite shear viscosity (measured recently).

• For the QGP we have seen that experiment indicates $\frac{\eta}{s} \sim 1-2$ in the same units.

This seems correlated with the fact that it is a strongly coupled system.

- QGP is so far the fluid with the minimum ratio $\frac{\eta}{s}$
- There is a new silver medal winner: ultra-cold strongly-coupled fermion gasses
- In normal units for the QGP :

$$\eta \simeq \frac{\hbar}{4\pi} \ s \sim \frac{10^{-27} \text{ erg} \cdot \text{s}}{(10^{-13} \text{ cm})^3} \sim 10^{14} \text{ cp}$$

The QGP is almost a "glass" \rightarrow Quark-Gluon Glasma

• Is there a theory that gives such low values for the shear viscosity/entropy ratio?

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Gauge theories with many colors

• Gauge theories with N-colors (SU(N) gauge group) have a single continuous parameter: the gauge coupling constant g_{YM} .

• When N is large ($N \to \infty$) there is another way of reorganizing the theory:

$$N
ightarrow \infty$$
 , keep $\lambda \equiv g_{YM}^2 N$ fixed

• The expansion in powers of 1/N is similar to the topological expansion of a string theory with

$$g_{ ext{string}} \sim rac{1}{N}$$

• When $N \to \infty$ and $\lambda \to 0$ we can use perturbation theory to calculate.

• When $N \to \infty$ and λ is large, we are at strong coupling.

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The gauge-theory/gravity duality

• The gauge-theory/gravity duality is a duality that relates a string theory with a (conformal) gauge theory.

• The prime example is the AdS/CFT correspondence

Maldacena 1997

• It states that N=4 four-dimensional SU(N) gauge theory (gauge fields, 4 fermions, 6 scalars) is equivalent to ten-dimensional IIB string theory on $AdS_5 \times S^5$

$$ds^{2} = \frac{\ell_{AdS}^{2}}{r^{2}} \left[dr^{2} + dx^{\mu} dx_{\mu} \right] + \ell_{AdS}^{2} \ (d\Omega_{5})^{2}$$

This space (AdS_5) is non-compact and has a single boundary, at r = 0.

• The string theory has as parameters, g_{string} , ℓ_{string} , ℓ_{AdS} . They are related to the gauge theory parameters as

$$g_{YM}^2 = 4\pi \ g_{\text{string}}$$
, $\lambda = g_{YM}^2 \ N = \frac{\ell_{AdS}^4}{\ell_{\text{string}}^4}$

• As
$$N o \infty$$
, $g_{{
m string}} \sim rac{\lambda}{N} o 0$.

• As $N \to \infty$, $\lambda \gg 1$ implies that $\ell_{\text{string}} \ll \ell_{AdS}$ and the geometry is very weakly curved. String theory can be approximated by gravity in that regime and is weakly coupled.

• As $N \to \infty$, $\lambda \ll 1$ the gauge theory is weakly coupled, but the string theory is strongly curved.





- There is one-to-one correspondence between on-shell string states $\Phi(r, x^{\mu})$ and gauge-invariant (single-trace) operators $O(x^{\mu})$ in the sYM theory
- In the string theory we can compute the "S-matrix", $S(\phi(x^{\mu}))$ by studying the response of the system to boundary conditions $\Phi(r = 0, x^{\mu}) = \phi(x^{\mu})$
- $\bullet\,$ The correspondence states that this is equivalent to the generating function of c-correlators of $O\,$

$$\langle e^{\int d^4x \ \phi(x) \ O(x)} \rangle = e^{-S(\phi(x))}$$

Gubser+Klebanov+Polyakov, Witten, 1998

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The gauge-theory at finite temperature

• The finite temperature ground state of the gauge theory corresponds to a different solution in the dual string theory: the AdS-Black-hole solution *E. Witten, 1998*

$$ds^{2} = \frac{\ell_{AdS}^{2}}{r^{2}} \left[\frac{dr^{2}}{f(r)} + f(r)dt^{2} + dx^{i}dx_{i} \right] + \ell_{AdS}^{2} (d\Omega_{5})^{2} \quad , \quad f(r) = 1 - (\pi T)^{4}r^{4}$$

- The horizon is at $r = \frac{1}{\pi T}$
- As the temperature increases, the horizon size increases, reaching the boundary at $T = \infty$.

Boost-invariant expansion and AdS/CFT

• Bjorken has guessed correctly in 1983 that a heavy-ion collision will be described in its later stages as a boost invariant expansion of a relativistic fluid: densities will depend only on $\tau^2 = (x^0)^2 - (x^3)^2$.

• Moreover, from scale invariance $\rho = T_{00} \sim \tau^{-\frac{4}{3}}$, instead of the freestreaming option $\rho = T_{00} \sim \tau^{-1}$. This implies that $T \sim \tau^{-\frac{1}{3}}$ and that the entropy remains approximately constant.

• This symmetric late time behavior was first justified by finding it as a (non-singular) solution of the dual gravitational equations corresponding to a bulk black hole where the horizon position shrinks with time.

Subleading terms in this gravitational solution indicated the presence of small viscosity.

• Finally it was shown in general that large-wavelength solutions to the AdS Einstein equations generate the relativistic Navier-Stokes equation with an infinite series of "viscous" corrections that generate dissipation. Bhattacharya+Hubeny+Minwalla+Rangamani, 2008

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The viscosity calculation

• The viscosity is obtained in the gauge theory by the Kubo formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d^3 x \ e^{i\omega t} \langle [T^{xy}(t, \vec{x}), T^{xy}(0, \vec{0})] \rangle$$

• In the gravitational theory it is obtained by the absorption cross-section of a very-low energy transverse graviton from the black hole.

• General theorems in gravity tell us that the low energy limit for the absorption of lowenergy gravitons is given by the geometrical horizon area.

- The Hawking-Bekenstein principle also tells us that the entropy of the system is proportional to the area of the horizon.
- This eventually leads to

$$rac{\eta}{s} = rac{\hbar}{4\pi} \left[1 - \mathcal{O}\left(rac{1}{\lambda^{rac{3}{2}}}
ight)
ight],$$

in the strong coupling limit.

Policastro, Starinets, Son, 2001

- This is universal in all strongly-coupled theories that have weakly coupled gravity duals.
- It is conjectured that this is an absolute lower bound!

Kovtun, Son, Starinets, 2003

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Holographic QCD

- A precise holographic model for QCD is notoriously difficult to pin down.
- Using input from string theory and QCD we can construct holographic "models" for QCD Polchinski+Strassler,2001, Erlich+Katz+Son+Stephanov, DaRold+Pomarol, 2005 Gursoy+Kiritsis+Nitti, 2007
- The most sophisticated one contains bulk fields $g_{\mu\nu}$ dual to $T_{\mu\nu}$ and ϕ dual to $Tr[F^2]$, generating an Einstein-dilaton system with a potential in 5 dimensions (improved holographic QCD).
- The 't Hooft coupling of QCD is given by $\lambda = e^{\phi}$. The action is

$$S = M^{3} \int d^{5}x \left[R - \frac{4}{3} \left(\frac{\partial \lambda}{\lambda} \right)^{2} + V(\lambda) \right]$$

• By using the following UV and IR asymptotics of the potential

$$V(\lambda) = \frac{12}{\ell^2} \left[1 + v_0 \lambda + v_1 \lambda^2 + \cdots \right] \quad , \quad V(\lambda) \sim \lambda^{\frac{4}{3}} \sqrt{\log \lambda} + \cdots$$

we obtain agreement on both T = 0 properties (glueball spectra) and finite temperature static properties.

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Thermodynamic variables



Figure 4: (Color online) Same as in fig. 1, but for the s/T^3 ratio, normalized to the SB limit. From M. Panero, arXiv:0907.3719

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Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4 .

From M. Panero, arXiv:0907.3719

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The speed of sound



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The bulk viscosity



The shear viscosity is the same as in CFT $\frac{\eta}{s} = \frac{\hbar}{4\pi}$

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A new paradigm for gravity

• Holographic ideas suggest that gravity may be an emergent force, that is semiclassical in the limit of a large number of particle species.

• The thermodynamic properties seen in gravity, are in agreement with such a picture

• In this sense gravity is universal because it is a sort of thermodynamics for QFT. If the QFT is strongly coupled, gravity is weakly coupled and vice versa.

• This picture and the relationship to strongly coupled gauge theories explains the black-hole microstates responsible for the Bekenstein-Hawking entropy.

• This new paradigm may have fundamental implications for cosmology

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Lessons for cosmology

• Untill now the QGP filling the Universe before 10^{-6} sec, was considered weakly coupled, so nothing of interest.

- New effects are in order, at strong coupling
- At very low viscosity the cosmological flow may be (most probably) turbulent.

Turbulence can have effects on perturbations, generating stochasticity, and perturbing existing fluctuations.

A new analysis of the deconfinement phase transition period is necessary (cross over+ turbulence)

• A framework must be found to describe strongly-coupled holographic matter coupled to usual gravity. This is provided by asymptotically AdS gravity with a UV cutoff, dual to Randall-Sundrum cosmology!

Binetruy+*Deffayet*+*Langlois*

• This can also address (relatively) cold "gravitational nuclei", ie neutron stars.

• In the long term, the AdS/CFT correspondence may shed light into the nature of cosmological/spacelike singularities.

In particular, the cosmological evolution may be viewed as an avatar of the entropic evolution of matter. If correct that may give a completely different perspective to the underlying principles of cosmology.

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- More data and improvements are expected to come from RHIC, in particular concerning heavy quarks.
- \bullet A new era for heavy-ion experiments starts with $\ensuremath{\mathsf{LHC}}$
- We have direct experimental access to "little bangs": tiny models of expanding hot matter as it happened microseconds after the big bang.
- Important theoretical challenges for holographic strong-coupling techniques wait ahead for ensembles with a large baryon chemical potential.
- They are expected to shed light into what is going on inside neutron stars and other more exotic conglomerations of nuclear matter (strange-stars).
- It may provide the largest set of universality classes of quantum critical behavior at finite density, relevant for adressing strongly correlated electroins in real and designer materials (strange metals, high- T_c superconductors, graphene, etc).
- It may change radically our view of the gravitational interaction and its avatars.
- This is an exciting domain which brings together, hydrodynamics, statistical mechanics, quantum field theory, gravity/string theory, and eventually astrophysics and cosmology.
- Despite the progress, it seems we are still far from understanding strongly coupled dense matter and/or gravity.

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Collaborators

My Collaborators

- Umut Gursoy (Utrecht \rightarrow CERN)
- Liuba Mazzanti (Santiago de Compostella)
- George Michalogiorgakis (Purdue)
- Fransesco Nitti (APC)
- Angel Paredes (Barcelona)

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- Three LHC experiments will investigate heavy-ion physics at 5.5 TeV/nucleo (30 times more than RHIC): ALICE, CMS, ATLAS
- New regimes will be explored and many more heavy quarks are expected to be produced.



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QCD phase diagram



(from Ruester et al., hep-ph/0503184)

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The Bjorken Relation

• Consider that after the collision of the nuclear pancakes a lot of particles are produced at $t = \tau$. These are confined in a slice of longitudinal width dz and transverse area A.

- The longitudinal velocities have a spread $dv_L = \frac{dz}{\tau}$.
- Near the middle region $v_L
 ightarrow 0$

$$\frac{dy}{dv_L} = \frac{d}{dv_L} \left[\frac{1}{2} \log \frac{1 + v_L}{1 - v_L} \right] = \frac{1}{1 - v_L^2} \simeq 1$$

• We may now write

$$dN = dv_L \; rac{dN}{dv_L} \simeq rac{dz}{ au} \; rac{dN}{dy} \quad
ightarrow \quad rac{dN}{dz} \simeq rac{1}{ au} \; rac{dN}{dy}$$

• If $\langle E_T \rangle \simeq \langle m_T \rangle$ is the average energy per particle then the energy density in this area at $t = \tau$ is given by the Bjorken formula:

$$\langle \epsilon(\tau) \rangle \simeq \frac{dN \langle m_T \rangle}{dz \ A} = \frac{1}{\tau} \frac{dN}{dy} \frac{\langle m_T \rangle}{A} = \frac{1}{\tau} \frac{dE_T^{\text{total}}}{dy}$$

• It is valid if (1) τ can be defined meaningfully (2) The crossing time $\ll \tau$.

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Viscous elliptic flow:PHOBOS data



Elliptic flow data from PHOBOS as a function of centrality compared to relativistic hydrodynamics calculations with non-zero shear viscosity. RETURN

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Glauber initial conditions

• We model the nucleus with the Woods-Saxon density distribution

For Au, A = 197, $R \simeq 6.4$ fm, $\chi = \simeq 0.54$ fm, and $\int d^3x \rho(\vec{x}) = A$.

• The nuclear thickness function is defined as

$$T_a(x_\perp) = \int_{-\infty}^{\infty} dz \
ho(ec{x})$$

• We can calculate the number density of nucleons participating in the collision as

$$n_{\text{part}}(x,y,b) = T_A\left(x+\frac{b}{2},y
ight) [1-P(x,y)] + (b o -b) \quad , \quad P(x,y) = 1 - \left(1 - \frac{\sigma T_A\left(x-\frac{b}{2},y\right)}{A}\right)^A$$

P is the probability of finding at least one nucleon of the second nucleus in position (x,y) and σ is the nucleon-nucleon cross-section. The number density of binary collisions

$$n_{\text{coll}}(x, y, b) = \sigma T_A\left(x + \frac{b}{2}, y\right) T_A\left(x - \frac{b}{2}, y\right)$$

(The two nuclei are at (b/2,0) and (-b/2,0).)

• The centrality is determined by the total number of participating nucleons, $N_{\text{part}}(b) = \int d^2x n_{\text{part}}$ and the initial energy density from

$$\epsilon(\tau = \tau_0, x, y, b) = \text{constant} \cdot n_{\text{coll}}(x, y, b)$$

• The constant is fitted to the data.

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The Color Glass Condensate initial conditions

• The number density of gluons, produced during the collision of two nuclei is given by

$$\frac{dN}{d^2x_T dY} = \mathcal{N} \int \frac{d^2p_T}{p_T^2} \int^{p_T} d^2k_T \, \alpha_s(k_T) \phi_A(x_1, (\vec{p}_T + \vec{k}_T)^2/4, \vec{x}_T) \phi_A(x_1, (\vec{p}_T - \vec{k}_T)^2/4, \vec{x}_T)$$

 \vec{P}_T and Y are the transverse momentum and rapidity of produced gluons, $x_{1,2} = p_T e^{\pm Y} / \sqrt{s}$ is the momentum fraction of colliding gluon ladders. \mathcal{N} is fitted to data.

• The gluon distribution function is

$$\phi_A(x, k_T^2, \vec{y}) = \frac{1}{\alpha_s(Q_s^2)} \; \frac{Q_s^2}{max[Q_s^2, k_T^2]} \; P_T(\vec{y}) \; (1-x)^4$$

and P is the probability of finding at least one nucleon in position \vec{y}

$$P_T(\vec{y}) = 1 - \left(1 - \frac{\sigma T_A(\vec{y})}{A}\right)^A$$

• The saturation scale is taken to be

$$Q_s^2(x, \vec{y}) = 2 \text{ GeV}^2 \left(\frac{T_A(\vec{y})/P_T(\vec{y})}{1.53/fm^2}\right) \left(\frac{0.01}{x}\right)^{\lambda}$$

with $\lambda \simeq 0.288$

• The initial energy density is given by

$$\epsilon(\tau = \tau_0, \vec{y}, b) = \text{constant} \times \left[\frac{dN}{d^2 x_T dY}\right]^{\frac{2}{3}}$$

The (most) perfect fluid,

Kinetic viscosity

• From fluid mechanics the shear stress τ is the rate of change of velocity with distance perpendicular to the direction of movement per unit area:

$$\tau = \frac{F}{A} = \eta \ \frac{du_y}{dx}.$$

Interpreting shear stress as the time rate of change of momentum, p, per unit area (rate of momentum flux) of an arbitrary control surface gives

$$\tau = \frac{\dot{p_y}}{A} = \frac{\dot{m}}{A} \ u_y.$$

Further manipulation will show

$$\dot{m} = \rho \bar{u_x} A$$
 and $\Delta u_y \simeq \lambda \frac{du_y}{dx}$

so that

$$\tau \simeq \underbrace{\rho \bar{u} \lambda}_{\eta} \cdot \frac{du}{dx} \quad \Rightarrow \quad \eta \simeq \rho \bar{u} \lambda$$

RETURN

The (most) perfect fluid,

The bulk viscosity: theory

• It is one of the important parameters for QGP hydrodynamics (along with the shear viscosity).

- It is related to entropy production (measurable at RHIC and LHC)
- It is defined from the Kubo formula

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} Im \ G_R(\omega) \quad , \quad G_R(\omega) \equiv \int d^3x \int dt \ e^{i\omega t} \theta(t) \ \langle 0|[T_{ii}(\vec{x},t),T_{ii}(\vec{0},0)]|0\rangle$$

Using a parametrization $ds^2 = e^{2A}(fdt^2 + d\vec{x}^2 + \frac{dr^2}{f})$ in a special gauge $\phi = r$ the relevant metric perturbation decouples Gubser+Nellore+Pufu+Rocha

$$h_{11}'' = -\left(-\frac{1}{3A'} - A' - \frac{f'}{f}\right)h_{11}' + \left(-\frac{\omega^2}{f^2} + \frac{f'}{6fA'} - \frac{f'}{f}A'\right)h_{11}$$

with

$$h_{11}(0) = 1$$
 , $h_{11}(r_h) \simeq C \left| e^{i\omega t} \right| \log rac{\lambda}{\lambda_h} \Big|^{-rac{i\omega}{4\pi T}}$

The correlator is given by the conserved number of h-quanta

$$Im \ G_R(\omega) = -4M^3 \mathcal{G}(\omega) \quad , \quad \mathcal{G}(\omega) = \frac{e^{3A}f}{4A'^2} |Im[h_{11}^*h_{11}']|$$

finally giving

$$\frac{\zeta}{s} = \frac{C^2}{4\pi} \left(\frac{V'(\lambda_h)}{V(\lambda_h)}\right)^2$$

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The (most) perfect fluid,



T. Schäfer, Phys. Rev. A **76**, 063618 (2007). *A. Turlapov, J. Kinast, B. Clancy, Le Luo, J. Joseph, J. E. Thomas, J. Low Temp. Phys.* **150**, 567 (2008)

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The (most) perfect fluid,

N=4 sYM (CFT) vs QCD

- We have seen that the strongly-coupled CFT fluids obey relativistic hydrodynamics with $\frac{\eta}{s} = \frac{\hbar}{4\pi}$ at low energies.
- Experimental data on QGP indicate an $\frac{\eta}{s} = 1 2$ in units of $\frac{\hbar}{4\pi}$.
- This is remarkable but..... QCD is certainly different from N=4 sYM.

• QCD is not conformally invariant, and has only gluons and quarks. However experience with holographic theories suggests it might not be that far away in the region probed by the experiments 100 MeV < T < 500 MeV.

• The bulk viscosity ζ of any CFT is identically zero.

• All simulations of QGP so far use $\zeta = 0$ because (a) ζ is essentially not known for QCD (b) the approximate conformal invariance of QCD suggest that it might be negligible.

• For almost all quantities computed so far in QCD the difference between N = 3 and $N = \infty$ is at most 5%.

The (most) perfect fluid,



• In p-p and in d-Au collisions high- p_T jets appear back-to-back.



- This is not the case in Au-Au central collisions
- This is strong evidence for jet-quenching



• R_{AA} is the ratio of π^0 cross section at mid-rapidity in Au+Au central or d-Au collisions to that in p-p collisions corrected for the multiplicity.

• R_{AA} is small in Au+Au because the medium strongly interacts and reduces the rate of production of pions for the same momentum.

The (most) perfect fluid,

Detailed plan of the presentation

- Title page 0 minutes
- Quantum Chromodynamics 1 minutes
- Confinement 3 minutes
- Deconfinement 5 minutes
- A weakly coupled plasma? 7 minutes
- The experimental saga 9 minutes
- RHIC 10 minutes
- RHIC head-on collision 12 minutes
- RHIC collision video 14 minutes
- The mid-rapidity range 17 minutes
- Phases of a collision 20 minutes
- Is there thermal equilibrium? 22 minutes
- Ellipticity 23 minutes
- Elliptic flow 25 minutes
- Hydrodynamic elliptic flow 28 minutes
- Elliptic flow in ultracold gases 30 minutes

- Lessons from the RHIC data 31 minutes
- Viscosity 33 minutes
- Viscosities of fluids 35 minutes
- Liquid Viscosity can be large 36 minutes
- Viscosity over entropy density 37 minutes
- η/s for real materials 39 minutes
- Gauge theories with many colors 41 minutes
- The gauge-theory/gravity duality 45 minutes
- The gauge-theory at finite temperature 46 minutes
- Boost-invariant expansion and AdS/CFT 48 minutes
- The viscosity calculation 50 minutes
- Holographic QCD 52 minutes
- Thermodynamic variables 53 minutes
- Equation of state 54 minutes
- The speed of sound 55 minutes
- The bulk viscosity 56 minutes
- Lessons for cosmology 58 minutes
- Outlook 59 minutes
- LHC 60 minutes
- Collaborators 60 minutes
- Bibliography 61 minutes

- The QCD Phase Diagram 63 minutes
- The Bjorken Relation 65 minutes
- Viscous elliptic flow: PHOBOS data 67 minutes
- Glauber initial conditions 69 minutes
- Color Glass Condensate initial conditions 71 minutes
- Kinetic viscosity 73 minutes
- The bulk viscosity: theory 76 minutes
- Viscosity in ultracold gases 78 minutes
- N=4 sYM (CFT) vs QCD 80 minutes
- Jet Quenching 84 minutes