Higgs in the light of Hadron Collider limits: impact on a 4th generation

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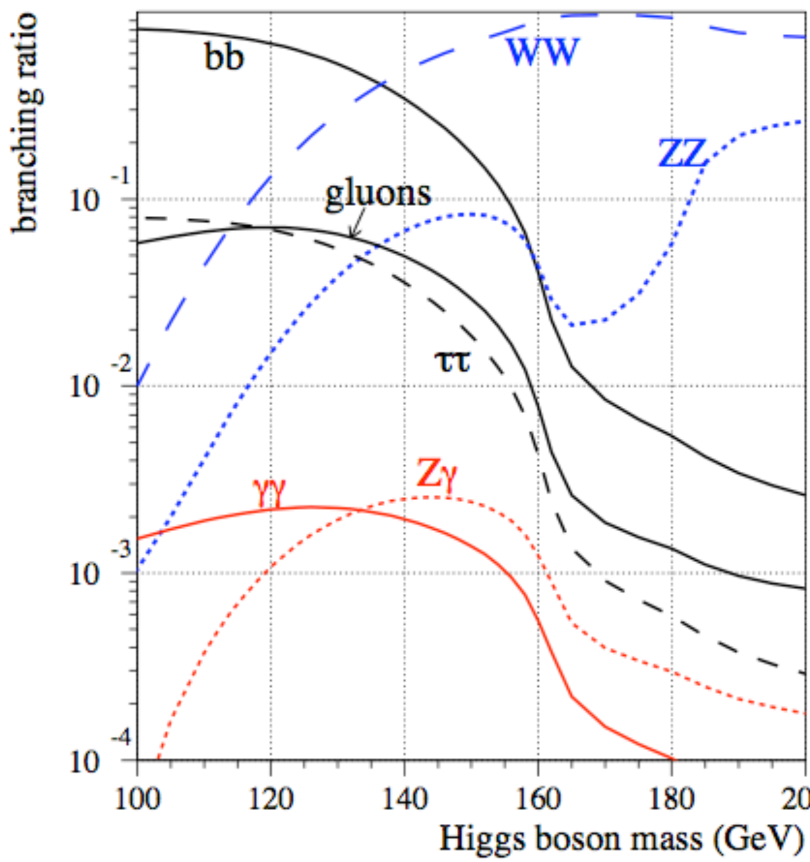
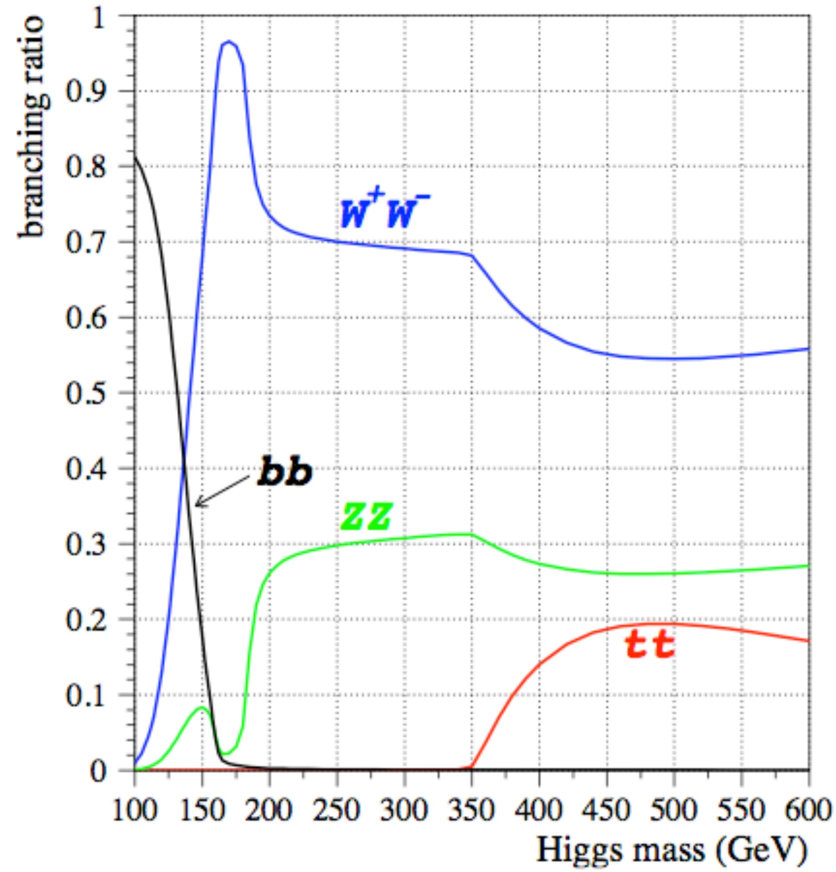
Outline

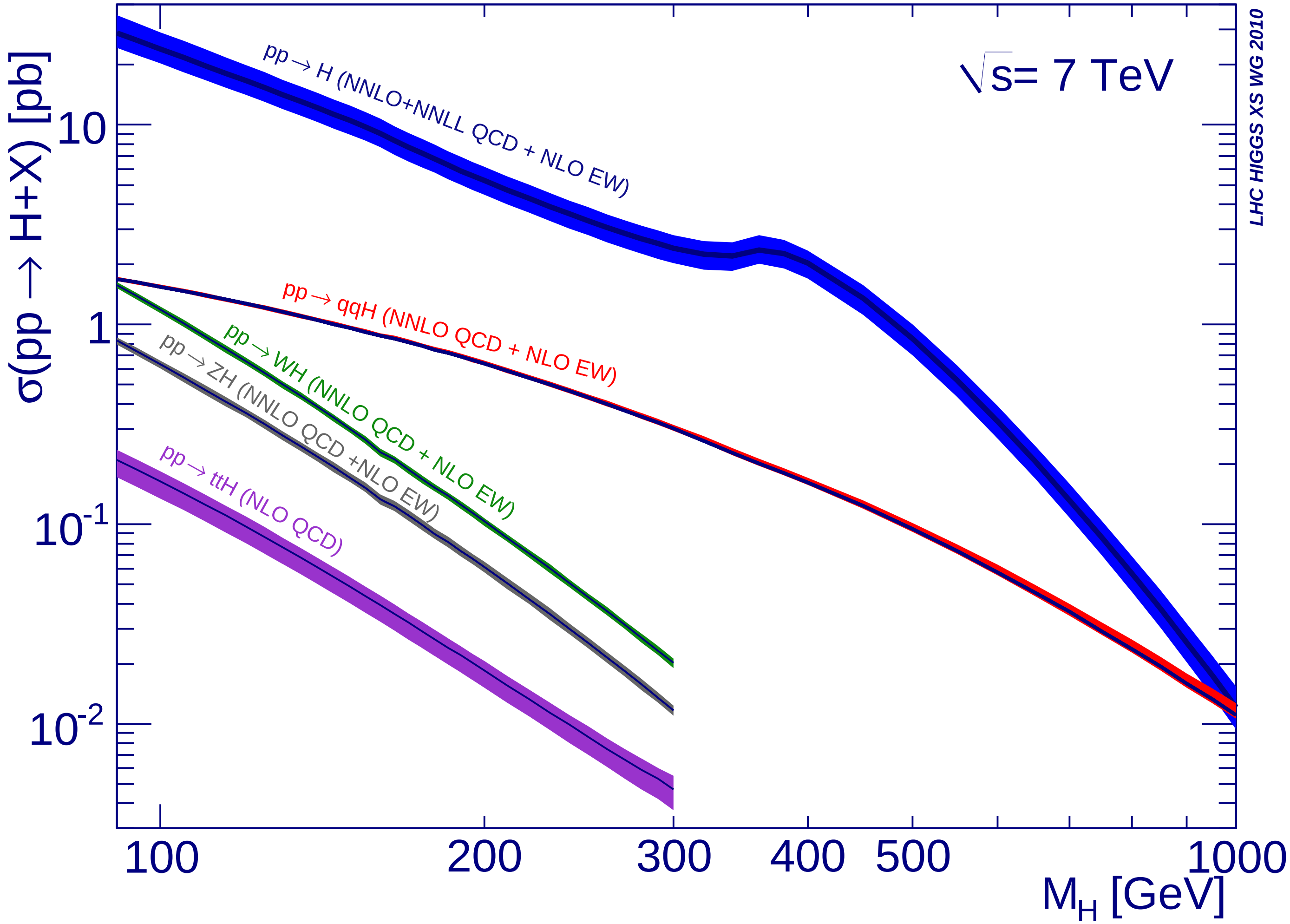
- Review of Experimental Status.
- Eliminating a 4th generation for a light SM-like Higgs.
- Eliminating a sequential W for a light SM-like Higgs.
- Eliminating a 4th generation if there is a light 2HDM A.
- Eliminating a 4th generation using the MSSM Higgs sector.

SM Higgs Cross Sections and Branching Ratios

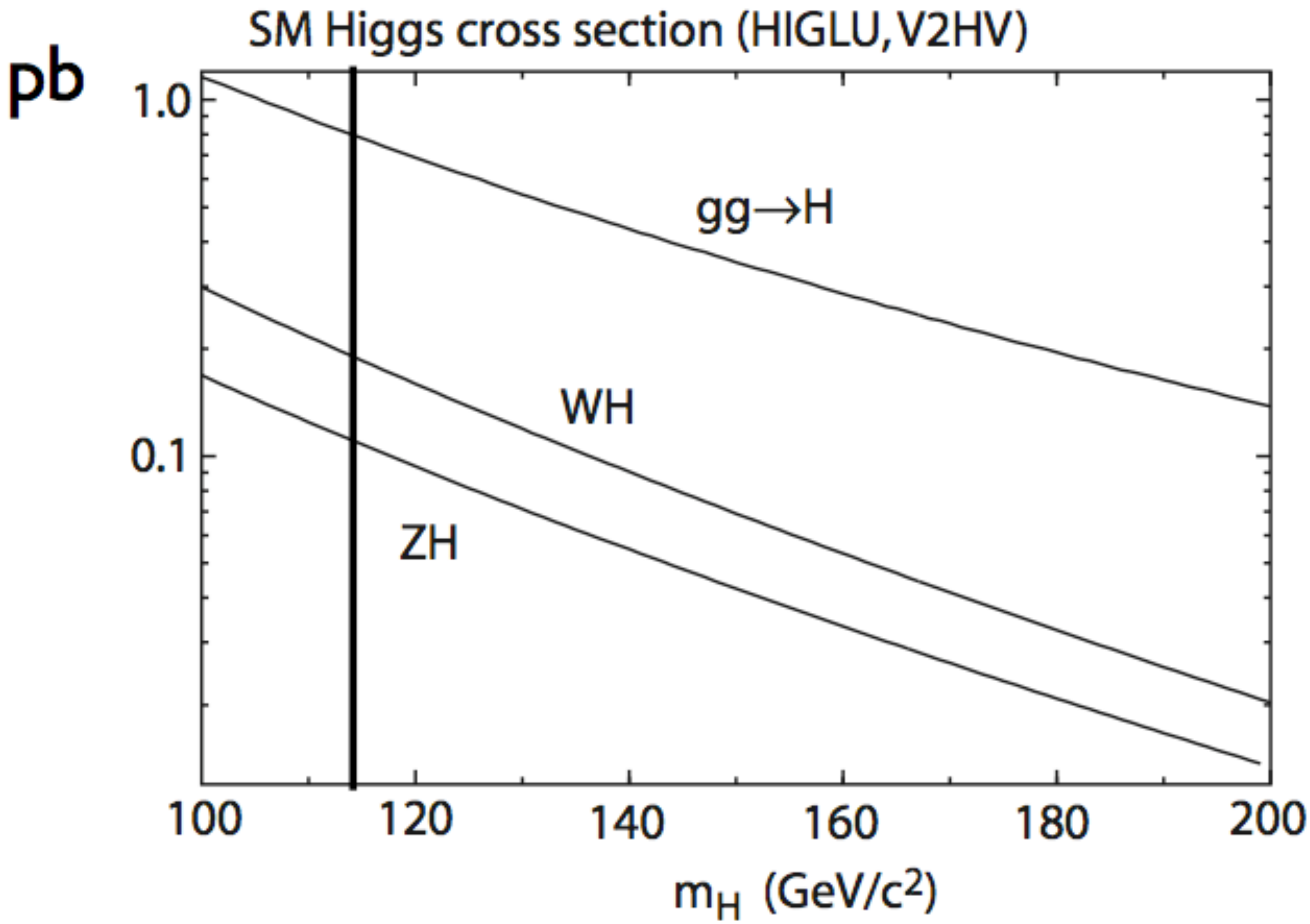




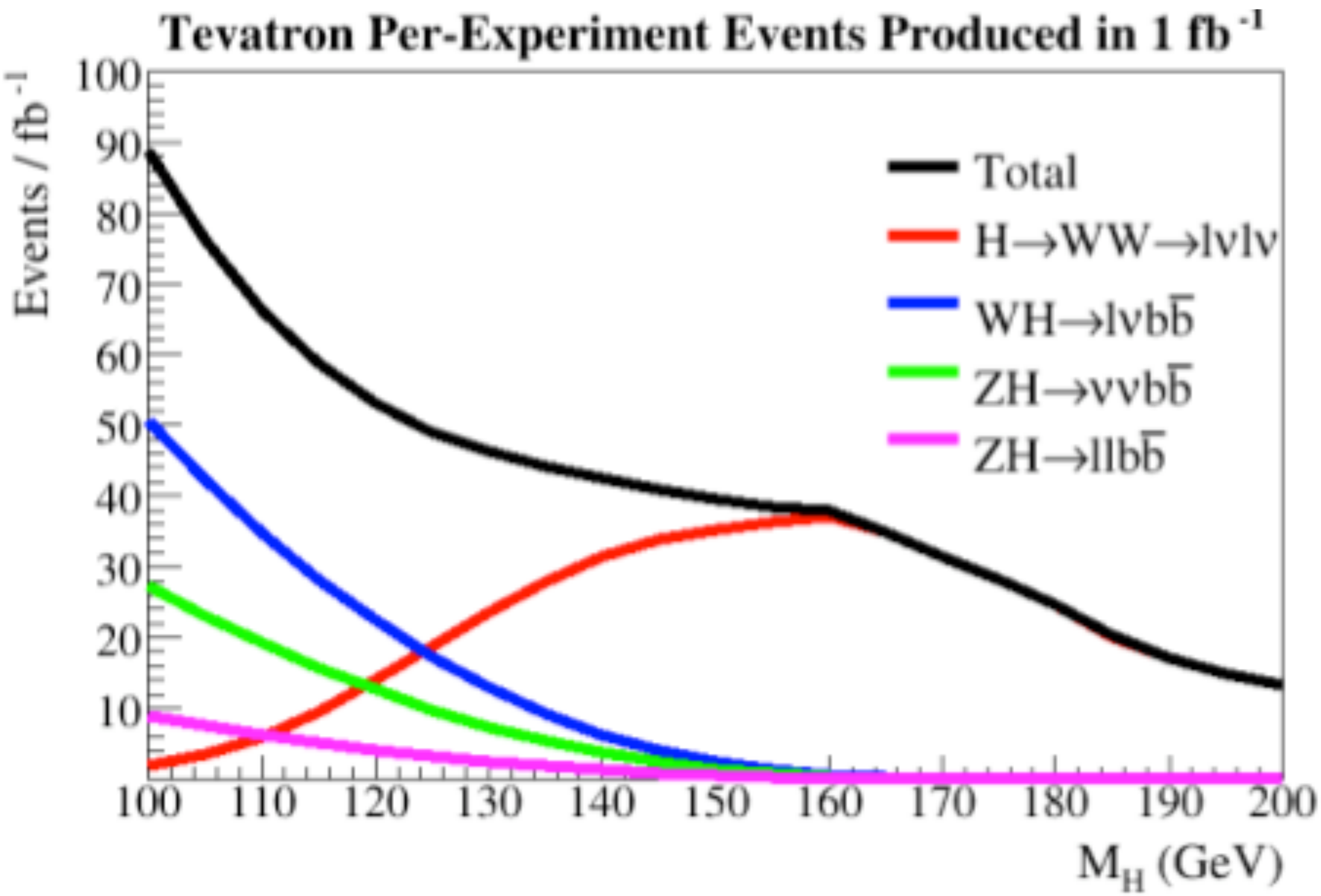
Main thing to note is $B(h_{\rm SM} \to \gamma\gamma) < 10^{-4}$ for $m_{h_{\rm SM}} > 180~{\rm GeV}$, due to onset of WW, ZZ decays. We will see what happens for a CP-odd Higgs for which such decays are absent (at tree level).



Up to 300 GeV, $\sigma(gg \to h_{\rm SM}) > 2$ pb.



At $m_{h_{\rm SM}} \sim 200$ GeV, $\sigma(gg \to h_{\rm SM}) \sim 0.15$ pb.



Translation into numbers of events per $1\ \mathrm{fb}^{-1}$. In particular, note that at $m_{h_{\rm SM}} \sim 120\ \mathrm{GeV}$ there will be of order 70 events for $L = 5\ \mathrm{fb}^{-1}$ in $gg \to h_{\rm SM} \to WW \to \ell\nu\ell\nu$. This means they have decent constraints on this channel down to $120\ \mathrm{GeV}$.

SM Status and a 4th Generation

- Let us first recall where a 4th generation can enter.

1. $gg \to h$ depends on quark loops. The more loops the better and heavy quark loops are maximally potent.
   Note, however, that 4th generation quark loops do not significantly affect $B(h \to WW)$.
2. $h \to \gamma\gamma$ depends on quark loops and W loops. Adding a 4th generation and/or a sequential W' will certainly change the expected branching ratio.
   Recall that quark loops enter with opposite sign to W loops. So adding a 4th generation reduces $B(h_{\rm SM} \to \gamma\gamma)$ while adding a W' will strongly increase the branching ratio.
3. If you are interested in $gg \to h_{\rm SM} \to \gamma\gamma \propto \Gamma_{gg} B(h_{\rm SM} \to \gamma\gamma)$ then adding a 4th generation increases the 1st factor but decreases the 2nd factor. For smaller $m_{h_{\rm SM}}$ these effects nearly cancel leaving you with little change.

- **Limits on a SM Higgs are getting strong.**

  Using all channels gives the following plot.

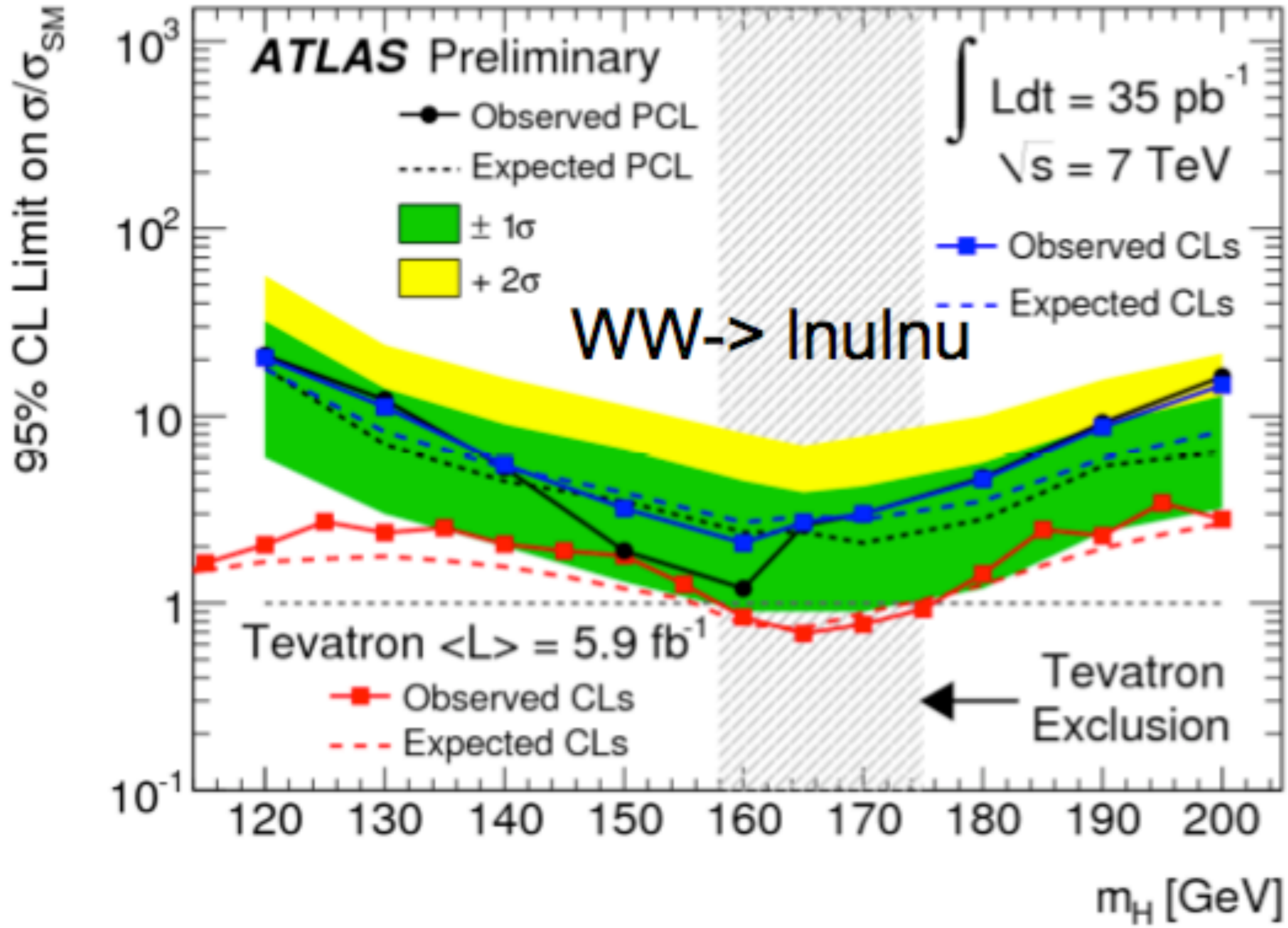


Figure 1: Limits on SM Higgs cross section relative to expectation. Tevatron currently places stronger limits than LHC.

If we focus just on the $h_{\rm SM} \to \gamma\gamma$ signal, we get the following limits after combining CDF and D0 data.

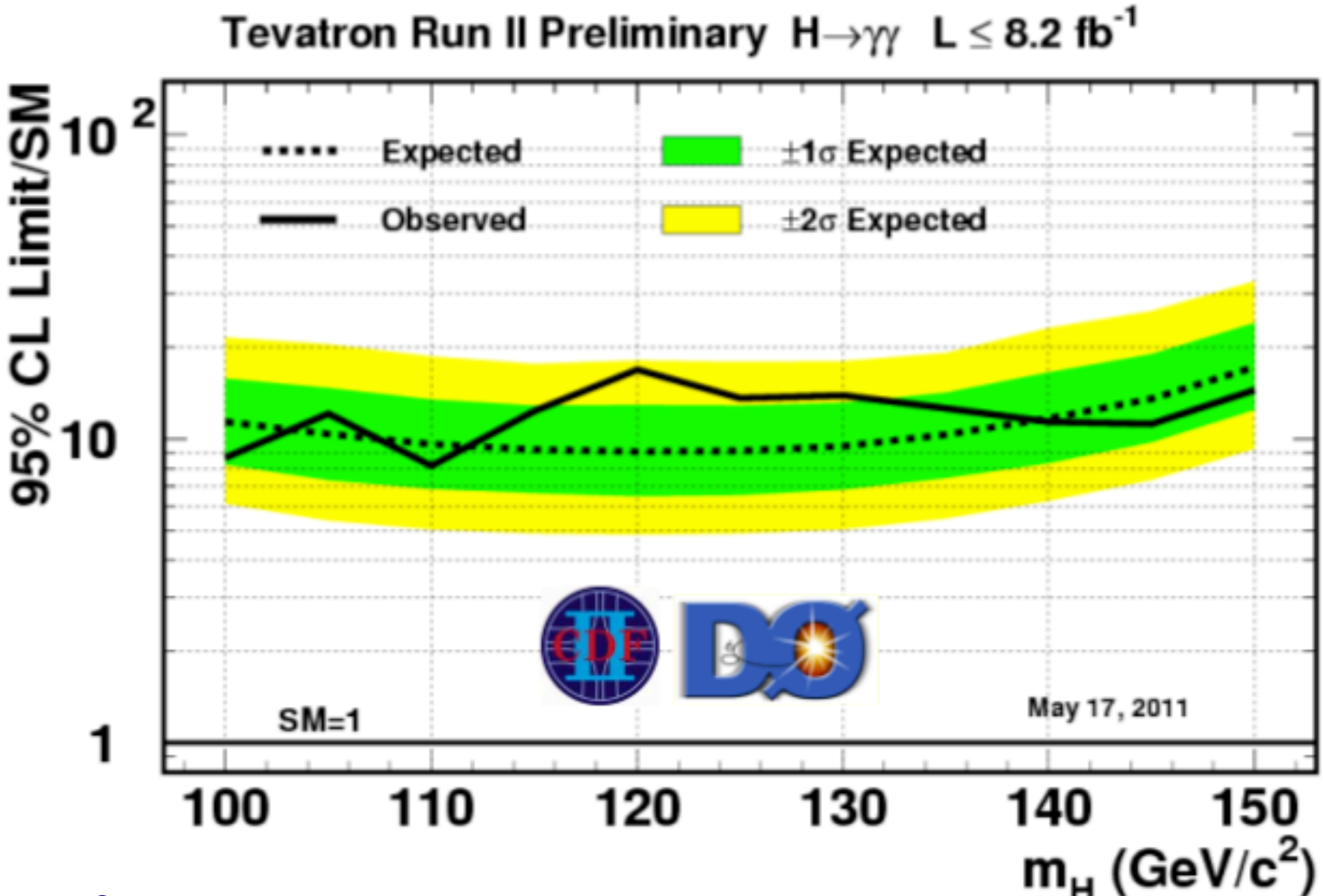


Figure 2: Limits on Higgs to $\gamma\gamma$ channel relative to SM expectation. Tevatron currently places stronger limits than LHC.

At the LHC, only ATLAS has presented a result. $L = 131\ \mathrm{pb}^{-1} \Rightarrow$

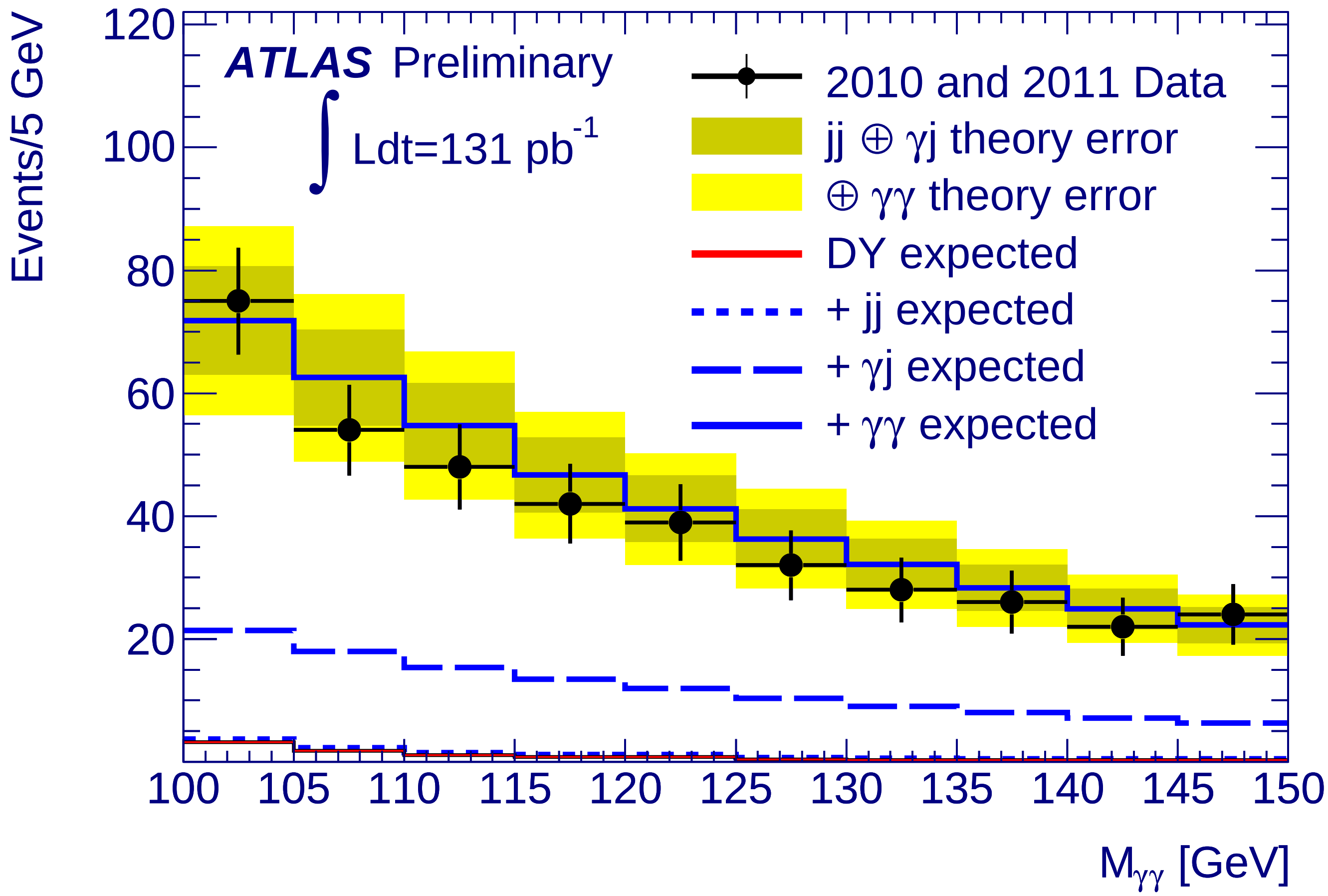


Figure 3: Limits on Higgs to $\gamma\gamma$ channel relative to SM expectation. Limits are at about $8 - 10 \times SM$. This is a little better than Tevatron for some masses.

- Meanwhile, a 4th generation and/or W' will have a significant impact on the expected level of the signals.

- To quantify further, it is useful to define the ratios:

$$R^h_X \equiv \frac{\Gamma^h_{gg} B(h \to X)}{\Gamma^{h_{\rm SM}}_{gg} B(h_{\rm SM} \to X)} \qquad (1)$$

  where the denominator is always computed for 3 generations and no additional W'

- From the following plot, you will see that the experimental limit on $R^h_{\gamma\gamma}$ is not currently strong enough to probe the 4th generation only possibility nor the W' only possibility, but does eliminate the 4th-generation + W' possibility.

- As regards the W' only case, if the h is light then $R^h_{\gamma\gamma} \sim 5$, a level that will soon be probed. But once, $m_h > 2m_W$ $R^h_{\gamma\gamma} \sim 2.5 - 3$ a level that will take a bit longer to probe.

Of course, we fully expect to reach $R^h_{\gamma\gamma} \sim 1$ for $m_h < 2m_W$ before too long.

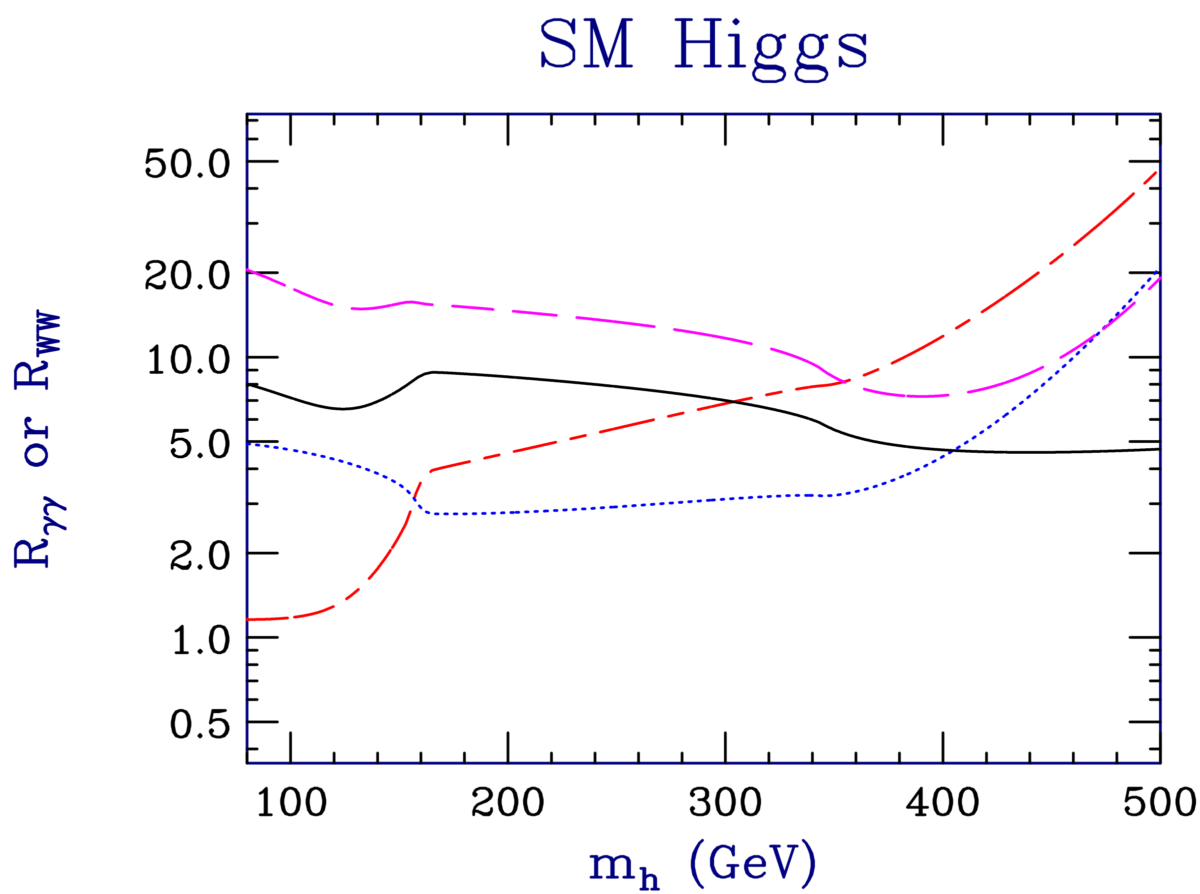


Figure 4: **The solid black curve shows R_{WW} in the presence of a 4th generation. For $R_{\gamma\gamma}$: the long-dash – short-dash red curve is for a 4th generation only; the dotted blue curve is for a sequential W' only; the long-dash magenta curve is for a 4th generation plus a sequential W'. All curves are for a Higgs boson with SM-like couplings and SM final decay states.**

- **Direct plots of R^h_{WW} have not been prepared by the Tevatron groups, but they do have plots of limits on $\sigma(gg \to h \to WW)$ that can then be compared to the SM expectation.**

  Focusing on just the $gg \to h \to WW$ process, the current CDF situation is that 4th generation is excluded for $m_{h_{\rm SM}} \gtrsim 124~{\rm GeV}$, as summarized by

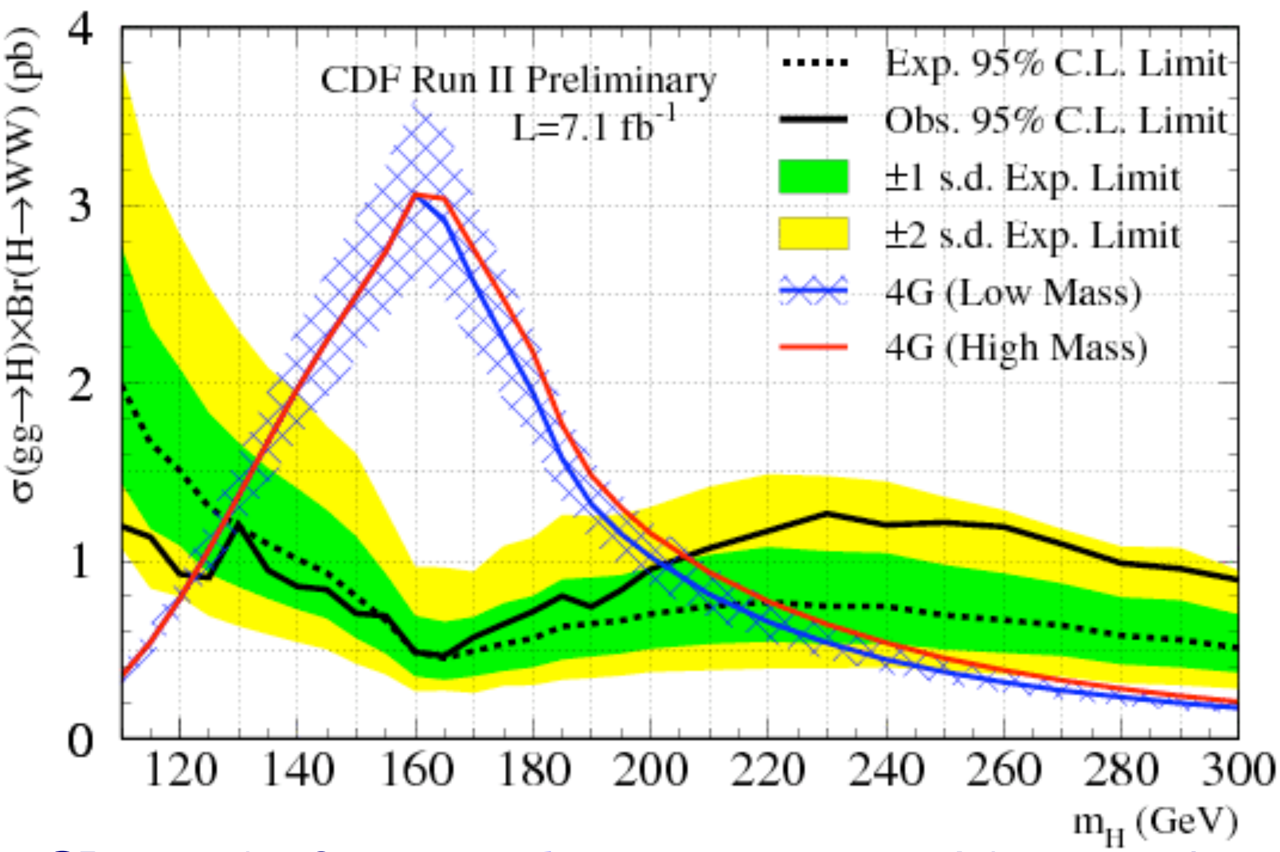


Figure 5: CDF results for $gg \to h_{\rm SM} \to WW$ vs. 4th generation prediction.

If D0 has similar results, a 4th generation is excluded if there is a SM-like Higgs with $m_{h_{\rm SM}} \gtrsim 115~{\rm GeV}$ (*i.e.* down to the LEP limit).

At the moment, although this will soon change, the Tevatron limits on the Higgs are stronger than the LHC limits which currently excludes only down to $\sim 140~{\rm GeV}$.

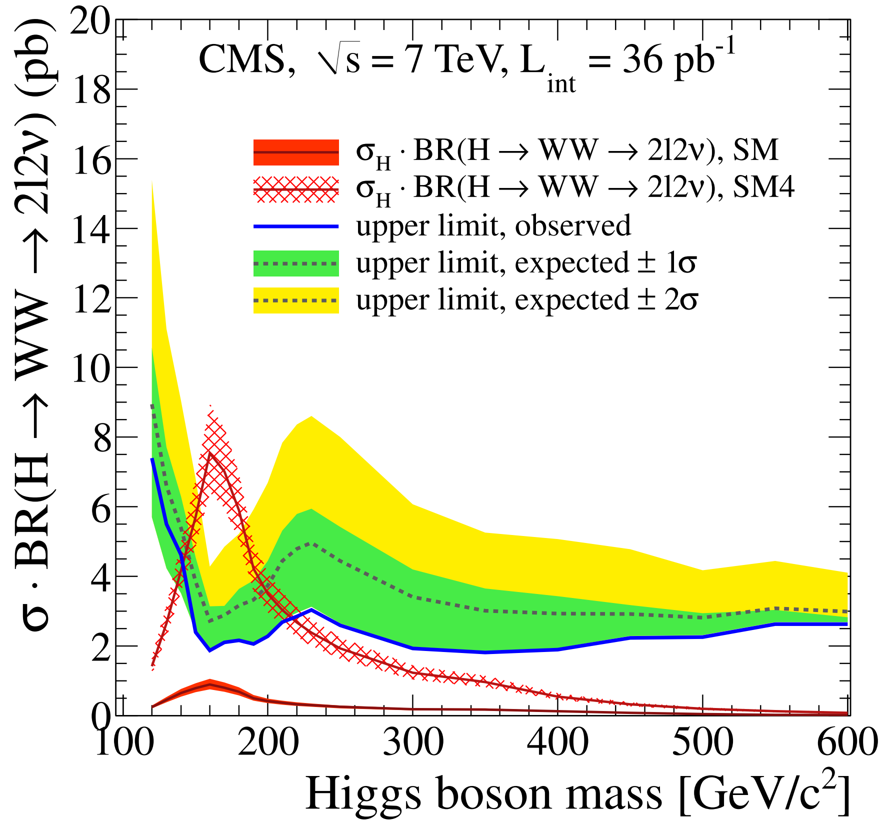


Figure 6: CMS results for $gg \to h_{\rm SM} \to WW$ vs. 4th generation prediction.

In addition, there is the $gg \to h \to \tau^+\tau^-$ process which also depends only on the $gg \to h$ loop-induced coupling. Although this channel does not significantly help at high mass, it apparently does make a non-negligible contribution to limits on the h at lower masses and might further strength the case against a 4th generation.

- I have made the request to the experimentalists at Blois 2011 to provide a limit on all $gg \to h$ processes (other than $\gamma\gamma$) relative to the SM, assuming that the branching ratios for the main h decay channels are as expected.

The Two-Higgs Doublet Model

- In the 2HDM there are only two possible models for the fermion couplings that naturally avoid flavor-changing neutral currents (FCNC), Model I and Model II.

Table 1: Summary of 2HDM quark couplings in Model I and Model II.

	Model I			Model II		
	h	H	A	h	H	A
$t\bar{t}$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$-i\gamma_5\cot\beta$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$-i\gamma_5\cot\beta$
$b\bar{b}$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$i\gamma_5\cot\beta$	$-\frac{\sin\alpha}{\cos\beta}$	$\frac{\cos\alpha}{\cos\beta}$	$-i\gamma_5\tan\beta$

(2)

In both Model I and Model II the WW, ZZ couplings of the h and H are given by $\sin(\beta-\alpha)$ and $\cos(\beta-\alpha)$, respectively, relative to the SM values.

And, very importantly, there is no coupling of the A to WW, ZZ at tree level.

If the λ_i of the Higgs potential are kept fully perturbative, the decoupling limit, in which $m_H \to m_A$ and $\sin^2(\beta-\alpha) \to 1$, sets in fairly quickly as m_A increases

- 4 Generations in the 2HDM are fairly constrained.

  A few issues are the following:

1. If we adopt the same values of $m_{t'} \sim m_{b'} \sim 400~\mathrm{GeV}$ and use the standard formulae

$$\lambda_{t'} = \frac{m_{t'}\sqrt{2}}{v\sin\beta}, \quad \lambda_{b'} = \frac{m_{b'}\sqrt{2}}{v\cos\beta} \tag{3}$$

   where $v = 246~\mathrm{GeV}$, then we find that $\frac{\lambda_{b'}^2}{4\pi} = 1.4$ for $\tan\beta = 1.5$. Similarly, $\frac{\lambda_{t'}^2}{4\pi} = 1.4$ for $\cot\beta = 1.5$.
   Relaxing strict perturbativity by allowing $\frac{\lambda_{t',b'}^2}{4\pi} = 4$ would allow $1/3 < \tan\beta < 3$ for 4th family masses of $400~\mathrm{GeV}$.
   I will explore a much larger range of $\tan\beta$ values. From an agnostic

point of view, non-perturbative couplings may still represent a real physics possibility as some limit of a technicolor-like theory.

2. The $\lambda_{t',b'}$ will grow as one evolves up from the 4th generation mass scale and become even more non-perturbative.

   This means that our ability to compute in the theory and the point at which some sort of ultraviolet completion is needed would need to lie below $10-100~\mathrm{TeV}$.

3. Unitarity for 4th generation $f\overline{f}'$ scattering amplitudes will be on the border line for masses above $700~\mathrm{GeV}$.

   For a colored doublet with masses m_1, m_2 the bounds read

$$m_1^2 < \sin^2\beta \frac{4\sqrt{2}\pi}{3G_F}, \quad m_2^2 < \cos^2\beta \frac{4\sqrt{2}\pi}{3G_F}, \tag{4}$$

   where $\frac{4\sqrt{2}\pi}{3G_F} \sim 733~\mathrm{GeV}$.

   Thus, $\tan\beta = 3$, which means $\cos^2\beta = 1/10$ would require $m_{b'} < 232~\mathrm{GeV}$.

   Once again, one can, and I will, ignore this issue in the spirit of allowing an effective 2HDM in some technicolor approach.

4. Precision electroweak constraints are not affected provided the t' and b' are sufficiently degenerate.
   Of course, there is the possibility that the masses of the CP-even Higgs bosons could be quite large provided $m_{t'} - m_{b'}$ is of appropriate size to bring the model back within the $S - T$ ellipse.
   This works for Higgs masses up to about $600~\mathrm{GeV}$. Of course, this remark also applies in the purely SM case.

- Let us focus on the CP-odd A^0.

  Note: if the h is SM-like, $i.e.$ $\sin^2(\beta - \alpha) \sim 1$, then the H^0 will give very similar results to those for the A, yielding a doubling if the $m_A \sim m_H$ decoupling limit applies.

  Only $\gamma\gamma$ decays are relevant and these are not influenced by a possible W' since it will not couple to the A (or the H if h is SM-like).

  $R^A_{\gamma\gamma}$ is plotted as a function of m_A in Fig. 7 for the 3 generation case.

  Enhanced $\gamma\gamma$ signals, $R^A_{\gamma\gamma} > 1$, are only possible for low $\tan\beta$ values.

Enhanced signals are possible for $\tan\beta < 1$ also in Model I.

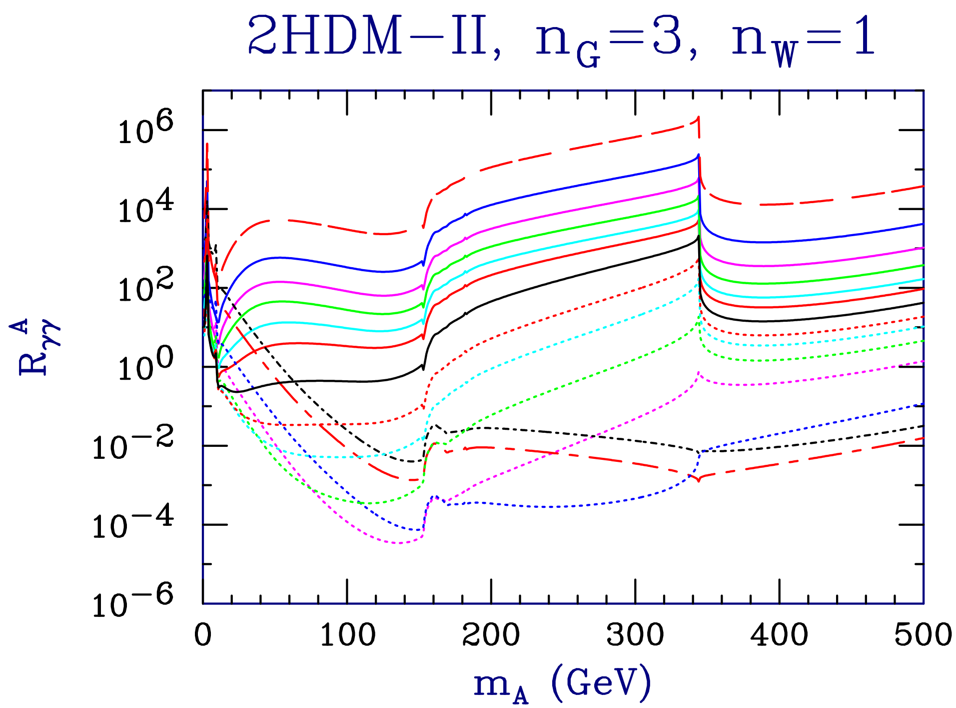


Figure 7: $R_{\gamma\gamma}$ for the 2HDM-II A. The legend is as follows: solid black→ $\tan\beta = 1$; red dots→ $\tan\beta = 1.5$; solid red→ $\tan\beta = 1/1.5$; cyan dots→ $\tan\beta = 2$; solid cyan→ $\tan\beta = 1/2$; green dots→ $\tan\beta = 3$; solid green→ $\tan\beta = 1/3$; magenta dots→ $\tan\beta = 1/5$; solid magenta→ $\tan\beta = 5$; blue dots→ $\tan\beta = 10$; solid blue→ $\tan\beta = 1/10$; long red dashes plus dots→ $\tan\beta = 30$; pure long red dashes→ $\tan\beta = 1/30$; black dotdash→ $\tan\beta = 50$. This and subsequent figures must be viewed in color in order to resolve the different $\tan\beta$ cases.

- The impact of a fourth generation on the two-doublet results depends strongly on whether or not the model is Model I or Model II.

1. In particular, a 4th generation does not affect $R^A_{\gamma\gamma}$ in the case of Model-I. This is because the t' and b' of the 4th generation couple to the A with opposite signs but equal coefficients — see Table 2.
2. In contrast, the results for a Model-II A are changed dramatically: the 4th family case is illustrated in Fig. 8.

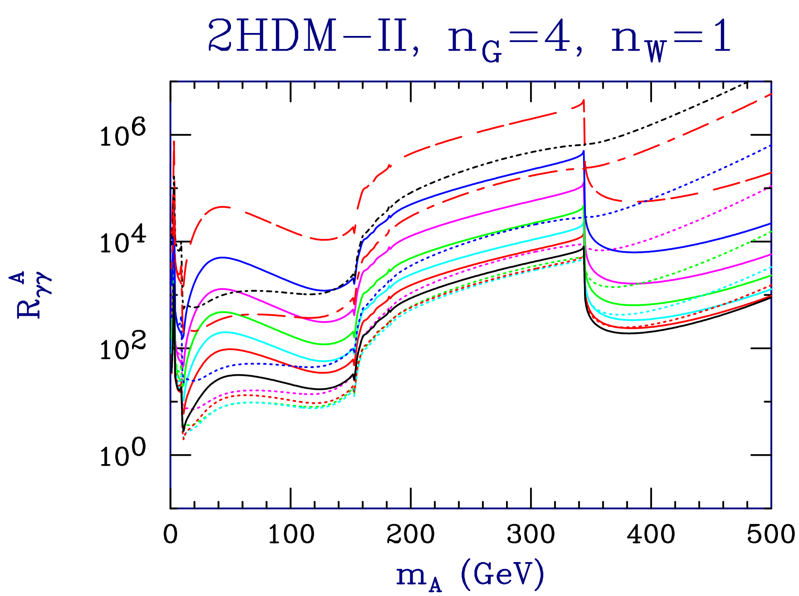


Figure 8: $R_{\gamma\gamma}$ for the 2HDM-II A after inclusion of 4th generation loops in gg production and in $A \longrightarrow \gamma\gamma$ decays. The legend is as in Fig. 7.

(a) Regardless of $\tan\beta$, one predicts large $R^A_{\gamma\gamma}$, the smallest values occurring at low m_A for moderate $\tan\beta \in [1,5]$, for which $R^A_{\gamma\gamma} \sim 10$ for $m_A \in [30, 150]$ GeV.

(b) Looking back at the combined Tevatron limit, we observe that it lies in the range $10 - 20 \times$ SM.

And, the CMS plot is probably at the level of 10 or lower times SM out to 150 GeV.

(c) One should keep in mind that $\tan\beta$ should be neither very small nor very large in order that the 4th family couplings remain perturbative.

The range $1/3 < \tan\beta < 3$ is singled out by this requirement.

(d) $R^A_{\gamma\gamma}$ increases dramatically for $m_A > 2m_W$ because of the drop in $B(h_{\rm SM} \to \gamma\gamma)$.

What is really happening is that both Γ^A_{gg} and $B(A \to \gamma\gamma)$ are remaining fairly constant or increasing at any given m_A.

The cross section will drop because of decreasing gg luminosity, but not very precipitously.

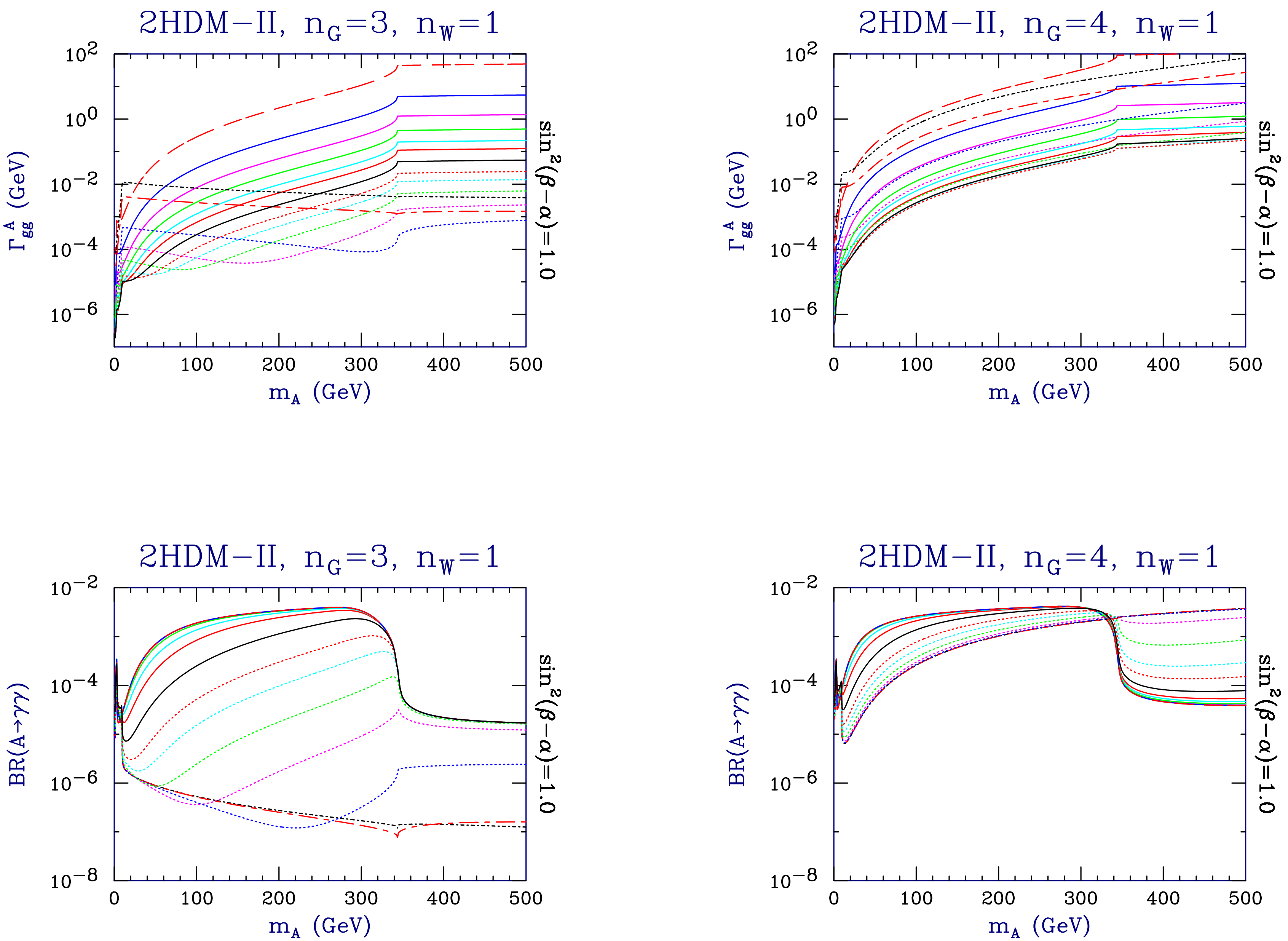


Figure 9: Top: Comparison of Γ^A_{gg} for 3 and 4 generations. Bottom: Comparison of $B(A \to \gamma\gamma)$ for 3 and 4 generations.

A useful plot is the following. Simply take $L \times \sigma_{SM}$ times the ratio plotted to get the number of events in the $\gamma\gamma$ final state.

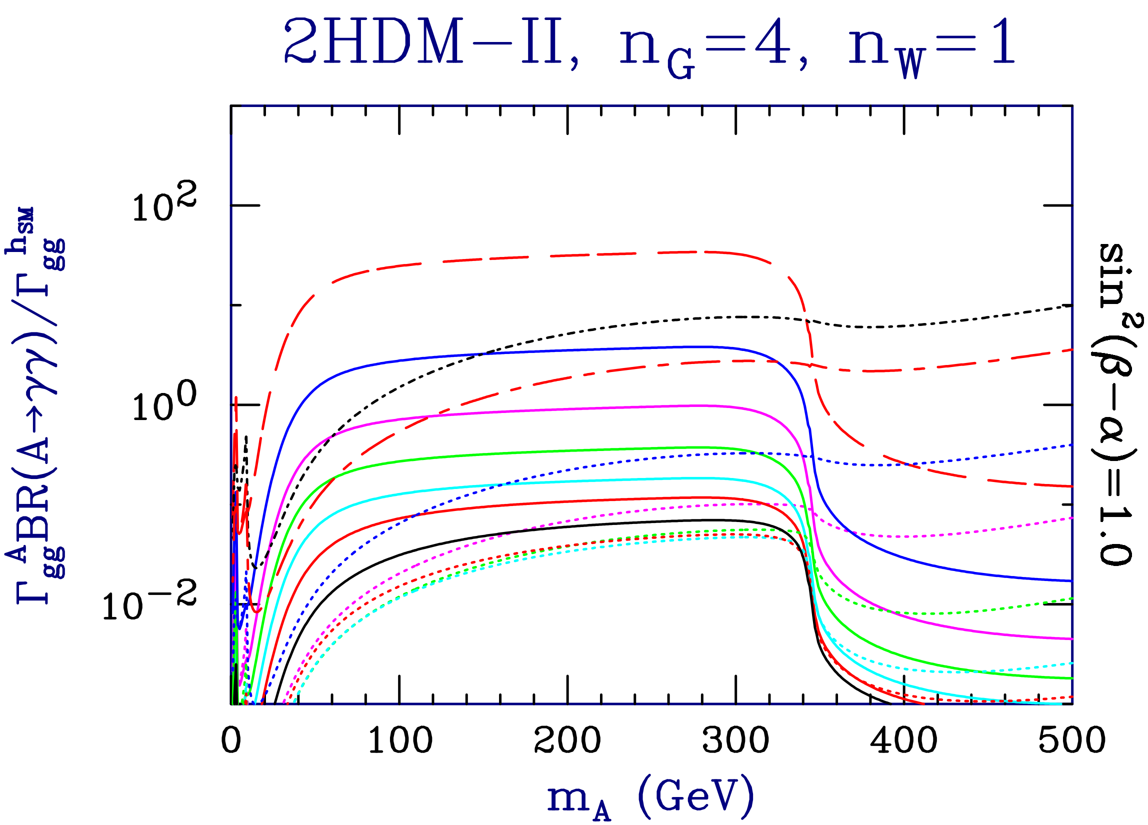


Figure 10: Multiply plotted quantity by $L \times \sigma_{SM}$ to get event rate. Tevatron example: $m_A = 200$ GeV, $\sigma_{SM} = 0.15$ pb, $L = 5$ fb^{-1}, $R \times B = 0.05 \Rightarrow 37$ events. LHC example: $m_A = 150$ GeV, $\sigma_{SM} = 10$ pb, $L = 131$ pb^{-1}, $R \times B = 0.04 \Rightarrow 52$ events, clearly visible above background of $20\ evts/5$ GeV for ATLAS plot.

3. One does need to check if the narrow width approximation is good when

assessing the observability of a signal.

A plot of $\Gamma^A_{\rm tot}$ for $m_A \leq 500~{\rm GeV}$ is given as Fig. 11 for the 4 generation case.

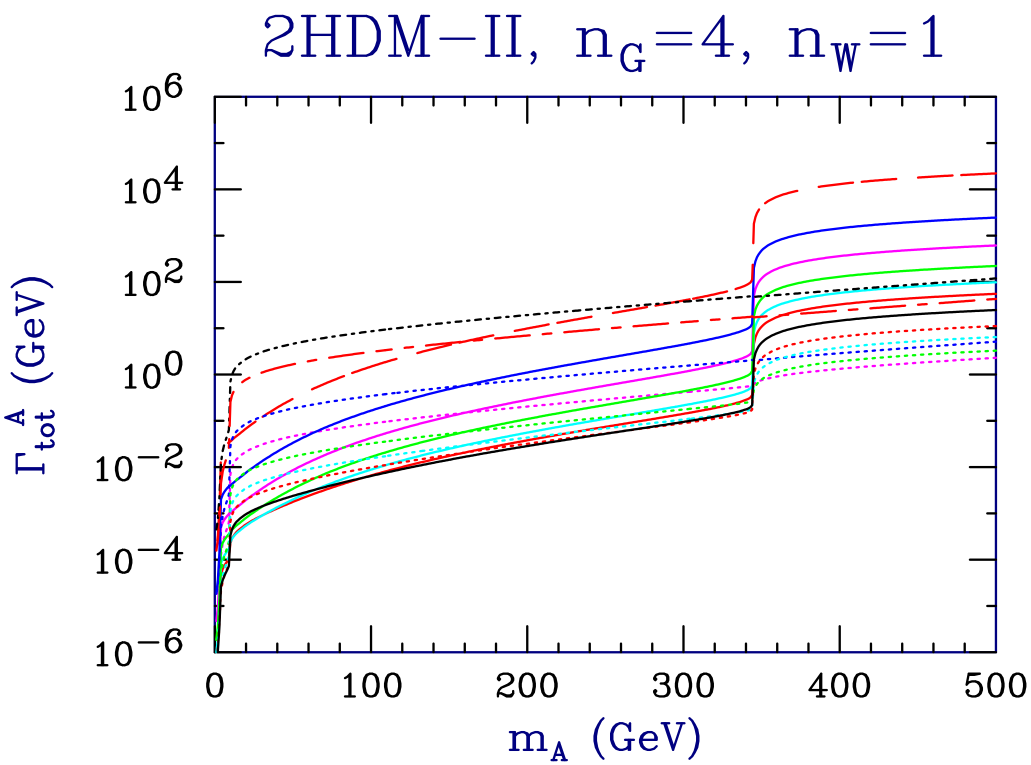


Figure 11: $\Gamma^A_{\rm tot}$ for Model II after inclusion of 4th generation loops for $A \to gg, \gamma\gamma$ decays. The legend is as in Fig. 7.

As noted earlier, a rough estimate using the latest ATLAS plot suggests $R_{\gamma\gamma} \lesssim 10$ for $M_{\gamma\gamma} \leq 150~{\rm GeV}$.

This estimate assumed a narrow resonance.

For $m_A < 150 \text{ GeV}$, the narrow width approximation only breaks down for $\tan\beta \geq 30$.

At $m_A = 150 \text{ GeV}$, $\Gamma_{\rm tot}^A = 5 \text{ GeV}, 13 \text{ GeV}$ for $\tan\beta = 30, 50$, respectively.

For such total widths, limits would then be weaker than naively estimated using the narrow resonance assumption. However, we should note that $\tan\beta > 30$ is excluded by LHC data for $m_A \lesssim 170 \text{ GeV}$ [1] using the $A \to \tau^+\tau^-$ decay mode and just $L = 35 \text{ pb}^{-1}$ of data.

Once $m_A > 2m_t$ the A total width increases dramatically; a study of the feasibility of detecting a highly enhanced broad $\gamma\gamma$ signal above the continuum $\gamma\gamma$ background is needed to determine the level of sensitivity.

4. A final note:

The enhanced values of $R^A_{\gamma\gamma}$ are least likely to be depleted by A decays to non-SM final states, most particularly $A \to hZ, H^\pm W^\mp$, when m_A is not large.

Of course, since the t' and b' masses are larger than $m_A/2$, direct decays

[1] This assumes the A and H are not degenerate.

to 4th generation quarks do not occur, but the 4th generation quarks do influence the loop-induced decays to gg (and $\gamma\gamma$).

- In passing, we note that $R^h_{\gamma\gamma}$ and $R^H_{\gamma\gamma}$ for the CP-even Higgs bosons are less robust as indicators of a 4th generation — in particular, they depend significantly on $\sin^2(\beta-\alpha)$ and are often below 1 (especially for the Yukawa-perturbativity-preferred modest $\tan\beta$ values) when $\sin^2(\beta-\alpha)$ is not close to 1.

- Conversely, it is important to note the complementarity of R^h_{WW} and $R^A_{\gamma\gamma}$ in the decoupling limit of $\sin^2(\beta-\alpha)=1$.

  In this limit, it is R^h_{WW} that currently does and $R^A_{\gamma\gamma}$ that shortly could rule out a 4th generation scenario if the h is relatively light and if the A is not too heavy, respectively.

- Many possible scenarios at the LHC can be envisioned.

1. For example, as L increases it could be that a light A ($m_A < 200~\mathrm{GeV}$) is observed in the $\tau^+\tau^-$ mode with rate corresponding to a modest $\tan\beta$

value (presumably below 30 given current limits).

If there is no sign of a $\gamma\gamma$ peak for the given L it could easily happen that the limit on $R_{\gamma\gamma}$ will exclude a 4th family in the Model II context.

2. If, on the other hand, no A is detected in the $\tau^+\tau^-$ mode a limit on $\tan\beta$ significantly below 30 in the $m_A < 200~\mathrm{GeV}$ mass region is likely.

   In this case, we could only conclude that there can be no 4th generation if we assume the 2HDM Model II structure and that $m_A < 200~\mathrm{GeV}$.

   But, of course, no contradiction would arise if m_A is significantly larger or if the 2HDM Model II is not the right model.

The MSSM

- **The MSSM with a 4th generation becomes highly problematical. This has been review by Dawson and Jaiswal (arXiv:1009.1099) and recently revisited by Cotta et al. (arXiv:1105.0039).**

  1. **The first major issue is perturbativity, as we have already discussed.**
  2. **The Higgs sector becomes very strange.**

     Recall that in the MSSM, the Higgs potential at tree-level is completely fixed in terms of gauge couplings and $\tan\beta$.

     At one loop, one must add the contributions of all quarks and squarks to the mass matrix.

     With just the t and b, this adds $> 20~\mathrm{GeV}$ to m_h if squarks have masses $\lesssim 1~\mathrm{TeV}$.

     With a 4th generation, the additional increase depends upon the new quark masses and associated squark masses and mixings.

     Recall that the 1-loop additions go like $(mass)^4$.

For $m_{t'} \sim m_{b'} = 400$ GeV, $m_h > 400$ GeV is expected with $m_H, m_{h^\pm}$ being even larger. I will give a plot shortly.

Indeed, only the A has a chance of being light.

3. Because the CP-even Higgs bosons must be so heavy, issues concerning FCNC and precision electroweak constraints arise.

   Precision electroweak constraints basically force one to have a situation where the Heavy Higgs contribution is compensated by an isospin-splitting $\Delta T > 0$ so as to get back into the $S - T$ plane ellipse.

   This is only possible if $m_h < 500$ GeV or so.

   As a result it is necessary that the 4th generation masses lie below 500 GeV and that there be a closely tuned mass splitting $0 < m_{t'} - m_{b'} < 100$ GeV (depending upon the precise $\tan\beta$ value) if $m_{\rm SUSY} \sim 1$ TeV (for degenerate ℓ' and ν').

   As one lowers $m_{\rm SUSY}$, precision electroweak requires progressively lower t' and b' masses with virtually no solutions if $m_{\rm SUSY} < 500$ GeV.

- To assess the situation regarding the A, I constructed an extended version of HDECAY to include a 4th generation.

- **For the 4th generation squarks, I adopted identical parameters to those for the 3rd generation.**

  I employed the "default" HDECAY values which basically assume that all squarks are quite heavy (~ 1 TeV).

  The resulting values of m_h as a function of m_A are plotted in Fig. 12.

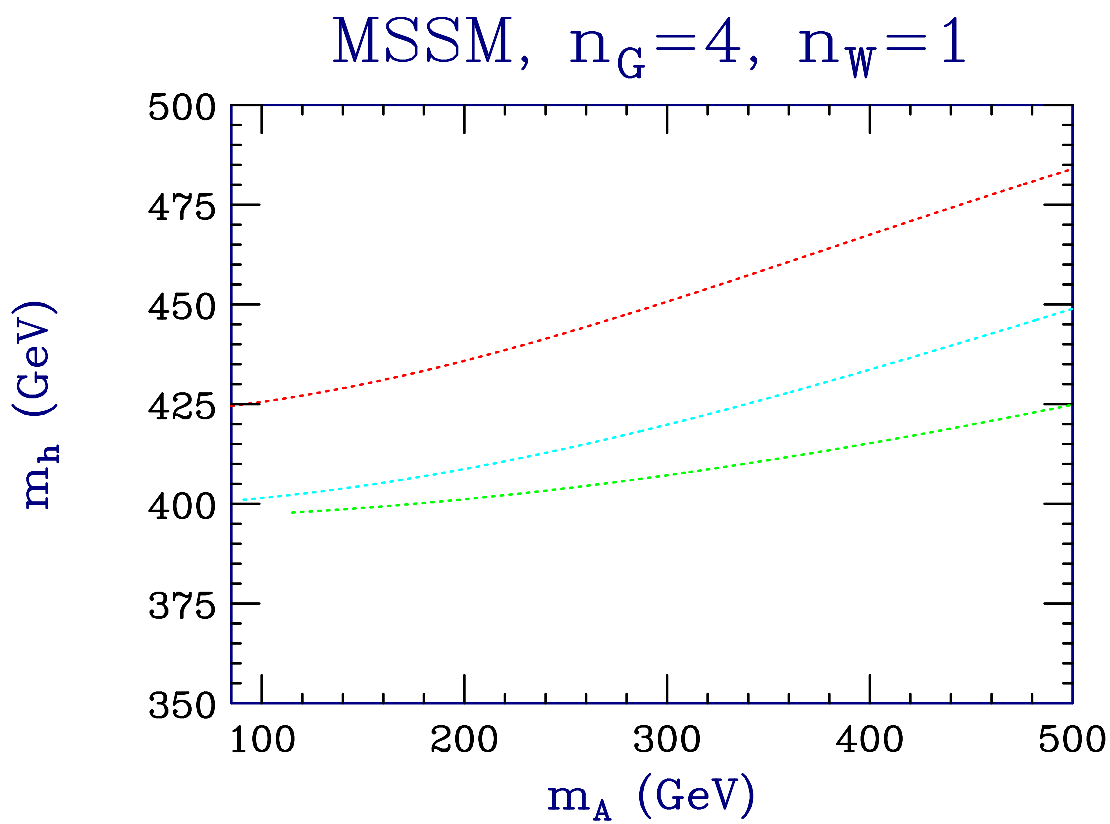


Figure 12: m_h **vs.** m_A **for** $\tan\beta = 1.5$, 2 **and** 3 **(red, green, cyan, respectively).**

- Once again, strong constraints on the possible presence of the 4th generation can arise from considering R^h_{WW} and $R^A_{\gamma\gamma}$. The relevant plots appear in Fig. 13.

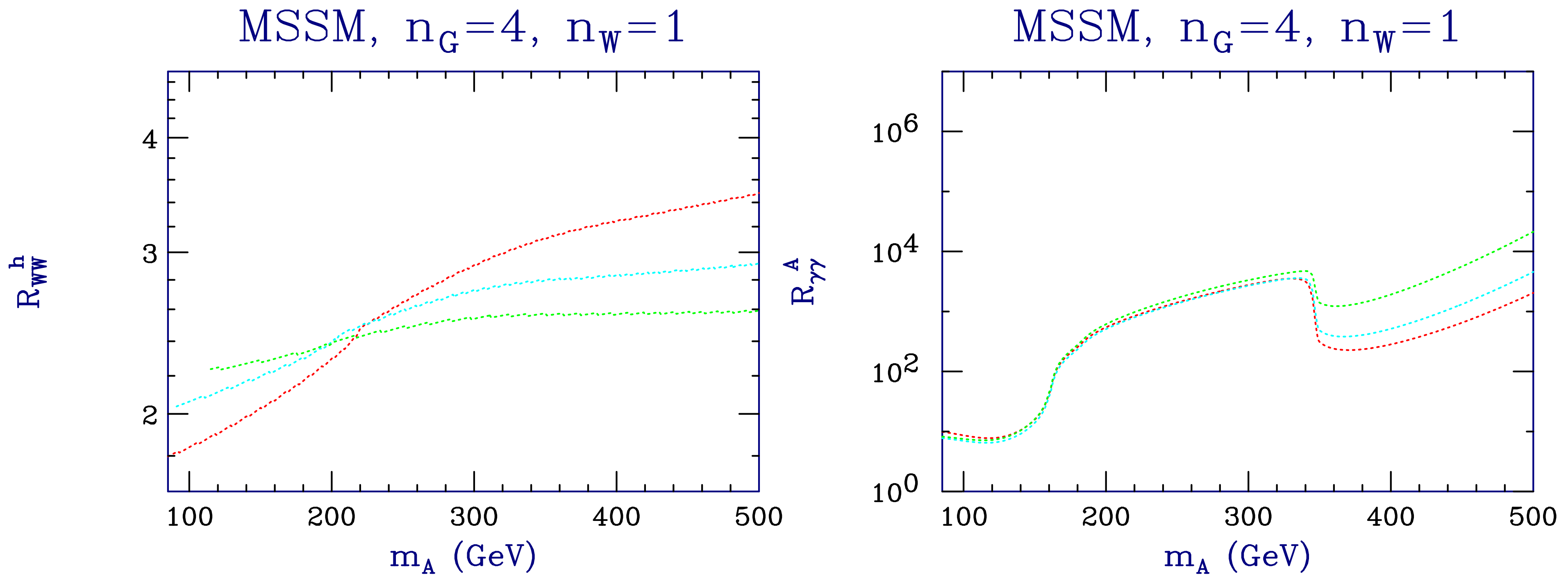


Figure 13: MSSM plots for $\tan\beta = 1.5$, 2 and 3 (red, green, cyan, respectively). Top: R^h_{WW} vs. m_A. Bottom: $R^A_{\gamma\gamma}$ vs. m_A.

These plots include loop effects from both the fermions and the sfermions of the 4th generation, but the sfermion and other parameters of the default `hdecay.in` are such that all sparticles are heavy and do not contribute significantly to the h or A decays for $m_A < 500~\mathrm{GeV}$.

- Some remarks:

  1. The smallest values of $R^A_{\gamma\gamma} \sim 6.5$ occur in the $m_A < 2m_W$ region.
  2. And, as for the 2HDM-II, for $m_A > 2m_W$ one finds $R^A_{\gamma\gamma} \geq 100$!
  3. R^h_{WW} is complementary in that for $m_A > 200~\mathrm{GeV}$, $R^h_{WW} > 2.4$, a value that will be probed even at the large $m_{WW} \sim m_h$ values of Fig. 12 given large enough L at the LHC.

- Thus, we have the following situation.

  – Analysis of LHC $\gamma\gamma$ spectrum data will probably soon place a limit of $R^A_{\gamma\gamma} < 6.5$ out to $m_A = M_{\gamma\gamma} \sim 2m_W$, in which case a 4th generation will be inconsistent with the MSSM for $m_A \lesssim 2m_W$, barring significant $A \to SUSY$ decays.

  – For $2m_W < m_A < 200~\mathrm{GeV}$ it seems likely that a limit below the minimum predicted value of $R^A_{\gamma\gamma} = 100$ will be achieved.

- Meanwhile, for 4 generations $R^h_{WW} > 2.4$ is predicted for all $m_A \geq 200~\mathrm{GeV}$ and will eventually be excludable in the relevant $m_h \sim 400-500~\mathrm{GeV}$ mass range.
- If sparticles are light, then hopefully the LHC will detect them and R^h_{WW} and $R^A_{\gamma\gamma}$ predictions can be corrected for substantial $B(h, A \to SUSY)$ values.

  In addition, predictions for $\Gamma^{h,A}_{gg} B(h, A \to SUSY)$ will be larger in the presence of a 4th generation than without. SUSY production through Higgs production would be substantially increased.

- Finally, we note that if there is a W', $R^A_{\gamma\gamma}$ is not affected (because of the absence of a tree-level $AW'W'$ coupling) while changes to R^h_{WW} are very tiny.

- Further, R^h_{WW} is only modestly influenced by sfermion loop contributions to Γ^h_{gg} and sfermion loops are not present for either $gg \to A$ or $A \to \gamma\gamma$.

- Thus, $R^A_{\gamma\gamma}$ and R^h_{WW} are quite robust tests for the presence of a 4th

generation and can potentially eliminate the possibility of 4 generations in the context of the MSSM even if no Higgs is observed.

- **Of course, by the time sufficient L is available to measure R^h_{WW} out to large m_h, direct observation or exclusion of the 4th-generation quarks may have occurred.**

Conclusions

- **If a Higgs sector exists, then it is very possible for hadron colliders to see some dramatic signals.**

- **In fact, the $\gamma\gamma$ final state signals are so dramatic in the case that a 4th generation exists, especially in the case of a Model II A, that it is not at all implausible that a $\gamma\gamma$ signal could emerge before we were expecting one!**

- **If no enhanced signal is seen, and a 4th generation exists (as we would learn from direct detection of the t' and b'), then we can be assured that m_A is large.**

Appendix on Higgs masses

The neutral Higgs masses are found from the eigenvectors of the matrix:

$$M^2 = \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix} \tag{5}$$

where $M_{ij} \equiv M_{ij,tree} + \Delta_{ij}$ The tree level values are (where $c_\beta = \cos\beta$ and $s_\beta = \sin\beta$),

$$\begin{aligned} M_{11,tree} &= M_A^2 s_\beta^2 + M_Z^2 c_\beta^2 \\ M_{12,tree} &= -(M_A^2 + M_Z^2) s_\beta c_\beta && (6) \\ M_{22,tree} &= M_A^2 c_\beta^2 + M_Z^2 s_\beta^2 && (7) \end{aligned}$$

At one-loop the effects of a heavy 4^{th} generation on the neutral Higgs masses, including only the leading logarithms and assuming no mixing in the sfermion

sector, are

$$\begin{aligned}
\Delta_{11} &= \hat{\epsilon}_b = \Sigma_{i=b',e'} \frac{N_c G_F}{2\sqrt{2}\pi^2} \frac{m_i^4}{c_\beta^2} \ln\left(\frac{\tilde{m}_{i1}^2 \tilde{m}_{i2}^2}{m_i^4}\right) \\
\Delta_{22} &= \hat{\epsilon}_t = \Sigma_{i=t',\nu'} \frac{N_c G_F}{2\sqrt{2}\pi^2} \frac{m_i^4}{s_\beta^2} \ln\left(\frac{\tilde{m}_{i1}^2 \tilde{m}_{i2}^2}{m_i^4}\right) \\
\Delta_{12} &= 0\,,
\end{aligned} \tag{8}$$

where $\tilde{m}_{i1}$ and $\tilde{m}_{i2}$ are the physical sfermion masses associated with f_i.

The neutral Higgs boson masses are then,

$$\begin{aligned}
m_{H,h}^2 &= \frac{1}{2}\Bigg\{M_A^2 + M_Z^2 + \hat{\epsilon}_b + \hat{\epsilon}_t \pm \Bigg[(M_A^2 + M_Z^2)^2 - 4c_{2\beta}^2 M_A^2 M_Z^2 \\
&\quad + (\hat{\epsilon}_b - \hat{\epsilon}_t)\left(2c_{2\beta}(M_Z^2 - M_A^2) + \hat{\epsilon}_b - \hat{\epsilon}_t\right)\Bigg]^{1/2}\Bigg\}.
\end{aligned} \tag{9}$$

The mixing angle (which we use to define the fermion and sfermion

couplings) is

$$sin2\alpha = \frac{2M_{11}^2}{\sqrt{(M_{11}^2 - M_{22}^2)^2 + 4M_{12}^4}} \tag{10}$$