Screened Modified Gravity

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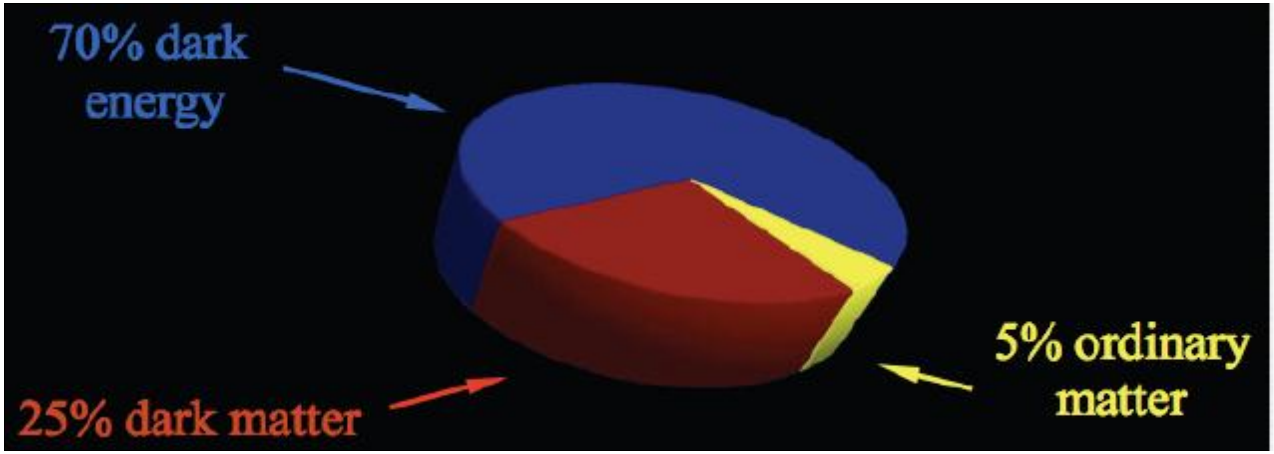
The Big Puzzle



Could be violated but more and more constrained

Result depending crucially on two assumptions:

The cosmological principle (isotropy and homogeneity on large scales)

Under scrutiny and merging with the idea of "dark energy"

The validity of General Relativity from the solar system to cosmological distances

The acceleration of the expansion of the Universe could have (at least) two dynamical explanations:

Dark Energy: a new form of matter

Modified law of gravity on large scales

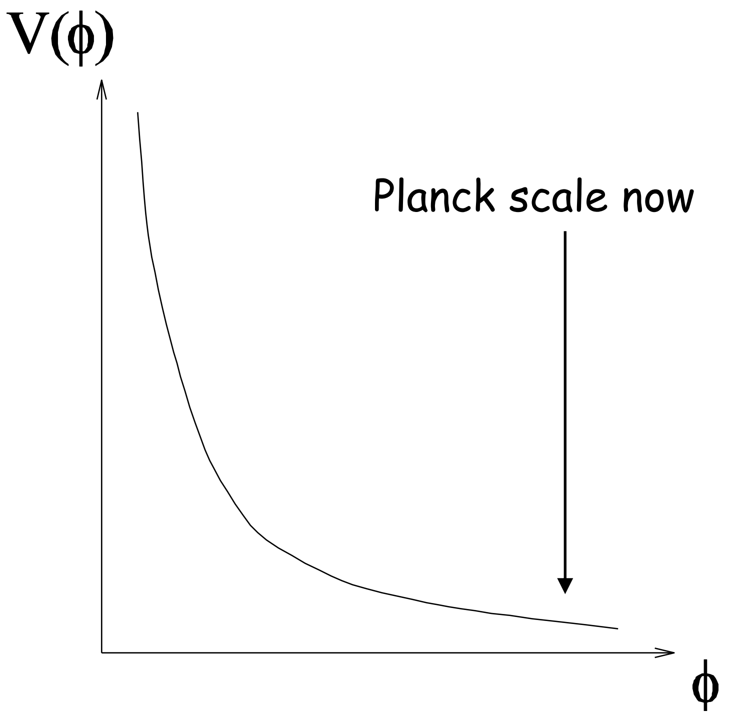
$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}(R - \mathcal{L}_{DE})$$

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}h(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$$

Surprisingly, both types of models involve scalar fields.

Dark Energy: a new form of matter

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} (R - \mathcal{L}_{DE})$$



$$\mathcal{L}_{\mathrm{DE}} = \frac{1}{2}(\partial\phi)^2 + V(\phi)$$

Field rolling down a runaway potential, reaching large values now (typically Planck scale)

Modifying Gravity

Modifying gravity is particularly difficult!

The simplest modification is massive gravity (Pauli-Fierz):

$$\delta\mathcal{L} = \frac{m_G^2}{4}(h^{\mu\nu}h_{\mu\nu} - h^2)$$

Pauli-Fierz gravity is ghost free (negative kinetic energy terms) . Unfortunately, a massive graviton carries 5 polarisations when a massless one has only two polarisations

$$h_{\mu\nu} = \frac{8\pi G_N}{p^2}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}) \to h_{\mu\nu} = \frac{8\pi G_N}{p^2 + m_G^2}(T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu})$$

The massless limit does not gives GR! (van Dam-Velman-Zakharov discontinuity). The extra polarization is lethal.

It has been argued that it becomes non-linear at short distance and cured by the Vainshtein mechanism (a lot more about this soon!)

Pauli- Fierz massive gravity has a ghost in curved backgrounds (cosmology).

New and complicated models of massive gravity avoid this problem (de Rham, Tolley et al.) ….they are very special as in general this is not the case!

An infinite class of modified gravity models can be considered:

$$S_{\mathrm{MG}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$$

These Lagrangian field theories fall within the category of higher derivative theories.

Ostrosgradski's theorem states that these theories are *generically* plagued with ghosts. Quantum mechanically, this implies an explosive behaviour with particles popping out of the vacuum continuously. In particular an excess in the gamma ray background.

A large class is ghost-free though, the f(R) models:

$$S_{\mathsf{MG}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}f(R)$$

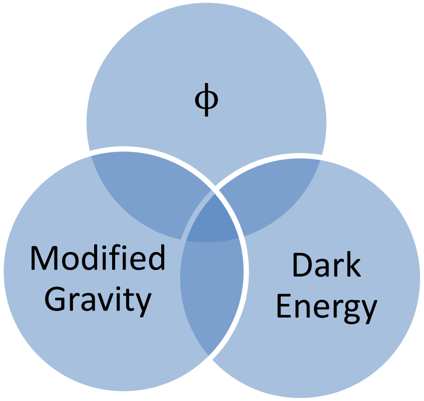
As promised, f(R) is totally equivalent to an effective field theory with gravity and scalars!

$$\boxed{S = \int d^4x \sqrt{-g}\left(\frac{1}{16\pi G_N}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\mathrm{Pl}}}g_{\mu\nu})\right)}$$

The potential V is directly related to f(R)

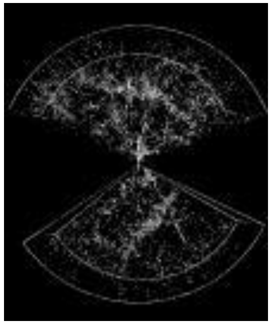
$$V(\phi) = m_{\mathrm{Pl}}^2 \frac{Rf' - f}{2f'^2}, \;\; f' = e^{-2\phi/\sqrt{6}m_{\mathrm{Pl}}}$$

The acceleration of the Universe could be due to either:



In both cases, current models use scalar fields. In modified gravity models, this is due to the scalar polarisation of a massive graviton. In dark energy, it is by analogy with inflation.

The fact that the scalar field acts on cosmological scales implies that its mass must be large compared to solar system scales.



The acceleration of the expansion leads to a strong constraint on the mass of the scalar field

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi),\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad\qquad w = \frac{p}{\rho}$$

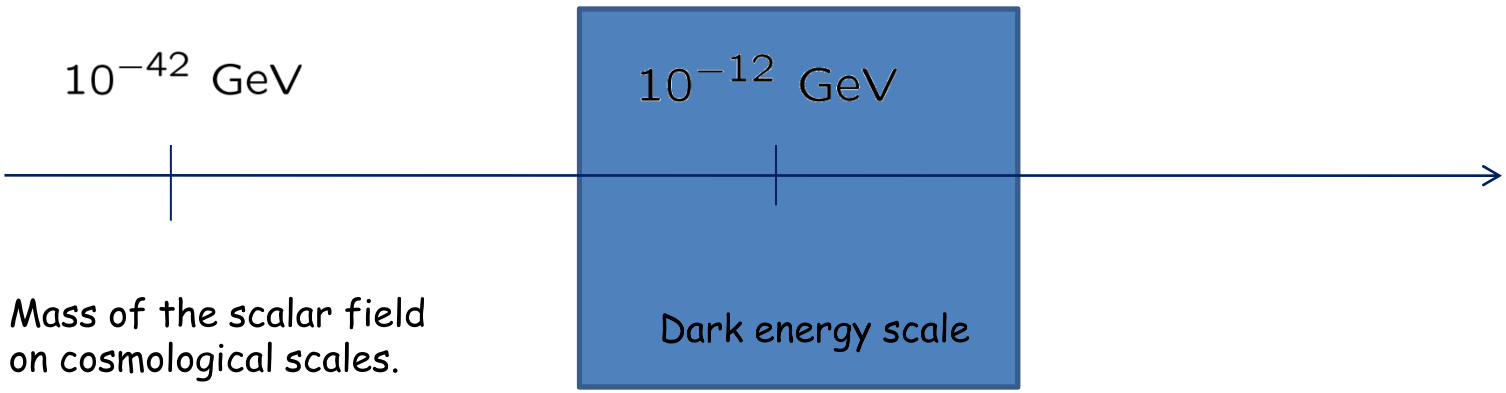
$$m \gg H_0 \longrightarrow w \approx 1$$

$$m \ll H_0 \longrightarrow w \approx -1$$

$$\boxed{m \approx H_0} \longrightarrow w \neq -1$$ dark energy

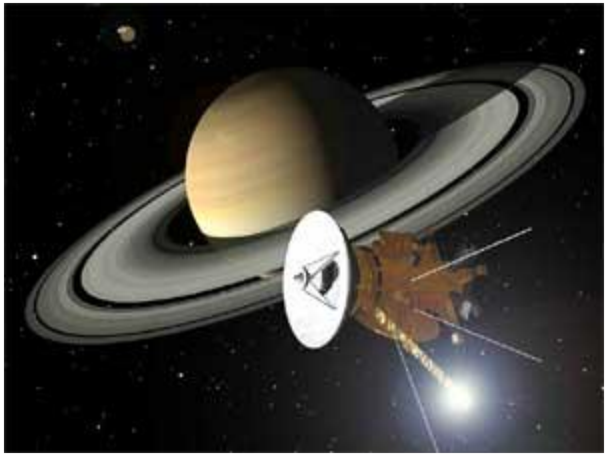
$$H_0 \approx 10^{-42}\text{GeV}$$

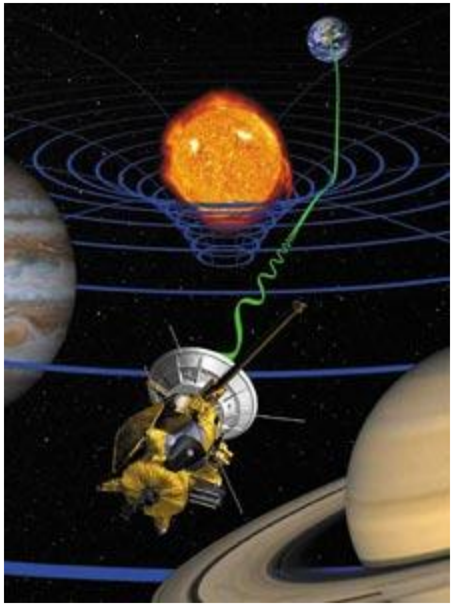
New Scales in Physics



New fifth force and violation of the equivalence principle

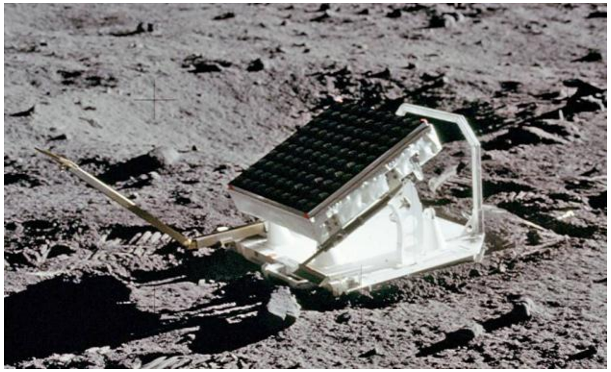
The tightest constraint on the existence of fifth forces comes from the Cassini probe:





Radiowaves sent around the sun feel the gravity of the sun and a possible deviation from General Relativity.

The tightest constraint on the equivalence principle comes from the lunar ranging experiment measuring the acceleration of the earth and the moon in the gravitational field of the sun.



$$\eta = \frac{|a_M - a_E|}{|a_E + a_M|}$$

Locally, deviations from Newton's law are parametrised by:

$$\phi_N = -\frac{G_N}{r}(1 + 2\beta_\phi^2 e^{-r/\lambda})$$

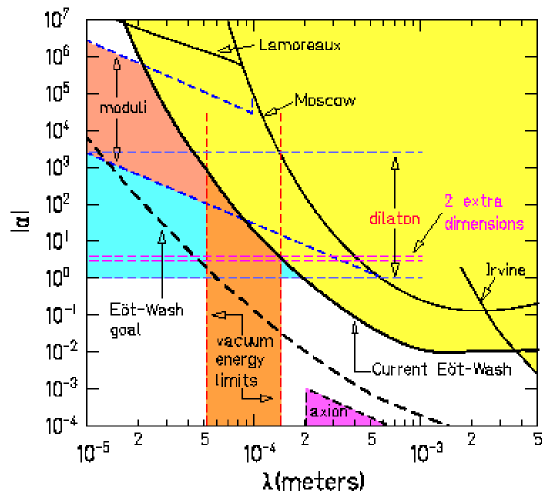
$$\beta_\phi^2 = \frac{1}{2\omega + 3}$$

The tightest constraint on β comes from the Cassini probe measuring the Shapiro effect (time delay):

$$\beta_\phi^2 \leq 1.2 10^{-5}$$

In the Brans-Dicke case, this translates to

$$\omega \geq 40000$$



Many modified gravity models seem to be ruled out, going back to f(R):

$$\boxed{S=\int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R-\frac{1}{2}(\partial\phi)^2-V(\phi)+\mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\rm Pl}}g_{\mu\nu})\right)}$$

A(ϕ)

The coupling is directly related to :

$$\beta_\phi = m_{\rm Pl}\frac{\partial A(\phi)}{\partial\phi}$$

$\beta_\phi = \frac{1}{\sqrt{6}}$ for f(R)

These models are not ruled out if the scalar force is screened in the solar system

There are three mechanisms to screen gravity, they can be easily understood using the Lagrangian of the scalar field fluctuation responsible for the fifth force:

Around a background configuration and in the presence of matter:

$$\mathcal{L} \supset -\frac{Z(\phi_0)}{2}(\partial\delta\phi)^2 - \frac{m^2(\phi_0)}{2}\delta\phi^2 + \frac{\beta(\phi_0)}{M_P}\delta\phi\delta T ,$$

The chameleon mechanism makes the range become smaller in a dense environment by increasing m

The Damour-Polyakov mechanism reduces β in a dense environment

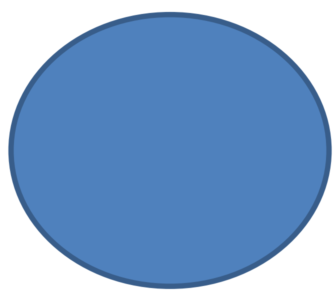
The Vainshtein mechanism reduces the coupling in a dense environment by increasing Z

Many models on the market, realising the three mechanisms:

K-mouflage, DGP, Galileons, massive gravity → **Vainshtein**

F(R), chameleons → **Chameleon**

Symmetrons, dilatons → **Damour-Polyakov**



Practically, one solves the field equations around a spherical overdensity as calculates the scalar force outside the object.

We will analyse models with a Vainshtein mechanism first, typically one would like to know if complex models like DBI from string theory could lead to the acceleration of the expansion and no gravitational incongruities.

$$\mathcal{L} = -T(\phi)\sqrt{1+\frac{(\partial\phi)^2}{T(\phi)}} - V(\phi) + T(\phi) + \frac{\phi}{M}\rho\,.$$

The Vainshtein mechanism is only present when no explicit ϕ dependence in T and V

There is an intrinsic link between the gravitational and the cosmological properties of these models.

They have interesting features astrophysically: a large Vainshtein radius for galaxy clusters.

A non-perturbative treatment is available for Lagrangians which may have a screening mechanism:

$$\mathcal{L} = M^4 f\left(\frac{2X}{M^4}\right) - \frac{\beta\phi}{M_P}T ,$$

The Klein-Gordon equation can be solved explicitly by defining x=r/R
and

$$\phi = M^2 R u(x) .$$

For any continuous function f, define the odd function g such that

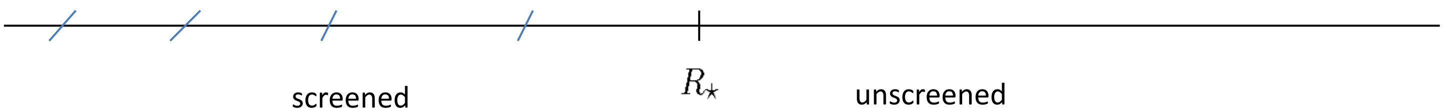
$$g(y)f'(g^2(y)) = y \ , g(y) = y\tilde{g}(y^2)$$

Outside the object the solution is

$$u = u(0) + \int_0^1 g\left(\left[\frac{R_\star}{R}\right]^2 \tilde{x}\right) d\tilde{x} + \int_1^x g\left(\left[\frac{R_\star}{R}\right]^2 \frac{1}{\tilde{x}^2}\right) d\tilde{x} . \qquad R_\star = \left(\frac{\beta M_c}{8\pi M_P M^2}\right)^{1/2} ,$$

Two limiting situations:

$$f(x) = x + \frac{x^m}{2^m} \to g(y) = (\frac{2^m y}{m})^{1/(2m-1)} \to \phi(r) = M^2 R(\frac{R_\star^2}{mR^2})^{1/(2m-1)}(\frac{r}{R})^{(2m-3)/(2m-1)}$$



and for the DBI case:

$$f(x) = \sqrt{1+x} \to g(y) = \frac{2y}{1-4y^2} \to \phi(r) = M^2 \frac{R^3}{2R_\star^2}(1 - \sqrt{1 - \frac{4R_\star^4 r^2}{R^6}})$$

The solution has a singularity inside the body!

The DBI model has no Vainshtein mechanism!

Let us look for non-singular models:

$$f(x) = \sqrt{1 + x^m} \rightarrow \frac{mg^{2m-1}}{2\sqrt{1 + g^{2m}}} = y$$

which has a singularity at finite distance only when m=1.

More generally, one may consider:

$$f(x) = \sqrt{1 + h(x)}, \ \ h(x) = \sum_{i=1}^{m} c_m x^m$$

Deep inside the body:

$$f(x) \sim \sqrt{1 + c_1 x}$$

The gradient of the field increases like in a DBI case and therefore inside the body and outside up to the radius $R_\star$, the highest power dominates

$$f(x) \sim \sqrt{c_m} x^{m/2}$$

which leads to a Vainshtein mechanism with radius around $R_\star$

This is a successful class of k-mouflage models

In a flat FRW background, the cosmological evolution of the scalar field is exactly known:

$$\dot{\phi} = -\frac{\beta M \rho_m t}{2M_P} \tilde{g}\left(-\left[\frac{\beta \rho_m t}{2MM_P}\right]^2\right) ,$$

The equation of state of the scalar fluid is

$$w_\phi = -\frac{M^4 f(-\frac{\dot{\phi}^2}{M^4})}{-\frac{\dot{\phi}\beta\rho_m t}{M_P} + M^4 f(-\frac{\dot{\phi}^2}{M^4})} .$$

In the late time Universe, these models are such that the scalar field slows down , the energy density becomes of order M^4 and the equation of state goes to -1.

The previous models defined by the square of a polynomial in X of degree m>1 satisfy this requirement. In the late time Universe:

$$\dot{\phi} = -\frac{\beta \rho_m t}{2c_1 M_P} .$$

The time when the equation of state becomes very close to -1 depends on c1. After this instant, matching with the cosmological constant requires :

$$M \sim 10^{-3} \text{ eV}$$

Earlier in the Universe, the higher power of the polynomial h dominate. Taking m=2 we find :

$$\frac{\rho_\phi}{\rho_m} = \frac{\beta^2 \rho_m t^2}{4\sqrt{c_2} M_P^2} \, .$$

In the matter dominated era we have:

$$\frac{\rho_\phi}{\rho_m} = \text{constant}$$

In the radiation era, the energy density of the scalar field increases with time implying that, if it is subdominant during the matter era, it is even more subdominant in the radiation era: **cosmological screening of the scalar field.**

Hence at the background level, these models are screened models of dark energy where the equation of state reaches -1 dynamically in the recent past of the Universe.

For structure formation, these models may have interesting consequences as the Vainshtein radius of large clusters is around:

$$R_\star \sim 10 \text{ Mpc}$$

Another class of models generalise f(R) gravity and lead to interesting phenomena: scalar-tensor theories

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R-\frac{1}{2}(\partial\phi)^2-V(\phi)+\mathcal{L}_m(\psi_m,A^2(\phi)g_{\mu\nu})\right)$$

The coupling to matter (before screening) is:

$$\beta_\phi=m_{\mathrm{Pl}}\frac{d\ln A}{d\phi}$$

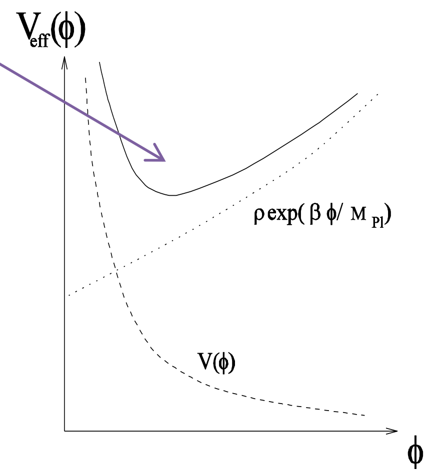
Different choices of V and A leads to chameleons, dilatons or symmetrons.

The effect of the environment

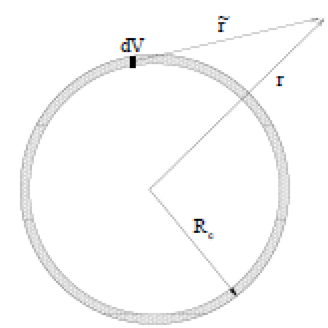
When coupled to matter, scalar fields have a matter dependent effective potential

$$V_{eff}(\phi) = V(\phi) + \rho_m (A(\phi) - 1)$$

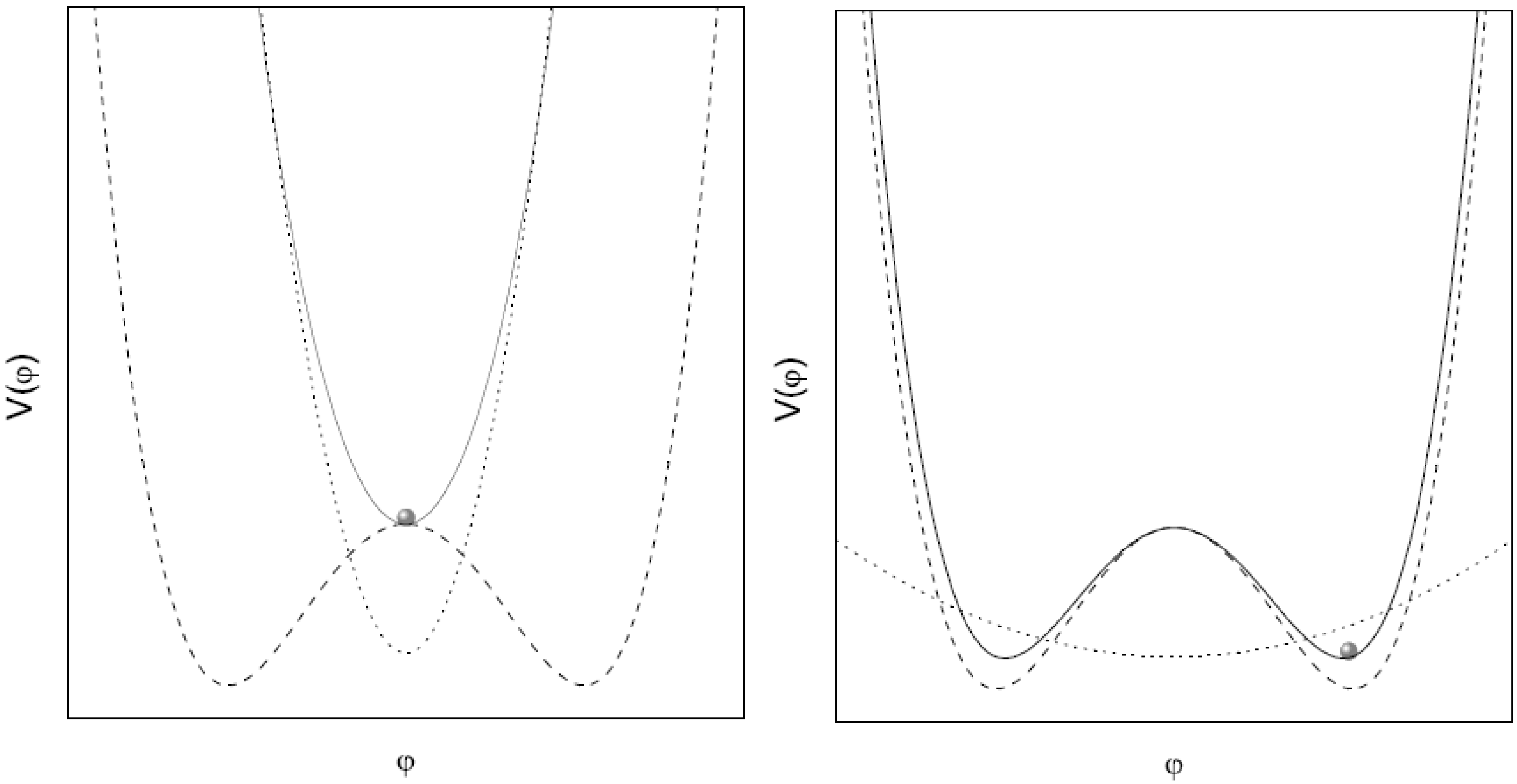
Environment dependent minimum



Chameleon=constant coupling

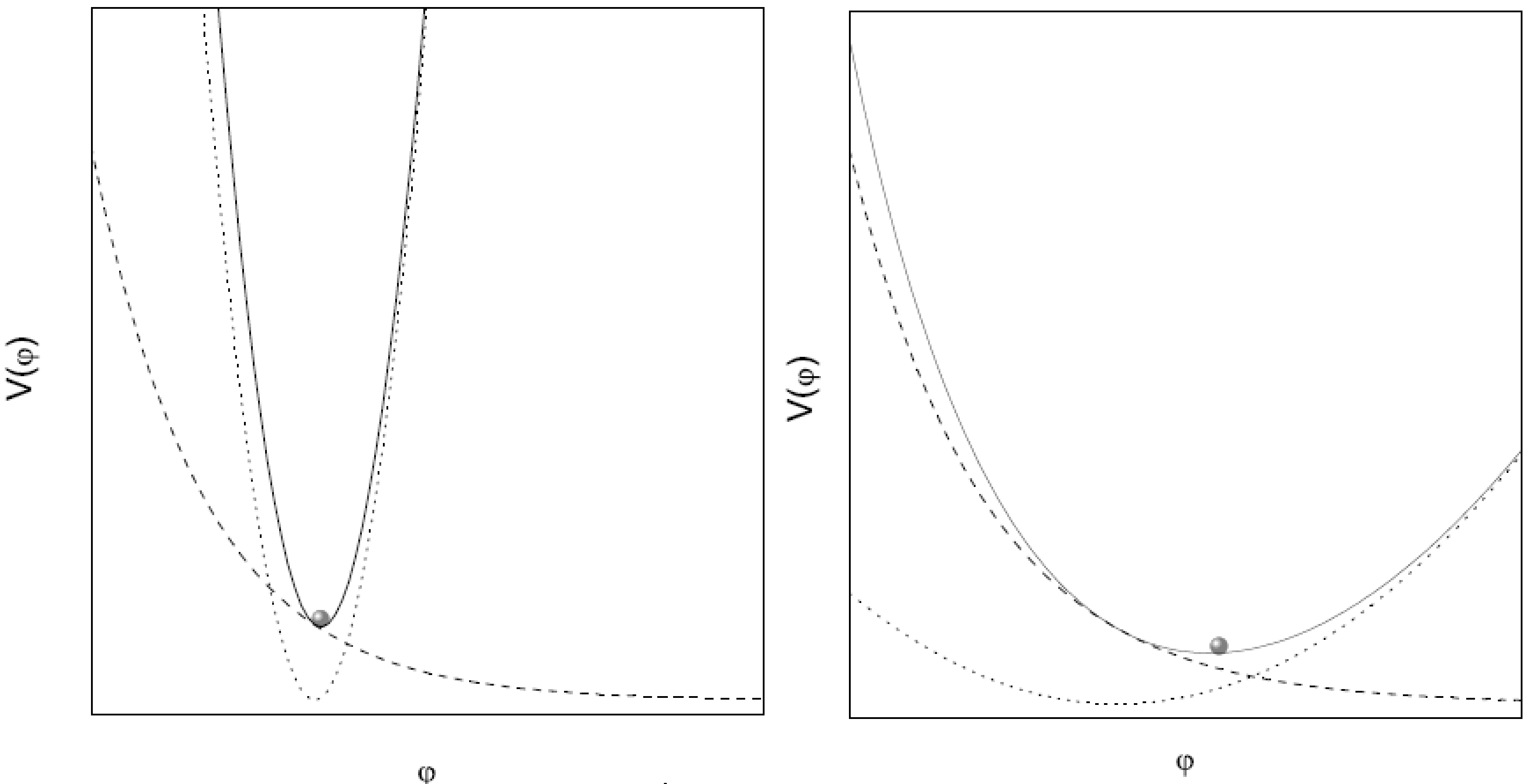


The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.



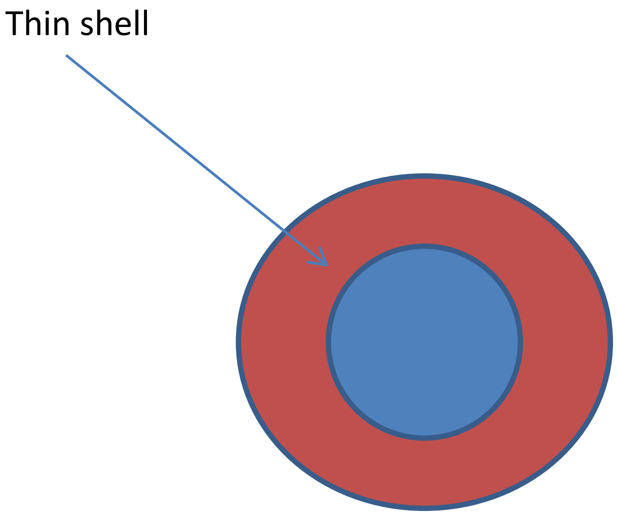
Symmetron

$$V(\phi) = V_0 - \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^2,\ \ A(\phi) = 1 + \frac{A_2}{2m_{\mathrm{Pl}}^2}\phi^2$$

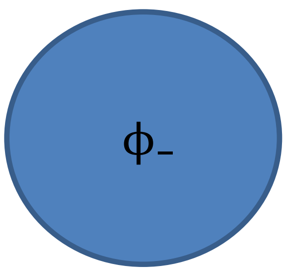


Dilaton

$$V(\phi) = V_0 e^{-\phi/m_{\rm Pl}}, \quad A(\phi) = 1 + \frac{A_2}{2m_{\rm Pl}}(\phi - \phi_\star)^2$$



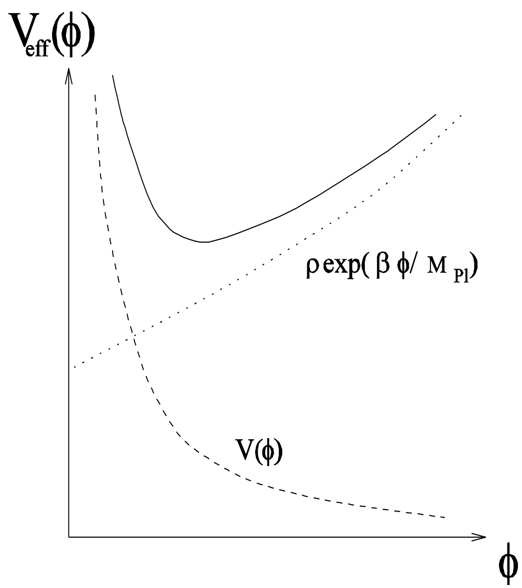
For chameleons, when objects are big enough/dense enough, the field is screened outside. Inside it is nearly constant apart from inside thin shell whose size is inversely proportional to Newton's potential at the surface.



$$Q = \frac{\phi_\infty}{\Phi}$$

For all chameleon, dilaton, symmetron models where either the potential and/or the coupling β is a non-linear function of ϕ, dense bodies are screened when their gravitational charge Q is small. This has strong implications cosmologically.

The family of screened modified gravity models (excluding Vainshtein) can be much more easily analysed using a reconstruction procedure.



The existence of a minimum for a medium of density ρ allows one to define a mapping between ρ and the value of the field and the value of the potential at the minimum. This implicit way of defining V(ϕ) is very useful as ρ itself can be parameterised by the expansion rate of the Universe

This implicit definition of the models depends only on the mass m(a) and the coupling β(a) at the minimum.

The non-linear potential of these models and the values of the field can be evaluated using:

$$\begin{aligned}\phi(a) &= \phi_i + \frac{3}{m_{\rm Pl}}\int_{a_i}^{a} da \frac{\beta(a)\rho(a)}{am^2(a)} \\ V(a) &= V_i - \frac{3}{m_{\rm Pl}}\int_{a_i}^{a} da \frac{\beta^2(a)\rho^2(a)}{am^2(a)}\end{aligned}$$

The full non-linear dynamics is reconstructed parametrically using the mass and the coupling function as a function of redshift! It works explicitly for chameleons, f(R), dilatons, symmetrons and one can invent new models!

As the Universe evolves from pre-BBN to now, the density of matter goes from the density of ordinary matter (10g/cm3) to cosmological densities. The minimum of the effective potential experiences all the possible minima from sparse densities (now) to high density (pre-BBN).

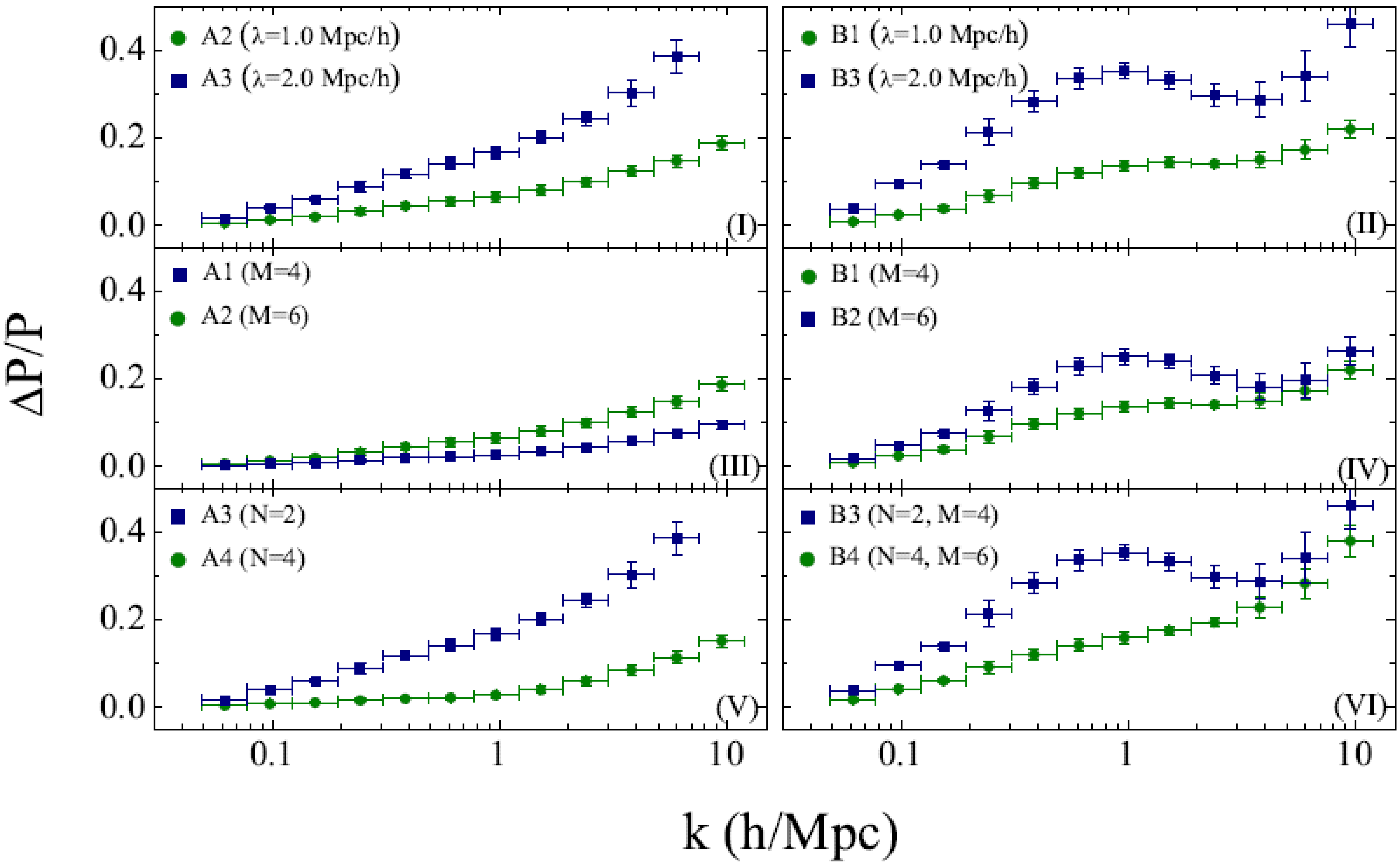
The loosest screening conditions requires that the Milky way is marginally screened, this corresponds to the absence of disruption of the galactic halo dynamics:

$$\frac{9\Omega_{m0}H_0^2}{m_0^2}\left(\int_{a_G}^{1}\frac{da}{a^4}\frac{\beta(a)}{\beta_0}\frac{m_0^2}{m^2(a)}\right)\leq 2\Phi_G$$

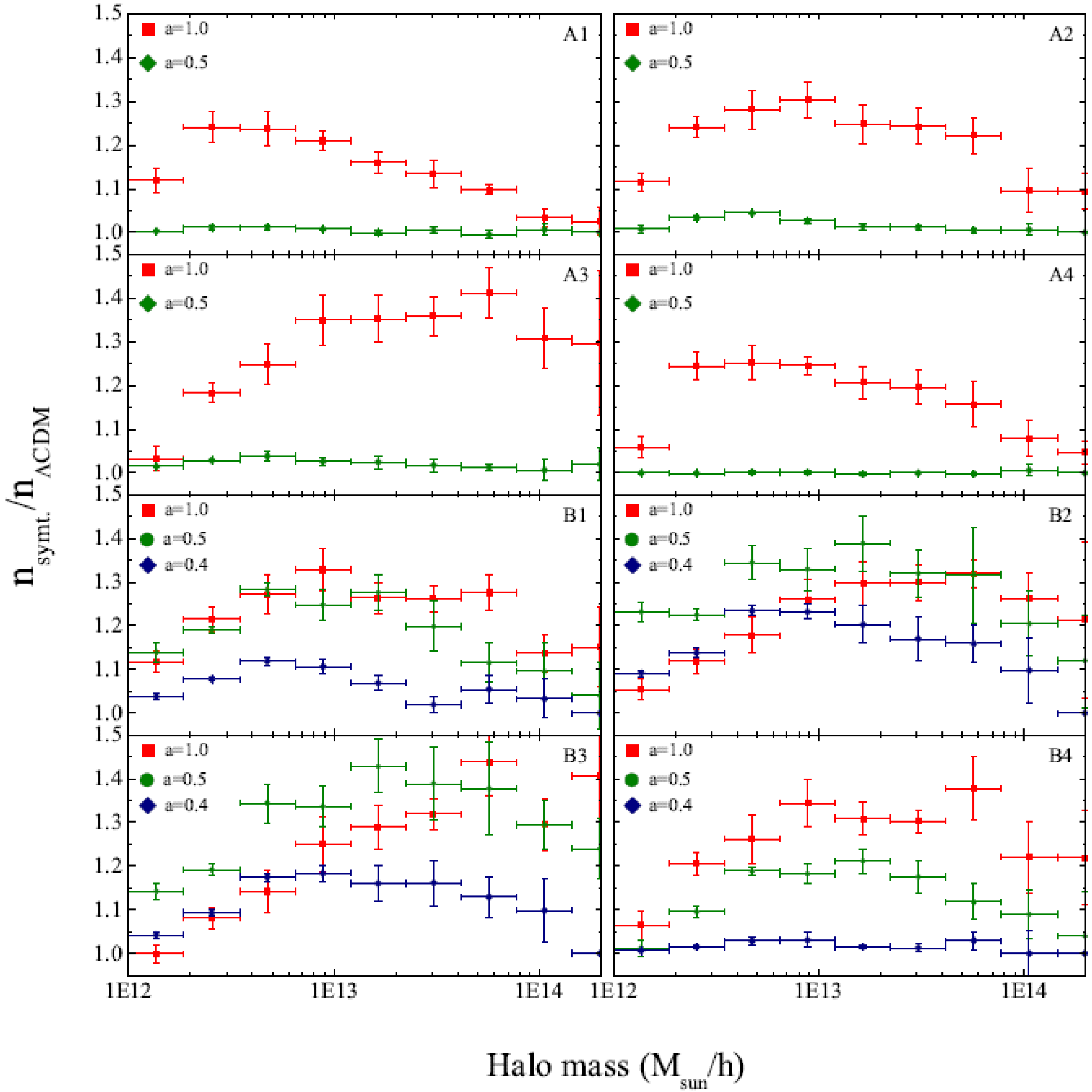
This implies the crucial bound:

$$\boxed{\frac{m_0}{H_0}\geq 10^3}$$

Effects of modified gravity can appear at most on the Mpc scale. Similar bound obtained from pulsar-white dwarf system (less loose and tied to the scalar emission of screened bodies).



Power Spectrum



Mass function

Depending on the models, one can see very clearly on the power spectrum the influence of modified gravity on marginally non-linear scales.

Could there be an effect on linear scales characterising the CMB (and then on BAO)?

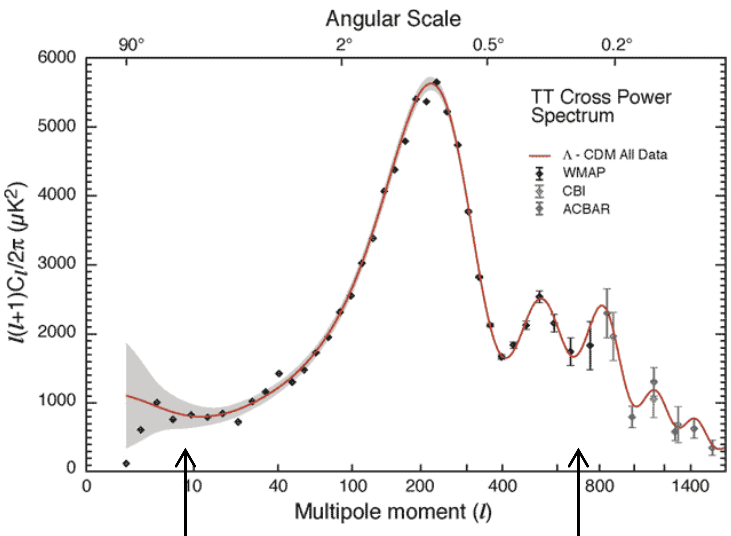
This depends on the precise way linear perturbations are modified by the presence of a scalar field.

PLANCK will give us very precise information on early universe physics.

What would happen if gravity were modified ?

In general, Newton's potential would not be constant since last scattering: ISW effect.

Effects on the peak structures for modes entering the horizon early enough if gravity is modified.



ISW

Early modification of gravity

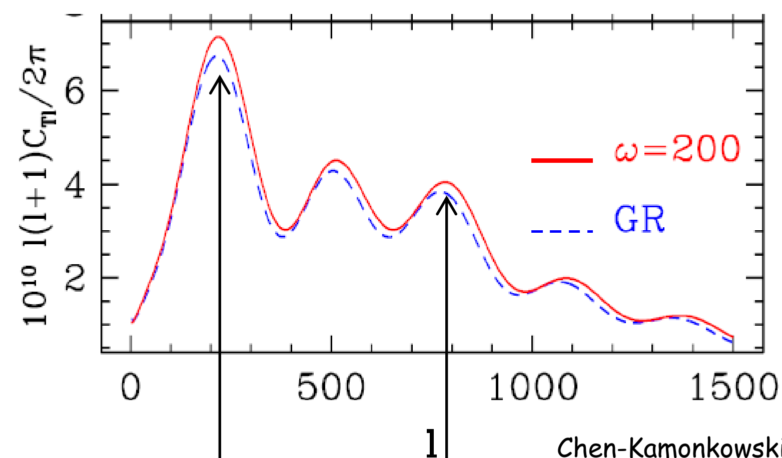
A simple modification of gravity : Brans-Dicke theory

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(\phi R - \frac{\omega}{\phi}(\partial\phi)^2 + L_m(\psi)\right)$$

BBN bound: $\omega \geq 32$ Damour-Pichon '99

CMB bound: $\omega \geq 1000$ Ali-Gannouji-Sami '10
Acquaviva et al '04

Planck sensitivity: $\omega \leq 3000$ Chen-Kamonkowski '99



Chen-Kamonkowski '99

Increase of the amplitude

Shift of the peaks

At the perturbation level where the CDM density contrast evolves like:

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2 \frac{\rho_c}{\rho_c + \rho_b + \rho_\gamma + \rho_\phi}\left(1 + \frac{2\beta_c^2}{1 + \frac{m^2 a^2}{k^2}}\right)\delta = 0$$

The new factor in the brackets is due to a modification of gravity depending on the comoving scale k.

This is equivalent to a scale dependent Newton constant.

The growth of structures depends on the comoving Compton length:

$$\lambda_c = \frac{1}{ma}$$

Gravity acts in an usual way for scales larger than the Compton length (matter era)

$$\delta \sim a$$

Gravity is modified inside the Compton length with a growth (matter era):

$$\boxed{\delta \sim a^{\frac{\nu}{2}},} \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta_c^2)}}{2}$$

For a given scale k, if am(a) decreases, the CDM density contrast first grows logarithmically in the radiation era, then like in GR before entering the Compton radius and the modified gravity regime resulting in an anomalous growth.

General Relativity → Modified gravity

$$z = z_k$$

This type of modification of gravity on large scales can have an influence on the CMB as the baryonic density contrast and Newton's potential may be affected.

Newton's potential at the last scattering surface (LS) is modified for scales having grown anomalously after matter-radiation equality and before last scattering . For $k_1 < k < k_c$

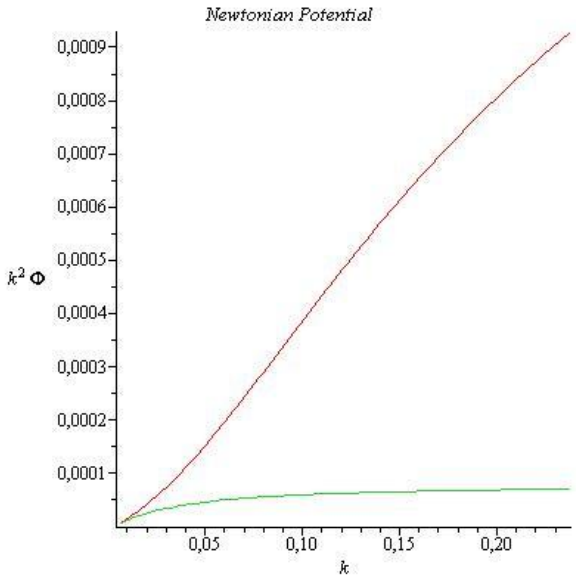
$$k^2\Phi \sim (\frac{k}{k_1})^{(1-\nu/2)/(1+r)}(1 + 2\ln\frac{a_{\rm eq}}{a_k})$$

$$k_1 = a_{LS} m(a_{LS})$$

For smaller and larger scales, the growth is only logarithmic.

We have illustrated this fact with:

$$m(a) = m(a_{LS})(\frac{a}{a_{LS}})^r$$



$$k_1 \sim 0.035h \cdot \text{Mpc}^{-1}, \quad k_c \sim 0.1h \cdot \text{Mpc}^{-1}$$

$$\beta_c = 100, \beta_b = 0$$

Baryons couple to the scalar field implying a modification of the speed of sound:

$$\tilde{c}_s^2 = c_s^2(1 - \frac{9\Omega_b \beta_b^2 A_b^2 R \mathcal{H}^2}{k^2 + m^2 a^2})$$

The baryon density contrast at the last scattering surface oscillates with a modified sound horizon. In the tight binding approximation, the Sach-Wolfe temperature receives new contributions:

$$\Theta = \frac{\tilde{\delta}}{3(1+R)^{1/2}} + \frac{R}{R+1}(1 + \frac{9\Omega_b \mathcal{H}^2}{k^2}\tilde{\beta}_b^2 + 2\tilde{\beta}_b\tilde{\beta}_c)\frac{\Phi}{3\tilde{c}_s^2}$$

where:

$$\tilde{\delta} = \frac{3}{2}(1+R_k)^{1/4}(1+2R_k)\Phi(0)(1+R)^{1/4}\cos\tilde{r}_s k$$

New sound horizon

The change of the speed of sound due to the coupling to baryons and the modified Newtonian potential due to the coupling to CDM imply that the peaks of both CMB and BAO are modified.

If modified gravity operates only up to a certain redshift larger than 100, gravity tests are essentially irrelevant and large couplings are allowed. If not the range of the scalar force now is constrained and couplings cannot be very large.

We will illustrate with a model:

$$m(a) = CH(a), \ \beta_b = \beta, \ \beta_c =, \ z < z_{\text{trans}}$$

If the transition is now, then C>1000, if the transition is earlier then this is relaxed. For a late transition, no effects on the CMB but still effects on BAO.

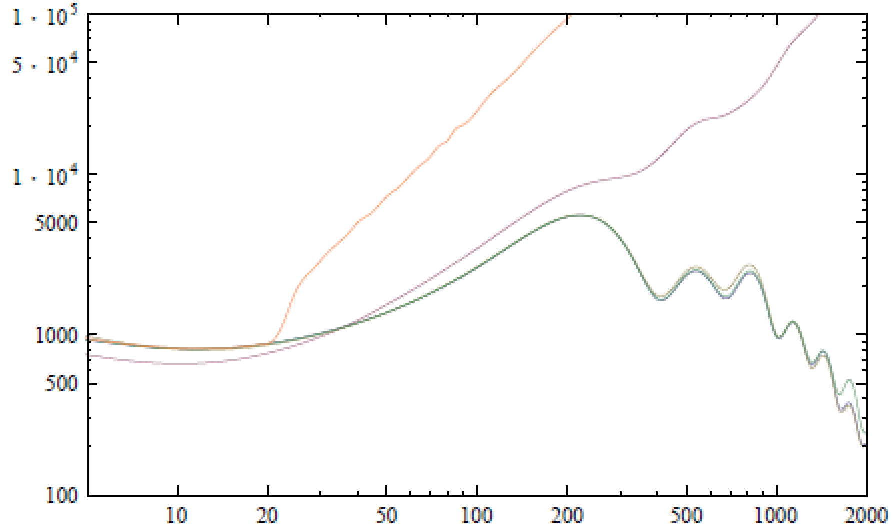


FIG. 7: CMB angular power spectrum for Model1, for LCDM (Blue) $z_{\rm trans} = 1100, \beta = 100, c = 100$ (red), $z_{\rm trans} = 1100, \beta = 20, c = 100$ (yellow), $z_{\rm trans} = 100, \beta = 20, c = 100$ (orange), (notice that in the case $z_{\rm trans} = 100, \beta = 100$, the Cls are pushed up by a crasy factor, something like 10^{40}), $z_{\rm trans} = 100, \beta = 10, c = 100$ (green)

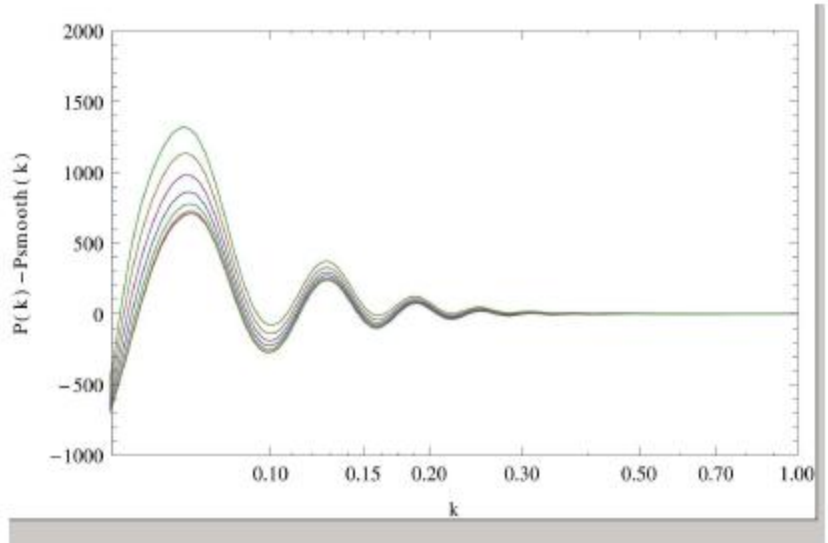


FIG. 11: The corresponding BAOs for the same parameters, $c = 100$, $z_{\rm trans} = 1100$ and $\beta_b = 0, 2, 5, 10, 15, 20, 25, 30$ from bottom to top.

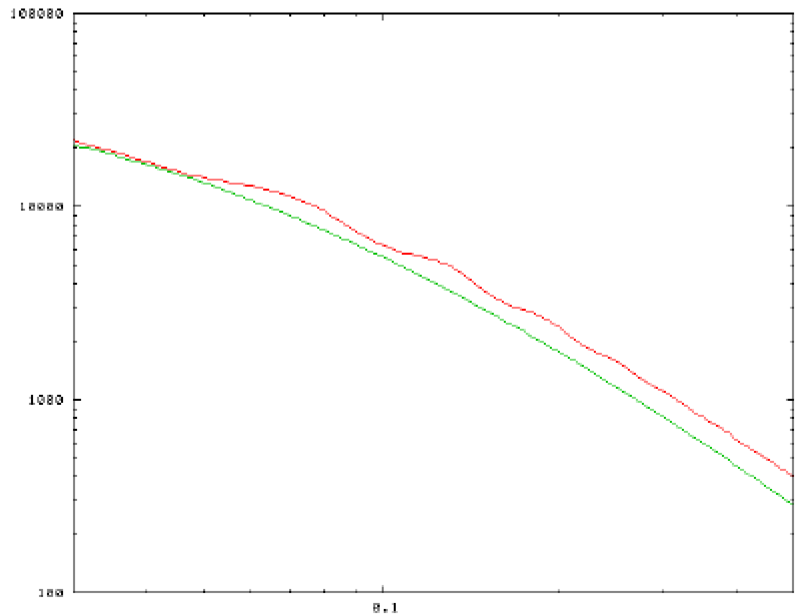


FIG. 2: $P(k)$ (red) and $P_{smooth}(k)$ (green) for $m = 100H$ and $\beta_b = 1$. $P_{smooth}(k)$ generated by cubic fit to ΛCDM $P(k)$.

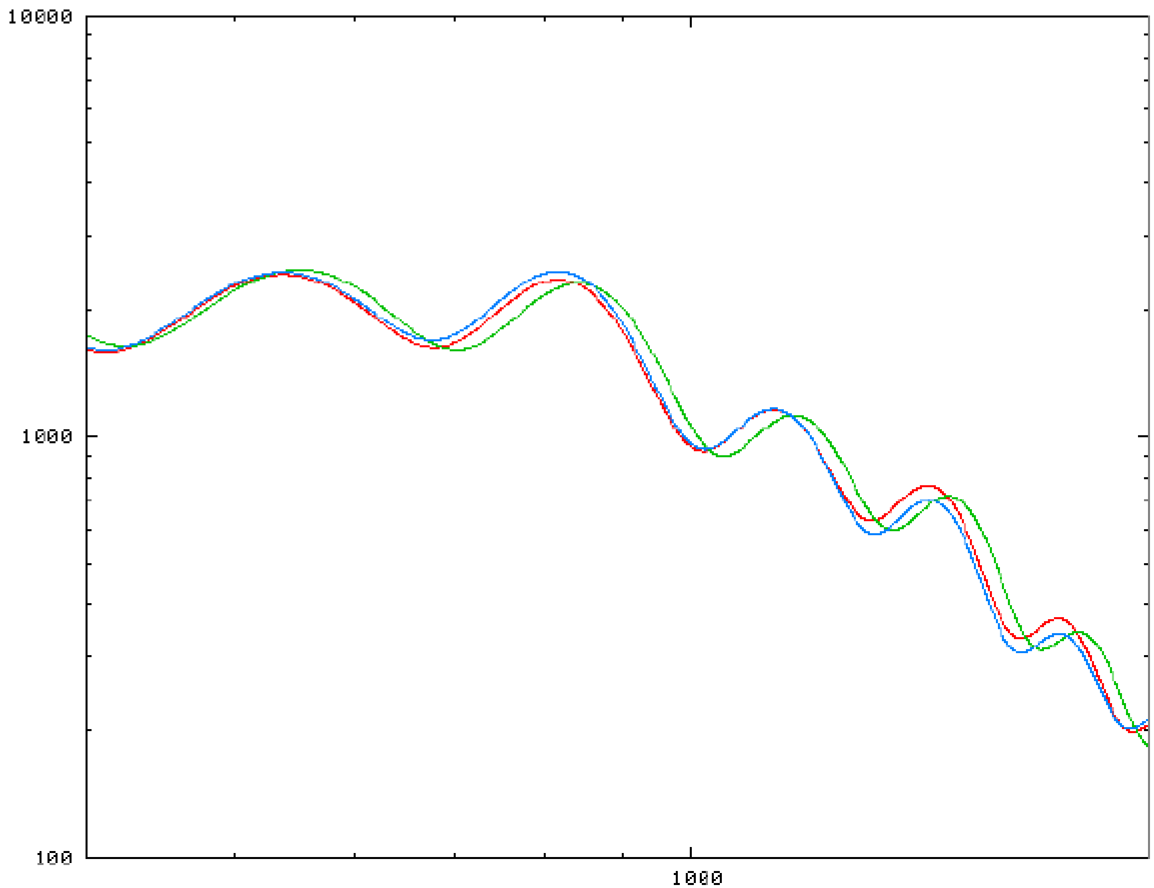


FIG. 1: ΛCDM (red), $C = 100$, $\beta_b = 50$ for $z > 10000$ (blue), $N_{eff} = 4$ (green).

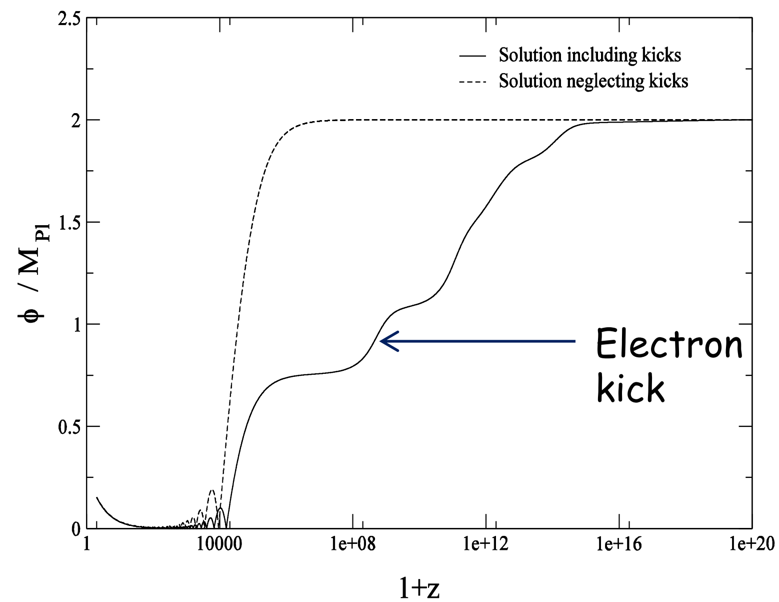
Modified gravity and dark energy are most of the time two faces of the same phenomenon: the possible existence of long range scalar forces.

Very long range forces are motivated by the acceleration of the expansion of the Universe. They need to be screened in the solar system.

These forces lead to changes in the growth of structure in the linear and non-linear regimes.

CMB and BAO are affected when gravity is modified around the last scattering epoch.

The scalar field must have converged towards the minimum of the effective potential before BBN otherwise large variation of particle masses due to the electron decoupling.



The scalar field follows the attractor (m>>H) from before BBN, where the matter density is similar to dense bodies now, to the present era with a tiny critical density.

$$10\ \mathrm{g}\cdot\mathrm{cm}^{-3} \rightarrow 10^{-29}\ \mathrm{g}\cdot\mathrm{cm}^{-3}$$