



A Panorama of Modified Gravity

Philippe Brax
IPhT Saclay



The Universe accelerates: why?



Maybe a landscape of Universes?

Or not?

The acceleration of the Universe could also be due to many mundane causes:

A calculable cosmological constant and/or vacuum energy

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi G_N \Lambda^4 g_{\mu\nu}$$

A modification of General Relativity (GR)

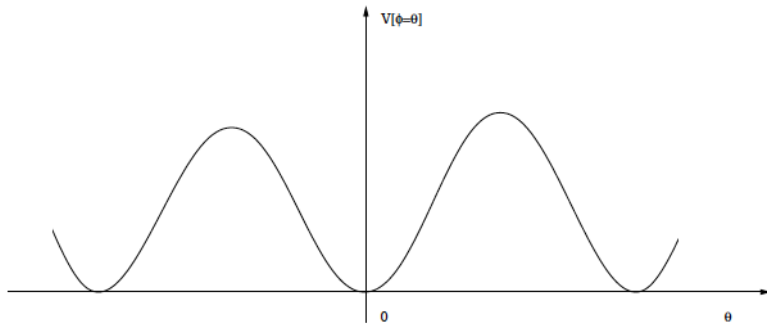
$$F(R_{\mu\nu}, g_{\mu\nu} \dots) = 8\pi G_N T_{\mu\nu}$$

The cosmological dynamics of fields (new matter)

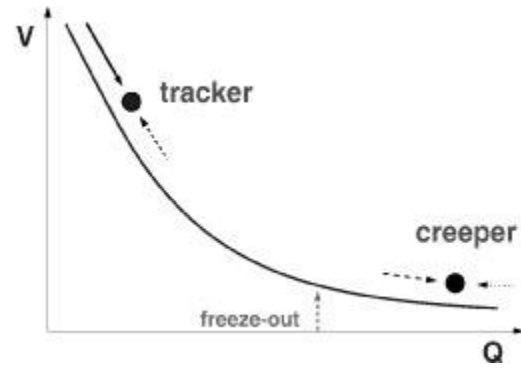
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{\text{new}}$$



DARK
ENERGY



Thawing



Freezing

A Simple Example: Pseudo-Goldstone Models

Global U(1) symmetry broken at scale f :

$$\Phi = f e^{i\varphi/\sqrt{2}f}$$

Symmetry broken by small term:

$$\mathcal{L}_{\text{breaking}} = \mu^4 \frac{\Phi}{2f} + \text{cc}$$

Low energy potential:

$$V(\varphi) = \mu^4 \cos \frac{\varphi}{\sqrt{2}f}$$

Cosmologically, if initially the field is small compared to f , it is frozen there until its mass becomes larger than the Hubble rate. Tuning this event to be in the recent past of the Universe and requiring that the dark energy is due to the low energy potential implies:

$$\mu \sim \Lambda, \quad f \sim m_{\text{Pl}}$$

A major drawback of these models is that the phase transition must be at the Planck scale, this requires embedding this model in theories dealing with quantum gravity... maybe embedding it in string theory although no global symmetries there...

On the other hand, this model is useful to see that low energy dark energy fields couple to matter:

$$\mathcal{L}_{\text{int}} = \frac{\partial\Phi\partial\bar{\Phi}}{f^3}\bar{\psi}\psi \quad \longrightarrow \quad \mathcal{L}_{\text{int}} \sim \frac{(\partial\phi)^2}{f^3}\bar{\psi}\psi$$

The derivative interaction implies no effect on static tests of gravity vs fifth forces. This an example of a disformal coupling to matter.

This is one example of coupling to matter, we will also consider conformal couplings which are tightly constrained.



Modified gravity

Massive Gravity

Mass term for a graviton

The simplest modification is massive gravity (Pauli-Fierz):

$$\delta\mathcal{L} = \frac{m_G^2}{4}(h^{\mu\nu}h_{\mu\nu} - h^2)$$

Pauli-Fierz gravity is ghost free (negative kinetic energy terms) . Unfortunately, a massive graviton carries 5 polarisations when a massless one has only two polarisations. In the presence of matter, the graviton wave function takes the form:

$$h_{\mu\nu} = \frac{8\pi G_N}{p^2}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}) \rightarrow h_{\mu\nu} = \frac{8\pi G_N}{p^2 + m_G^2}(T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu})$$

The massless limit does not give GR! (van Dam-Velman-Zakharov discontinuity). The extra polarization is lethal. Solved by Vainshtein mechanism (non-linearity).

Ghost in curved space-time!

Bimetric Gravity

One way to cure these problems is to consider a non-linear version of massive gravity with two dynamical metrics:

$$S = \int d^4x \left(e_1 \frac{R_1}{16\pi G_N} + e_2 \frac{R_2}{16\pi G_N} + \Lambda^4 \sum_{ijkl} m_{ijkl} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e_\mu^{ai} e_\nu^{bj} e_\rho^{ck} e_\sigma^{dl} \right)$$

where the graviton mass is of order:

$$m_g^2 \sim \frac{\Lambda^4}{m_{\text{Pl}}^2}$$

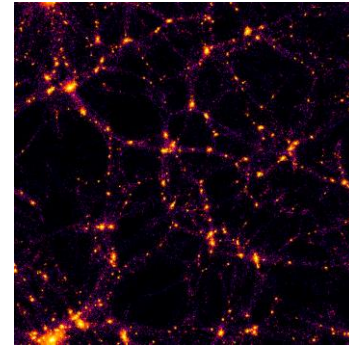
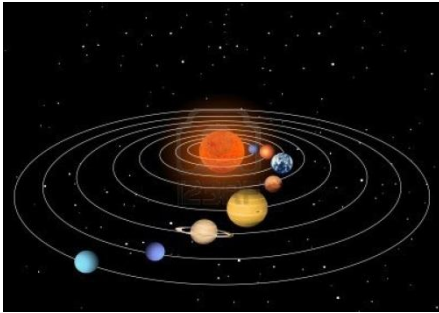
What we have learnt in the last ten years is that such extensions involve light scalar fields, why?:

Massive gravitons have a **scalar** part

$$5 = 2 + 2 + \textcircled{1}$$

Their interaction with matter generates **fifth forces** which would have been seen in the laboratory and the solar system.

GRAVITY ACTS ON ALL SCALES



Looking for extensions of General Relativity valid from small to large scales.

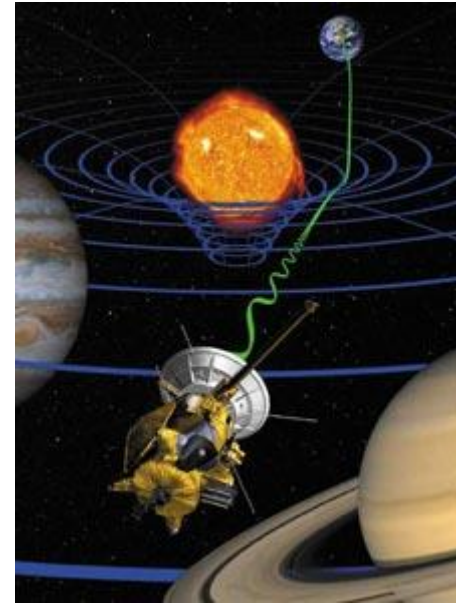
In general, these theories require a fine-tuning of the cosmological constant but have unexpected field theoretic properties which go beyond the usual framework of field theory: irrelevant operators dominate, UV completions may not be required (classicalisation)... The most general form of these theories is known in some cases (Horndeski for one scalar) but one must go into the details of the models to make them work from the solar system (or the laboratory) to large cosmological scales.

Deviations from Newton's law are parametrised by:

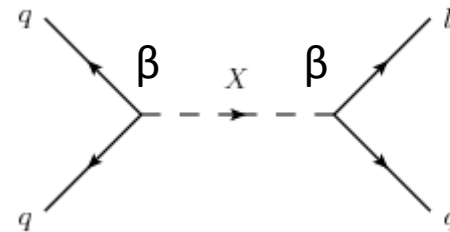
$$\phi_N = -\frac{G_N}{r}(1 + 2\beta^2 e^{-r/\lambda})$$

For large range forces with large λ , the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay):

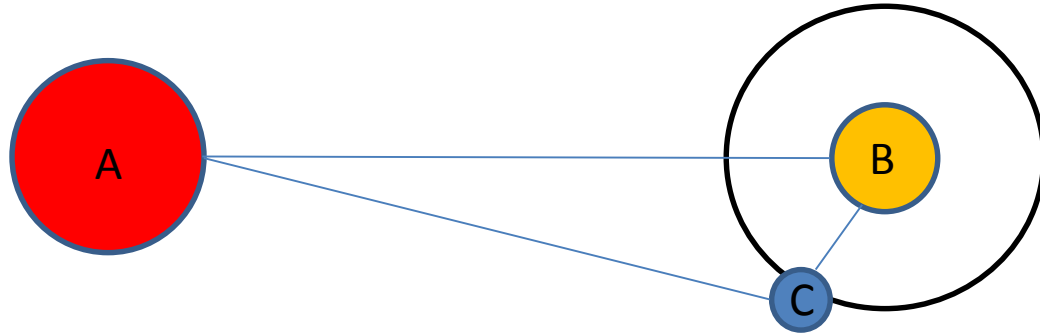
$$\beta^2 \leq 4 \cdot 10^{-5}$$



Bertotti et al. (2004)



VIOLATION OF THE STRONG EQUIVALENCE PRINCIPLE



$$\eta_{BC} \equiv \left| \frac{a_C - a_B}{a_C + a_B} \right|$$

$$\eta_{\text{moon-earth}} \leq 10^{-13}$$

Tight bound on the perihelion advance of the moon too (see later)



Lunar ranging experiment

How do we hide light scalar fields ?

SCREENING

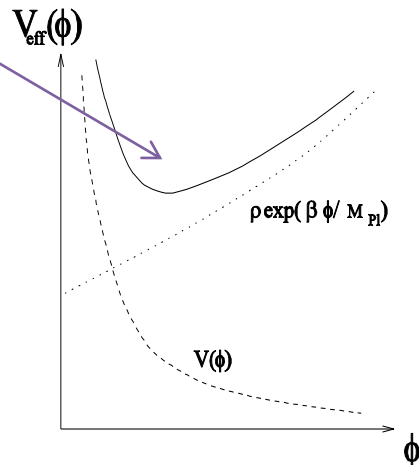
The effect of the environment

When conformally coupled to matter, scalar fields have a **matter dependent effective potential**

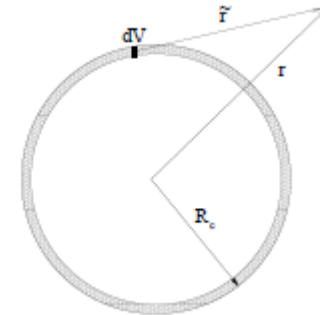
$$V_{eff}(\phi) = V(\phi) + \rho_m(A(\phi) - 1)$$

Environment
dependent
minimum

$$\beta(\phi) = m_{Pl} \frac{d \ln A(\phi)}{d\phi}$$



Chameleon



The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.

One example: f(R) gravity

$$S_{\text{MG}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} f(R)$$

f(R) is totally equivalent to a field theory with gravity and a scalar

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G_N} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\text{Pl}}} g_{\mu\nu}) \right)$$

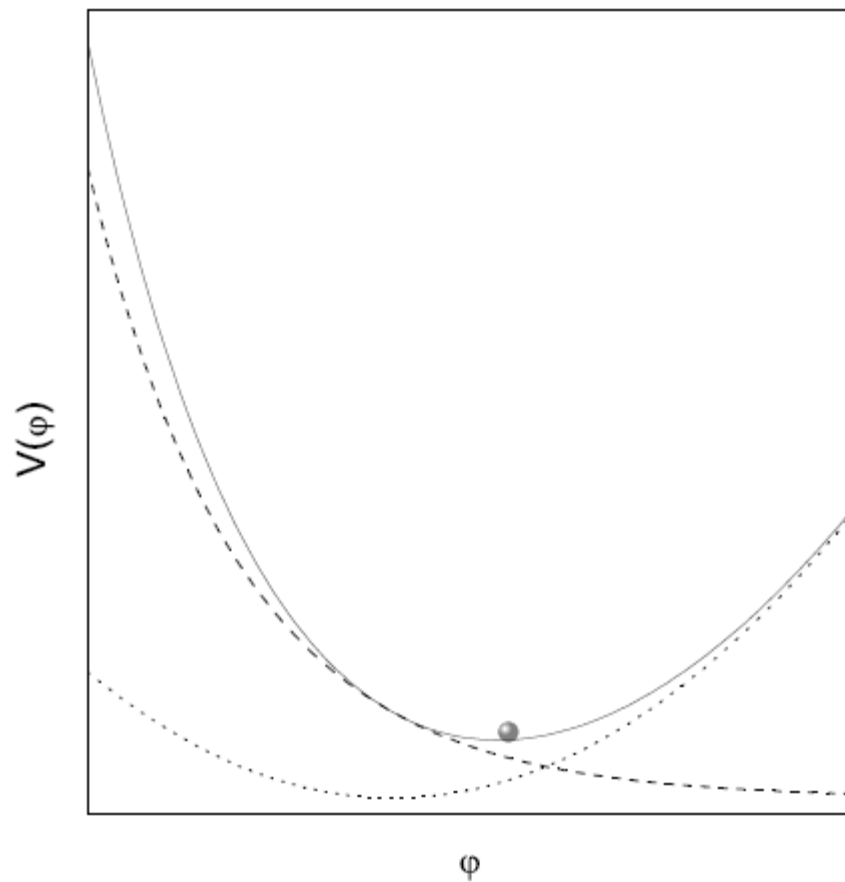
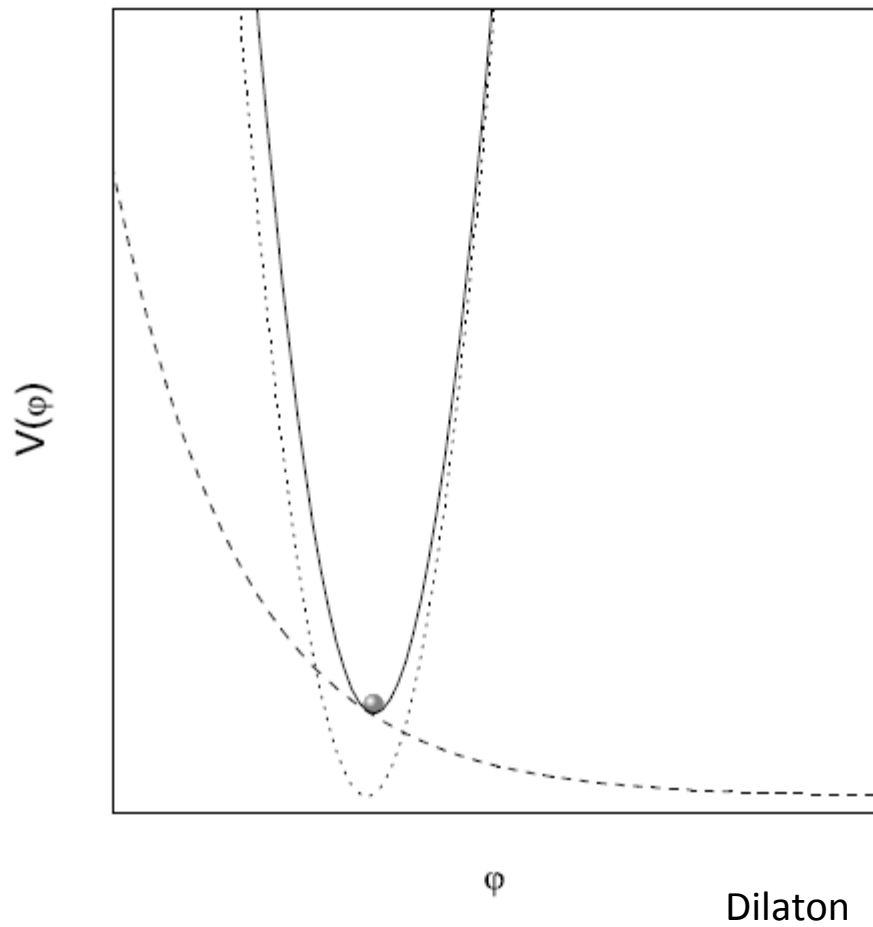
The potential V is directly related to f(R)

Crucial coupling between matter and the scalar field

$$V(\phi) = m_{\text{Pl}}^2 \frac{Rf' - f}{2f'^2}, \quad f' = e^{-2\phi/\sqrt{6}m_{\text{Pl}}}$$

Would be ruled out if no chameleon effect

Damour-Polyakov mechanism

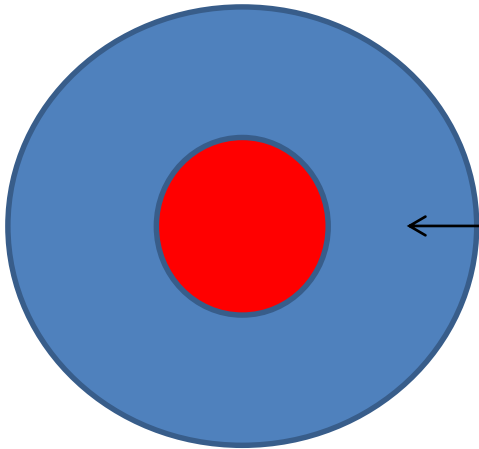


$$V(\phi) = V_0 e^{-\phi/m_{\text{Pl}}}, \quad A(\phi) = 1 + \frac{A_2}{2m_{\text{Pl}}^2} (\phi - \phi_*)^2$$

Another simple example: **the CUBIC GALILEON**

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2\Lambda^3}(\partial\phi)^2\partial^2\phi + \frac{\beta\phi}{M_P}T .$$

$$\Lambda^3 = m^2 m_{\text{Pl}} \quad m \text{ graviton mass}$$



$$\frac{F_\phi}{F_N} = 2\beta^2 \left(\frac{r}{R_V} \right)^{3/2} .$$

Well inside the Vainshtein radius, Newtonian gravity is restored. Well outside gravity is modified.

The Vainshtein radius is very large for stars, typically 0.1 kpc for the sun, and a mass for the graviton of the order of the Hubble rate .

K-mouflage models

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} + M^4 K(\chi) \right) + S_m(\psi, A^2(\phi) g_{\mu\nu}) \quad \chi = -\frac{(\partial\phi)^2}{2M^4}$$

M is the dark energy scale for cosmologically interesting models. Examples that one may consider:

$$K(\chi) = -1 + \chi + K_0 \chi^m$$

Particles have modified trajectories compared to Newton's law in this background:

$$\frac{d^2 r}{dt^2} = -\frac{G_N m}{r^2} \left(1 + \frac{2\beta^2}{K'} \right)$$

Screening happens inside the K-mouflage radius where $K' \gg 1$. Still the tiny deviations depend on r and can lead to an anomalous perihelion in the earth-moon system.

In fact, around a background configuration and in the presence of matter, the Lagrangian of such extensions can be linearised and the main screening mechanisms can be schematically distinguished :

$$\mathcal{L} \supset \frac{Z(\phi_0)}{2} (\partial\delta\phi)^2 - \frac{m^2(\phi_0)}{2} \delta\phi^2 + \frac{\beta(\phi_0)}{M_P} \delta\phi\delta T ,$$

The **Vainshtein** mechanism reduces the coupling by increasing Z . The **K-mouflage mechanism** has the same effect ... while the **Damour-Polyakov mechanism** suppresses β and the **chameleon** property increases m .

The **Vainshtein** and **K-mouflage** mechanisms can be easily analysed:

Effective Newtonian potential:

$$\Psi = \left(1 + \frac{2\beta^2(\phi)}{Z(\phi)}\right)\Phi_N$$

For theories with second order equations of motion:

$$Z(\phi) = 1 + a(\phi)L^2 \frac{D^\mu D_\mu \phi}{m_{\text{Pl}}^2} + b(\phi) \frac{(\partial\phi)^2}{M^4} + \dots$$

Vainshtein

K-mouflage

$$M^4 \sim 3H_0^2 m_{\text{Pl}}^2, \quad L \sim H_0^{-1}$$

Cosmological choice

Vainshtein

Newtonian gravity retrieved when the curvature is large enough:

$$\nabla^2 \Phi_N \geq \frac{1}{2\beta L^2}$$

On large cosmological scales, this tells us that overdensities such as galaxy clusters are screened :

$$\delta \geq \frac{1}{3\Omega_{m0}\beta}$$

On small scales (solar system, galaxies) screening only occurs within the Vainshtein radius:

$$R_V = \left(\frac{3\beta m L^2}{4\pi m_{\text{Pl}}^2} \right)^{1/3}$$

K-mouflage

Newtonian gravity retrieved when the gravitational acceleration is large enough:

$$|\nabla\Phi_N| \geq \frac{M^2}{2\beta m_{\text{Pl}}}$$

On large cosmological scales, this tells us that overdensities such as galaxy clusters are not screened :

$$\frac{k}{H_0} \leq \beta\delta$$

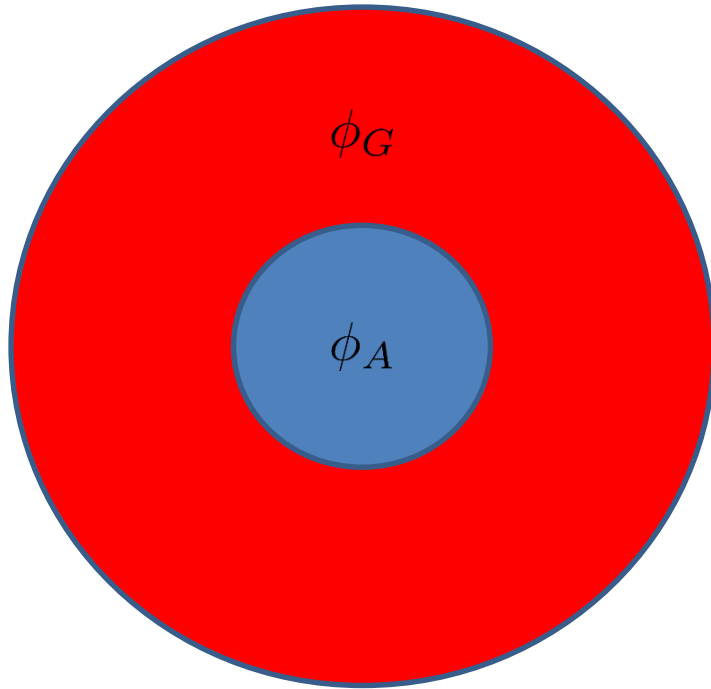
On small scales (solar system, galaxies) screening only occurs within the K-mouflage radius:

$$R_K = \left(\frac{\beta m}{4\pi m_{\text{Pl}} M^2}\right)^{1/2}$$

Dwarf galaxies are not screened.

Chameleons:

The screening criterion for an object **BLUE** embedded in a larger region **RED** expresses the fact that the **Newtonian potential of an object must be larger than the variation of the field**:



Scalar charge: $Q_A = \frac{|\phi_G - \phi_A|}{2m_{\text{Pl}}\Phi_A}$

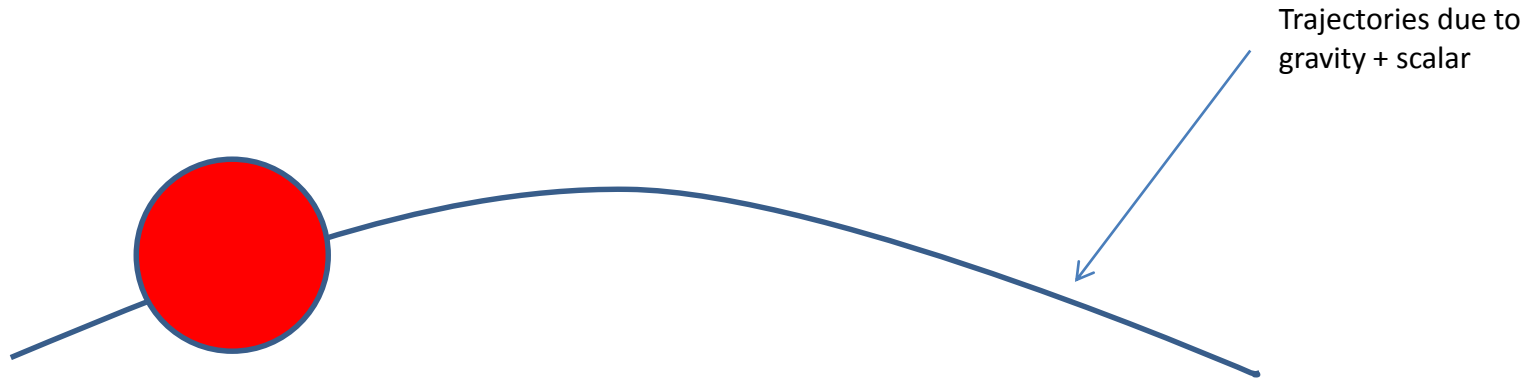
$$Q_A \leq \beta_G$$

Self screening: large Newton potential

Blanket screening: due to the environment G

Φ_A Newton's potential at the surface

Summary: the motion of massive objects in modified gravity theories:



$$\ddot{X} = -\nabla\Phi_N - Q\nabla\phi_{\text{ext}}$$

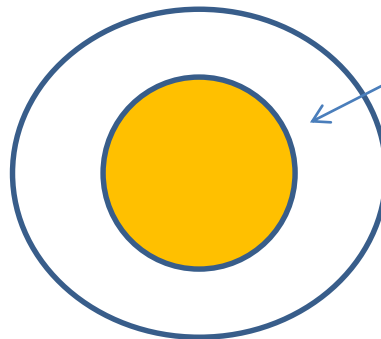
Chameleons: $Q \ll \beta$

Unlike chameleons et al., K-mouflage and Vainshtein do not affect the charge Q :

$$Q = \beta$$



Outside the Vainshtein-Kmouflage radius, the object feels the scalar force



$$\nabla\phi_{\text{ext}} \ll 2\beta\nabla\Phi_N$$

$$\nabla\phi_{\text{ext}} = 2\beta\nabla\Phi_N \text{ outside the Vainshtein-Kmouflage radius}$$

One can also change the type of coupling to matter!

Matter couples to a metric which can differ from the Einstein metric involved in the Einstein-Hilbert term with a constant Newton constant:

$$S = \int d^4x \sqrt{-g} \frac{R(g)}{16\pi G_N} + S_m(\psi^i, \tilde{g}_{\mu\nu})$$

Bekenstein (1992) showed that causality and the weak equivalence principle restricts the form of the auxiliary metric:

$$\tilde{g}_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

$$X = -\frac{1}{2}(\partial\phi)^2$$

What is the physics associated with the disformal coupling $B(\phi, X)$?

Expanding the effect of the disformal coupling to leading order, we obtain a direct coupling with the energy-momentum tensor:

$$\mathcal{L} \supset \frac{1}{M^4} \partial_\mu \phi \partial_\nu \phi T^{\mu\nu}$$

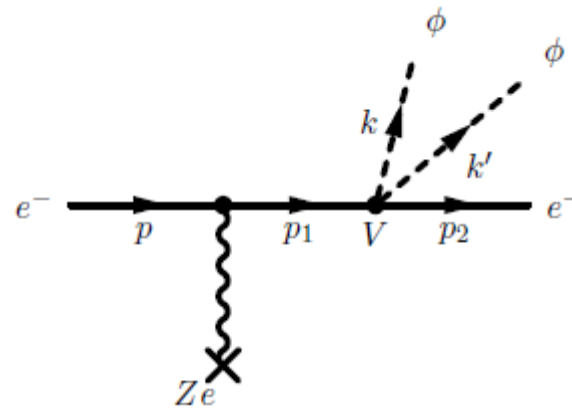
Gravity tests involving static objects are not affected by the disformal coupling: **SCREENING**

$$T^{00} = \rho \quad \rightarrow \quad \frac{\dot{\phi}^2}{M^4} \rho \equiv 0$$

As the axions, the disformally coupled scalars can have an effect at high density and high temperature inside the inferno at the core of stars. From the gentle burning of main sequence stars, to helium burning stars and then the explosion of supernovae, the processes involve higher energies (hence shorter distances) and electromagnetic to strong interaction processes.



Bremsstrahlung is one of the most common processes in stars. Here two scalars can be emitted from one electron in the electric field of a nucleus.



$$\epsilon = \frac{Z^2 \alpha^2 m_e}{210 \pi^3 A m_p} \rho \left(\frac{T}{M} \right)^8 \frac{1}{(2\pi m_e T)^{3/2}} g \left(\frac{m_D^2}{2m_e T} \right),$$

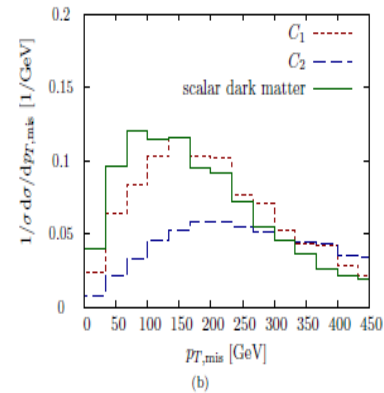
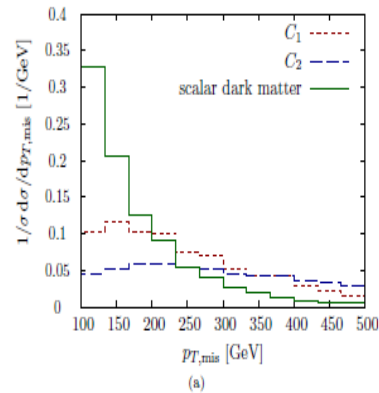
$$g(x) = \int_0^\infty du \frac{u^9 e^{-u}}{(u+x)^2}.$$

The strongest constraint comes from stars for which helium burns: $\epsilon_{HB} \leq 10 \text{ erg/s} \cdot \text{g}$

$$M \geq 173 \text{ MeV}$$

The most constraining test of the disformal coupling is at colliders and comes from missing energy, for instance monojets or t-tbar.

$$\mathcal{L}_1 : 2\phi + t\bar{t} \quad M \geq 237.4\text{GeV}(ATLAS), \quad M \geq 192.4\text{GeV}(CMS)$$

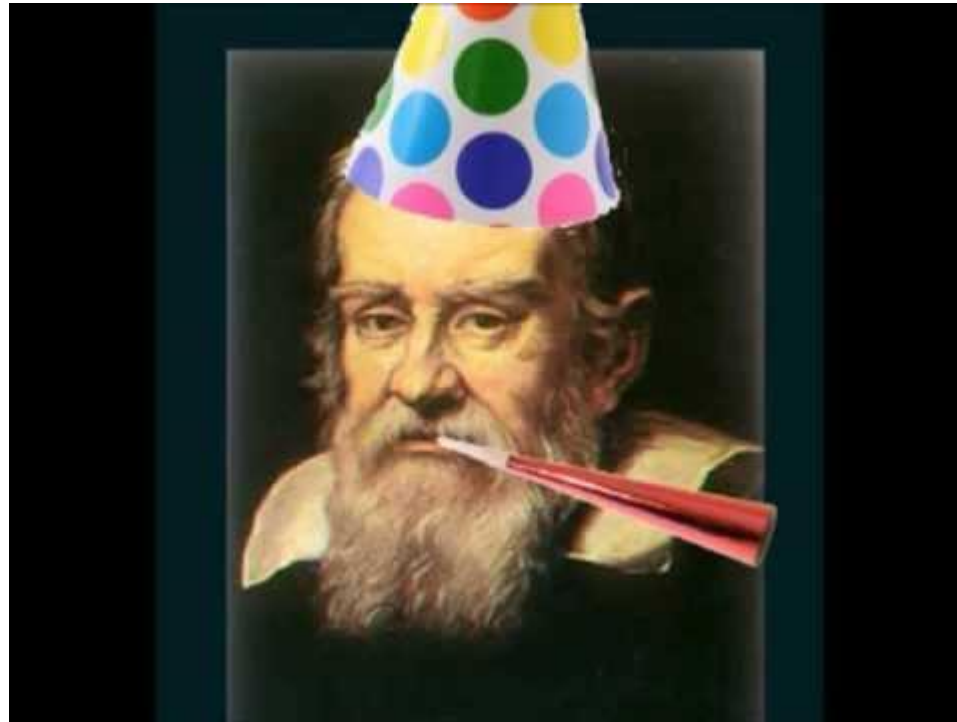


The missing pT distribution can be significantly different from a dark matter signal (in particular for jets).

$$\mathcal{L}_2 : 2\phi + jet \quad M \geq 693.4\text{GeV}(ATLAS), \quad M \geq 822.8\text{GeV}(CMS)$$

$$\mathcal{L}_2 : 2\phi + t\bar{t} \quad M \geq 461.2\text{GeV}(ATLAS), \quad M \geq 399.8\text{GeV}(CMS)$$

A worked out example: more about Galileons:



The Galileon models are motivated by the requirement that the effective scalar field theory is invariant under a symmetry which prevents the existence of a potential term generalising a cosmological constant. The aim is to get **self-accelerated Universes** without a cosmological constant. The acceleration of the Universe would have to be self-generated dynamically:

$$\partial_\mu \phi \rightarrow \partial_\mu \phi + c_\mu$$

This is a **Galilean invariance** of the action. The scalar Lagrangian involves only a small number of building blocks:

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X)D^2\phi + G_4(\phi, X)R + G_{4,X}((D^2\phi)^2 - (D_\mu D_\nu \phi)^2) - \frac{1}{6}G_{5X}((D^2\phi)^3 - 3D^2\phi(D_\mu D_\nu \phi)^2 + 2D^\mu D_\alpha \phi D^\alpha D_\beta \phi D^\beta D_\mu \phi)$$

where X is the kinetic term of the scalar field. The scalar field ϕ is screened by the Vainshtein mechanism.

The coupling functions are not arbitrary but specified by 4 real coefficients:

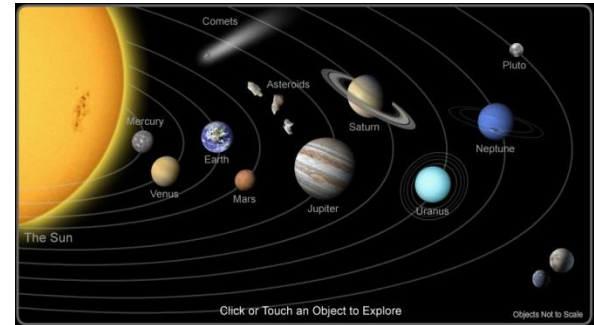
$$K(\phi, X) = c_2 X, \quad G_3(\phi, X) = -\frac{2c_3}{\Lambda^3} X, \quad G_4(\phi, X) = \frac{2c_4}{\Lambda^6} X^2, \quad G_5(\phi, X) = -\frac{6c_5}{\Lambda^9} X^2$$

The simplest example: the cubic Galileon. The scale Λ is the strong coupling scale of the model. These models have a cosmological interest when:

$$\Lambda^3 = m_{\text{Pl}} H_0^2$$

Screening for Galileon theories happens inside the Vainshtein radius. Still the tiny deviations depends on r and can lead to an anomalous perihelion in the earth-moon system.

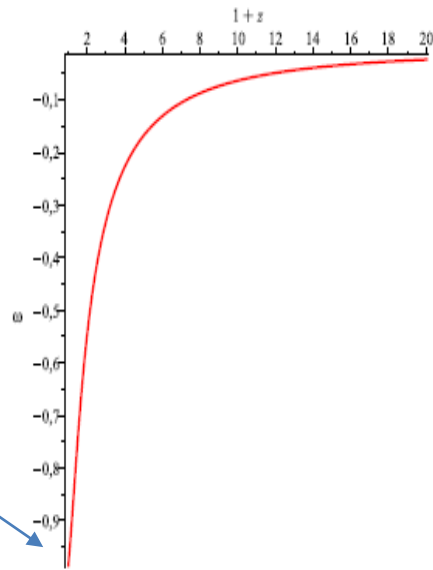
$$|\delta\theta| = \pi r \frac{d}{dr} \left(r^2 \frac{d}{dr} \left(\frac{\epsilon}{r} \right) \right) \leq 10^{-11}$$



$$\frac{\beta^{1/3} \Lambda}{c_3^{1/3}} \leq 1.6 \cdot 10^{-22} \text{ GeV}$$

This forces us to consider cosmological Galileons for couplings to matter of order one

Equation of state -1 now



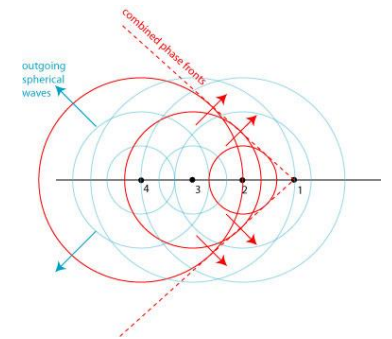
This requires tuning of some of the parameters of the models! See later for a comparison with data (Neveu et al. 2016).

Another possible effect of modified gravity is... to change the speed of gravitons compared to the speed of light.

Severe constraints must be taken into account:

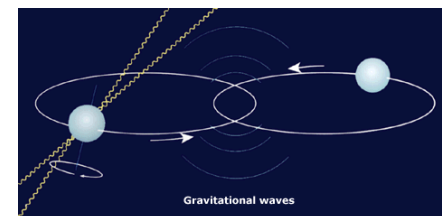
If the gravity waves travel slower than the speed of light, relativistic cosmic rays may emit gravitons by the Cerenkov effect. This would suppress them heavily unless:

$$1 - \frac{c_T}{c} \leq 10^{-17}$$



If the gravity waves travel faster than the speed of light, this would affect the period of binary pulsars unless:

$$\frac{c_T}{c} - 1 \leq 10^{-2}$$



Modified gravity models can have effects on the speed of gravity waves when the term Einstein-Hilbert term is modified in the significant way:

$$G_{4,X} \neq 0$$

Typically in astrophysical situations, we are interested in the emission of spherical waves in a time-dependent cosmological background. In this context the wave equation for gravitons takes the form:

$$\omega^2(G_4 - G_{4,X}\dot{\phi}^2) - 2\omega k\dot{\phi}\partial_r\phi - k^2(G_4 + G_{4,X}(\partial_r\phi)^2) = 0$$

The speed of gravity wave is “screened”, i.e. hardly modified, if the terms in $G_{4,X}$ can be neglected. Could it be that the “Vainshtein mechanism” plays a role here too and screens the speed of gravity waves to a level such that the tight bounds from the Cerenkov effect and the timing of pulsars could be satisfied?

Inside the Vainshtein radius of quartic Galileons, **IF** spatial gradients are larger than time derivatives:

$$c_T^2 = 1 - \frac{2XG_{4,X}}{G_4}$$

Where the gradient is essentially constant... and very small... implying that the speed of gravity waves would be screened...

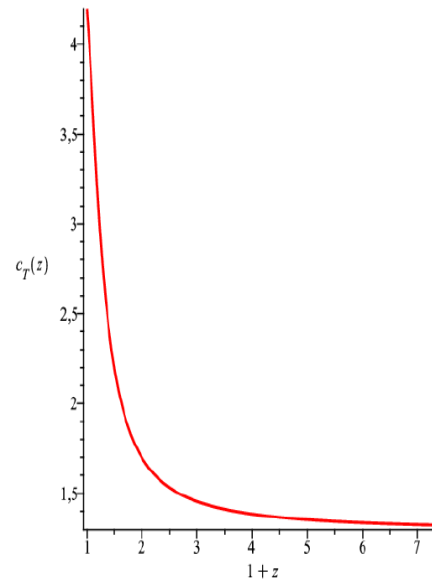
$$X = -\frac{\Lambda^4}{2} \left(\frac{c_{0b}M}{8\pi m_{\text{Pl}}c_4} \right)^{2/3}, \quad |\Delta c_T^2| \leq 10^{-30}$$

Unfortunately, the time derivatives are smaller than the gradients when the following condition is verified:

$$R_V H_0 \gg 1$$

This is violated for masses of around one solar mass as the Vainshtein radius is much smaller than the cosmological horizon!

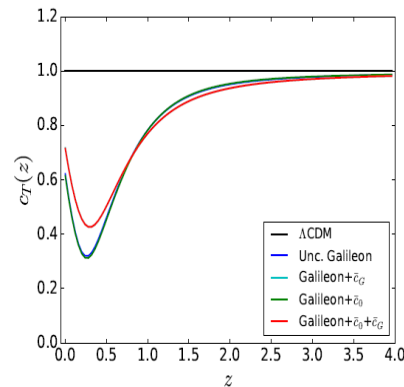
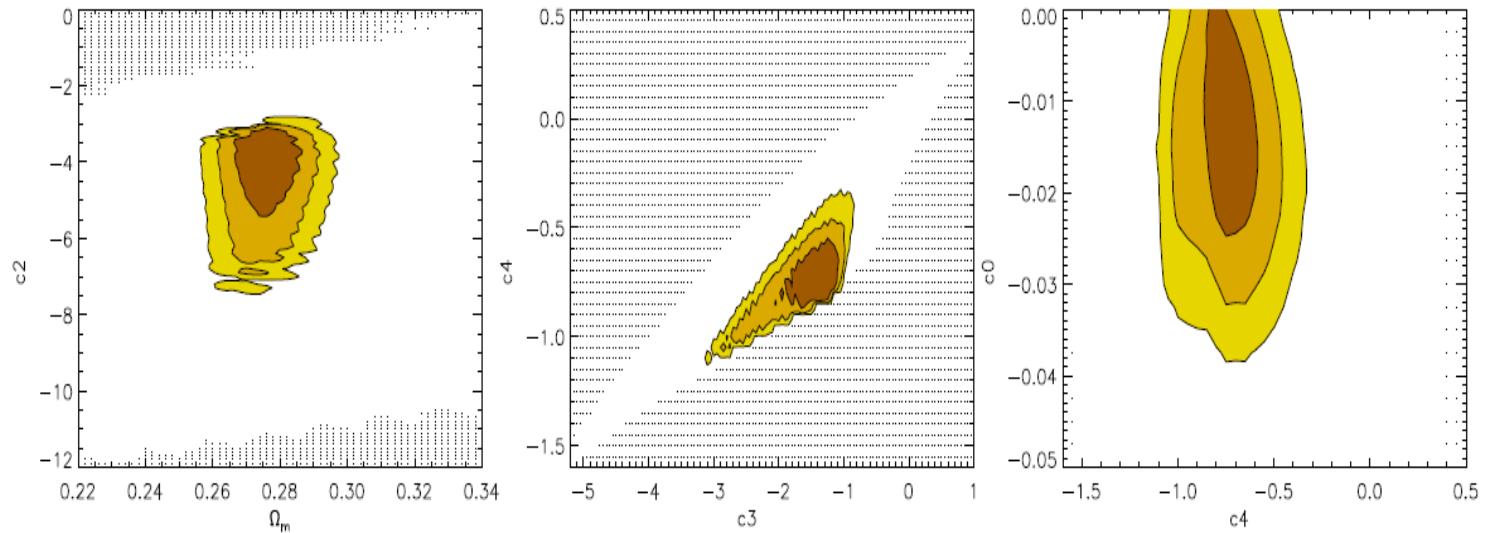
$$R_V H_0 \sim 10^{-7}$$



For Galileon models with an equation of state of -1 now, the speed of gravity waves is way too large... Can be cured using only cubic Galileons... but they have an equation of state -1 only when $c_2 < 0$.

This is a major stumbling block as this entails that these Galileons cannot be embedded in more fundamental models such as 4-branes moving in 5d. This would require a negative tension brane, which is unstable and crumples eventually.

Combined constraints from all present cosmological observables favour a Universe with $c_2 < 0$ and a smaller than unity $c = 0.7$. The negative value of c_2 is allowed because the models are stable in a FRW background. The small value of c can only be envisaged as the Cerenkov constraint from cosmic rays might not apply as beyond the domain of validity of the theory, i.e. beyond its cut-off where the models are in a regime which is beyond our understanding (and using a 5d picture cannot serve as a UV completion as $c_2 < 0$).



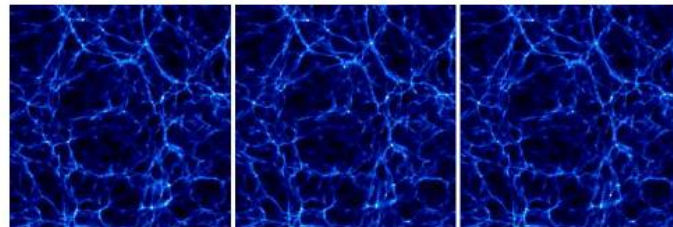
Are there distinguishable features of the screening mechanisms?

At the background level?

Sandage effect?

At the perturbative level?

Linear scales? Non-linear?



The Sandage effect:

$$\frac{\Delta v}{c} \equiv \frac{\Delta z}{z+1} = \frac{\Delta t}{1+z} ((1+z)H_0 - H(z))$$

An observer measuring the variation of the redshift of a distant source at two different times will see a drift which can be interpreted as a Doppler effect due to the « spectroscopic velocity ». This is essentially a fancy way of doing cosmography.

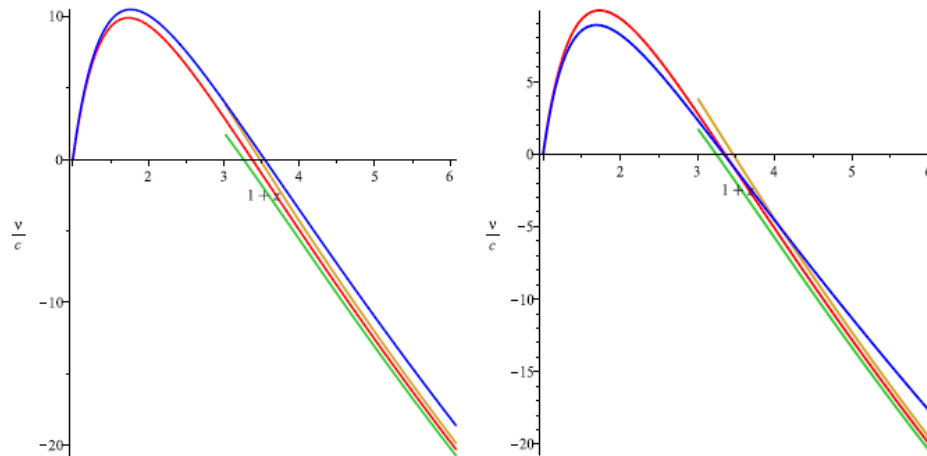
The spectroscopic velocities can be calculated for typical models subject to the three screening mechanisms:

F(R) and
dilatons

Cubic K-
mouflage

Quartic
Galileons

The precision of forthcoming experiments such as HIRES-EI can be estimated for quasar absorption system at redshifts $z > 2$, depending essentially on the number of years of observations and the number of objects. For 30 years of tracking and 100 objects, K-mouflage (left) and Galileons (right) deviate at 2-sigma for $z > 4$.



Cubic K-mouflage

Quartic Galileons

$$\sigma = 1.35 \left(\frac{N_{QSO}}{30} \right)^{-1} \left(\frac{S/N}{2370} \right)^{-1} (1+z)^{-1.7}$$

Large Scale Structure?

These three types of theories affect the growth of large scale structure in different ways:

Chameleons:



K-mouflage:



For [Vainshtein](#), even clusters are screened.

Chameleons:

The cosmological background evolves like in the concordance model. The main difference coming from the modification of gravity arises at the perturbation level where the Cold Dark Matter density contrast evolves like:

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}\Omega_m\mathcal{H}^2\left(1 + \frac{2\beta^2}{1 + \frac{m^2 a^2}{k^2}}\right)\delta = 0$$

Modified gravity



The new factor in the bracket is due to a modification of gravity depending on the comoving scale k . Most effects in the quasi-linear regime around Mpc scales, i.e. requires N-body or semi-analytical methods to go beyond linear perturbation theory.

This is now available for most models.

The growth of structures depends on the comoving **Compton length**:

$$\lambda_c = \frac{1}{ma}$$

Gravity acts in an usual way for scales larger than the Compton length (matter era)

$$\delta \sim a$$

Gravity is modified inside the Compton length with **MORE** growth (matter era):

$$\delta \sim a^{\frac{\nu}{2}}, \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta_c^2)}}{2}$$

When the coupling is a function of the scalar field, the growth is not power-like but still anomalous growth.

Lensing is also affected:

$$\Phi_{WL} = \Phi_N$$

No modification of the lensing potential

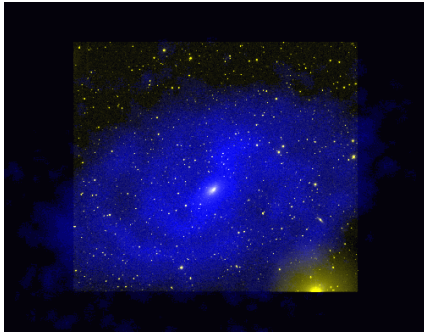
But the density contrast is modified and the effective Newton constant varies with time:

$$\Delta\Phi_N = 4\pi\bar{A}(t)G_N a^2 \delta\rho$$

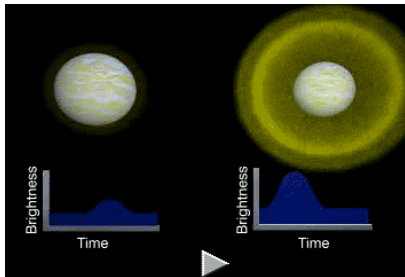
Growth and lensing are modified in all these models. Still linear scales more reliable. Comparison with available data of large scale structure and lensing has been performed giving constraints on $f(R)$ and other models such as dilatons which are loosely restricted. For $f(R)$, the cosmological bounds are not as good as the bounds from astrophysics but subject to fewer uncertainties.

Astrophysical tests:

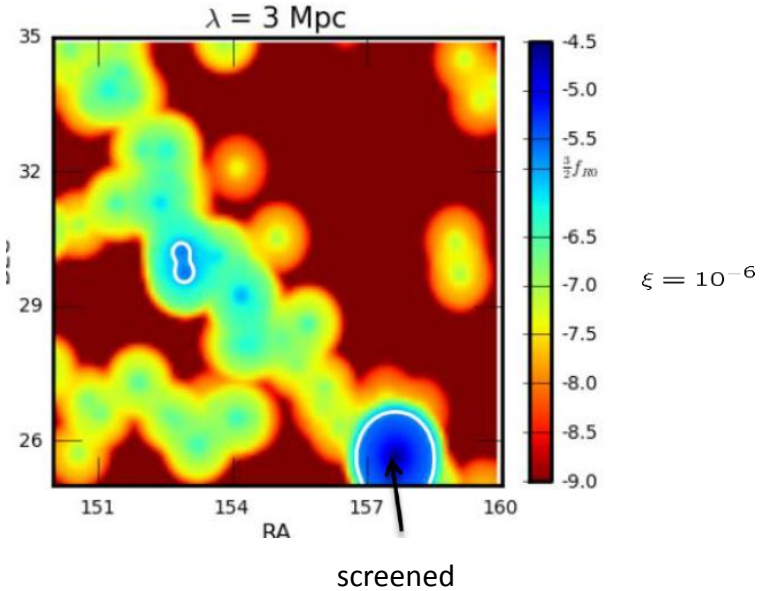
Motion of screened stars different from unscreened HI gas in unscreened dwarf galaxies.



Distance indicators for cepheids and TRGB stars in screened and unscreened dwarf galaxies are different as their luminosities vary



$$\Phi_N \sim 10^{-7}$$

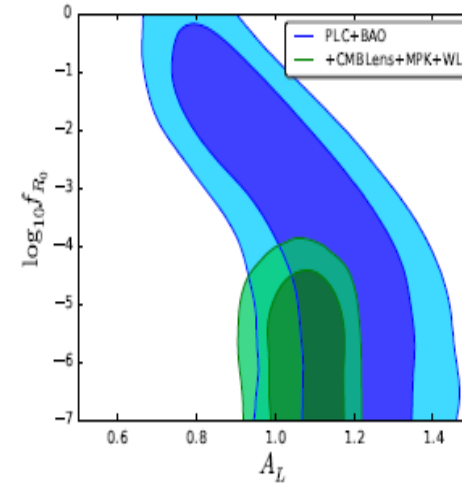
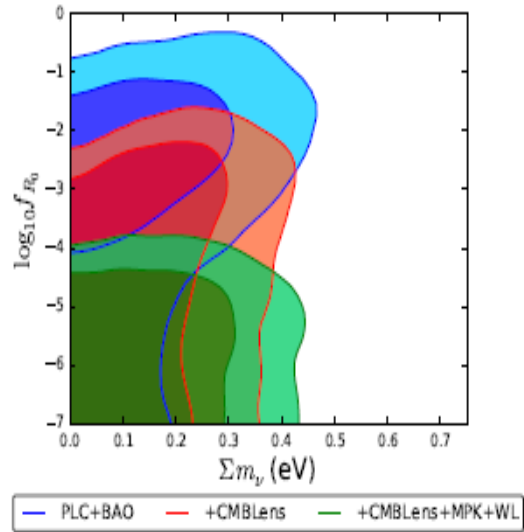


SDSS catalogue, within 200 Mpc, scalar range 1 Mpc

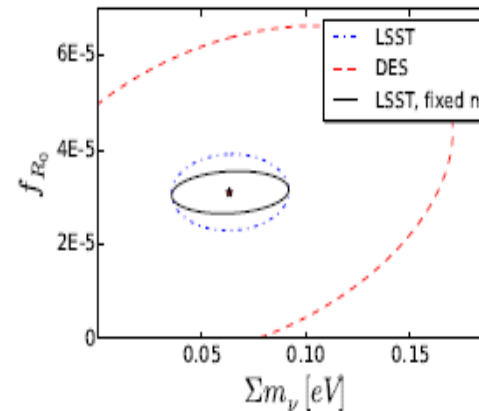
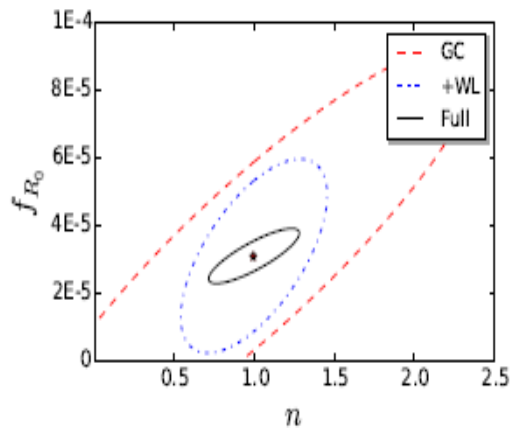
No effects measured so far: bound on the range of the scalar interaction.

$$|f_{R_0}| \leq 10^{-7}$$

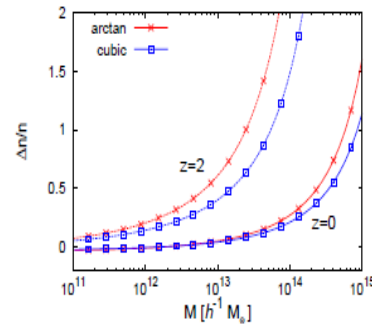
The latest contours on $f(R)$ from cosmological observables



$f(R)+A_L$, fixed Σm_ν		$f(R)+A_L$, varying Σm_ν		
f_{R_0}	A_L	f_{R_0}	A_L	Σm_ν
$3(8) \times 10^{-6}$	$1.08^{+0.07(0.12)}_{-0.05(0.13)}$	$0.4(1.0) \times 10^{-4}$	$1.11^{+0.10(0.16)}_{-0.06(0.15)}$	$0.30(0.38)$



One of the main stumbling blocks of all these analyses is the difficulty of finding clear features which could distinguish models. One of those would be the fact that the physics of large clusters is largely affected for K-mouflage models passing the solar system tests.



More clusters for K-mouflage models on large scales

Much more analytical and numerical work will be required to chart the different modified gravity models which are compatible with local gravity tests.

Conclusions

Light scalar fields could be what remains from massive gravity, string theory They need to be screened in the local environment otherwise tight bounds would be violated.

This prompts one to study the screening mechanism from a bottom-up approach irrespective of their UV completion, if ever needed.

For conformally coupled scalars, there are only three main mechanisms: Vainshtein, K-mouflage and chameleon+Damour-Polyakov.

Must find distinguishing features in view of future missions such as Euclid.