

NE290D: Rare Neutral Particle Detection in Fundamental and Applied Physics:

Reactor antineutrino Spectra & Neutrino Safeguards

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- NE290D Lecture #10 :
 - Reactor antineutrinos spectra

- NE290D Lecture #11:
 - Neutrino Safeguards

NE290D Lecture #10

- Nuclear Reactors
- Reactor Antineutrino Flux
- Fuel Burn-up & Neutrino Flux
- Reactor Antineutrino Energy Spectra
 - Ab-Initio Computation using Nuclear Databases
 - Computation using Experimental Integral β -Data

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Nuclear Reactors

▪ Power Nuclear Stations:

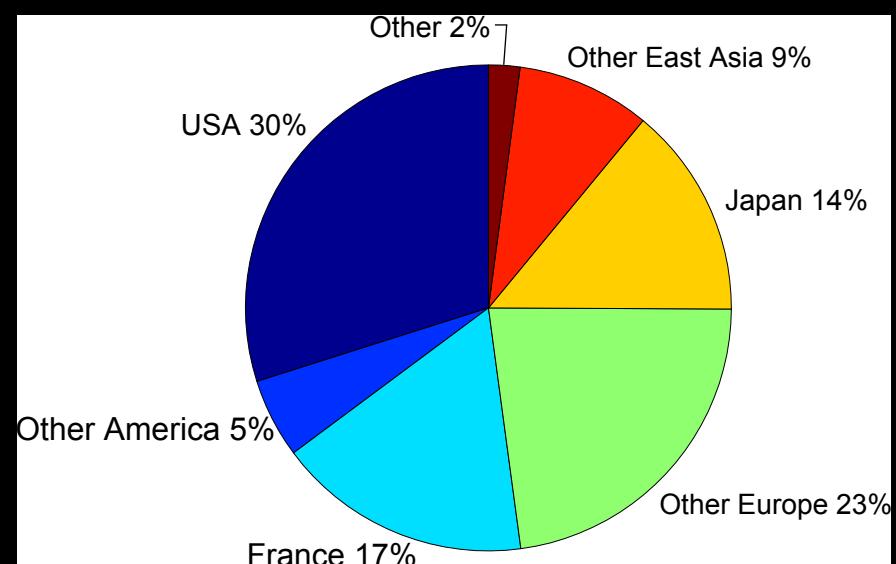
- 201 in the world (most of them having multiple units)
- Total thermal power = 1134 GWth total thermal power
- Mixed fuel ($^{235,238}\text{U}$ & $^{239,241}\text{Pu}$)
- Thermal neutron flux (0.025 eV)
- Extended neutrino source:
 - 3-4m diameter, 4m high

▪ Reactor Types

- Pressurized Light Water Reactor (PWR)
- Boiling Water Reactor (BWR)
- CANDU (heavy water)
- Naval
- Research
- Weapons Production
- New Technology

▪ Non-Power Nuclear Reactors:

- Research reactors used as neutron source
- Reactors used for propulsion
- Highly enriched in ^{235}U
- Thermal neutron flux
- Extended/Compact neutrino source (0.6mx0.6m possible)



Nuclear Fuel

▪ Uranium based fuel

- Mainly ^{238}U (99.2745%, $T_{1/2}=4.47 \cdot 10^9$ y)
- Naturally: 0.7% of ^{235}U (fissile, $T_{1/2}=703.8 \cdot 10^6$ y)
→ fuel in form of UO_2



▪ ^{238}U

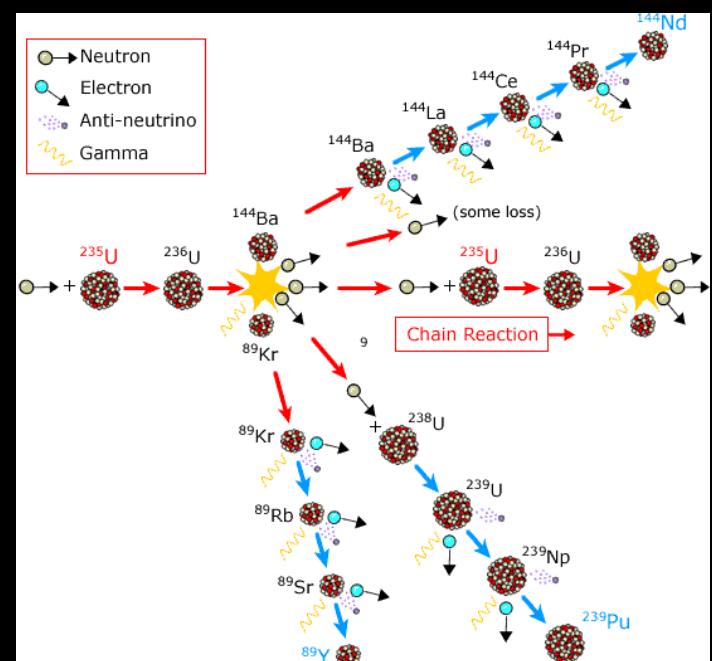
- High neutron capture threshold (0.8 MeV)
- No fission with thermal neutrons
- Fissions induced by fast neutrons

▪ Enrichment in ^{235}U (3.5% in PWR)

- Fission chain reactions induced by thermal neutrons on ^{235}U

▪ But other reactions:

- ^{238}U capture neutrons
- in-situ production of ^{239}Pu , ^{241}Pu
- Proliferate

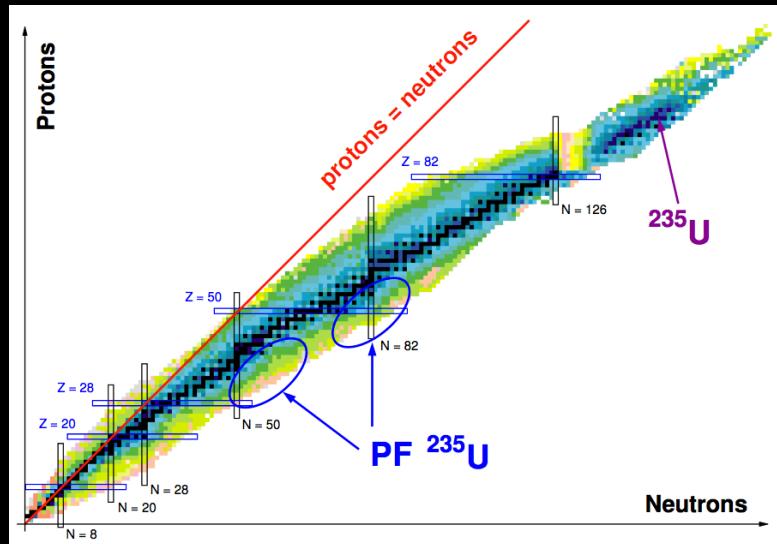


Fission Products & Fission Yields

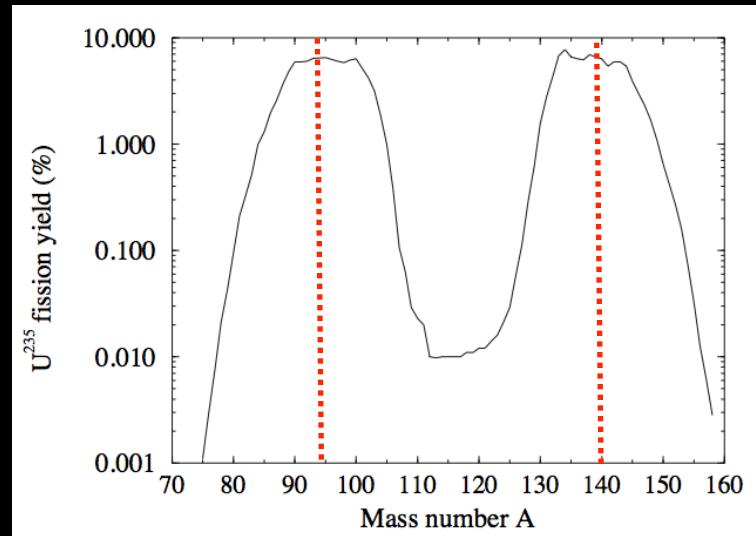
- Fission of ^{235}U



- X, Y are called ‘Fission Fragments’ or ‘Fission Products’
- Highest fission yields for the couple: Zr(94,40) and Ce(140,58)
 - X+Y: $40+58=98$ protons & $94+140=234$ neutrons
 - On average 6 neutrons have to β -decay to 6 protons to reach stability $\rightarrow 6 \nu$
 - On average 1.5 ν (25%) are emitted with $E_\nu > 1.8$ MeV



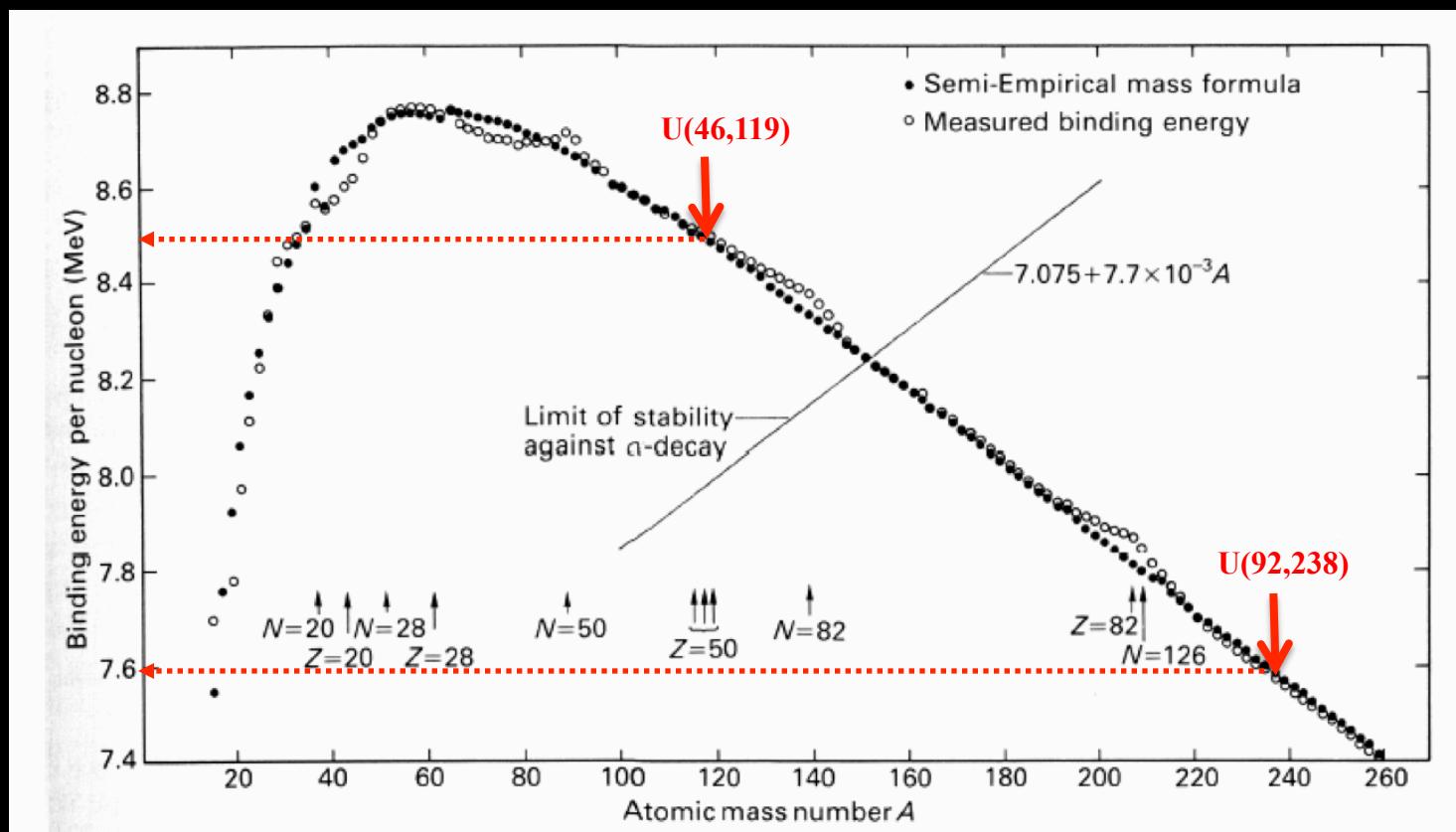
Two ‘Mass’ Bumps: $Z=86-104$, $Z=130-148$



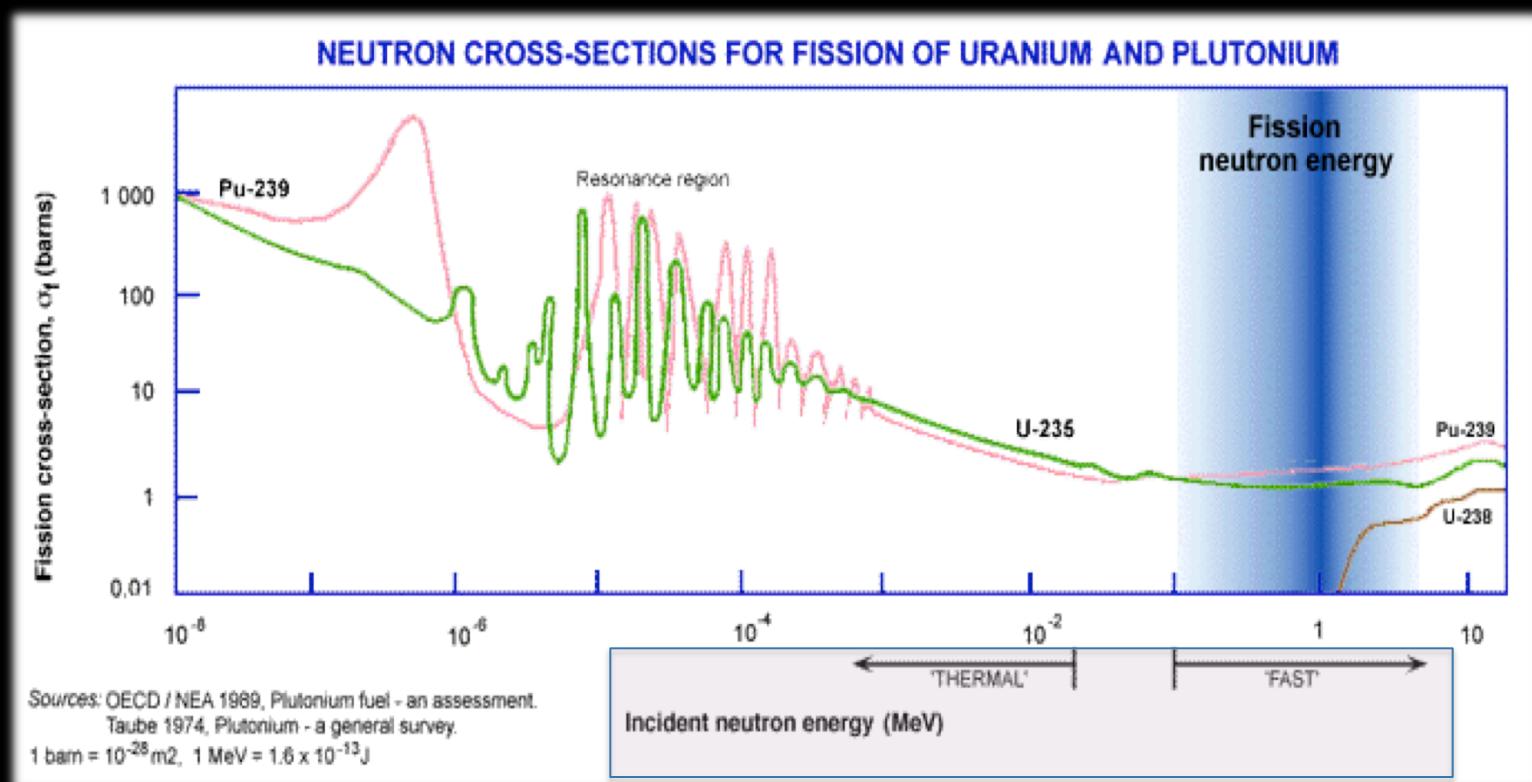
Energy Released per Fission

■ Illustrative example: $^{238}_{92}\text{U} \rightarrow ^{119}_{46}\text{Cs} + ^{119}_{46}\text{Cs}$

- U(92,238) : $B(Z,A)/A=7.6 \text{ MeV/nucleon}$
- Cs(46,119): $B(Z,A)/A=8.5 \text{ MeV/nucleon}$
- Energy released = $\Delta m.c^2 = 238 \times 8.5 \text{ MeV} - 2 \times 119 \times 7.6 \text{ MeV} \approx 215 \text{ MeV}$



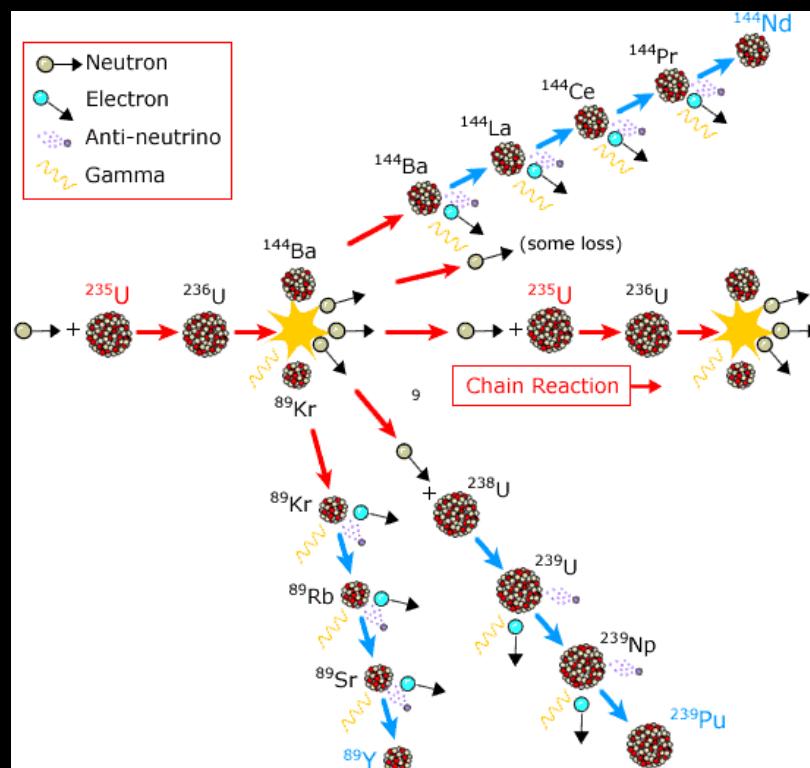
U/Pu Fission Cross Sections



Nuclear Chain Reaction & Neutrinos



- Nuclear reactors are copious, isotropic sources of electron antineutrinos
- Neutrinos come from β -fission fragments, not directly from the fission
- Fission of ^{235}U , ^{238}U , ^{239}Pu , ^{241}Pu
- β -decay of neutron rich fission fragments
 - $X(A,Z) \rightarrow Y(A,Z+1) + e^- + \text{anti-}\nu_e + Q$
 - $Q \approx 200 \text{ MeV} / \text{fission released}$
 - Fission rate $\approx 4 \text{ GW}/200 \text{ MeV} \sim 2 \cdot 10^{20} / \text{s}$
 - 6 anti- ν_e emitted per fission
 - $7.5 \cdot 10^{20} \text{ anti-}\nu_e/\text{s}$ for a 4 GW nuclear core
- Antineutrino spectrum is **time dependent** as the beta daughters come into equilibrium



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Reactor neutrino Flux: Guesstimate

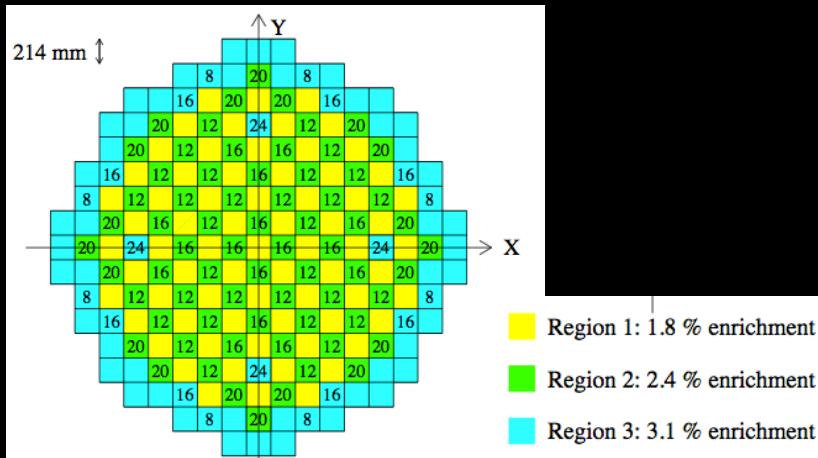
10^{20} fissions in a 4 GW nuclear core (estimate)

200 MeV/fission et 6 v/fission of U²³⁵, Pu^{239,241}

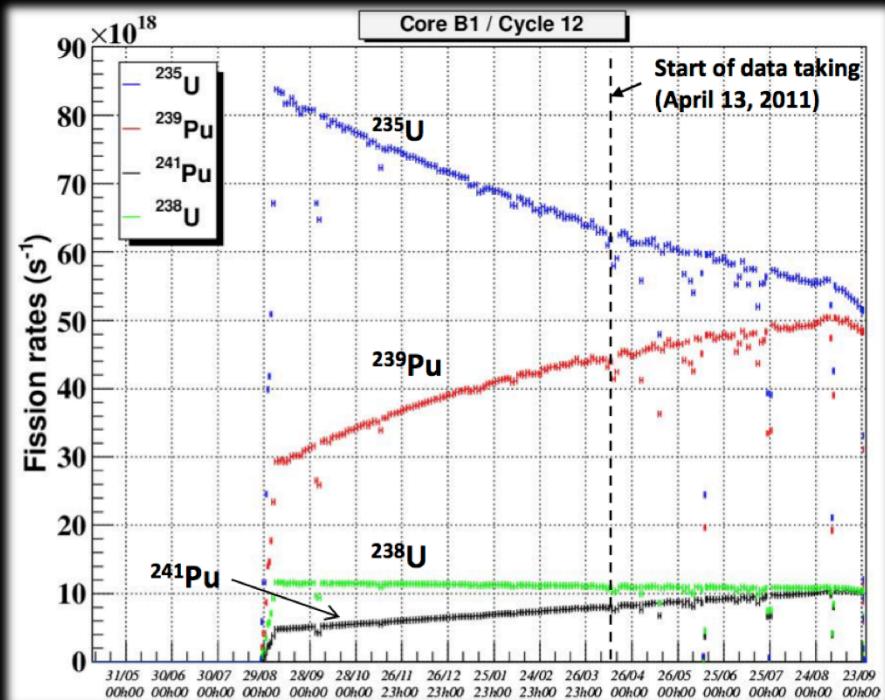
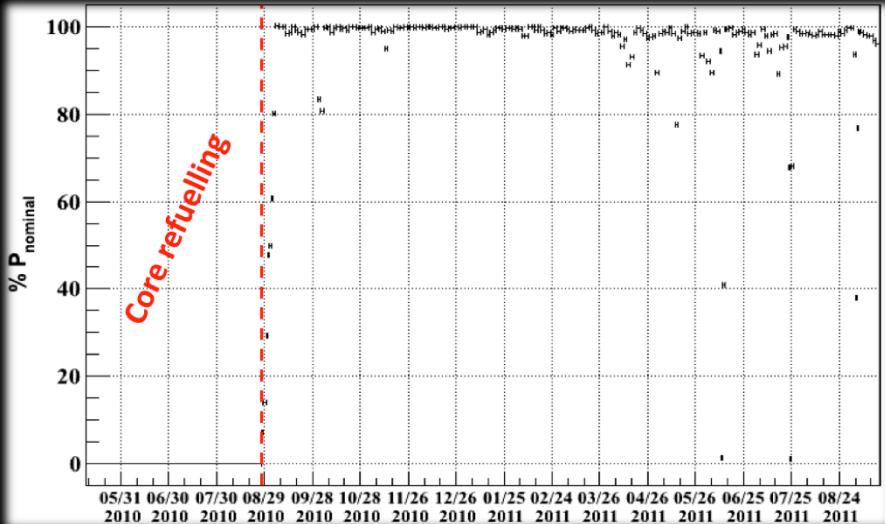
- 200 MeV released per fission:
 $\rightarrow 200 \text{ MeV} = 200 * 10^6 * 1,6 \cdot 10^{-19} \text{ J} = 3,2 \cdot 10^{-11} \text{ Joules}$
- Thermal Power:
 $\rightarrow 4 \text{ GW} = 4 \cdot 10^9 \text{ W (J / s)}$
- Electron anti-neutrinos
 $\rightarrow 4 \cdot 10^9 \text{ W} / 3,2 \cdot 10^{-11} \text{ fissions/sec} * 6 \text{ neutrinos/fission}$
 $\rightarrow (\phi_v)^R = 7.5 \cdot 10^{20} \text{ neutrinos / s}$

Typical Reactor Running Cycle

- Example: Double Chooz experiment
- 3-6 week shutdown every 12-18 months
- 1/4-1/3 of fuel assemblies are replaced, remaining fuel repositioned



- 6 months shutdown every 10 years



Production of Plutonium in reactors

- $^{238}\text{U} + \text{n}_{\text{fast}} \rightarrow ^{239}\text{U} + \gamma$
 $\hookrightarrow ^{239}\text{Np} + \text{e}^- + \bar{\nu}_e \quad (23.45 \text{ m})$
 $\hookrightarrow ^{239}\text{Pu} + \text{e}^- + \bar{\nu}_e \quad (2.36 \text{ d})$
- $^{239}\text{Pu} + \text{n} \rightarrow ^{240}\text{Pu} + \gamma$
- $^{240}\text{Pu} + \text{n} \rightarrow ^{241}\text{Pu} + \gamma$
- Plutonium 239, 240, and 241 are being produced as U is burnt
- ^{238}U and ^{240}Pu have small cross sections for *fast* neutron induced fission
- ^{239}Pu , ^{241}Pu are fissile isotopes (thermal neutrons)
 - Beta decays \rightarrow electron antineutrino emission

- Fission rate of isotope k: $f_k(t) = \Phi_n(t) \sigma_k(t) N_k(t)$ in fissions/s
 - k : ^{235}Pu , ^{238}U , ^{239}Pu , ^{241}Pu
 - $\Phi_n(t)$: neutron flux (cm^2/s)
 - $\sigma_k(t)$: energy averaged fission cross section (cm^2)
 - $N_k(t)$: Number of isotope k (from the mass of isotope k)
 - t is the time running during the fuel cycle
- The evolution of the core isotopic content is ruled by the Bateman equations
- Fission fraction: $F_k = \frac{f_k}{\sum_k f_k}$, with $\sum_k F_k = 1$
- Burn-up:
 - Measure the number of fissions that occur per unit of mass
 - Equivalent to the amount of energy extracted from a nuclear core
 - Unit: Joules/ton or more commonly MWd/t

Fuel Evolution in a PWR (burn-up)



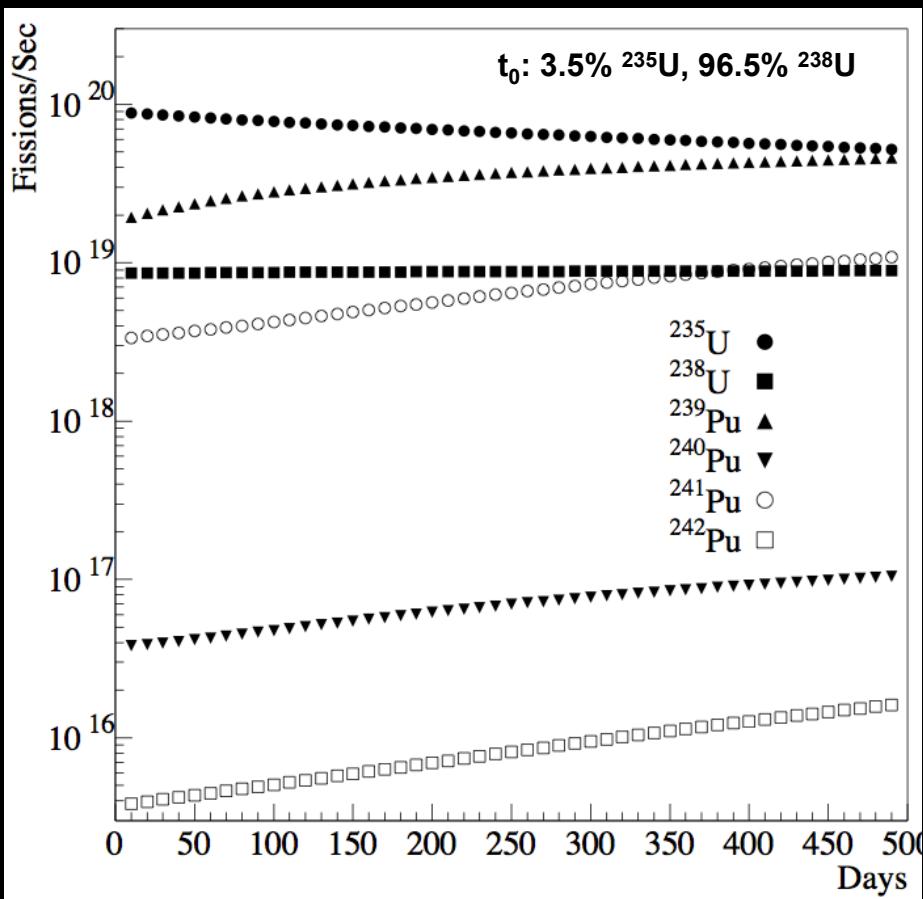
- Fuel evolve with time
- A typical cycle last 500 days
then 1/3 of the fuel is being replaced

- Average fission fractions
 - $\langle F(^{238}\text{U}) \rangle$: 53.8%
 - $\langle F(^{239}\text{Pu}) \rangle$: 32.8%
 - $\langle F(^{238}\text{U}) \rangle$: 7.8%
 - $\langle F(^{241}\text{Pu}) \rangle$: 5.6%
 - Others <0.1%

- 250 kg of plutonium produced during
a fuel cycle, mainly ^{239}Pu

- Evolution of the antineutrino flux
 - $N (\text{s}^{-1}) = a \cdot (1+k) P (\text{GW})$
 - k: burn-up factor (Pu/U fraction)
 - k: <10% correction

Information from the operator
or
Your reactor core simulation



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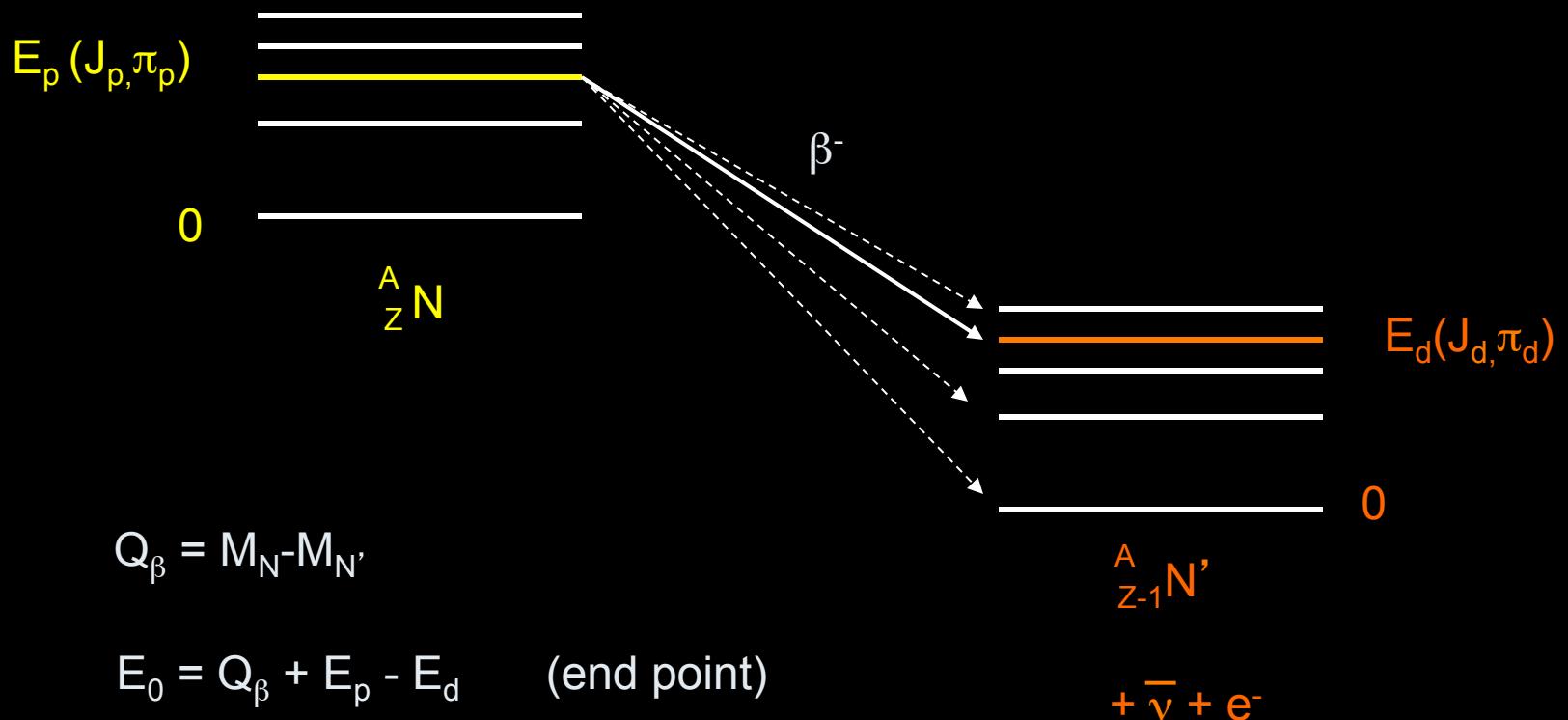
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Reactor Neutrino Yields



- Thermal power, P_{th} in Watt
 - $P_{th}(t) = \sum f_k p_k$
 - f_k : fission rate for the fission isotope k (mainly $^{235,238}U$, $^{239,241}Pu$)
 - p_k : thermal energy released in one fission of the isotope k
- Total neutrino flux from a reactor
 - $\Phi(E,t) = \sum f_k(t) S_k(E)$
 - $S_k(E,t)$: neutrino yield for the fission isotope k
- Energy dependent neutrino yield $S_k(E)$
 - $S_k(E,t) = \sum Y_i^k(t) v_i(E)$
 - $Y_i^k(t)$: cumulative yield for each fission fragment i of fission isotope k
 - $v_i(E)$: neutrino spectra for a given fission fragment i
 - i runs over ≈ 800 isotopes
- Neutrino spectrum $v_i(E)$
 - Never directly measured, even for a given fission fragment i
 - Has to be inferred from the corresponding β -spectrum (e^-)
 - Challenging since β -decay is a complicated process...

β^- decay



β -decay: Classification

Classification	L	S	$\Delta J = L+S $	$\Delta \pi$	Log ft
Allowed	0	0,1	0, 1 ($0^+ \rightarrow 0^+$)	No	4-6
1 st forbid. non-unique	1	0,1	0, 1	Yes	6-10
1 st forbid. unique	1	1	2	Yes	7-10
2 nd forbid. non-unique	2	0,1	2	No	11-14
2 nd forbid. unique	2	1	3	No	14
3 rd forbid. non-unique	3	0,1	3	Yes	17-19
3 rd forbid. unique	3	1	4	Yes	18

- All non-unique transitions involve several nuclear matrix elements with complex E dependence → spectrum shape is unknown
- Forbidden decays are less probable because they contain an orbital angular momentum change (higher log ft)

β -decay: Fermi theory & Corrections



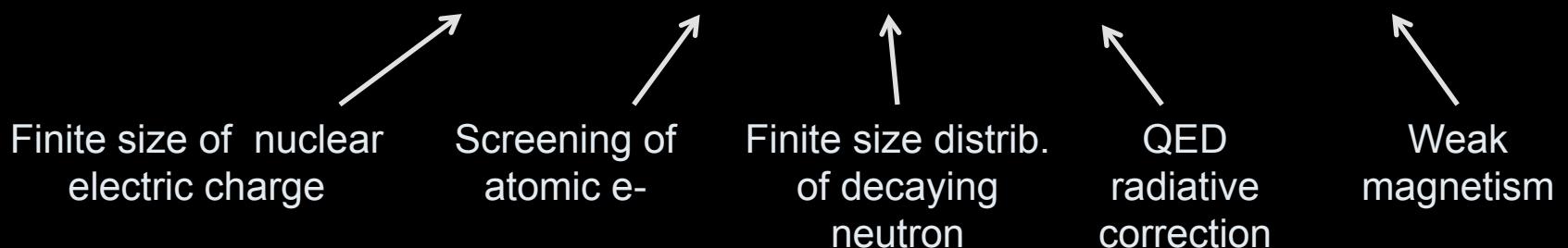
Fermi theory:

$$N_{\beta}^F(W) = K \underbrace{p^2}_{\text{Phase space}} \underbrace{(W - W_0)^2}_{\text{Fermi function}} \underbrace{F(Z, W)}_{\text{Shape factor}} C_{Shape}(W)$$

- Unit of m_e , $W=E/m_e$
- $W = e^-$ total energy
- W_0 = end-point
- $p = e^-$ momentum
- Z = Nuclear charge

Corrections:

$$N_{\beta}(W) = N_{\beta}^F(W) L_0(Z, W) S(Z, W) C(Z, W) G_{\beta}(Z, W) (1 + \delta_{WM} W)$$



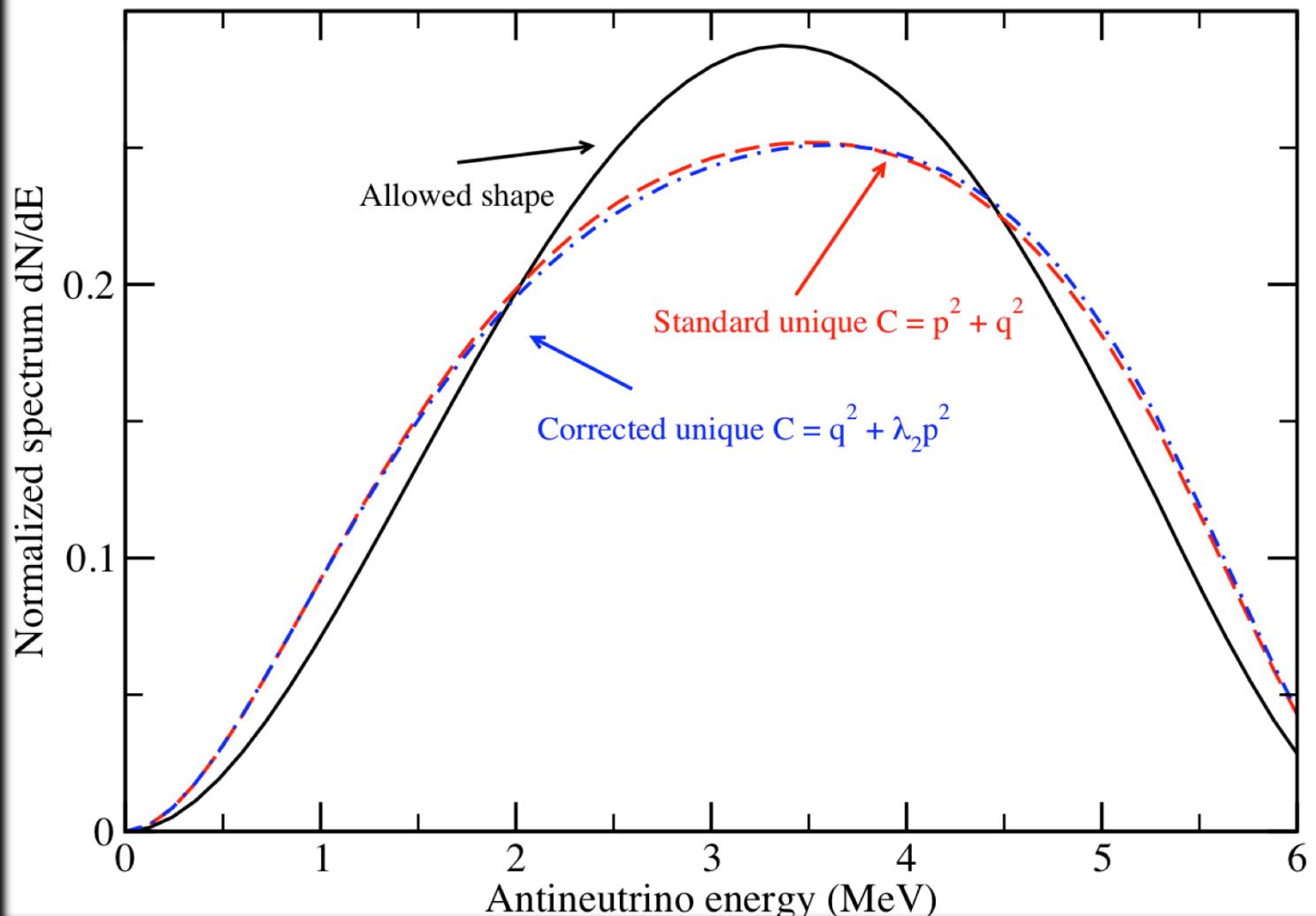
Neutrino branch obtained by replacing:

$$W_v \rightarrow W_0 - W_{\beta}, G_{\beta} \rightarrow G_v$$

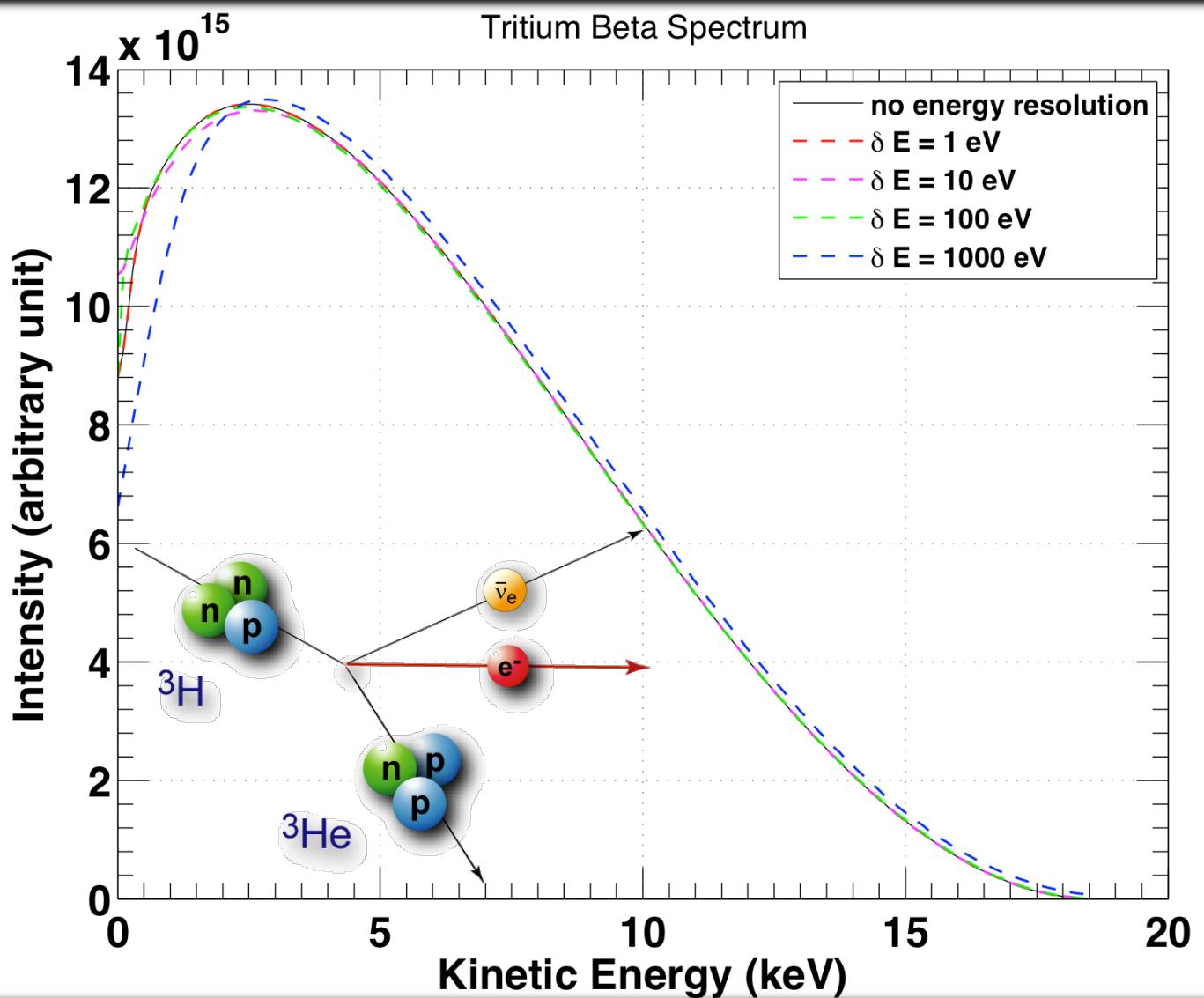
β -decay: shape factor (C_{shape})

Antineutrino spectra in the unique first forbidden decay

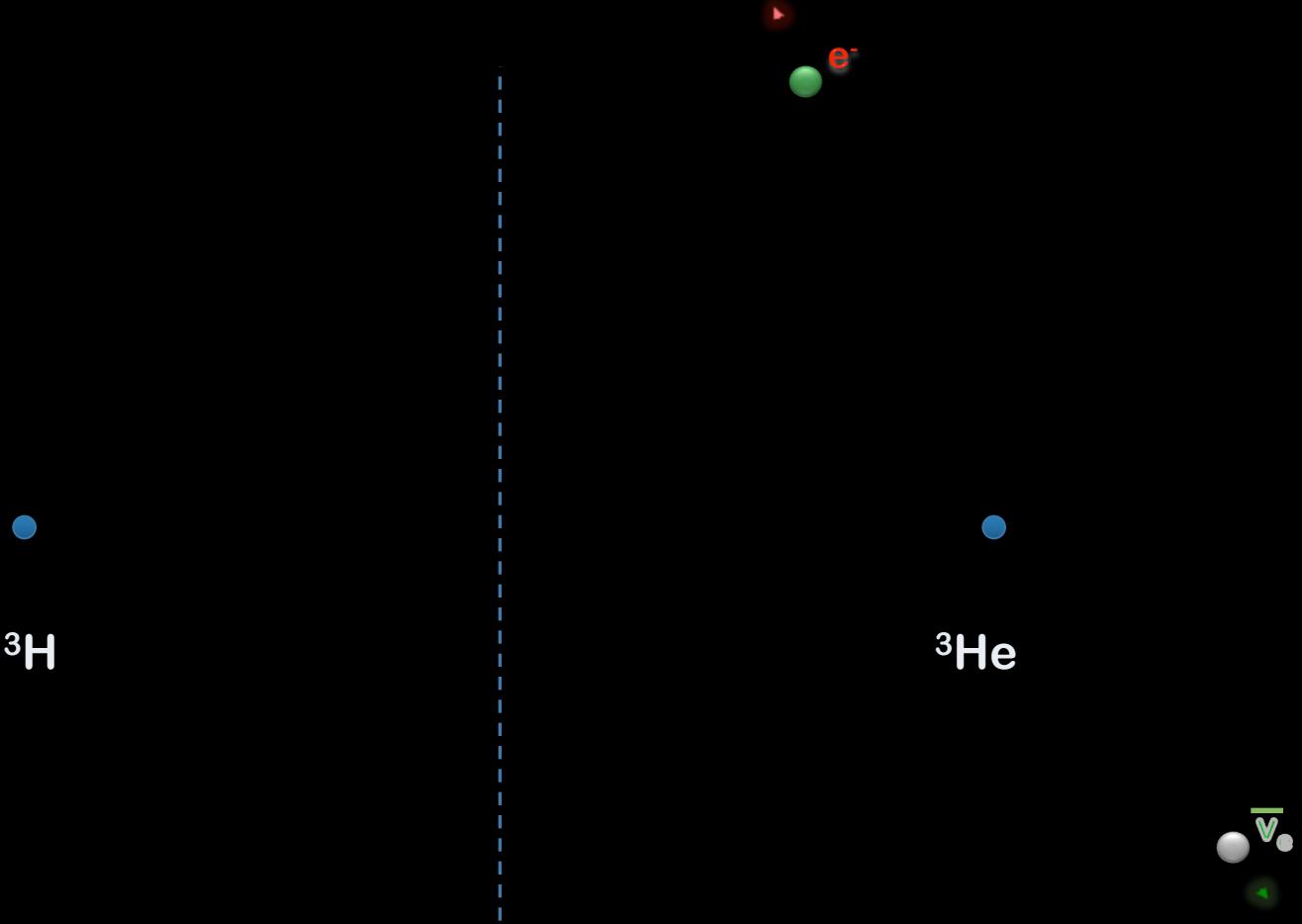
Example for $Z = 46$, $Q = 6$ MeV



β -decay: Tritium academic example

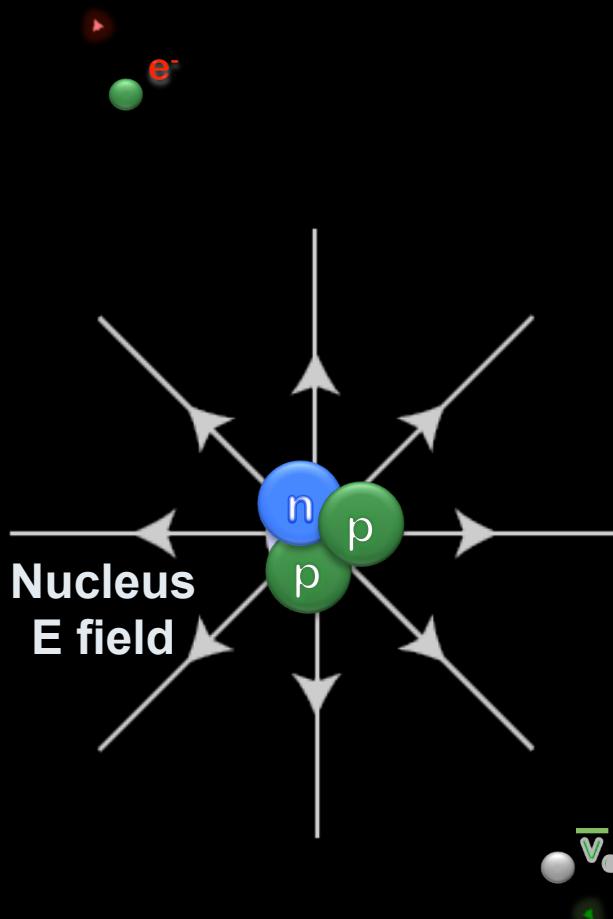


Phase space factor

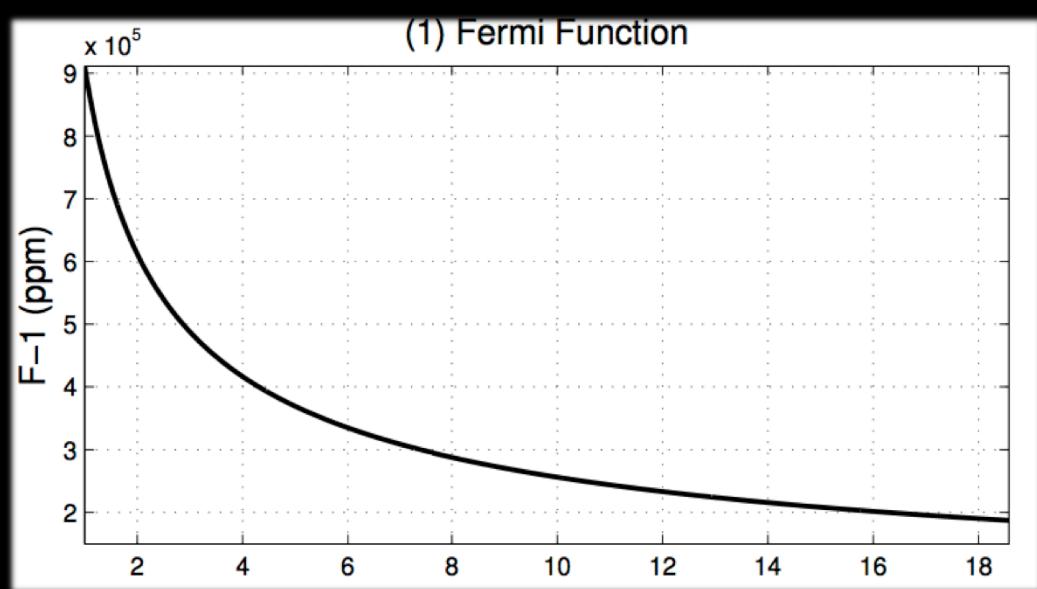


$$\frac{d\Gamma}{dE_e}(m_{\nu_i}) = C \cdot p_e E_e \cdot \sqrt{(E_e - E_0)^2 - m_{\nu_i}^2} \cdot (E_e - E_0)$$

Coulomb field of the daughter He nucleus



$$\frac{d\Gamma}{dE_e}(m_{\nu_i}) = C \cdot p_e E_e \cdot \sqrt{(E_e - E_0)^2 - m_{\nu_i}^2} \cdot (E_e - E_0) \cdot F(E_e, Z)$$



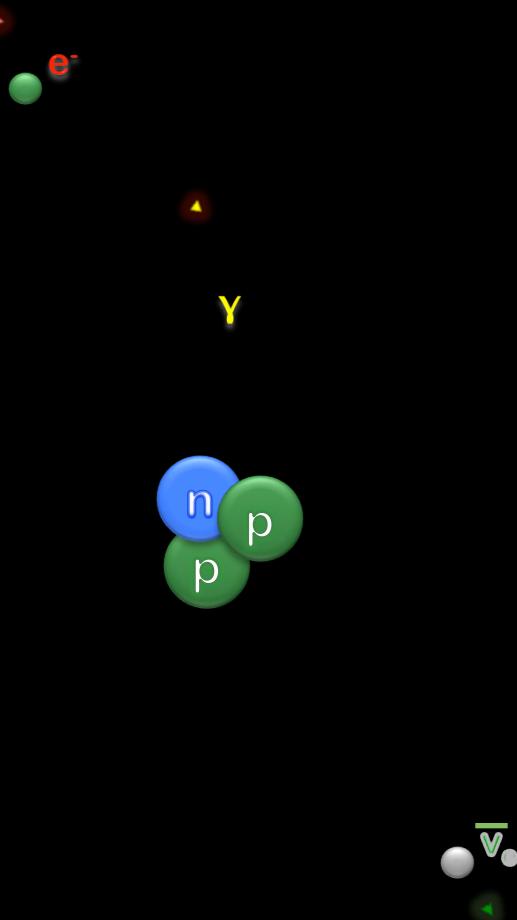
2nd order corrections to Fermi theory



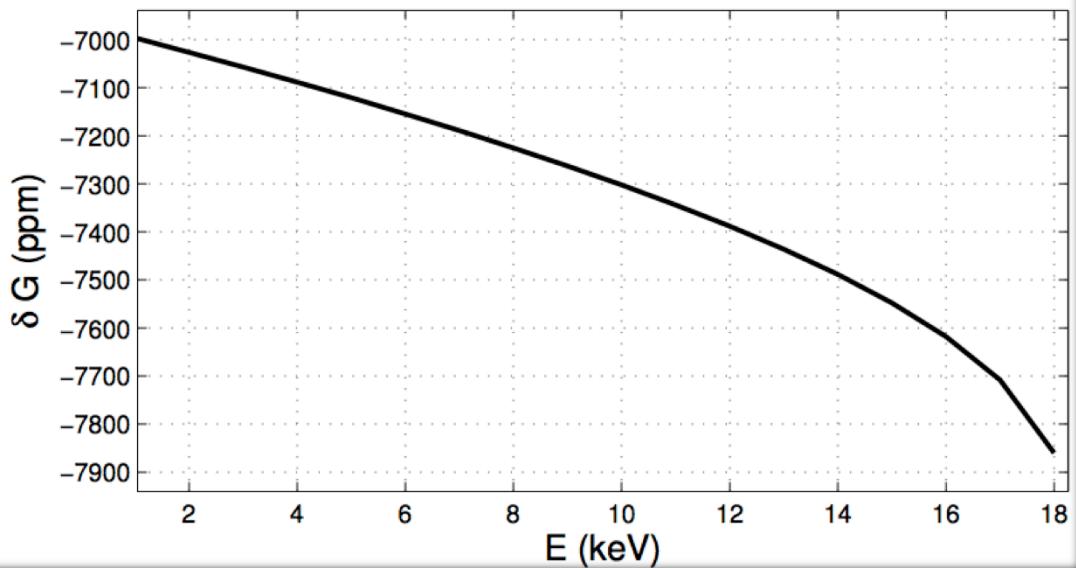
$$\left(\frac{d\Gamma}{dE_e} \right)^{corr} = \frac{d\Gamma}{dE_e} \cdot \left[\prod_{\Psi=L_0, S, E, Q, R, G} \Psi(E_e, Z) \right]$$

- Screening Correction (S)
- He recoil corrections (R)
- Recoiling Coulomb field (Q)
- Finite extension of the nucleus
- Radiative corrections (G)

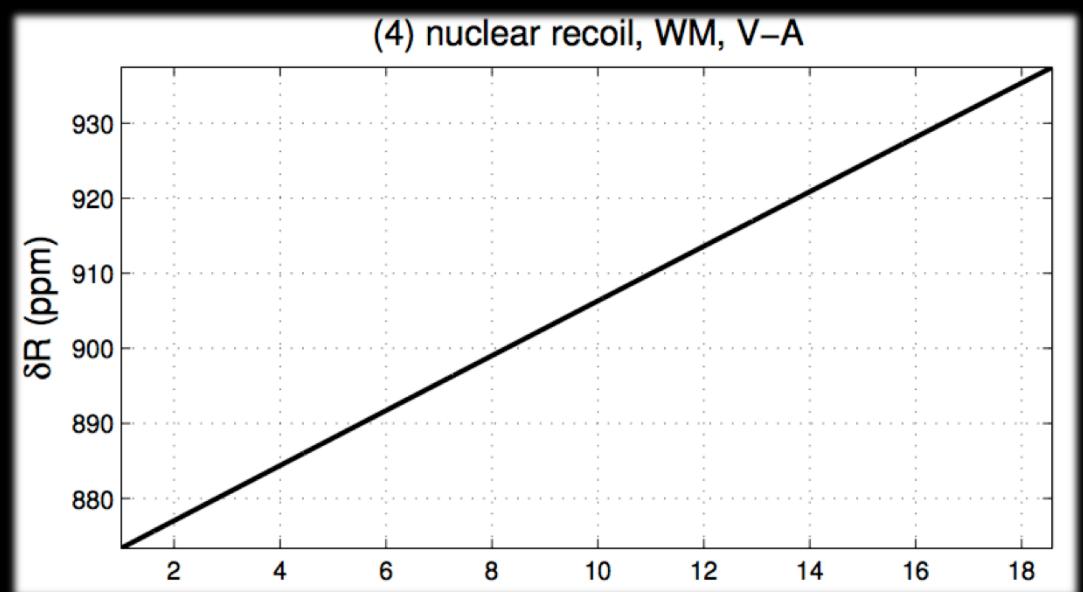
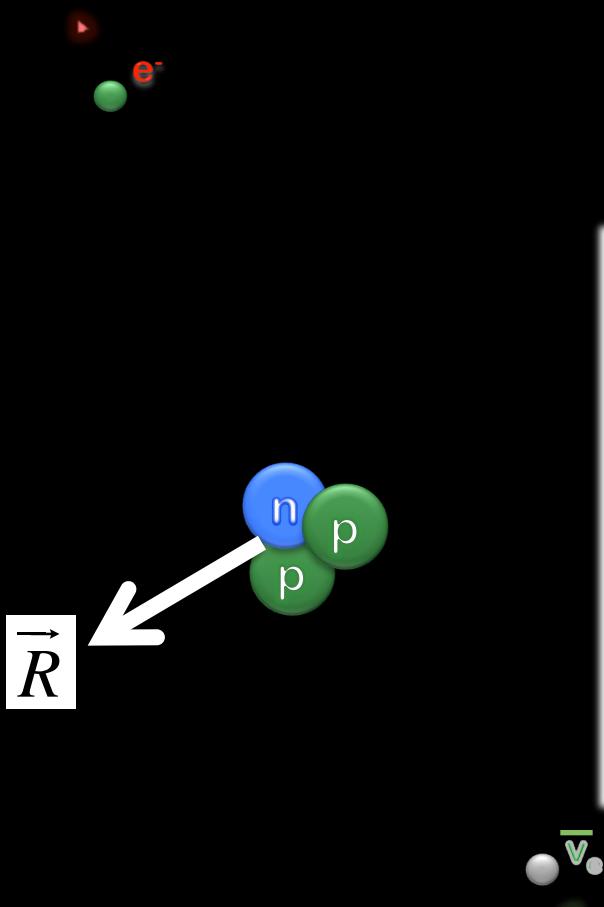
Radiative Correction



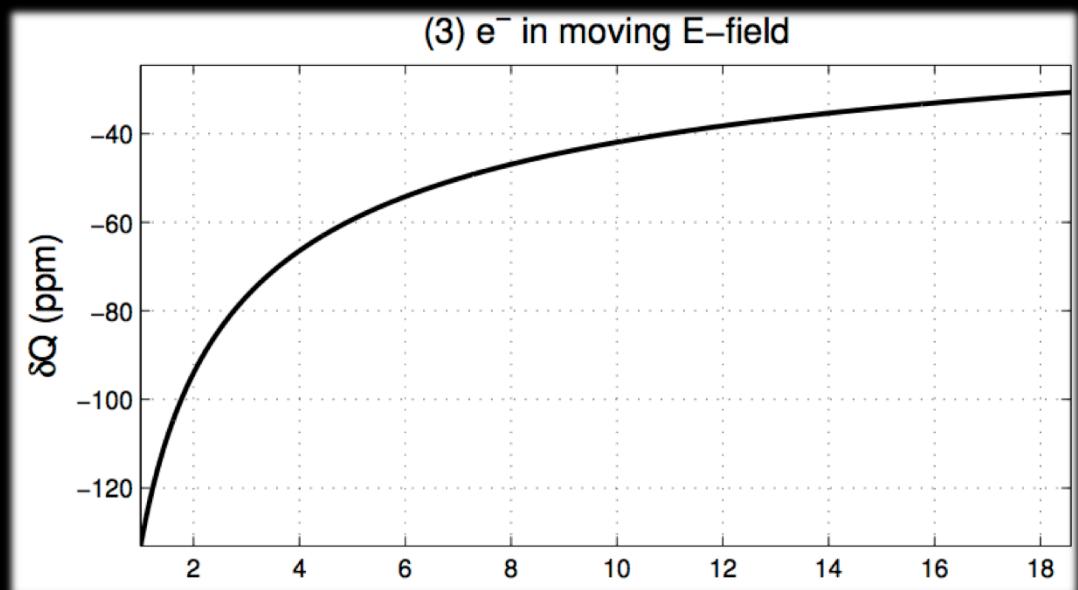
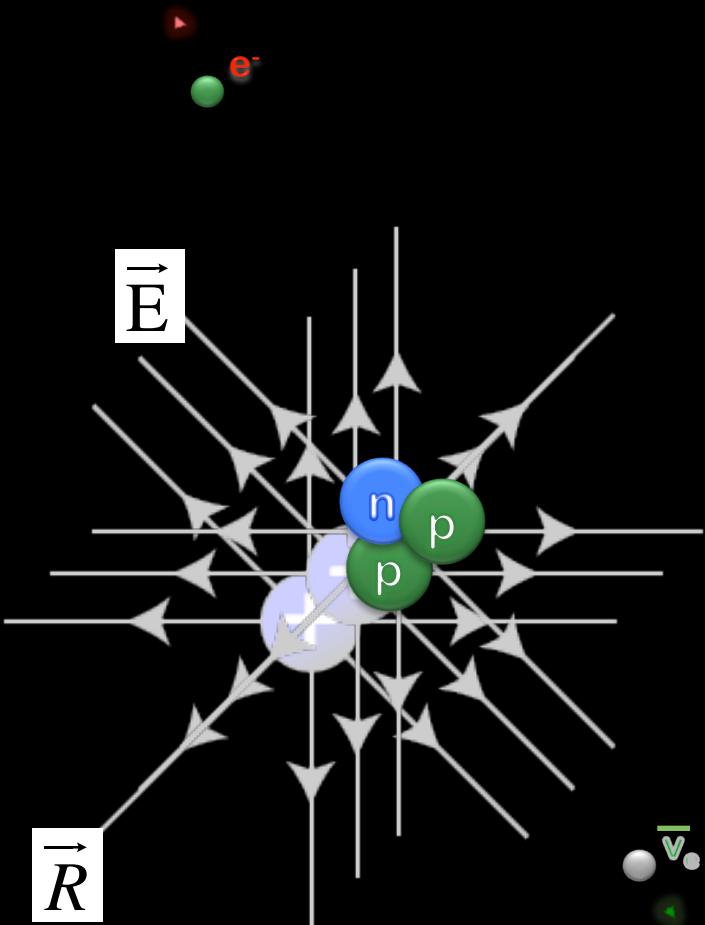
(8) radiative correction



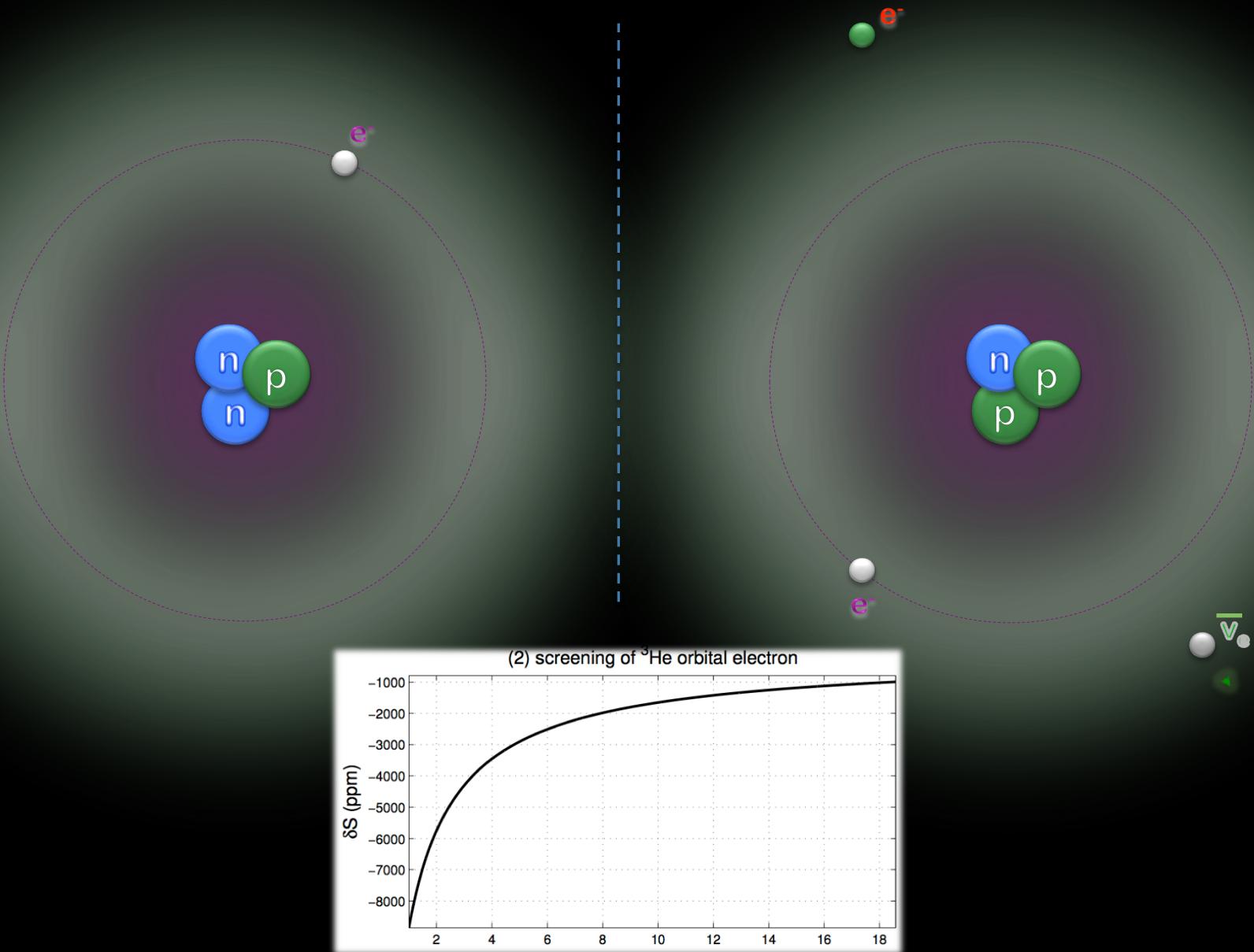
Recoil of the daughter nucleus (+ Weak Magnetism)



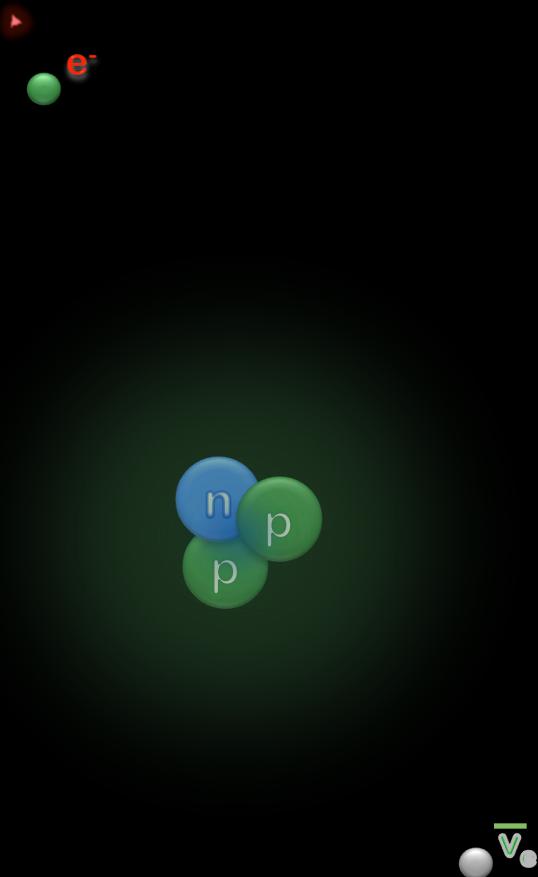
Recoiling Coulomb field of ${}^3\text{He}$



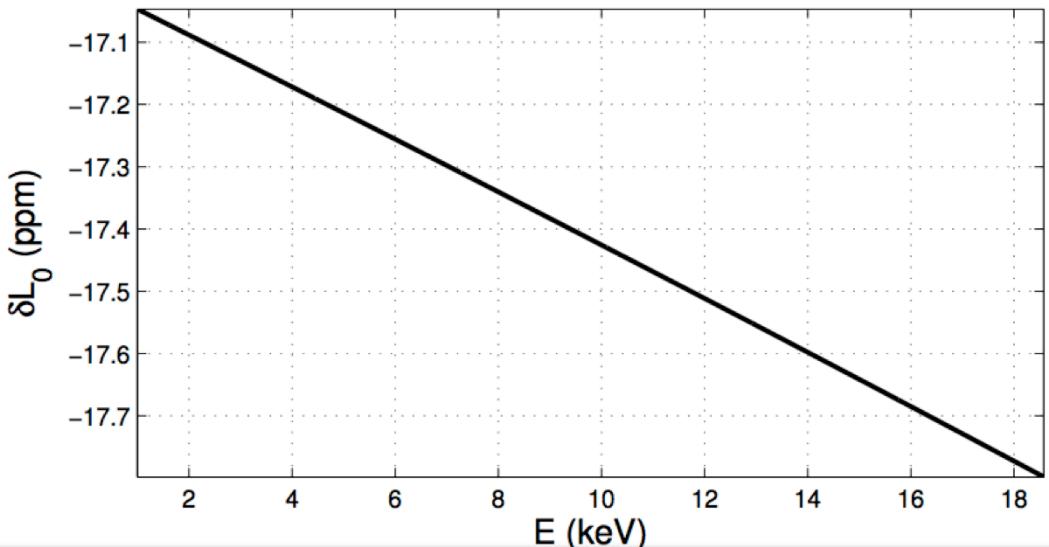
Screening of the orbital electron



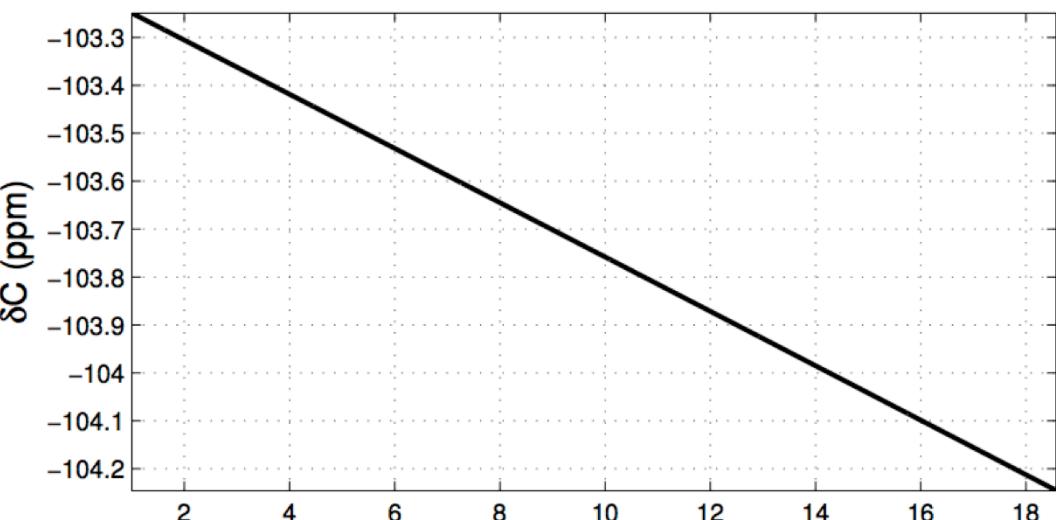
Finite Extension of the nucleus



(7) extension of nucleus charge



(6) weak interaction finite size



Reactor Neutrino Flux/Spectra

▪ Antineutrino flux

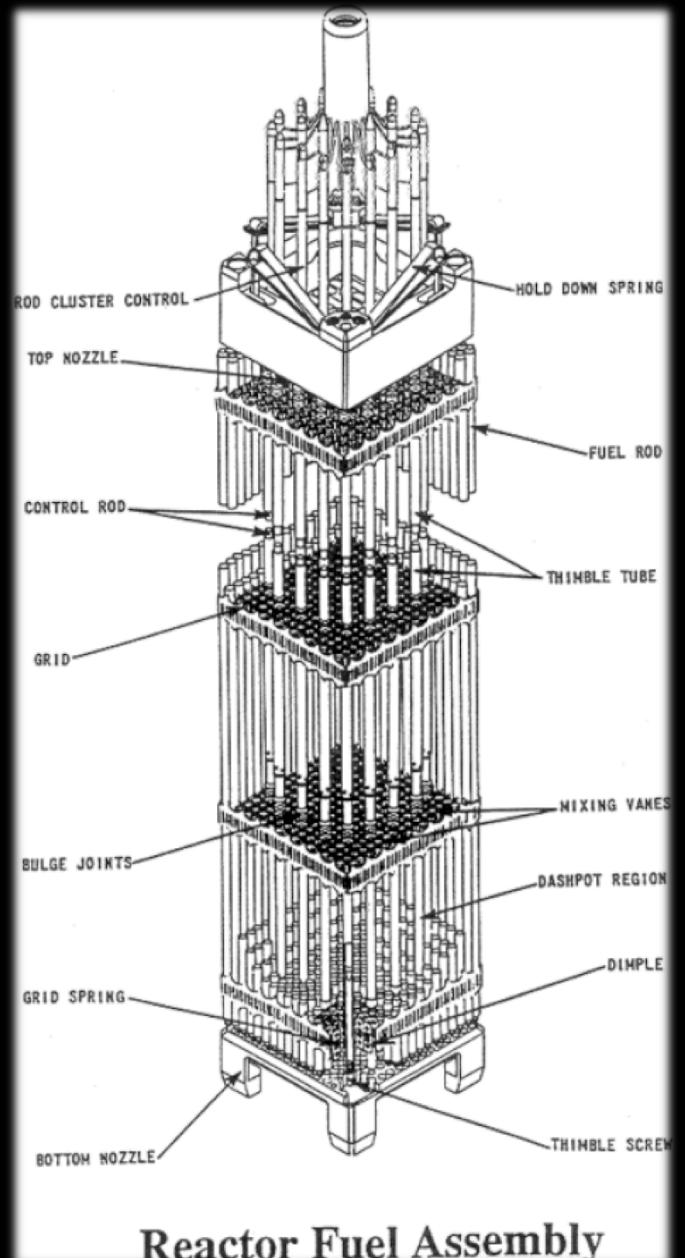
- >99.9% antineutrinos produced by ^{235}U , ^{239}Pu , ^{238}U , ^{241}Pu
- >90% antineutrinos produced by ^{235}U , ^{239}Pu
- ^{235}U , ^{239}Pu , ^{241}Pu , and ^{238}U
- Case A: ab-initio method – 15% uncertainties
 - computation based on nuclear databases
 - old computation (Vogel et al., 1980's)
 - new computation (Mueller/Huber, 2011, +10% w.r. old)
- Case B: Conversion method – 3% uncertainties
 - electron spectrum measurement
 - electron → neutrino spectrum conversion
 - old conversion (Schreckenbach et al., 1980's)
 - new conversion (Mueller/Huber et al., 2011, +3.5%, w/r old)

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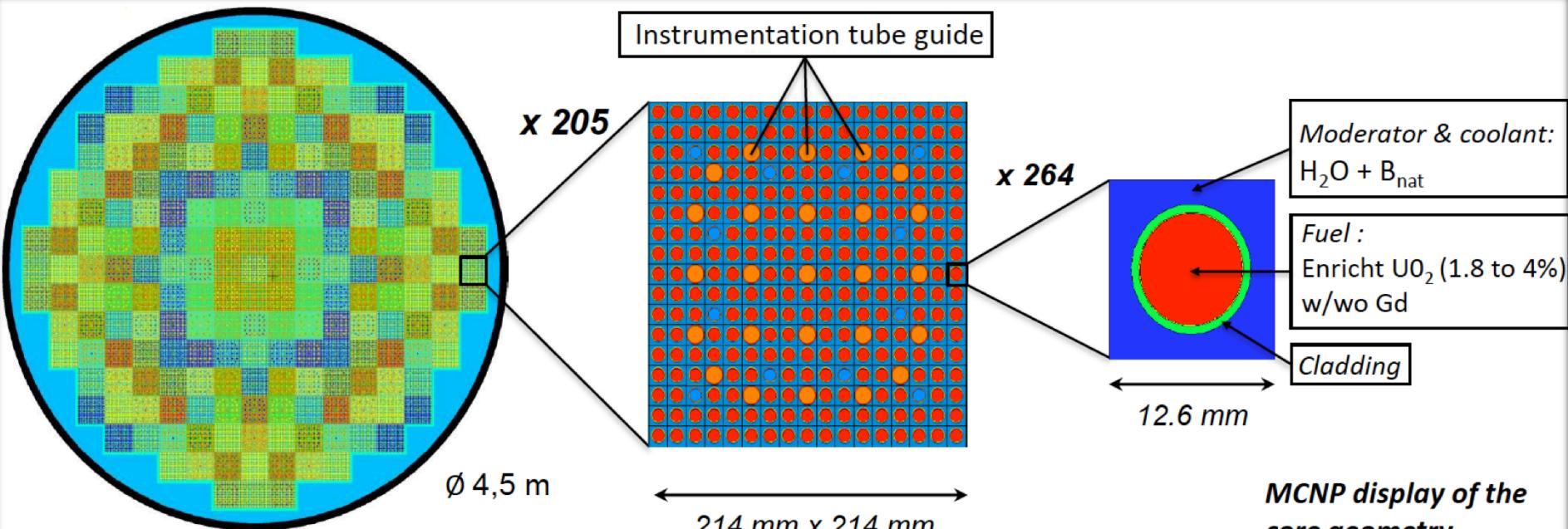
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Nuclear Fuel Assemblies

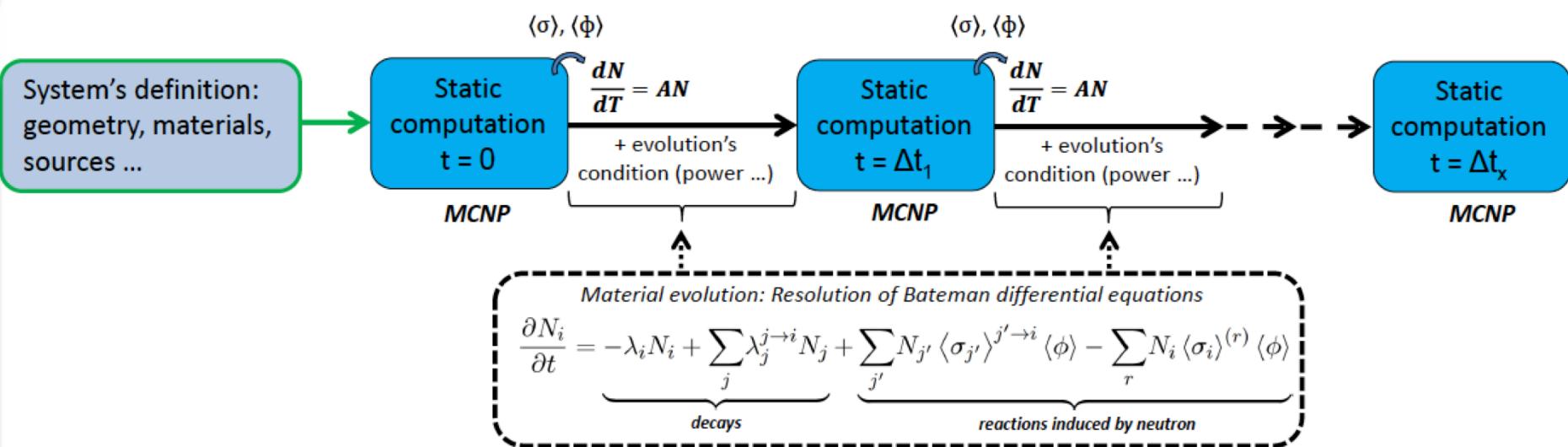
- Fuel in N4-type reactors (Chooz, France)
 - 120 tons of UO_2
 - $^{235}\text{U} \approx 3.45\% : 3.60$ tons
- 205 fuel assemblies
 - 264 rods per assembly
 - 272 pellets per rods
 - 8 g per pellets
- Loading/unloading
 - by one/third
 - every 1.5 years
- Typical energy extracted
 - $45 \text{ GW.d/ton} = 3.89 \cdot 10^{15} \text{ J/ton}$
 - Called ‘burn-up’



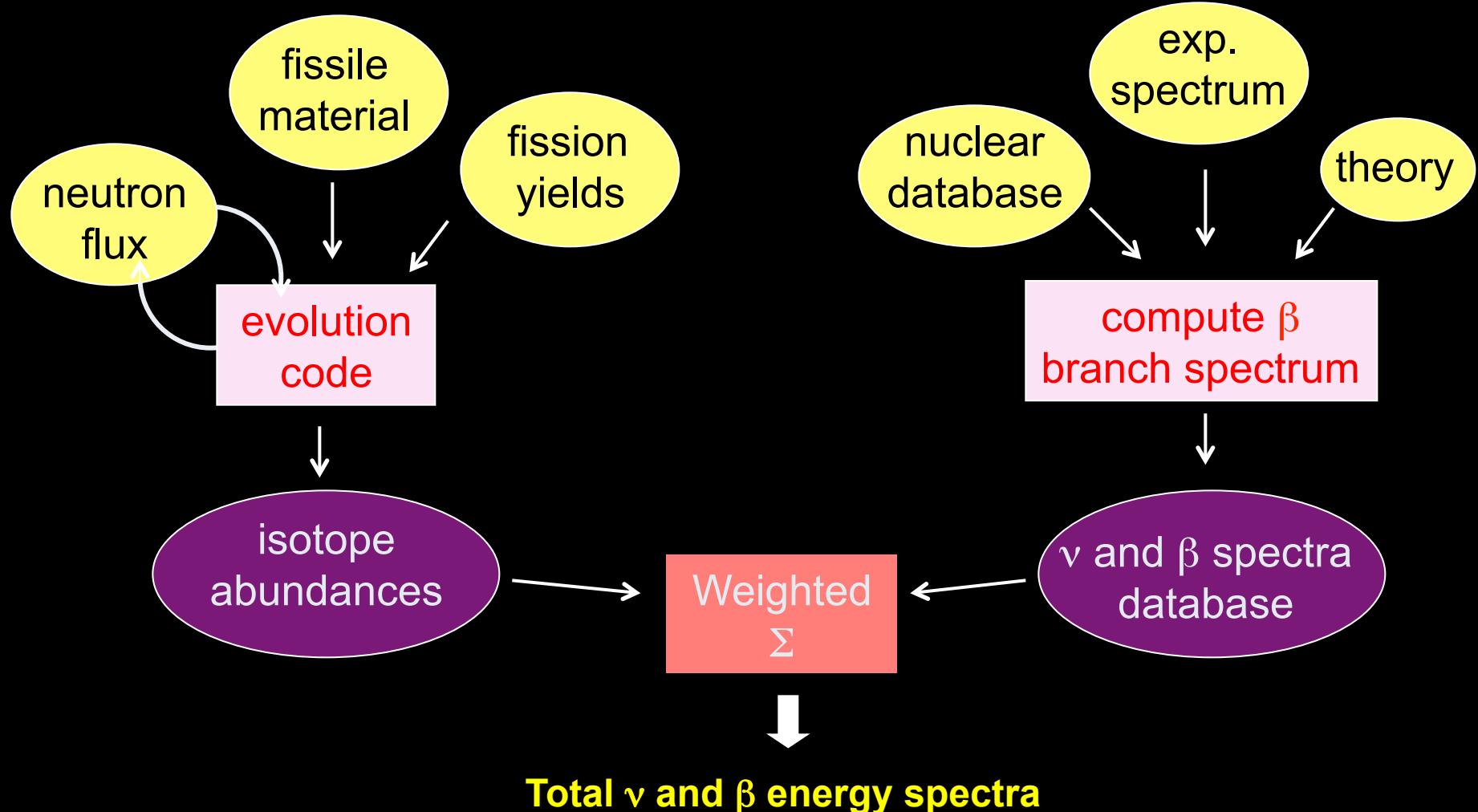
A) Reactor Core Evolution Simulation



MCNP display of the core geometry



A) Reactor Neutrino Flux/Spectra



Reactor neutrino spectrum: notations

- $S_{\text{tot}}(E)$: integrated neutrino spectrum

$$S_{\text{tot}}(E) = \sum_{k=^{235}\text{U},^{238}\text{U},^{239}\text{Pu},^{241}\text{Pu}} \alpha_k \times S_k(E)$$

- $S_k(E)$: neutrino yield normalized to 1 fission of the isotope k

- E: electron energy
 - α_k : number of fission of the isotope k, at a given time

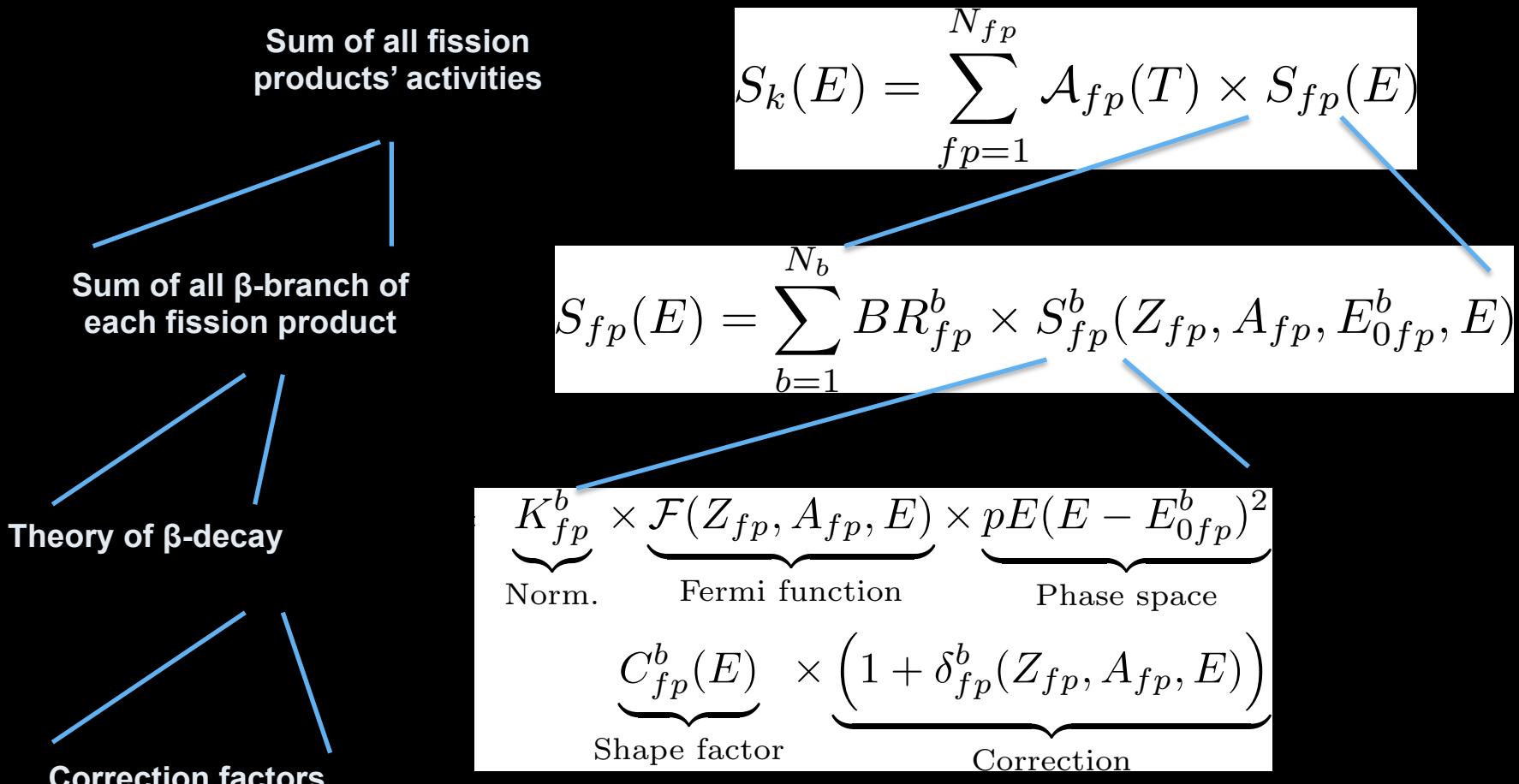
- Each isotope k undergo fission producing fission products ‘fp’

- $A_{\text{fp}}(t)$: activity of the fpth fission product normalized to 1 fission of isotope ‘k’

- Each fission product decays, via N_b branches connecting the ground state of the parent nucleus to the exited states of the daughter nucleus

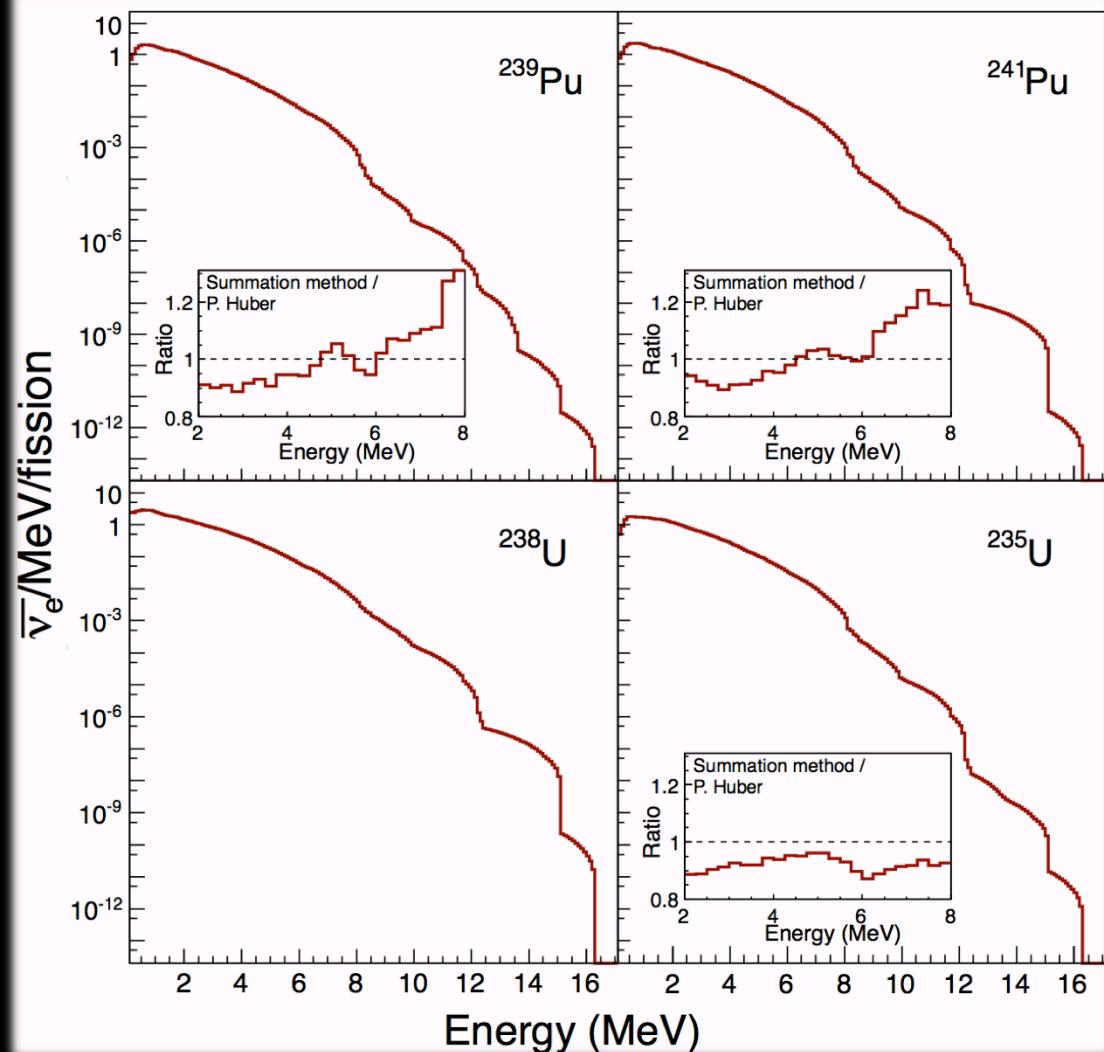
- BR_{fb}^p : branching ratio of the bth branch of the fpth fission product
 - $E_{0\text{fp}}^p$: end-point energy of the bth branch of the fpth fission product.
 - Z_f : charge of the parent nucleus.
 - A_f : atomic number of the parent nucleus.

$S_k(E)$: neutrino yield for isotope k



$$\delta_{fp}^b = G_{\nu(QED)} + L_{0(\text{coulomb size})} + C_{(\text{weak size})} + S_{(\text{screening})} + \delta_{WM(\text{weak magnetism})}$$

A) ab-initio reactor neutrino spectra

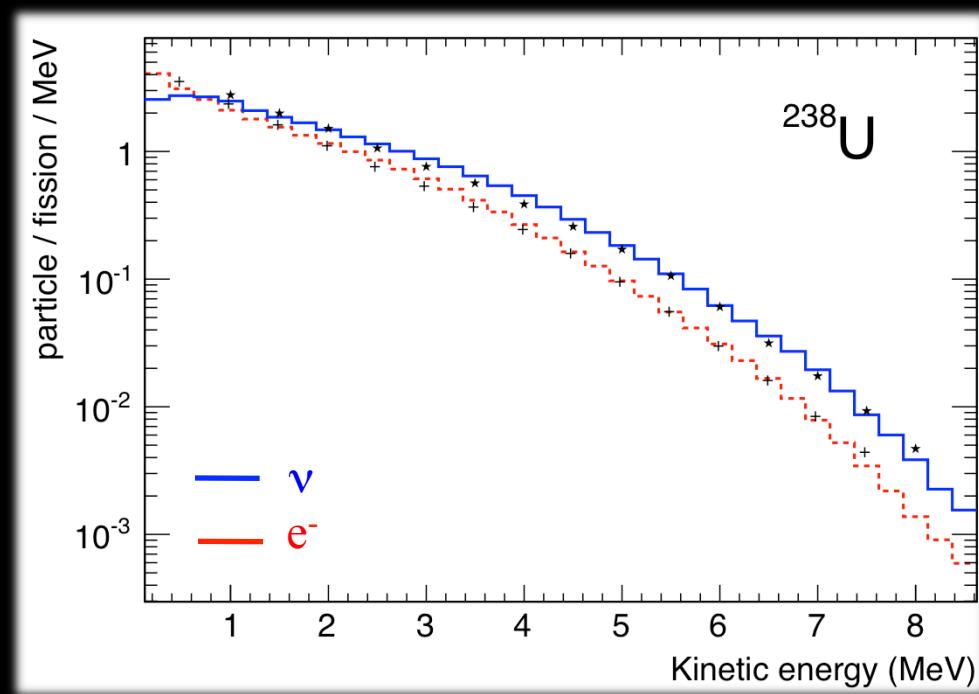


- Ab-initio computation
 - based on nuclear databases
 - reactor core evolution simulation
 - 800 nuclei
 - 10 000 decay branches
- γ -spectroscopy yields energy levels and branching fractions, but with experimental limitations
- 10-20% discrepancy with the ILL β -spectrum data
- SAME uncertainty for neutrino spectra

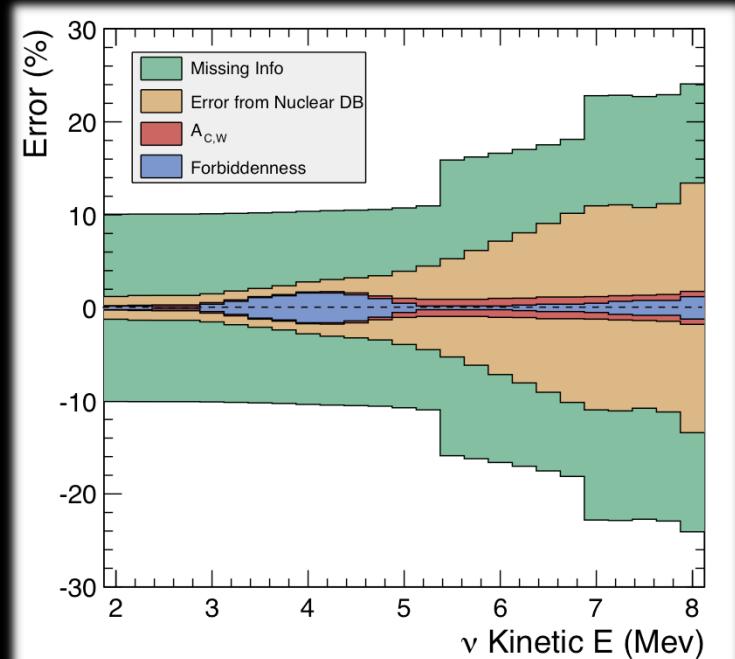
A) Ab-Initio Approach: Results

- MURE: Reactor core evolution
- BESTIOLE: β - ν conversion code: database of ~800 nuclei and 10000 β -branches

^{238}U spectrum prediction (not a fit to data)



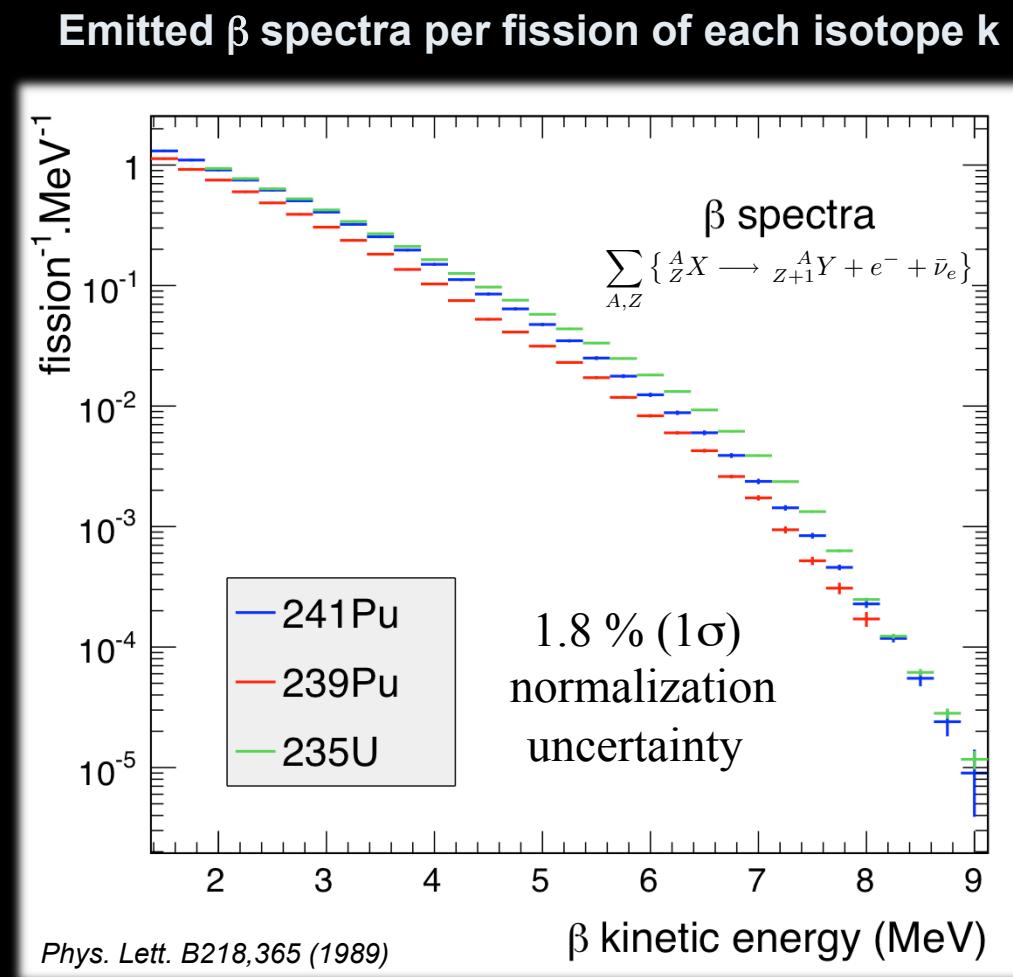
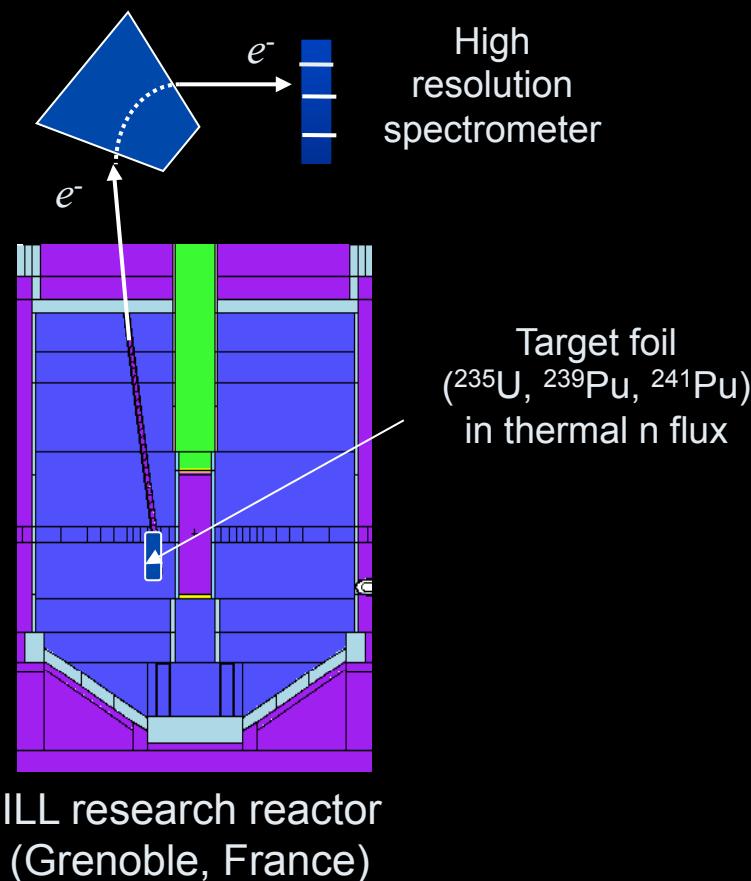
Stack of the uncertainties



- Total error in the 10-20% range.
- Dominated by systematics of nuclear databases & missing informations

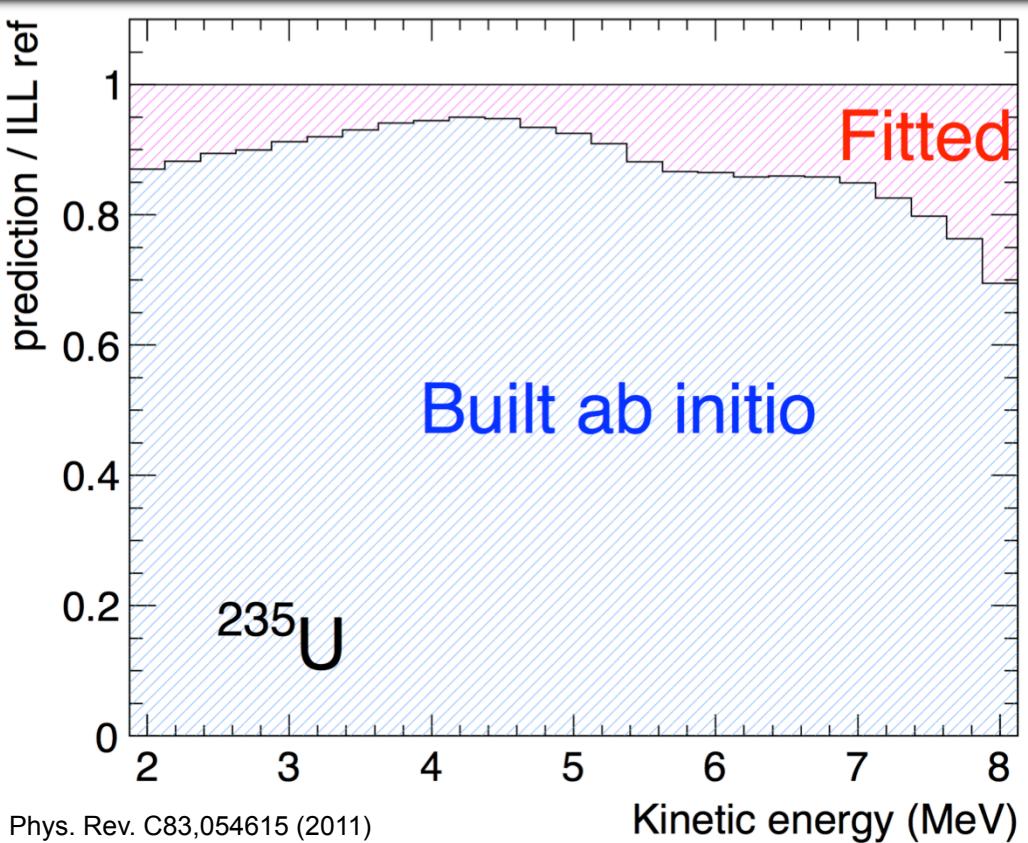
ILL β -spectrum data (e^-)

A measurement of the cumulated beta spectrum from ^{235}U $^{239,241}\text{Pu}$ fission products was performed with a magnetic spectrometer at the ILL reactor (France) in the 1980's



A) Ab-Initio Approach: Uncertainty

Comparison of the ILL β spectra (electron data) with the ab-initio computation



- Built ab initio
 - 90% of the total β spectra of ^{235}U and $^{239,241}\text{Pu}$ are described by the sum of measured β -decays \times fission yields.
- Fitted:
 - 10% missing information has to be inferred (fit by 5 virtual β -branches)
 - Main uncertainty

NE290D Lecture #10

- Nuclear Reactors
- Reactor Antineutrino Flux
- Fuel Burn-up & Neutrino Flux
- Reactor Antineutrino Energy Spectra
 - Ab-Initio Computation using Nuclear Databases
 - Computation using Experimental Integral β -Data

B) Conversion method (1)

- The total β -spectrum, N_β , is a sum of all decay branches

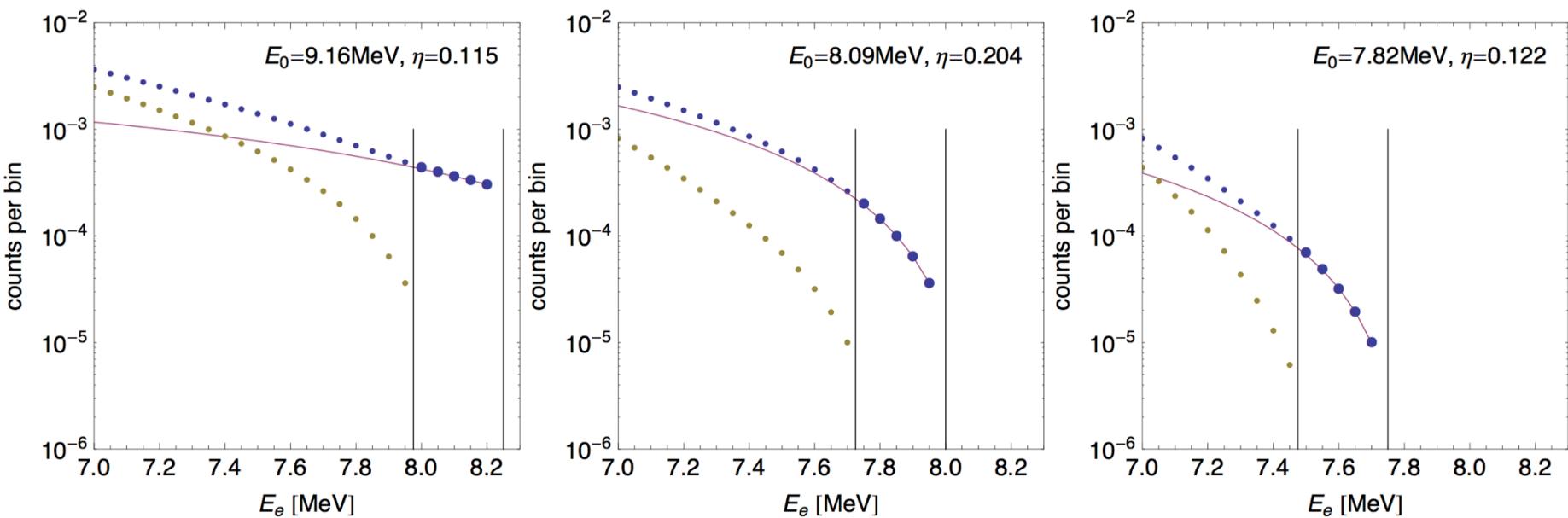
$$N_\beta(W) = \int N_\beta(W, W_0; Z_{\text{eff}}) \eta(W_0) dW_0$$

- W : electron energy, in unit of m_e
- W_0 : end-point energy
- Z_{eff} : effective nuclear charge
- $\eta_0(W)$: underlying (weighter) distribution of all end-points

- The ILL spectrum can be fitted with N_{vb} effective virtual branches
- Use ILL electron data - Fitting Procedure
 - 1 – fit an allowed β -spectrum the last s data points with free normalization η and endpoint energy W_0
 - 2 – delete the last s data points
 - 3 – subtract the fitted spectrum from the data
 - 4 – goto 1
- Invert each virtual branch using energy conservation into a neutrino spectrum and add them all

B) Conversion method (2)

- Fit ILL electron spectrum with a sum of $N_{vb}=30$ effective branches

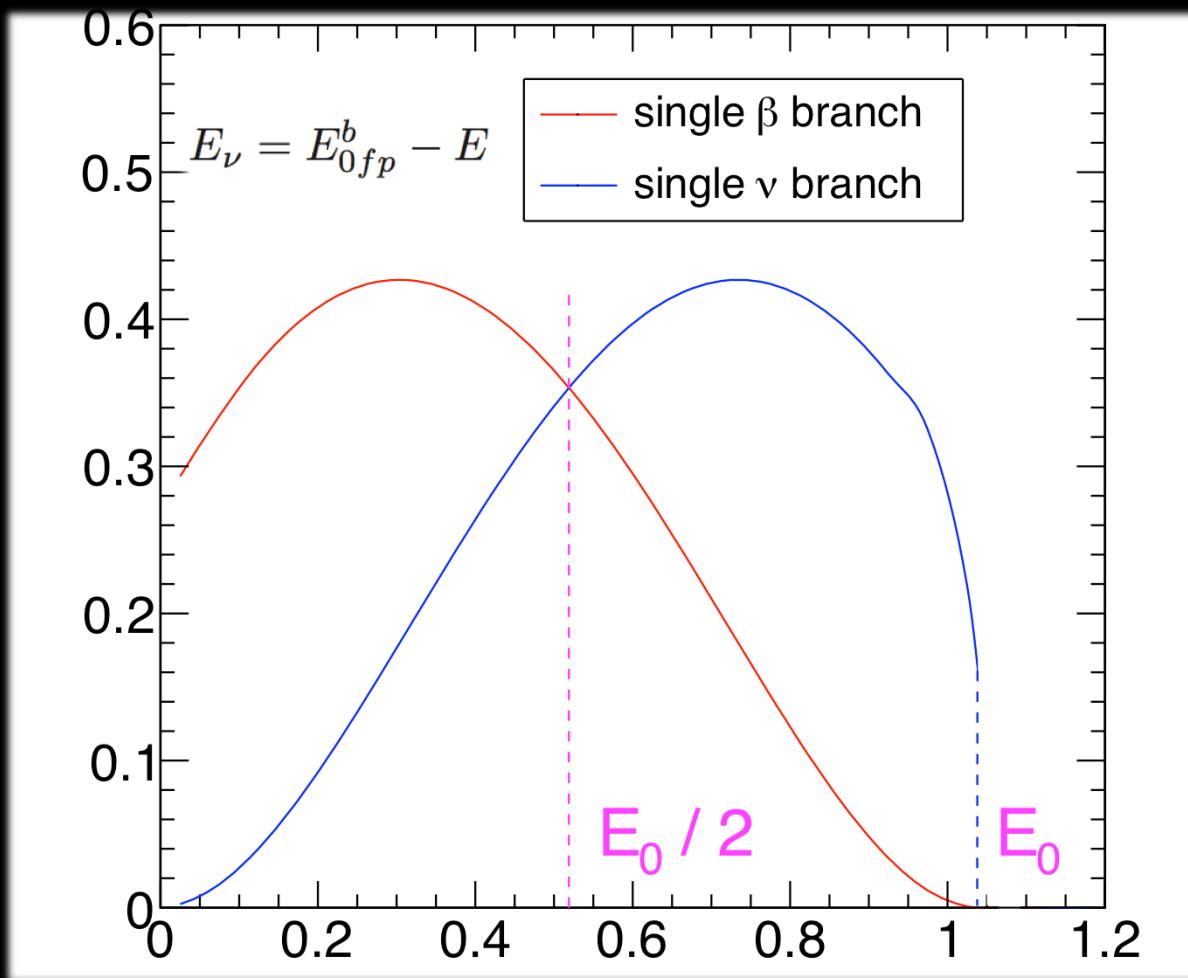


- Conversion of the effective branches to ν spectra (next slide)
- a Z_{eff} associated to each virtual branch: mean fit on nuclear data Z=f(W₀)

$$Z(W_0) \approx 49.5 - 0.7E_0 - 0.09E_0^2, \quad Z \geq 34$$

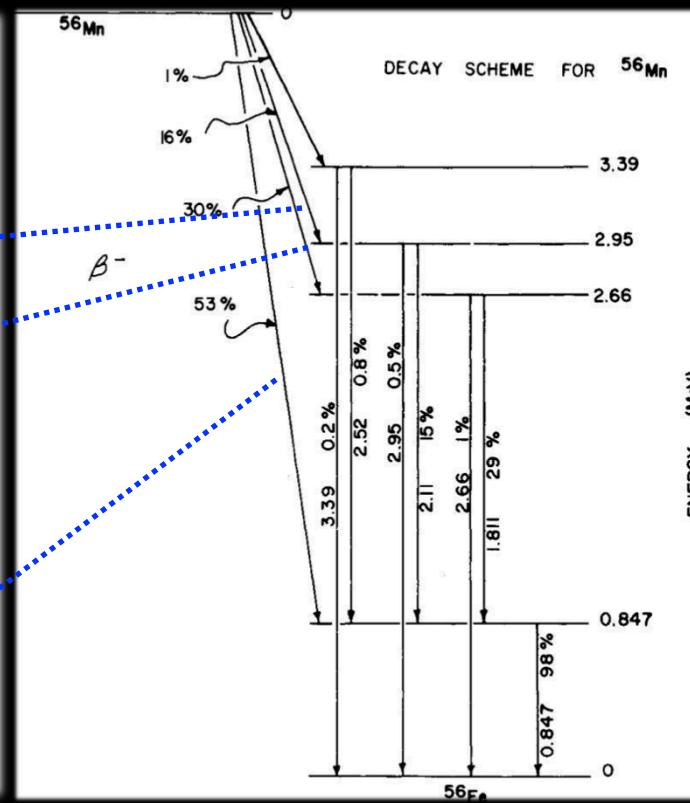
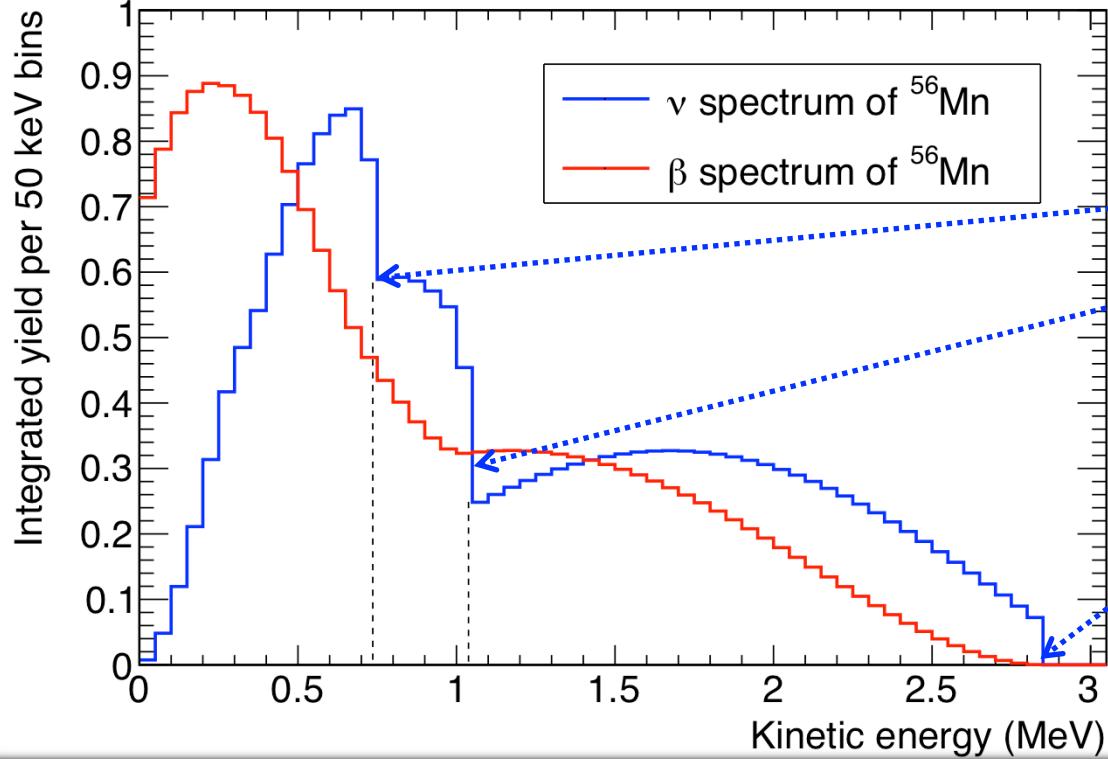
From e^- to ν : a single branch

For a single branch energy conservation implies a one-to-one correspondence between β and ν spectrum



From e^- to ν : multiple branches

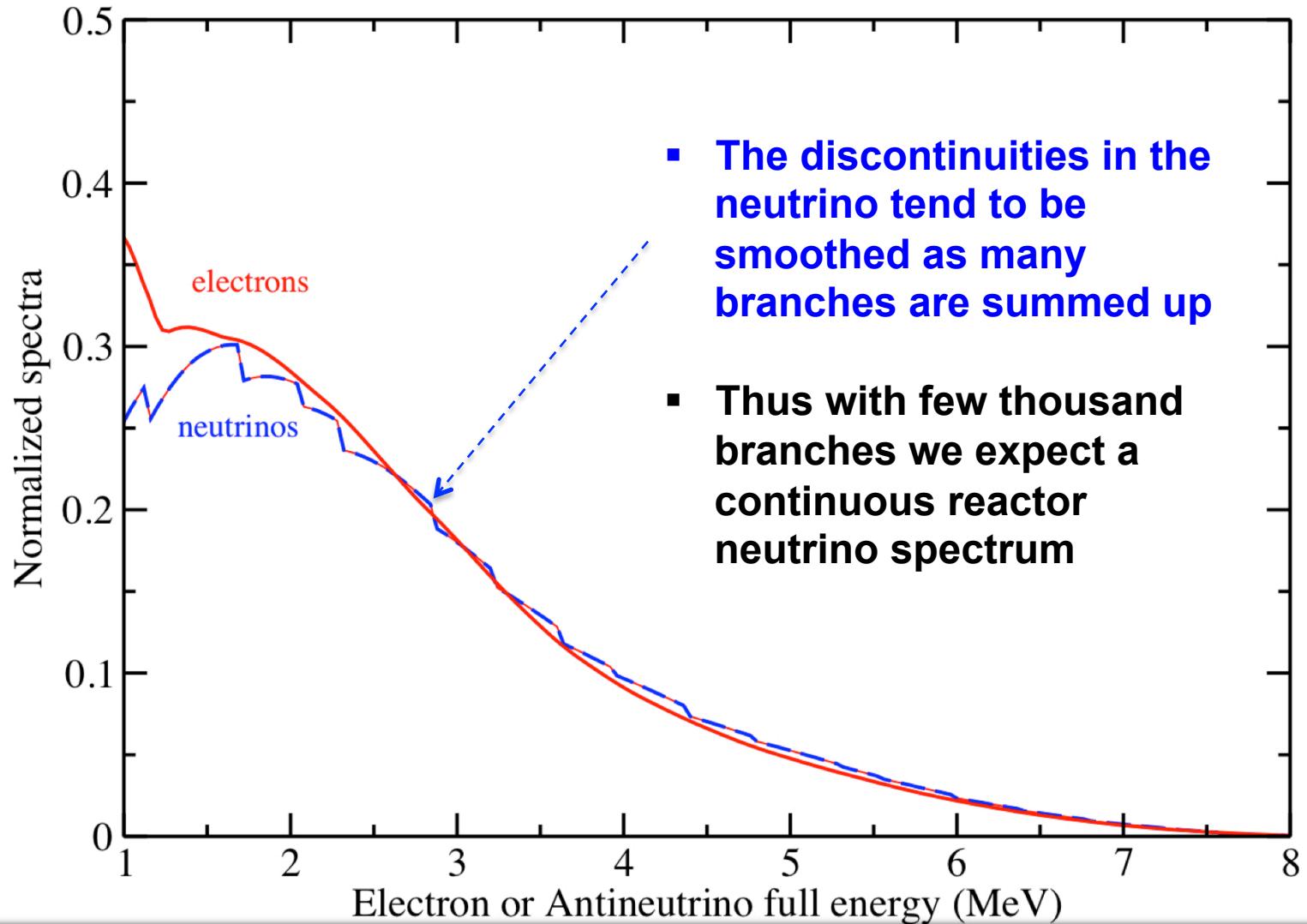
For multiple branches the conversion has to be applied separately for each branch



But not all the branches properties are well known...

From e^- to ν : 20 branches

$Z=46$, 20 random endpoints, random branching ratios



Apply corrections to Fermi theory

Applied to each virtual branch

Example for a single branch with $Z=46$, $A=117$, $E_0=10$ MeV

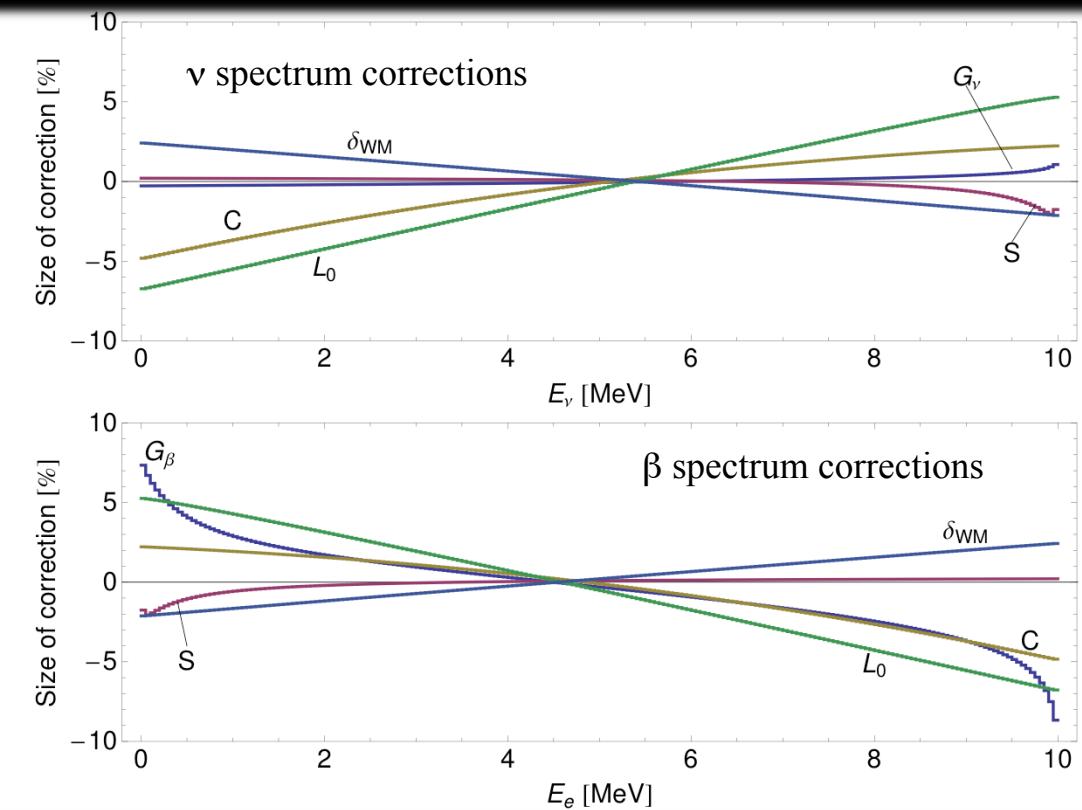
L_0 : finite size of nuclear charge

G: QED radiative correction

S: screening of atomic e-

C: finite size distrib. of decaying neutron

δ_{WM} : weak magnetism

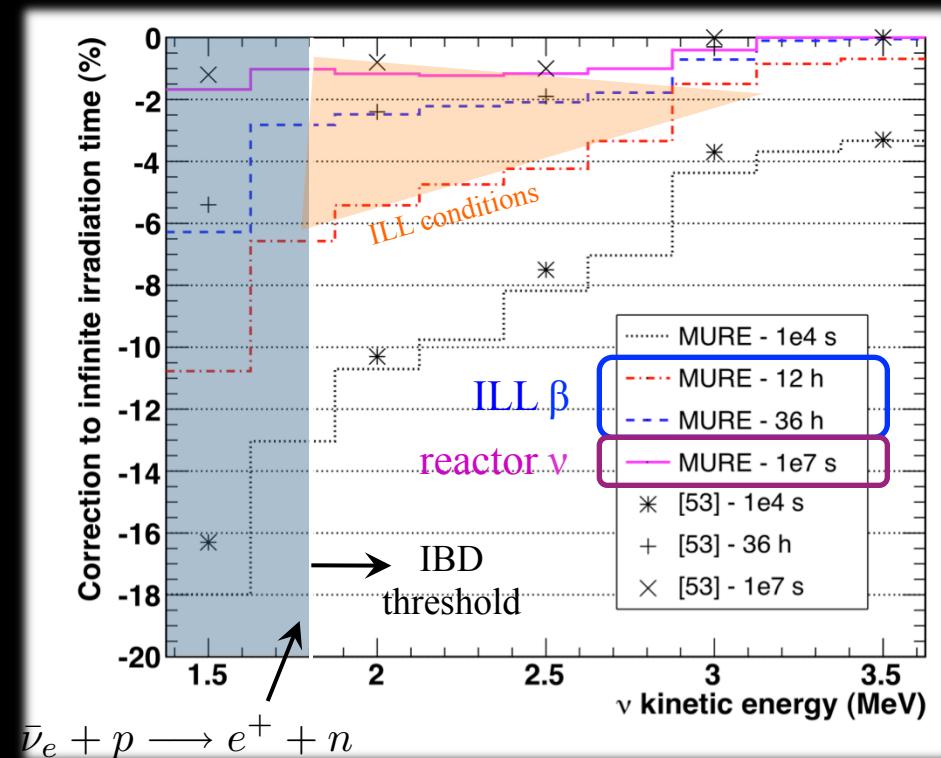


→ Corrections up to 10% - apply to both neutrino/beta spectra

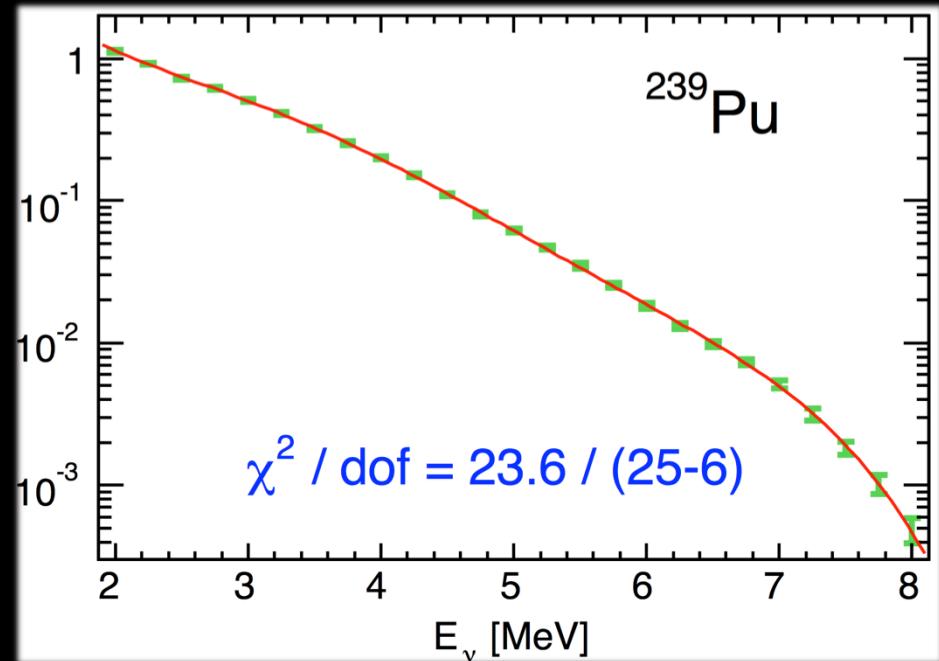
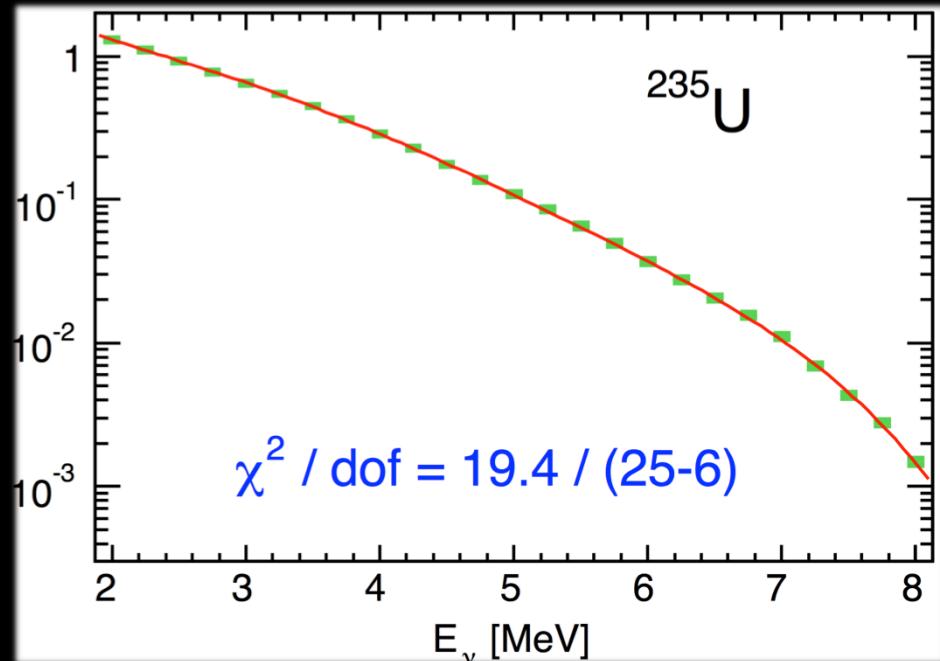
Off-equilibrium effects

- ILL electron reference spectra : 12 to 40 hours irradiation time
- Neutrino reactor experiments irradiation time >> months
- **BUT 10% of fission products have a β -decay life-time long enough to keep accumulating after several days**
- need a correction through reactor core evolution simulation

- Relative change of ν spectrum w.r.t. infinite irradiation time
- 1% correction at $E < 3$ MeV
- Can be computed accurately

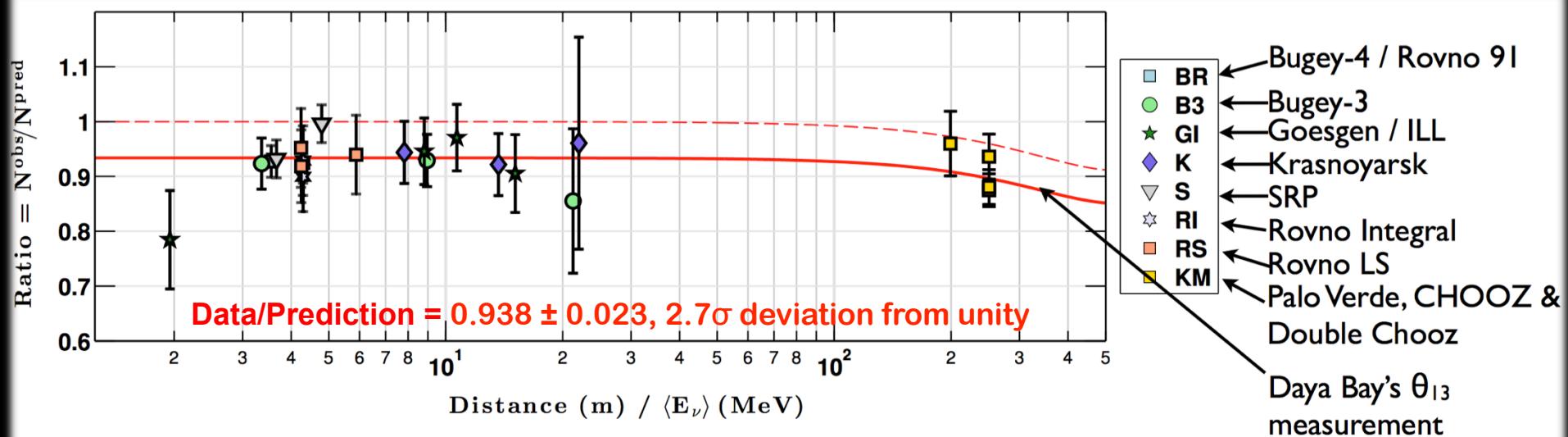
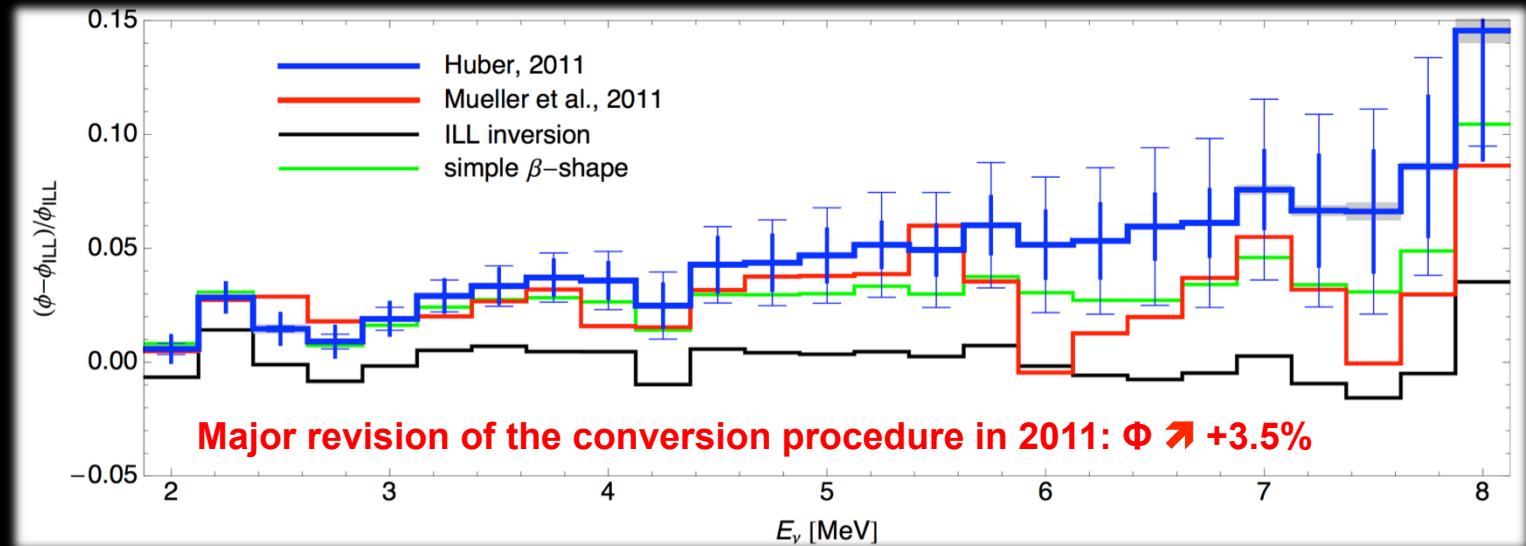


B) Reactor Neutrino Spectra



- **Useful parameterization:** $S_{k,\text{fit}}(E_\nu) = \exp \left(\sum_{p=1}^6 \alpha_{pk} E_\nu^{p-1} \right)$
- **α parameters can be found in**
 - T. Mueller et al., Phys. Rev. C83, 054615 (2011)
 - P. Huber, Phys. Rev. C84, 024617(2011)

The Reactor Anomaly: still a mystery



Comparison of A & B

A) Ab initio calculation

Complete simulation of nuclear reactor core evolution

- Fuel loading, geometry, n-capture and fission physics

→ Fission product inventory

Description of all β -decays

- Nuclear databases
- Fermi theory + corrections
- Nuclear models

→ β and ν total spectra from some 10^4 β -branches

→ 10-20% uncertainty

B) Conversion of total β spectra

Total β spectra of fissile isotopes measured at ILL in the 80's

→ Accurate reference electron spectra

Conversion to antineutrinos

- Use of "virtual" β -branches
- Fermi theory + corrections
- Control of approximations

→ Reference ν spectra per isotope to be combined with prediction of fissions rates

→ ≈3% uncertainty

Summary: reactor ν spectrum

The prediction of reactor ν spectrum is the dominant source of systematic error for single detector reactor neutrino experiments

Reactor data
Thermal power, $\delta P_{th} \leq 1\%$

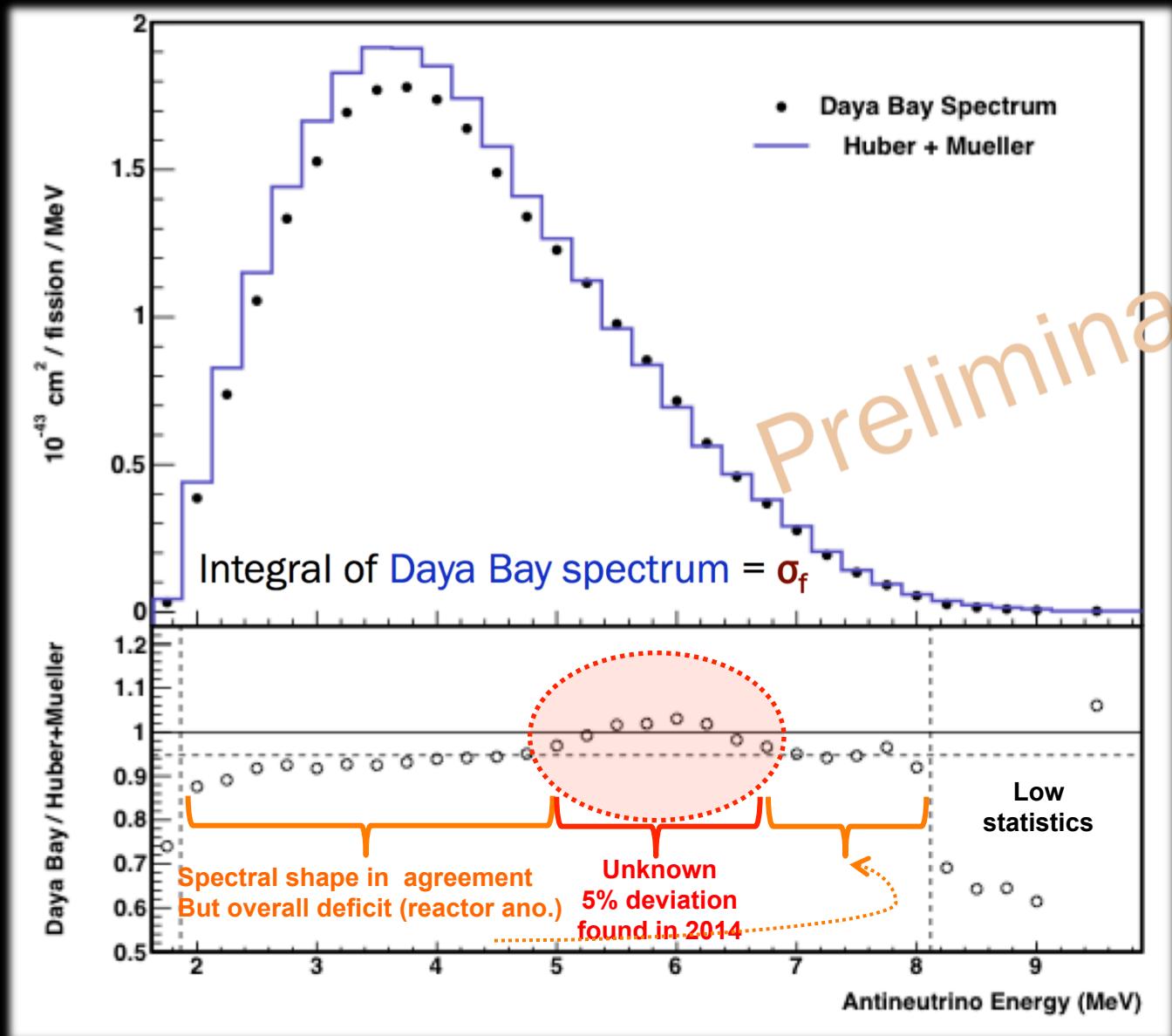
Reactor evolution codes
Fraction of fissions from isotope k , $\delta \alpha_k = \text{few } \%$
but large anti-correl @ fixed P_{th}

$$\Phi_\nu(E, t) = \frac{P_{th}(t)}{\sum_k \alpha_k(t) E_k} \times \sum_k \alpha_k(t) S_k(E)$$

Nuclear databases
 E released per fissions of isotope k ,
 $\delta E_k \approx 0.3\%$

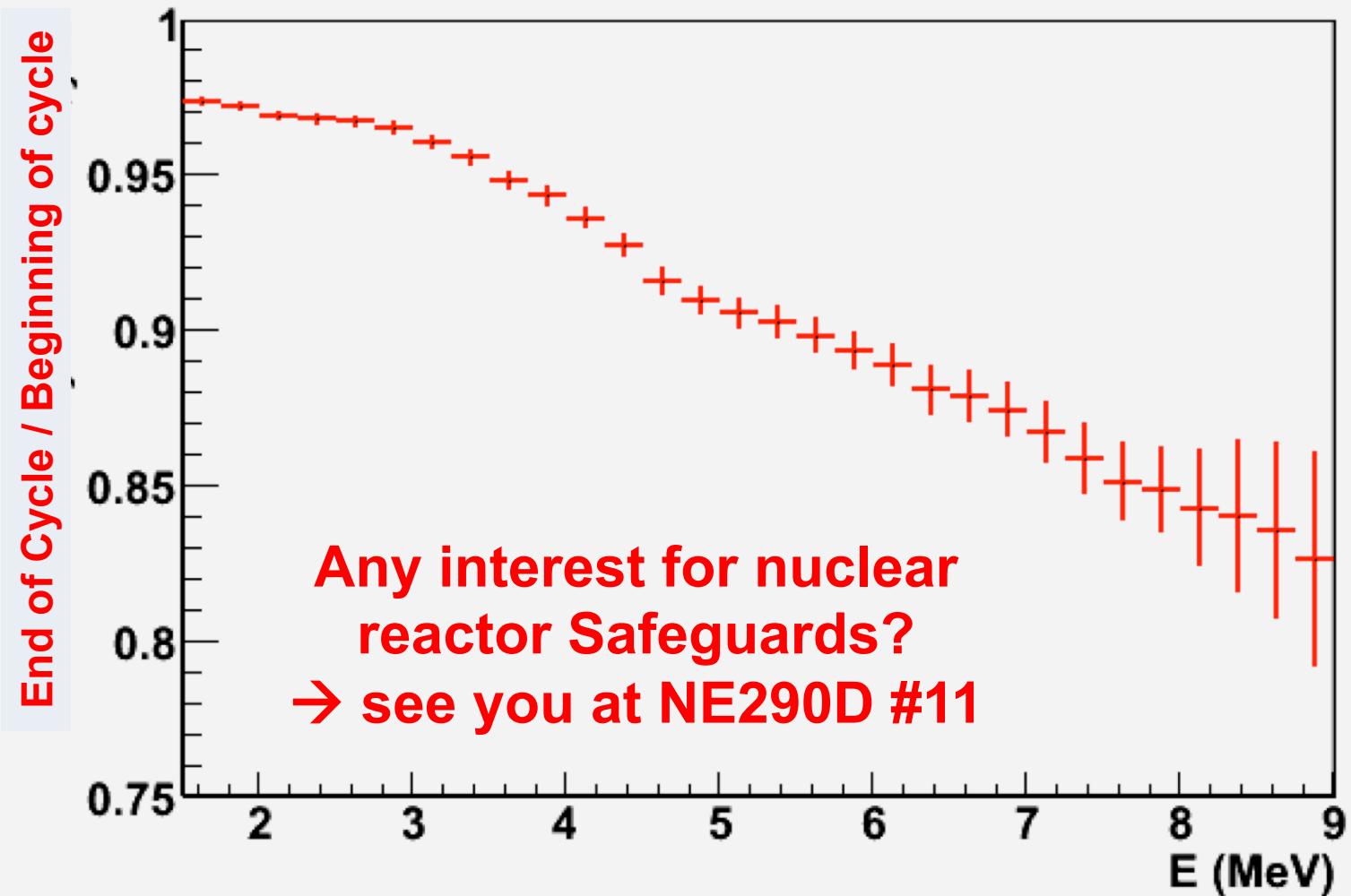
ν spectrum per fission
 $k = {}^{235}\text{U}, {}^{238}\text{U}, {}^{239}\text{Pu}, {}^{241}\text{Pu}$

Absolute Neutrino Spectrum measurement



Neutrino Spectrum shape & burnup

The energy dependent neutrino yields vary with the burn-up





BACKUP SLIDES

Spent-fuel repository



- Fuel stored for a few years on power plant site in cooling pools
- Potential emitter of antineutrinos
- Main isotopes with Q>1.8 MeV: ^{140}Ba , ^{144}Ce , ^{106}Ru , ^{90}Sr
- Typically add $(\phi_v)^{\text{sf}} < 0.5\% (\phi_v)^{\text{R}}$ due to long half life and low Q

Isotope	$T_{1/2}$	Fission Yield			β EndPoint	$\frac{M \times \langle \sigma \rangle}{T_{1/2}}$
		^{235}U	^{239}Pu	Mass (kg)		
^{131}I	8.02 d	$2.88 \cdot 10^{-2}$	$3.84 \cdot 10^{-2}$		0.971	-----
$^{140}\text{Ba}/^{140}\text{La}$	12.752 d	$6.12 \cdot 10^{-2}$	$5.59 \cdot 10^{-2}$	6.15	3.762	
^{141}Ce	32.501 d				0.581	-----
^{89}Sr	50.53 d				1.495	-----
$^{95}\text{Zr}/^{95}\text{Nb}$	64.02 d				1.16	-----
$^{144}\text{Ce}/^{144}\text{Pr}$	284.893 d	$5.26 \cdot 10^{-2}$	$3.73 \cdot 10^{-2}$	5.44	2.997	
$^{106}\text{Ru}/^{106}\text{Rh}$	373.59 d	$4.02 \cdot 10^{-3}$	$4.28 \cdot 10^{-2}$	3.06	3.678	
$^{147}\text{Pm}/^{147}\text{Sm}$	2.6234 y	$2.09 \cdot 10^{-2}$	$2.04 \cdot 10^{-2}$		0.224	-----
$^{90}\text{Sr}/^{90}\text{Y}$	28.79 y	$5.90 \cdot 10^{-2}$	$2.10 \cdot 10^{-2}$	3.81	2.280	
^{137}Cs	30.07 y	$6.27 \cdot 10^{-2}$	$6.55 \cdot 10^{-3}$		1.176	-----
^{99}Tc	$0.21 \cdot 10^6 \text{ y}$					
^{93}Zr	$1.5 \cdot 10^6 \text{ y}$					
^{135}Cs	$2.0 \cdot 10^6 \text{ y}$					
^{129}I	$16. \cdot 10^6 \text{ y}$					

Masses are given for the full load of Uranium
after a combustion at 45 GW . d / ton