A short review of the Dark energy problem

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Outline

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The homogeneous Universe: the cosmological principle

In the late time Universe, matter distribution appears:

- Homogeneous on average at scales larger than ~ 100 Mpc (Yadav & al, MNRAS 2005)
- Highly structured at smaller scales (galaxies, clusters, filaments, walls, voids)

Friedmann Universe: the strong cosmological principle

- Ignore the inhomogeneities: the Universe is locally homogeneous and isotropic.
- Maximally symmetric, homogeneous and isotropic space:
  $ds^2 = -dt^2 + a^2(t)dl^2$; spatial section of constant curvature k.
- $a(t)$: scale factor that describes the expansion of length.
- Dynamics: second order differential equations for $a(t)$; the so-called Friedmann equations.
- Universe filled with a perfect fluid $(\rho(t), p(t))$.

The homogeneous Universe: Friedmann equations

$$
\begin{aligned}
H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho(t) + \frac{\Lambda}{3} - \frac{k}{a^2} \\
\frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho(t) + 3p(t)) + \frac{\Lambda}{3} \\
\rho(t) + 3H(t)(\rho(t) + p(t)) &= 0\,.
\end{aligned}
$$

Often written:

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1 \quad ; \quad q = \frac{1}{2}(1 + 3w_b)\Omega_m - \Omega_\Lambda$$

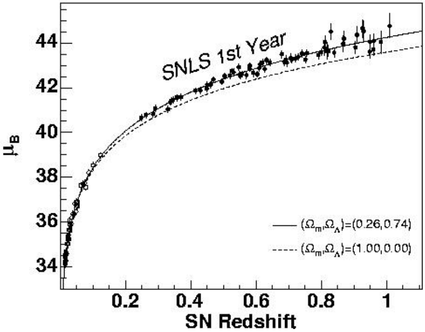
$$\Omega_m = \frac{8\pi G\rho}{3H^2}\,,\ \Omega_\Lambda = \frac{\Lambda}{3H^2} \quad ; \quad \Omega_k = -\frac{k}{a^2H^2}\ ;\ q = -\frac{\ddot{a}}{aH^2}\ ;\ w_b = \frac{p(t)}{\rho(t)}\,.$$

Until the 90's: $\Lambda = 0$ and in the late time Universe: $p = 0$ (dust fluid). This is the Einstein-de Sitter Universe: $(\Omega_m, \Omega_\Lambda) = (1, 0)$.

$\Longrightarrow \ddot{a} < 0$: matter slows down the expansion.

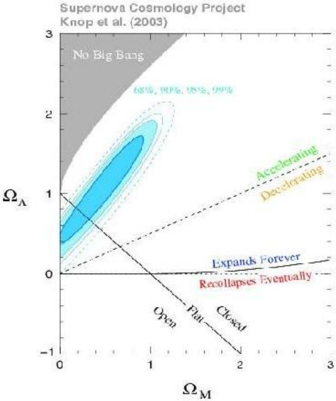
Accelerated expansion: the surprise from Supernovae

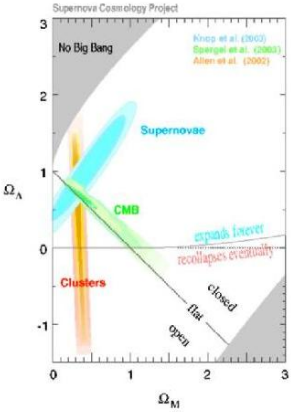
But, in 1998, Perlmutter & al. (Bull.Am.Astron.Soc.) and Schmidt & al (ApJ) have measured that: $\ddot{a} > 0$ by considering SN1a as standard candles!



$$\Lambda \neq 0$$

The concordance model





Outline

Why is the concordance model not enough?

Cosmological constant problem(s):

- Why is it so small?
  Λ corresponds to a fluid with $p = -\rho$: this can be quantum vacuum fluctuations.
  But:
  - $\rho_\Lambda^{planck} \sim 10^{76}$ GeV^4; $\rho_\Lambda^{GUT} \sim 10^{64}$ GeV^4; $\rho_\Lambda^{QCD} \sim 10^{-6}$ GeV^4
  - $\rho_\Lambda^{concord} \sim 10^{-47}$ GeV^4!!!

  More generally, fine-tuning problem.
- Why is it becoming dominant when structures form?
  Coincidence problem.

$\Longrightarrow$ Importance to test alternative origins.

Exotic component vs modification of gravity

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- Acceleration is possible iff $w_b = p/\rho < -1/3$.
  This is exotic matter (dust: $w_b = 0$, radiation: $w_b = 1/3$), violating the strong energy condition.
- But this equation may be wrong if gravity is not exactly described by GR on cosmological scales: GR is well tested only on solar-system scales (PPN constraints).

Standard quintessence: a model for exotic matter

If DE comes from exotic matter, the most simple model is a scalar field (Wetterich, Nucl. Phys. B 1988; Ratra & Peebles, PRD 1988):

$$\mathcal{L} = -\frac{1}{2} g_{\mu\nu} \partial^\mu \varphi \partial^\nu \varphi - V(\varphi)$$

- Equation of state:

$$\boxed{w_b = \frac{\dot{\varphi}^2/2 - V(\varphi)}{\dot{\varphi}^2/2 + V(\varphi)}}$$

- Acceleration is possible if $\dot{\varphi}^2 < V(\varphi)$.
- Inflation-like mechanism: $\dot{\varphi}^2 \ll V(\varphi)$. $\Rightarrow$ Flat potential. Example: $V(\varphi) \propto 1/\varphi^\alpha$.
- In general $w_b = w_b(t)$.

Main features

- Highly homogeneous field (Jeans length $\sim cH_0^{-1}$).
- Purely gravitating field: no direct interaction with matter.

Standard quintessence: a model for exotic matter

Pros and cons (Copeland & al, Int. J. Mod. Phys. 2006)

- Pros:
  - Tracking solutions: the initial conditions need not be fine-tuned.
  - The scalar field mass can be compatible with the particle physics hierarchy.
- Cons:
  - How to generate a convenient potential from first principles?
  - In particular: why no interaction with ordinary matter?

In the same family:

- Phantom fields (non positive definite kinetic energy).
- K-essence (non standard kinetic term): avoid the problem of the potential.

Modification of gravity

What if there is only ordinary matter?
Then Friedmann equations cannot be true anymore.
$\Rightarrow$ Dark Energy: imprint of a modification of gravity?

- Long range additional force and violation of the equivalence principle: coupled quintessence; scalar-tensor gravity: $(g_{\mu\nu}, \phi)$. (Amendola, PRD 2000; Uzan, PRD 1999; Perotta & al, PRD 2000; Fuzfa & Alimi, PRD 2006; etc.)
- $f(R)$ gravity. (Capoziello & al, Int. J. Mod. Phys. 2003; Carrol & al, PRD 2004; etc.)
- Higher dimensional gravity: braneworlds. (Randall & Sundrum, Phys. Rev. Lett. 1999; Dvali & al, Phys. Lett. B 2000; Deffayet & al, PRD 2002; Sahni & Shtanov, JCAP 2003; etc.)

Each of these (numerous) models can fit the SN1a data!

An example: the AWE hypohesis

Modified gravity can explain Dark Energy without exotic matter.
An example: the Abnormally Weighting Energy (Fuzfa & Alimi, PRD 2006).

- Three components in the late-time Universe: baryons, DM, and gravity described by $(g_{\mu\nu}, \phi)$.
- Introduce a violation of the weak equivalence principle between ordinary (baryonic) matter and Dark Matter: Baryons and DM does not couple in the same way to geometry: they don't weight identically.
- Explain the acceleration by a variation of the gravitational coupling only.
- Clustering may be drastically changed, becuse of the WEP violation.

A third way to Dark Energy: the backreaction effect

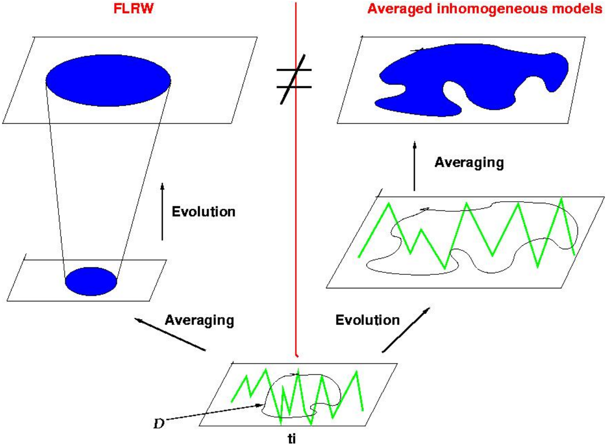
The weak cosmological principle

- We retain the averaged homogeneity on large scales: the large scale observables can be deduced from purely time-dependent functions.
- But, no assumption on the local structure of space-time and matter distribution (in particular no perturbative expansion).
- Local dynamics obeys the general Einstein equations.

- Defining the homogeneous model: averaging procedure?
- Dynamical equations?

Evolving the average or averaging the evolution?

$$[\partial_t \cdot, \langle \cdot \rangle_{\mathcal{D}}] \neq 0$$



Defining the averaged dynamics

Einstein equations in 3+1 splitting:

- Constraints equations: $R + K^{i\,2}_{i} - K^{i}_{j}K^{j}_{i} = 16\pi G\rho$; $\nabla_i K^{i}_{j} - \partial_j K^{i}_{i} = 0$
- Evolution equations: $\partial_t g_{ij} = -2K_{ij}$; $\partial_t K^{i}_{j} = K^{k}_{k}K^{i}_{j} + R^{i}_{j} - 4\pi G\rho\delta^{i}_{j}$
- Conservation of energy-momentum: $\partial_t \rho + \theta\rho = 0$

Averaging procedure (Buchert, GRG, 2000 and 2001)

- Choose a spatial domain $\mathcal{D}$
- Volume of $\mathcal{D}$: $V_{\mathcal{D}}(t) = \int_{\mathcal{D}} J d^3x$, with $J = \sqrt{\det(g_{ij})}$
- Effective volume scale factor: $a_{\mathcal{D}}(t) = \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}_\mathrm{i}}}\right)^{1/3}$
- Averaging operator for scalars: $\langle \Upsilon \rangle_{\mathcal{D}} = \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} \Upsilon J d^3x$

We apply this averaging procedure to the scalar part of the equations.

Effective averaged equations

$$\begin{aligned} H_{\mathcal{D}}^2 &= \frac{8\pi G}{3}\langle\rho\rangle_{\mathcal{D}} - \frac{1}{6}(\mathcal{Q}_{\mathcal{D}} + \langle R\rangle_{\mathcal{D}}) \\ 3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} &= -4\pi G\langle\rho\rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}} \\ \langle\rho\rangle_{\mathcal{D}} &= \rho_{\mathcal{D}_\mathrm{i}} a_{\mathcal{D}}^{-3} \\ \partial_t \mathcal{Q}_{\mathcal{D}} + 6H_{\mathcal{D}} &= -\partial_t\langle R\rangle_{\mathcal{D}} - 2H_{\mathcal{D}}\langle R\rangle_{\mathcal{D}} \end{aligned}$$

where:

- $H_{\mathcal{D}} = \frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}$: effective Hubble parameter
- $\langle R\rangle_{\mathcal{D}}$: averaged 3-curvature
- $\mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left\langle(\theta - \langle\theta\rangle_{\mathcal{D}})^2\right\rangle_{\mathcal{D}} - \left\langle\sigma^{ij}\sigma_{ij}\right\rangle_{\mathcal{D}}$: fluctuation term called kinematical backreaction.

The averaged system is not closed:
the cosmological model requires a closure condition.

A remark on curvature

One can define a constant curvature $k_{\mathcal{D}_{\mathsf{i}}}$ à la Friedmann for the averaged system:

$$\frac{k_{\mathcal{D}_{\mathsf{i}}}}{a_{\mathcal{D}}^2} = \frac{\langle\mathcal{R}\rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}}}{6} + 2\frac{1}{3a_{\mathcal{D}}^2}\int_1^{a_{\mathcal{D}}} a\mathcal{Q}_{\mathcal{D}}(a)da$$

A priori, the averaged 3-curvature is not a Friedmannian constant curvature: General Relativistic effect: coupling between averaged 3-curvature and backreaction.

The morphon field

Identification of $\mathcal{Q}_{\mathcal{D}}$ and $\langle \mathcal{R} \rangle_{\mathcal{D}}$ with a scalar field:

$$-\frac{1}{8\pi G}\mathcal{Q}_{\mathcal{D}} = \epsilon \dot{\Phi}_{\mathcal{D}}^2 - U(\Phi_{\mathcal{D}}) \; , \; -\frac{1}{8\pi G}\langle \mathcal{R} \rangle_{\mathcal{D}} = 3U(\Phi_{\mathcal{D}}) \qquad (1)$$

Then, the averaged equations become (Buchert, Larena & Alimi, CQG, 2006):

$$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3}\left(\langle \rho \rangle_{\mathcal{D}} + \frac{\epsilon}{2}\dot{\Phi}_{\mathcal{D}}^2 + U(\Phi_{\mathcal{D}})\right)$$

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3}\left(\langle \rho \rangle_{\mathcal{D}} + 2\epsilon \dot{\Phi}_{\mathcal{D}}^2 - 2U(\Phi_{\mathcal{D}})\right)$$

$$\ddot{\Phi}_{\mathcal{D}} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\dot{\Phi}_{\mathcal{D}} + \epsilon\frac{\partial U(\Phi_{\mathcal{D}})}{\partial \Phi_{\mathcal{D}}} = 0$$

All the features of the scalar field are fixed by the (classical) fluctuations in matter and geometry.

A homogeneous model on large scales for the Universe naturally leads to an additional scalar field source for the evolution of the cosmological scale factor.

This could be the origin of cosmological quintessence.

Late time accelerated expansion: the morphon as Dark Energy

- $a_{\mathcal{D}}$ accelerates iff $\mathcal{Q}_{\mathcal{D}} > 4\pi G \langle \rho \rangle_{\mathcal{D}}$.
- $\mathcal{Q}_{\mathcal{D}} V_{\mathcal{D}}$ grows with the fluctuations in the expansion rate.
- Late time Universe:
  - Highly underdense and overdense regions.
  - The fluctuations in the expansion rate are important and growing.
- Acceleration may be possible if this growth is faster than the growth of the volume.
- It can provide a solution to the coincidence problem.
- A perturbative calculation (Li & Schwarz, astro-ph/0702043) shows that at second order, a quintessence mode appears. This is very encouraging.

A thermodynamical analogy

The scalar field description allows to identify:

$$E_{kin}^{\mathcal{D}} = \frac{\epsilon}{2}\dot{\Phi}_{\mathcal{D}}V_{\mathcal{D}}$$
$$E_{pot}^{\mathcal{D}} = -U_{\mathcal{D}}V_{\mathcal{D}}$$

$$\frac{E_{kin}^{\mathcal{D}}}{E_{pot}^{\mathcal{D}}} = -\frac{3}{2}\left(\frac{\mathcal{Q}_{\mathcal{D}}}{\langle R\rangle_{\mathcal{D}}} + \frac{1}{3}\right)$$

When there is no fluctuation ($\langle R\rangle_{\mathcal{D}} \propto a_{\mathcal{D}}^{-2}$), we then have a 'virial' condition: $2E_{kin}^{\mathcal{D}} + E_{pot}^{\mathcal{D}} = 0$

$\Rightarrow$ Backreaction causes deviations from 'equilibrium'.

The scaling solutions

(Buchert, Larena & Alimi, CQG, 2006)
$\mathcal{Q}_{\mathcal{D}} = \mathcal{Q}_{\mathcal{D}_\mathrm{i}} a_{\mathcal{D}}^{n}$ and $\langle\mathcal{R}\rangle_{\mathcal{D}} = \mathcal{R}_{\mathcal{D}_\mathrm{i}} a_{\mathcal{D}}^{p}$.

$$a_{\mathcal{D}}^{-6}\partial_t\left(a_{\mathcal{D}}^{6}\mathcal{Q}_{\mathcal{D}}\right) = -a_{\mathcal{D}}^{-2}\partial_t\left(a_{\mathcal{D}}^{2}\langle\mathcal{R}\rangle_{\mathcal{D}}\right)$$

Two types of solutions:

2 types of solutions:

- $n \neq p$:

$$\mathcal{Q}_{\mathcal{D}} = \mathcal{Q}_{\mathcal{D}_\mathrm{i}} a_{\mathcal{D}}^{-6}\ ;\ \langle\mathcal{R}\rangle_{\mathcal{D}} = \mathcal{R}_{\mathcal{D}_\mathrm{i}} a_{\mathcal{D}}^{-2}$$

  Backreaction and averaged curvature are decoupled.
  Quasi-Friedmannian Universe: $\Omega_{\mathcal{R}}^{\mathcal{D}} + \Omega_{\mathcal{Q}}^{\mathcal{D}} \sim \Omega_{k}^{\mathcal{D}}$ when $a_{\mathcal{D}} \to +\infty$.

- $n = p$:

$$\mathcal{Q}_{\mathcal{D}} = r\langle\mathcal{R}\rangle_{\mathcal{D}} = r\mathcal{R}_{\mathcal{D}_\mathrm{i}} a_{\mathcal{D}}^{n}\ ,\ n = -2(1+3r)/(1+r),\ r \neq -1$$

  $r = \frac{1}{3}\frac{1+3w_{\mathcal{D}}}{1-w_{\mathcal{D}}} =$ cst is the conversion rate between kinematical backreaction and averaged curvature.
  Purely General Relativistic effect.

Reconstruction of the morphon field potential

From the correspondence:

$$\dot{\Phi}^2_{\mathcal{D}} = -\epsilon \frac{\mathcal{R}_{\mathcal{D}_i}}{8\pi G}\left(r + \frac{1}{3}\right) a^n_{\mathcal{D}}$$
$$U(\Phi_{\mathcal{D}}) = -\frac{\mathcal{R}_{\mathcal{D}_i}}{24\pi G} a^n_{\mathcal{D}}$$

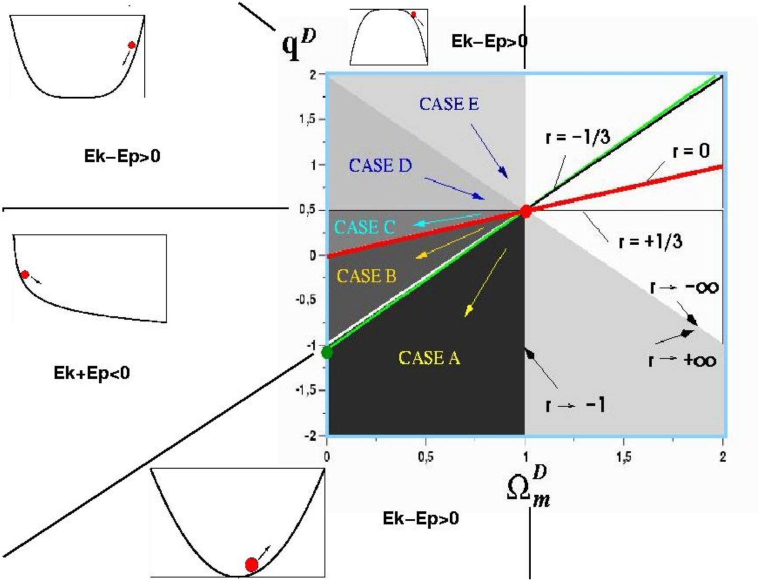
So,

$$U(\Phi_{\mathcal{D}}) = \alpha\left(r, \mathcal{R}_{\mathcal{D}_i}, \langle\varrho\rangle_{\mathcal{D}_i}\right) \sinh^{-4\frac{1+3r}{1-3r}}\left(\beta\left(r, \mathcal{R}_{\mathcal{D}_i}, \langle\varrho\rangle_{\mathcal{D}_i}\right)\frac{\Phi_{\mathcal{D}}}{G}\right)$$

(Sahni & al, JETP Lett., 2003; Sahni & al, Int. J. Mod. Phys., 2000, Copeland & al, hep-th/0603057)

- The scalar field parameters are fixed by the initial averaged quantities.
- Solution to the coincidence problem?
- Constant equation of state: $w^{\mathcal{D}}_{\Phi} = -\frac{1}{3}\frac{1-3r}{1+r}$.

The space of solutions



Outline

Distance measurements

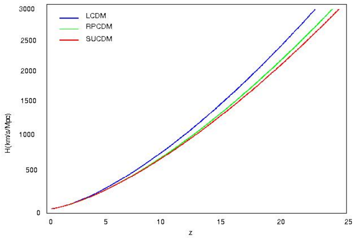
- Historical test with SN1a; Limited in redshifts (up to z$\sim$ 1).
  Future (see the DETF report astro-ph/0609591):
  - Testing the standard candle hypothesis
  - Taking care of the environment of SN1a (extinction by dust clouds...)
  - Reducing the systematics (calibration...)
- Possibility to use GRBs: Allow to extend the redshift range.

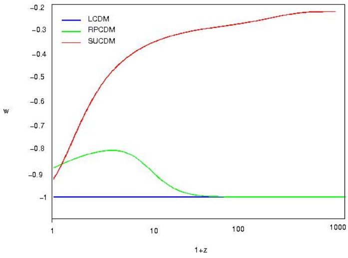
But, standard candles only test the homogeneous properties of DE:

$$w(t) = p_{DE}(t)/\rho_{DE}(t)$$

Distance measurements (II)

This is not sufficient to discriminate between the models:





⇒ We need more information at higher redshifts.

The clustering of matter

Impacts on the clustering of matter differ:

- In ΛCDM only $H(z)$ is affected
- In standard quintessence:
  - $H(z)$ is affected.
  - $c_s^2 = \delta p_{DE}/\delta\rho_{DE} \sim 1$: power spectrum enhanced by DE clustering at scales greater than the sound horizon. But, sound horizon $\sim$ cosmological horizon.
- In coupled quintessence (or modified gravity):
  - $H(z)$ is affected.
  - $c_s^2 < 1$: power spectrum enhanced by DE clustering at scales greater than the sound horizon.
  - Clustering of matter itself is affected by the variation of the gravitational coupling: $\Delta\Phi \sim 8\pi G(\phi)\delta\rho$. If different couplings for different sources (like in AWE): non-linear bias
- In averaged cosmologies: **?**: Work in progress.
  - $H(z)$ affected, then affecting distances measurements, but clustering maybe not affected at all?
  - Correct way to measure distances: light cone averaging?
  - Understanding the scale dependence (Renormalization group: Ricci flow).

Clustering properties

What can be measured?

- ISW: correlation CMB/galaxies survey
- Clusters number count
- BAO
- Weak lensing

To be compared with numerical simulations in each context.

Outline

Conclusion

- "Dark Energy": many very different origins are possible:
  - Cosmological constant
  - "Exotic" matter
  - Modification of gravity
  - Inadequacy of the Friedmannian context in the late-time Universe.
- This is not only refinements: it can change our view on physics.
- Predictions vary with the context.
- The models essentially differ at high redshifts and in their predictions for the matter distribution.
  $\Rightarrow$ Importance of the inhomogeneous Universe
- Current and future important observations:
  - SN1a: SNLS
  - CMB: PLANCK
  - Galaxies survey: SKA (Torres-Rodrigues & Cress, astro-ph/0702113)
  - Weak lensing and SN1a: DUNE (Schimd & Tereno, astro-ph/0612022)
- In parallel: a great work to understand and compute precisely (numerical simulations) the properties of structure formation in the various contexts.