Large distance modification of gravity: the example of "massive gravity"

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1. Why massive gravity?

From massless gravity to the DGP model: an « invitation au voyage ».

2. Generic properties and problems of massive gravity.

3. Some recent progresses and open issues.

FP7/2007-2013
« NIRG » project no. 307934

1.1. Introduction: why « massive gravity » ?

➡ Theoretical challenge !

➡ One way to modify gravity at « large distances »
… and get rid of dark energy (or dark matter) ?

$$H^2 = \frac{8\pi G}{3}\rho$$

Changing the dynamics
of gravity ?

Dark matter
dark energy ?

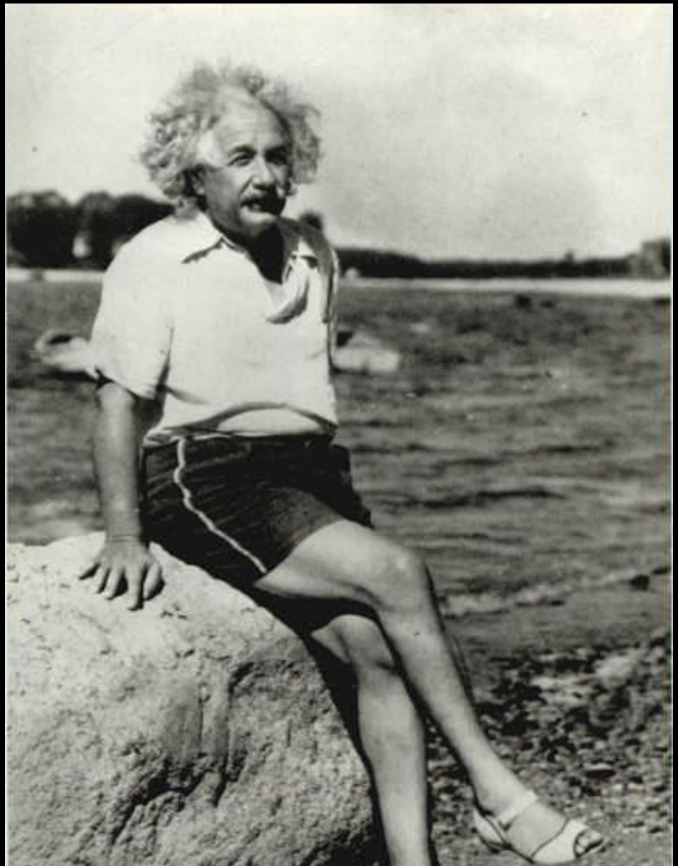
for this idea to work…

I.e. to « replace » the cosmological constant by a non vanishing graviton mass…

NB: It seems one of the Einstein's motivations to introduce the cosmological constant was to try to « give a mass to the graviton »

⇒ One obviously needs a very light graviton (of Compton length of order of the size of the Universe)

(see « Einstein's mistake and the cosmological constant » by A. Harvey and E. Schucking, Am. J. of Phys. Vol. 68, Issue 8 (2000))



1.2. Some properties of « massless gravity » (i.e. General Relativity – GR)

In GR, the field equations (Einstein equations) take the same form in all coordinate system (« general covariance »)

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Einstein tensor: Second order non linear differential operator on the metric $g_{1\,o}$

Newton constant

Energy momentum tensor: describes the sources

The Einstein tensor obeys the identities $\nabla^\mu G_{\mu\nu} = 0$

In agreement with the conservation relations $\nabla^\mu T_{\mu\nu} = 0$

Einstein equations can be obtained from the action

$$M_P^2 \int d^4x \sqrt{-g}\,(R + \mathrm{matter})$$

With
$$\begin{cases} G_N \propto M_P^{-2} \\ R = -G_{\mu\nu} g^{\mu\nu} \\ g = |\det g_{\mu\nu}| \end{cases}$$

NB: Einstein equations $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$ are highly non linear

If one linearizes Einstein equations around e.g. a flat metric ´$_{1\,o}$ one obtains the field equations for a « graviton »

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

given by

$$\underbrace{\partial_\mu \partial_\nu h + \Box h_{\mu\nu} - \nabla_\mu \partial_\rho h^\rho_\nu - \partial_\nu \partial_\rho h^\rho_\mu + \eta_{\mu\nu}(\partial^\rho \partial^\sigma h_{\rho\sigma} - \Box h)}_{K_{\mu\nu\rho\sigma} \;\; h^{\rho\sigma}} \propto T_{\mu\nu}$$

Kinetic operator of the graviton h$_{1\,o}$:
does not contain any mass term / (undifferentiated h$_{1\,o}$)

The masslessness of the graviton is guaranteed by the gauge invariance (general covariance) $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Which also results in the graviton having 2 = (10 – 4 £ 2) physical polarisations (cf. the « photon » A$_1$)

General Relativity (GR) is a very « optimal » and « rigid » theory, quite well tested.

Modifications to GR result in general to new propagating fields (or polarizations) which make the new theory differ drastically from GR

Need for a « screening » mechanism

1.3. Kaluza-Klein gravitons

Massive gravitons (from the standpoint of a 4D observer) are ubiquitous in models with extra space-time dimensions in the form of « Kaluza-Klein » modes (in **common interaction**).

Consider first a **massless** scalar-mediated force in 4D.

It is obtained from the Poisson equation
(e.g. for an electrostatic or a gravitationnal potential)

$$\begin{cases} \Delta \Phi_E = - \rho/\varepsilon_0 \\ \Delta \Phi_N = 4 \pi G_N \rho_m \end{cases}$$

Yielding a force between two bodies / $1/r^2$

A force mediated by a **massive** scalar would instead obey the modified Helmholtz equation

$$\Delta \Phi - \Phi / \lambda_C^2 \ / \ \text{source}$$

Compton length

$\lambda_C = \sim / m\, c$

This comes from a **quadratic (m^2 ©^{2})**

mass term in the Lagrangian

And results in the finite range Yukawa potential

$$\Phi(r) \ / \ \exp(-r/\lambda_C) \ / \ r$$

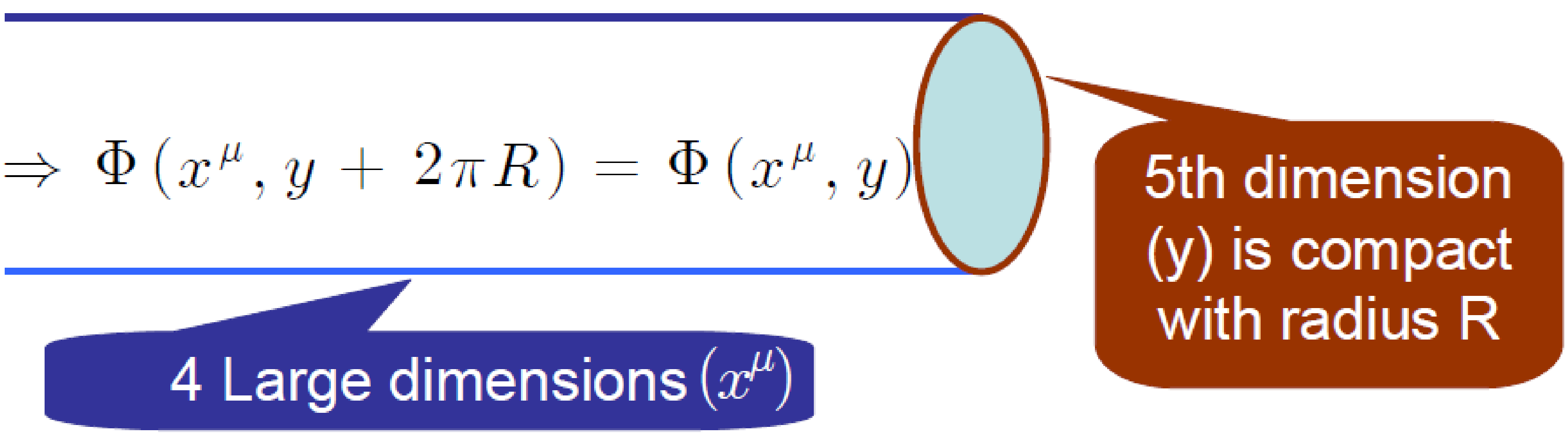
Now consider a massless scalar field $\Phi(x^\mu, y)$ in a 4 (x^μ) + 1 (y) dimensional space-time.

It obeys the field equations (in the absence of sources)

$$\underbrace{(\partial^\mu \partial_\mu + \partial^y \partial_y)}_{\text{5D d'Alembertian}} \Phi = 0$$

Assume now the 5th dimension to be "compact"

$$\Rightarrow \Phi(x^\mu, y + 2\pi R) = \Phi(x^\mu, y)$$



$$\Rightarrow \Phi(x^\mu, y) = \sum_k \Phi_k(x^\mu) \exp\left(\frac{iky}{R}\right)$$

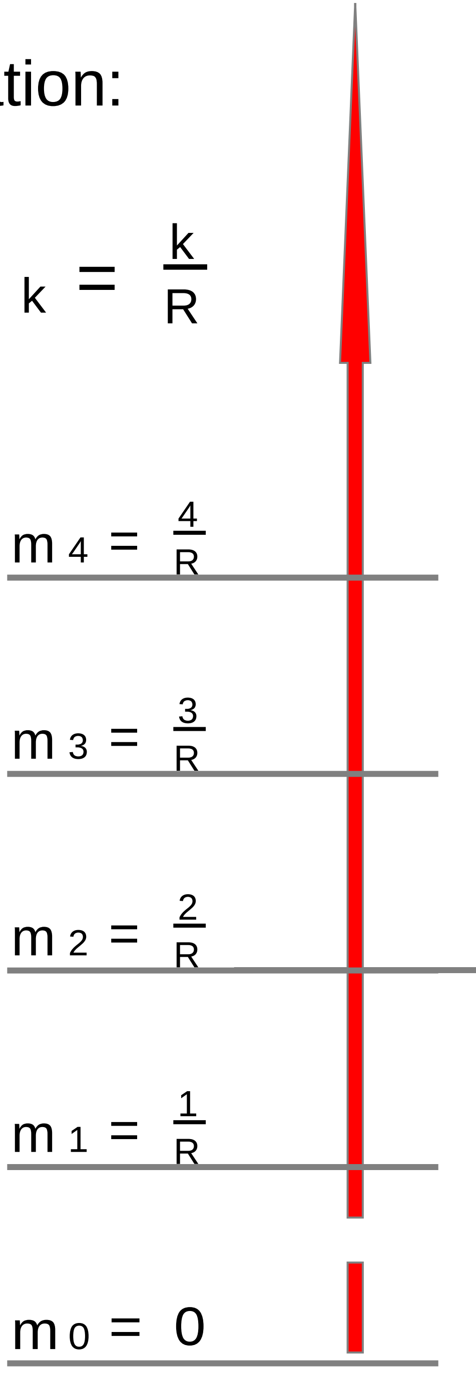
Inserting this into the 5D massless field equation:

$$(\partial^\mu \partial_\mu + m_k^2)\Phi_k = 0$$

with $m_k = \frac{k}{R}$

Field equation for a 4D scalar field of mass m_k

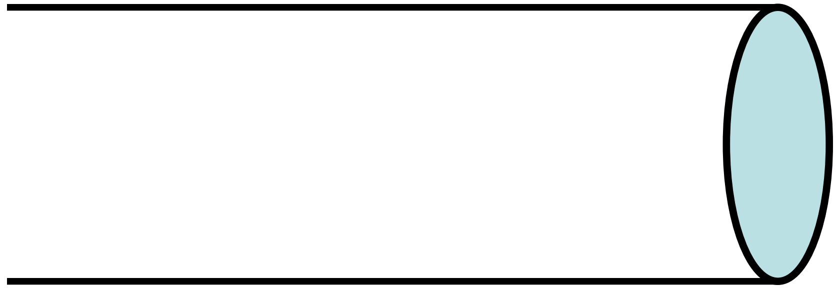
A 5D massless scalar appears as a



Experiments at energies much below m_1 only see the massless mode

Low energy effective theory is four-dimensional

The same reasoning holds for the graviton:



Small perturbation in the vicinity of a reference "cylinder" :

with $g_{\mu\nu}(x^{\mu},y) = \eta_{\mu\nu} + h_{\mu\nu}(x^{\mu},y)$

Metric describing the 4+1D space-time

Flat metric describing the reference cylinder

Decomposed in terms of a Fourier serie :

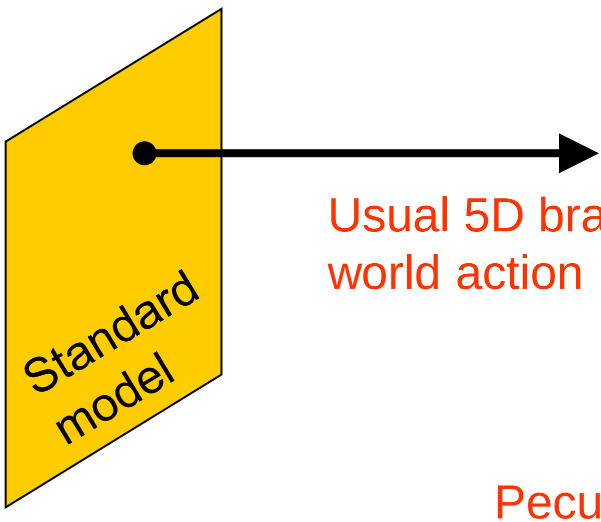
² One massless graviton

² A tower of massive "Kaluza-Klein" graviton

Theories with extra dimensions appear to contain **infinitely many interacting massive** gravitons ?

1.4. The DGP model

Dvali, Gabadadze, Porrati, 2000



Usual 5D brane world action

$$S = M_{(5)}^3 \int d^5x \sqrt{g} \left(\tilde{R} + \cdots \right) + \int_{\text{brane}} d^4x \sqrt{g} \mathcal{L}_{\text{matter}}$$

Peculiar to DGP model

$$+ M_P^2 \int_{\text{brane}} d^4x \sqrt{g} \left(R + \cdots \right)$$

- Brane localized kinetic term for the graviton
- Will generically be induced by quantum corrections

A special hierarchy between $M_{(5)}$ and M_P is required to make the model phenomenologically interesting

DGP model

- <u>Phenomenological interest</u>
  A new way to modify gravity at large distance, with a new type of phenomenology … The first framework where cosmic acceleration was proposed to be linked to a large distance modification of gravity (C.D. 2001; C.D., Dvali, Gababadze 2002)

  (Important to have such models, if only to disentangle what does and does not depend on the large distance dynamics of gravity in what we know about the Universe)

- <u>Theoretical interest</u>
  Consistent (?) non linear massive gravity …

- Still many open questions !

- <u>Intellectual interest</u>
  Lead to many subsequent developments (massive gravity, Galileons, …)

Theories with extra dimensions appear to contain (infinitely many massive gravitons) …

… some with interesting phenomenology related to cosmology

Can one build a consistent theory for a **<u>single</u>** (or finitely many) **<u>massive graviton</u>** ?

2. Generic properties and problems of massive gravity

2.1. Quadratic massive gravity: the Pauli-Fierz theory and the vDVZ discontinuity

Pauli-Fierz action: second order action for a massive spin two

$$\int d^4x \underbrace{\sqrt{g} R_g} + m^2 \int d^4x h_{\mu\nu} h_{\alpha\beta} \left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

second order in $h_{\mu\nu}$ ´ $g_{\mu\nu} - \eta_{\mu\nu}$

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Only Ghost-free (quadratic) action for a massive spin two Pauli, Fierz 1939

(NB: $h_{1\,o}$ is TT: **5 degrees of freedom**)

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vDVZ discontinuity
(van Dam, Veltman;
Zakharov; Iwasaki 1970)

The propagators read

propagator for $m=0$ $\quad D_0^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2p^2} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{2p^2} + \mathcal{O}(p)$

propagator for $m\neq 0$ $\quad D_m^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2(p^2-m^2)} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{3(p^2-m^2)} + \mathcal{O}(p)$

Coupling the graviton with a conserved energy-momentum tensor

$$S_{int} = \int d^4x \, \sqrt{g} \, h_{\mu\nu} T^{\mu\nu}$$

$$h^{\mu\nu} = \int D^{\mu\nu\alpha\beta}(x - x') T_{\alpha\beta}(x') d^4x'$$

The amplitude between two conserved sources T and S is given by

$$\mathcal{A} = \int d^4x \, S^{\mu\nu}(x) h_{\mu\nu}(x)$$

For a massless graviton: $\mathcal{A}_0 = \left(\hat{T}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\hat{T}\right)\hat{S}^{\mu\nu}$

For a massive graviton: $\mathcal{A}_m = \left(\hat{T}_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}\hat{T}\right)\hat{S}^{\mu\nu}$

In Fourier space

e.g. amplitude between two non relativistic sources:

$$\left.\begin{array}{l}\hat{T}^{\mu}_{\nu} \propto \mathrm{diag}(\hat{m}_1, 0, 0, 0) \\ \hat{S}^{\mu}_{\nu} \propto \mathrm{diag}(\hat{m}_2, 0, 0, 0)\end{array}\right\} \quad \mathcal{A} \sim \tfrac{2}{3}\hat{m}_1\hat{m}_2 \quad \text{Instead of} \quad \mathcal{A} \sim \tfrac{1}{2}\hat{m}_1\hat{m}_2$$

Rescaling of Newton constant $G_{\mathrm{Newton}} = \frac{4}{3} G_{(4)}$

defined from Cavendish experiment

appearing in the action

but amplitude between an electromagnetic probe and a non-relativistic source is the same as in the massless case (the only difference between massive and massless case is in the trace part) ⇨ wrong light bending! (factor ¾)

N.B., the PF mass term reads

$$M_P^2 m^2 \int d^4x \, (h_{ij}h_{ij} - 2h_{0i}h_{0i} - h_{ii}h_{jj} + 2h_{ii}h_{00})$$

h_{00} enters linearly both in the kinetic part and the mass term, and is thus a Lagrange multiplier of the theory…

… which equation of motion enables to eliminate one of the a priori 6 dynamical d.o.f. h_{ij}

By contrast the h_{0i} are not Lagrange multipliers

5 propagating d.o.f. in the quadratic PF

$h_{\mu\nu}$ is transverse traceless in vacuum.

2.2. Non linear Pauli-Fierz theory and the « Vainshtein Mechanism »

Can be defined by an action of the form

Isham, Salam, Strathdee, 1971

$$S = \int d^4x \sqrt{-g} \ \left(\frac{M_P^2}{2} R_g + L_g \right) + S_{int}[f, g],$$

Einstein-Hilbert action for the *g* metric

Matter action (coupled to metric *g*)

Interaction term coupling the metric *g* and the <u>non dynamical</u> metric *f*

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The interaction term $S_{int}[f, g]$, is chosen such that

- It is invariant under diffeomorphisms
- It has flat space-time as a vacuum
- When expanded around a flat metric ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $f_{\mu\nu} = \eta_{\mu\nu}$)
  It gives the Pauli-Fierz mass term

- Some working examples

$$S^{(2)}_{int} = -\frac{1}{8} m^2 M_P^2 \int d^4x \sqrt{-f}\, H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau}\right)$$

(Boulware Deser)

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(Arkani-Hamed, Georgi, Schwartz)

with $H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$

- Infinite number of models with similar properties
- Have been investigated in different contexts
  - « f-g, strong, gravity » Isham, Salam, Strathdee 1971
  - « bigravity » Damour, Kogan 2003
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  - de Rham, Gabadadze,Tolley 2010, 2011

~~Generically: a 6th ghost like degree of freedom propagates~~ (Boulware-Deser 1972)

Look for static spherically symmetric solutions with the ansatz (not the most general one)

$$\begin{cases} g_{AB}\,dx^A dx^B = -J(r)dt^2 + K(r)dr^2 + L(r)r^2 d\Omega^2 \\ f_{AB}\,dx^A dx^B = -dt^2 + dr^2 + r^2 d\Omega^2 \end{cases}$$

Gauge transformation

$$\begin{cases} g_{\mu\nu}\,dx^\mu dx^\nu = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2 \\ f_{\mu\nu}\,dx^\mu dx^\nu = -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2 \end{cases}$$

Which can easily be compared to Schwarzschild

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Which can easily be compared to Schwarzschild

Then look for an expansion in
G_N (or in $R_S / G_N M$) of the would-be solution

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2$$

$$\nu(R) = -\frac{R_S}{R}(1 + O(1)^2$$ With $$\epsilon = \frac{R_S}{m^4 R^5}$$

$$\lambda(R) = +\frac{1}{2}\frac{R_S}{R}(1 + O(1)^2$$

Vainshtein 1972
In « some kind »
[Damour et al. 2003]
of non linear PF

Wrong light bending!

This coefficient equals +1
in Schwarzschild solution

Introduces a new length scale R_v in the problem
below which the perturbation theory diverges!

For the sun: bigger than solar system!

with $R_v = (R_S m^{-4})^{1/5}$

So, what is going on at smaller distances?

Vainshtein 1972

There exists an other perturbative expansion at smaller distances, defined around (ordinary) Schwarzschild and reading:

$$\left.\begin{array}{l} \nu(R) = -\frac{R_S}{R}\left(1 + O\left(R^{5/2}/R_v^{5/2}\right)\right) \\ \lambda(R) = +\frac{R_S}{R}\left(1 + O\left(R^{5/2}/R_v^{5/2}\right)\right) \end{array}\right\} \quad \text{with} \quad R_v^{-5/2} = m^2 R_S^{-1/2}$$

- This goes smoothly toward Schwarzschild as *m* goes to zero
- This leads to corrections to Schwarzschild which are non analytic in the Newton constant

The **Vainshtein mechanism** is widely used in various attempts to modify gravity in the IR

- DG
- Ma
- De
- Ca
- Ga
- GI
- k-I

Goo

e.g. in DGP:

Various arguments in favour of a working Vainshtein mechanism,

Including
• some exact cosmological solutions
C.D., Dvali, Gabadadze, Vainshtein ‘02
• Sphericall symmetric solution on the brane
Gabadadze, Iglesias ‘04
• Approximate solutions
Gruzinov ‘01, Tanaka ‘04

… However no definite proof (up to recently) that this is indeed the case !

2.3.The crucial properties (and possible sickness) of massive gravity can all be seen taking its « decoupling limit »

Originally proposed in the analysis of Arkani-Hamed, Georgi and Schwartz (2003) using « Stückelberg » fields …

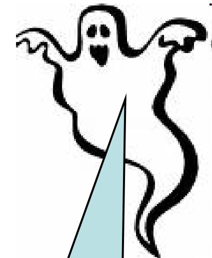
and leads (For a generic theory in the PF universality class) to the cubic action in the scalar sector (helicity 0) of the model

$$\frac{1}{2}\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu} - \frac{1}{M_P}\tilde{\phi}T - \frac{1}{\Lambda^5}\left\{\alpha(\Box\tilde{\phi})^3 + \beta\left(\Box\tilde{\phi}\,\tilde{\phi}_{;\mu\nu}\tilde{\phi}^{;\mu\nu}\right)\right\}$$

« Strong coupling scale »
(hidden cutoff (

• 4. Superluminality !

Problems with causality ?



$$\frac{1}{2}\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu} - \frac{1}{M_P}\tilde{\phi}T - \frac{1}{\Lambda^5}\left\{ ® (\Box Á)^3 + ¯ (\Box Á Á_{;1 o} Á^{;1 o}) \right\}$$

With $\Lambda = (m^4 M_P)^{1/5}$ and ® and ¯ model dependent coefficients

In the de[...] limit, the Vainshtein radius is kept [...] and
o[...] is

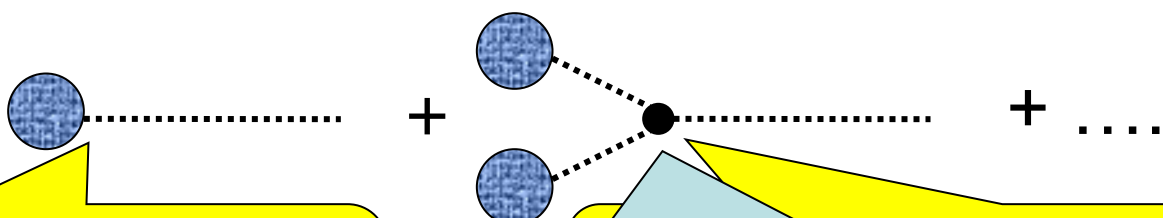
• 3. Low Strong Coupling scale

Can one have a higher cutoff ?

• 2. Boulware Deser ghost

Can one get rid of it ?

E.g. around a heavy source: of mass M



Interaction M/M_P of
t
W

• 1. vDVZ discontinuity

Cured by the Vainshtein mechanism ?

interaction above generates
t $R=R_v= (R_S m^{-4})1/5$

Exemplifies standard properties of attempts to modify GR

- Existence of extra modes/polarizations (needed to be screened)

- Instabilities, causality issues

- UV completion issues

3. Recent progresses and open issues

3.1. The Vainshtein mechanism.

3.2. Getting rid of the Boulware-Deser ghost.

3.3. Strong coupling, UV completion and other issues (back to DGP like models ?) .

3.1. The Vainshtein mechanism

To summarize: 2 regimes

with $\epsilon = \frac{R_S}{m^4 R^5}$

$$\nu(R) = -\frac{R_S}{R}\left(1 + \mathcal{O}(1)\epsilon + \cdots\right)$$

Valid for $R \gg R_V$

with $R_V = \left(R_S m^{-4}\right)^{1/5}$

Standard perturbation theory around flat space

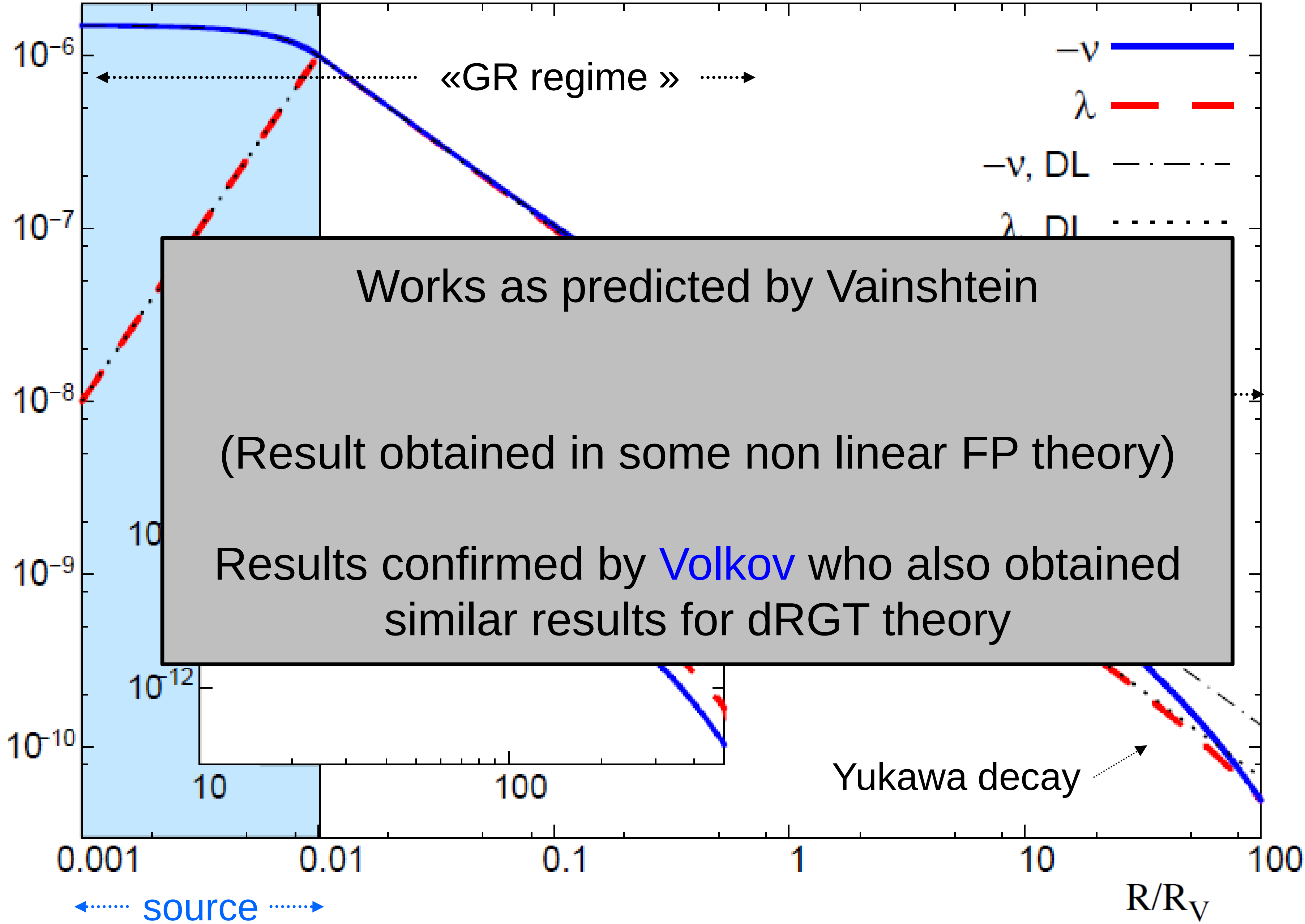
Crucial question: can one join the two regimes in a single existing non singular (asymptotically flat) solution? (Boulware Deser 72)

Expansion around Schwarzschild solution

$$\nu(R) = -\frac{R_S}{R}\left(1 + \mathcal{O}\left(R^{5/2}/R_V^{5/2}\right)\right)$$

Valid for $R \ll R_V$

Numerical investigations: Babichev, C.D., Ziour, 2009, 2010



Works as predicted by Vainshtein

(Result obtained in some non linear FP theory)

Results confirmed by Volkov who also obtained similar results for dRGT theory

Solutions were obtained for very low density objects. We did (and still do) not know what is happening for dense objects (for BHs we now do know) or other more complicated solutions.

(Standard) Vainshtein mechanism does not work for black holes.

C.D.,T. Jacobson, 2012

9 obstructions to have two metrics on the same manifold which do not share a common Killing horizon…

e.g. a the dynamical metric g and the non dynamical flat metric f of non linear Fierz-Pauli theory (applies to the case where metrics are commonly diagonal)

⇒ The (standard) Vainshtein mechanism does not work for Black Holes

End point of gravitational collapse ?

3.2. Getting rid of the Boulware Deser ghost : dRGT theory

de Rahm, Gabadadze; de Rham, Gababadze, Tolley 2010, 2011

Elementary Symmetric polynomials

general massive gravity (in the sense above) devoid
ser ghost is given by the 3 (4 counting ¤)
f theories:

$$= M_g^2 \int d^4x \sqrt{|g|} \left[R(g) - 2m^2 V(S;\beta_n) \right]$$

Where $V(S;\beta_n) = \sum^{3} \beta_n e_n(S)$

$$\begin{cases} e_1(\mathbf{S}) = tr\,\mathbf{S} \\ e_2(\mathbf{S}) = \frac{1}{2}\left((tr\,\mathbf{S})^2 - tr\,\mathbf{S}^2\right) \\ e_3(\mathbf{S}) = \frac{1}{6}\left((tr\,\mathbf{S})^3 - 3\,tr\,\mathbf{S}\,tr\,\mathbf{S}^2 + 2\,tr\,\mathbf{S}^3\right) \end{cases}$$

$S^1{}_o$ is a matrix square root

$$S^\mu{}_\sigma S^\sigma{}_\nu = g^{\mu\sigma} f_{\sigma\nu} = \mathfrak{F}^\mu{}_\nu$$

The absence of ghost is first seen in the decoupling limit

(using the observations of C.D., Rombouts 2005; Creminelli, Nicolis, Papucci, Trincherini 2005)

Which instead of the generic

$$\frac{1}{2}\tilde{\phi}\Box\tilde{\phi} + \frac{1}{\Lambda^5}\left\{\alpha\left(\Box\tilde{\phi}\right)^3 + \beta\left(\Box\tilde{\phi}\right)\tilde{\phi}_{,\mu\nu}\tilde{\phi}^{,\mu\nu}\right\}$$

With $\Lambda = (m^4 M_P)^{1/5}$

Looks like (de Rham, Gabadadze, 2010)

$$\begin{aligned}\frac{1}{2}\tilde{\phi}\Box\tilde{\phi} \quad &+ \quad \frac{1}{\Lambda_3^3}\tilde{\alpha}\left(\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu}\right)\Box\tilde{\phi} \\ &+ \quad \frac{1}{\Lambda_3^6}\tilde{\beta}\left(\tilde{\phi}^{,\mu}\tilde{\phi}_{,\mu}\right)\left(\tilde{\phi}_{,\mu\nu}\tilde{\phi}^{,\mu\nu} - \left(\Box\tilde{\phi}\right)^2\right) \\ &+ \quad \cdots\end{aligned}$$

With $\Lambda_3 = (m^2 M_P)^{1/3}$

The absence of ghost in the full theory has been heavily debated

Gabadadze, de Rham, Tolley;
Alberte, Chamseddine, Mukhanov;
Hassan, Rosen, Kluson, Alexandrov…

Easier to see using vierbeins

Hinterblicher, Rosen arXiv:1203.5783.
C.D., Mourad, Zahariade arXiv:1207.6338, 1208.4493

(Even though the metric and vierbein formulations are not totally equivalent
C.D., Mourad, Zahariade 2012; Bañados, C.D., Pino, 2014)

The mass term

$$S = M_P^2 m^2 \int d^4x \sqrt{-g} \sum_{n=1}^{3} \beta_n e_n(\mathbf{K})$$

with $\begin{cases} g^{\mu\nu} = \eta^{AB} e^\mu_A e^\nu_B \\ f_{\mu\nu} = \eta_{AB} \omega^A_\mu \omega^B_\nu \end{cases}$

Can be written as Linear Combinations of

$$\left\{ \begin{array}{l} M_P^2 m^2 \beta_0 \int \epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge e^{A_4} \\ M_P^2 m^2 \beta_1 \int \epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge \omega^{A_4} \\ M_P^2 m^2 \beta_2 \int \epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge \omega^{A_3} \wedge \omega^{A_4} \\ M_P^2 m^2 \beta_3 \int \epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge \omega^{A_2} \wedge \omega^{A_3} \wedge \omega^{A_4} \end{array} \right.$$

Using the « symmetric vierbein condition »:
(or « Deser- van Nieuwenhuizen
gauge condition »)

$$e^\mu_A \omega_{B\mu} = e^\mu_B \omega_{A\mu}$$

Rich phenomenology (self acceleration in particular) currently under investigation.

3.4. Strong coupling and UV completion

A crucial question for the sake of massive gravity and also …. for the DGP model:

Find a proper UV completion of the model .

For DGP model, ⇨ Yes/ May be ?

Antoniadis, Minasian, Vanhove; Kohlprath, Vanhove; Kiritsis, Tetradis, Tomaras; Corley, Lowe, Ramgoolam.

String theory? ⇨ No/ May be not ?

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi.

For massive gravity ?

(hope: Galileon duality, classicalization ?)

Conclusions

- Massive gravity is a nice arena to explore large distance modifications of gravity.

- A first, possibly consistent (?), non linear theory has recently been proposed (after about 10 years of progresses following the DGP model)…

  … with many things still to be explored (in particular stability issues).