

Testing General Relativity and the Copernican principle

Jean-Philippe UZAN



Interpretation of cosmological data

The interpretation of the dynamics of the universe and its large scale structure relies on the hypothesis that gravity is well described by General Relativity

Galaxy rotation curves

Introduction of *Dark Matter*

Einsteinian interpretation

Most of the time Newtonian interpretation

Acceleration of the cosmic expansion

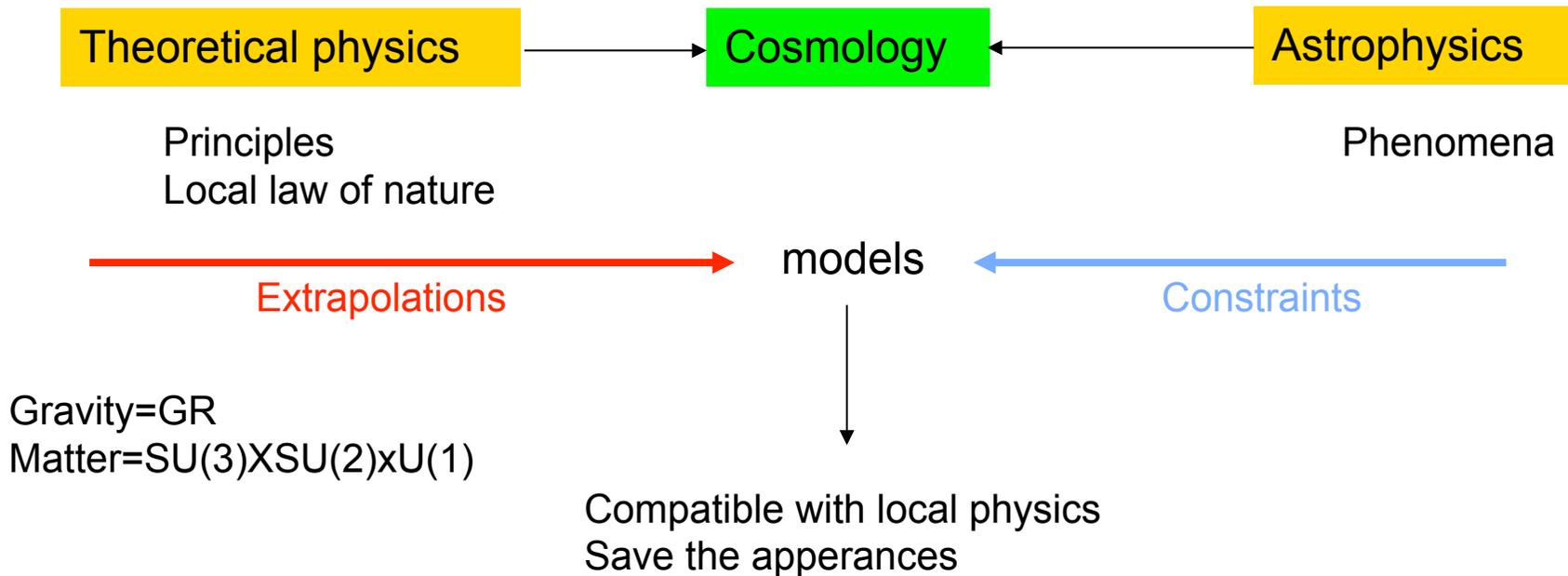
Introduction of *Dark Energy*

Einsteinian interpretation

But more important Friedmanian interpretation

This raises many questions concerning our cosmological model.

Cosmological models



Its construction relies on 4 hypothesis

1. Theory of gravity [General relativity]
 2. Matter [Standard model fields + CDM + Λ]
 3. Symmetry hypothesis [Copernican Principle]
 4. Global structure [Topology of space is trivial]
- New physics with simple cosmological solution
- Standard physics with more involved solution

In agreement with all the data.

Underlying hypothesis

The standard cosmological model lies on 3 hypothesis:

H1- Gravity is well described by general relativity

H2- Copernican Principle

On large scales the universe is homogeneous and isotropic

Consequences:

- 1- The dynamics of the universe reduces to the one of the scale factor
- 2- It is dictated by the Friedmann equations

$$3 \left(H^2 + \frac{K}{a^2} \right) = 8\pi G\rho$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

H3- Ordinary matter (standard model fields)

Consequences:

- 3- On cosmological scales: pressureless + radiation
- 4- The dynamics of the expansion is dictated by

$$\Omega \equiv \frac{8\pi G\rho}{3H^2}$$

$$H^2(z)/H_0^2 = \Omega_m^0(1+z)^3 + \Omega_r^0(1+z)^4 + \Omega_K^0(1+z)^2$$

Implications of the Copernican principle

Independently of any theory (**H1, H3**), the Copernican principle implies that the geometry of the universe reduces to $a(t)$.

Consequences:

$$\bullet \quad 1 + z = \frac{E_{rec}}{E_{em}} \stackrel{H_2}{=} \frac{a_0}{a(t)}$$

$$\bullet \quad a(t) = a_0 \left[1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2(t - t_0)^2 + \dots \right]$$

so that

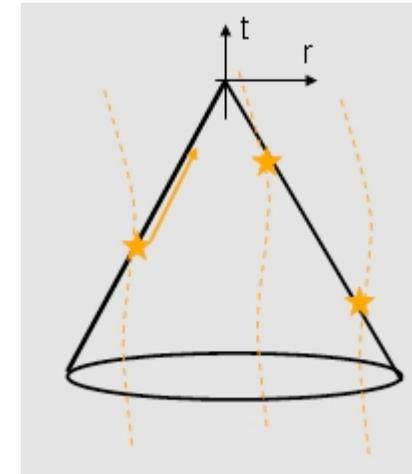
$$H^2(z)/H_0^2 = 1 + (q_0 + 1)z + \mathcal{O}(z^2)$$

- **Hubble diagram** gives
 - H_0 at small z
 - q_0

Supernovae data (1998+) show

$$q_0 < 0$$

The expansion is now **accelerating**



$$q_0 = \Omega_{m0}/2$$

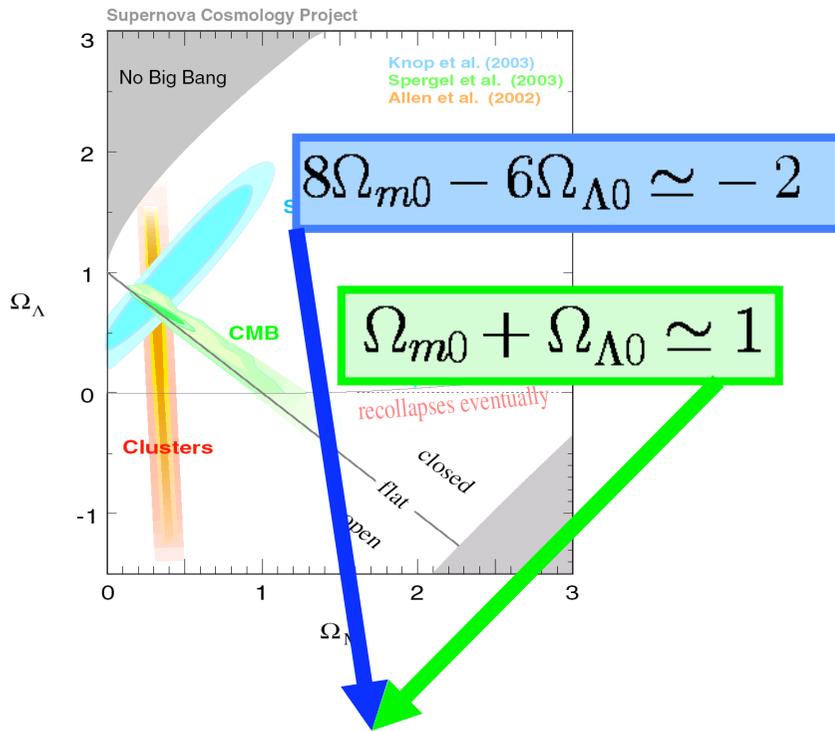
No hypothesis on gravity at this stage.

Λ cdm (reference) model

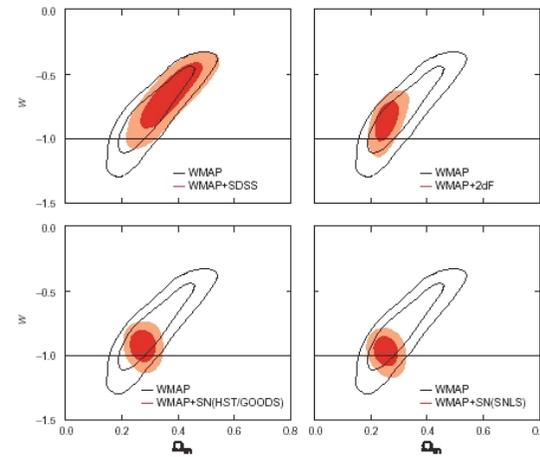
The simplest extension consists in introducing a cosmological constant

- constant energy density
- well defined model and completely predictive

$$\rho_{\Lambda} = \frac{\Lambda}{8\pi G} = -P_{\Lambda}$$



$$\Omega_{m0} \sim 0.3, \quad \Omega_{\Lambda0} \sim 0.7$$



Spergel et al., astro-ph/0603449

$$P_{de} = w\rho_{de}$$

Λ CDM consistent with all current data

Observationally, very good
Phenomenologically, very simple
But: cosmological constant problem

Λ CDM: mater content

$$\Omega_b = 4\%$$

visible

can form structures

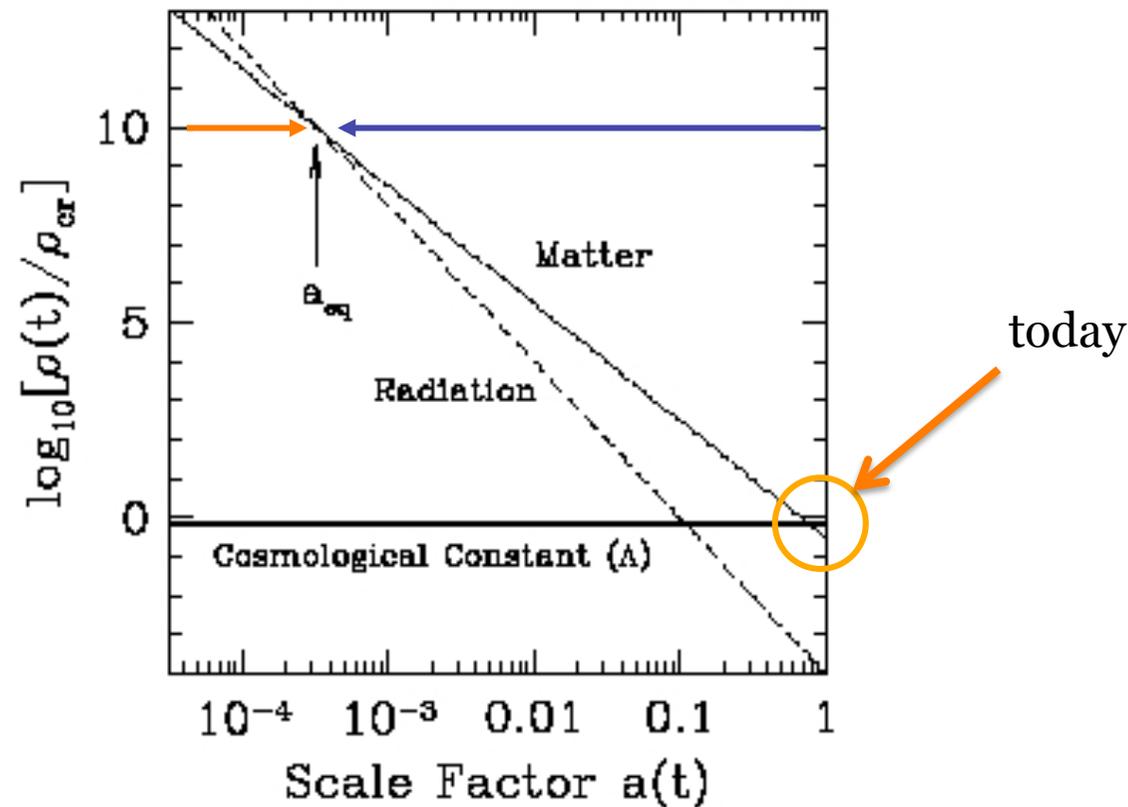
$$\Omega_{\text{cdm}} = 23\%$$

invisible

$$\Omega_\Lambda = 73\%$$

invisible

homogeneous



Λ : problem

Classically

No problem !
New constant in the theory - measured.

Quantumly

Interpretation in terms of vacuum energy

$$\rho_{\Lambda,obs} = \frac{\Lambda}{8\pi G} = H_0^2 M_p^2 = 10^{-47} \text{GeV}^4$$
$$\rho_{\Lambda,th} = M_{\text{fondamental}}^4 > 10^{12} \text{GeV}^4$$

Cosmological constant problem

$$\rho_{\Lambda} > 10^{59} \rho_{\Lambda,obs} !!$$

$$\Lambda_{\text{obs}} = \Lambda_E + \Lambda_Q \ll \Lambda_Q$$

The current interpretation of the cosmological data requires the need for a dark sector with

$$\Omega_r : \Omega_b : \Omega_m : \Omega_\Lambda \sim 10^{-3} : 1 : 5 : 14$$

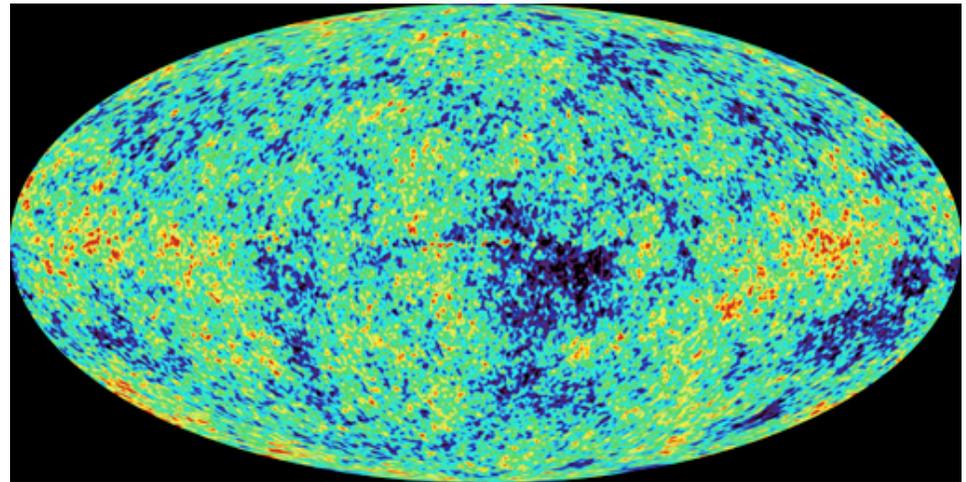
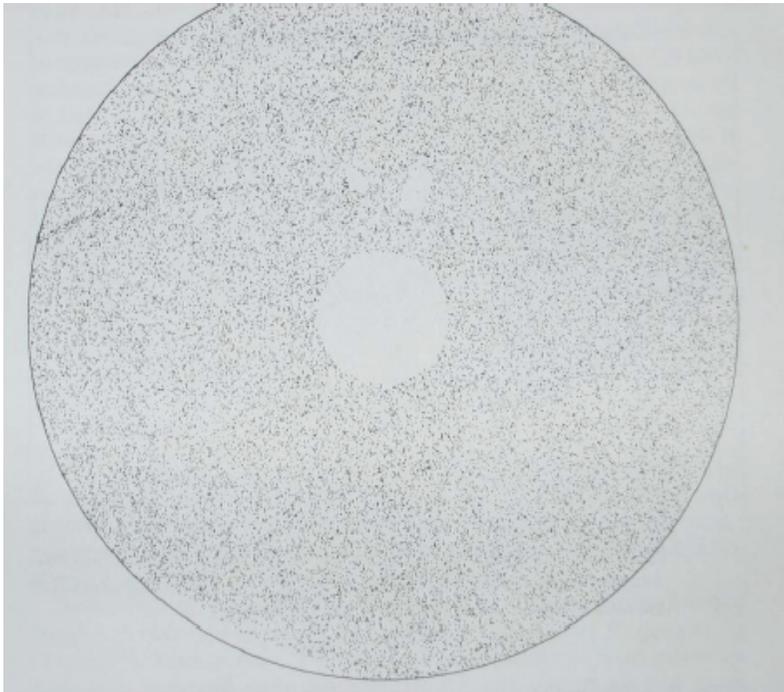
This conclusion relies heavily on our hypothesis.

- Test of the Copernican principle
- New degrees of freedom [Theory]
- Test of general relativity

Testing
the
Copernican Principle

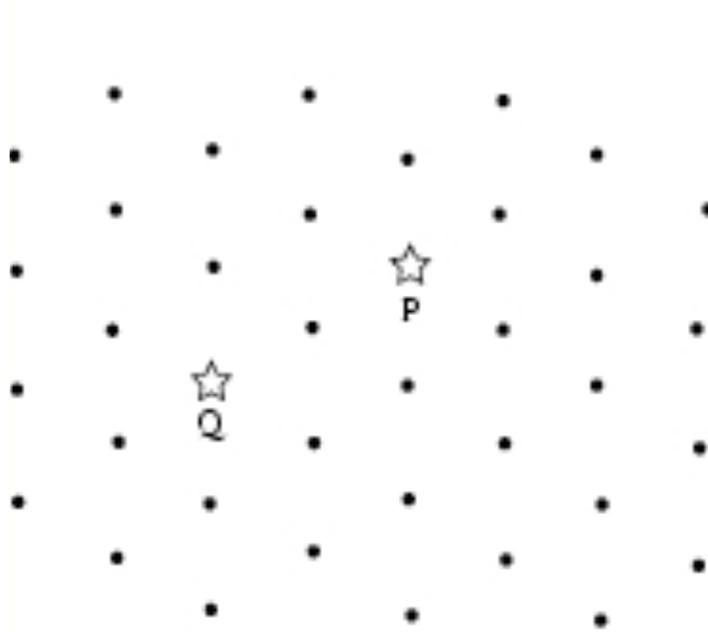
Isotropy

Observationally, the universe seems very isotropic around us.

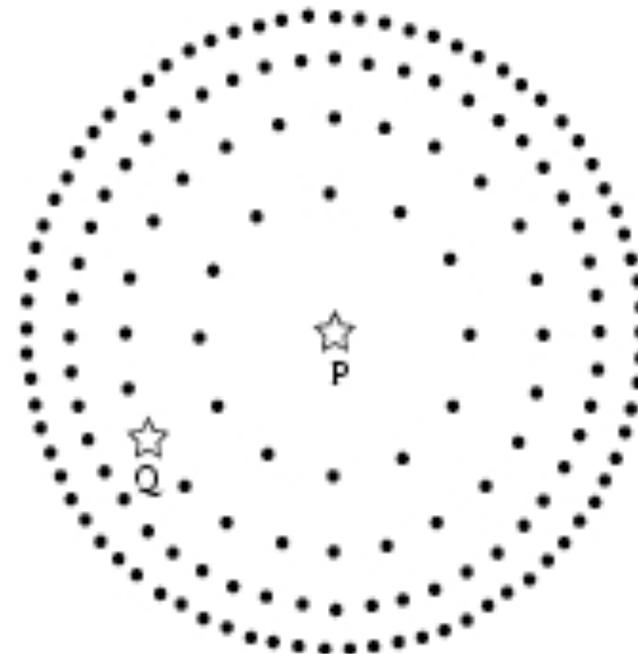


Uniformity principle

Two possibilities to achieve this:



Spatially homogeneous & isotropic



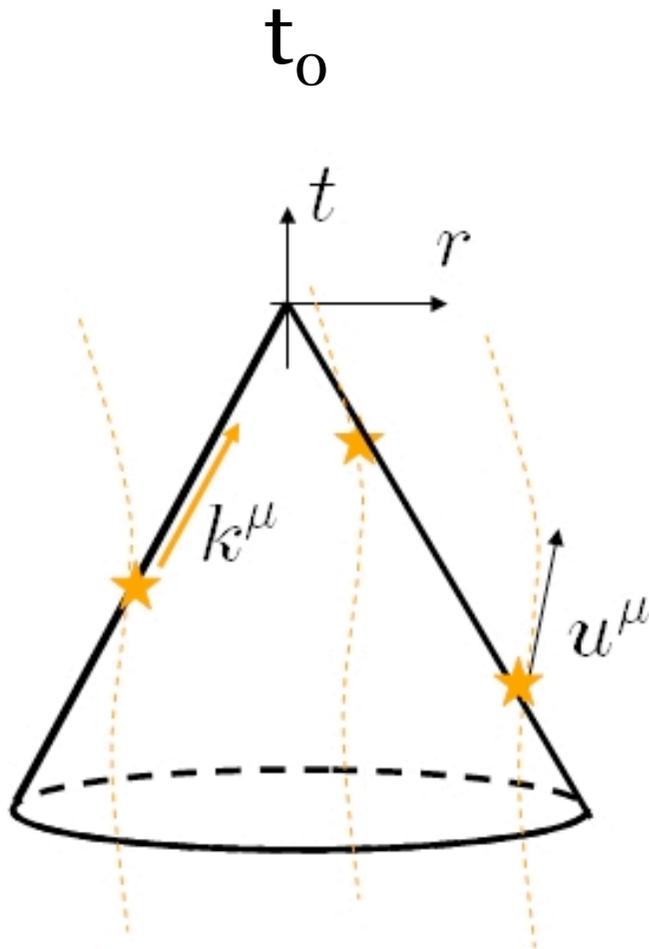
Spherically symmetric
Universe has a center

Copernican Principle: we do not occupy a particular spatial location in the universe

Test of the Copernican principle

Redshift:

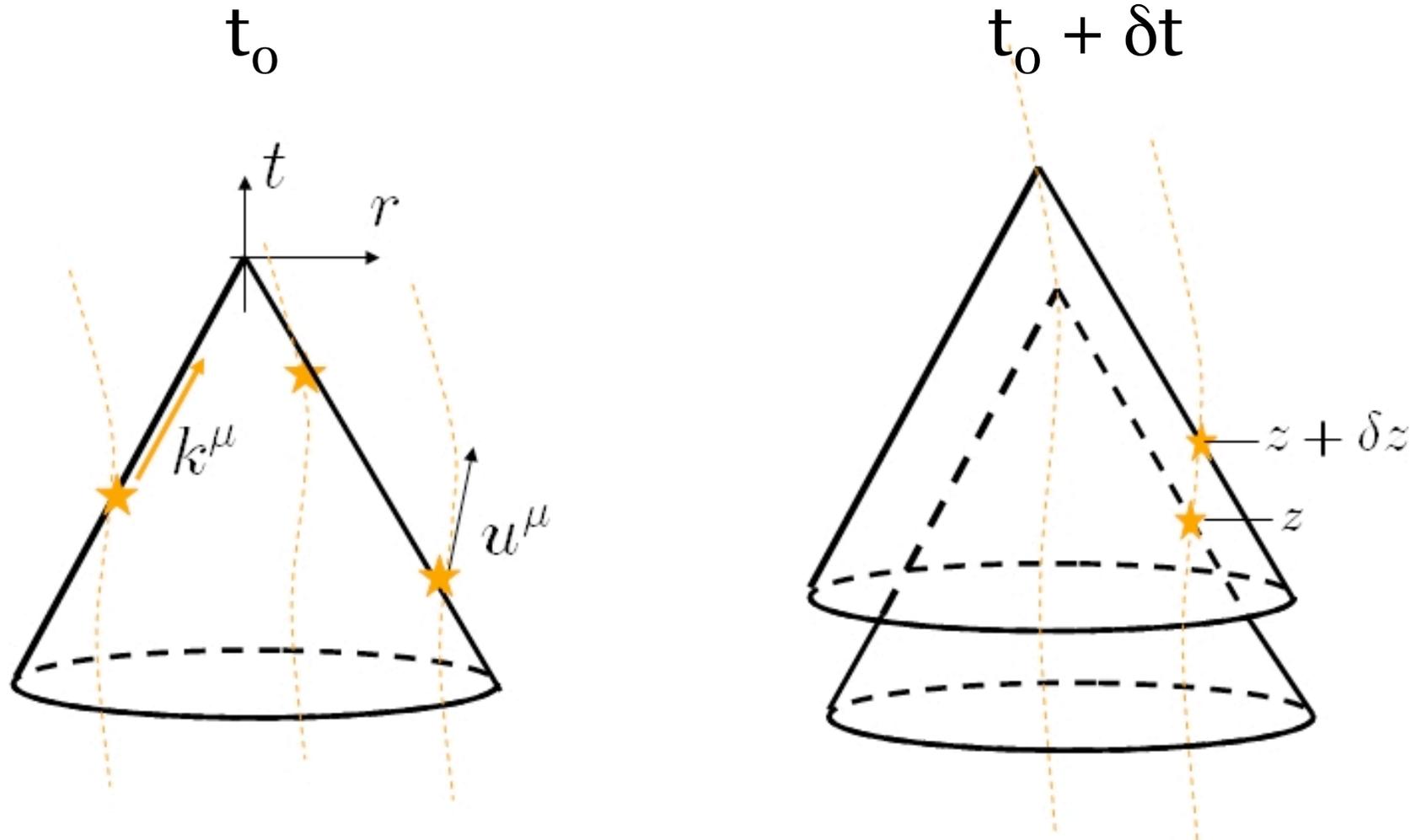
$$1 + z = \frac{\lambda_{\text{rec}}}{\lambda_{\text{em}}} = \frac{a_0}{a}$$



Test of the Copernican principle

Redshift:

$$1 + z = \frac{\lambda_{\text{rec}}}{\lambda_{\text{em}}} = \frac{a_0}{a}$$



Time drift of the redshifts

An interesting observable is the time drift of the redshift

Homogeneous and isotropic universe

$$\dot{z} = H_0(1 + z) - H(z)$$

[Sandage 1962, McVittie 1962]

Typical order of magnitude ($z \sim 4$)

$$\delta z \sim -5 \times 10^{-10} \quad \text{on} \quad \delta t \sim 10 \text{ yr}$$

Measurement of $H(z)$

Inhomogeneous universe

$$\dot{z} = H_0(1 + z) - H(z) + \frac{1}{\sqrt{3}}\sigma(z)$$

[JPU, Clarkson, Ellis, PRL (2008)]

ELT

At a redshift of $z=4$, the typical order of magnitude is

$$\delta z \sim -5 \times 10^{-10} \quad \text{sur} \quad \delta t \sim 10 \text{ ans}$$

Variance

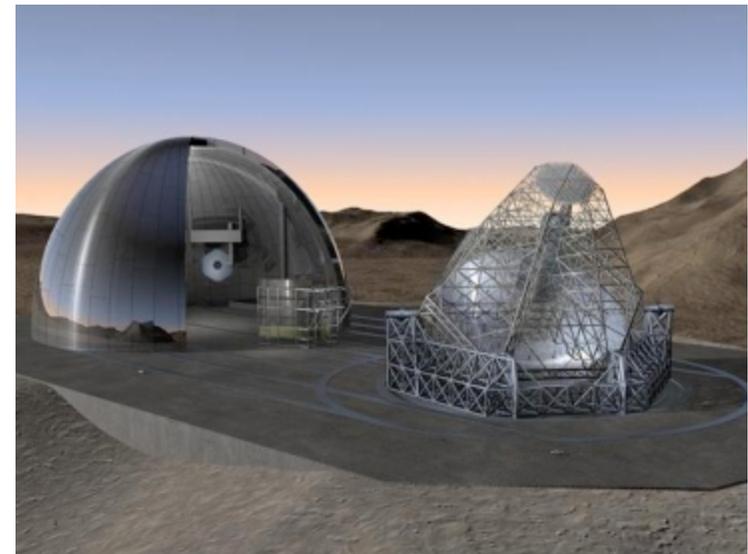
[JPU, Bernardeau, Mellier, PRD (2007)]

Beyond what we can measure today **BUT**

ELT project:

- 60 meters of diameter
- ultrastable high resolution spectrograph (CODEX)
- 25 yrs ?
- 10 yrs of observation !

[see, Pasquini et al. (2005)]



How sensitive can such a test be?

« Popular » universe model: Lemaître-Tolman-Bondi

- spherically symmetric but inhomogeneous spacetime
- i.e. spherical symmetry around one worldline only : *the universe as a center*

$$ds^2 = -dt^2 + \frac{X^2(r, t)}{1 + 2E(r)} dr^2 + R^2(r, t) d\Omega^2$$

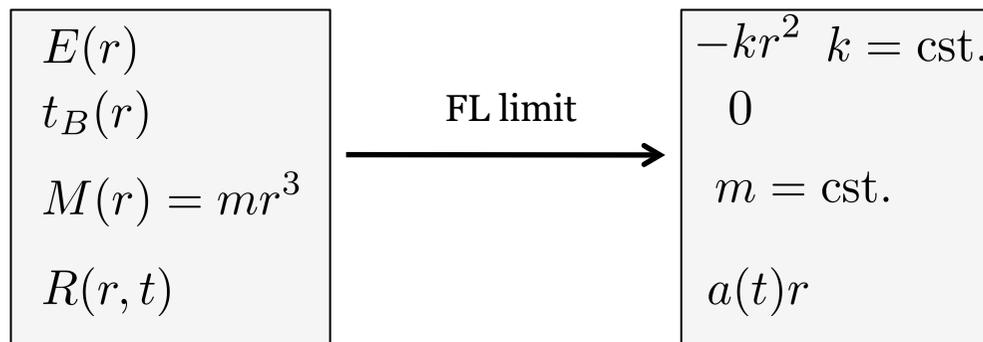
Two expansion rates, a priori different

[for an off-center observer, the universe does not look isotropic]

$$H_{\perp} \equiv \frac{\dot{R}}{R}, \quad H_{\parallel} \equiv \frac{\dot{X}}{X} = \frac{\dot{R}'}{R'}$$

The solution depends on 2 arbitrary functions of r

$$3 - 1 = 2$$



How sensitive can such a test be?

R can be interpreted as the angular diameter distance so that, evaluated on the past light-cone:

$$R[t_*(z), r_*(z)] = D_A(z)$$

This allows to fix one of the free functions IF $D_A(z)$ is known.

There exist a class of LTB models reproducing the FL- $D_A(z)$, i.e. the FL- $D_L(z)$, observation.

Full reconstruction requires an extra set of independant data.

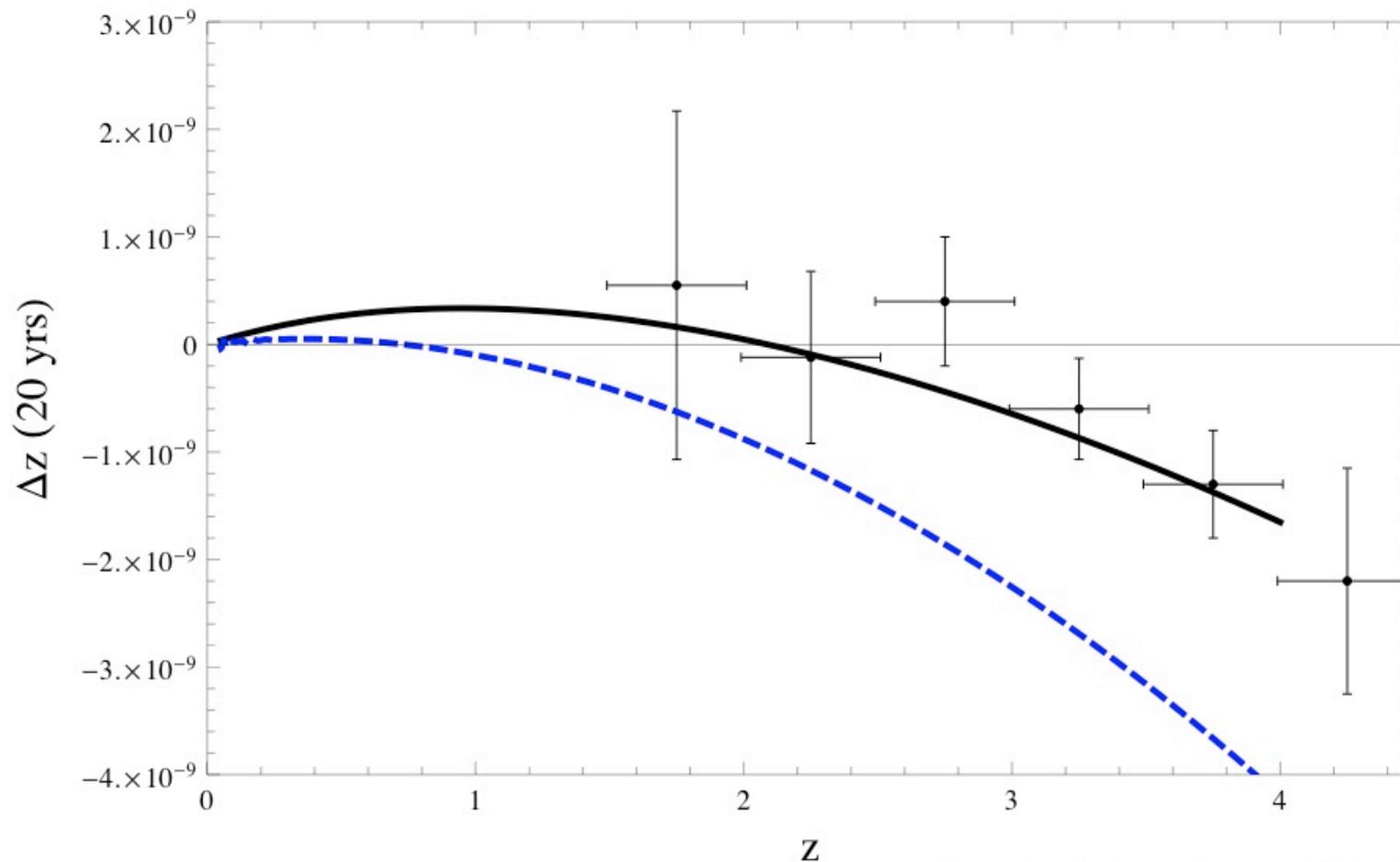
In that class of models, we have $\dot{z} = (1 + z)H_0 - H_{\perp}(z)$.

- $D_A(z)$ and $\delta z(z)$ allow to fully reconstruct the LTB
- Give acces to H_{\parallel} and H_{\perp}

How sensitive can such a test be?

We assume that $8\pi G\rho(z) = 8\pi G\rho_{FL}(z) = 3\Omega_{m0}H_0^2(1+z)^3$

i.e. same $D_L(z)$ & same matter profile BUT NO cosmological constant



[Dunsby,Goheer,Osano,JPU, 1002.2397]

Prospective

- The time drift of the cosmological redshift is potentially a good way to constrain the Copernican principle.

[It gives access to some information outside the light-cone]

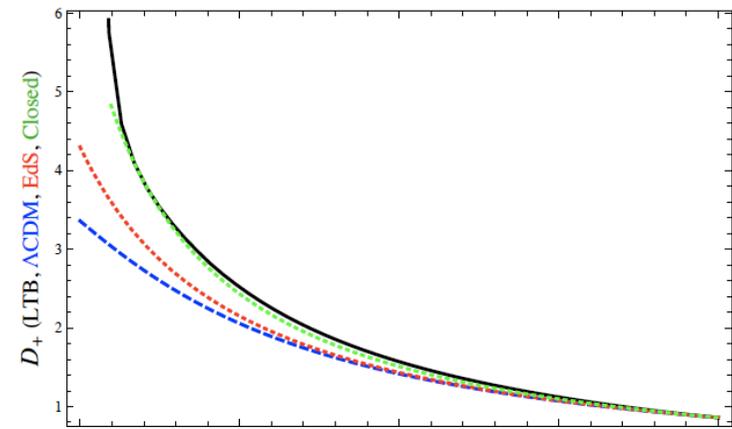
- Other possibilities in the literature:

- CMB polarisation [Goodman (1995), Caldwell & Stebbins (2009), Abramo & JPU (2010)]
- Measurement of the curvature [Clarkson, Bassett & Lu (2008)]

- Recently:

Investigation of the evolution of perturbation shows that the growth rate of the large scale structure is also very sensitive

[depends on the spacetime structure inside the light-cone.]



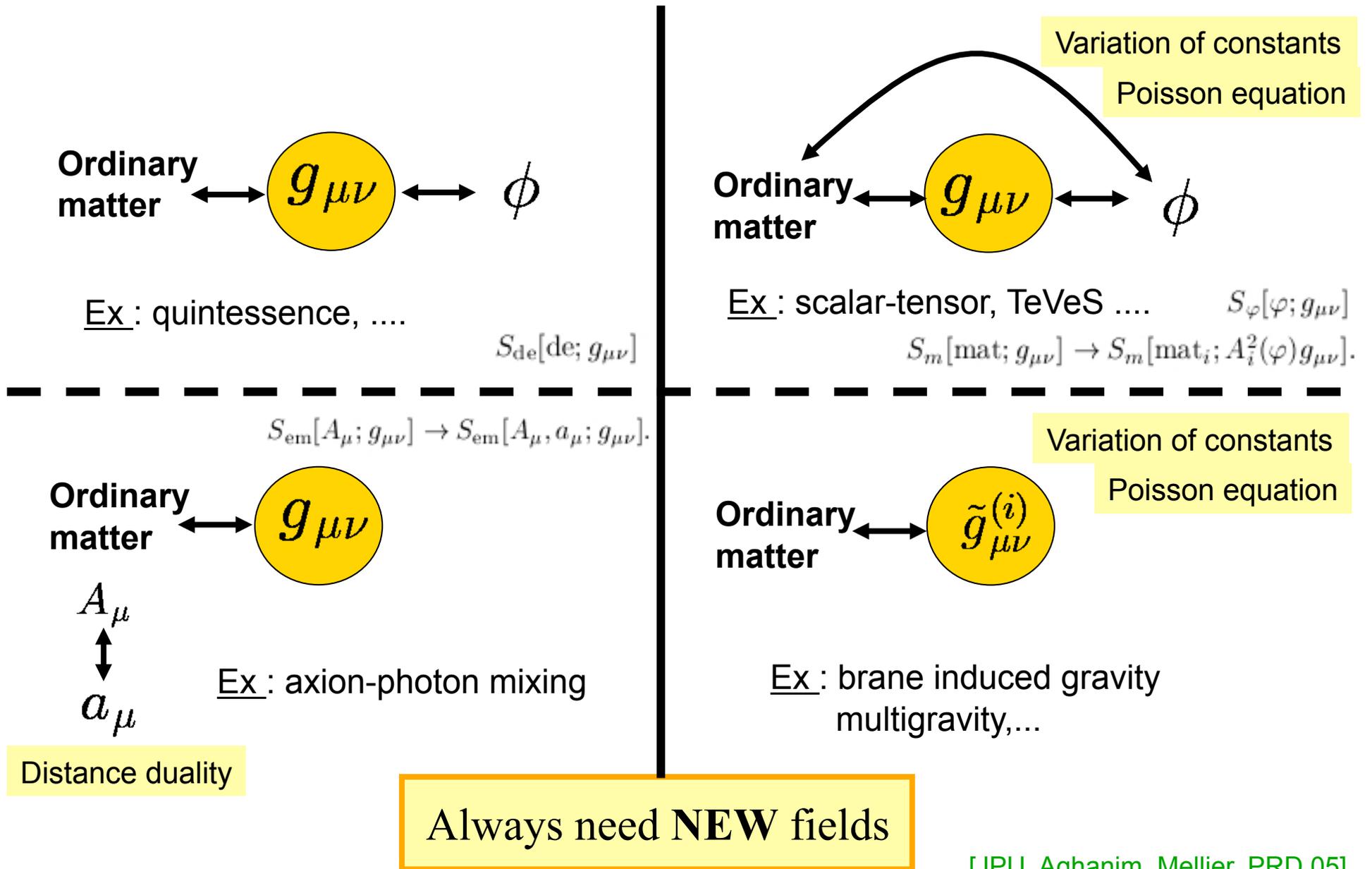
[Dunsby, Goheer, Osano, JPU, 1002.2397]

Introducing
new physical
degrees of freedom

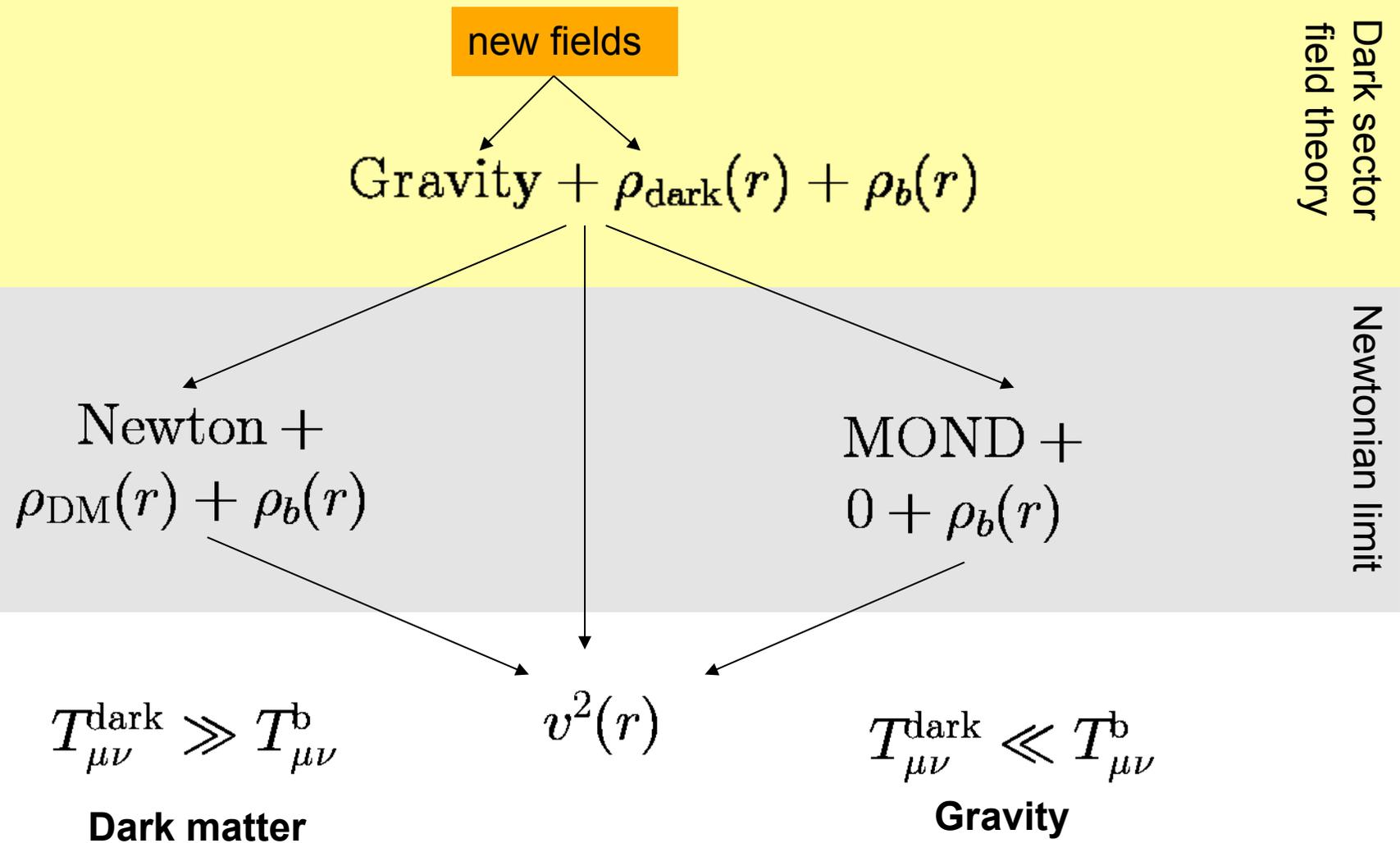
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Some theoretical insight

Universality classes of extensions



New matter vs modification of GR



Extensions

Any of these extensions requires new-degrees of freedom

we always have new matter fields

distinction matter/gravity is a Newtonian notion

MATTER: amount imposed by initial conditions

This matter dominates matter content and triggers acceleration (**dark energy**)

This matter clusters and generates potential wells (**dark matter**)

GRAVITY: ordinary matter « generates » an effective dark matter halo

« induces » an effective dark energy fluid

We would like to determine

the nature of these degrees of freedom

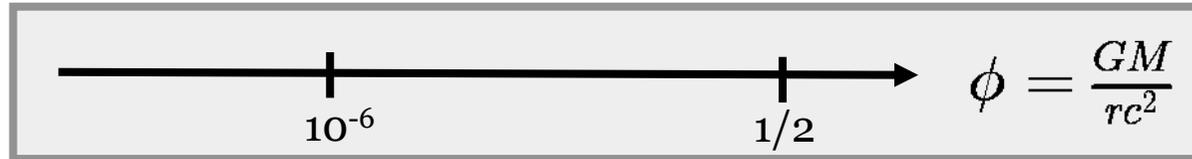
the nature of their couplings

If they are light and if they couple to ordinary matter

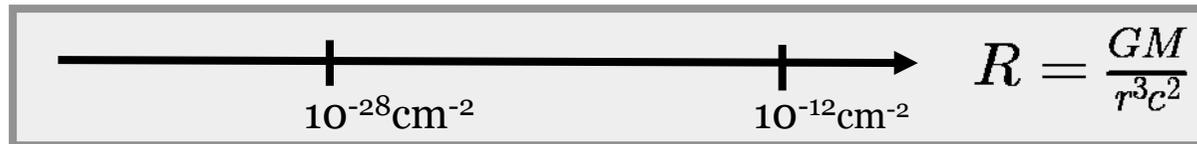
responsible for a long range interaction

In which regime

- Usually, we distinguish *weak-strong field* regimes



- Corrective terms in the action have to be compared to R



Also discussed in distinguishing *large-small distances*

Static configuration:

these limits are related because main dependence is (M,r)
acceleration may also be the best parameter (e.g. rotation curves)

Cosmology:

background level: R increases with z

perturbation: always in weak field

but at late time, we can have high curvature corrections

Parameter space

Tests of general relativity on astrophysical scales are needed

- galaxy rotation curves: low acceleration
- acceleration: low curvature

Solar system:

$$\frac{R}{\phi^3} = \frac{c^4}{G^2 M_{\odot}^2}$$

Cosmology:

$$R = 3H_0^2 \{ \Omega_m (1+z)^3 + 4\Omega_{\Lambda} \}$$

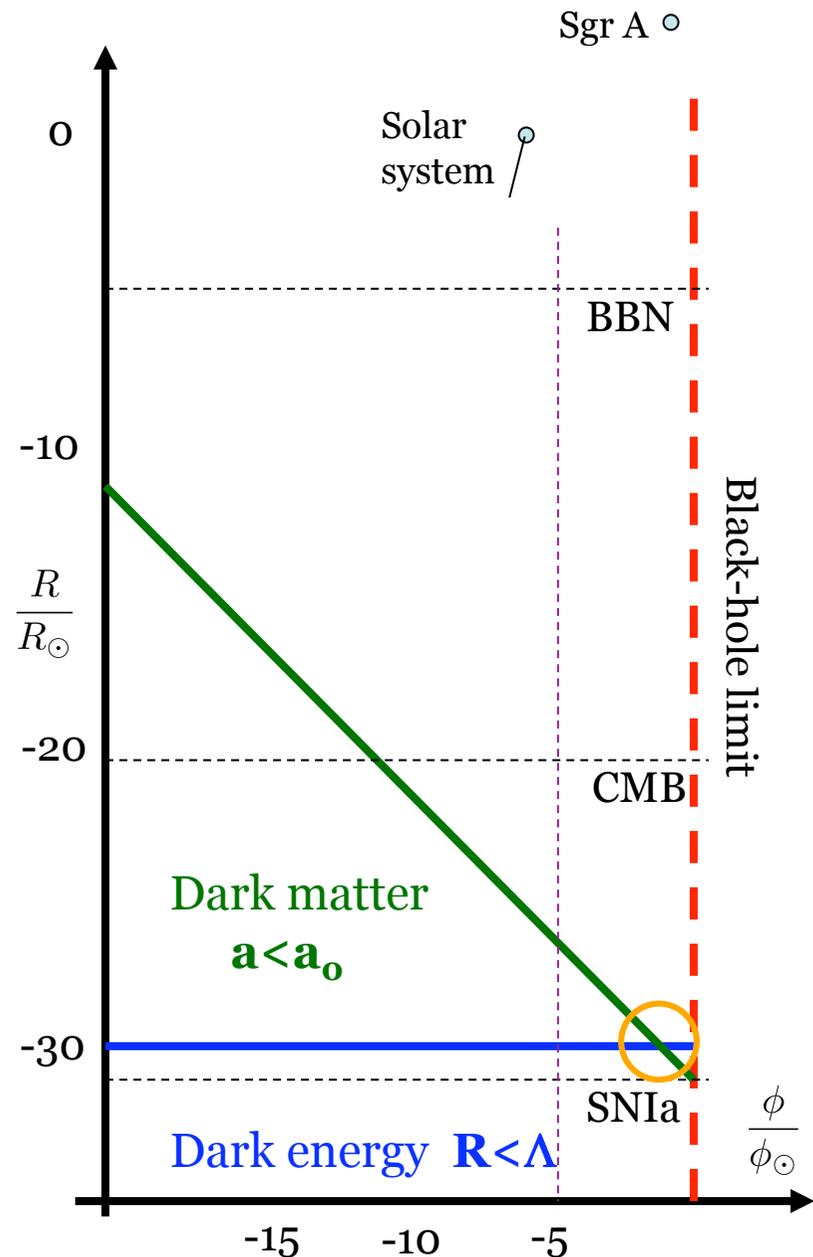
Dark energy:

$$R < R_{\Lambda} = 12H_0^2 \Omega_{\Lambda}$$

Dark matter:

$$a < a_0 \sim 10^{-8} \text{ cm.s}^{-2}$$

$$a^2 = \phi R < a_0^2 \quad [\text{Psaltis, 0806.1531}]$$



Modifying GR

The number of modifications are numerous.

I restrict to field theory.

We can require the following constraints:

- Well defined **mathematically**
 - full Hamiltonian should be bounded by below*
 - no ghost ($E_{kinetic} > 0$)*
 - No tachyon ($m^2 > 0$)*
 - Cauchy problem well-posed*
- In agreement with existing **experimental** data
 - Solar system & binary pulsar tests*
 - Lensing by « dark matter » - rotation curve*
 - Large scale structure – CMB – BBN - ...*
- Not pure fit of the data!

Design

The regimes in which we need to modify GR to explain DE and DM are different.

DM case: *we need a force $\sim 1/r$*

a priori easy:

- consider $V(\varphi) = -2a^2e^{-b\varphi}$ [**Not bounded from below**]
- static configuration: $\Delta\varphi = V'(\varphi)$ and thus $\varphi = (2/b)\ln(abr)$

But:

The constant $(2/b)$ has to be identified with $M^{1/2}$!!

DE case:

[see PRD76 (2007) 124012]

Coincidence problem

ST: 2 free functions that can be determine to reproduce $H(z)$ and $D_+(z)$.

	bgd	bgd + Newt. pert.	bgd + Newt. pert. + Solar syst.
DGP vs quintessence	Y	N	N
DGP vs scalar-tensor	Y	?	N

Example: higher-order gravity...

At quadratic order

$$S_g = \frac{c^3}{16\pi G} \int (R + \alpha C_{\mu\nu\rho\sigma}^2 + \beta R^2 + \gamma GB) \sqrt{-g} d^4x$$

- $GB = R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2$ does not contribute to the field eqs.
- $\alpha C_{\mu\nu\rho\sigma}^2$ theory contains a ghost [Stelle, PRD16 (1977) 953]

$$\frac{1}{p^2 + \alpha p^4} = \frac{1}{p^2} \ominus \frac{1}{p^2 + \alpha^{-1}}$$

massless graviton

massive degrees of freedom with $m^2 = 1/\alpha$
carries negative energy
 $\alpha < 0$: it is also a tachyon.

- βR^2 equivalent to positive energy massive scalar d.o.f

...and beyond

These considerations can be extended to $f(R, R_{\mu\nu}, R_{\mu\nu\alpha\beta})$

[Hindawi et al., PRD**53** (1996) 5597]

Generically contains massive spin-2 ghosts but for $f(R)$

These models involve generically higher-order terms of the variables.

the Hamiltonian is then generically non-bounded by below

[Ostrogradsky, 1850]

[Woodard, 0601672]

Argument does not apply for an infinite number of derivative

non-local theories may avoid these arguments

Only allowed models of this class are $f(R)$.

Scalar-tensor theories

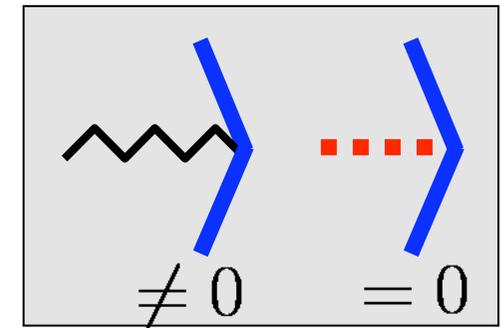
$$S = \frac{c^3}{16\pi G} \int \sqrt{-g} \{ R - 2(\partial_\mu \phi)^2 - V(\phi) \} + S_m \{ \text{matter}, \tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} \}$$

Maxwell electromagnetism is conformally invariant in $d=4$

$$\begin{aligned} S_{em} &= \frac{1}{4} \int \sqrt{-\tilde{g}} \tilde{g}^{ab} \tilde{g}^{cd} F_{ac} F_{bd} d^d x \\ &= \frac{1}{4} \int \sqrt{-g} g^{ab} g^{cd} F_{ac} F_{bd} A^{d-4}(\phi) d^d x \end{aligned}$$

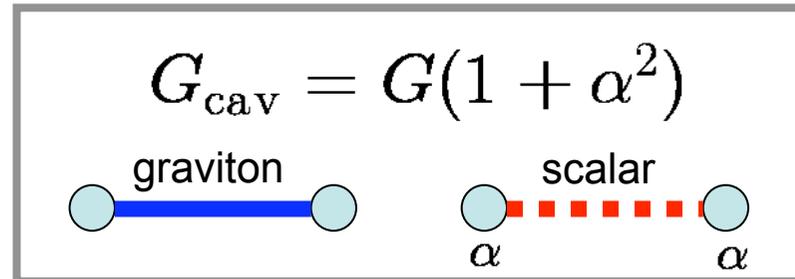
Light deflection is given as in GR

$$\delta\theta = \frac{4GM}{bc^2}$$



What is the difference?

The difference with GR comes from the fact that massive matter feels the scalar field



$$\alpha = d \ln A / d\phi$$

Motion of massive bodies determines $G_{\text{cav}}M$ **not** GM .

Thus, in terms of observable quantities, light deflection is given by

$$\delta\theta = \frac{4G_{\text{N}}M}{(1+\alpha^2)bc^2} \leq \frac{4GM}{bc^2}$$

which means

$$M_{\text{lens}} \leq M_{\text{rot}}$$

Cosmological features of ST theories

Close to GR today

assume light scalar field

Can be attracted toward GR during the cosmological evolution.

[Damour, Nordtvedt]

Dilaton can also be a quintessence field

[JPU, PRD 1999]

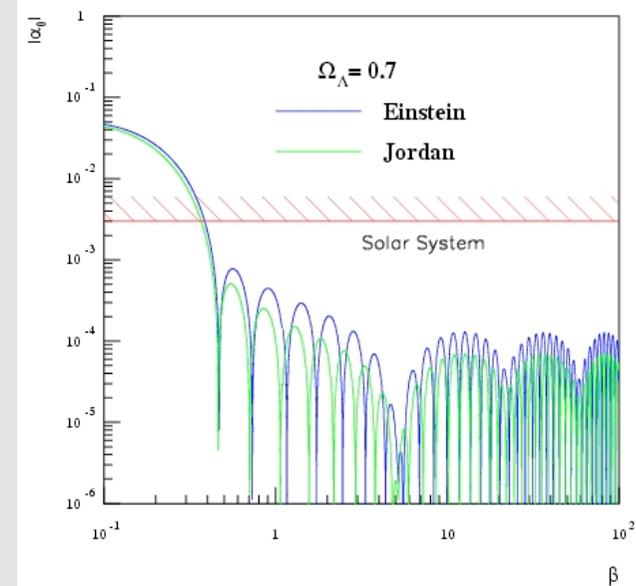
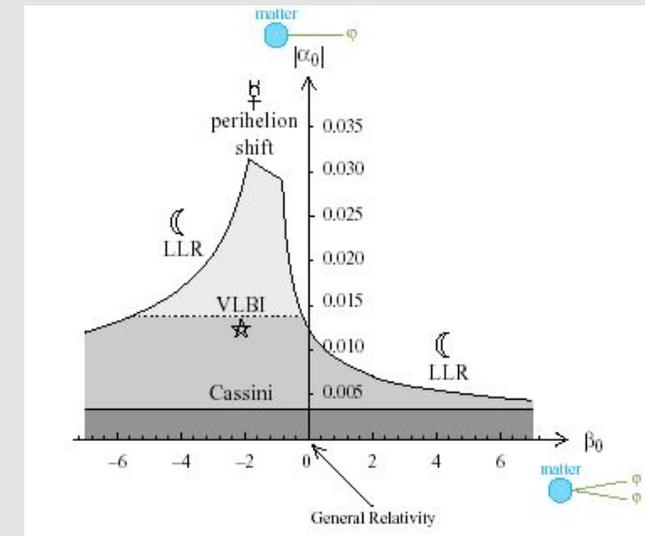
Equation of state today

$$3\Omega_{de0}(w_0 + 1) \simeq 2(1 - \beta_0)\phi_0'^2 - 2\alpha_0\phi_0''$$

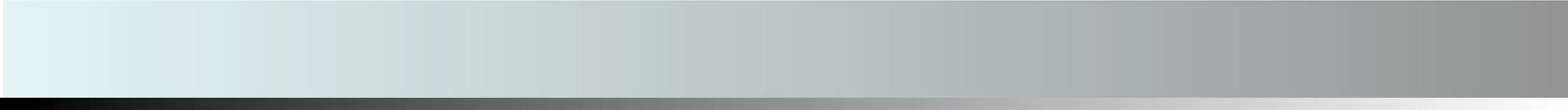
[Martin, Schmid, JPU, 0510208]

Cosmological predictions computable
(BBN, CMB, WL,...)

[Schimd et al., 2005; Riazuelo JPU, 2000,
Coc et al., 2005]



[Coc et al, 0601299]



Astrophysical tests
of
General Relativity

Models!

Λ

Quintessence

TeV*S*

Multigravity

K-essence

DGP

Chameleon

Extended quintessence

Tachyon

Quintom

Cardassian

$f(R)$

Chaplygin gaz

AWE

Chaplygin gaz

Cosmon

Two approaches

TESTING MODELS

- *too numerous*
- *contain the cosmological constant as a CONTINUOUS limit!*

TESTING THE HYPOTHESIS

- Negative : increase the domain of validity of the theory and thus the credence in our cosmological model
- Positive: class of models that enjoy this particular **NEEDED** deviation

WHAT TO TEST

- Copernican principle (already discussed)
- General relativity
- Other [topology, Maxwell,...]

General relativity in a nutshell

Equivalence principle

- Universality of free fall
- Local Lorentz invariance
- Local position invariance

Dynamics

$$S_{grav} = \frac{c^3}{16\pi G} \int \sqrt{-g_*} R_* d^4x$$

Relativity

$$g_{\mu\nu} = g_{\mu\nu}^*$$

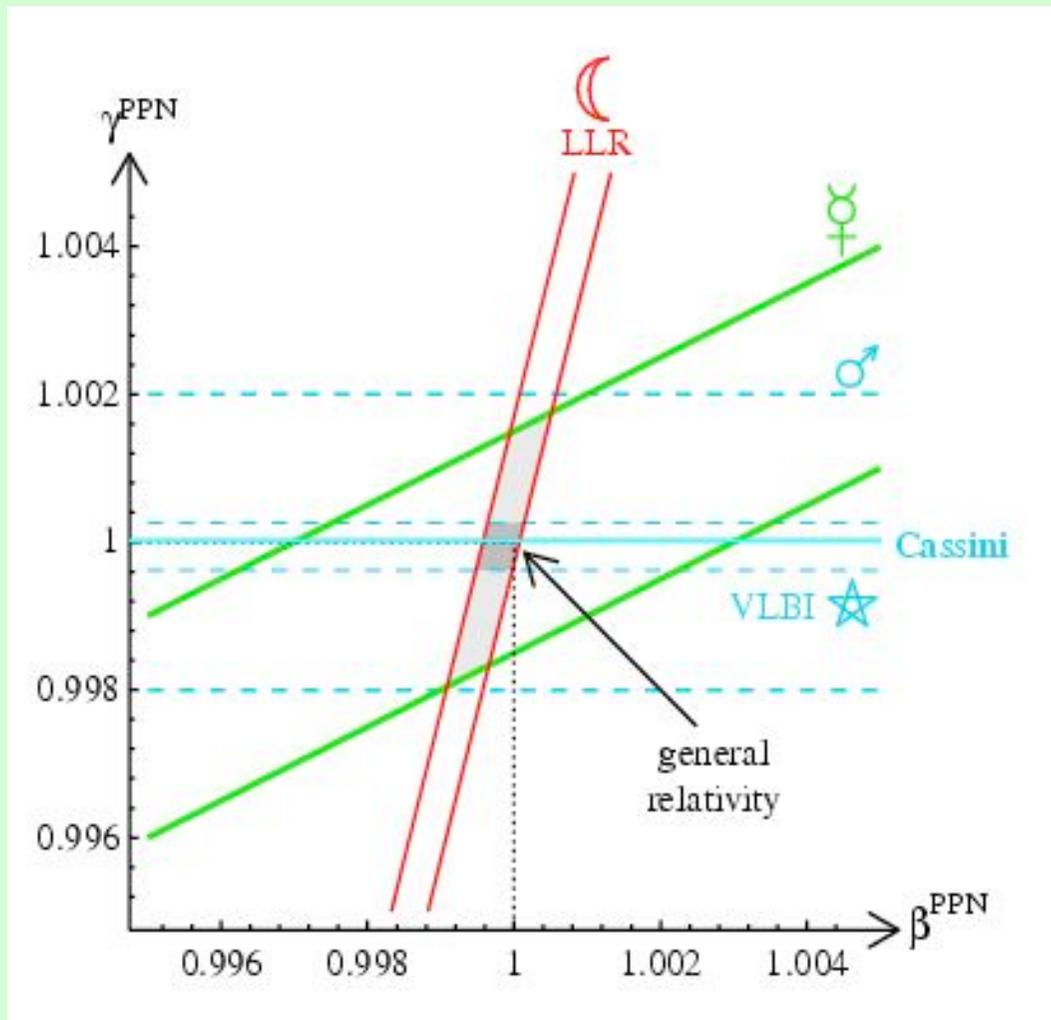
RelativitField equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$S_{matter}(\psi, g_{\mu\nu})$$

General relativity: validity

Courtesy of G. Esposito-Farèse

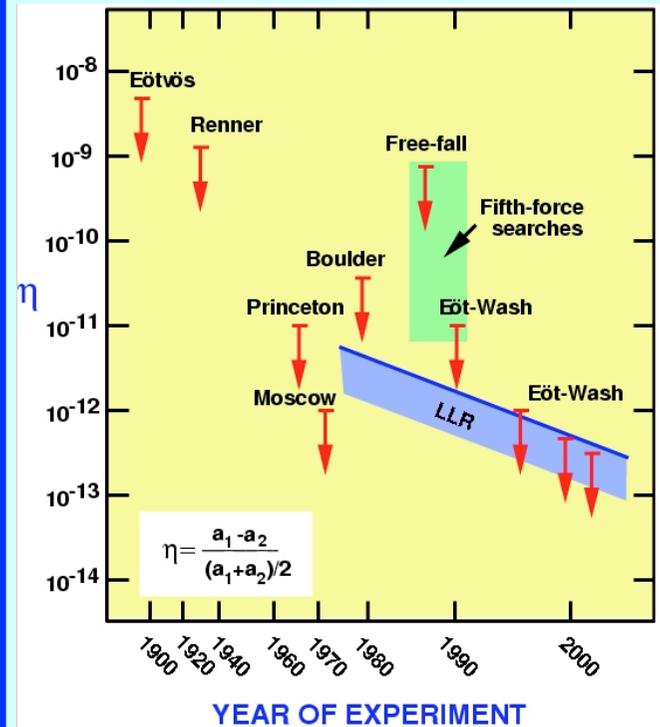


$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

$$|2\gamma - \beta - 1| < 3 \times 10^{-3}$$

Universality of free fall

$$\eta = 2 \frac{|a_1 - a_2|}{|a_1 + a_2|}$$

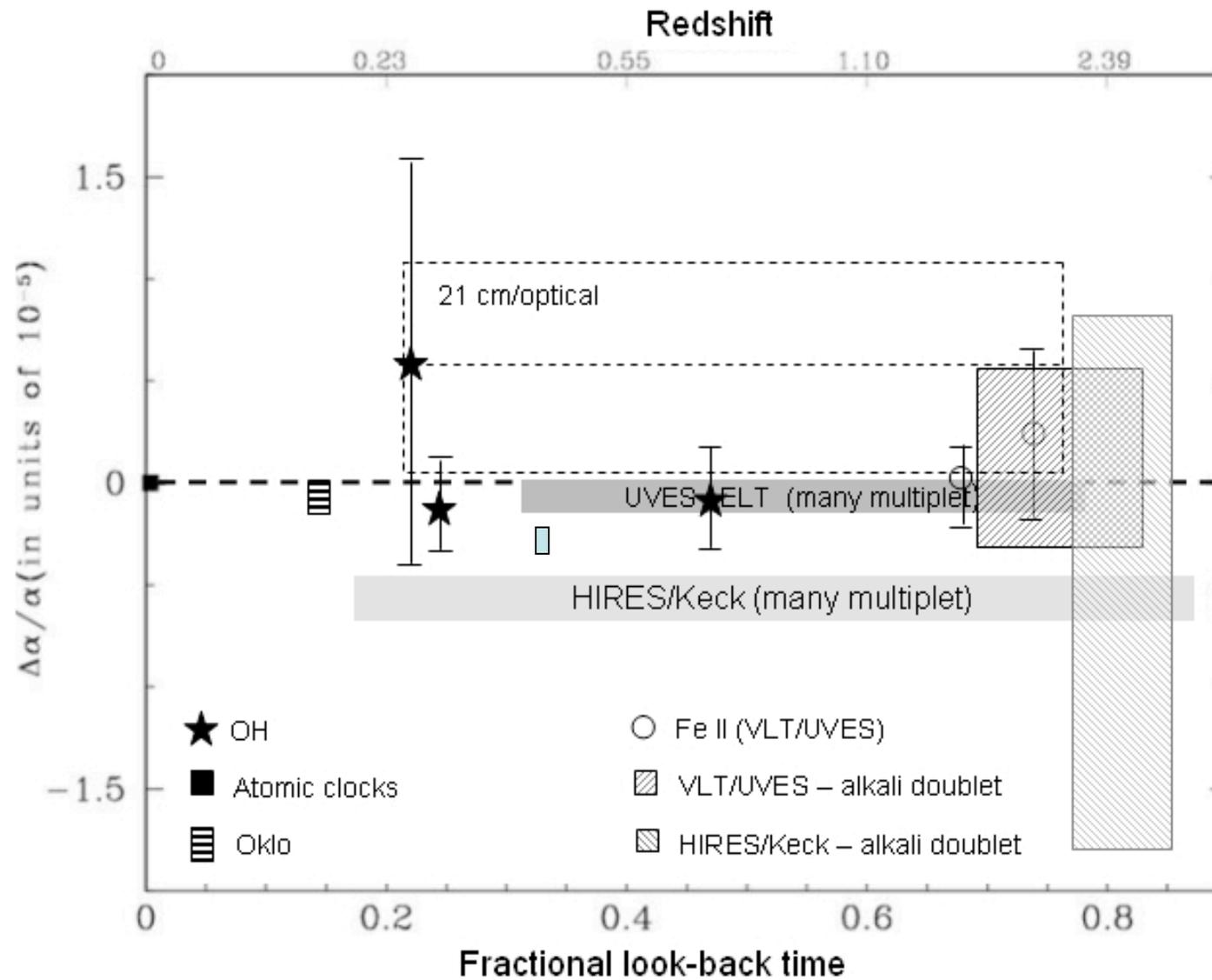


C. Will, gr-qc/0510072

«Constancy» of fundamental constants

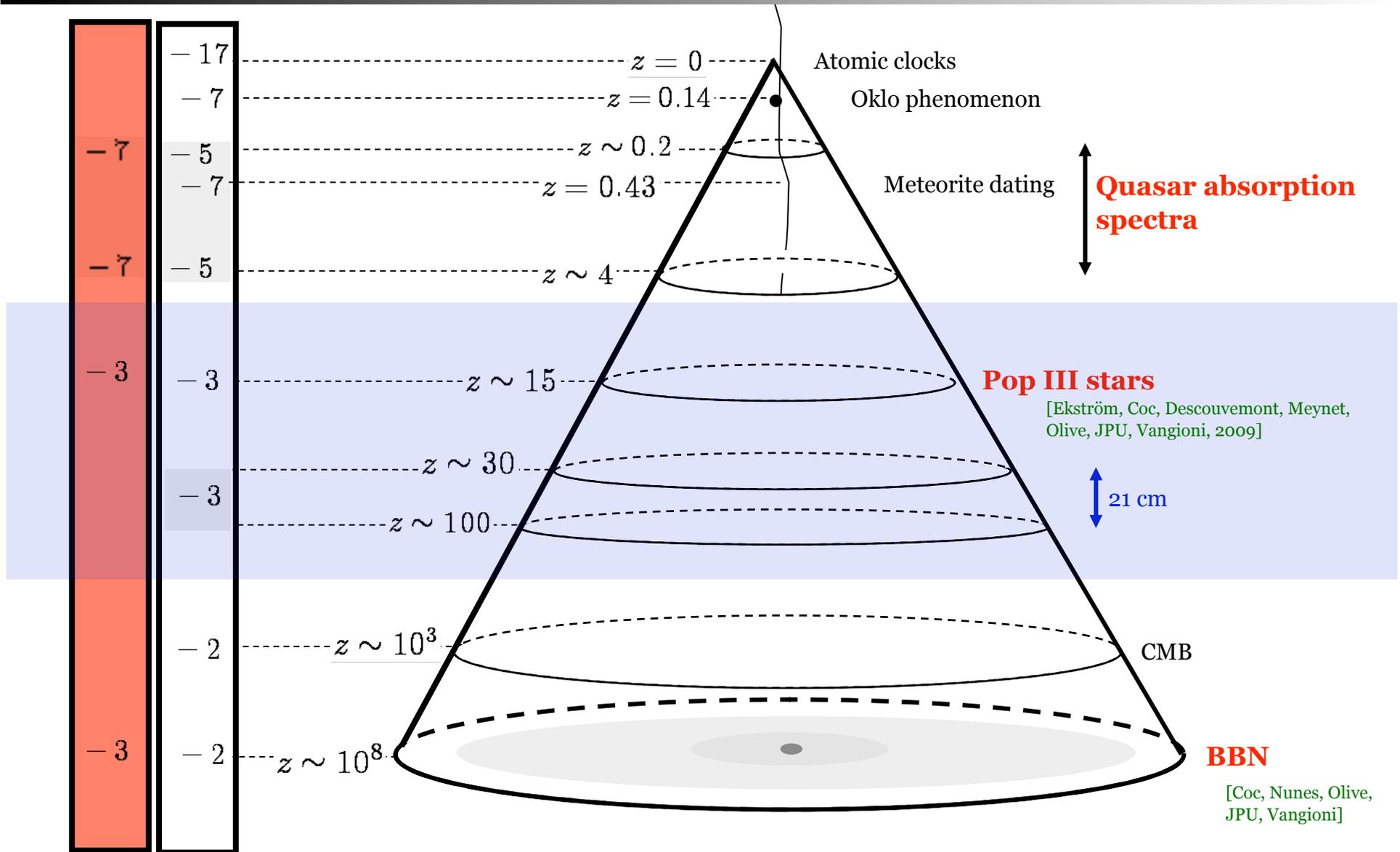
JPU, RMP (2003)

Constraints

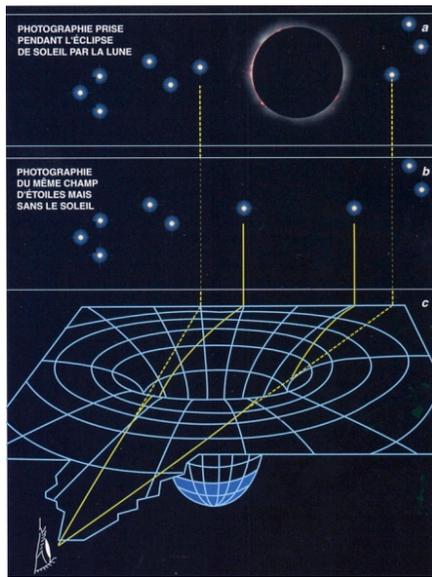


JPU, RMP (2003);
arXiv:09XX.XXXX

Future evolution

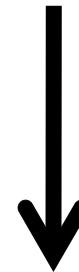


Testing relativity



Light deflection

$$\Delta\theta_{\text{GR}} = \frac{4GM_{\odot}}{bc^2}$$



2 measurements of M

$$GM_{\odot}^{\text{lens}} = \frac{bc^2 \Delta\theta}{4}$$

$$GM_{\odot}^{\text{dyn}} = \frac{2\pi a^3}{P}$$

Dynamics

$$\frac{P}{2\pi} = \frac{a^3}{GM_{\odot}}$$



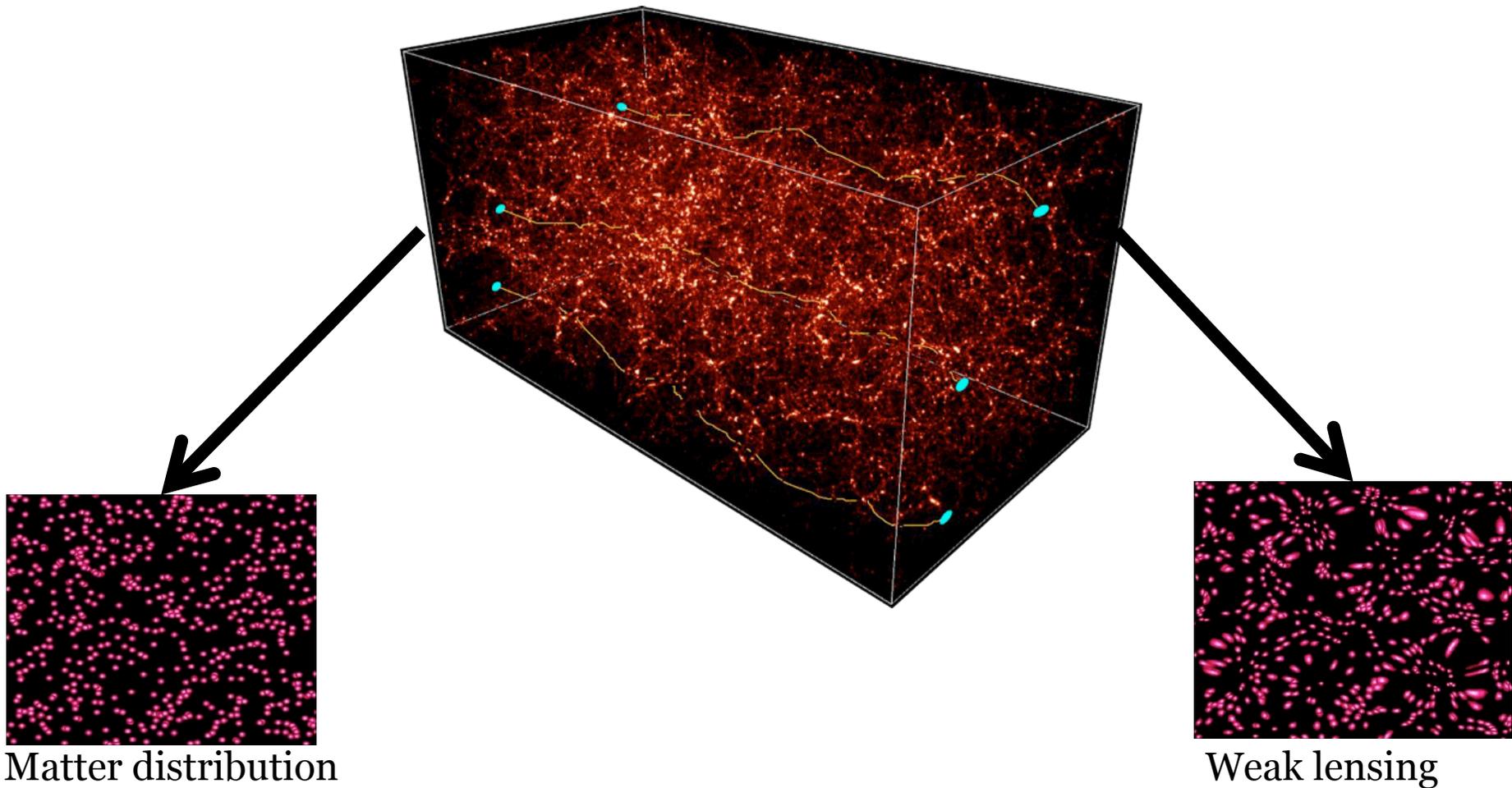
Have to agree if GR is a good description of gravity.

Testing GR on large scales

One needs at least **TWO** independent observables

Large scale structure

[Uzan, Bernardeau (2001)]



Structure in Λ CDM

Restricting to low- z and sub-Hubble regime

$$ds^2 = a^2(\eta)[- (1 + 2\Phi)d\eta^2 + (1 - 2\Psi)\gamma_{ij}dx^i dx^j]$$

Background

$$H^2/H_0^2 = \Omega_m^0(1+z)^3 + (1 - \Omega_m^0 - \Omega_\Lambda^0)(1+z)^2 + \Omega_\Lambda^0$$

Sub-Hubble perturbations

$$\Phi = \Psi$$

$$\Delta\Psi = 4\pi G\rho a^2\delta$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi$$

$$\theta \propto -f\delta$$

$$f \propto \Omega_{\text{mat}}^{0.6}$$

This implies the existence of **rigidities** between different quantities

Original idea

On sub-Hubble scales, in weak field
(typical regime for the large scale structure)

$$\Delta\Phi = 4\pi G\rho a^2\delta$$

Weak lensing

$$\delta\theta = \frac{2}{c^2} \int \nabla_{\perp} \Phi \, d\lambda$$

$$\langle \Phi(\theta) \Phi(\theta + n) \rangle$$

Distribution of the gravitational
potential

Galaxy catalogues

$$n_{gal}(\mathbf{x})$$

$$\xi(r) = \langle \delta(\mathbf{x}) \delta(\mathbf{x} + \mathbf{r}) \rangle$$

Distribution of the matter

Compatible?

[JPU, Bernardeau (2001)]

Example of some rigidity

In the linear regime, the growth of density perturbation is then dictated by

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_{\text{mat}}\delta = 0$$

This implies a *rigidity* between the growth rate and the expansion history

Bertschinger, astro-ph/0604485,
JPU, astro-ph/0605313

It can be considered as an equation for $H(a)$

Chiba & Takahashi, astro-ph/0703347

$$(H^2)' + 2 \left(\frac{3}{a} + \frac{\delta''}{\delta'} \right) H^2 = 3 \frac{\Omega_0 H_0^2 \delta}{a^5 \delta'}$$

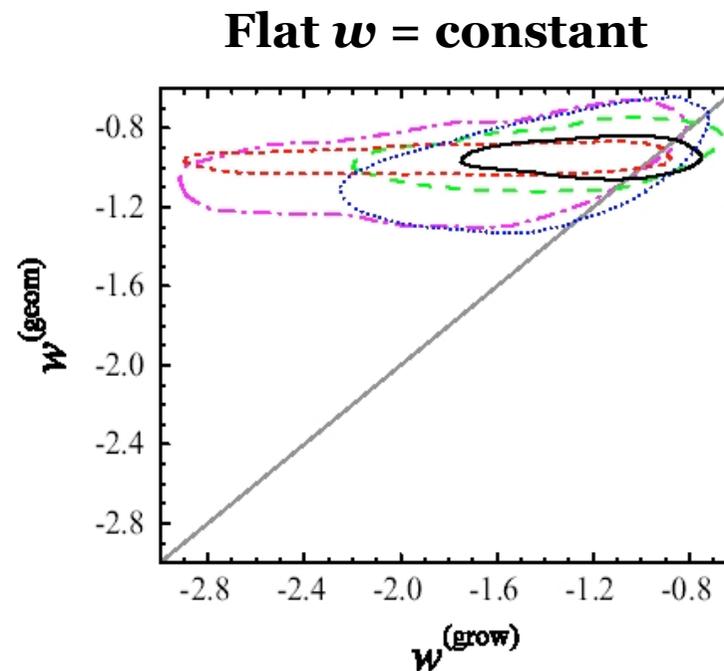
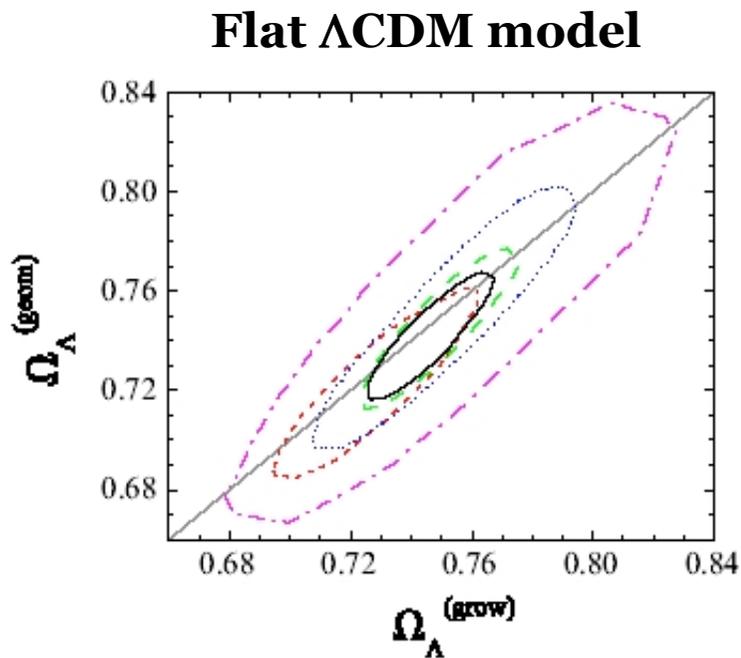
$$\frac{H^2}{H_0^2} = 3\Omega_{m0} \frac{(1+z)^2}{\delta'(z)^2} \int_z \frac{\delta}{1+z} (-\delta') dz$$

$H(a)$ from the background (geometry) and growth of perturbation have to agree.

Growth factor: example

SNLS – WL from 75 deg² CTIO – 2dfGRS – SDSS (luminous red gal)
CMB (WMAP/ACBAR/BOOMERanG/CBI)

Wang *et al.*, arViv:0705.0165



Consistency check of any DE model within GR with non clustering DE
Assume Friedmannian symmetries! (see e.g. [Dunsby, Goheer, Osano, JPU, 2010](#))

To go beyond we need a parameterization of the possible deviations

Post- Λ CDM

Restricting to low- z and sub-Hubble regime

$$ds^2 = a^2(\eta)[- (1 + 2\Phi)d\eta^2 + (1 - 2\Psi)\gamma_{ij}dx^i dx^j]$$

Background

$$H^2/H_0^2 = \Omega_m^0(1+z)^3 + (1 - \Omega_m^0 - \Omega_\Lambda^0)(1+z)^2 + \Omega_{de}(z)$$

Sub-Hubble perturbations

$$\Delta(\Phi - \Psi) = \pi_{de}$$

$$-k^2\Phi = 4\pi G_N F(k, H) \rho a^2 \delta + \Delta_{de}$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi + S_{de}$$

Λ CDM

$$(F, \pi_{de}, \Delta_{de}, S_{de}) = (1, 0, 0, 0)$$

[JPU, astro-ph/0605313;
arXiv:0908.2243]

Data and tests

DATA

Weak lensing

Galaxy map

Velocity field

Integrated Sachs-Wolfe

OBSERVABLE

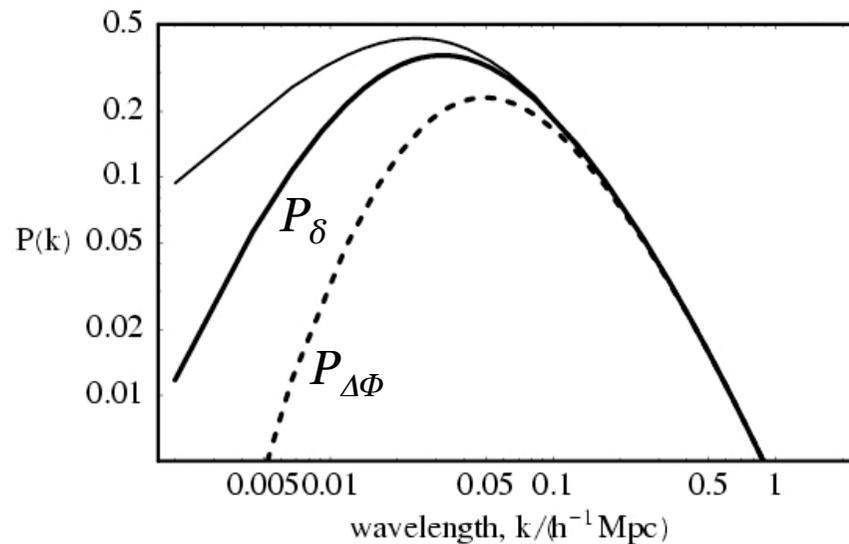
$$\kappa \propto \Delta(\Phi + \Psi)$$

$$\delta_g = b \delta$$

$$\theta = \beta \delta$$

$$\Theta_{SW} \propto \dot{\Phi} + \dot{\Psi}$$

Various combinations of these variables have been considered



JPU and Bernardeau, Phys. Rev. D **64** (2001)

EUCLID: ESA-class M-phase A

Data and tests

Large scale structure $\delta_g = \frac{\delta n_g}{n_g}$ $\delta_g = b_1 \delta + b_2 \delta^2$

$$P_{gg}^z(k, \mu) = P_{gg}(k) + 2 \frac{\mu^2}{aH} P_{g\theta_g}(k) + \frac{\mu^4}{a^2 H^2} P_{\theta_g \theta_g}(k)$$

Lensing

-weak lensing: $P_{\Phi+\Psi, \Phi+\Psi}$

-galaxy-galaxy lensing: $P_{g, \Phi+\Psi}$

In a Λ CDM, all these spectra are related

$$P_{g\theta_g} = aH \frac{f}{b} P_{gg} \quad P_{\theta_g \theta_g} = a^2 H^2 \frac{f^2}{b^2} P_{gg}$$

One needs to control the biases.

Biais

$$\begin{array}{c} \text{velocity map} \\ \langle \delta_g \theta \rangle = b\beta \langle \delta^2 \rangle \\ \uparrow \\ \text{Galaxy map} \\ \langle \delta_g \kappa \rangle \propto b \langle \delta \Delta (\Phi + \Psi) \rangle \propto 8\pi G \rho a^2 b \langle \delta^2 \rangle \\ \downarrow \\ \text{weak lensing} \end{array}$$

Λ CDM

The ratio of these 2 quantities is independent of the bias

Zhang et al, arXiv:0704.1932

Assume - no velocity bias ($S_{DE}=0$)
- no clustering of DE ($\Delta_{DE}=0$)

Conclusions

Our cosmological model requires dark sector.

The understanding of this sector calls for tests of the hypothesis of the model.

Underlying idea:

*Any hypothesis implies that some quantities are related;
We can test these rigidities [Consistency tests].*

Copernican principle:

*Time drift of redshift vs distance measurements.
Good test that allows to distinguish models that have the same light-cone properties.*

Modification of gravity:

difficult to construct models that are theoretically well-defined

Any modification from the LCDM:

*modifies the prediction (growth rate, background dynamics)
and more important: violation of SOME of the rigidities.*

Data analysis:

*Parametrisations: (w , γ) [!!have to be compatible!!]
Not yet in the spirit. Attempt with CFHTLS [Doré et al].
Requires: matter and velocity distribution + lensing [Tomography].*

Other datasets: *weakly NL regime / Gravity waves/ constants...*