Higgs mass implications on the fate of the electroweak vacuum

Gino Isidori
[*INFN, Frascati & CERN*]

- Introduction
- The Higgs potential at high energies
- Stability and metastability bounds
- Vacuum stability at NNLO
- Speculations on Planck-scale dynamics
- Conclusions

Introduction

All known phenomena in particle physics (*leaving aside a few cosmological observations*) can be described with good accuracy by a remarkably simple (*effective*) theory:

$$\mathscr{L}_{\rm SM} = \mathscr{L}_{\rm gauge}(A_a, \psi_i) + \mathscr{L}_{\rm Symm.\ Break.}(\phi, A_a, \psi_i)$$

$$\mathscr{L}_{\rm gauge} = \Sigma_a - \frac{1}{4g_a^2}(F_{\mu\nu}^a)^2 + \Sigma_\psi \Sigma_i \bar{\psi}_i i\not{D}\psi_i$$

- *Natural*
- Experimentally tested with high accuracy
- Stable with respect to quantum corrections (UV insensitive)
- Highly symmetric
  - $SU(3)_c \times SU(2)_L \times U(1)_Y$ *local symmetry*
  - *Global flavor symmetry*

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- Stable with respect to quantum corrections (UV insensitive)
- Highly symmetric [*gauge + favor symmetries*]

- *Ad hoc*
- Necessary to describe data [*the electroweak symmetry forbid masses for all the elementary particles observed so far...*]
- Not stable with respect to quantum corrections (UV sensitive)
- Origin of the flavor structure of the model [*and of all the problems of the model...*]

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- *Natural*
- Experimentally tested with high accuracy

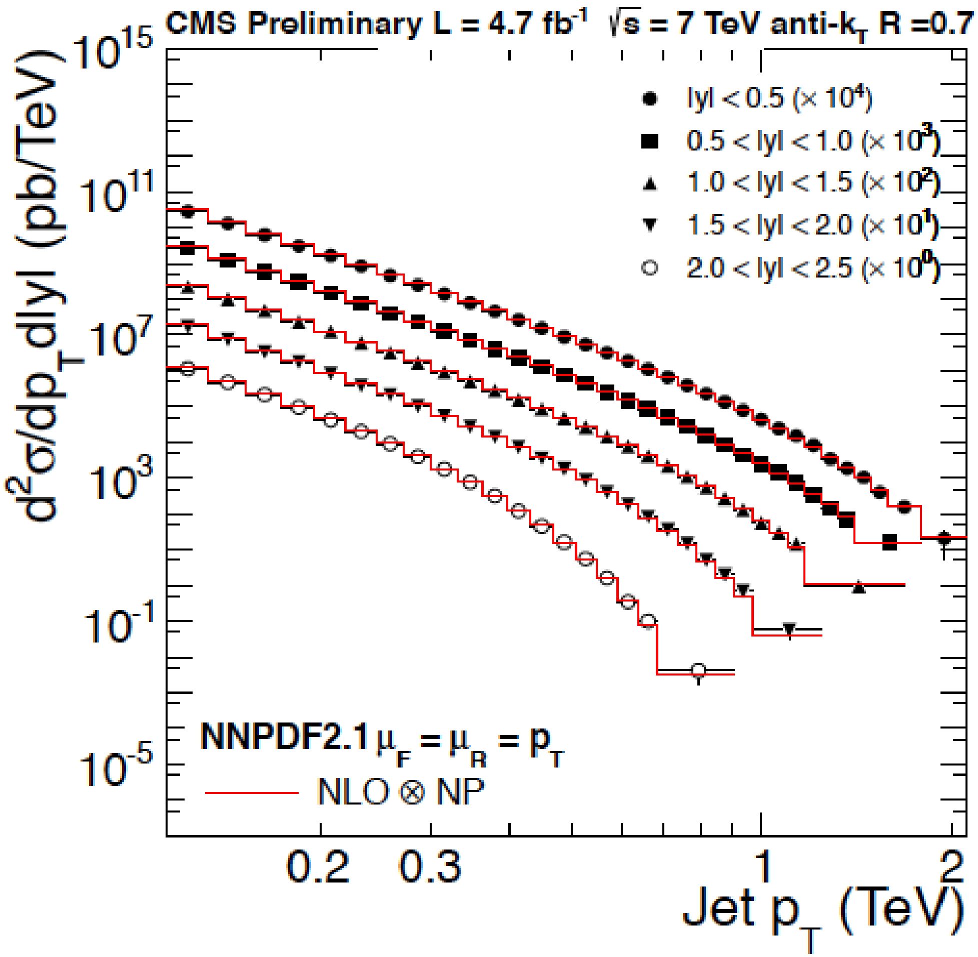
Elegant & stable, but also a bit boring...

- *Ad hoc*
- Necessary to describe data [*we couldn't live in a fully symmetric world...*]

Ugly & unstable, but is what makes nature interesting...!

Introduction

LHC experiments have confirmed once more that we understand very well gauge interactions...



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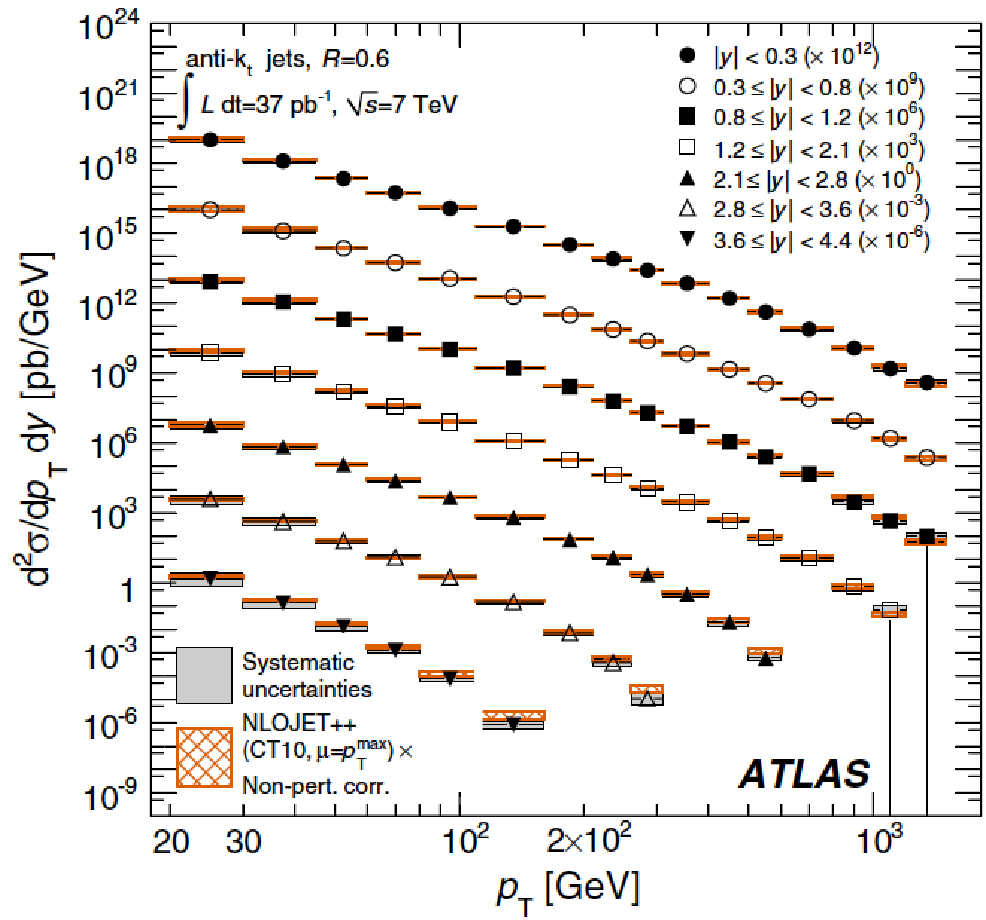

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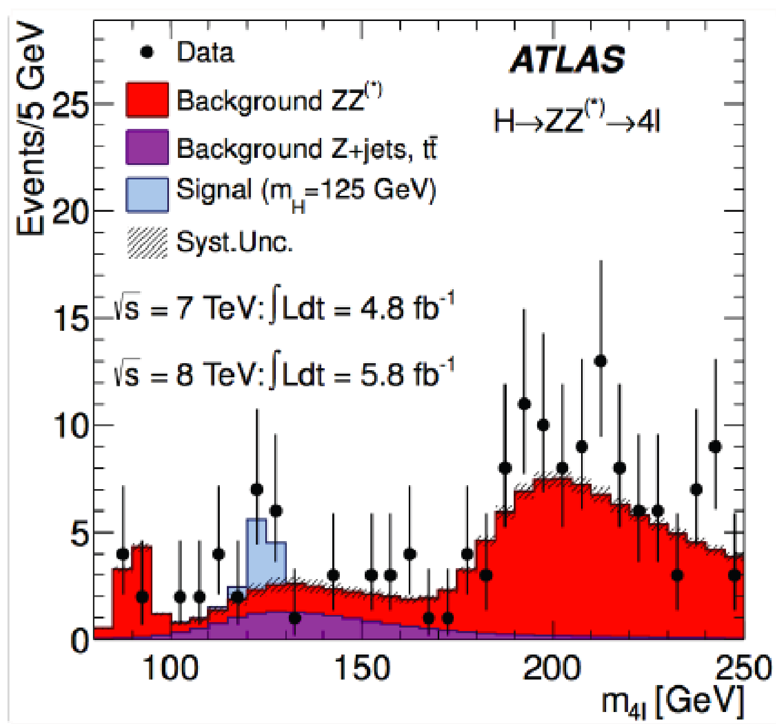
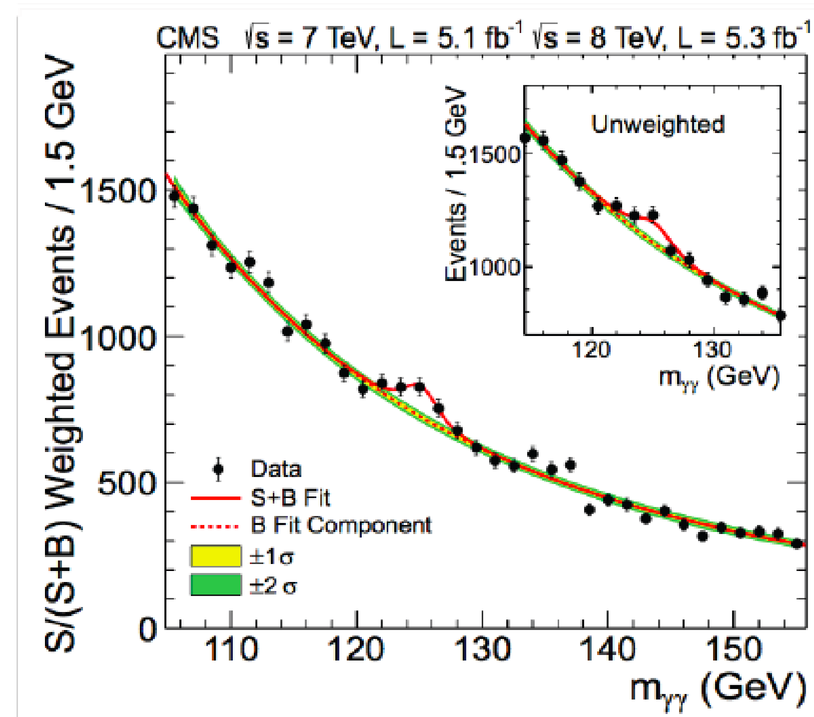


arXiv:1207.7214v1 [hep-ex]



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Clear evidence of a new particle compatible with the properties of the Higgs boson

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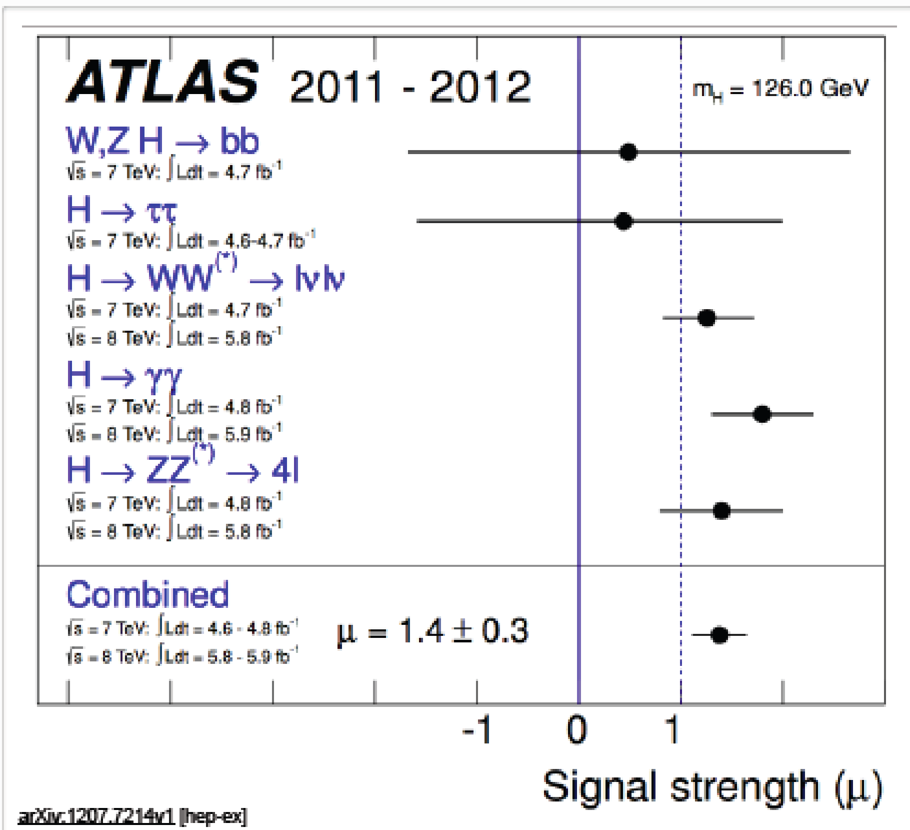
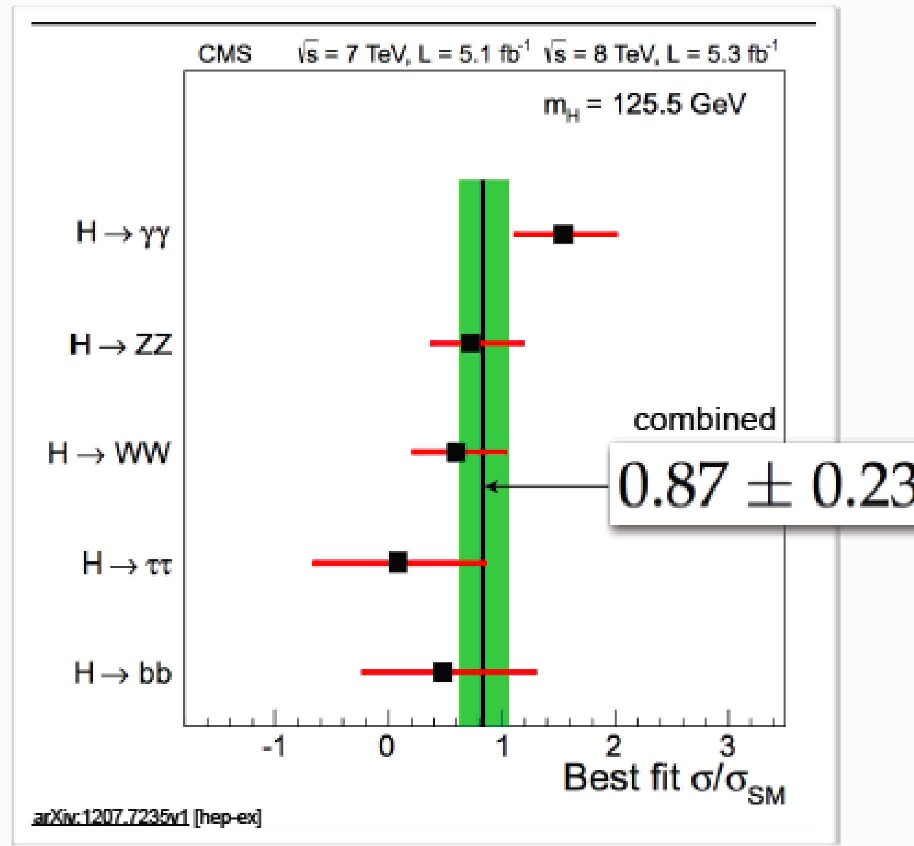
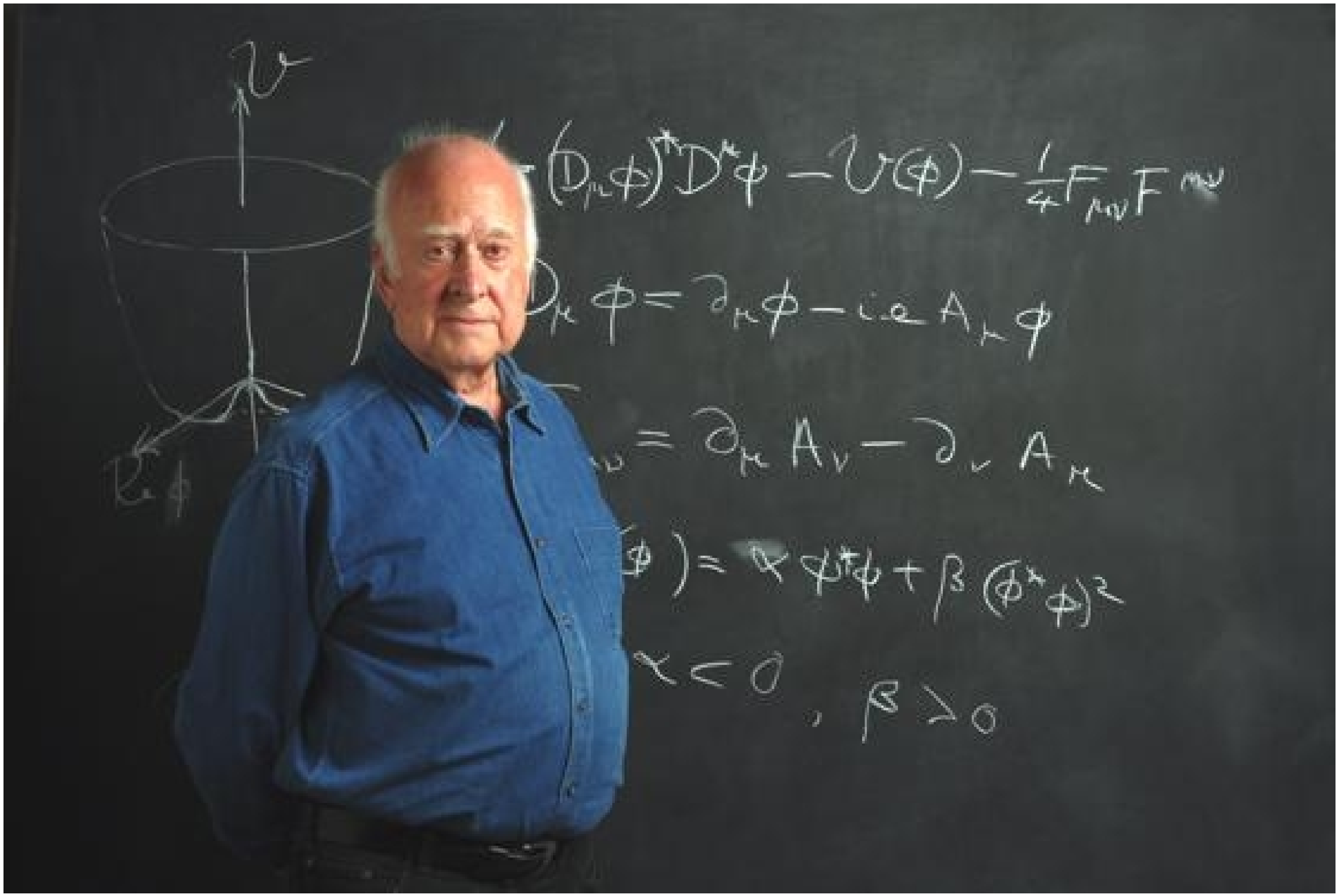




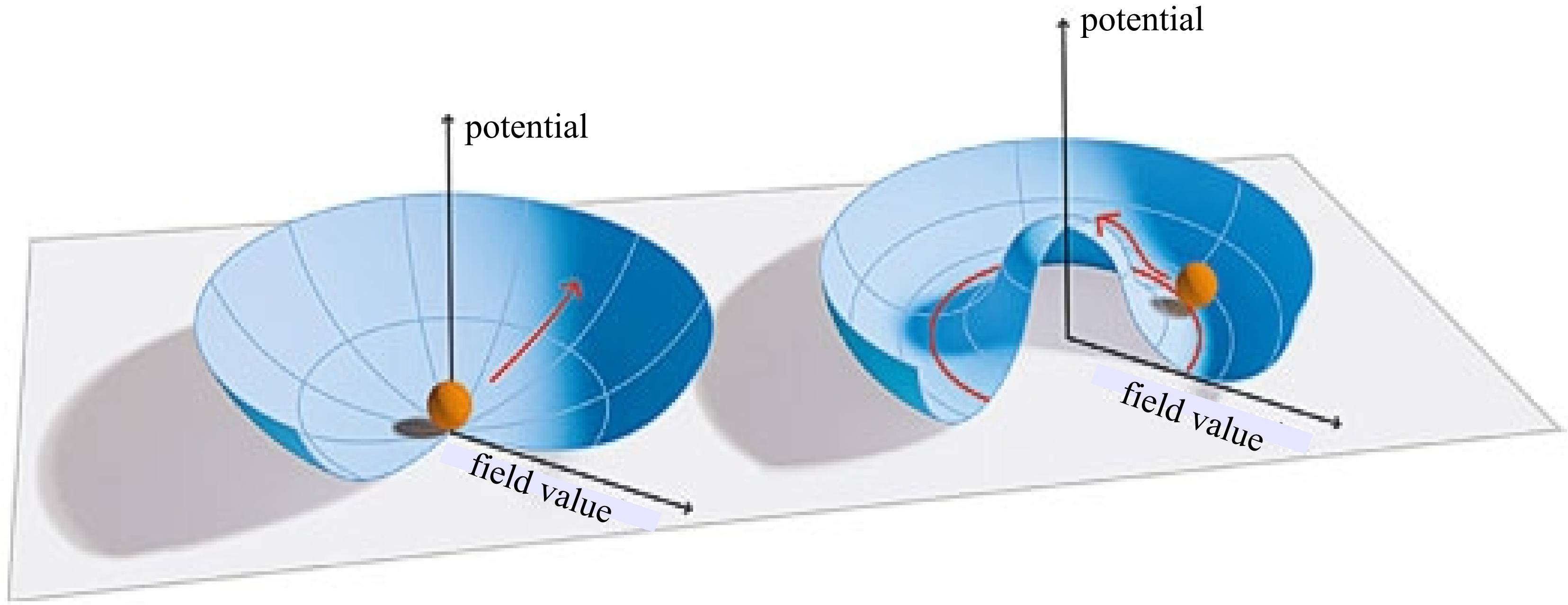
Clear evidence of a new particle compatible with the properties of the Higgs boson

The SM Higgs sector



The SM Higgs sector

The Higgs mechanism, namely the introduction of an elementary $SU(2)_L$ scalar doublet, with ϕ^4 potential, is the most economical & simple choice to achieve the spontaneous symmetry breaking of both gauge [$SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$] and flavor symmetries that we observe in nature.



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$$\mathcal{L}_{\rm higgs}(\phi, A_a, \psi_i) = D\phi^+ D\phi - V(\phi)$$

$$V(\phi) = -\mu^2 \phi^+\phi + \lambda(\phi^+\phi)^2 + Y^{ij}\, \psi_L{}^i\, \psi_R{}^j\, \phi$$

Till very recently only the ground state determined by this potential (*and the corresponding Goldstone boson structure*) was tested with good accuracy:

$$v = \langle \phi^+\phi \rangle^{1/2} \sim 246 \text{ GeV} \quad [\, m_W = \tfrac{1}{2}\, g\, v \,]$$

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The situation has substantially changed a few weeks ago, with the observation of the 4th degree of freedom of the Higgs field (or its *massive excitation*):

$$\lambda_{\rm (tree)} = \tfrac{1}{2}\, m_h^{\ 2} / v^2 \sim 0.13$$

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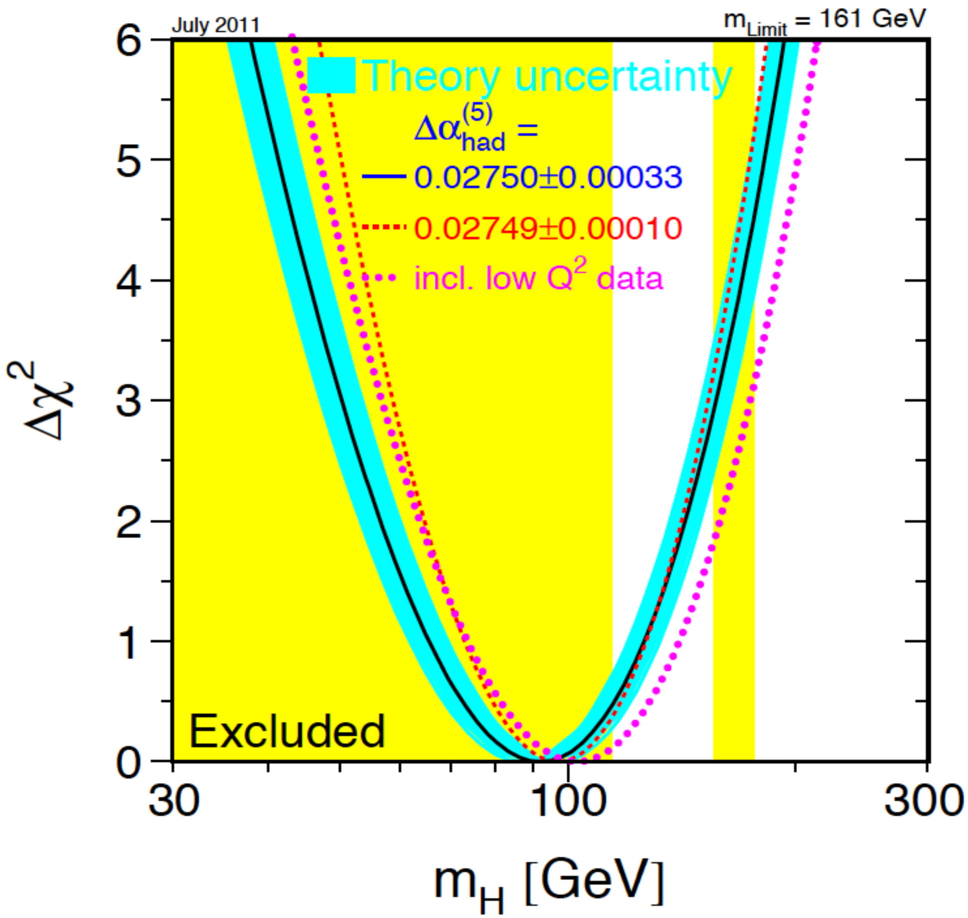
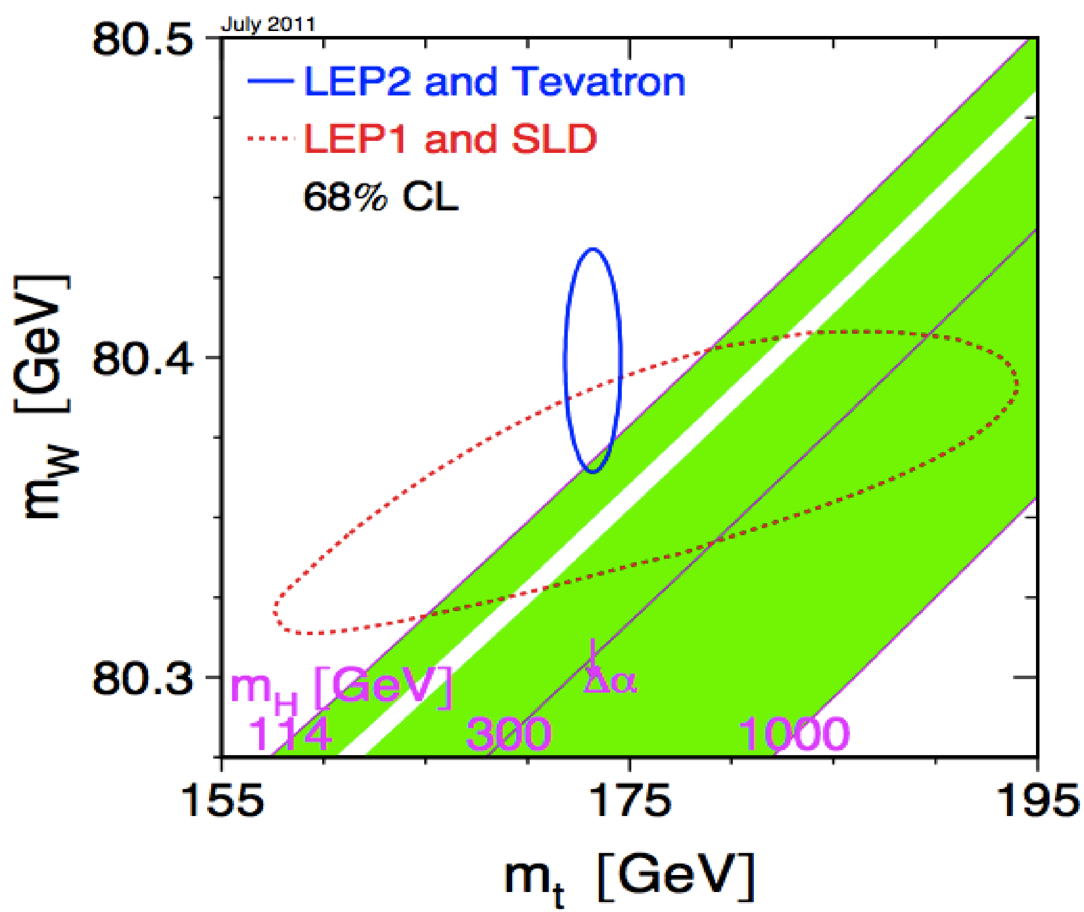
Actually some information about the Higgs mass was already present in the e.w. precision tests (*assuming the validity of the SM up to high scales*):





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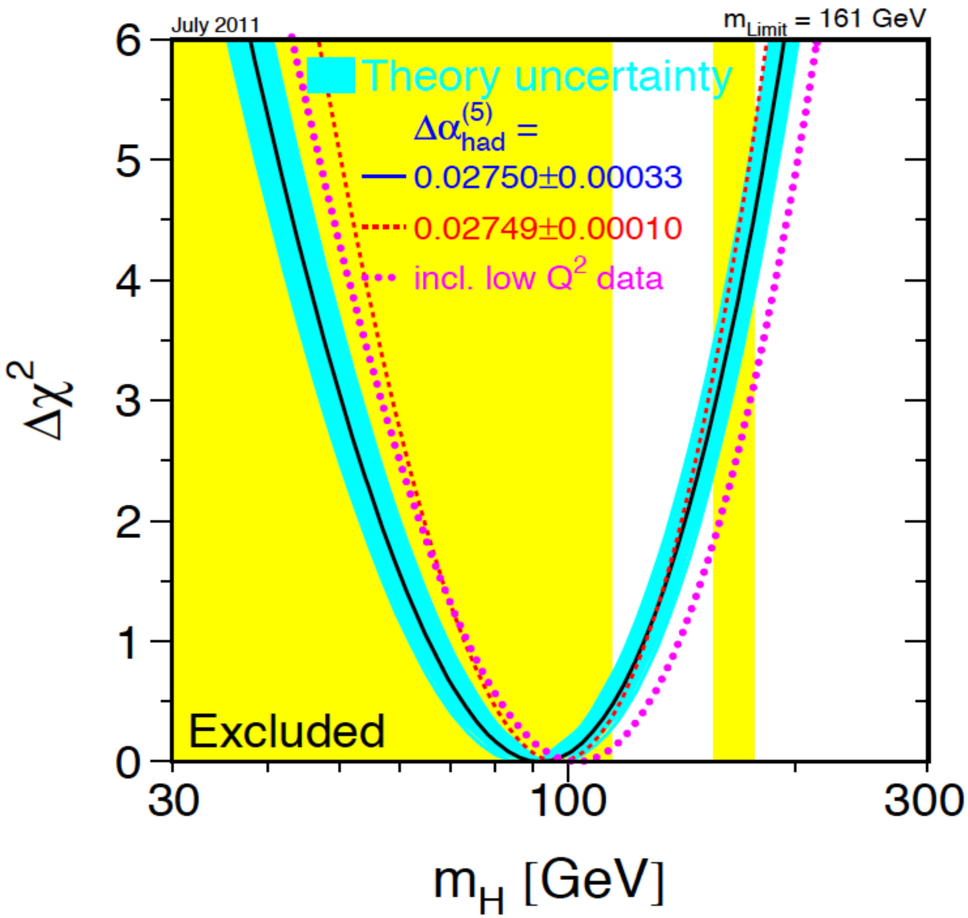

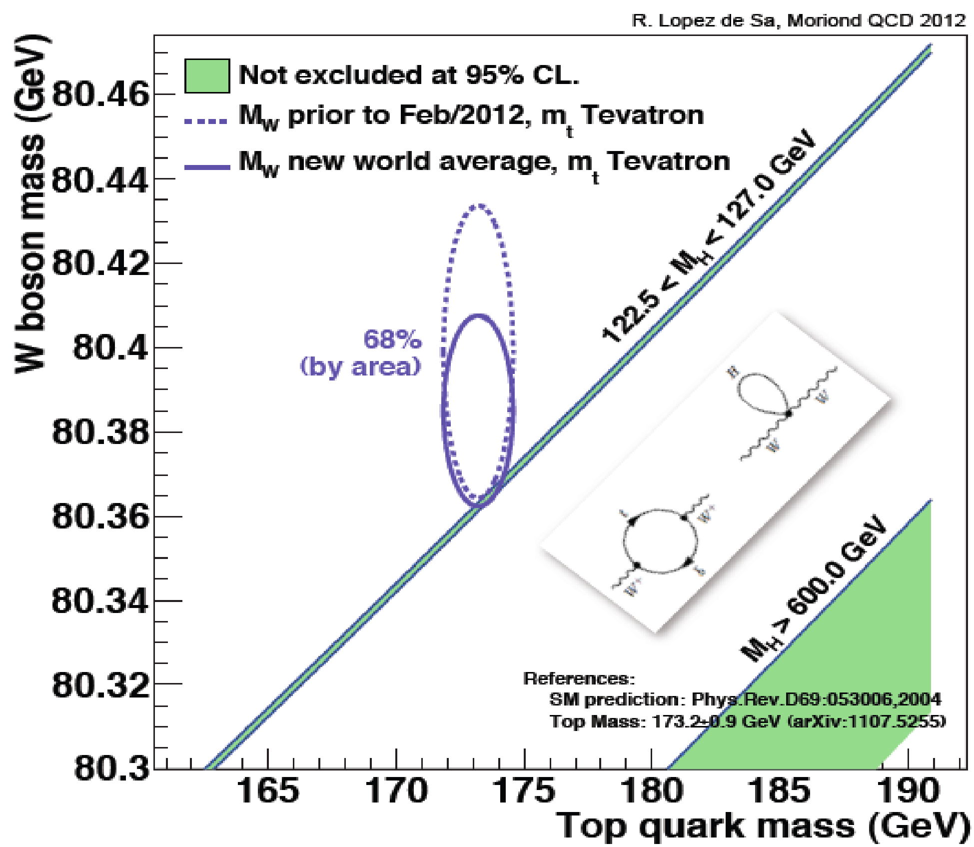

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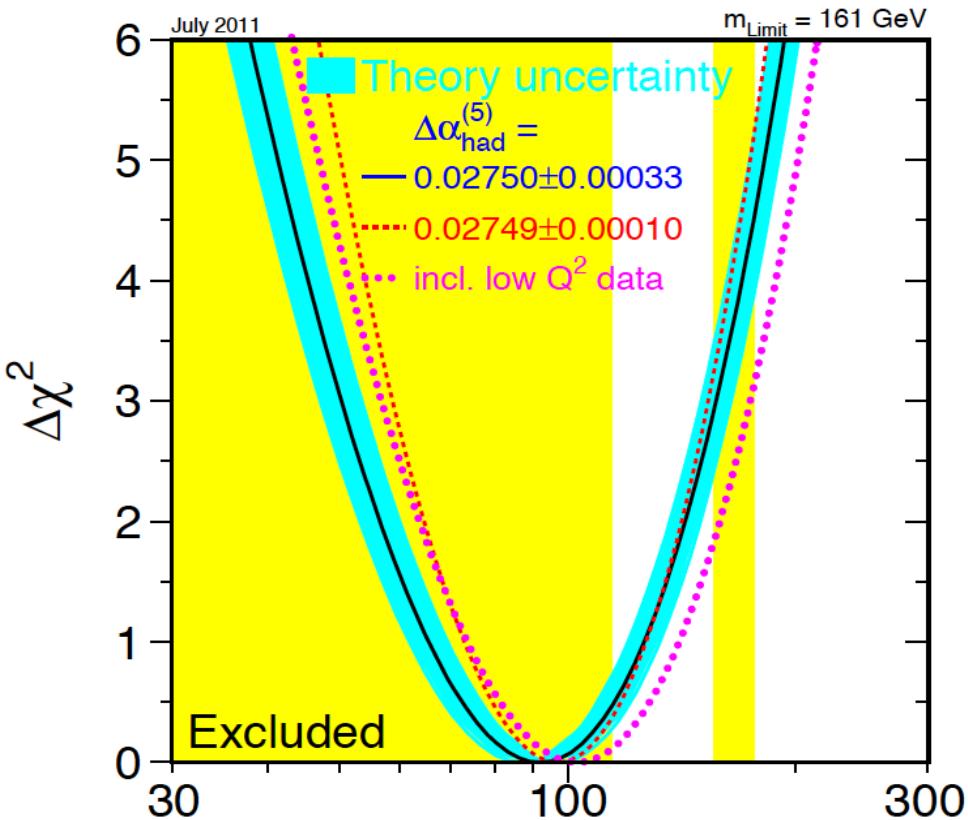

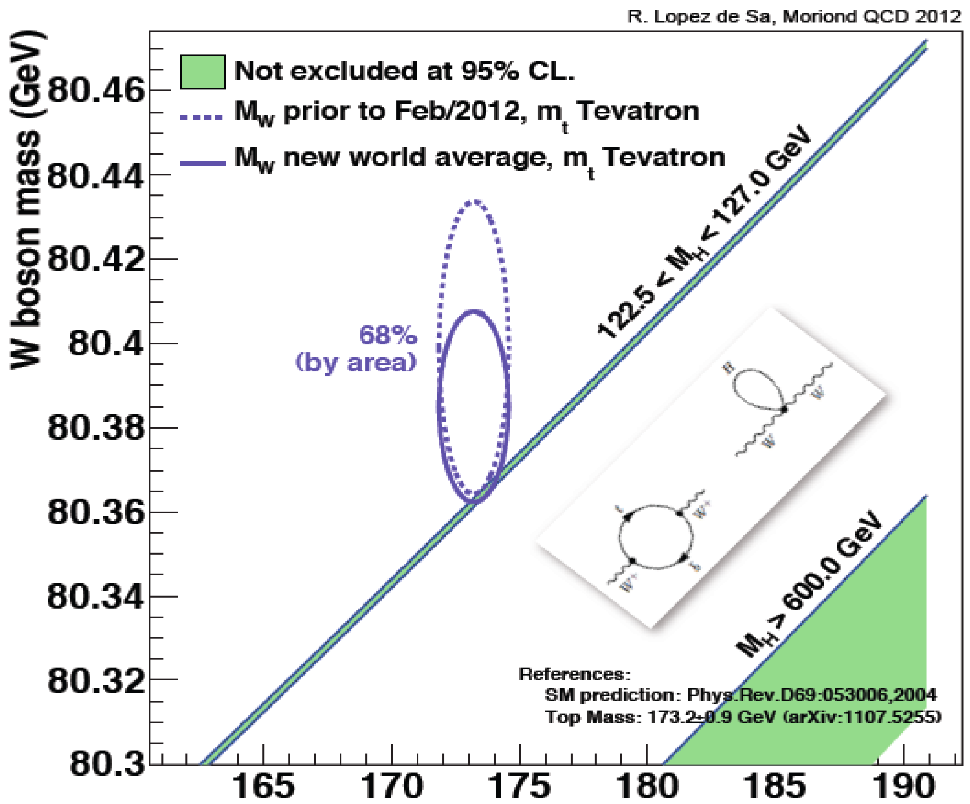

Message n.1: The observation of the physical Higgs boson with m_h well consistent with the (indirect) prediction of the e.w. precision tests is a *great success of the SM !*

▶ *The SM Higgs sector*

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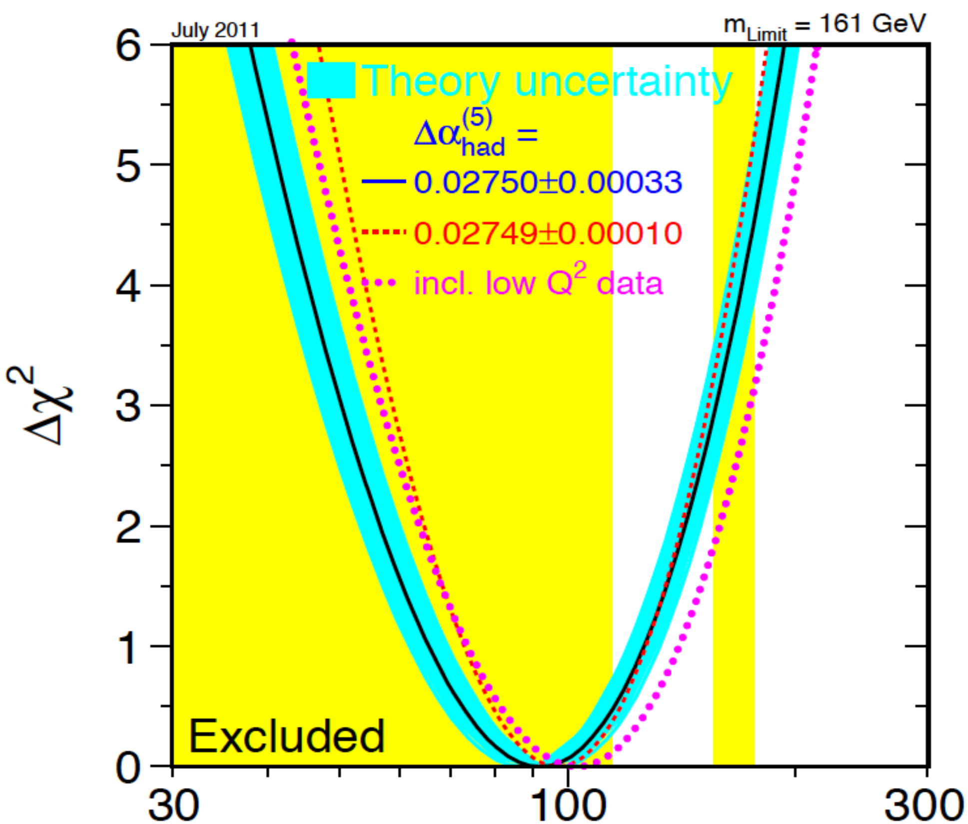

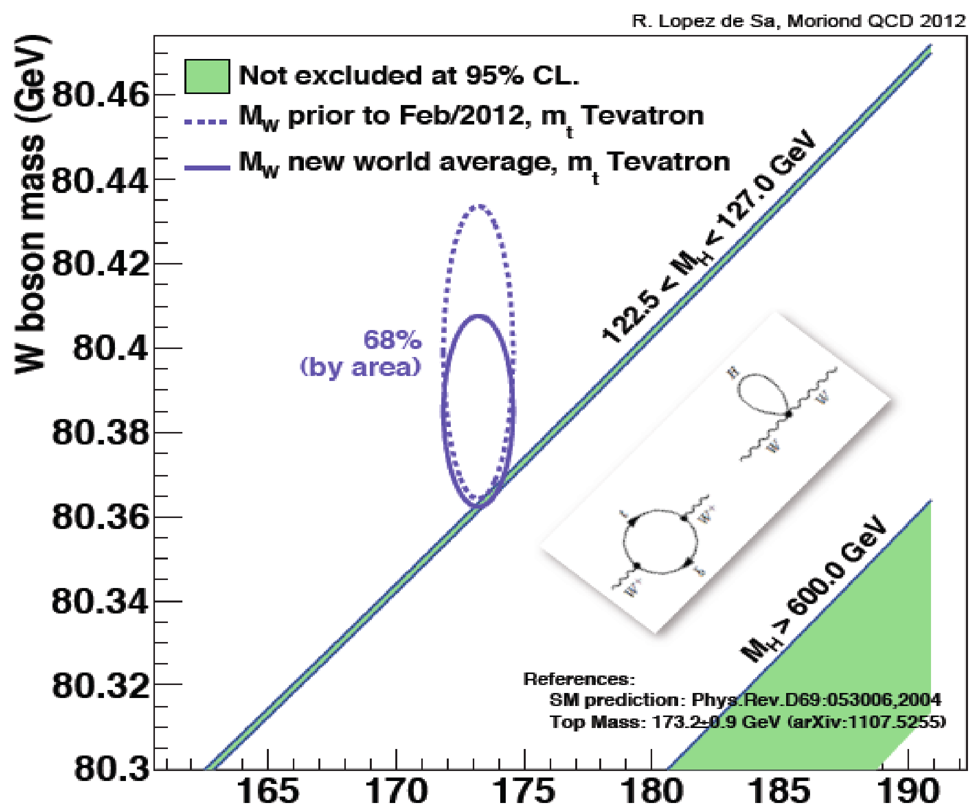

More generally, we have a strong indication that the symmetry breaking sector of the theory has a *minimal* and *weakly coupled* structure (at least around the TeV scale)

The SM Higgs sector

Still, the SM Higgs potential is "ugly" and hides the most serious *theoretical problems* of this highly successful theory:

$$V(\phi) = - \mu^2 \phi^+\phi + \lambda (\phi^+\phi)^2 + Y^{ij} \psi_L^{\,i} \psi_R^{\,j} \phi$$

vacuum instability

possible internal inconsistency of the model ($\lambda < 0$) at large energies

[*key dependence on* m_h]

Quadratic sensitivity to the cut-off

$$\Delta\mu^2 \sim \Delta m_h^{\,2} \sim \Lambda^2$$

(indication of *new physics* close to the electroweak scale ?)

SM flavor problem

(unexplained span over several orders of magnitude and strongly hierarchical structure of the Yukawa coupl.)

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effective neutrino mass term

Cosmological constant prob.

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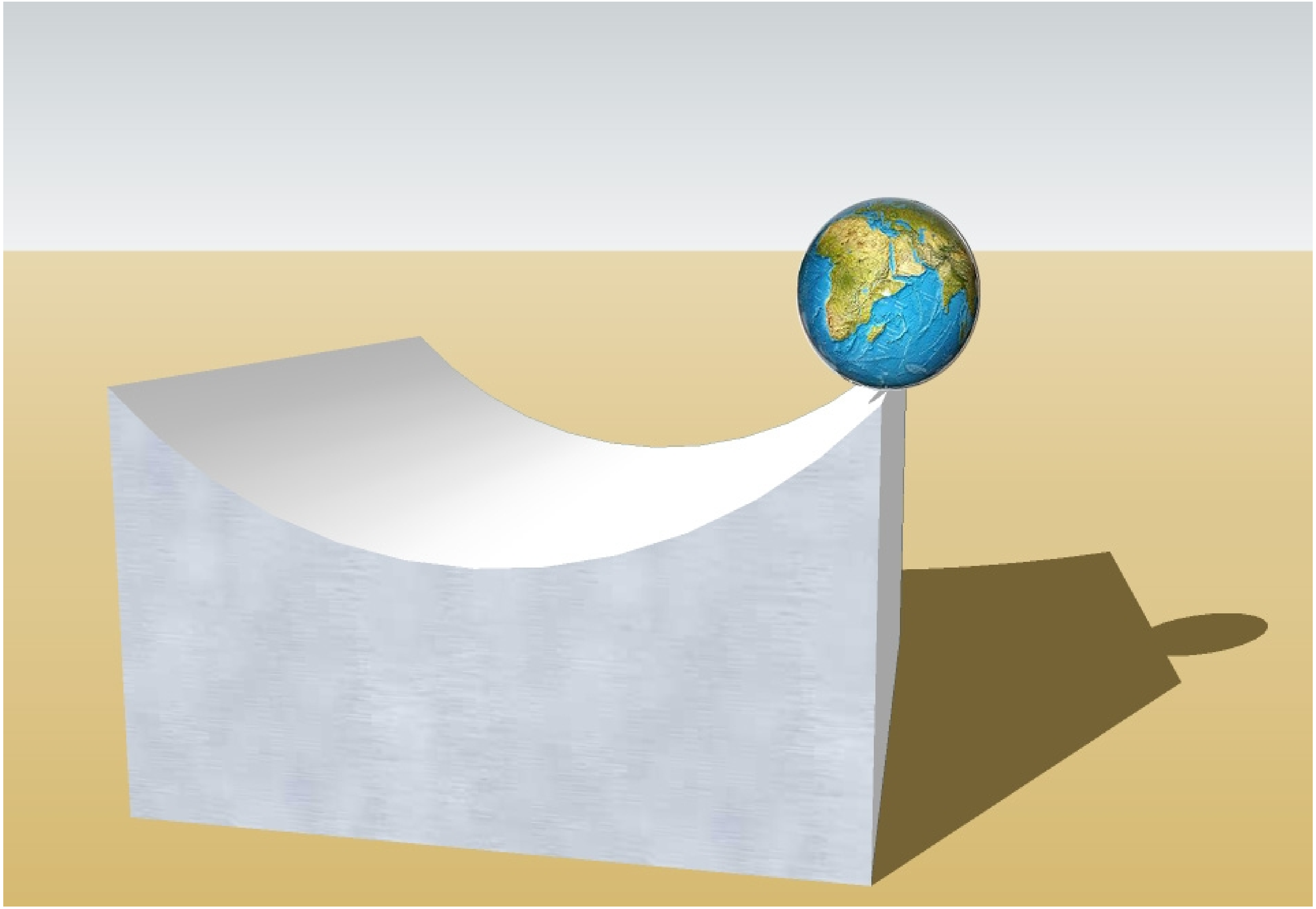
Stability and metastability bounds

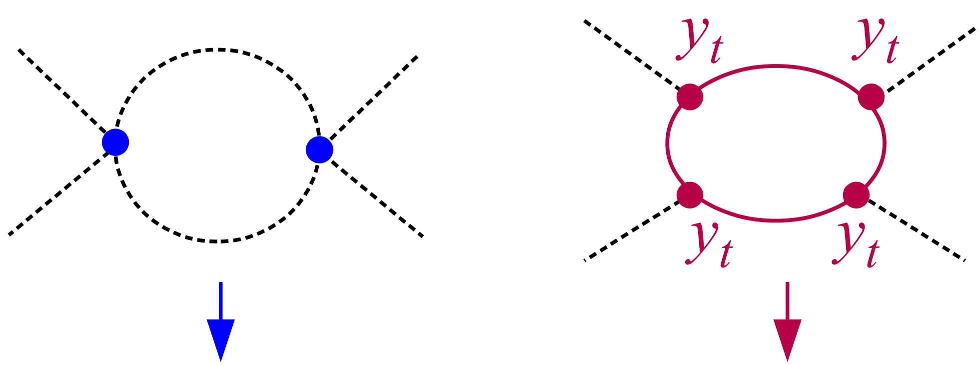


Stability and metastability bounds

At large field values the shape of the Higgs potential is determined by the RGE evolution of the Higgs self coupling:

$$V_{eff}(\ |\phi| \gg\ v\ )\ \approx\ \lambda(|\phi|) \times |\phi|^4\ +\ O(v^2|\phi|^2)$$

The evolution of λ is determined by two main effects:



growing λ at large energies

decreasing λ

$$\lambda(v)\ \propto\ \frac{m_h^2}{v^2}$$

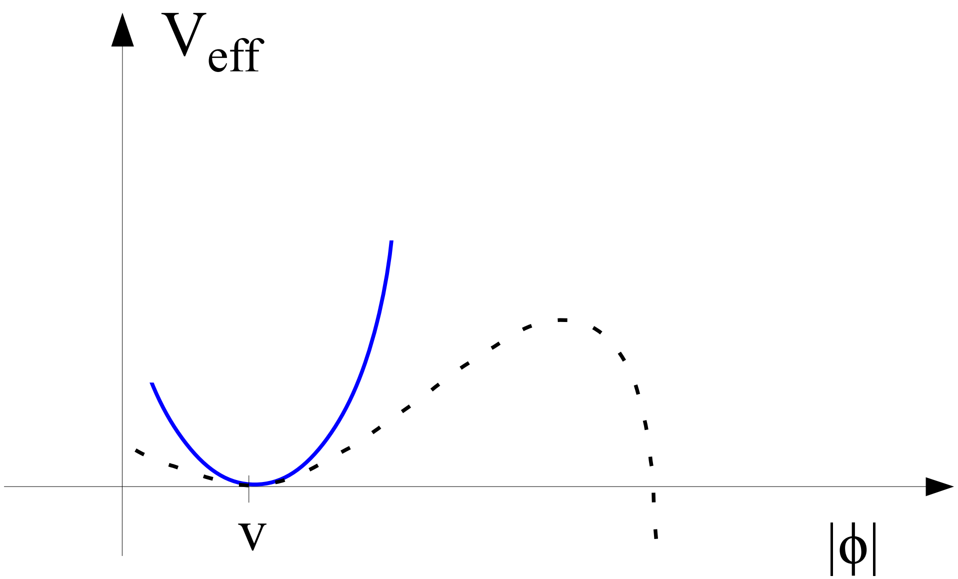
$$y_t(v) \propto \frac{m_t}{v}$$

Given the large value of y_t, the destabilization due to top-quark loops is quite relevant

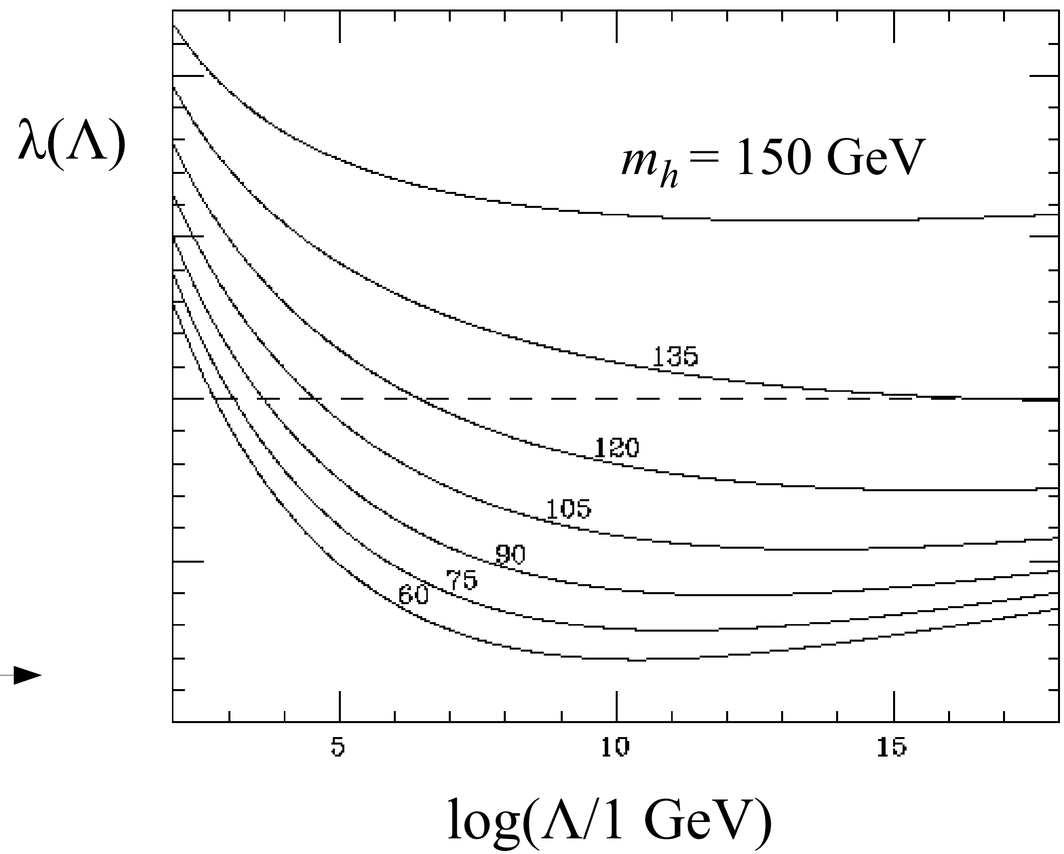
Stability and metastability bounds

At large field values:

$$V_{eff}(|\phi|) \approx \lambda(|\phi|) \times |\phi|^4$$



Altarelli, G.I, '94

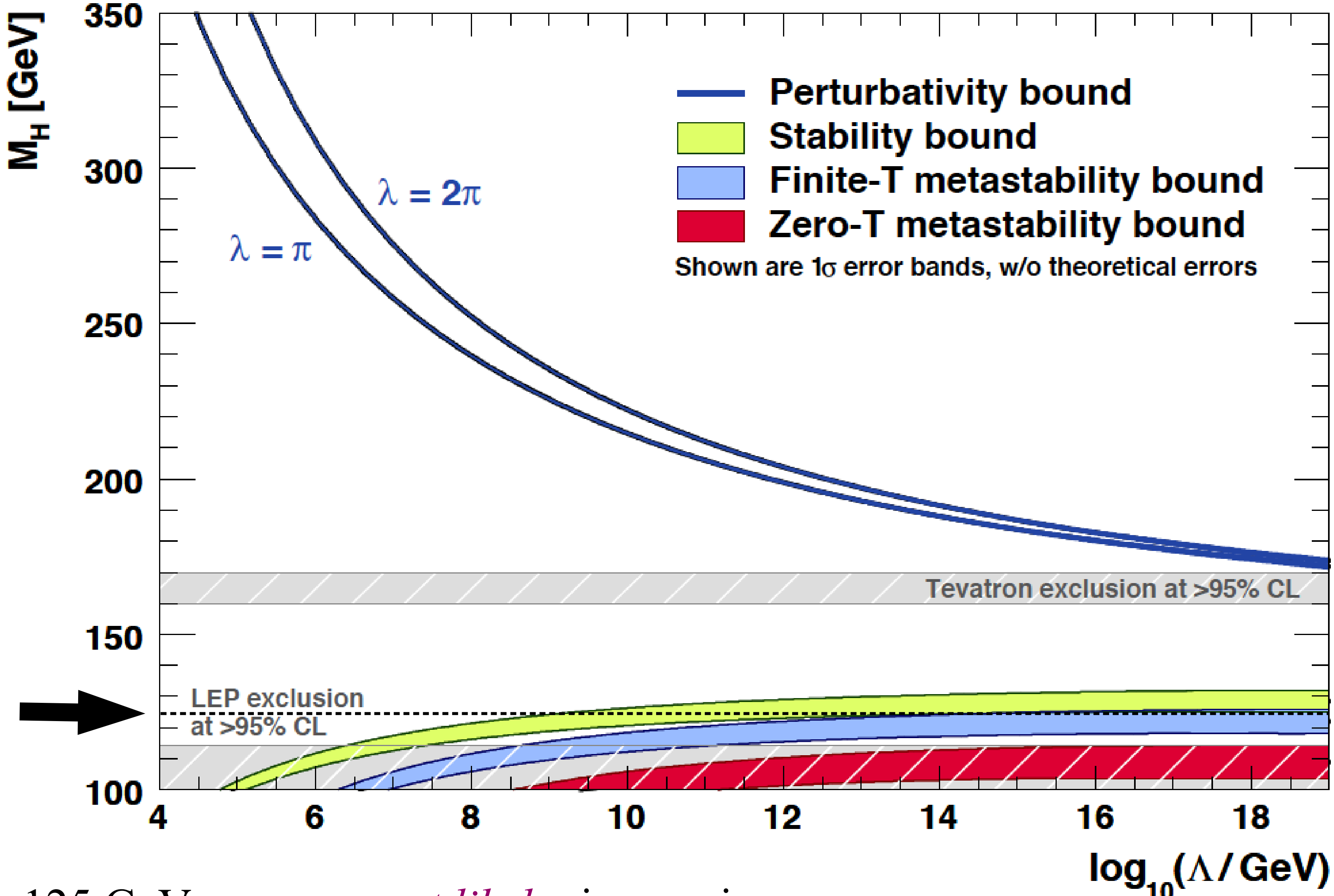


The problem was well-known since a long time, but now for the first time we can "quantify it", knowing the Higgs mass

Cabibbo, Maiani, Parisi, Petronzio, '79; Hung '79; Lindner 86; Sher '89; ....

Stability and metastability bounds

Ellis et al. '09



For $m_h \sim 125$ GeV we are -*most likely*- in a region where the Higgs potential is not absolutely stable

⭑ *The metastability condition:*

Can we rule out the model (and determine an upper bound on the new-physics scale Λ) if there is a second (deeper) minimum at large field values ?

Not really: The model could still be consistent if the lifetime of the (unstable) e.w. minimum is sufficiently long (i.e. longer than the age of the Universe)

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The e.w. minimum is destabilized by:

quantum fluctuations (at T=0)

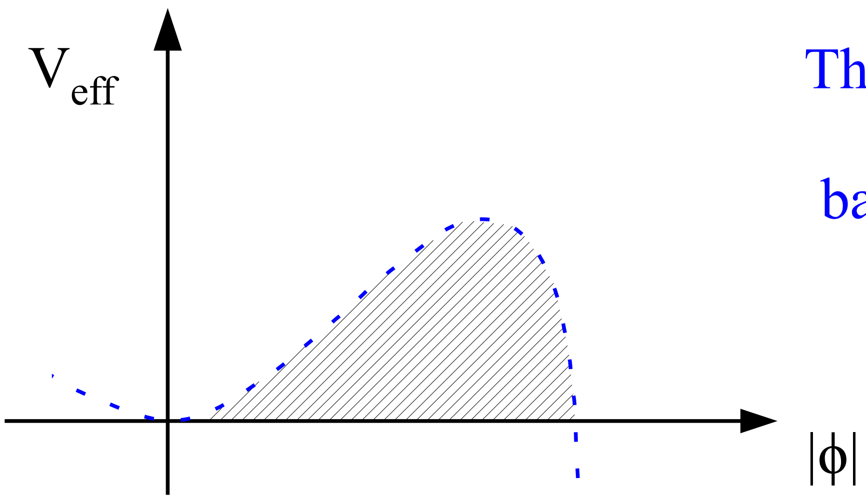
computable in a model-independent way

thermal fluctuations

strongly dependent on the thermal history of the universe & competing with quant. fluctuations only for very high T

The most conservative bound is obtained by considering the stability under quantum fluctuations at zero temperature

✯ The quantum-tunneling rate:



The quantum tunneling occurs via bubble formation in the homogeneous background of the false (e.w.) minimum

Coleman '79

At the semi-classical level, the tunneling probability can be written as:

$$\boxed{p \approx K\, e^{-S_0[h]}}$$

Volume factor

$K \propto T_U^4$ not exactly calculable within the semi-classical approx.

Euclidean action

$$\int \frac{1}{2}(\partial_\mu h)^2 + V(h)$$

Bounce

solution of the e.o.m. that interpolates between the false and the true vacuum

N.B.: within a QFT (system with infinite d.o.f.) the tunneling is suppressed even in absence of a potential barrier (kinematic barrier due to the boundary conditions)

✯ *The quantum-tunneling rate:*

If we neglect the mass term, the tree-level Higgs potential is scale invariant & its bounces have a rather simple form:

$$h(r) = \left(\frac{2}{|\lambda|}\right)^{1/2} \frac{2R}{r^2+R^2}$$

$r = x_\mu x_\mu$ $O(4)$ invariant bounces minimize the action

$R =$ arbitrary scale parameter

$$\Downarrow$$

$$S_0[h] = \frac{8\pi^2}{3|\lambda|} \quad \Rightarrow \quad p_{semicl.} \approx (T_U/R)^4\, e^{-8\pi^2/3|\lambda|}$$

If λ remains sufficiently small, the tunneling rate can be very suppressed

N.B.: the tunneling rate is a pure non-perturbative phenomenon - cannot be computed to any finite order in "ordinary" perturbation theory [*wrong choice of the vacuum*]

✯ The quantum-tunneling rate:

To go beyond the semi-classical level we need to take into account the quantum fluctuations around the (non-constant) bounce solution

Callan, Coleman '79

Non-trivial problem which has been solved (semi-analytically) in the SM case:

G.I., Ridolfi, Strumia '01

- Quantum corrections break scale invariance
- The tunneling is dominated by bounces of size R, such that λ(1/R) reaches its minimum value:

$$p = \max_R \frac{V_U}{R^4} \exp\left[-\frac{8\pi^2}{3|\lambda(\mu)|} - \Delta S(\mu R) \right]$$

μ independent

$\Delta S \approx 0$ if we set $\mu = 1/R$

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⬇

- Quantum corrections break scale invariance
- The tunneling is dominated by bounces of size R, such that λ(1/R) reaches its minimum value
- The critical R determine the reference scale of the volume pre-factor:

$$p \approx \max_R \frac{V_U}{R^4} \exp\left[-\frac{8\pi^2}{3|\lambda(1/R)|}\right]$$

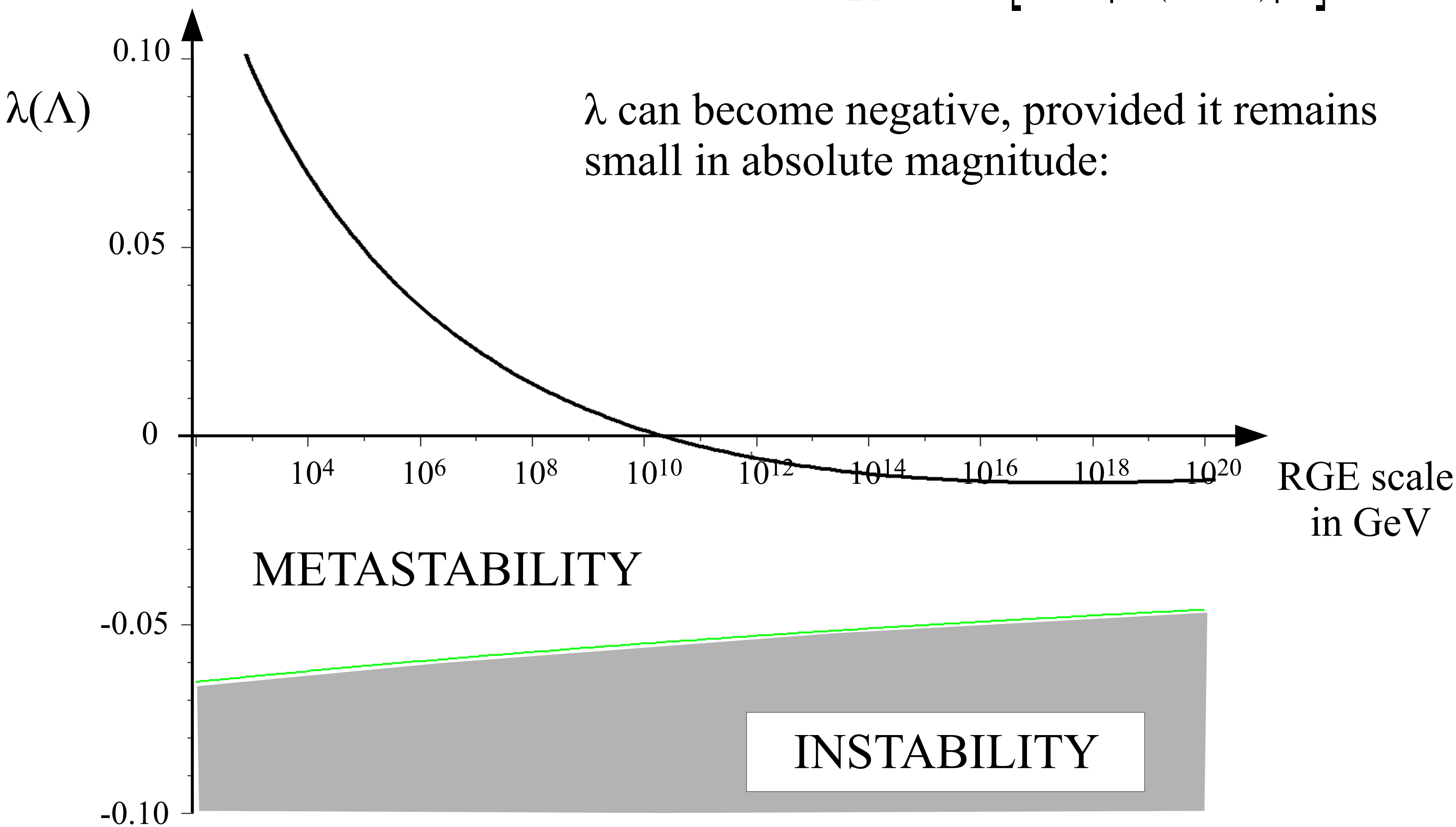
The leading gravitational effects are also calculable when 1/R is not far from (but below) M_{pl}

G.I., Rychkov, Strumia, Tetradis '08

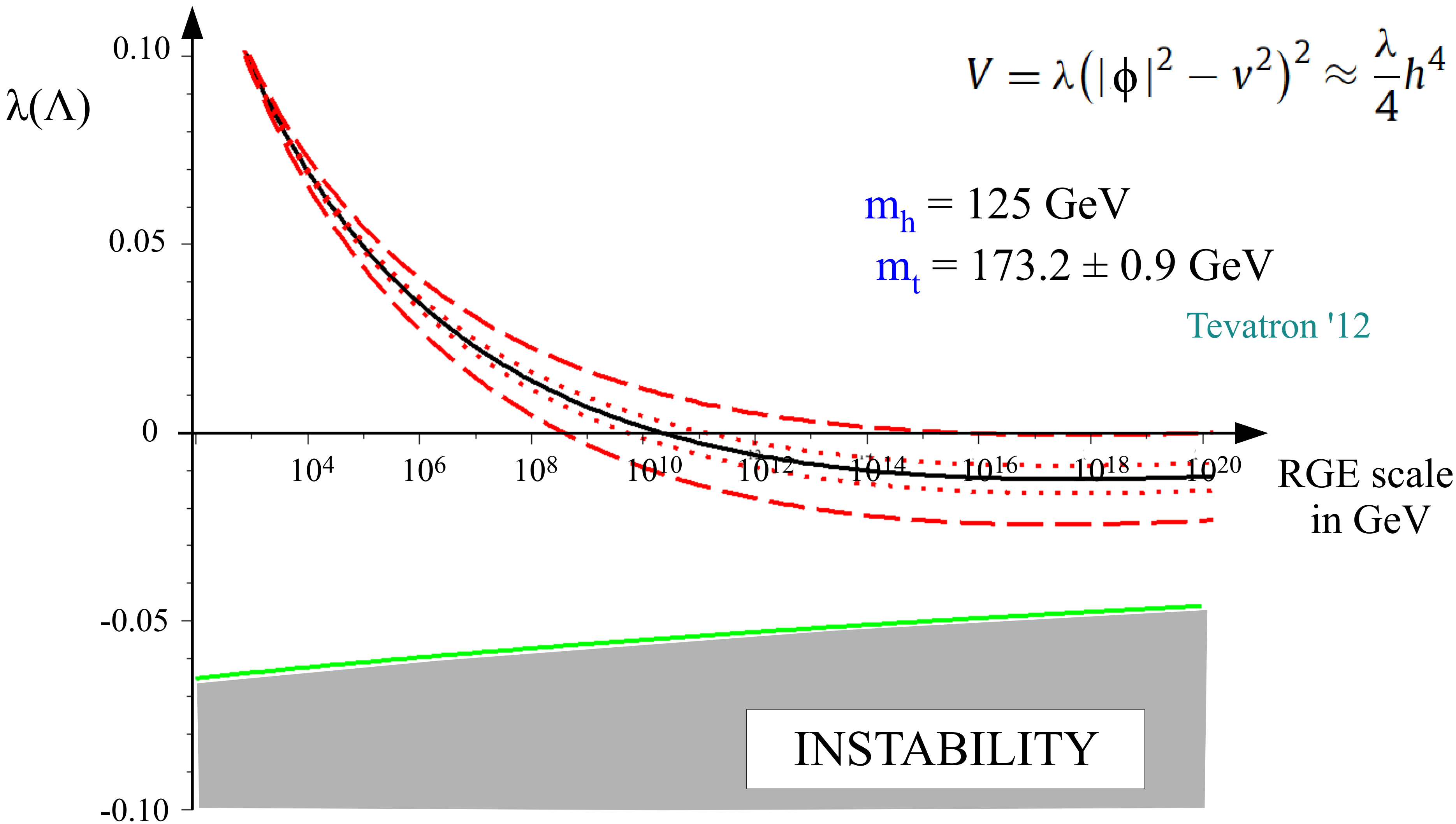
The metastability condition:

$$p \approx \max_R \frac{V_U}{R^4} \exp\left[-\frac{8\pi^2}{3|\lambda(1/R)|} \right]$$

λ can become negative, provided it remains small in absolute magnitude:



Message n.2: For m_h =125 GeV and the present central value of m_{top}, the SM vacuum is unstable but sufficiently long-lived, compared to the age of the Universe



$$V = \lambda\left(|\phi|^2 - v^2\right)^2 \approx \frac{\lambda}{4} h^4$$

m_h = 125 GeV

m_t = 173.2 ± 0.9 GeV

Tevatron '12

▶ *Vacuum stability at NNLO* (for m_h ~125 GeV)

For m_h =125 GeV and the present central value of m_{top}, the SM vacuum is unstable but sufficiently long-lived, compared to the age of the Universe

How "precise" is this statement?

A full NNLO analysis has recently become possible:

- Two-loop potential — Ford, Jack, Jones '92, '01
- Three-loop beta functions — Mihaila, Salomon, Steinhauser 1201.5868; Chetyrkin, Zoller, 1205.2892
- Two-loop threshold corrections in relating $\lambda(v)$ to the Higgs mass:

$$\lambda(\mu) = \frac{G_F\, m_h^2}{\sqrt{2}} + \Delta\lambda(\mu)$$

(*dominant uncertainty*)

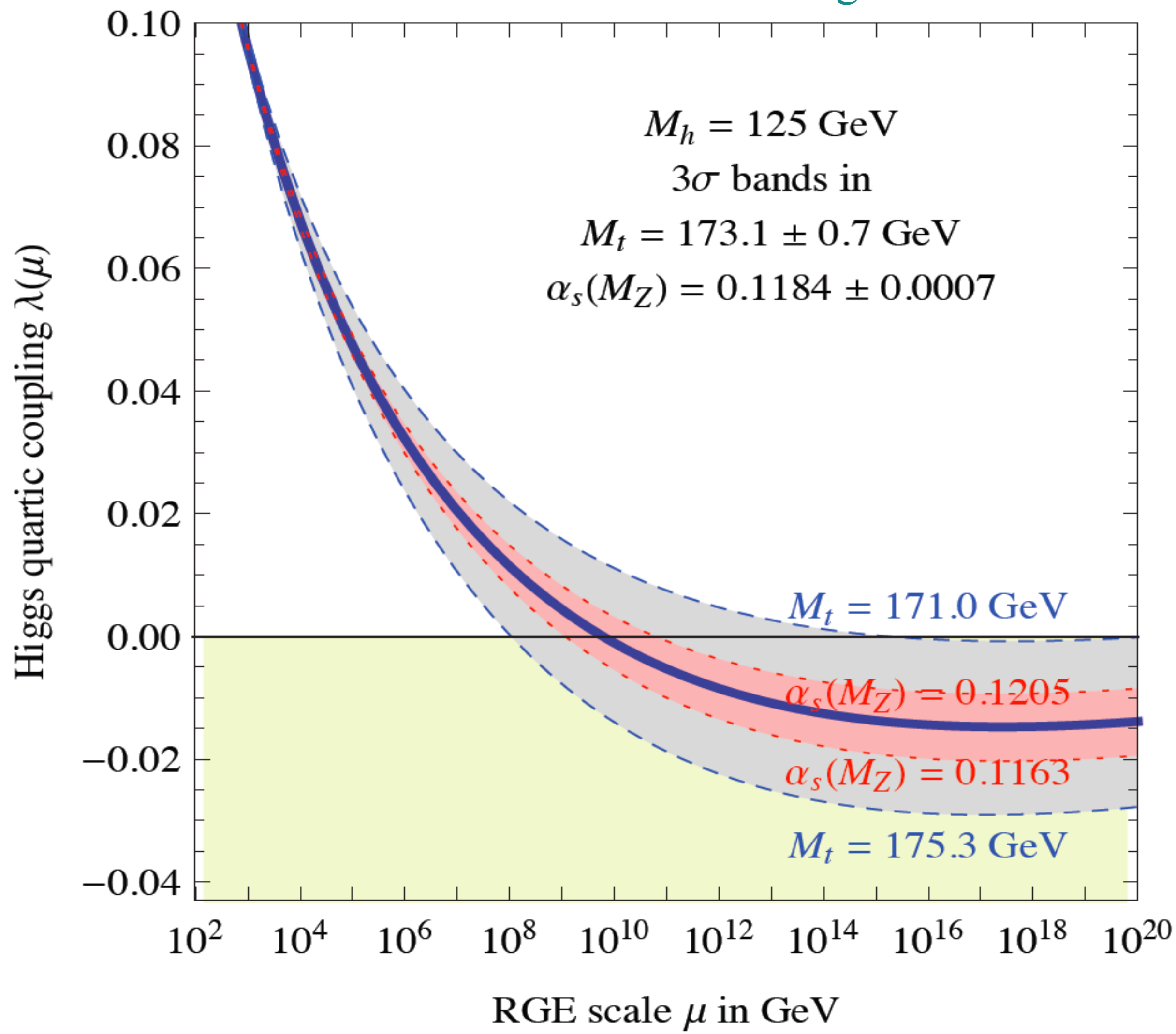
Yukawa×QCD — Bezrukov, Kalmykov, Kniehl, Shaposhnikov, 1205. 2893

Yukawa×QCD, Yuk.×Yuk. — Degrassi, Di Vita, Elias-Miro', Espinosa, Giudice, G.I., Strumia 1205.6497

Given the fast running of λ close to the e.w. scale, the dominant uncertainty comes from threshold (non-log enhanced) corrections at the electroweak scale (or in the precise evaluation of the initial condition).

While the smallness of λ (and the other couplings) at high energies imply that the 3-loop terms in the beta functions play a very minor role (useful to control the error).

Degrassi *et al.* '12



With the NNLO calculation we are able to derive a very precise relation between Higgs and top masses from vacuum stability:

Absolute stability:

$$M_h\ [\mathrm{GeV}] > 129.4 + 2.0\left(\frac{M_t\ [\mathrm{GeV}] - 173.1}{1.0}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\mathrm{th}}$$

Conservative th. error given the size of the shifts from NLO to NNLO:

$+\,0.6$ GeV due to the QCD threshold corrections to λ

$+\,0.2$ GeV due to the Yukawa threshold corrections to λ

$-\,0.2$ GeV from RG equation at 3 loops

$-\,0.1$ GeV from the effective potential at 2 loops.

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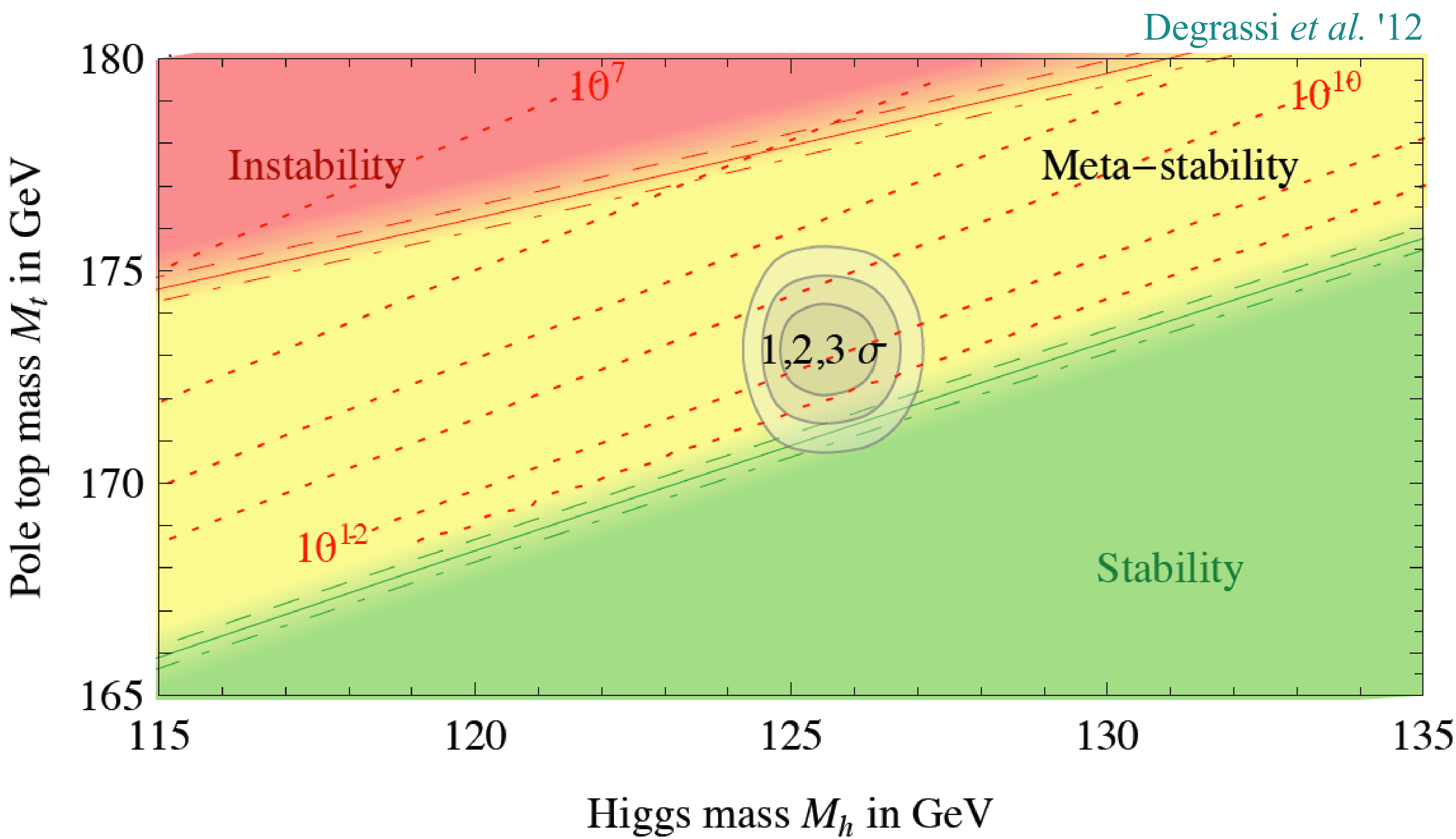

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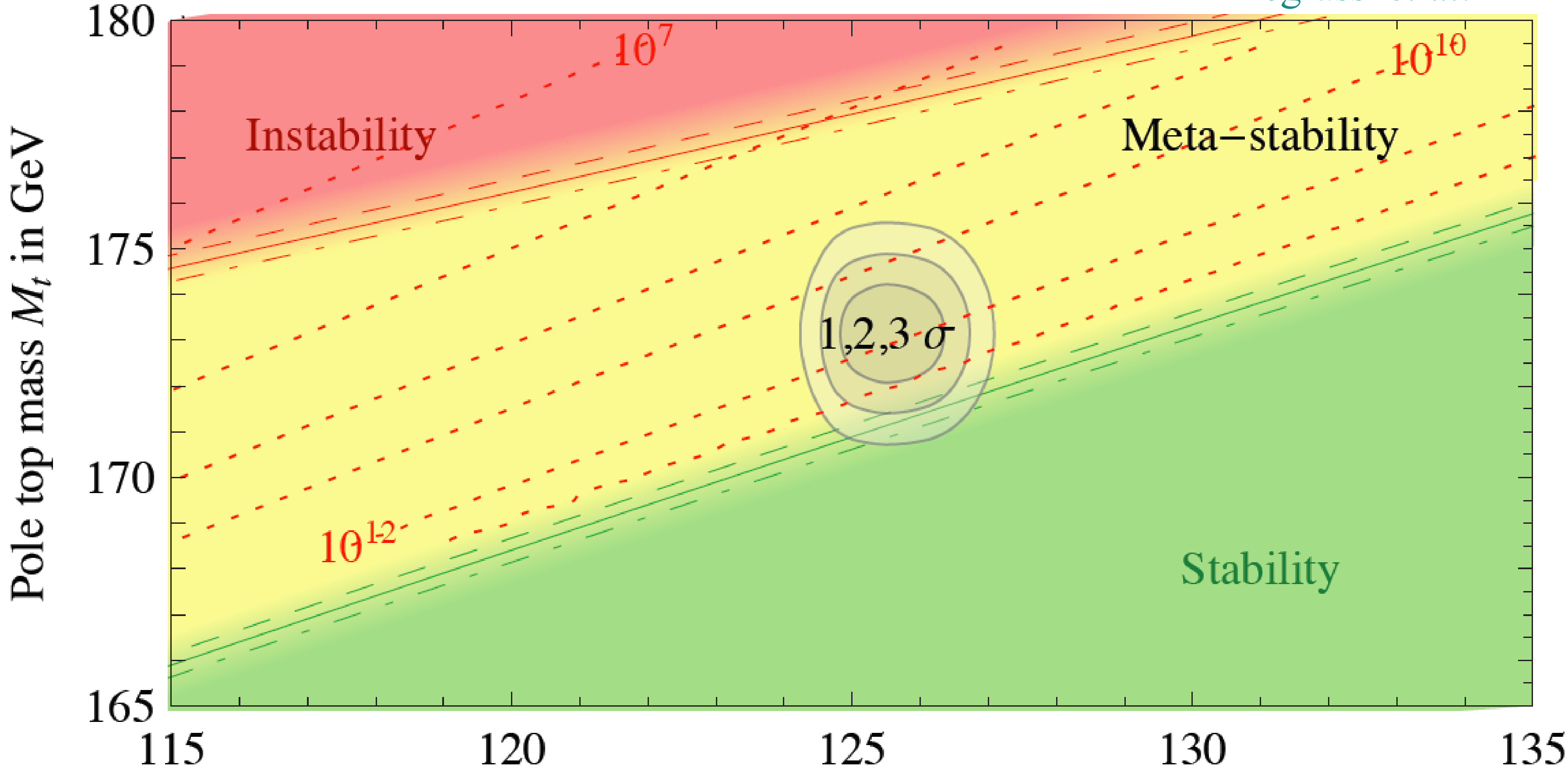

Assuming a precise determination of m_h by ATLAS & CMS in a short time, the main uncertainty will remain the top mass.

Note also that the m_t measured by Tevatron is not really the pole mass (*possible larger error...* Alekhin, Djouadi, Moch '12, Hoang & Stewart, '07-'08)

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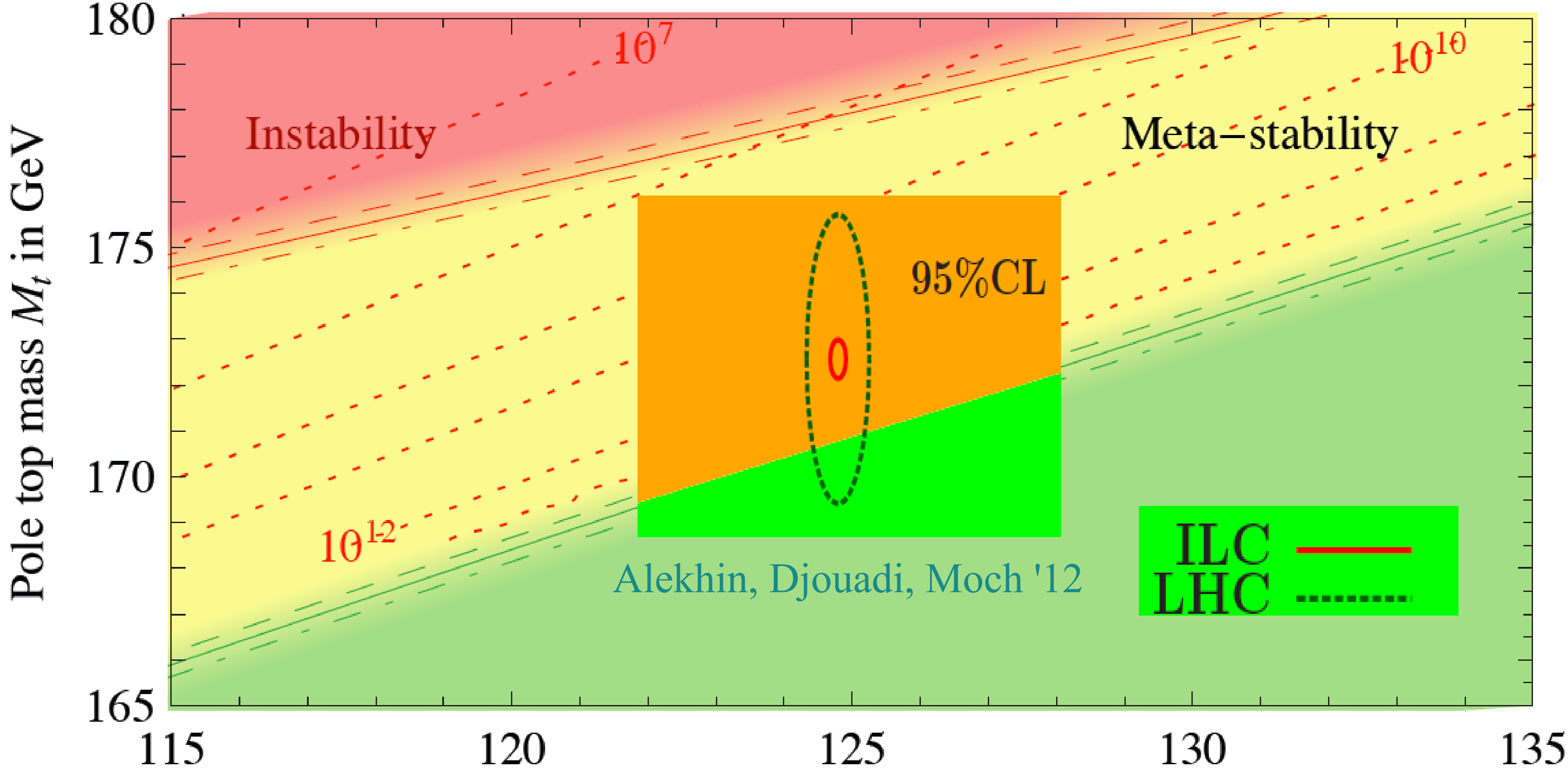

A linear collider would be the ideal machine to bring down this uncertainty, determining more precisely the fate of the SM vacuum (*if in the meanwhile we have not found anything else...!*)

Two additional remarks about the instability of the SM potential:

I. What about the instability because of thermal fluctuations?

II. What about adding to the model heavy right-handed neutrinos?

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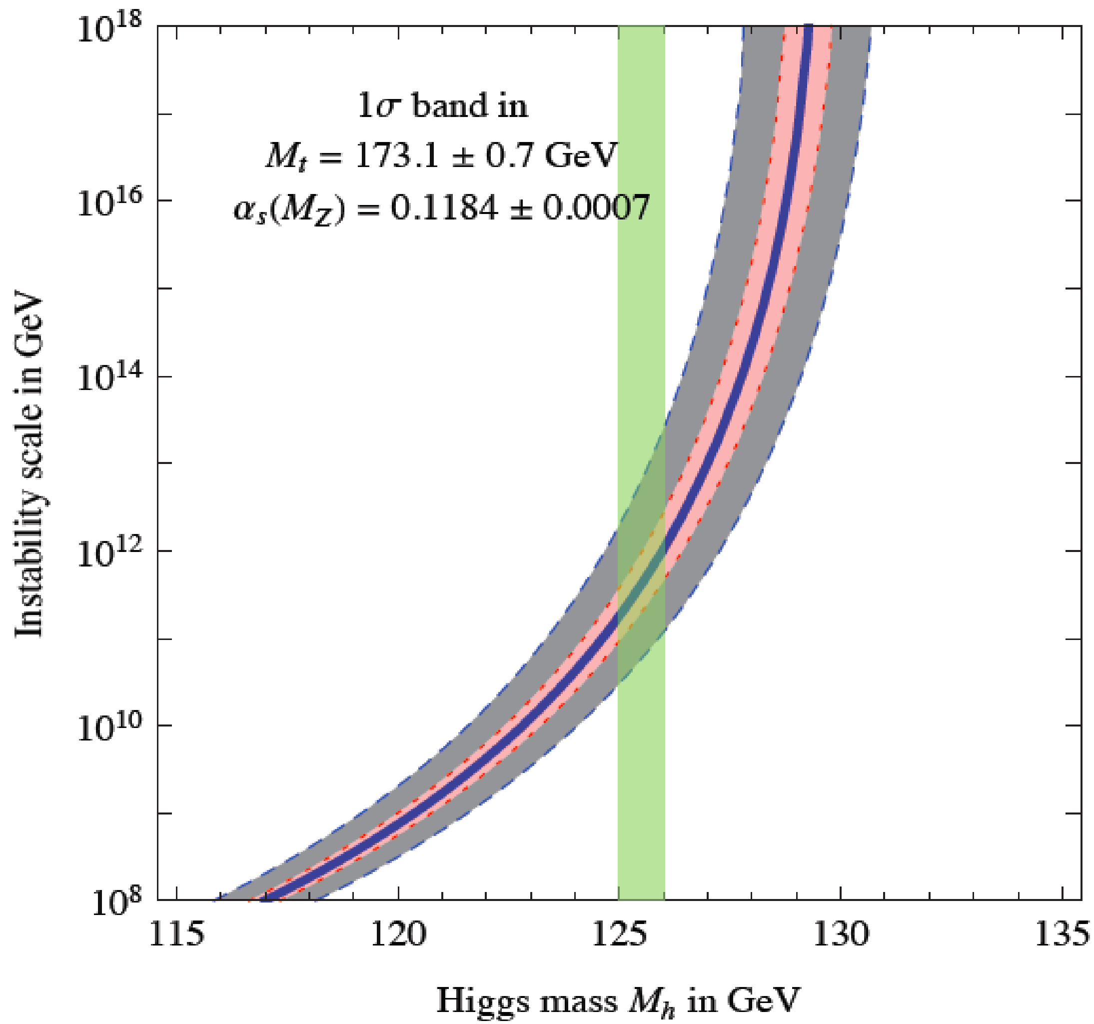

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II. What about adding to the model heavy right-handed neutrinos?

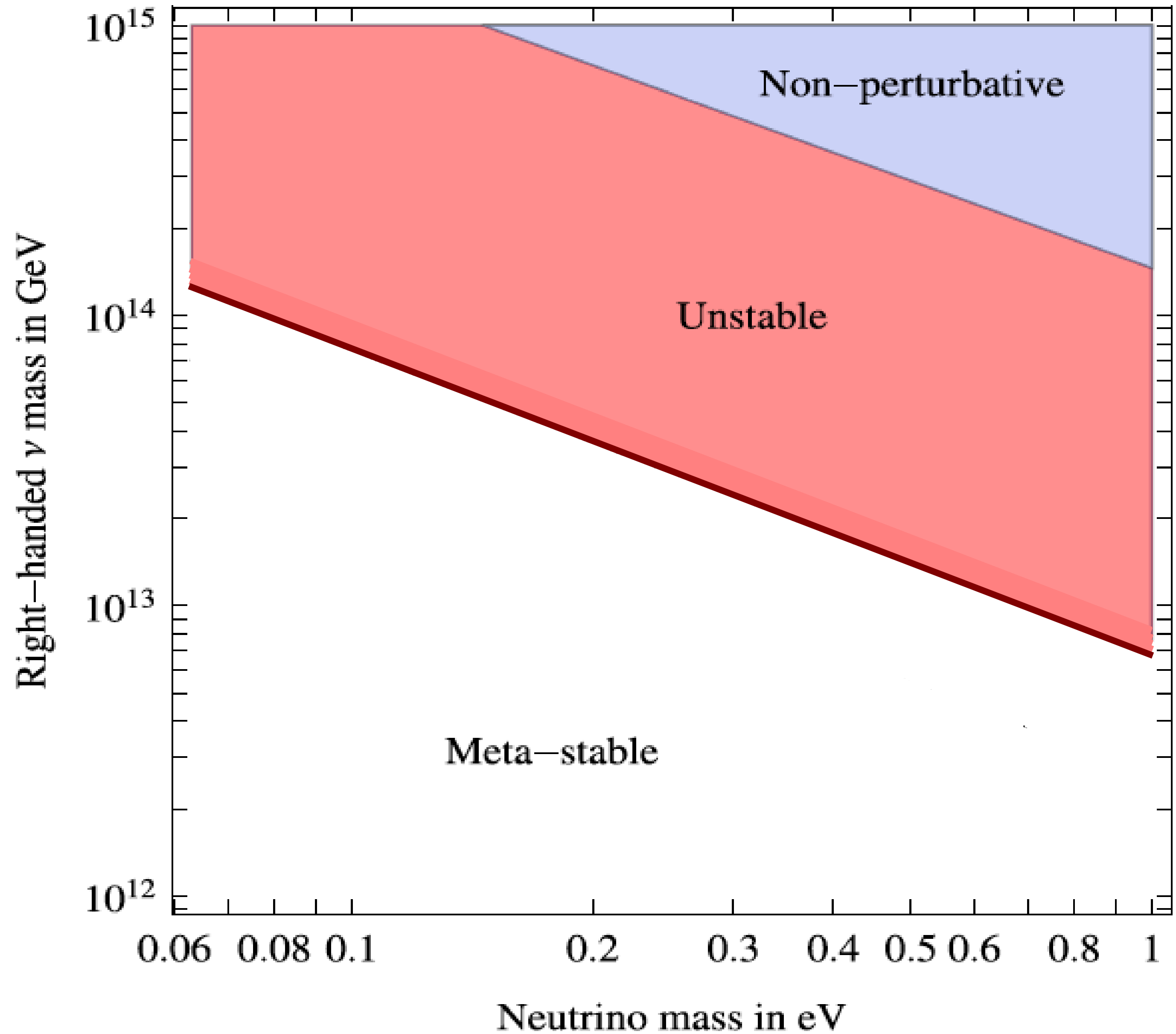
On general ground, adding new fermions may induce a further destabilization of the potential. However, the effect depend on the size of the new Yukawa couplings:

$$m_\nu \sim Y_n^T \frac{v^2}{M_R} Y_n \longrightarrow$$

Requiring a sufficiently stable Higgs potential allow us to derive an upper bound on M_R

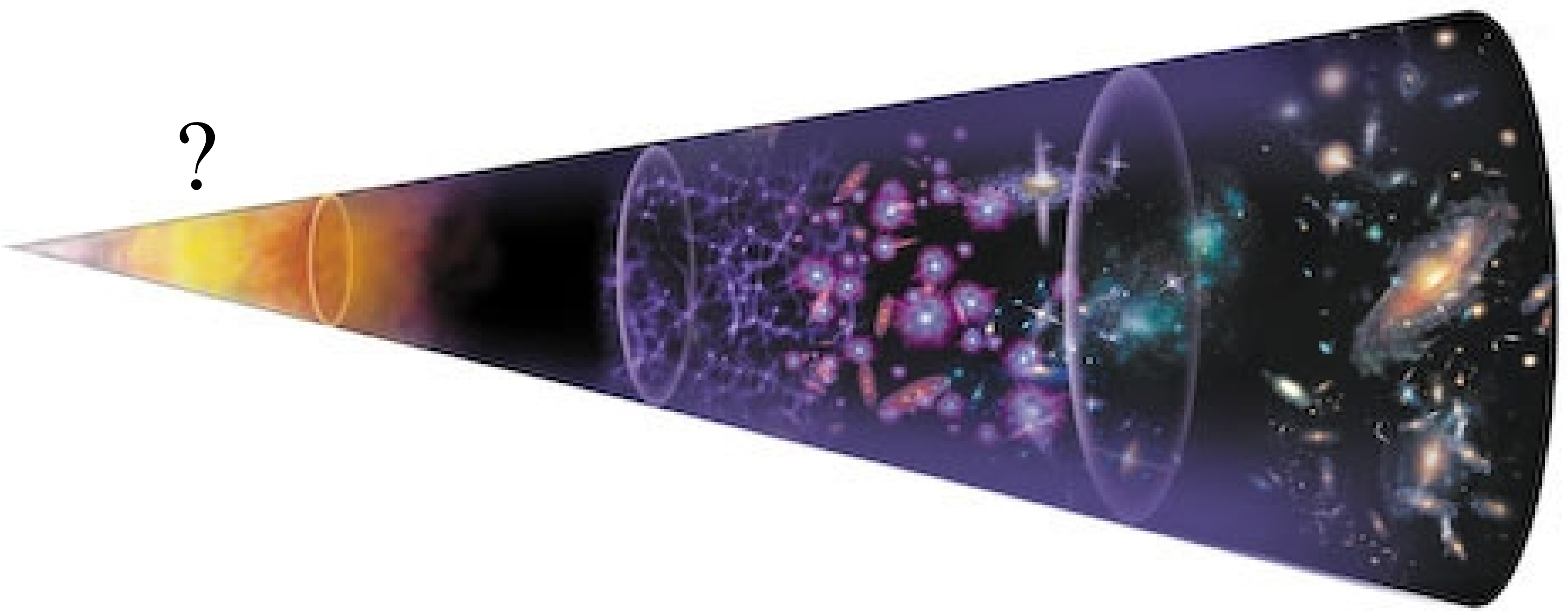
$$m_\nu \sim Y_n^T \frac{v^2}{M_R} Y_n$$

Elias-Miro *et al.* '11

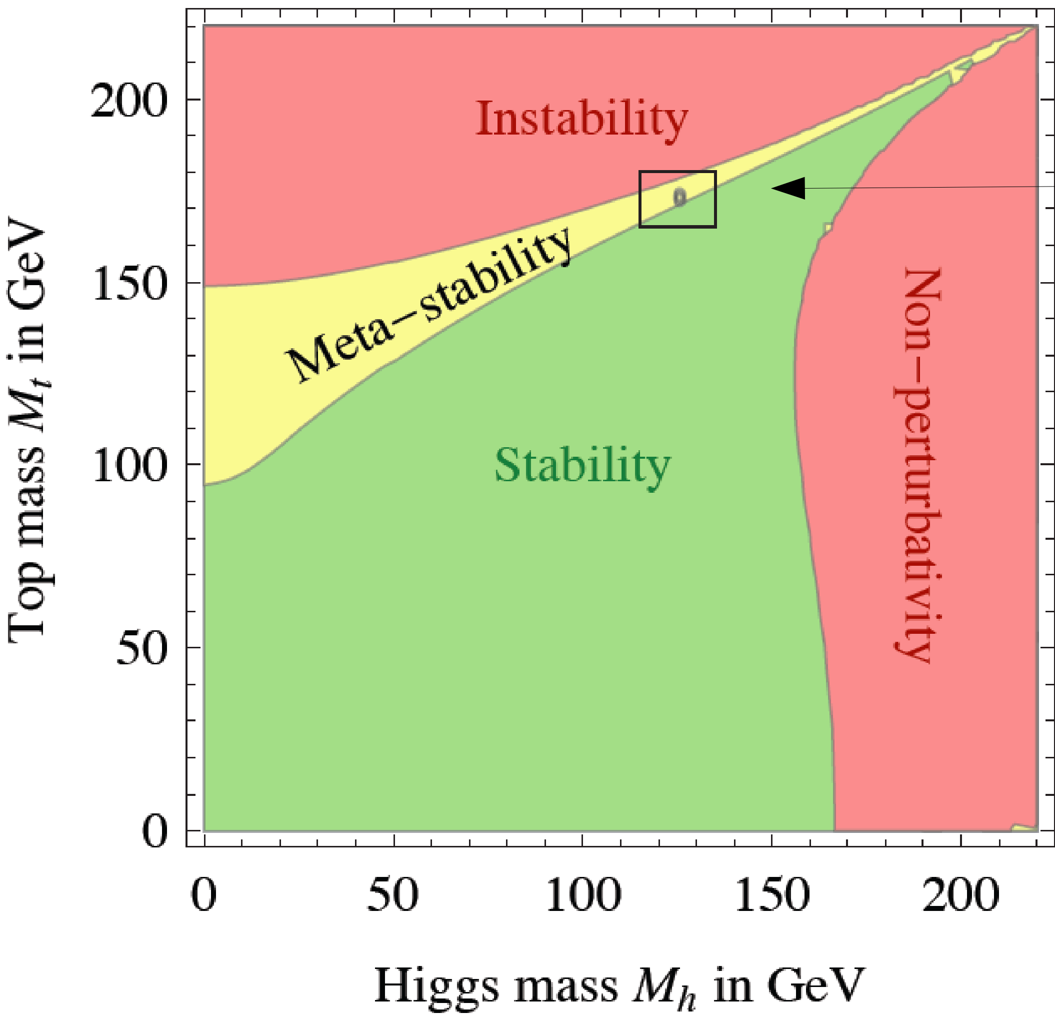


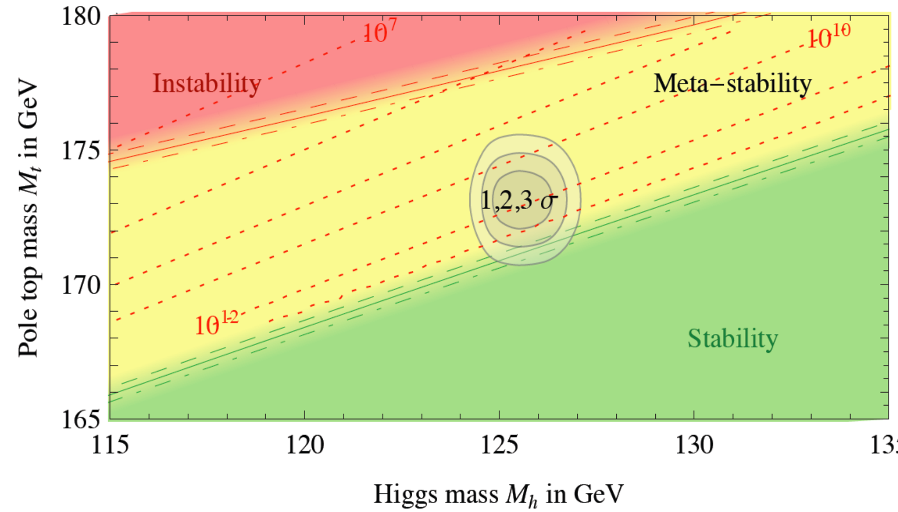
Still enough room for leptogenesis to take place.

Speculations on Planck-scale dynamics



Speculations on Planck-scale dynamics





Looking at the plane from a more distant perspective, it appears more clearly that "we live" in a quite "peculiar" region...

Moving m_t down by ~ 2 GeV, we reach the even more peculiar configuration where $\lambda(M_{pl})=0$

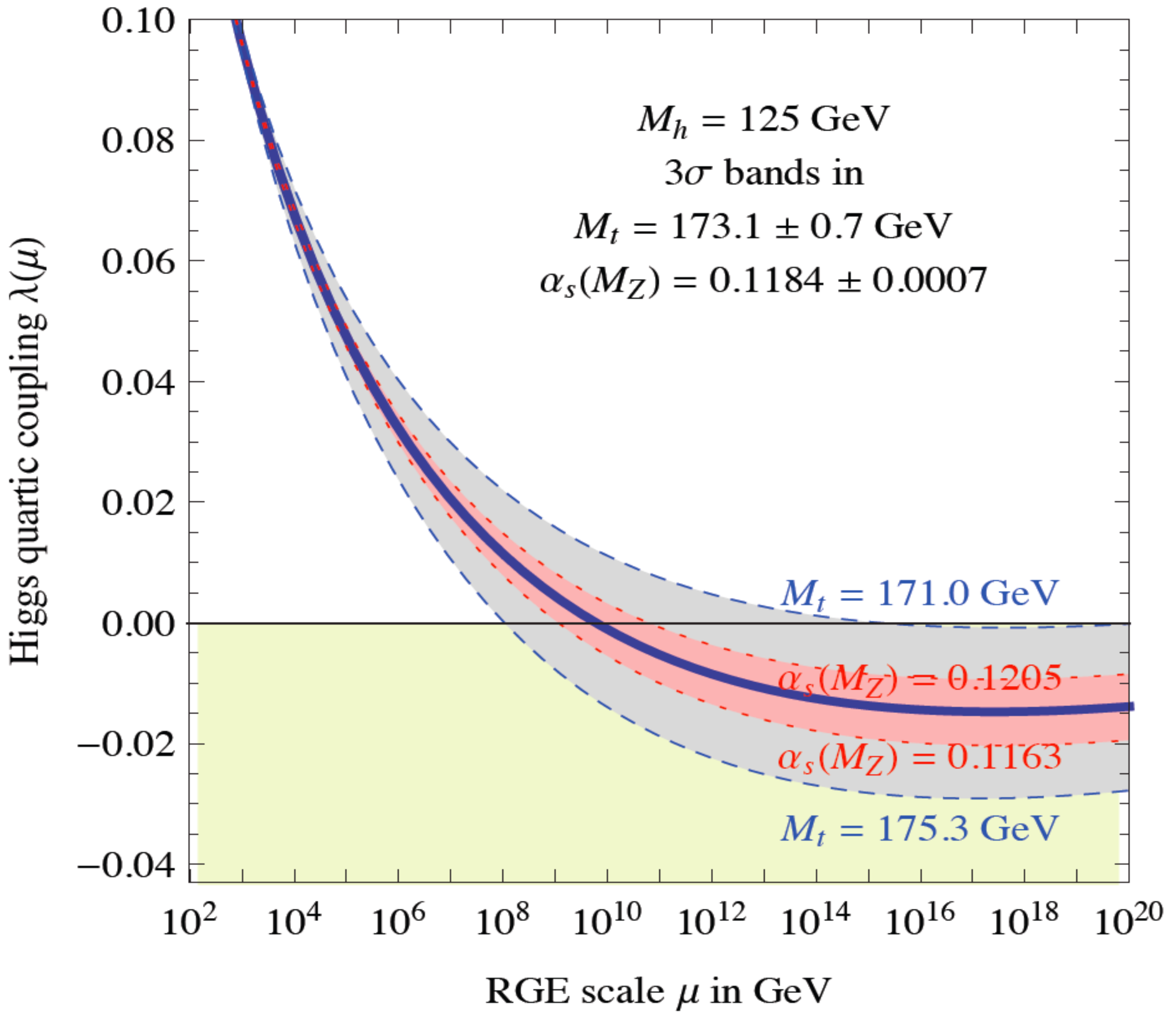
Froggatt, Nielsen, Takanishi, '01
Arkani-Hamed *et al.*, '08
Shaposhnikov, Wetterich, '10
...

Speculations on Planck-scale dynamics

What's special about $\lambda(M_{\rm pl})=0$?

Despite also the beta function vanishes, is not a true fixed point (other coupl. $\neq 0$).

Maybe more interesting the overall smallness of λ compared to the other couplings.



Speculations on Planck-scale dynamics

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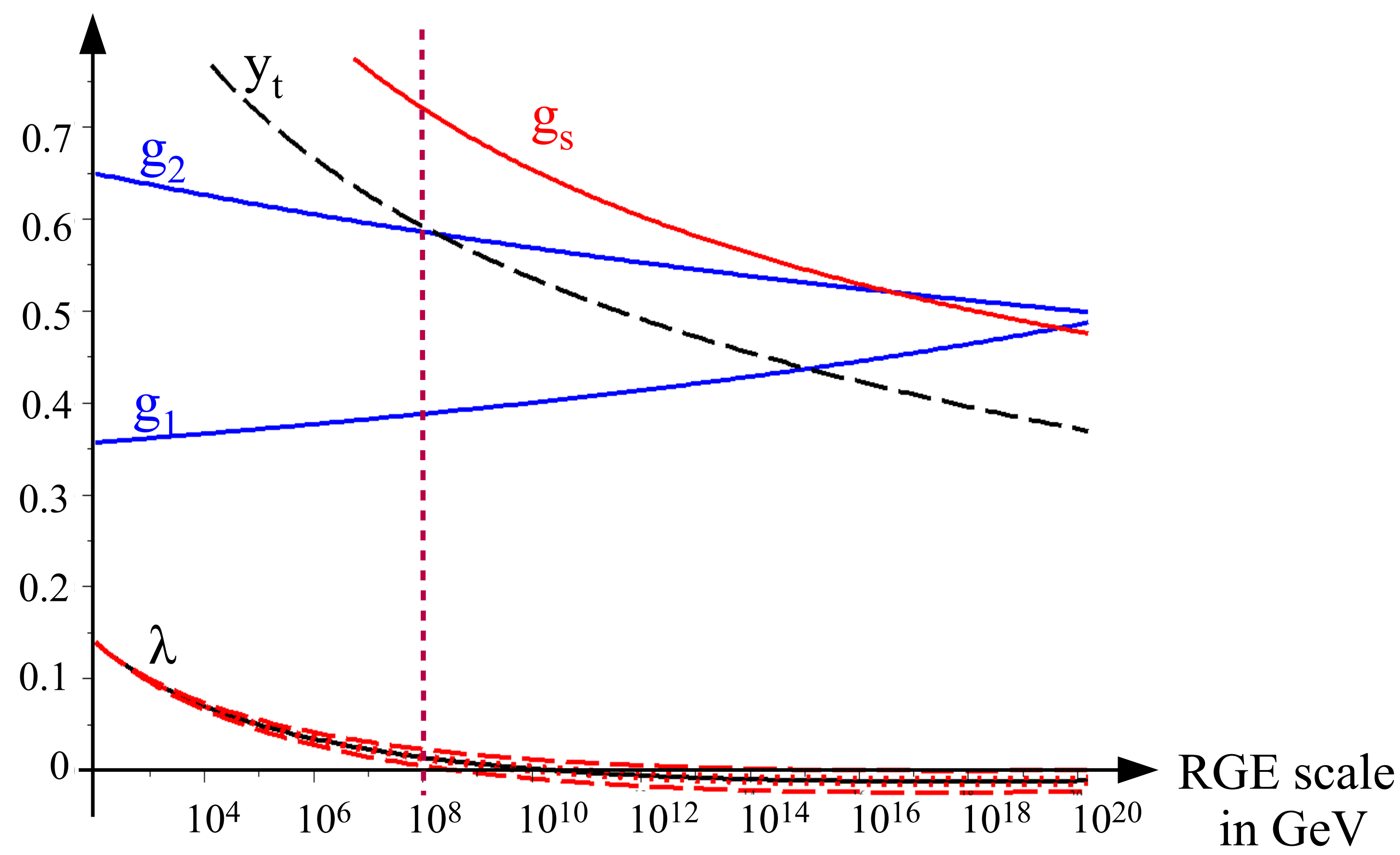
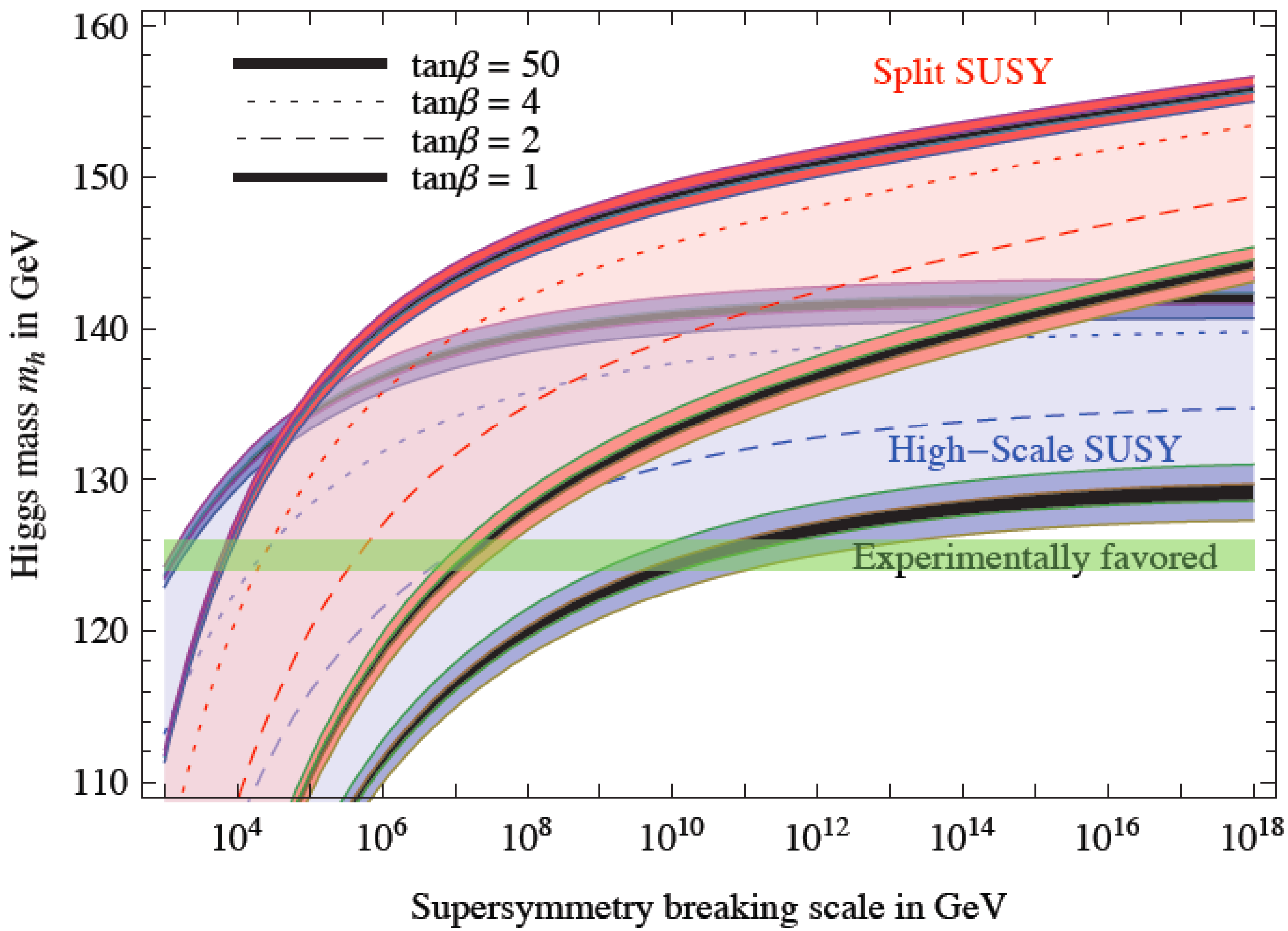
At a scale $\Lambda \gtrsim 10^8$ GeV λ becomes of the same order of its typical e.w. quantum corrections: *hints of a radiatively generated coupling*?



Speculations on Planck-scale dynamics

The smallness of λ certainly fits well with the possibility of a high-scale matching with a weakly coupled theory

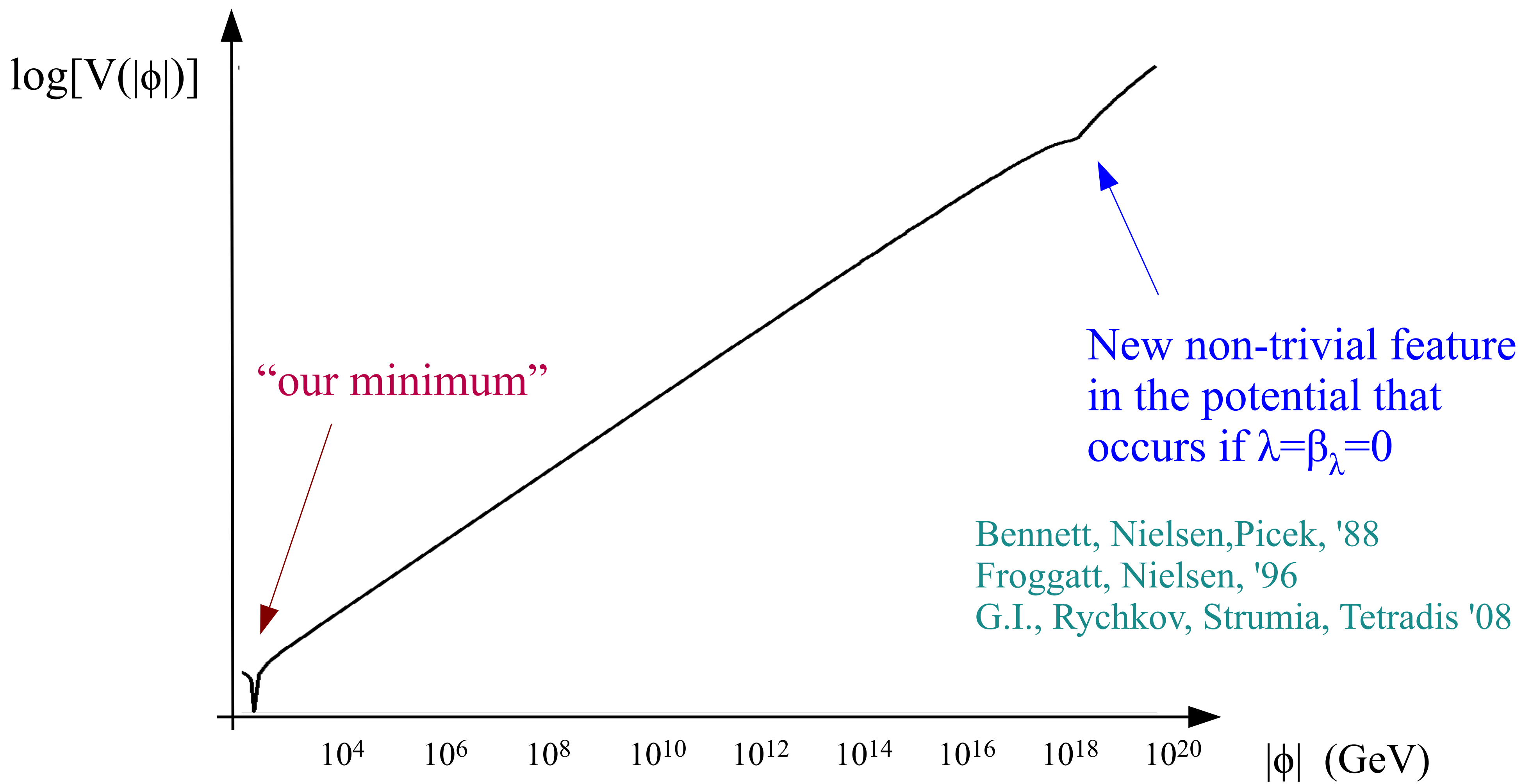
Giudice & Strumia '11-'12



Speculations on Planck-scale dynamics

Probably the most attractive feature of having $\lambda=0$ close to M_{pl} (*assuming no new physics below such scale*) is the possibility that the Higgs field has played some role in the early Universe, during inflation.

Bezrukov & Shaposhnikov, '08
Notari & Masina '11-'12



Bennett, Nielsen,Picek, '88
Froggatt, Nielsen, '96
G.I., Rychkov, Strumia, Tetradis '08

Speculations on Planck-scale dynamics

Probably the most attractive feature of having λ=0 close to $M_{\rm pl}$ (*assuming no new physics below such scale*) is the possibility that the Higgs field has played some role in the early Universe, during inflation.

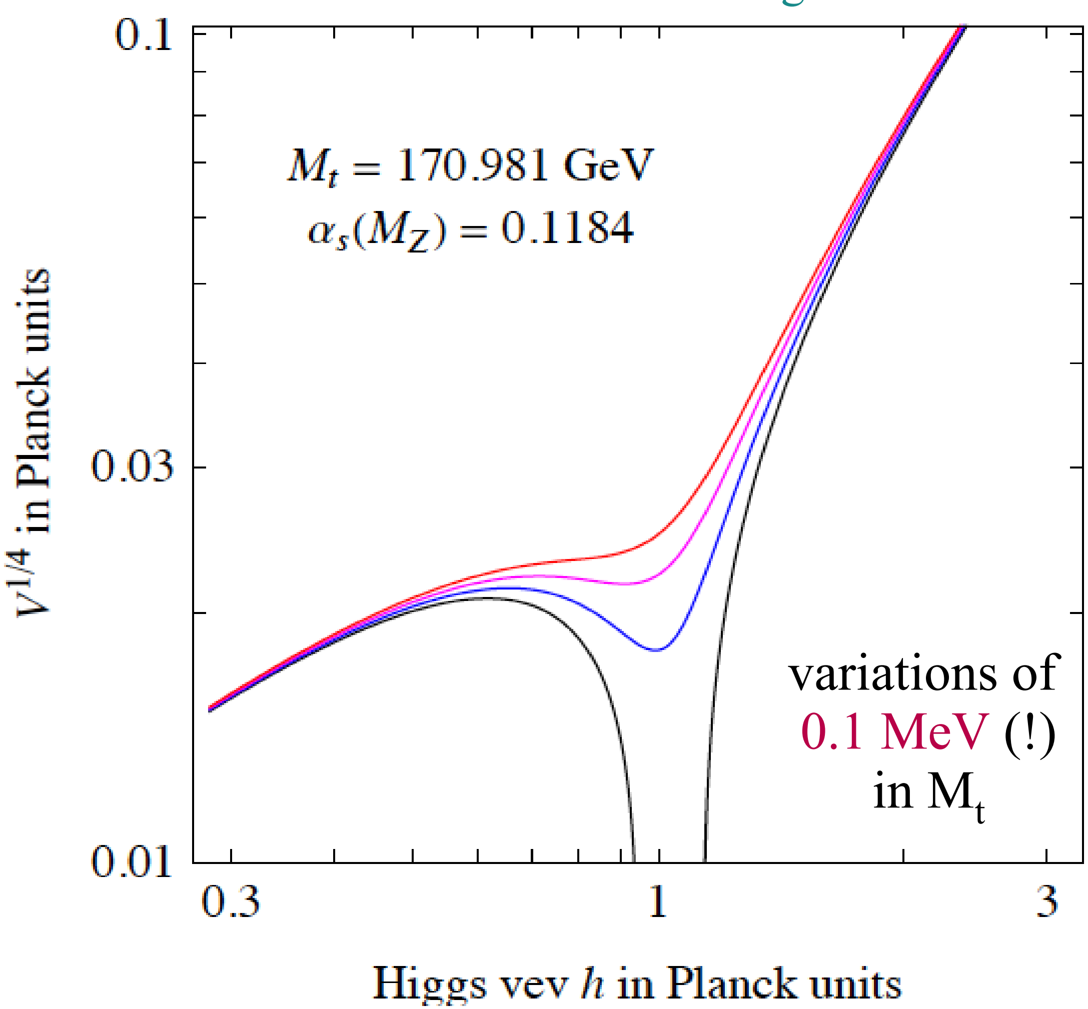
Degrassi *et al.* '12

The minimal set-up (SM only) does not work (*field trapped into the new minimum or too large fluctuations*)

But the problem can be solved with non-minimal couplings of the Higgs field to gravity and/or to other fields

Bezrukov & Shaposhnikov, '08
Notari & Masina '11-'12

The minimality of the scheme is lost, but it remains an intriguing possibility.



Conclusions

- A SM-like Higgs with $m_h \sim 125$ GeV does not allow us to derive model-independent conclusions about the scale of New Physics: the Higgs potential is most likely unstable, but the e.w. vacuum is certainly sufficiently long-lived.

Conclusions

- A SM-like Higgs with $m_h \sim 125$ GeV does not allow us to derive model-independent conclusions about the scale of New Physics: the Higgs potential is most likely unstable, but the e.w. vacuum is certainly sufficiently long-lived.

- Clear indication about a small, or even vanishing, Higgs self-coupling at high energies: if the SM is only an effective theory, we have to match it into a model where the Higgs

    - is a weakly interacting particle, if the matching occurs close to the e.w. scale [*as indicated by naturalness*]

    - may have a vanishing intrinsic self-coupling (trivial $\lambda\phi^4$, with gauge & Yukawa), if the matching occurs above $\sim 10^8$ GeV

- More precise determinations of both m_h & m_t would be very useful, especially in absence of other NP signals, to better investigate the structure of the Higgs potential at high energies