Detecting High-Frequency Gravitational Waves with Intense Magnetic Fields

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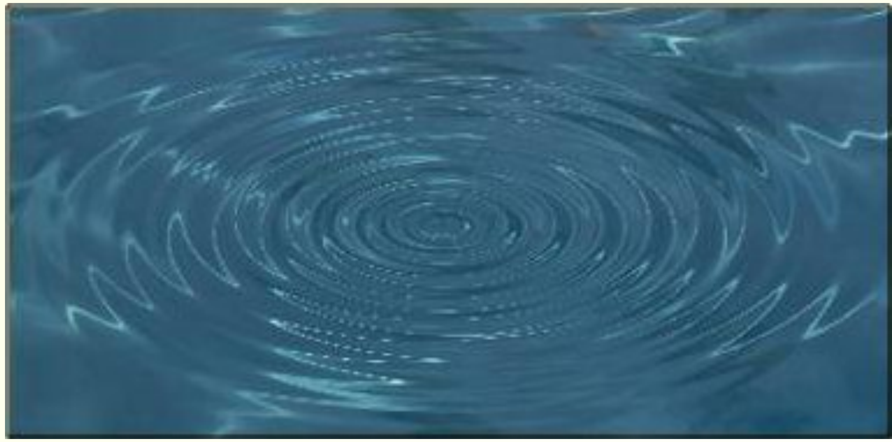
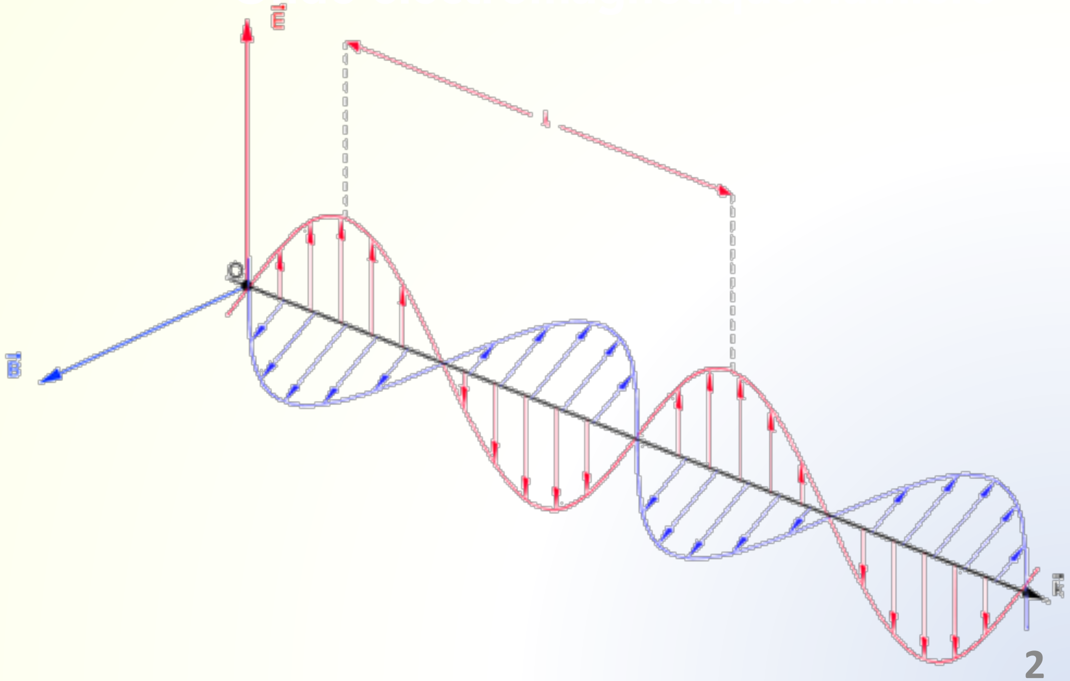
Gravitational Waves

Onde à la surface de l'eau
« onde de gravité »

- Not to be confused with « Gravity Waves »
  - Waves in matter distribution with gravity as driving force
  - ex.: deformation at the surface of water
  - Detection : oscillation of a buoy



- Light :
  - Wave of electromagnetic fields
  - Detection: oscillation of electric charges



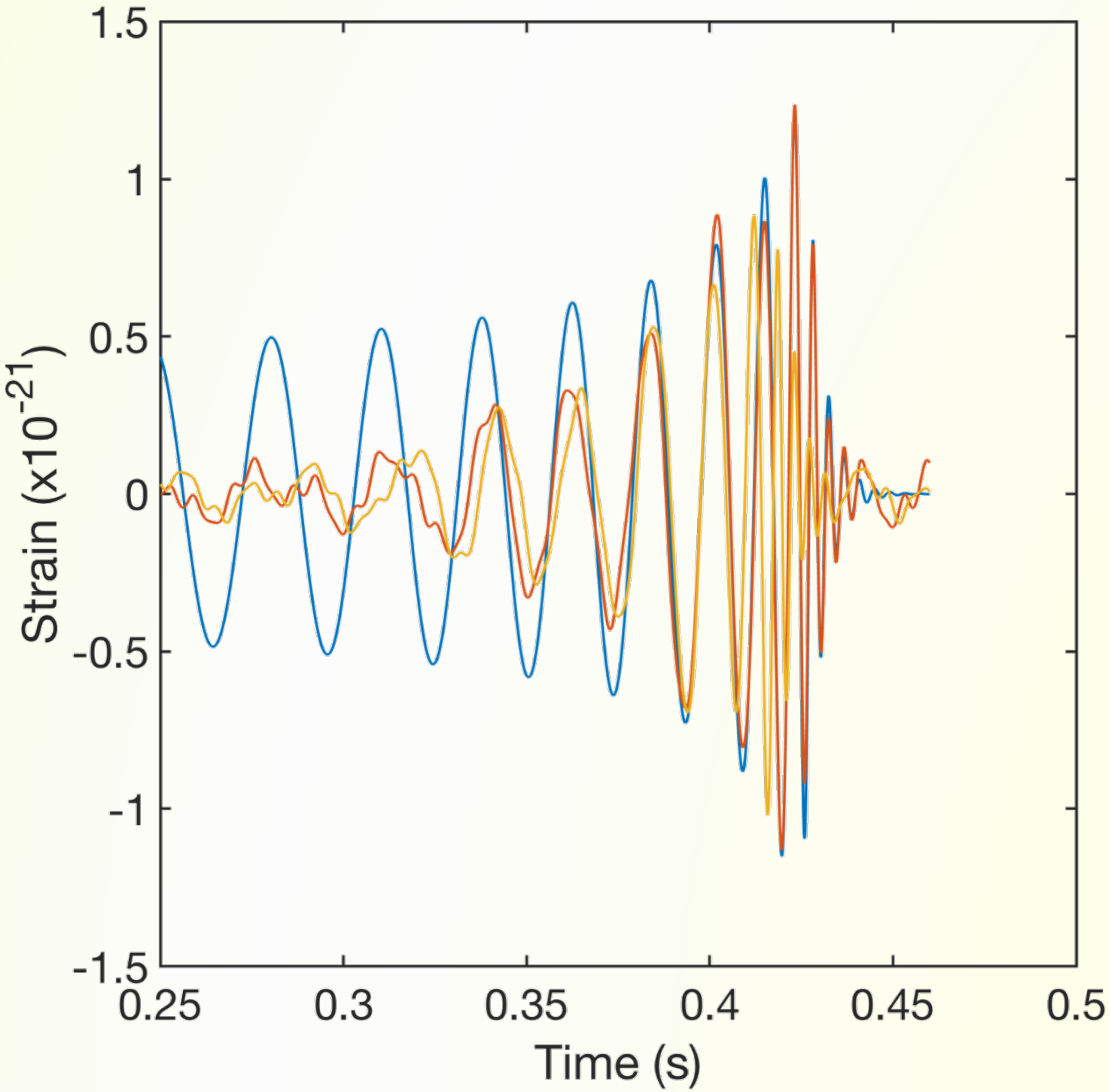
- Gravitational waves :
  - « gravitational light »
  - Wave of space-time deformation
  - Detection : oscillation of inertial masses

Selected facts on gravitational waves

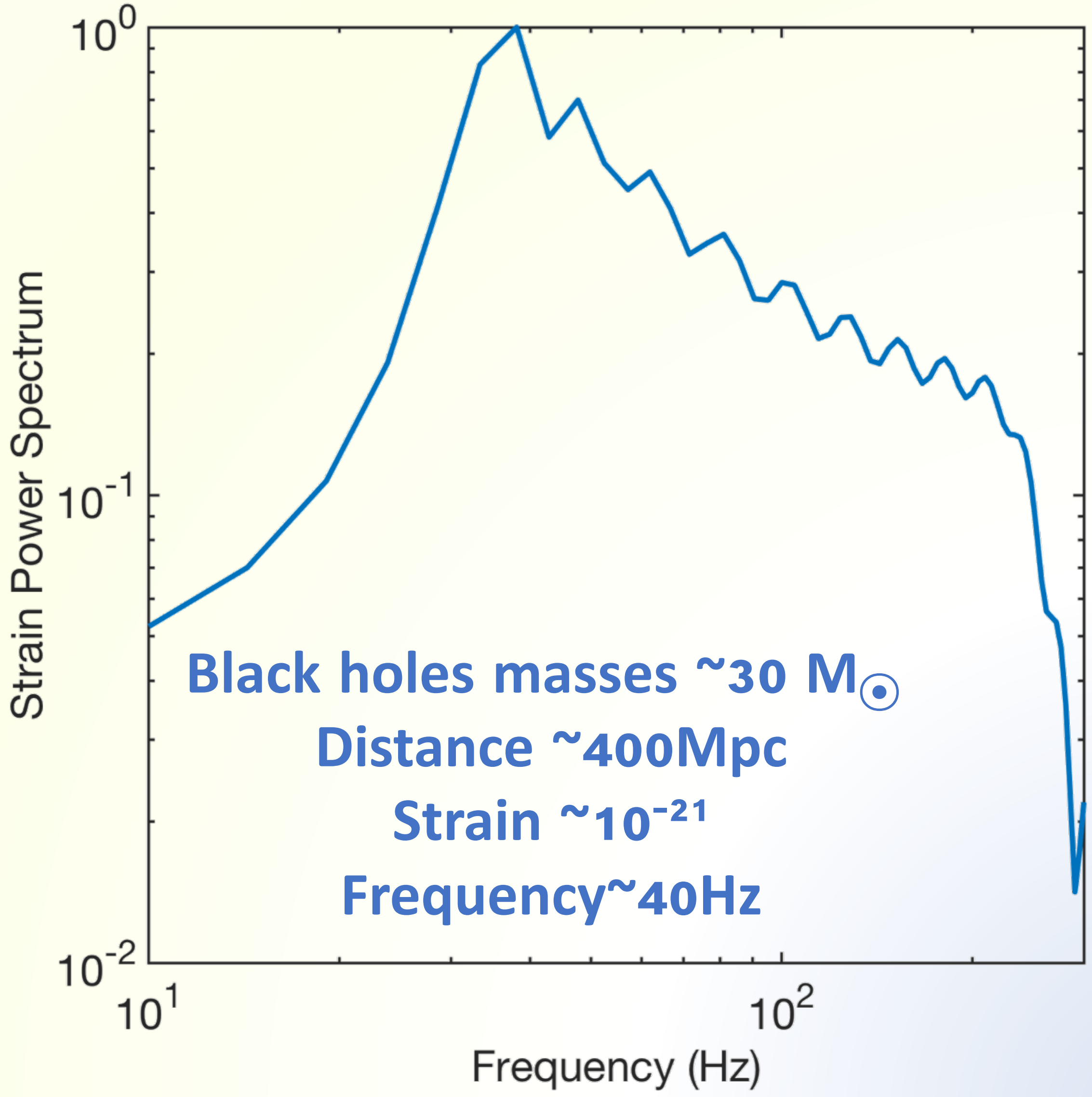
- Theoretical discovery: Einstein 1916
- First detection attempts: mechanical resonators by Weber 1960s
- Indirect detection (1974-1990) in binary pulsars (Nobel 1993)
- Concepts and prototypes of laser interferometer (1970-1980)
- LIGO first generation : runs 2002-2000 without detection
- upgrade to Advanced LIGO (second generation) : 2008-2015
- First run O1 (sept.2015-Jan. 2016) : three events of black hole mergers
- Physics Nobel Prize 2017
- now at 3 runs, 50 detections (black hole and neutron star mergers)
- Third generation : Einstein telescope

The first detection : GW150914

Theoretical waveform and LIGO signals



Power spectrum



High-frequency Gravitational waves

- Frequency range : from 10kHz to 10^{12}Hz !
- Astrophysical sources:
  - neutron star merger and quakes, primordial black hole merger, relics from very early universe (inflation, phase transitions, topological defects, …), braneworld, etc.
- Laboratory sources (for a gravitational Hertz experiment):
  - extremely weak but controllable ; possibility of long duration experiment
- Some high-frequency detector concepts (>10kHz):
  - Mechanical deformation of high Q microwave cavity (Reece, 1984)
  - Coupled superconducting cavities (P. Bernard et al., 1999) ~1MHz
  - Magnetic conversion of GW to EM waves (inverse Gerstenshtein effect)
  - GW effect on microwave beam polarization (Cruise, 1983-present)
  - Bulk acoustic wave resonators (Goryachev-Tobar, 2014) : 1MHz-1GHz
- Magnetic conversion similar to axion search with haloscopes
  - sensitivity of ADMX ~10^{-24} W of induced EM radiation at ~100MHz thanks to special SQUID receiver

Electromagnetism and gravitation

- Einstein's equivalence principle
- General relativity (Einstein's equations) :

Matter sources — Electromagnetic fields

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4}\left(T^{(\text{mat})}_{\mu\nu} + T^{(\text{em})}_{\mu\nu}\right)$$

Spacetime curvature

Equivalence:
Any types of energy, same constant strength

- Pure Einstein-Maxwell system ($T_{\mu\nu}^{(\text{mat})}=0$)

Einstein equations

$$R_{\mu\nu} = -\frac{8\pi G}{c^4} T^{(\text{em})}_{\mu\nu}$$

$$T^{(\text{em})} = 0 \Rightarrow R = 0$$

Maxwell equations on curved space

$$\nabla_\alpha F_{\beta\gamma} + \nabla_\gamma F_{\alpha\beta} + \nabla_\beta F_{\gamma\alpha} = 0$$

$$\nabla_\mu F^{\mu\nu} = \mu_0 J^\nu$$

Electromagnetism and gravitation

- Electromagnetic source (stress-energy tensor) :

$$T^{(\mathrm{em})}_{\mu\nu} = -\frac{1}{\mu_0}\left(g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

Electromagnetic stress-energy tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Faraday tensor with A_μ electromagnetic vector potential $A^\mu = \left(V/c, \vec{A}\right)$

- or $T_{\mu\nu} \sim -\begin{pmatrix} \epsilon_0 E^2 + B^2/\mu_0 & \vec{S}/c \\ \vec{S}/c & M_{3\times 3} \end{pmatrix}$

S : Poynting vector
M : Maxwell stress tensor

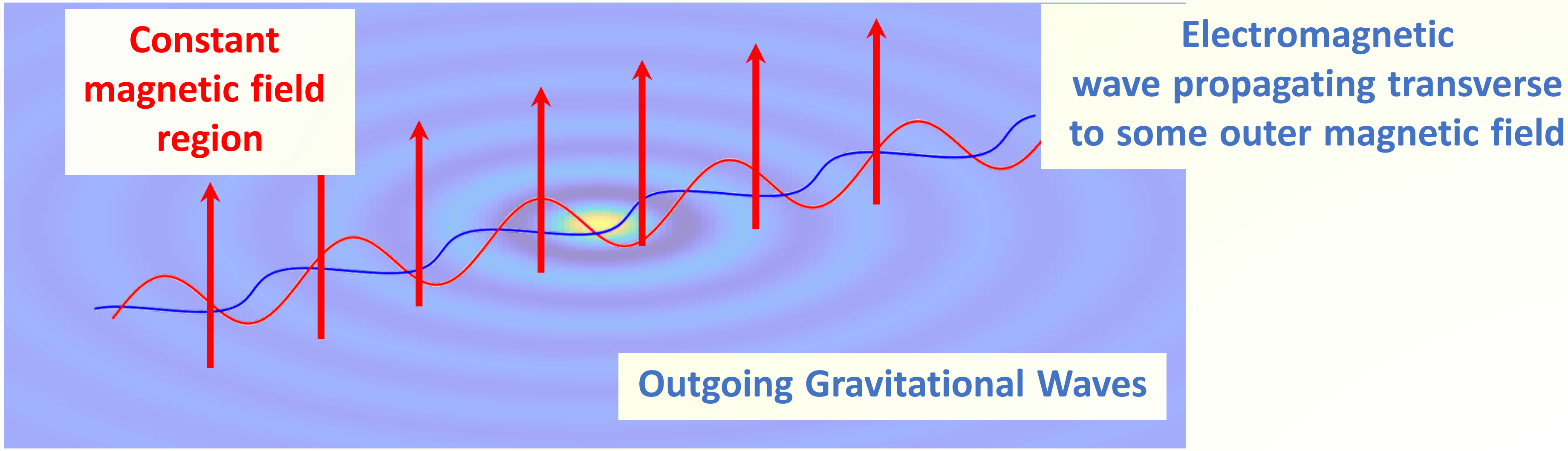
- Gravitation from electromagnetism: $F^{tot} = F^{static} + F^{wave}$
  - static-static terms in $T^{(em)}$ => static grav. fields (from coil, capacitor, etc.)
  - wave-wave terms => Gravitational Waves from EM waves (e.g. laser pulses)
  - static-wave terms => wave resonance : Gertsenshtein effect

Gravitation from electromagnetism: some proposed experiments

1. Static EM to static gravitational field :
   - Static gravitational field from coils (Füzfa, 2016)
2. EM Wave – GW :
   - high-intensity Laser pulses (Rätzel et al. 2016)
   - EM resonators as GW emitters/receivers
3. Static EM field -GW (Direct Gertenshtein effect) :
   - EM wave generation from static electric field in a capacitor (Lupanov, 1967)
   - Gravitational Hertz experiments
     - EM Resonators in external magnetic fields (Grischuk & Sazhin, 1974-2003)
     - With Fabry-Perot cavities into external magnetic field (Kolosnitsyn & Rudenko 2015)

Gravitational waves generation with wave resonance

- Direct Gertsenshtein effect (1962):



- Amplitude of generated Gravitational Wave :

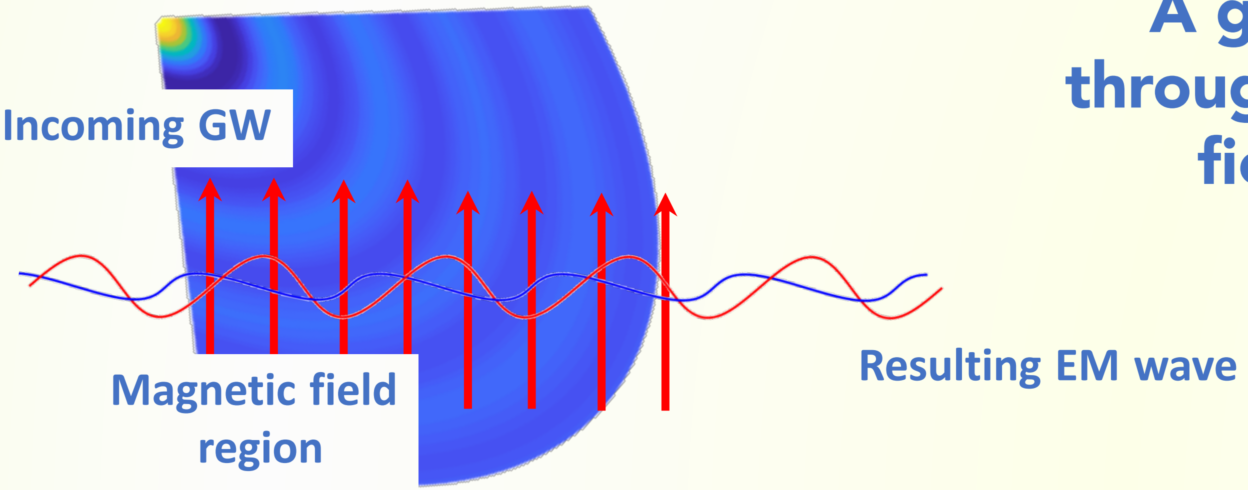
Of order of the Gerstenshtein number :

$$\mathcal{G}_Z = \frac{4GB_0E_0L^2}{c^5\mu_0}$$

G : Newton's constant
B_0: external magnetic field
E_0: TEM wave electric field
L: size of interacting region
c : speed of light in vacuum
μ_0: magnetic permeability

Direct and inverse Gertsenshtein effects

- Direct Gertsenshtein effect: GW from EM waves
  - Tiny coupling of GW generation : $\frac{4G}{c^5\mu_0} \approx 10^{-46}\,(T.V.m)^{-1}$
  - For generating a strain $h \sim 10^{-21}$ with external $B_0 \sim 10$ T ; $E_0 \sim 1$MV/m, one needs L ~120 lyr!
  - Astrophysical application (Zeldovich, 1973)
- « Inverse Gertsenshtein effect » : EM waves from GW !



A gravitational wave $h^{\alpha\beta}$ passing through a transverse constant magnetic field produces a faint EM wave

- Application to GW detection !

Inverse Gertsenshtein effect

- From Maxwell equations on curved space

$$\nabla_\mu F^{\mu\nu} = 0$$

(second group)

GW through a magnetic field

Total EM field = Background + Induced

$$F_{\mu\nu} = F^{(0)}_{\mu\nu} + F^{(1)}_{\mu\nu}$$

TEM wave

External Static Magnetic field

$$\partial_\mu F^{\mu\nu}_{(1)} = -\eta^{\mu\alpha}\partial_\mu h^{\beta\nu} F^{(0)}_{\alpha\beta} \equiv \mathcal{J}^\nu$$

- Modified Maxwell wave equations for electric & magnetic fields:

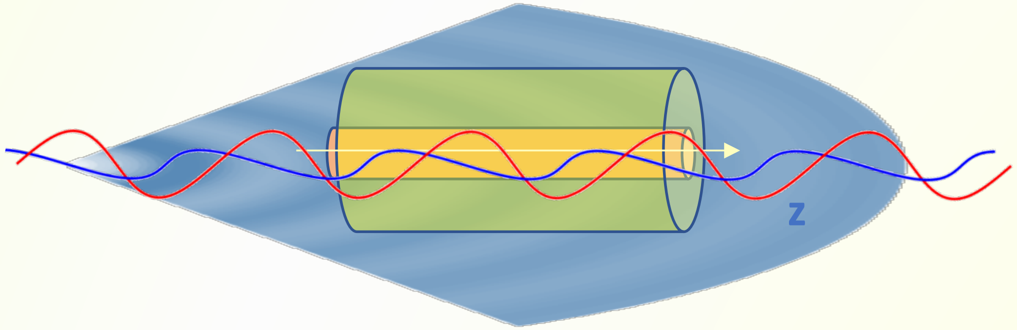
$$\left(\partial^2_{ct} - \vec{\Delta}\right) \vec{B}^{(1)} = B_0 \vec{S}(h, \partial^2 h)$$

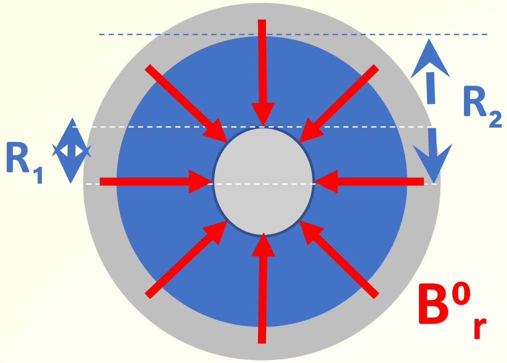
For the induced Magnetic field

- Magnetic field tranverse to a passing GW:
  - -> generation of TEM waves
  - -> same frequency spectrum than GW (wave resonance)
  - -> induced magnetic field $||B^{(1)}||\sim B_0$ * GW strain
  - -> Variation of EM energy : $\Delta E \approx \frac{B_0}{\mu_0}\int_V B^{(1)} dV + \frac{1}{2\mu_0}\int_V \left(B^{(1)}\right)^2 dV$

A case study of an electromagnetic GW detector

- TEM coaxial waveguide inside a radial magnetic field





- TEM mode & boundary conditions:
  - $E^{(1)}_z = B^{(1)}_z = 0$; $E^{(1)}_\varphi(R_1) = E^{(1)}_\varphi(R_2) = B^{(1)}_r(R_1) = B^{(1)}_r(R_2) = 0$
- Cylindrical harmonics decomposition (spectral method) :

$$B_r^{(1)} \approx \sum_{kmn} C_{kmn}(t) \cdot \mathcal{R}_{km}(r) \cdot \begin{pmatrix} \cos \\ \sin \end{pmatrix} (m\varphi) \cdot \begin{pmatrix} \cos \\ \sin \end{pmatrix} (\frac{2\pi n z}{L})$$

Linear combination of Bessel Functions of 1^{st} and 2^{d} kinds

- Maxwell wave equations => forced harmonic oscillators $C_{kmn}(t)$

Based on patents PCT/EP2018/086758 & PCT/EP2018/086760

A case study of an electromagnetic GW detector

- Energy variation inside the resonator: $\Delta E \approx \frac{B_0}{\mu_0}\int_V B^{(1)} dV$ **At first order in GW strain**

$$\Delta E \approx \frac{B_0}{\mu_0}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi}\int_{R_1}^{R_2} \boxed{B_r^{(1)}}.r.dr.d\varphi.dz$$

For external radial magnetic field B_0

- Induced radial magnetic field $B^{(1)}$ is boosted by external field B_0
- Only the m=n=0 radial modes contribute to ΔE at first order in GW strain

$$\Delta E(t) \sim 2\pi \underbrace{\frac{B_0^2 L^3 |h_+|}{\mu_0}}_{\text{Coil's magnetic energy x GW strain}} \underbrace{\sum_k C_{k00}(t)\frac{1}{L^2}\int_{R_1}^{R_2}\mathcal{R}_{k0}(r).r.dr}_{\text{Geometrical factors} \sim R_2 - R_1}$$

Coil's magnetic energy x GW strain

Geometrical factors ~R_2-R_1

Based on patents PCT/EP2018/086758 & PCT/EP2018/086760

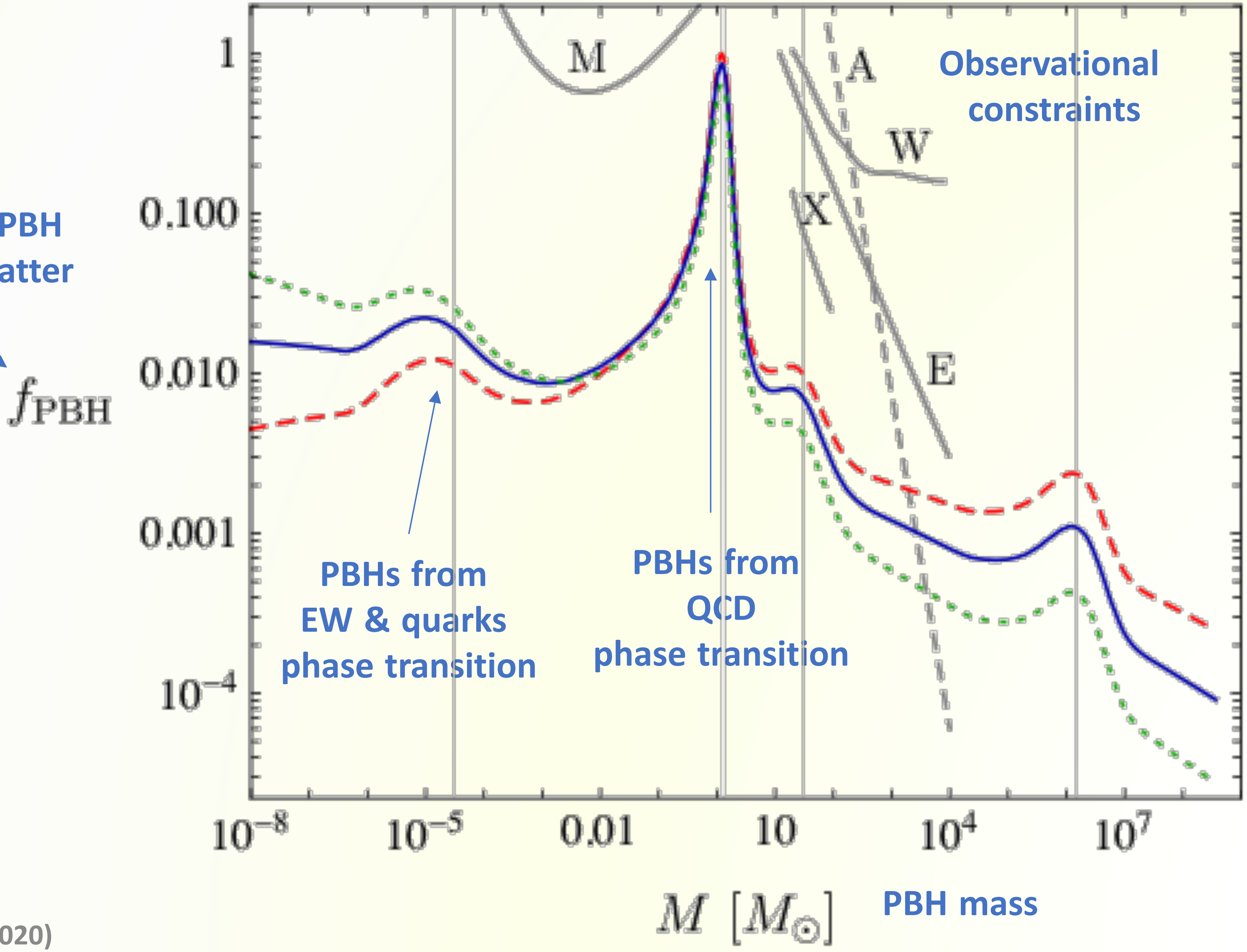
Application to detection of binary primordial black holes

- PBHs : relics of very early universe – formation from strong density fluctuations deep inside radiation dominated era
- Strong candidate for (fraction of) dark matter, seeds of supermassive and intermediate-mass black holes
- Mass far below stellar masses
- Mass range depends on formation epoch
- Evaporation by Hawking radiation after $t \sim 10^{-17}$ (mass/1kg)^3 sec
- Coalescence of binary PBHs system produce GW of frequencies above 10kHz, outside the reach of present laser interferometers

=> detection by electromagnetic detectors!

Application to detection of binary primordial black holes

Present fraction of PBH among cold dark matter

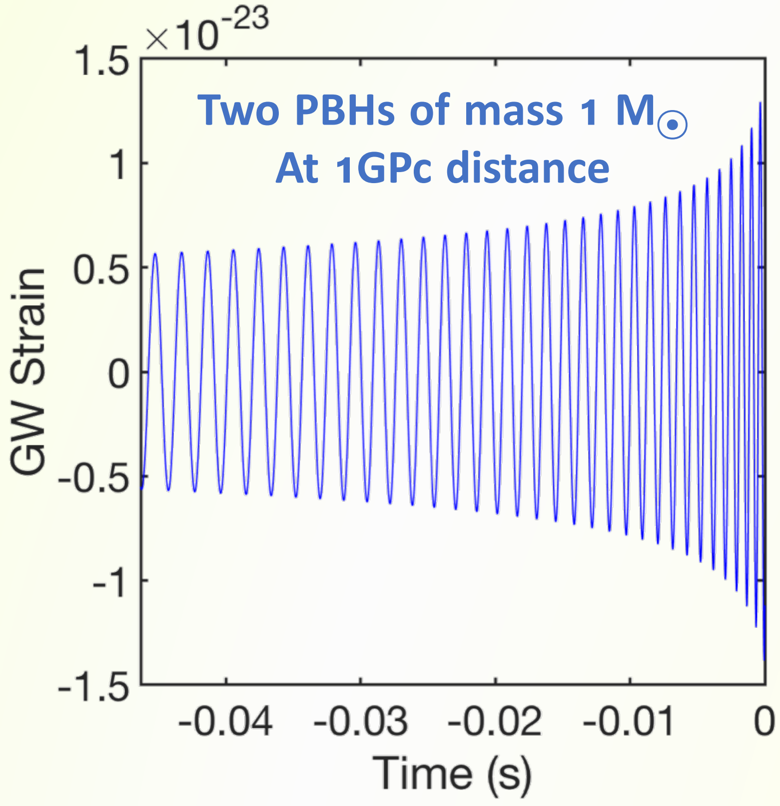


PBH mass

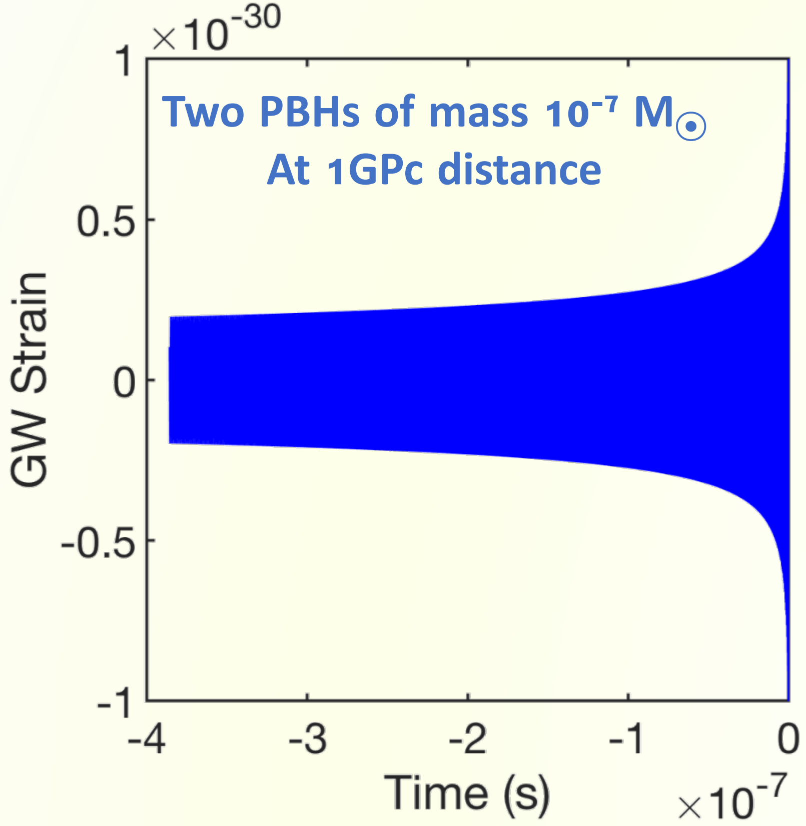
Spectral indexes: n_s=0.965, 0.97, 0975

From B. Carr et al., arXiv:1906.08217

Some possible signals : time domain



Mass (M☉)	Strain	Time scale (s)	Lower frequency (Hz)
10^{-7}	10^{-30}	10^{-7}	10^{9}
10^{-5}	10^{-28}	10^{-5}	10^{7}
10^{-4}	10^{-27}	10^{-4}	10^{6}
1	10^{-23}	10^{-2}	10^{3}



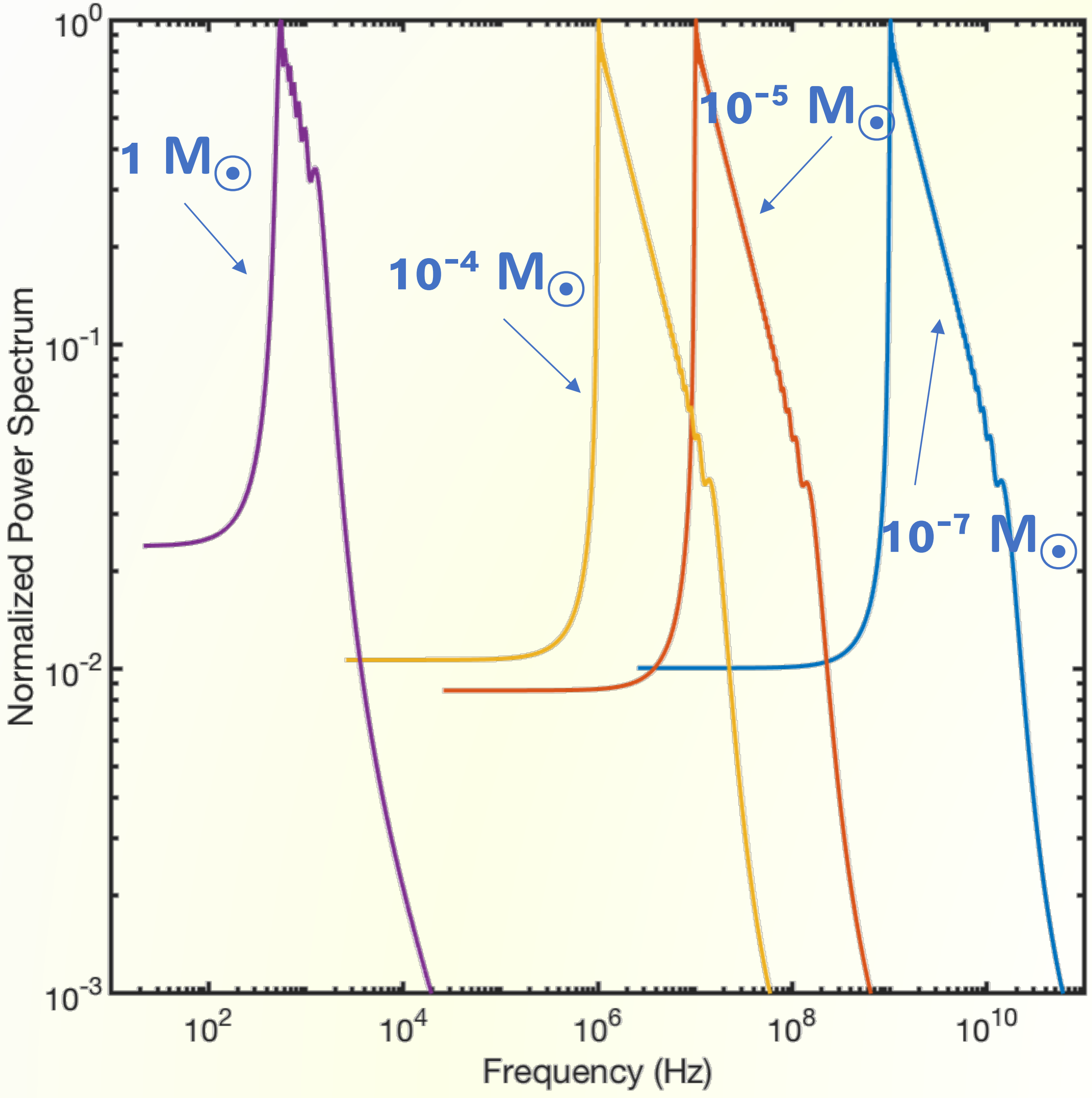
Each case at 1GPc distance

Signals of black hole coalescence from Post-Newtonian formalism, pre-merger phase

Some possible signals : frequency domain

PBHs at 10^{-5} $M_\odot$:

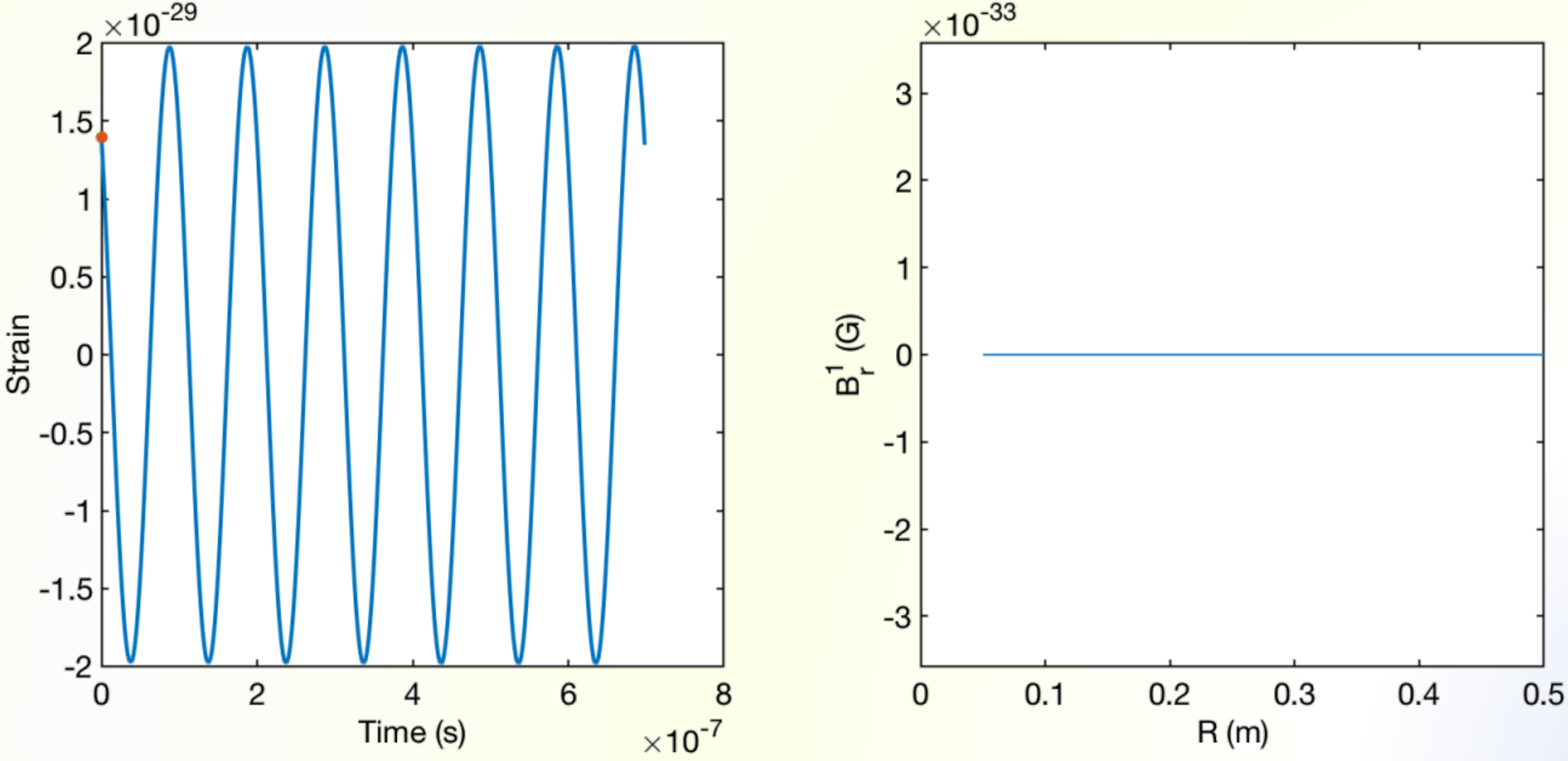
Formation when elementary particles become non-relativistic



Results: induced magnetic field

PBH merger $10^{-5}\ M_{\odot}$

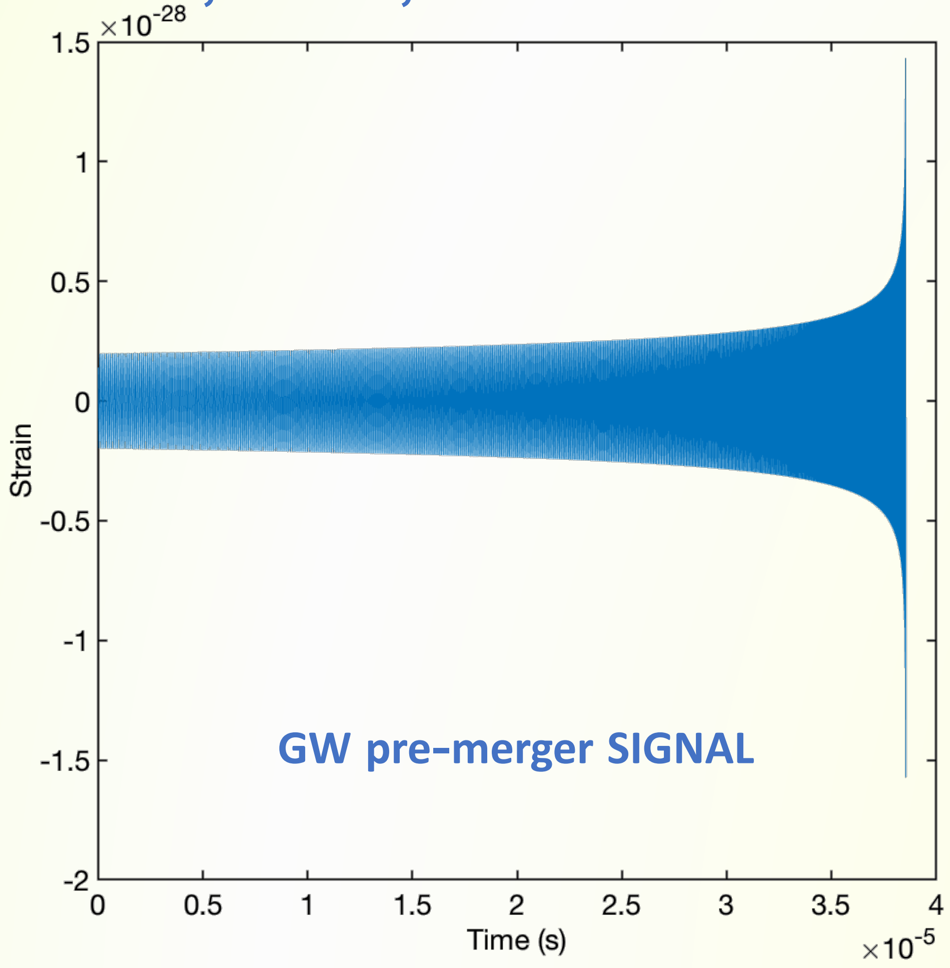
B0=5T ; L=10m ; Inner/Outer diameters : 10cm (1m)

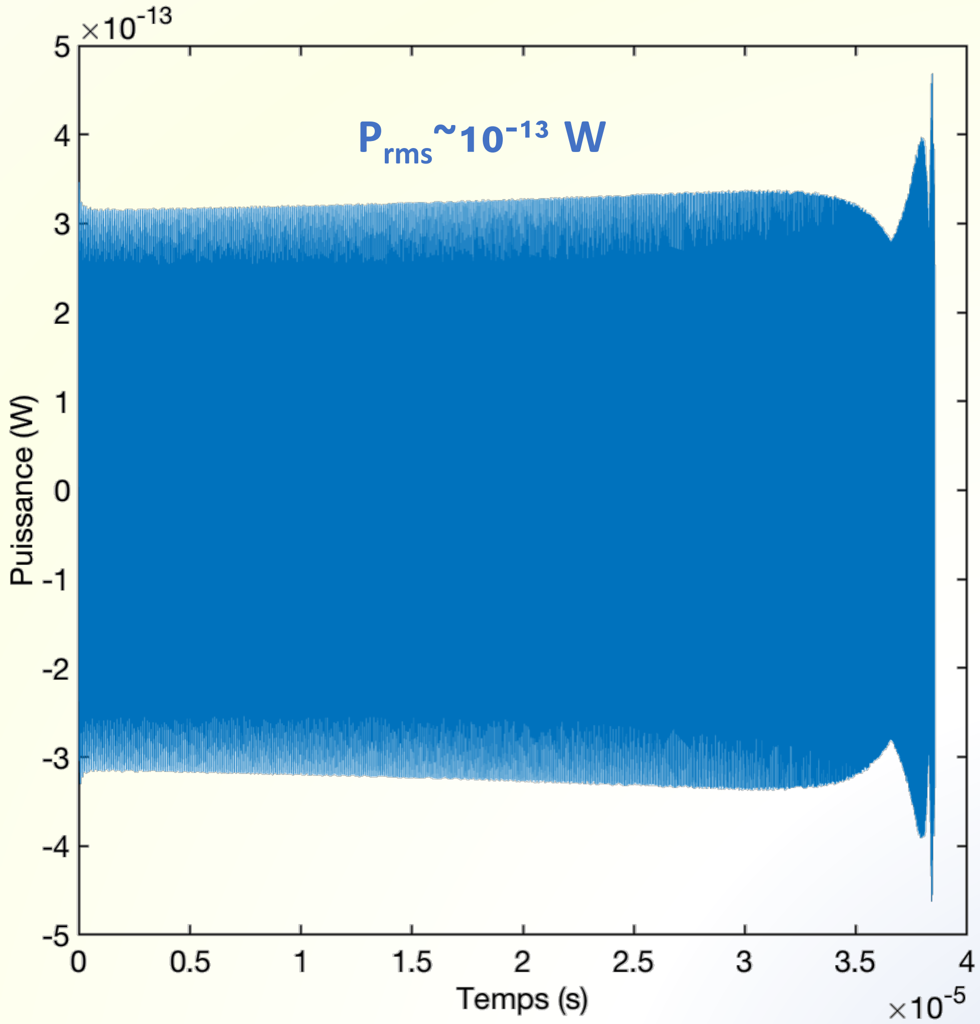


Results: energy variation and induced power

PBH merger $10^{-5}\ M_{\odot}$

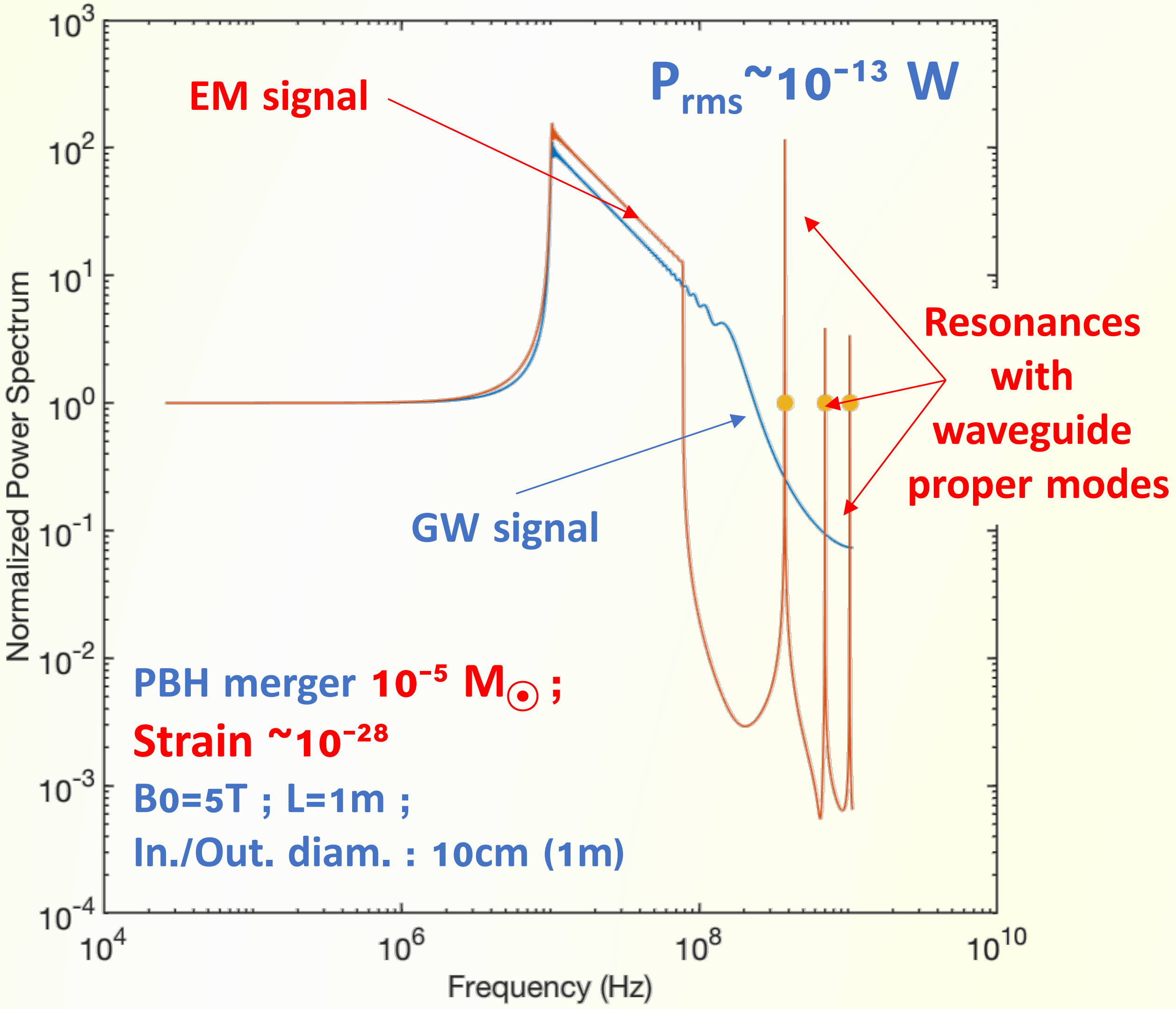
B0=5T ; L=1m ; Inner/Outer diameters : 10cm (1m)

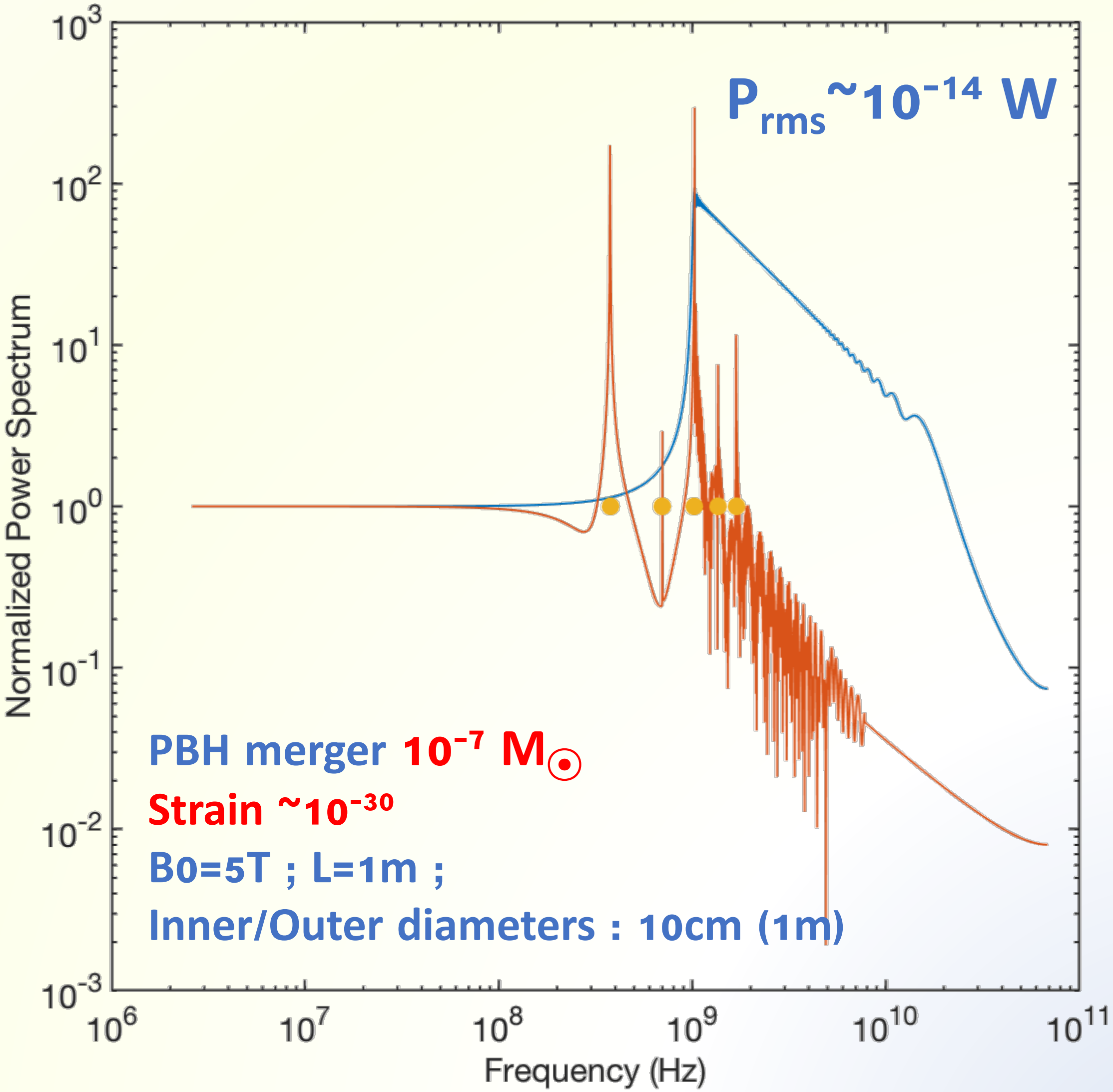




$$\Delta E \sim 10^{-21} J \sim 2\pi \frac{B_0^2 L^3 (R_2 - R_1) h_{GW}}{\mu_0}$$

Results: input and output power spectra

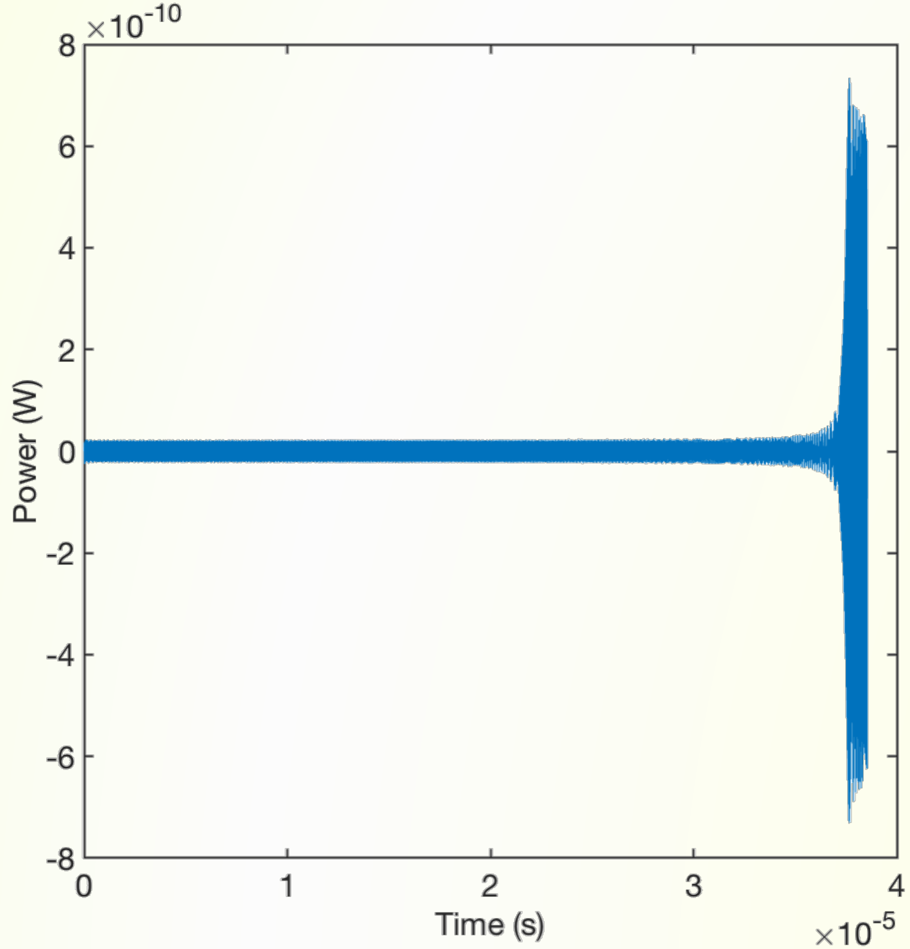




Results : waveguide in resonance with incoming GW

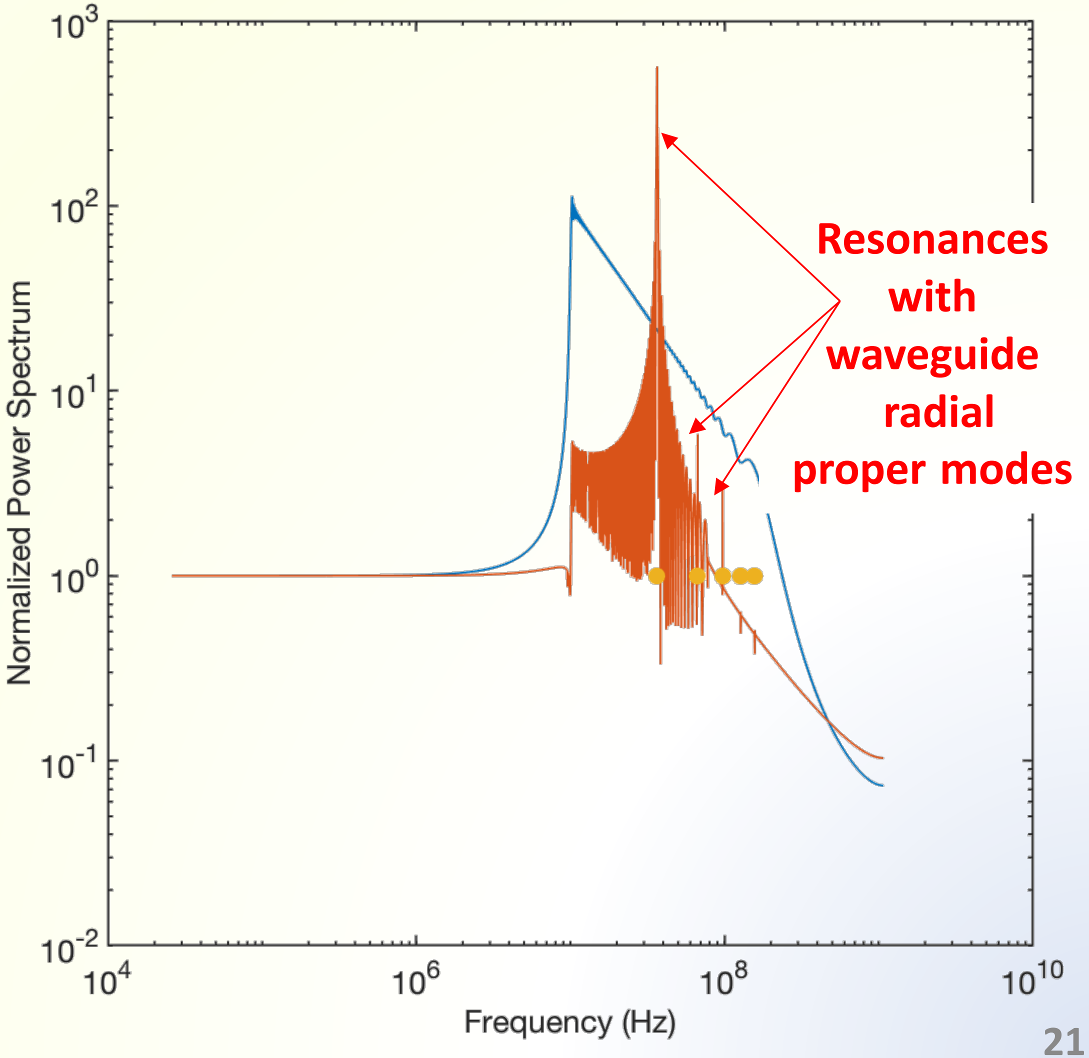
PBH merger 10^{-5} $M_\odot$

B0=5T ; L=1m ; Outer diameter : 10m (instead of 1m)



P_{rms}~7×10^{-11} W

ΔE_{rms}~3×10^{-19} J



Conclusions

- Detecting high-frequency GWs for the study of
  - Physics of neutron star interiors (starquakes)
  - Very early universe (inflation, cosmic defects)
  - Primordial black holes of mass << solar masses
- Detector concepts of high-frequency GWs involve intense magnetic fields
- Similar set-ups than haloscopes (searches for axion)
  - But coupling GW-magnetic field produce TEM waves
  - Energy variation is in second order in GW strain in cavities
- Patented concept :
  - TEM waveguide in a transverse magnetic field
  - Detection through measurement of the energy variation boosted by external magnetic field
  - Waveguide resonance enhances detection threshold for specific progenitors
  - PBHs of mass 10^{-5} $M_{\odot}$; GW strain $\sim 10^{-28}$ can produce radiation with $P_{rms} \sim 10^{-10}$ W (in resonance)
- Difficulty: detection of faint induced radiation in high-intensity magnetic field region

For more technical details, see patents on-line :

- PCT/EP2018/086758
- PCT/EP2018/086760

High-frequency Gravitational waves

- Some high-frequency detector concepts (>10kHz):
  - MiniGrail (Leiden)
  - Mechanical deformation of high Q microwave cavity (Reece)
  - Conversion of GW to EM waves (Gerstenshtein)
  - GW effect on EM wave polarization (Cruise: change of polarization of a microwave beam circulating in a closed loop)
  - Bulk acoustic wave resonators (Goryachev, Tobar)
  - Superconducting rings and Sagnac effect (Anandan, Chiao)
  - Resonant antenna with two superconducting spheres
  - Magnetic conversion detectors

Previous proposals of EM generators of GW

★ Grishchuk & Sazhin (1974):

→ Spherical electromagnetic hollow cavity with TE or TM modes, possibly in external static field

→ Rectangular EM hollow cavity (TE/TM modes)

→ Weaknesses:

- Strong isotropy of GW emission (Birkhoff theorem: spherical symmetry=> no GWs!)
- Requires strong EM energy density in the cavity
- TE or TM modes: less efficient in terms of GW polarization modes excited (only one polarization excited at once)
- Only part of the generator volume contributes to GW emission

★ Grishchuk & Sazhin (1975):

→ Toroidal TM/TE hollow cavity for the generation of standing GW on axis

→ Detection of GWs through excitation of hollow cavity modes

→ Weaknesses:

- Also requires strong EM energy density in the cavity
- Cavity detector not efficient for the generation of photons (TEM modes) induced by passing GWs