Exploring Three-Nucleon Forces in Lattice QCD

Takumi Doi

(CNS, Univ. of Tokyo)

for HAL QCD Collaboration

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9/9/2011

Seminar @ CEA Saclay

arXiv:1106.2276 [hep-lat]

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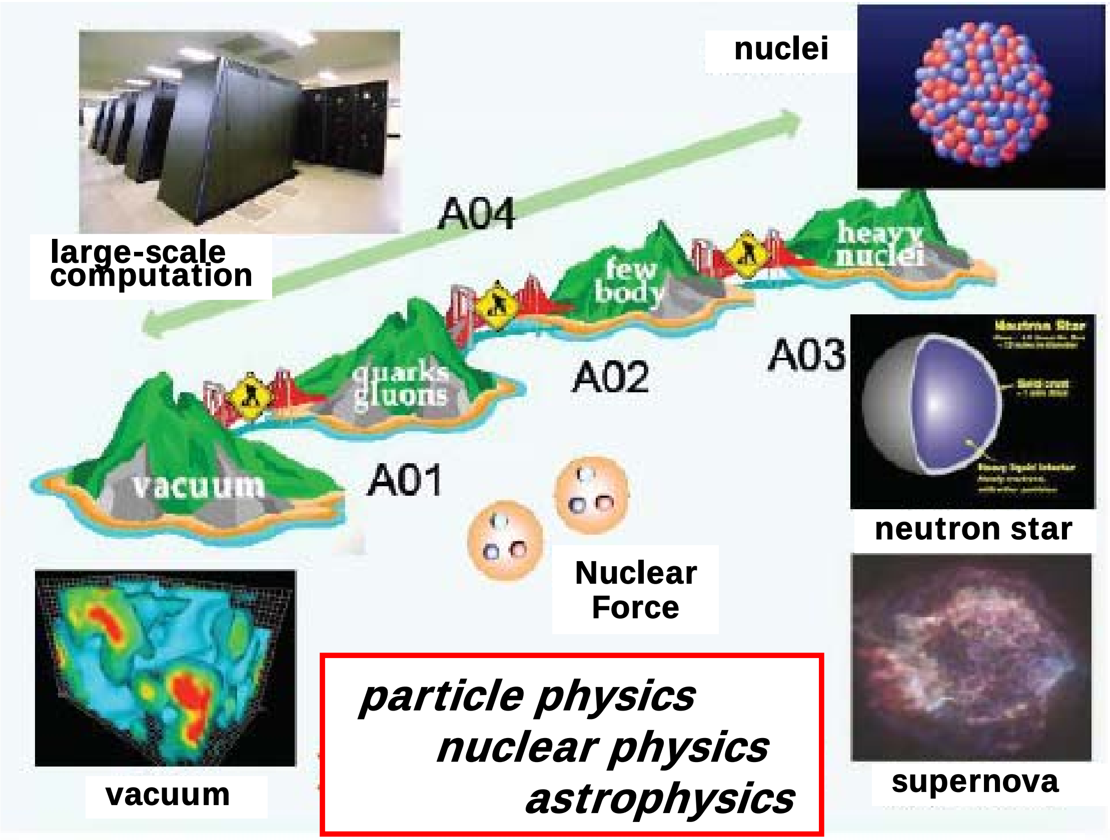
- Motivation
- Formulation for NN (and YN, YY) forces in Lattice QCD
- Three-Nucleon Forces (3NF)
  - The role of 3NF in nuclear physics and astrophysics
  - Framework to identify 3NF, the study of 3NF w/ linear setup
- Summary and Outlook

Motivation



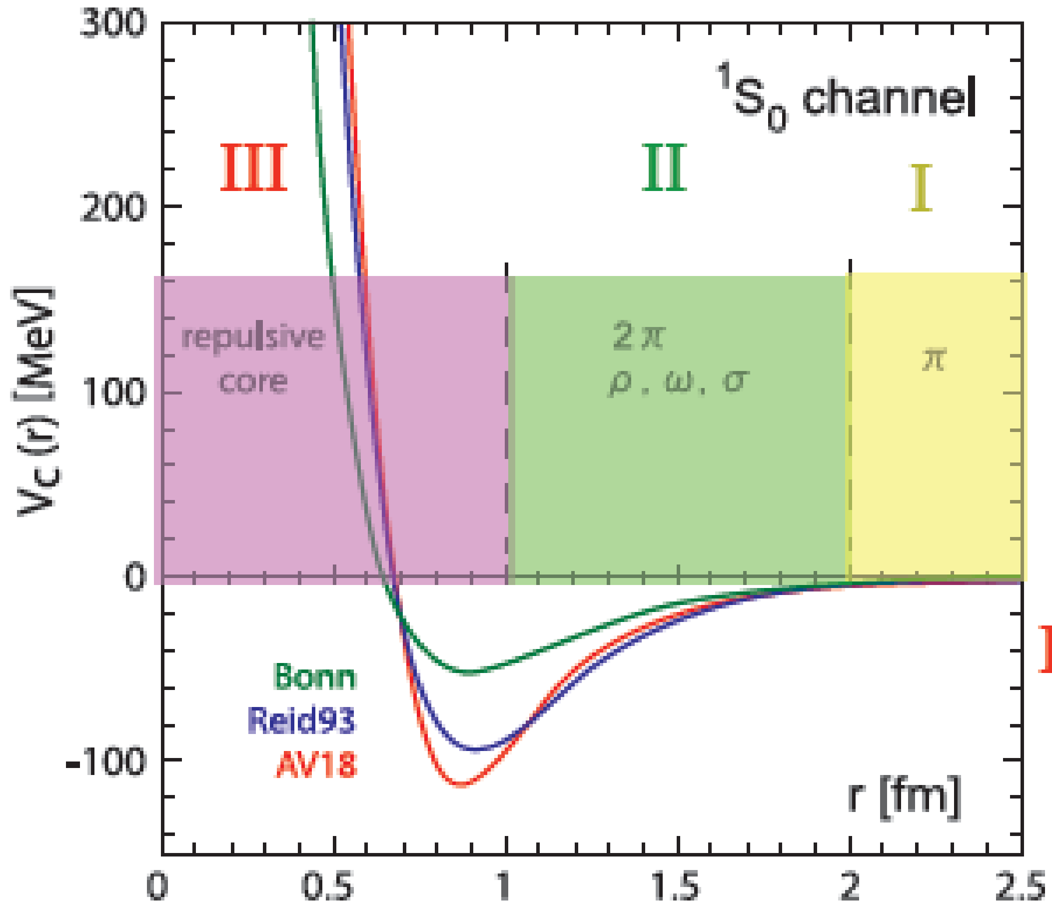
Understand the various phenomena from fundamental theory

- Nuclei
- Neutron star
- SuperNova

Nuclear Force is the key concept which **bridges** (effective) DOF in **different hierarchy**

Phenomenological NN potential

(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

Yiukawa(1935)

II Multi-pions

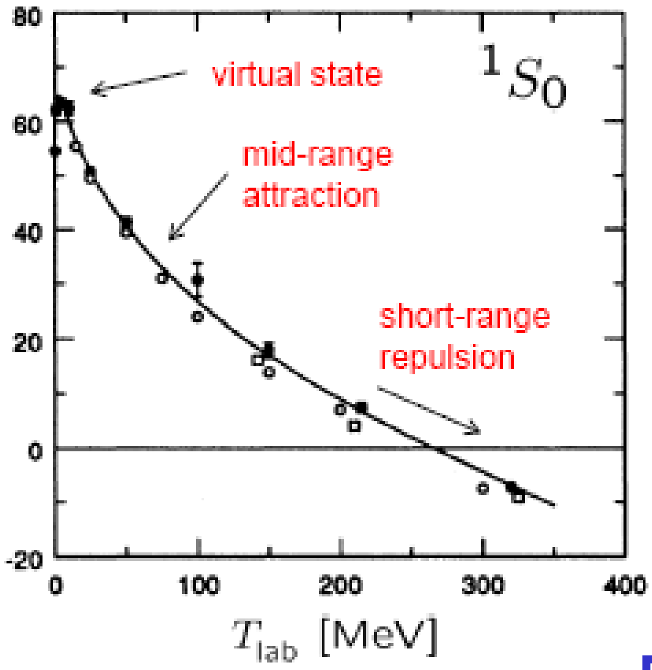
Taketani et al.(1951)

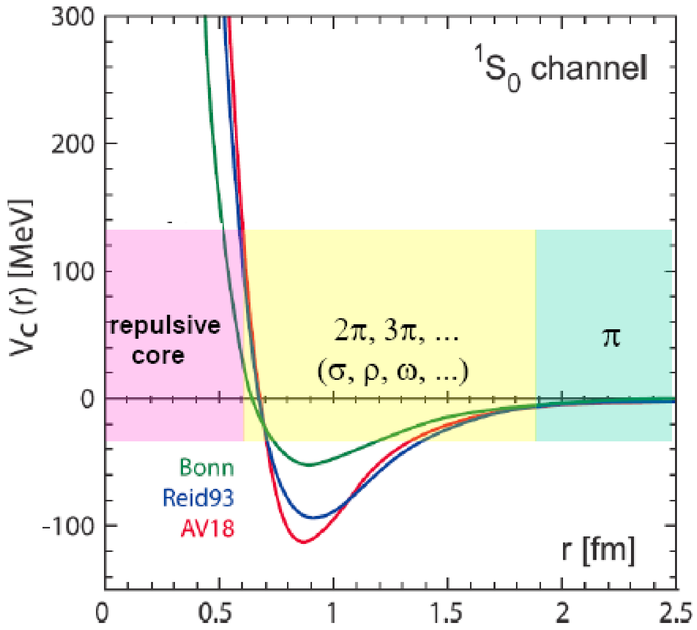
III Repulsive core

Jastrow(1951)

Nuclear Force from Experiments

- Potential is constructed so as to reproduce the NN phase shift (or, S-matrix)

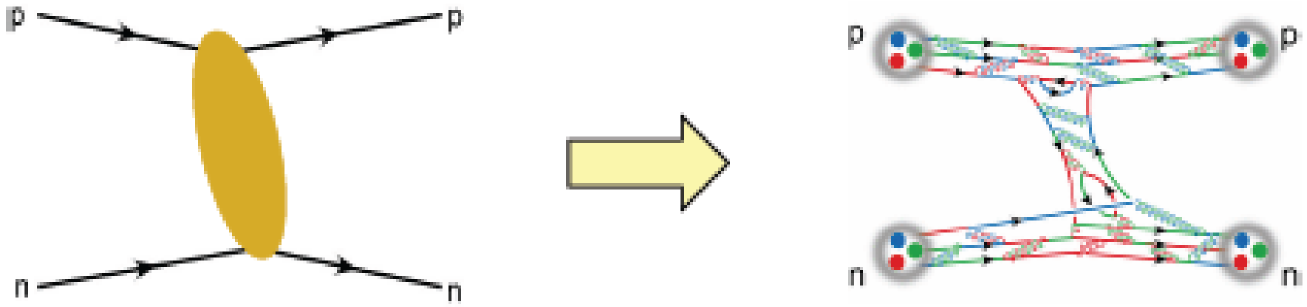




Various applications: few / many-body system of nuclei, EOS of Nuclear matter..

Nuclear Force from QCD

- First principle calculation of QCD

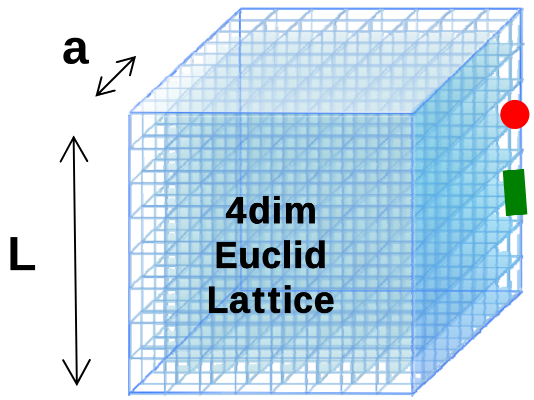


Y. Nambu, "Quarks : Frontiers in Elementary Particle Physics", World Scientific (1985)

"Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task."

Lattice QCD:
First-principle calculation of QCD

$$Z = \int dU\, dq d\bar{q}\ e^{-S_E}$$



quarks on sites

$$q(x)$$

gluons on links

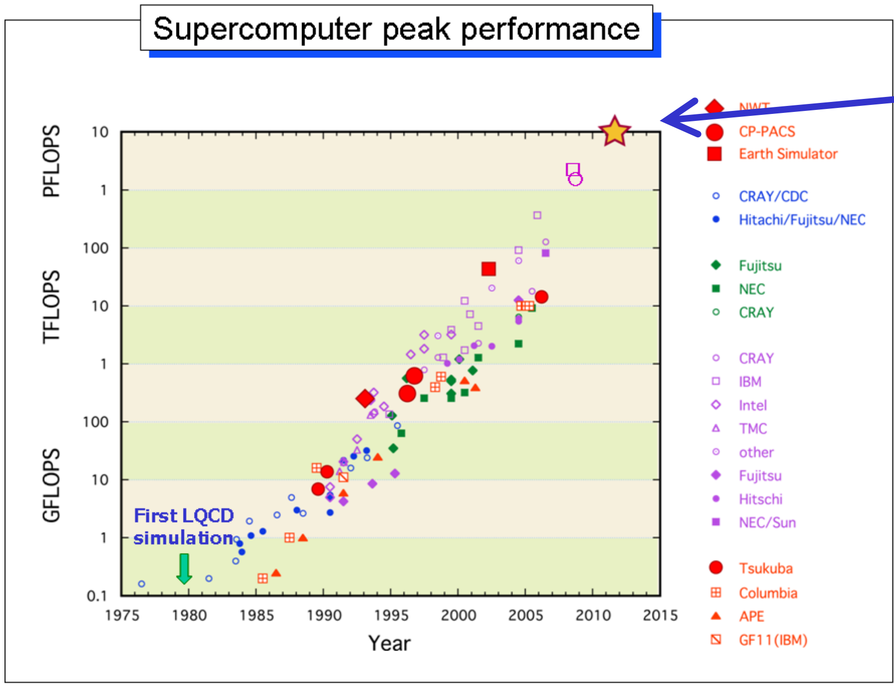
$$U_\mu(x, x+\mu) = \exp[-iaA_\mu]$$

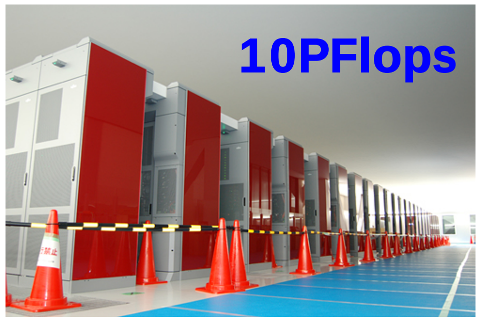
- **Well-defined reguralized system (finite a and L)**
- **Gauge-invariance manifest**
- **Fully-Nonperturbative**
- **DoF ~ 10^7 ➔ Monte-Carlo simulation w/ Euclid time**

Quenched QCD: w/o creation/annihilation q-qbar
Full QCD : w/ creation/annihilation q-qbar

“Exponential Progress” in Lat QCD

- Significant algorithmic development
  - Better scaling, better chirality, faster algorithm, …
- Hardware development (incl. by Lat QCD community)

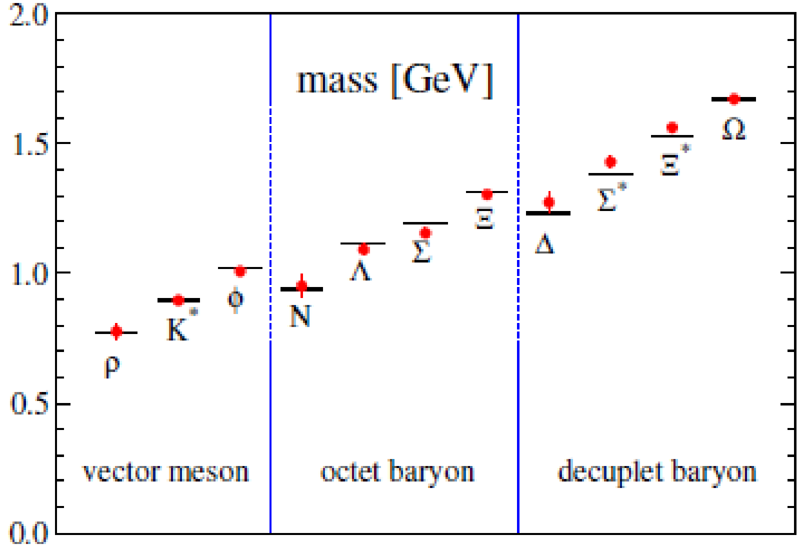




Status of Lattice QCD: hadron spectroscopy

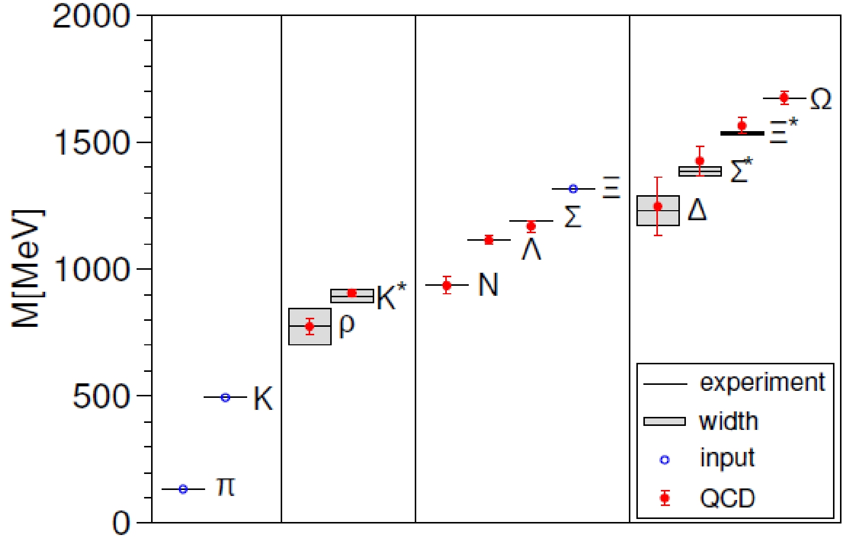
PACS-CS Collab., PRD79(2009)034503

Nf=2+1 clover fermion
L=2.9fm, a=0.09fm (1/a =2.18GeV)
mπ(min) = 156MeV



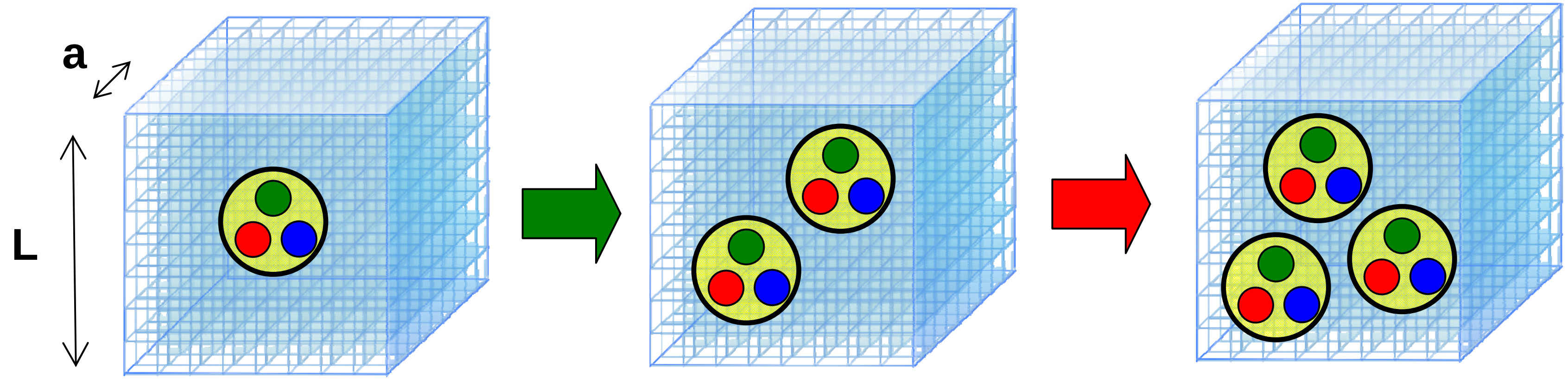
BMW Collab., Science 322 (2008) 1224

Nf=2+1 clover fermion
L= 2-4 fm, a=0.07-0.13fm
mπ(min) = 190MeV



➔ Physical mπ point configurations also generated !

Lattice QCD:
Towards Nuclear Physics



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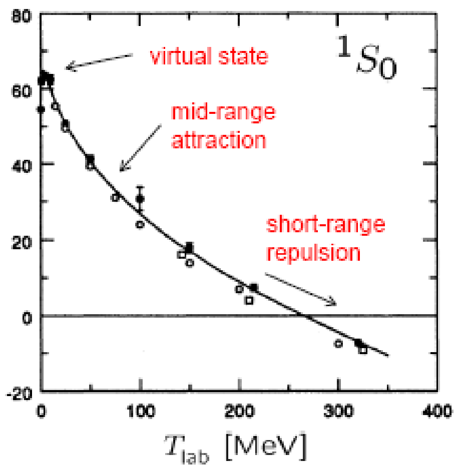
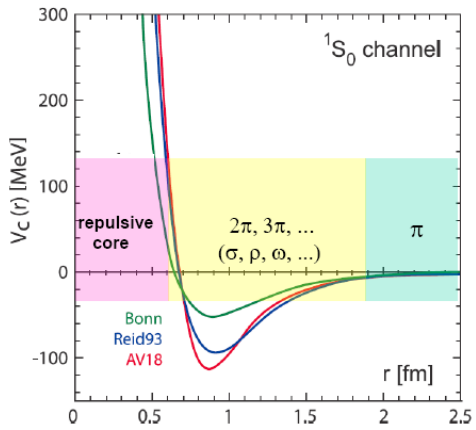
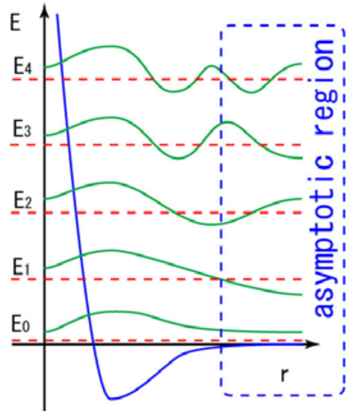
Nuclear Force from Lattice QCD [HAL QCD strategy]

- Potential is constructed so as to reproduce the NN phase shift (or, S-matrix)







Nuclear Force from Lattice QCD [HAL QCD strategy]

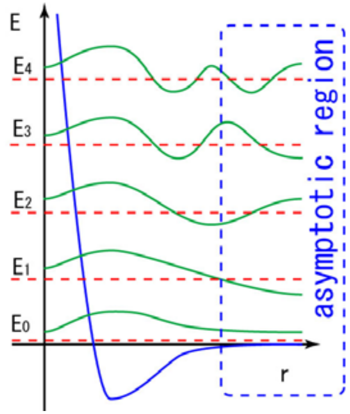
- Potential is constructed so as to reproduce the NN phase shift (or, S-matrix)
- Nambu-Bethe-Salpeter(NBS) wave function

$$\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r})N(\vec{x})|2N\rangle$$

  - Key concept: asymptotic region ←→ phase shift

$$(\nabla^2 + k_\delta^2)\psi(\vec{r}) = 0, \quad r > R$$

$$\psi(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$



Luscher, NPB354(1991)531

C.-J.Lin et al., NPB619(2001)467

CP-PACS Coll., PRD71(2005)094504

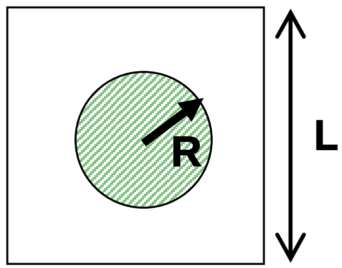
$$E = 2\sqrt{m^2 + \vec{k}^2}$$

4pt correlator

sink **source**

$$G(\vec{r}, t - t_0) = \sum_{\vec{x}} \langle 0|N(\vec{x}+\vec{r},t)N(\vec{x},t)\overline{NN}(t_0)|0\rangle$$

$$= \sum_{\vec{x}} \boxed{\langle 0|N(\vec{x}+\vec{r})N(\vec{x})|E_{2N}\rangle} \, e^{-E_{2N}(t-t_0)} \, \langle E_{2N}|\overline{NN}|0\rangle$$



Aoki-Hatsuda-Ishii PTP123(2010)89

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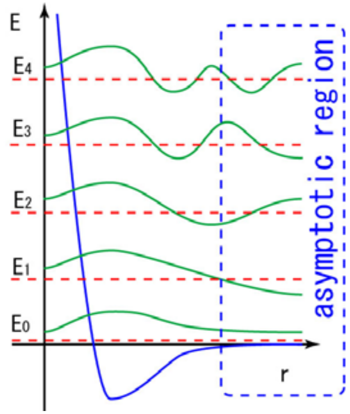


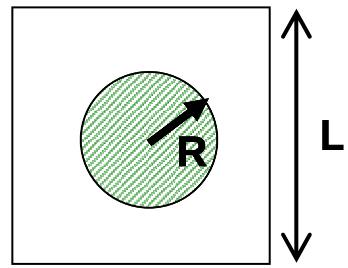
Luscher, NPB354(1991)531

C.-J.Lin et al., NPB619(2001)467
CP-PACS Coll., PRD71(2005)094504

- Define the potential at interaction region

$$(\nabla^2 + k_\delta^2)\psi(\vec{r}) = \int d\vec{r'}\, U(\vec{r},\vec{r'})\psi(\vec{r'}), \quad r < R$$

  - Non-local, but E-independent potential



- Velocity expansion Okubo-Marshak(1958)

$$U(\vec{r},\vec{r'}) = V_c(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}V_{LS}(r) + \mathcal{O}(\nabla^2)$$

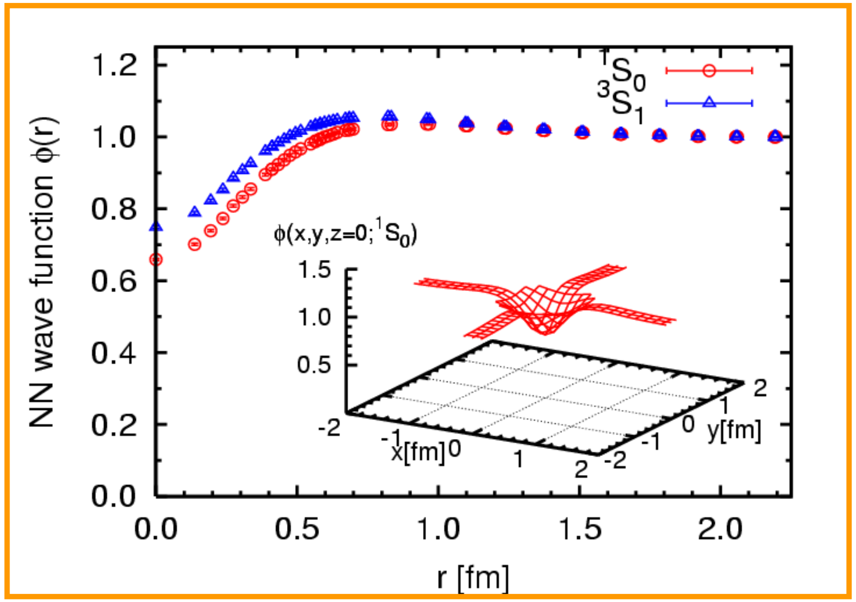
LO LO NLO NNLO

  - Truncation in expansion introduces E-dep (only practically), but we can **improve order by order**

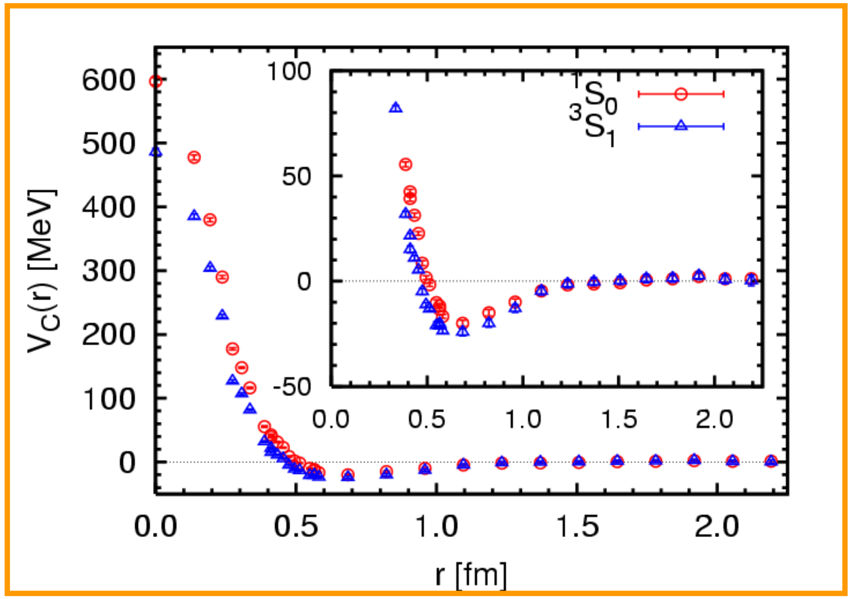
Aoki-Hatsuda-Ishii
PTP123(2010)89

Nuclear Potential (from Lat QCD)

NBS wave function



Nuclear Force

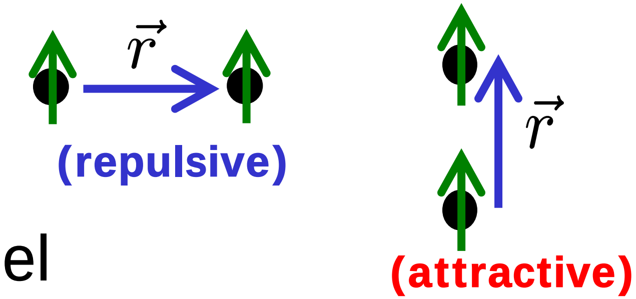


<u>Quenched</u> QCD
$m\pi$ = 530MeV, L=4.4fm

Ishii-Aoki-Hatsuda,
PRL99(2007)022001

Tensor Potential from Lat QCD

- Tensor operator $S_{12} = 3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})/r^2 - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
  - Essential to understand the nuclei
  - Responsible for deuteron binding
  - Hyper nuclei binding (ΛN-ΣN)



- Coupled channel study in $^3S_1$-$^3D_1$ channel

$$(H_0 + V_C + V_T S_{12})\psi = E\psi$$

$$\psi = \psi_S + \psi_D$$

$$\psi_S(\vec{r}) = P\psi(\vec{r}) = \frac{1}{24}\sum_{g\in O}\psi(g^{-1}\vec{r})$$

$$\psi_D(\vec{r}) = Q\psi(\vec{r}) = (1-P)\psi(\vec{r})$$

P	: projection to L=0
Q=(1-P)	: projection to L=2

$\Rightarrow$

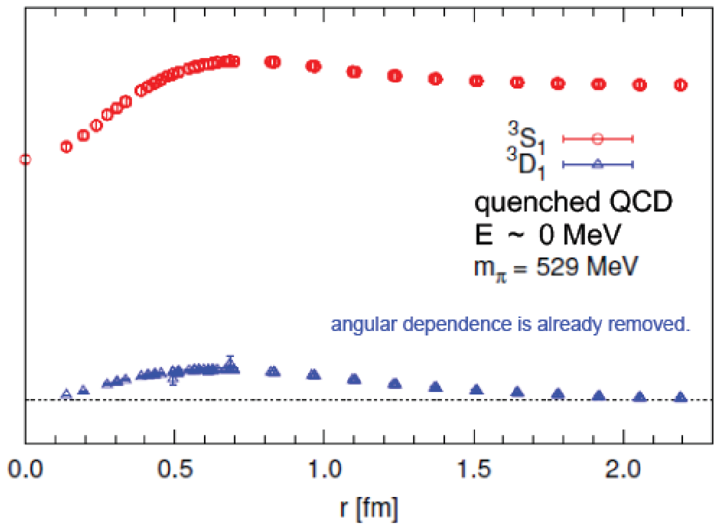
$$P(H_0 + \boxed{V_C} + \boxed{V_T} S_{12})\psi = EP\psi$$

$$Q(H_0 + \boxed{V_C} + \boxed{V_T} S_{12})\psi = EQ\psi$$

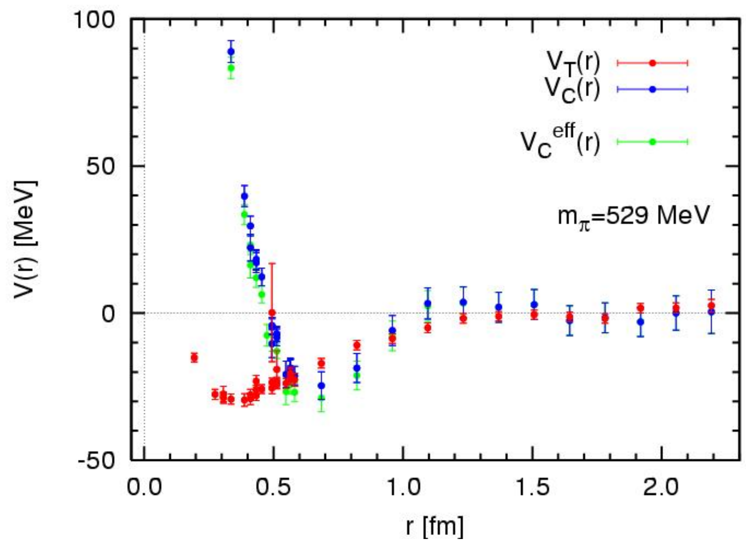
Tensor Potential from Lat QCD

- Coupled channel study in $^3S_1$-$^3D_1$ channel

Wave function



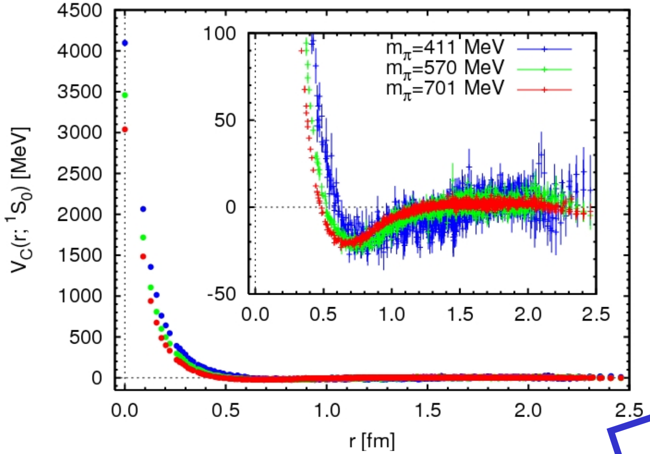
Potentials



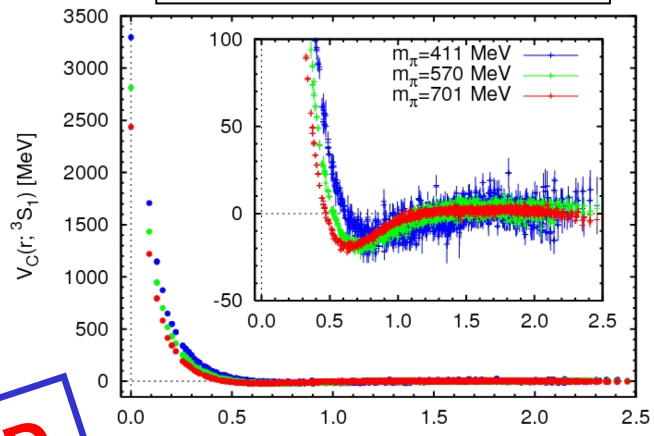
Aoki-Hatsuda-Ishii,
PTP 123 (2010) 89

Quark mass dependence

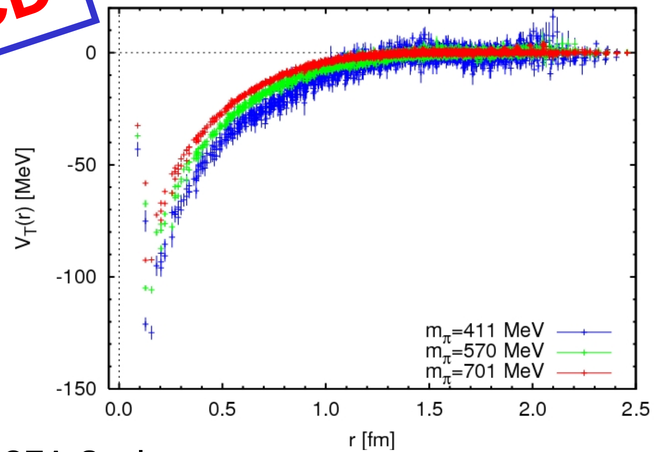
Central in $^1S_0$



$^3S_1$-$^3D_1$ channel



Central



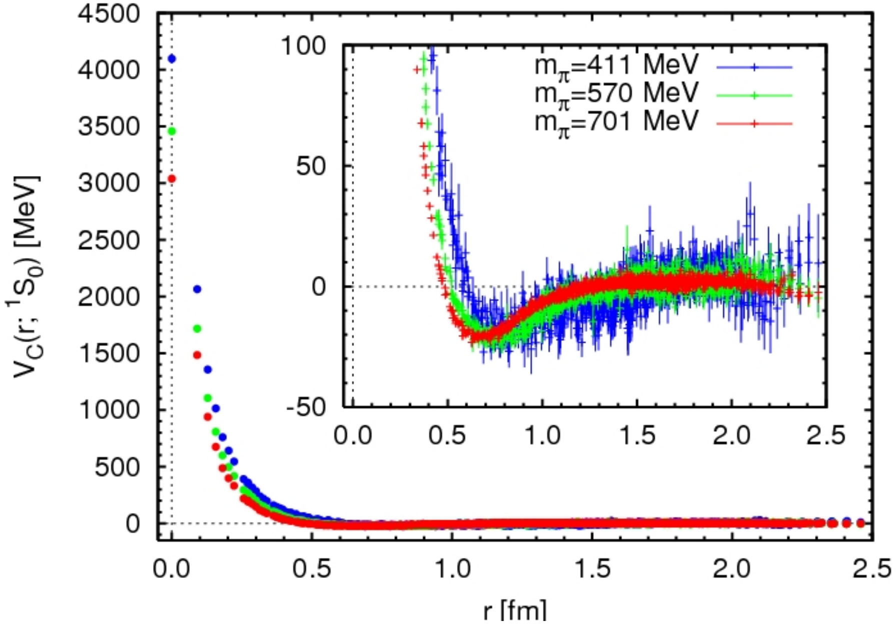
Tensor

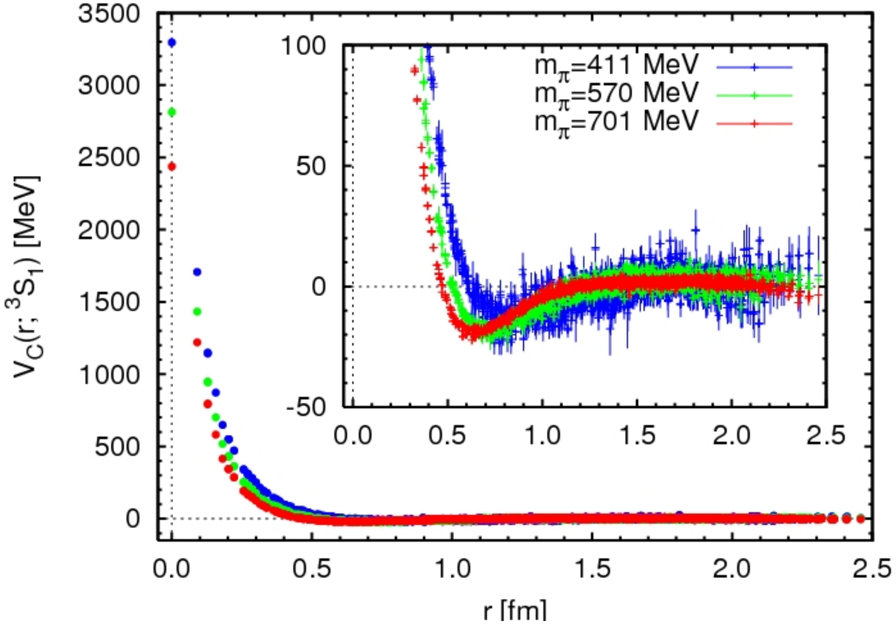
Full QCD

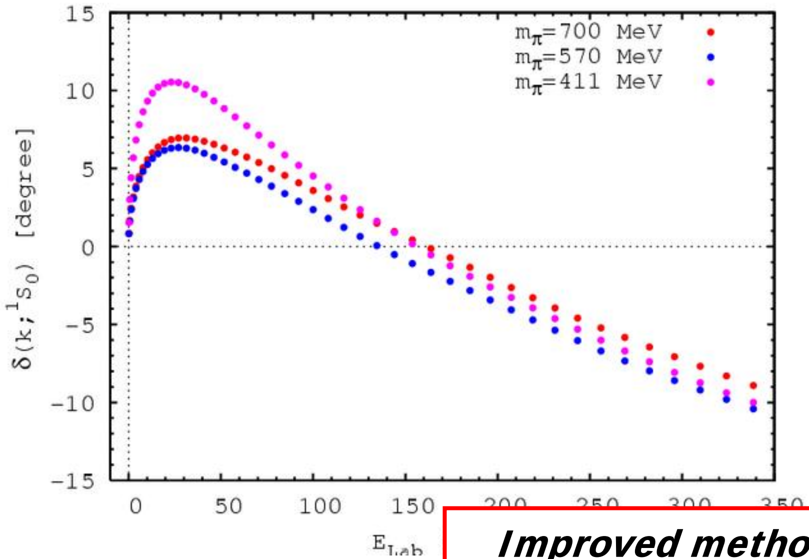
<u>Lighter mass</u> corresponds to…

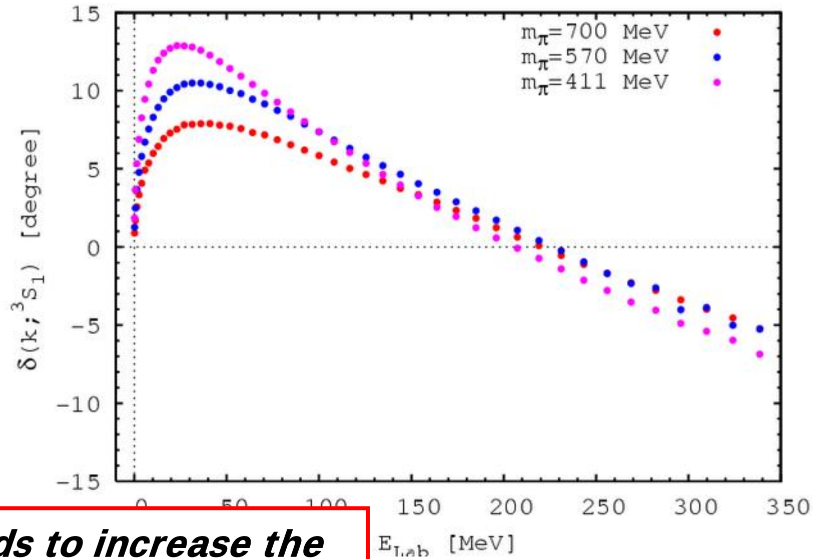
- **Longer interaction range**
- **Larger Repulsive Core**
- **Stronger Tensor Force**
- **(stronger attraction in Center Force)**

Phase shift from potential



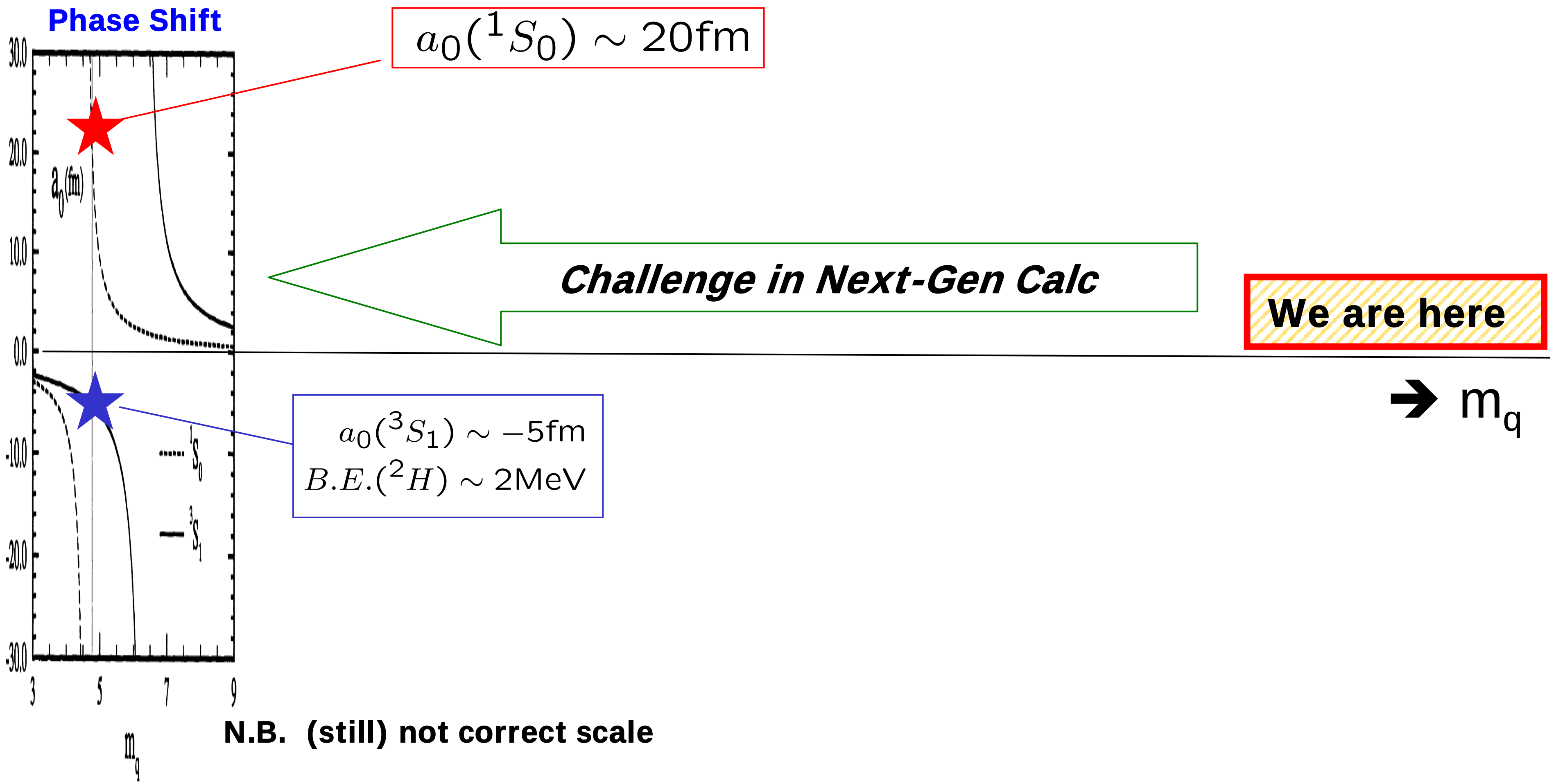






Improved method tends to increase the phase shifts, still we have no deuteron

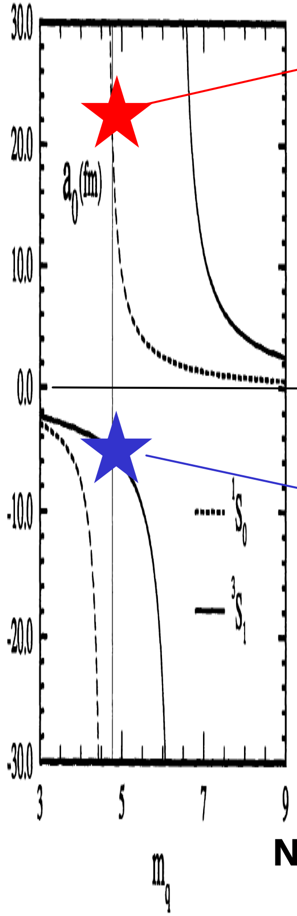
Challenge in Next-Gen Simulation



N.B. (still) not correct scale

Challenge in Next-Gen Simulation

Phase Shift



$a_0(^1S_0) \sim 20\text{fm}$

$a_0(^3S_1) \sim -5\text{fm}$

$B.E.(^2H) \sim 2\text{MeV}$

Temporal info (Luscher's formula)

$$\delta E = -\frac{2\pi a_0}{\mu L^3}\left(1 + c_1\frac{a_0}{L} + c_2\frac{a_0^2}{L^2}\right) + \mathcal{O}(L^{-6})$$

Spacial info (potential/phase shift)

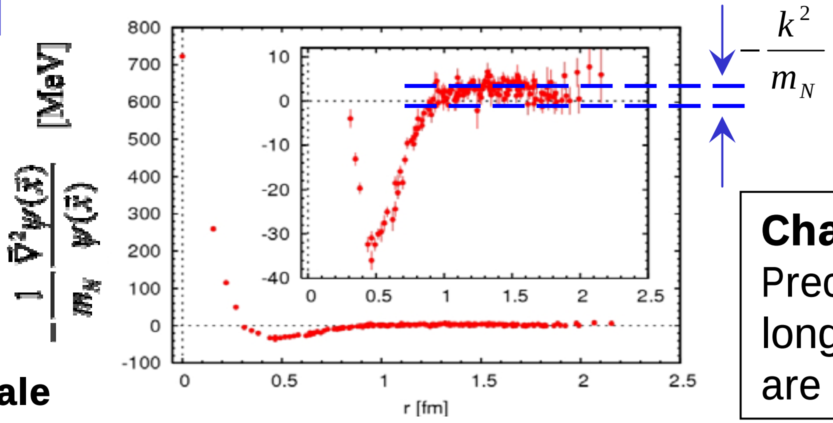
$$V(r \to \infty) \sim e^{-m_\pi r}/r$$

Localized !

Challenge in Next-Gen Calc

We are here

→ m_q

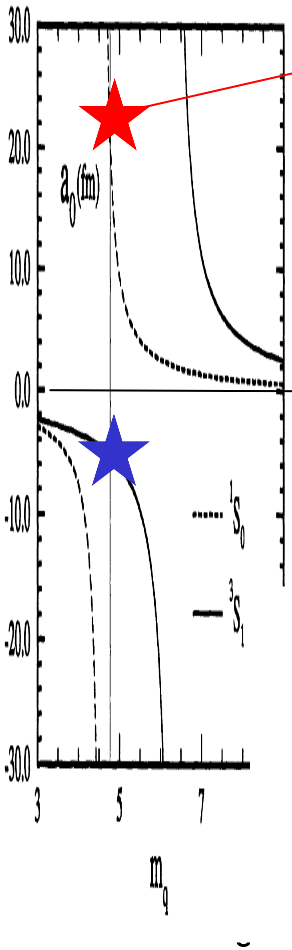


Challenge:
Precise results at long range part are necessary

N.B. (still) not correct scale

Challenge in Next-Gen Simulation

Phase Shift



$a_0(^1S_0) \sim 20\text{fm}$

- **np : -23.7 fm**
- **pp : -17.9 fm (w/o Coulomb)**
- **nn : -16.5 fm**

SU(2) breaking

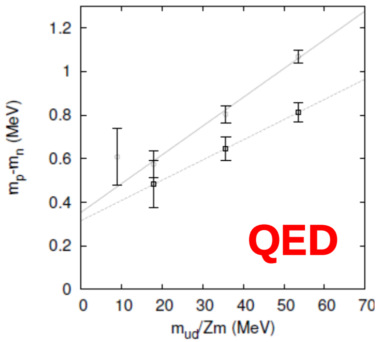
Challenge in Next-Gen Calc

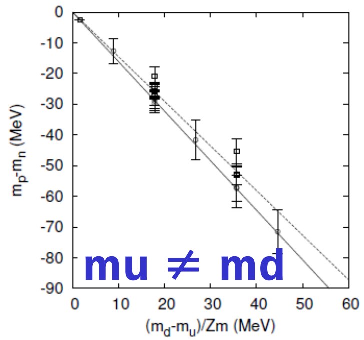
We are here

RBC(em)

T.Blum, R.Zhou, T.D., M.Hayakawa, T.Izubuchi, S.Uno, N.Yamada, PRD82(2010)094508

Nf= 2+1 DWF w/ quenched QED





	$m_p - m_n$(MeV)
QED	$+0.383(68)$
$m_u \neq m_d$	$-2.51(14)$
total	$-2.13(16)(70)$

$m(\pi^+)$-$m(\text{“}\pi^0\text{”})$[QED]=4.50(23)MeV
m_u=2.24(10)(34), m_d=4.65(15)(32)MeV

NEXT

Nf=1+1+1 w/ full QED, RBC(em), PACS-CS, …

Frequently Asked Questions

[Q1] Is potential observable ? Just give me phase shifts !

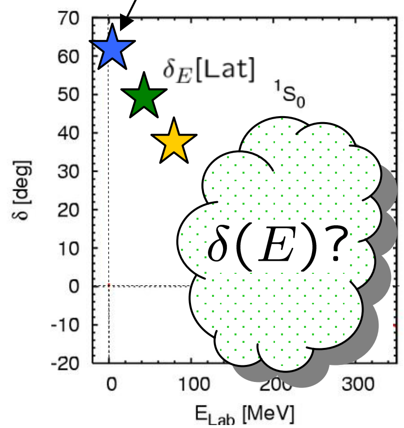
- Potential U(x,y) is NOT observable, and is NOT unique. However, combination of (Φ(x),U(x,y)) gives observable, which is unique.
  - Same situation for QM(Φ,U), QFT(Φ(asym),vertices), EFT(eff. dof, LECs) … Yet, we use “wave function Φ(x)” in QM, etc.
- We study potential (in addition to phase shift), because:
  - Convenient framework/concept to understand the physics
  - Potential is essential to study many-body systems
    - c.f. QM: Matrix mechanics vs. Wave mechanics

$$Lat \to \boxed{\delta_E \to U(x,y)} \to \text{many-body}$$

$$\boxed{Lat \to \to \to U(x,y)} \to \text{many-body}$$

  - It is very difficult to calculate phase shift at high energy
    - Lattice ➔ only ground state + a few excited energy states
  - Potential (hopefully) contains “useful” off-shell information
    - Sys. error by velocity expansion can be checked order by orde

$$\delta[\text{by Lat}] = \delta[\text{by } U(x)]$$



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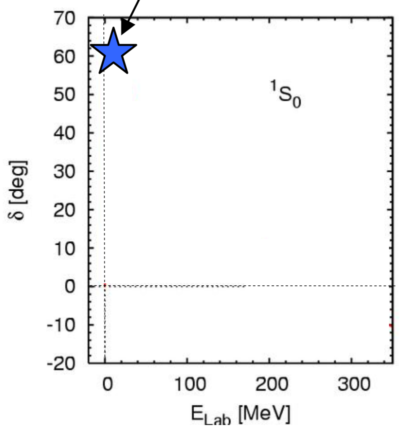

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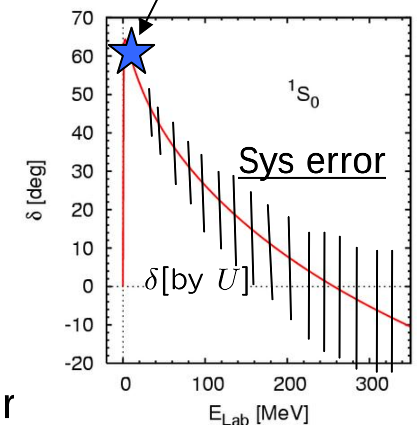


Frequently Asked Questions

[Q2] Isn't Potential dependent on the sink operator ?

- Yes, the potential is dependent on the choice of the sink operator, since Potential U(x,y) is NOT observable. (➔ go back to the 1st Q&A)
  - One can choose any sink opeartor, and the physical observables (at least phase shift) calculated from that potential remain same
  - We choose local operator as convenient choice for the reduction formula
  - Good operator ←→ small non-locality in potential
    - We check the velocity expansion convergence *a posteriori*

[Q3] How good is velocity expansion of potential ?

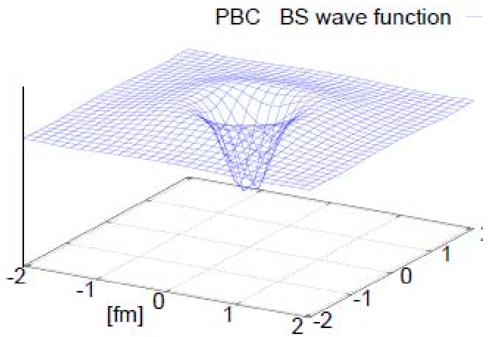
- We explicitly checked the validity of expansion using two methods:
  - By Energy dependence of LO potential $V_C(r)$
  - By L^2 dependence of $V_C(r)$

K.Murano et al.
PTP125 (2011) 1225

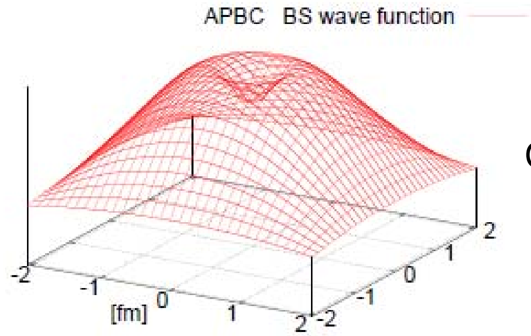
“Energy dependence” of LO $V_c(r)$ in velocity expansion

Anti-periodic BC to achieve $E \neq 0$

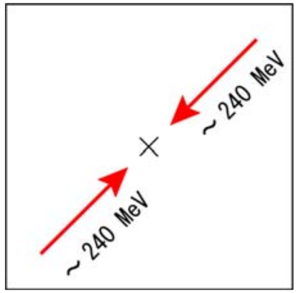
E ∼ 0 MeV

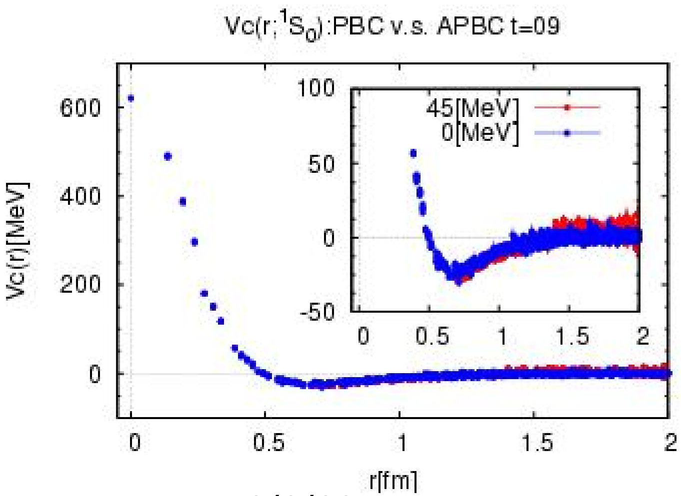


E ∼ 45 MeV



Quenched QCD
m_π =0.53GeV
a=0.137fm

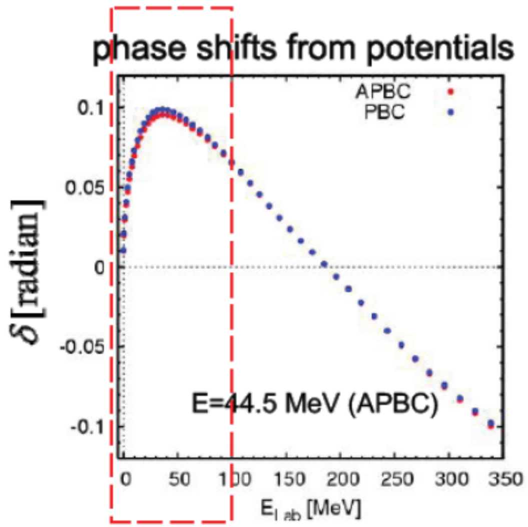




In our choice of wave function,
E-dependence of the local potential
turns out to be very small at low E.

➔ *Velocity expansion is good !*

K.Murano et al., PTP125 (2011) 1225



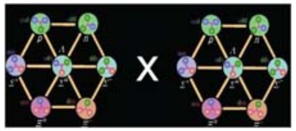
O.K. !

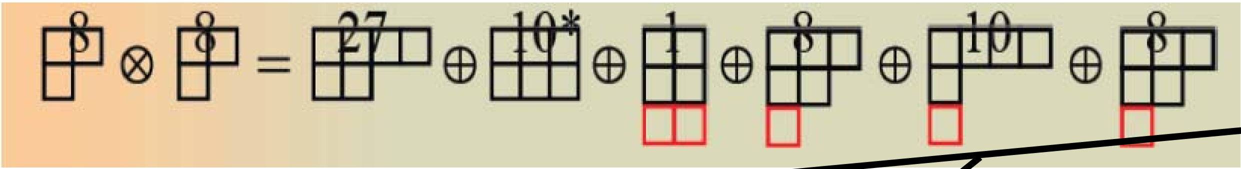
Towards the prediction from Lattice QCD

- "Realistic" NN potentials have achieved quite a good precision
  - ~40 parameters for ~5000 (high prec) phase shifts, χ^2/dof ~ 1
- Hyperon-Nucleon(YN), Hyperon-Hyperon(YY) potentials
  - Large uncertainties in YN, YY potentials, and theoretical predictions are highly awaited
    - Huge impact on EoS in high density, Neutron Star Core / Supernova
  - "Generalization" of the nuclear force
    - ➔ what is universal, what is not universal in hadron-hadron interactions ? (e.g. origin of repulsive core)
  - Quest for the H-dibaryon
- Three-Baryon Potentials
  - The Lattice study of Three-Nucleon Force (3NF)

Hyperon potentials (YN, YY)

- Generalized BB force





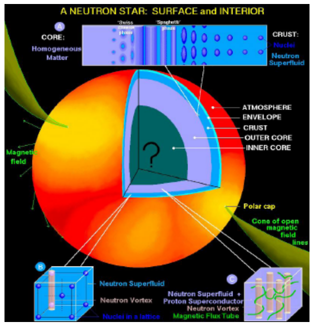
NN channel

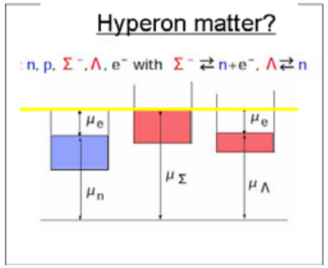
$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$

symmetric (27 + 8s + 1) **anti-symmetric** (10* + 10 + 8a)

"Strange World" is opening !

EOS of high density matter

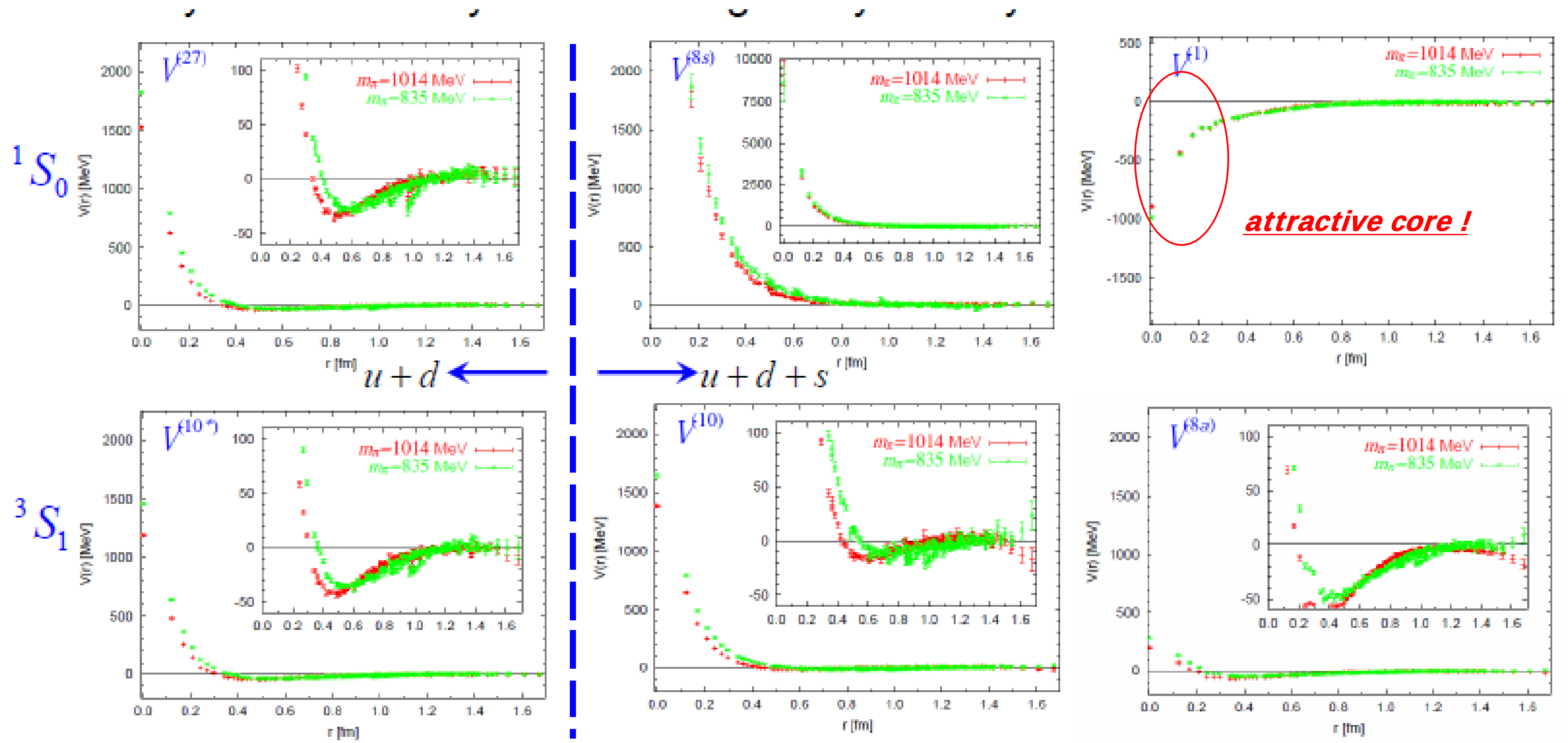




SU(3) study

BB potentials

a=0.12fm, L=2fm, m(PS)= 0.84, 1.01GeV



27,10*:
Same as NN

8s,10:
strong repulsive core

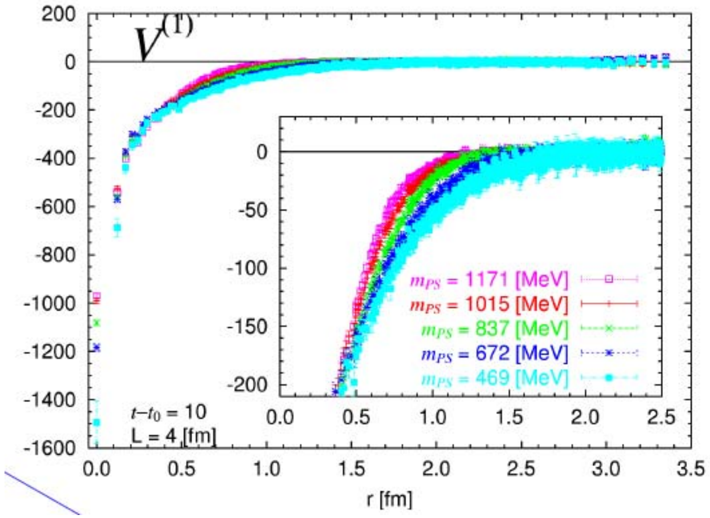
8a: weak repulsive core
1s: deep attractive pocket

Pauli principle at work !

T.Inoue et al., (HAL QCD Collab.), PTP124(2010)591, arXiv:1007.3559

Quest for the H-dibaryon

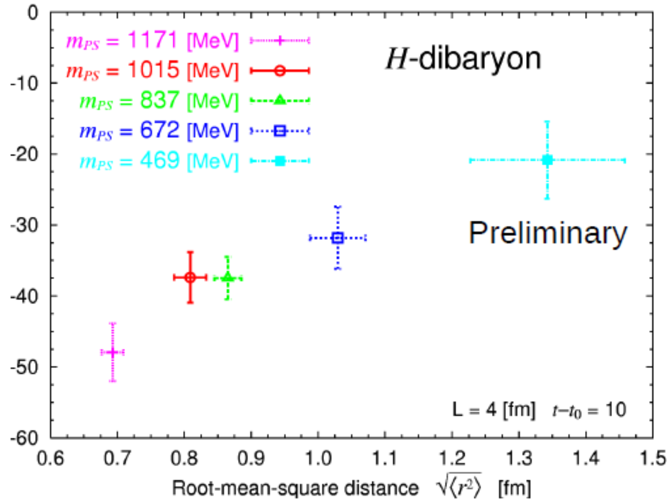
- H-dibaryon (uuddss) R.L. Jaffe, PRL38(1977)195
  - Extract the essence by the SU(3) limit study



Lat QCD potential

Solve Schrodinger eq. in infinite V

Bound state found !



Binding Energy and RMS

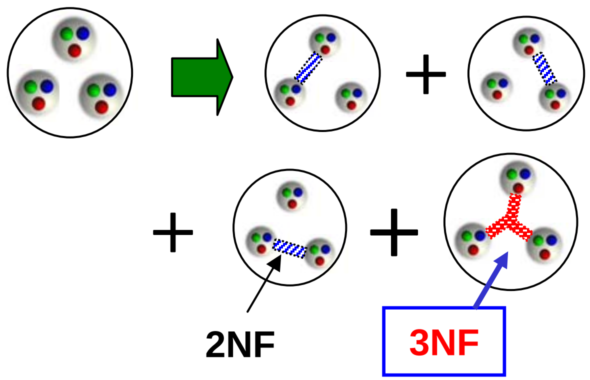
T.Inoue et al. [HAL QCD Coll], PRL106(2011)162002, talk @ Lat2011

Beyond SU(3) in progress

Three-Nucleon Force (3NF)

Importance of Three-Nucleon Force (3NF)

- **3NF** ← potential among 3N which **cannot** be reduced to pair-wise 2N potential



  - Important role in B.E. of light-nuclei
  - Nucleus-nucleus scattering, Ay puzzle ?
  - Saturation point of nuclear matter
  - **EoS** of high density matter → Neutron Star, SuperNova
  - Properties of neutron rich nuclei → Nucleosynthesis

Importance of Three-Nucleon Force (3NF)

- <u>Precise</u> few-body calc:
  - e.g. benchmark calc of ^{4}He by 7 methods (NN only)

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

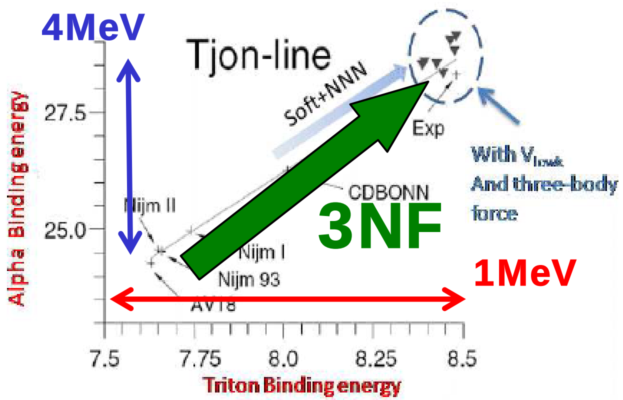
➔ **0.5% prec. for B.E.**

H.Kamada et al., PRC64(2001)044001

- 2N force <u>cannot</u> reproduce B.E.

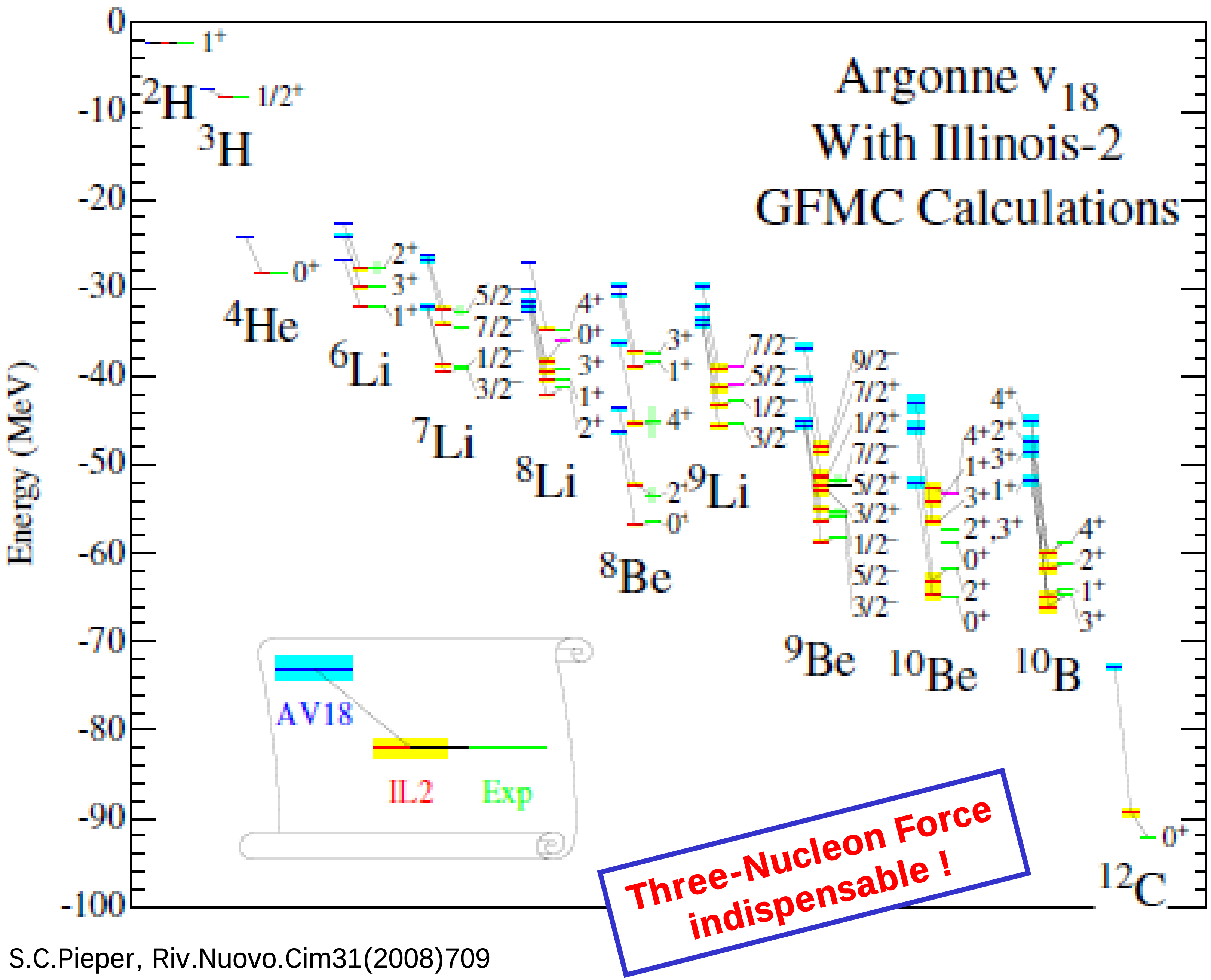
δB.E.= 0.5-1MeV for ^{3}H
δB.E.= 2-4 MeV for ^{4}He

<u>missing</u>



Nogga et al., PRL85(2000)944

➔ Attractive 3NF necessary

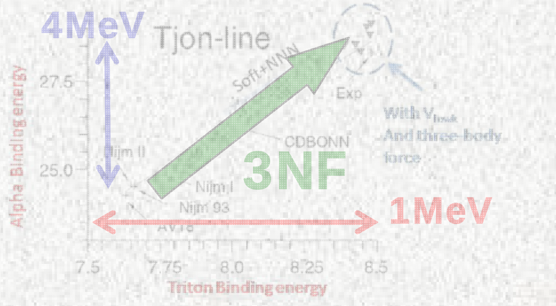


S.C.Pieper, Riv.Nuovo.Cim31(2008)709
arXiv:0711.1500

Importance of Three-Nucleon Force (3NF)

- Precise few-body calc: NN force cannot reproduce B.E.
  - δB.E.= 0.5-1MeV for ^{3}H
  - δB.E.= 2-4 MeV for ^{4}He

Attractive 3NF necessary



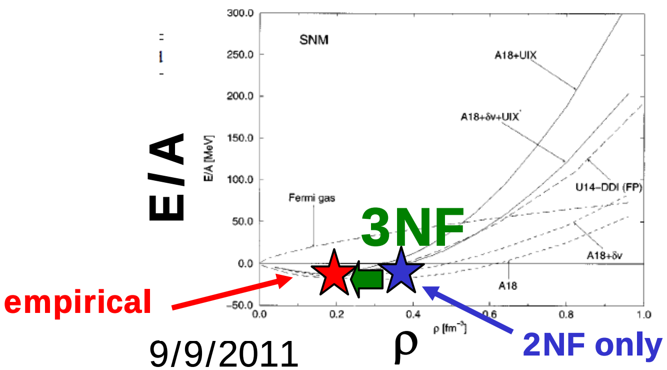
Nogga et al., PRL85(2000)944

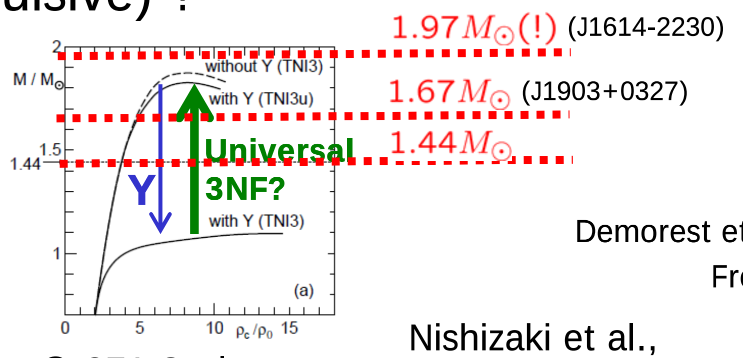
- Saturation density/energy of nuclear matter also requires 3NF
  - EOS of neutron star
    - Flavor universal 3NF (repulsive) ?

Repulsive 3NF also necessary

A.Akmal et al., PRC58(1998)1804

Takatsuka et al., PTPS174(2008)80



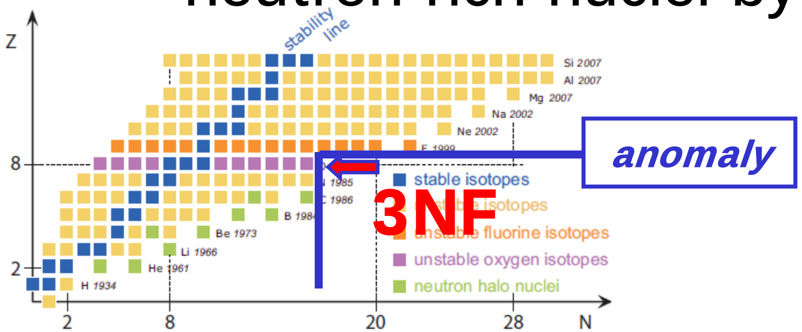


Demorest et al. (2010)
Freire (2009)

Nishizaki et al., PTP108(2002)703

Importance of Three-Nucleon Force (3NF)

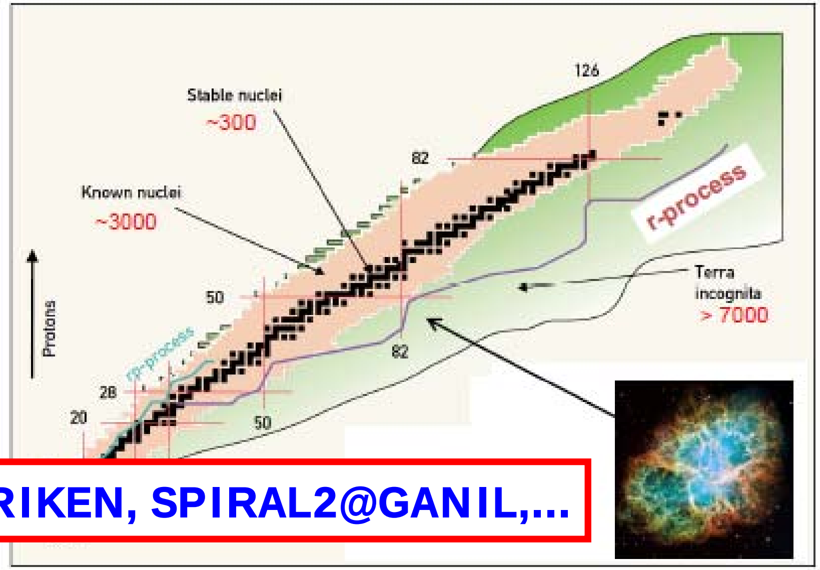
- The effect on the nuclear chart
  - Anomaly in drip line and nontrivial magic number in neutron rich nuclei by 3NF



drip line: $^{28}O \rightarrow {}^{24}O$

nontrivial magic number
N=28 for $_{20}Ca$

T.Otsuka et al., PRL105(2010)032501
J.D.Holt et al., arXiv:1009.5984

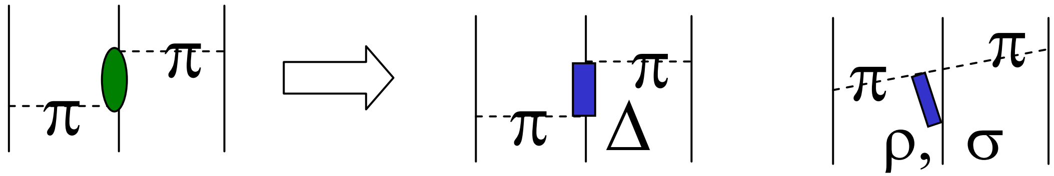


RIBF@RIKEN, SPIRAL2@GANIL,...

Nucleosynthesis by Supernova

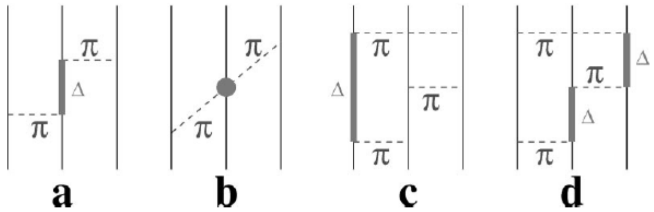
Three-Nucleon Force (3NF)

- It is natural to expect the existence of 3NF
- It is very nontrivial to determine 3NF from QCD
- 2πE-3NF Fujita-Miyazawa, PTP17(1957)360
  - Off-energy-shell πN scatt



- Phenomenological models
  - Fujita-Miyazawa, Tucson-Melbourne, Urbana/Illinois, …

Urbana/Illinois



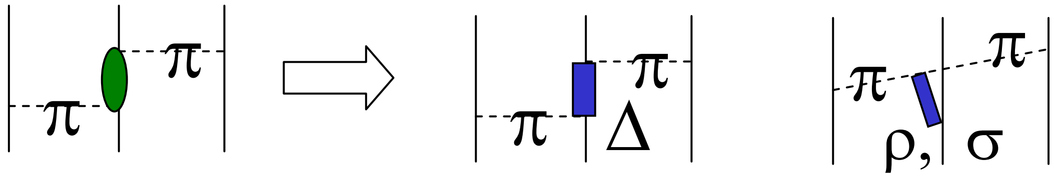
+ Repulsive 3NF

- Phenomenological short-range repulsion is necessary

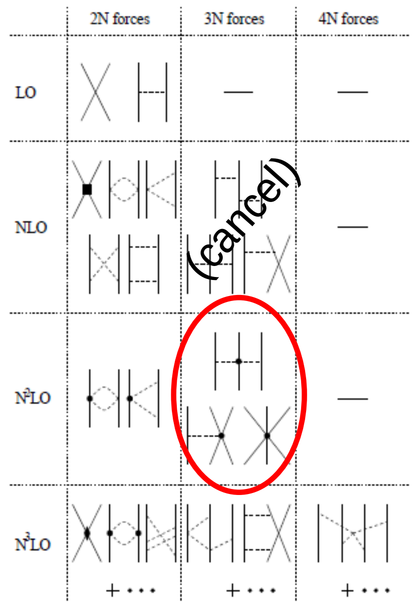
Pieper et al., PRC64(2001)014001

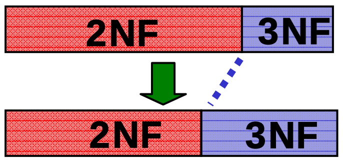
Three-Nucleon Force (3NF)

- It is natural to expect the existence of 3NF
- It is very nontrivial to determine 3NF from QCD
- 2πE-3NF Fujita-Miyazawa, PTP17(1957)360
  - Off-energy-shell πN scatt



- Phenomenological models
  - Fujita-Miyazawa, Tucson-Melbourne, Urbana/Illinois, …
- EFT expansion ➔ 3NF appear at NNLO
- N.B. the combination of (2NF, 3NF) ➔ observables
  - ← systematic determination by Lat QCD





U.v.Kolck, PRC49(1994)2932

Epelbaum, Prog.Part.Nucl.Phys.57(06)654

How can we tackle 3NF in Lattice QCD ?

c.f. pioneering lat calc of B.E. ^{3}He(=^{3}H), ^{4}He
T.Yamazaki et al., PRD81(2010)111504

- **In the case of 2N system…**
  - Calc 4pt func ➔NBS amp.

$$\psi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | 2N \rangle$$

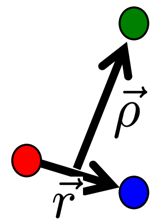
$$|2N\rangle = \bar{N}_{src}(t=0)\bar{N}_{src}(t=0)|0\rangle$$

➔ $(E - H_0)\psi(\vec{r})$

$$= [V_c(r) + S_{12}V_T(r) + \cdots]\psi(r)$$

- **Extention to 3N system**
  - Calc 6pt func ➔NBS amp. of 3N

$$\psi(\vec{r},\vec{\rho}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) N(\vec{x}+\vec{r}/2+\vec{\rho}) | 3N \rangle$$



  - Obtain 3NF through

$$(E - H_0^r - H_0^\rho)\psi(\vec{r},\vec{\rho}) = \left[\sum_{i<j} V_{ij}(\vec{r}_{ij}) + V_{3NF}(\vec{r},\vec{\rho})\right]\psi(\vec{r},\vec{\rho})$$

by 2N calc

  - Difficulty(1): volume factor
    - 2N: naïve O(L^6) calc ➔ O(L^3 log L^3)
    - 3N: naïve O(L^9) calc ➔ O(L^6 log L^6)

O(10^4-10^5) factor

  - Difficulty(2): naïve calc of quark dof grows in factorial (~N_u! N_d!)
    - 2N: O(L^3) X N_{wick} X color/spinor loops
    - 3N: O(L^6) X N_{wick} X color/spinor loops

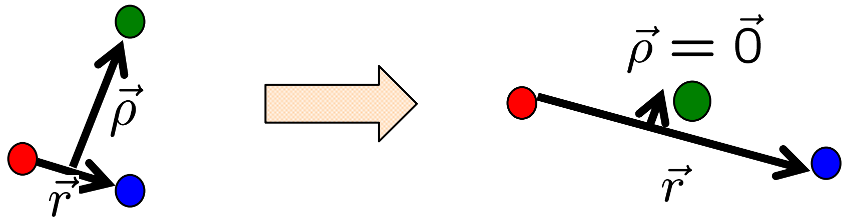
O(L^3) X O(4000) = O(10^7-10^8) factor

3NF is exceptionally challenging problem !

How can we tackle 3NF in Lattice QCD ? (cont'd)

- Calculation for **fixed 3D-configuration** of 3N system
  - **Direct access to 3NF is possible !**
  - ➔ We can explore the various features of 3NF (spin/isospin/spacial, etc.)
  - Huge calc cost (**O(10^2-10^3) factor** compared to 2N)
  - We study **linear setup**

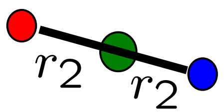
We consider Triton channel



$$(\vec{r}_2 \equiv \vec{r}/2)$$

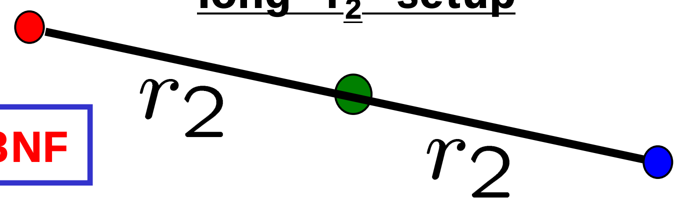
  - **Linear setup** with various distance **"r_2"**

short "r_2" setup



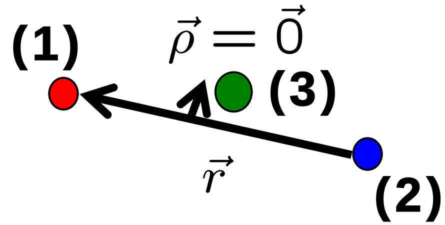
Study r_2-dependence of 3NF

long "r_2" setup



Features of Linear setup for ^{3}H

- Simplified coupled channel analysis possible
  - The vector to 3rd particle $\vec{\rho} = \vec{0}$
  - ➔ $L^{(1,2)\text{-pair}} = L^{\text{total}}$ = 0 or 2 only
  - ➔ Possible **bases are only three**, which can be labeled by **1S0, 3S1, 3D1 for (1,2)-pair**



Schrodinger Eq.

$$\hat{H}_0 \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} + \begin{pmatrix} V \\ (V_{2N} + V_{3NF}) \end{pmatrix} \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} = E \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix}$$

Kinetic energy

3x3 Matrix

Parity-odd potential Issue

- However, in order to determine 3NF in **3x3 coupled channel**, we need information of parity-odd potential
  - Although (1,2)-pair is L=even, (3,1),(2,3)-pair have L=odd components

- Parity-odd potential from lattice QCD is under R&D now
  - ➔3X3 channel, but unknown $V_C^{I,S=0,0}$,$V_C^{I,S=1,1}$,$V_T^{I,S=1,1}$,3NF(s)

$$\hat{H}_0 \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} + \left(\underset{(V_{2N} + V_{3NF})}{V} \right) \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} = E \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix}$$

$V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1}$: (P = even)

$V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1}$: (P = odd)

unknown

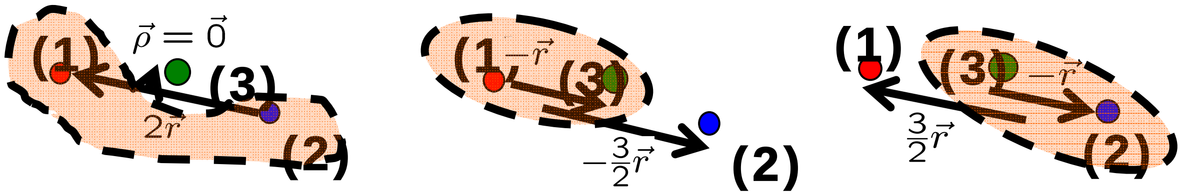
Target to be determined

Solution using "symmetric" wave function

- Rotate the basis $|\psi_{^1S_0}\rangle, |\psi_{^3S_1}\rangle, |\psi_{^3D_1}\rangle \longrightarrow |\psi_S\rangle, |\psi_M\rangle, |\psi_{^3D_1}\rangle$

$$|\psi_S\rangle = 1/\sqrt{2}\left(-|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right) \qquad |\psi_M\rangle = 1/\sqrt{2}\left(+|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)$$

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric



  - ➔ <u>**L=even for any 2N pair**</u> automatically guaranteed

$$\begin{aligned}
|\psi_S\rangle \;=\; 1/\sqrt{6}\Big[&-(p_\uparrow n_\uparrow - n_\uparrow p_\uparrow)n_\downarrow && (\leftarrow I=0, S=1)\\
&-(n_\uparrow n_\downarrow - n_\downarrow n_\uparrow)p_\uparrow && (\leftarrow I=1, S=0)\\
&+1/2(p_\uparrow n_\downarrow + n_\uparrow p_\downarrow - p_\downarrow n_\uparrow - n_\downarrow p_\uparrow)n_\uparrow && (\leftarrow I=1, S=0)\\
&+1/2(p_\uparrow n_\downarrow - n_\uparrow p_\downarrow + p_\downarrow n_\uparrow - n_\downarrow p_\uparrow)n_\uparrow\Big] && (\leftarrow I=0, S=1)
\end{aligned}$$

Solution using “symmetric” wave function

- Rotate the basis $|\psi_{^1S_0}\rangle, |\psi_{^3S_1}\rangle, |\psi_{^3D_1}\rangle \longrightarrow |\psi_S\rangle, |\psi_M\rangle, |\psi_{^3D_1}\rangle$

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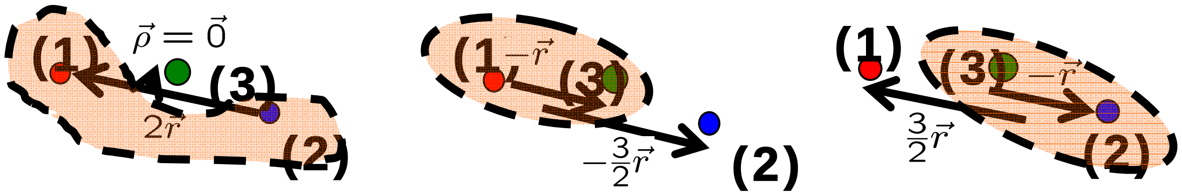
- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric



- ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed

All pair P=even

$$\hat{H}_0 \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \left(V_{2N} \right) \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \hat{V}_{3NF} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} = E \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix}$$

No V(P=odd)

Solution using "symmetric" wave function

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric
  - ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
- 3x3 coupled channel is reduced to
  - **<u>one channel with only 3NF unknown</u>**
  - two channels with $V_C^{I,S=0,0}$, $V_C^{I,S=1,1}$, $V_T^{I,S=1,1}$,(3NF) unknown

$$\begin{pmatrix} & & \\ & H_0 & \\ & & \end{pmatrix}\begin{pmatrix}\psi_S\\ \psi_M\\ \psi_{^3D_1}\end{pmatrix} + \begin{pmatrix} & & \\ & V_{2N} & \\ & & \end{pmatrix}\begin{pmatrix}\psi_S\\ \psi_M\\ \psi_{^3D_1}\end{pmatrix} + \begin{pmatrix} & & \\ & V_{3NF} & \\ & & \end{pmatrix}\begin{pmatrix}\psi_S\\ \psi_M\\ \psi_{^3D_1}\end{pmatrix} = E\begin{pmatrix}\psi_S\\ \psi_M\\ \psi_{^3D_1}\end{pmatrix}$$

No V(P=odd)

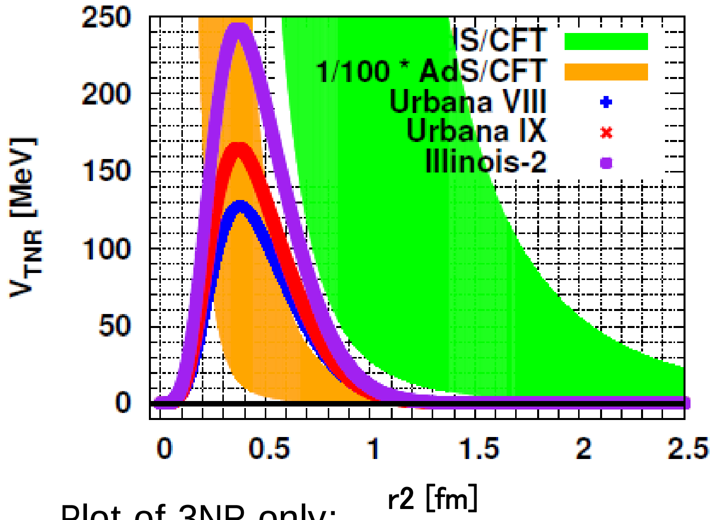
Target to be determined

(L^2-dep ignored)

- ➔ Even without parity-odd V, we can determine one 3NF
  - This methodology works for any fixed 3D-conf other than linear

Short-Range 3NF

- We determine 3NF effectively represented by a scalar/isoscalar functional form
  - c.f. phenomenological 3NF to reproduce saturation point of nuclear matter, etc.



Plot of 3NR only:
there is cancellation from 3NA

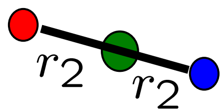
$$V_{3NF} = V_{2\pi E} + (V_{3\pi R}) + V_{3NR}$$

Urbana/Illinois

$$V_{3NR} = U_0 \sum_{cyc} T^2(r_{12}) T^2(r_{13})$$

$$T(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r} T_{cut}(r)$$

AdS/CFT: $V_{3NF} = +\text{const.} \cdot \frac{1}{r^4}$



K.Hashimoto, N.Iizuka
JHEP 1011 (2010) 058

Lattice QCD Calculations

Numerical
Setup & Results

BG/L@KEK

T2K@Tsukuba

SR16000
@YITP

「SR16000 モデル XM1」

Lattice calculation setup

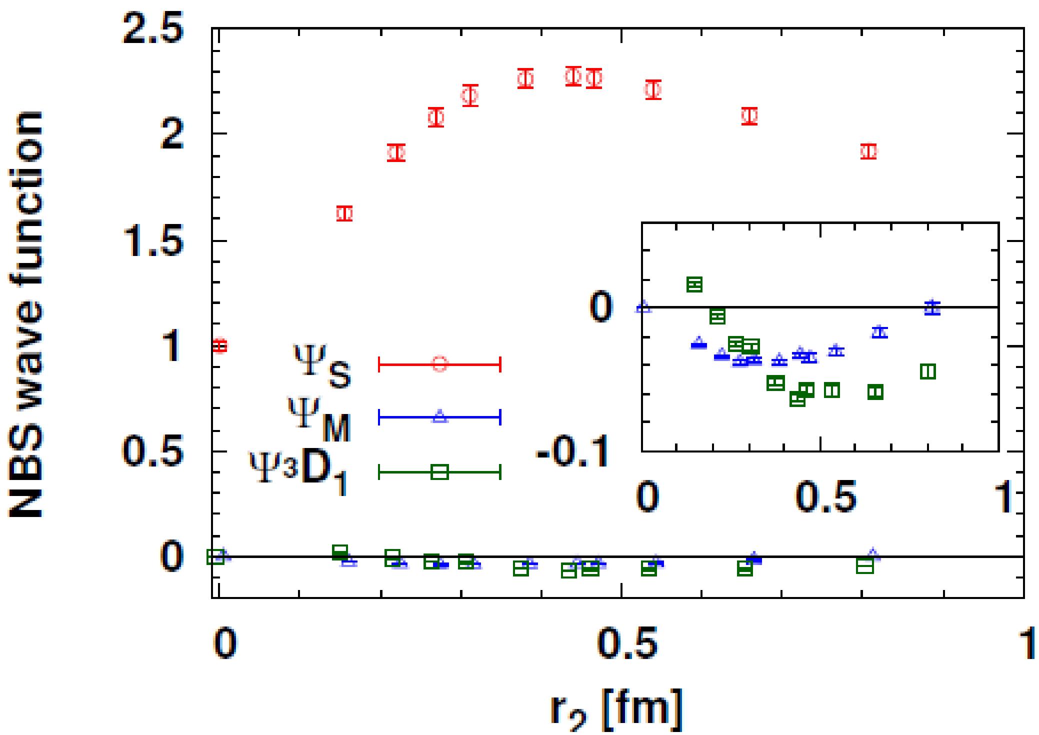
- Nf=2 dynamical clover fermion + RG improved gauge configs (CP-PACS)
  - 598 configs X 32 measurements
  - beta=1.95, (a^{-1}=1.27GeV, a=0.156fm)
  - 16^3 X 32 lattice, L=2.5fm
  - Kappa(ud)=0.13750
    - $M(\pi)$ = 1.13GeV
    - M(N) = 2.15GeV (Mπ L=14)
    - $M(\Delta)$ = 2.31GeV
  - Techniques
    - **Automatic Wick contraction code** to handle 4 up- and 5 down-quarks
    - **Non-rela limit op** is used to create 3N state **at source**

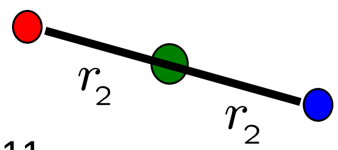
CP-PACS Coll. S. Aoki et al., Phys. Rev. D65 (2002) 054505

$$N^{src} = \epsilon_{abc}(u_a^T C\gamma_5 \frac{1+\gamma_4}{2} d_b)\frac{1+\gamma_4}{2} u_c$$

→ Factor of 2^3=8 faster

Results for wave functions



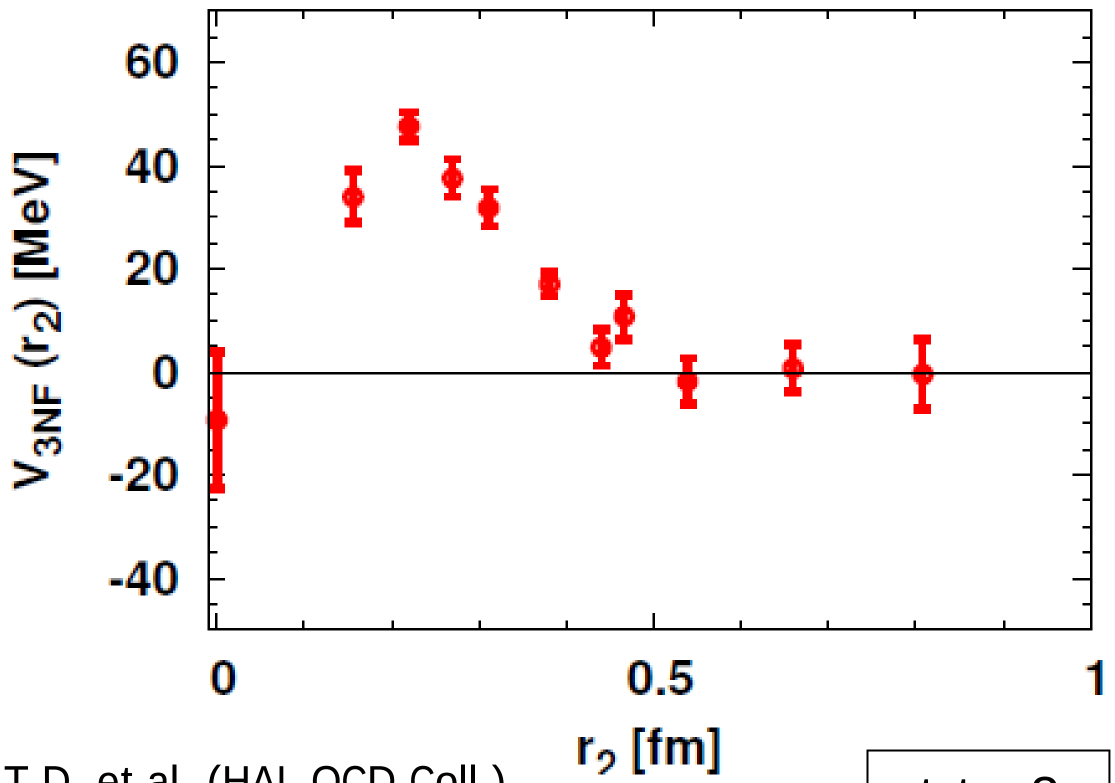


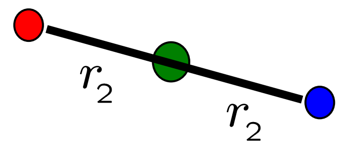
Red: Ψ_S
Blue: Ψ_M
Green: Ψ_{3D1}

Ψ_S overwhelms the wave function:

➔ Indication of the dominance of all S-wave component, higher waves suppressed

Genuine Three-Nucleon Force





short-range repulsive 3NF !

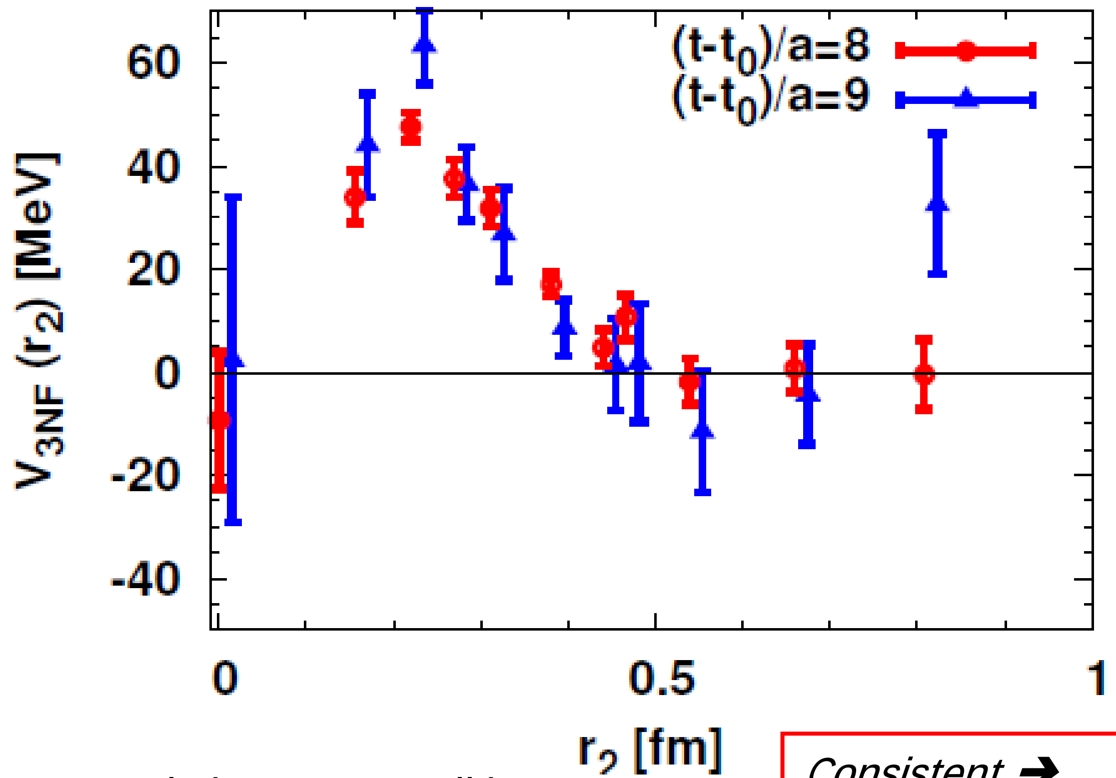
Huge Impact on physics of high density matters, EoS, Neutron Star, SuperNova, …

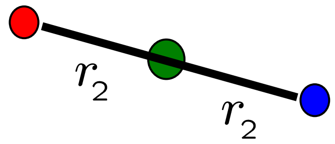
T.D. et al. (HAL QCD Coll.)
arXiv:1106.2276 [hep-lat]

$t\text{-}t_0=8$

$M(\pi)=1.13$GeV

Check on sink time dependence





short-range repulsive 3NF !

Huge Impact on physics of high density matters, EoS, Neutron Star, SuperNova, …

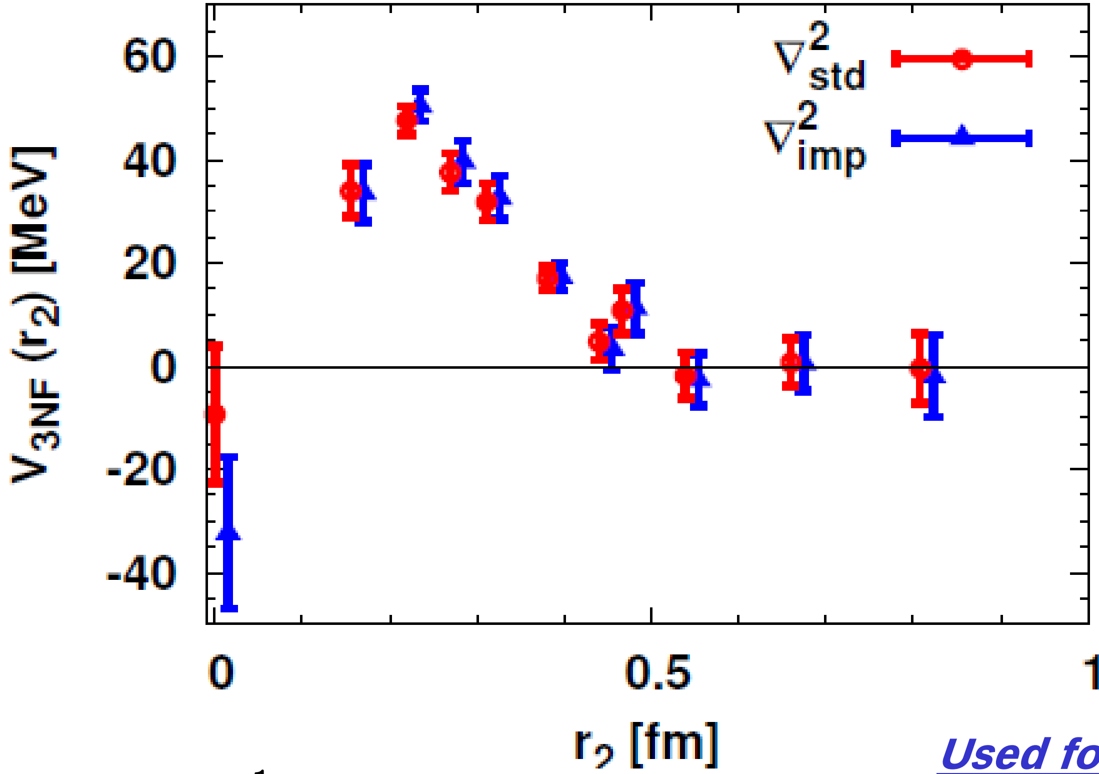
T.D. et al. (HAL QCD Coll.)
arXiv:1106.2276 [hep-lat]

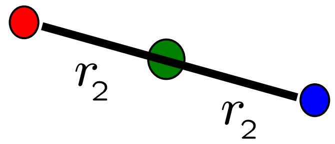
Consistent ➔
Saturated to G.S.

M(π)=1.13GeV

(further improvement in progress)

Studies on discretization error





Comparison with
Improved Laplacian op.

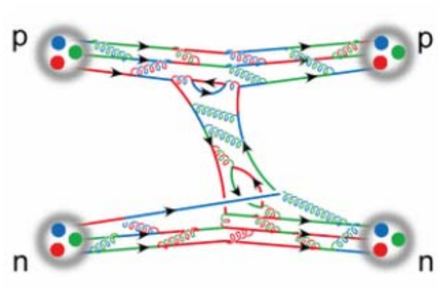
Discretization error in
Laplacian op. is small

Used for Kinetic energy

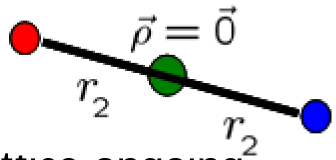
$$\nabla^2_{std} f(\vec{x}) = \frac{1}{a^2}\sum_i [f(\vec{x}+a_i) + f(\vec{x}-a_i) - 2f(\vec{x})] = \nabla^2 f(\vec{x}) + \mathcal{O}(a^2)$$

$$\nabla^2_{prc} f(\vec{x}) = \frac{1}{12a^2}\sum_i [-(f(\vec{x}+2a_i) + f(\vec{x}-2a_i)) + 16(f(\vec{x}+a_i) + f(\vec{x}-a_i)) - 30 f(\vec{x})] = \nabla^2 f(\vec{x}) + \mathcal{O}(a^4)$$

Summary/Outlook

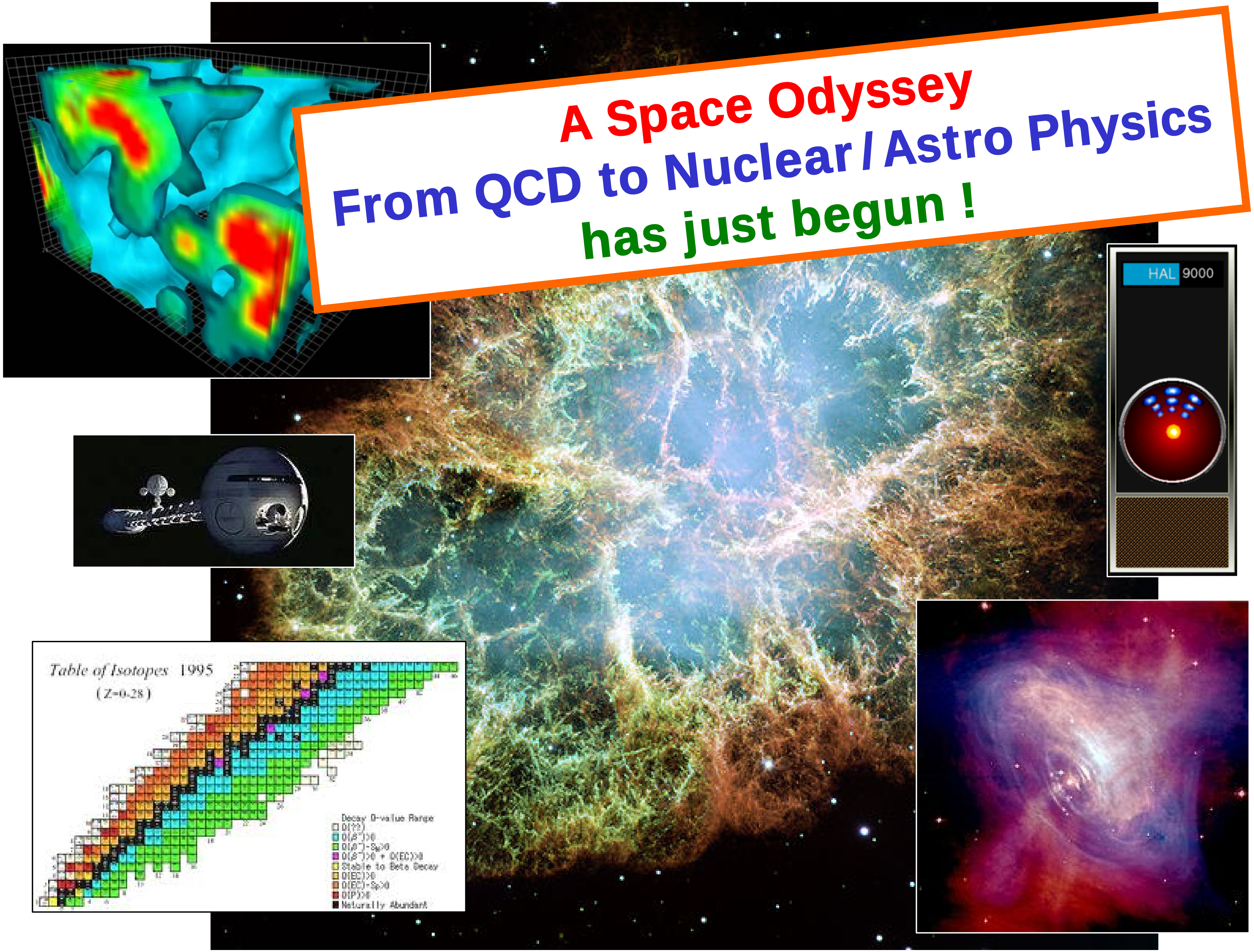


- **Potentials from Lattice QCD using NBS wave function**
  - Central and tensor potentials in parity-even channel
  - Qualitative features of NN potentials are reproduced, Velocity expansion checked
  - Significant step toward **Nuclear Physics from QCD**
- Lattice QCD can give **useful predictions** on unknown potentials
  - **YN, YY**: Strangeness physics, H-dibaryon, hyperon matter in neutron star, etc.
- **The First** calculation on **Genuine Three-Nucleon Force (3NF)** from Lattice QCD
  - Framework to subtract 2NF using only parity-even potentials
  - We have calculated the **linear setup** of 3N (^{3}H) system
    - System is reduced to 3x3 coupled channel
  - Nf=2 dynamical clover fermion at mπ = 1.13 GeV
    - **Repulsive 3NF at short distance**, further studies w/ finer lattice ongoing



- **Outlook**
  - Realistic potentials (and phase shifts) with **physically light masses w/ large volume**
  - More independent 3NF using parity-odd potentials ➔ FM, chEFT
  - Other 3D-conf of 3N, such as triangle ➔ spacial information of 3N
  - In future: other channel, I=3/2 [hard to access by scatt. exp]
    Extend the flavor space SU(2) ➔ SU(3) : Astrophysics (e.g., neutron star)

Ab-initio calculation using 2NF, 3NF from Lat QCD


A Space Odyssey
From QCD to Nuclear / Astro Physics
has just begun !
HAL 9000
Table of Isotopes 1995
(Z=0-28)
Decay Q-value Range
Stable to Beta Decay
Naturally Abundant