Lattice QCD:

a nonperturbative bridge from the pion mass to the Strong coupling at τ and Z^0 mass

by
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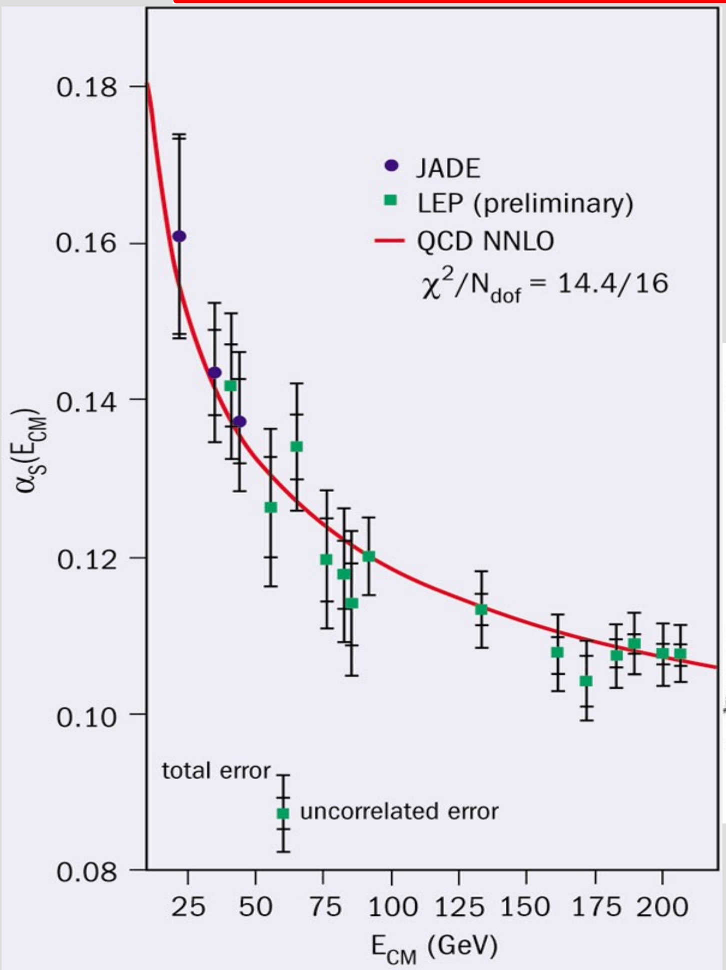
February 15, 2013; Saclay (France)

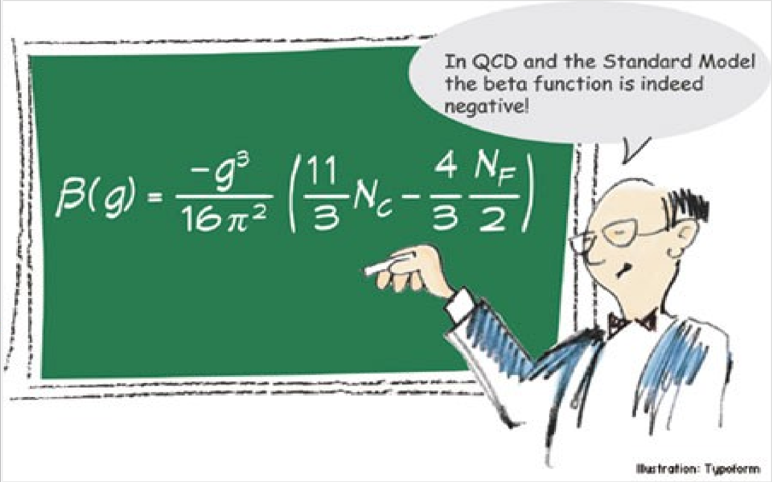
A task:
The running of ALPHA$_s$

$$\mathcal{L}_{\rm QCD} = \sum_q \left(i\bar{q}\not{D}\,q - m_q\bar{q}q\right) - \frac{1}{4}(D\times A)^2$$

$$(D^\mu \times A^\nu)_a \equiv \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g\sum f_{abc}\, A_b^\mu A_c^\nu$$

$$\left[\lambda^a,\lambda^b\right] = i f_{abc}\,\lambda^c$$





A task:

The running of ALPHA$_S$

4-loops perturbation theory[3]: $p \gg \Lambda_{QCD}$

$$\alpha_T(\mu^2) = \frac{4\pi}{\beta_0 t}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log(t)}{t} + \frac{\beta_1^2}{\beta_0^4}\frac{1}{t^2}\left(\left(\log(t) - \frac{1}{2}\right)^2 + \frac{\widetilde{\beta}_2\beta_0}{\beta_1^2} - \frac{5}{4}\right)\right)$$

$$+ \frac{1}{(\beta_0 t)^4}\left(\frac{\widetilde{\beta}_3}{2\beta_0} + \frac{1}{2}\left(\frac{\beta_1}{\beta_0}\right)^3\left(-2\log^3(t) + 5\log^2(t) + \left(4 - 6\frac{\widetilde{\beta}_2\beta_0}{\beta_1^2}\right)\log(t) - 1\right)\right), \; t = \ln\frac{\mu^2}{\Lambda_T^2}$$

$$\beta_T(\alpha_T) = \frac{d\alpha_T}{d\ln\mu^2} = -4\pi\sum_{i=0}\widetilde{\beta}_i\left(\frac{\alpha_T}{4\pi}\right)^{i+2}, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507-40N_f}{792-48N_f}} = 0.541449$$

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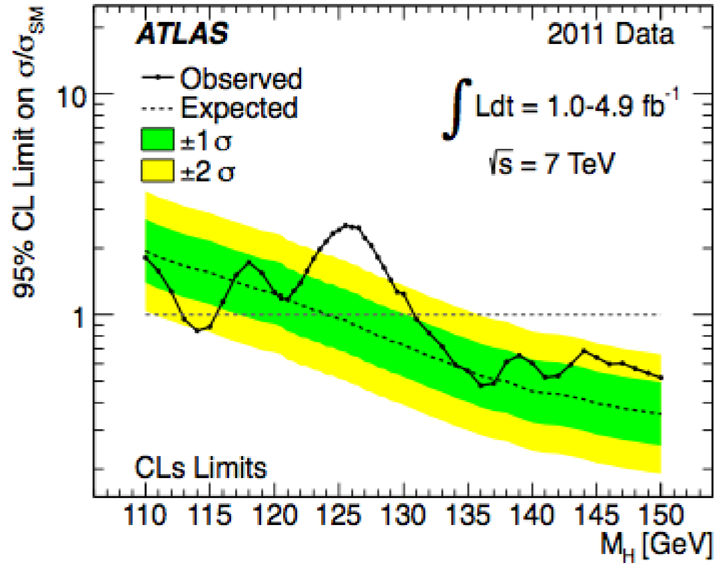
K.G. Chetyrkin, Nucl. Phys. B710 (2005) 499

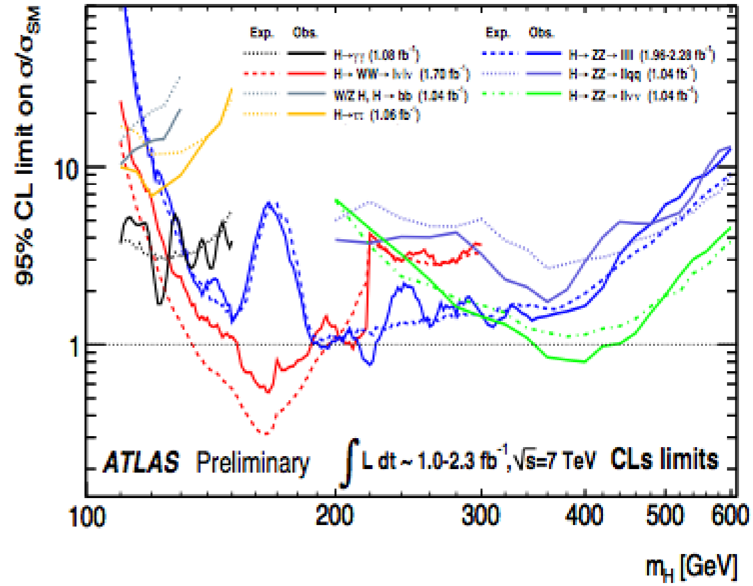
A task:
The running of ALPHA$_s$

The interest (one of them):

A major activity in Particle Physics is nowadays the search of Higgs boson, whose the existence might explain the spontaneous symmetry breaking of $SU(2)_W \times U(1)_Y$ predicted by the Standard Model and observed in Nature.

[ATLAS, '12; Lepton-Photon '11]





ATLAS has excluded at 95% of CL the region $131 < m_H < 238$ GeV (and also the mass range $251 < m_H < 466$ GeV). Hint of a signal around 125 GeV, both for ATLAS and CMS, in $h \to \gamma\gamma$ and $h \to 4l$, also for CDF and D0.

A task:
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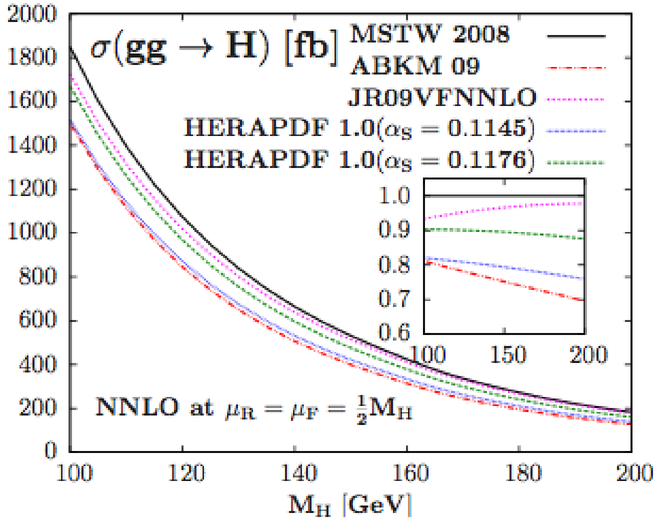
The interest (one of them):

Estimating as accurately as possible $\sigma^{th}_{gg\to H\to X}$ is an important ingredient to assess the detectors sensitivity to the Higgs physics. Several sources of uncertainty:

– NNNLO (QCD) and NNLO (EW) corrections
– factorisation scale uncertainties
– error on $H \to X$
– parton distribution functions and $\delta(\alpha_s)$

[J. Baglio et al, '11]

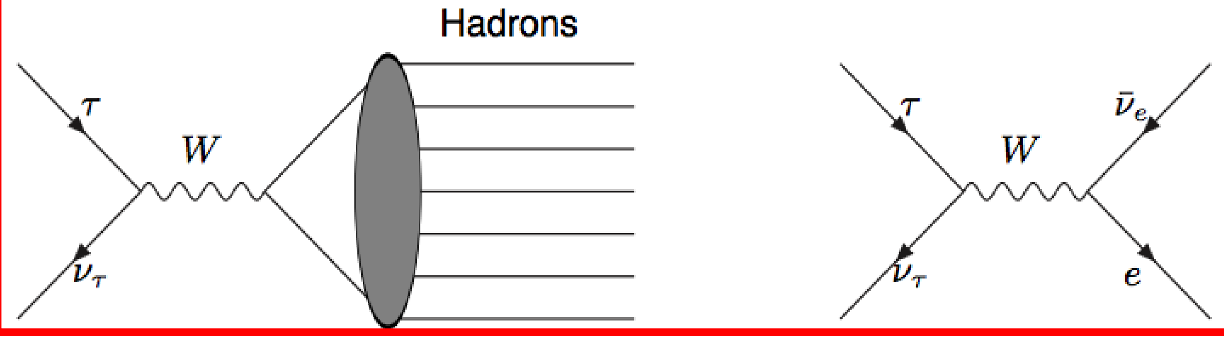


$\Delta\sigma^{NNLO}_{gg\to H\to X} \sim 20-25\%$ at LHC ($\sqrt{s} = 7$ TeV), with 2-3 % from $\delta\alpha_s$

A task:

The running of ALPHA$_s$

The interest (more): Extraction of the coupling from tau decays



$$R_\tau \equiv \Gamma[\tau^- \to \nu_\tau \, \text{hadrons}]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e]$$

$$R_{\tau,V+A} = N_c |V_{ud}|^2 S_{\rm EW}(1 + \delta_P + \delta_{NP}), \quad \delta_{NP} = -0.0059(14)$$ [Davier *et al*, '08]

$$\delta_P = \sum_n K_n A^{(n)}(\alpha_s) = \sum_n (K_n + g_n)\alpha^n(m_\tau)$$

$$A^{(n)} = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left(\frac{\alpha_s(-s)}{\pi} \right)^n \left(1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8} \right) = \alpha^n(m_\tau) + \mathcal{O}(\alpha^{n+1}(m_\tau))$$

Fixed Order Perturbation Theory (FOPT) vs Contour Improved Perturbation Theory (CIPT):

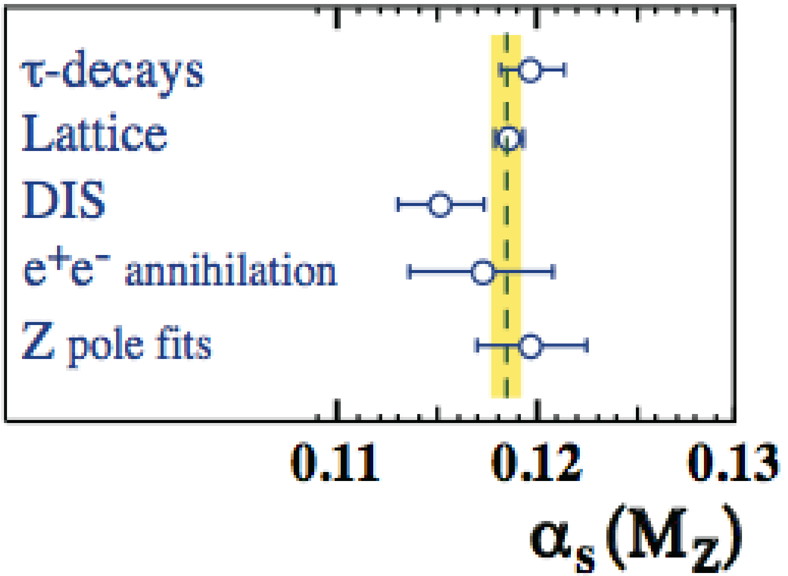
$\alpha_s(m_\tau)_{\rm CIPT} = 0.344(14)$ $\quad \alpha_s(m_\tau)_{\rm FOPT} = 0.321(15)$ [A Pich, '11]

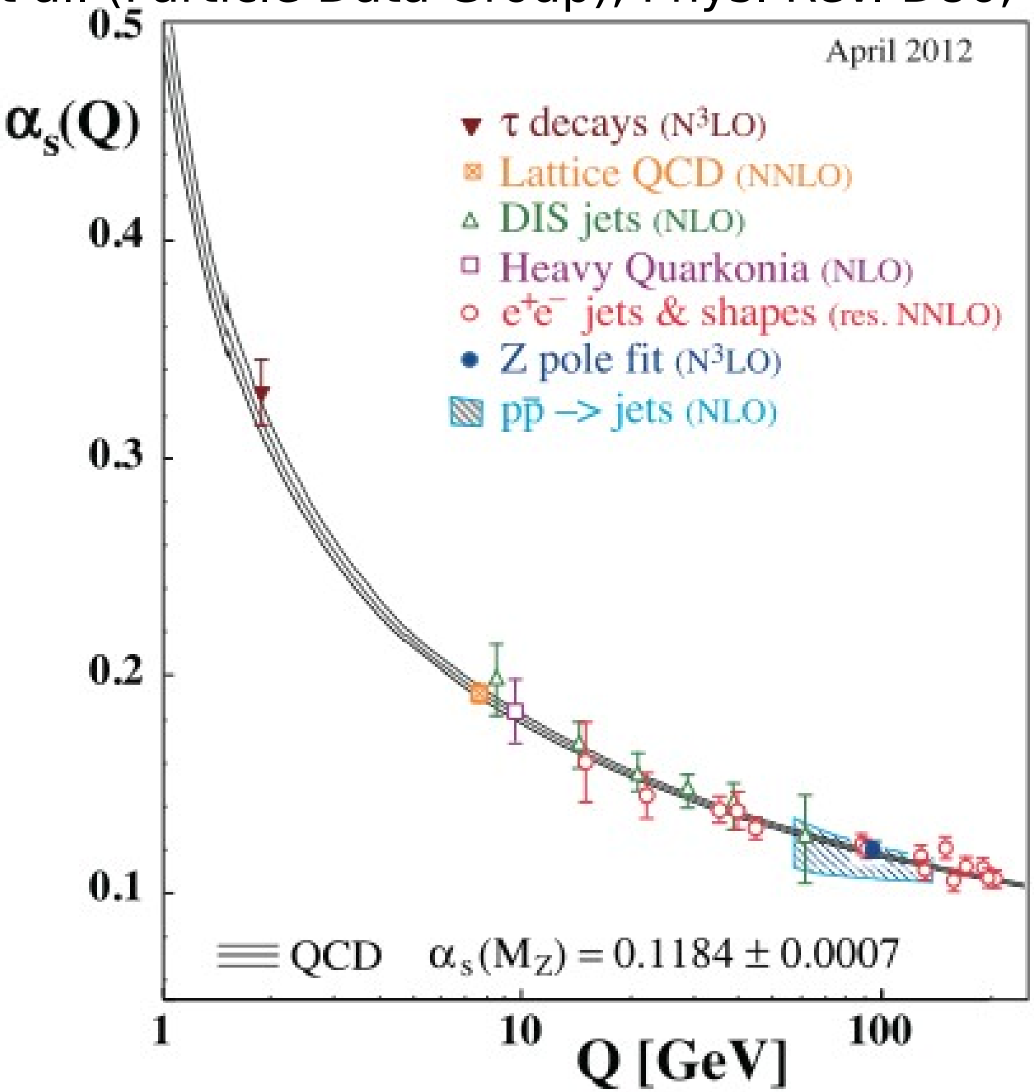
A task:

The running of ALPHA$_s$

J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010

The interest (more):
The QCD coupling at many scales and from many processes!!!



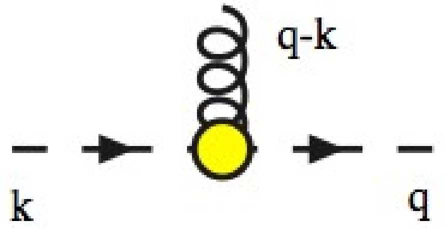


Main characters:

The ghost-gluon coupling

The ghost-gluon vertex:

$$\widetilde{\Gamma}_\nu^{abc}(-q,k;q-k) =$$



$$= i g_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

$$\widetilde{\Gamma}_R = \widetilde{Z}_1 \Gamma$$

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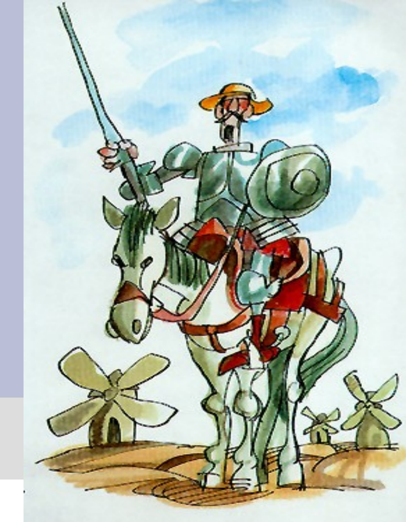

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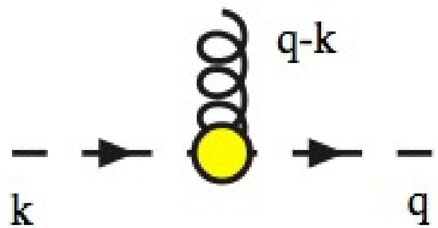

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$$g_R(\mu^2) = \lim_{\Lambda\to\infty} Z_g^{-1}(\mu^2,\Lambda^2) g_0(\Lambda^2)$$

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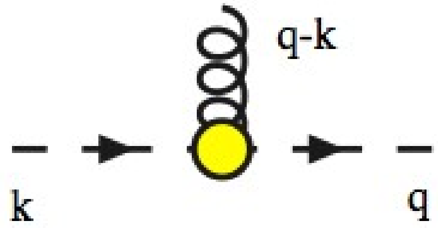

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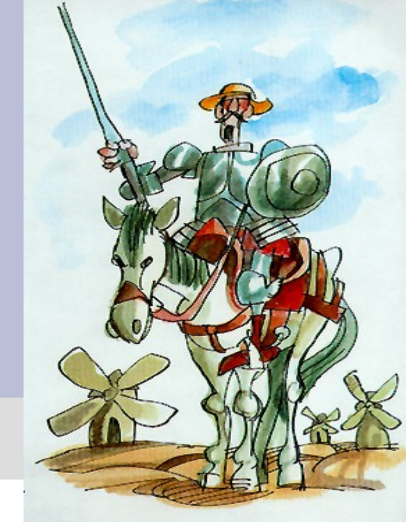

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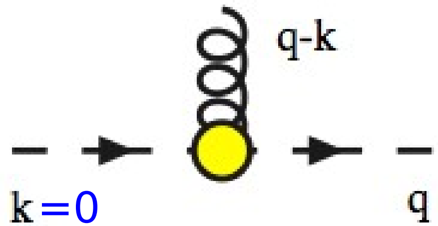

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In Taylor scheme

Main characters:

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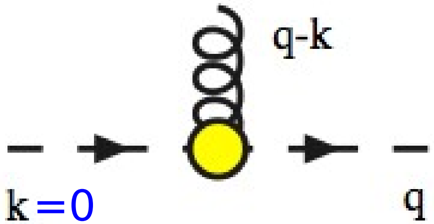

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$\widetilde{Z}_1(\mu^2,\Lambda^2) \rightarrow 1$

In Taylor scheme & Landau gauge

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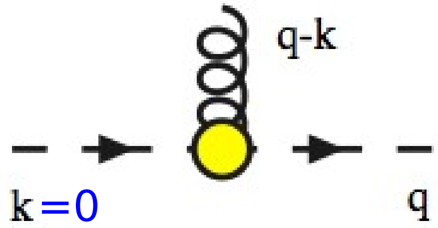

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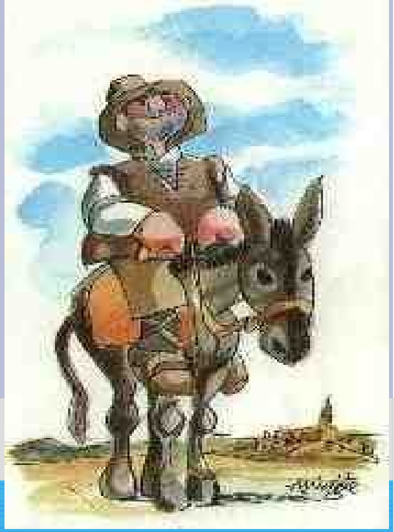
$\widetilde{Z}_1(\mu^2,\Lambda^2) \to 1$

In Taylor scheme & Landau gauge

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} Z_3(\mu^2,\Lambda^2)\widetilde{Z}_3^2(\mu^2,\Lambda^2)$$

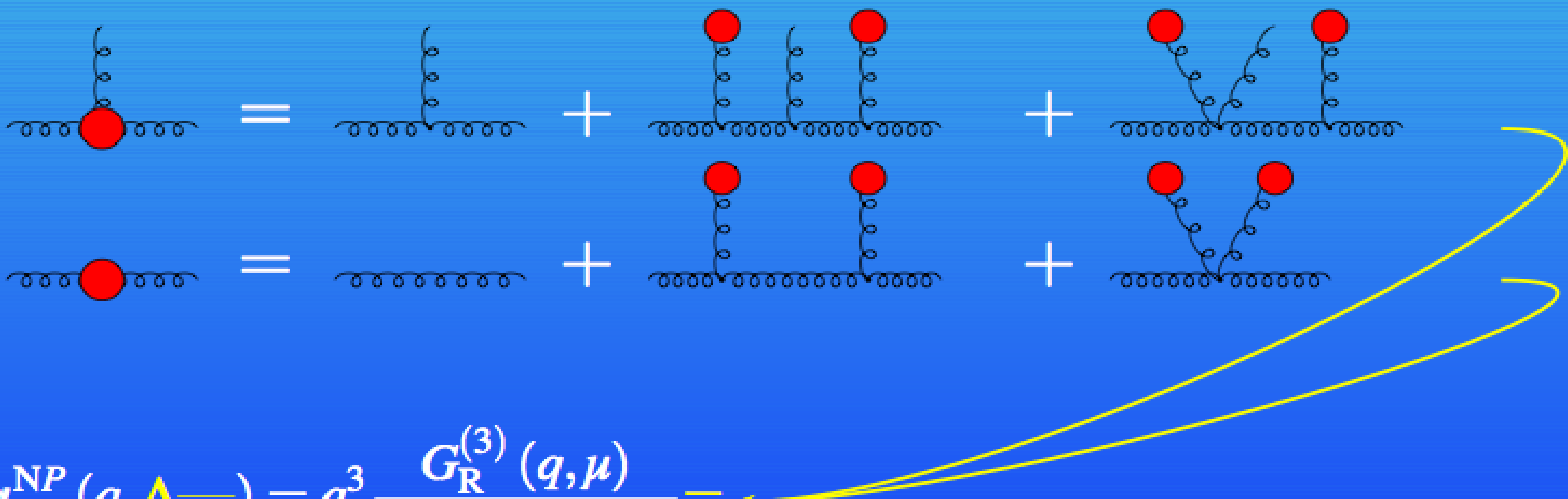
Main characters:

The ghost-gluon coupling & gluon condensate



OPE + SVZ :

(PLB493(2000)315, PRD63(2001)114003)



$$\alpha^{NP}(q,\Lambda_{\overline{MS}}) = q^3 \frac{G_R^{(3)}(q,\mu)}{\left[G_R^{(2)}(q,\mu)\right]^{\frac{3}{2}}} =$$

$$= \alpha\left(\ln\frac{q}{\Lambda_{\overline{MS}}}\right)\left(1+\frac{9g_R^2(\mu)\langle A^2\rangle_\mu}{4(N_C^2-1)}\,\frac{1}{q^2}\right)$$

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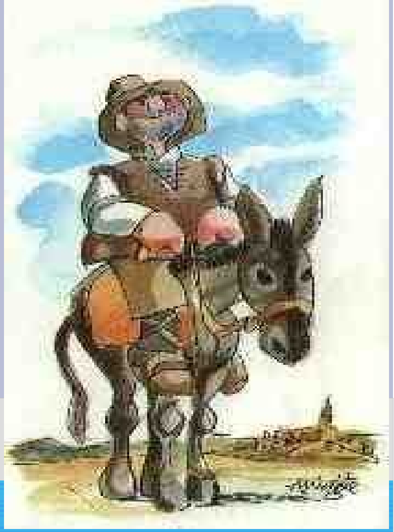

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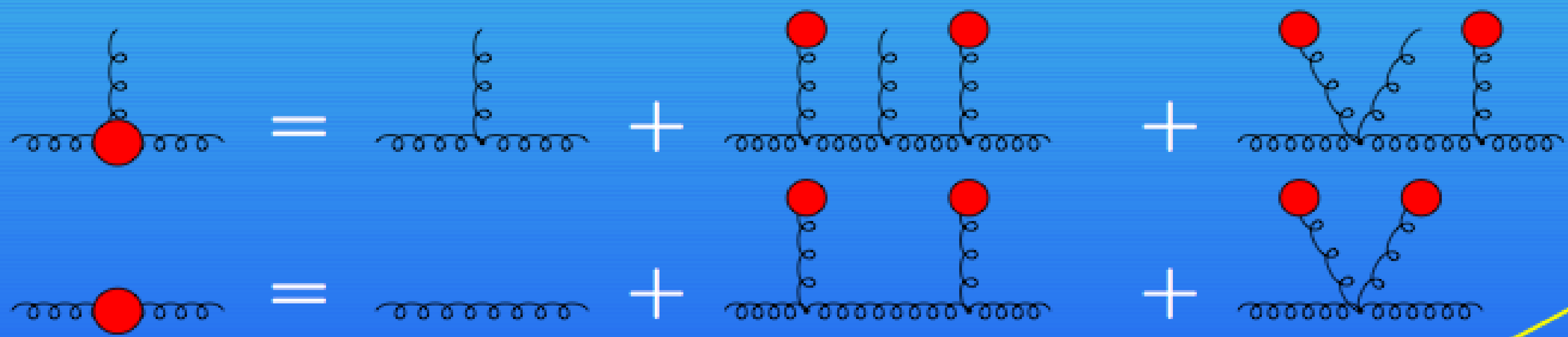

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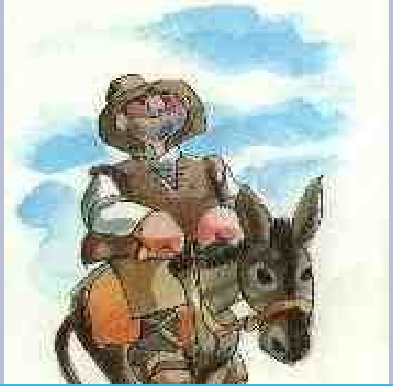


(JHEP04(2003)005,PRD70(2005)114503)

Instanton (BPST) $\rightarrow gA^a_\mu(x) = \dfrac{2\overline{\eta}^a_{\mu\nu}x^\nu\rho^2}{x^2(x^2+\rho^2)} \rightarrow$

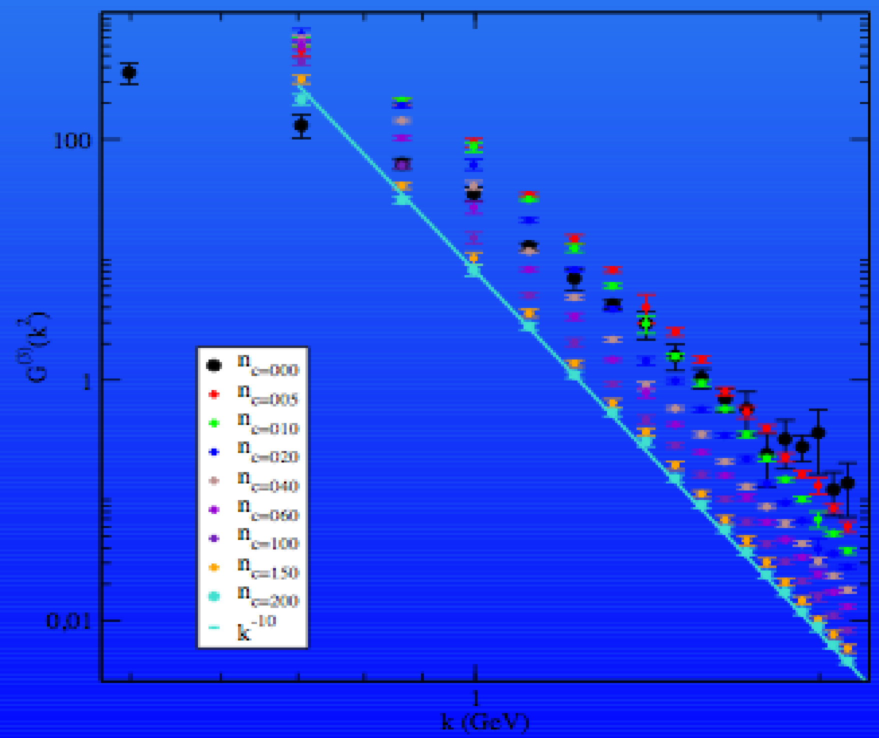
$$\langle A^2 \rangle \approx n \int d^4x\, A^a_\mu(x) A^a_\mu(x) = \frac{12 n \pi^2 \rho^2}{g^2}$$

ILM Shuryak,D&P ...

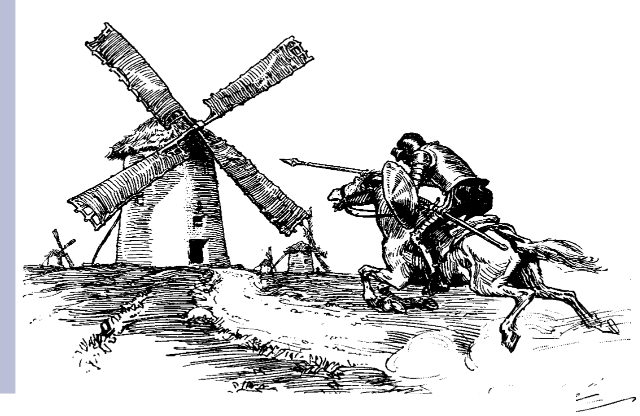
$$\alpha_{\mathrm{MOM}}(q) = \frac{1}{18\pi n} q^4$$

$n = 5.27(4)\ \mathrm{fm}^{-4}$

$g_R^2 \langle A^2 \rangle \simeq 2.3\ \mathrm{GeV}^2$

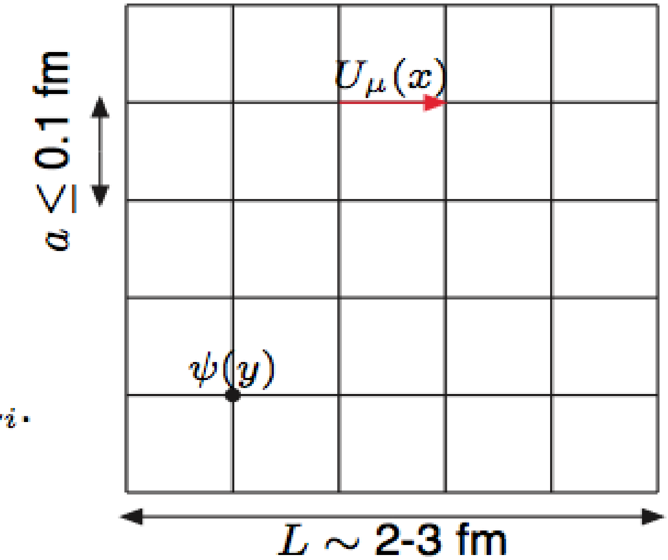


The weaponry:
Lattice QCD



Discretisation of QCD in a finite volume of Euclidean space-time.

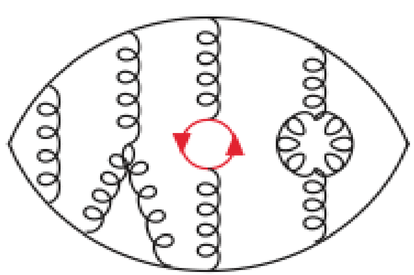
The lattice spacing a is a non perturbative UV cut-off of the theory.



Fields: $\psi^i(x)$, $U_\mu(x) \equiv e^{iag_0 A_\mu(x+\frac{a\hat{\mu}}{2})}$.

Inputs: bare coupling $g_0(a) \equiv \sqrt{6/\beta}$, bare quark masses m_i.

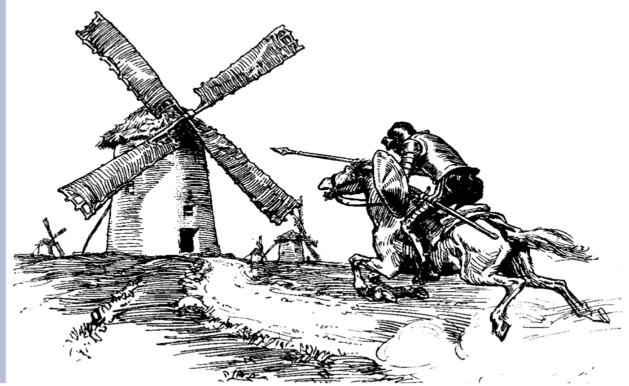
Computation of Green functions of the theory from first principles:



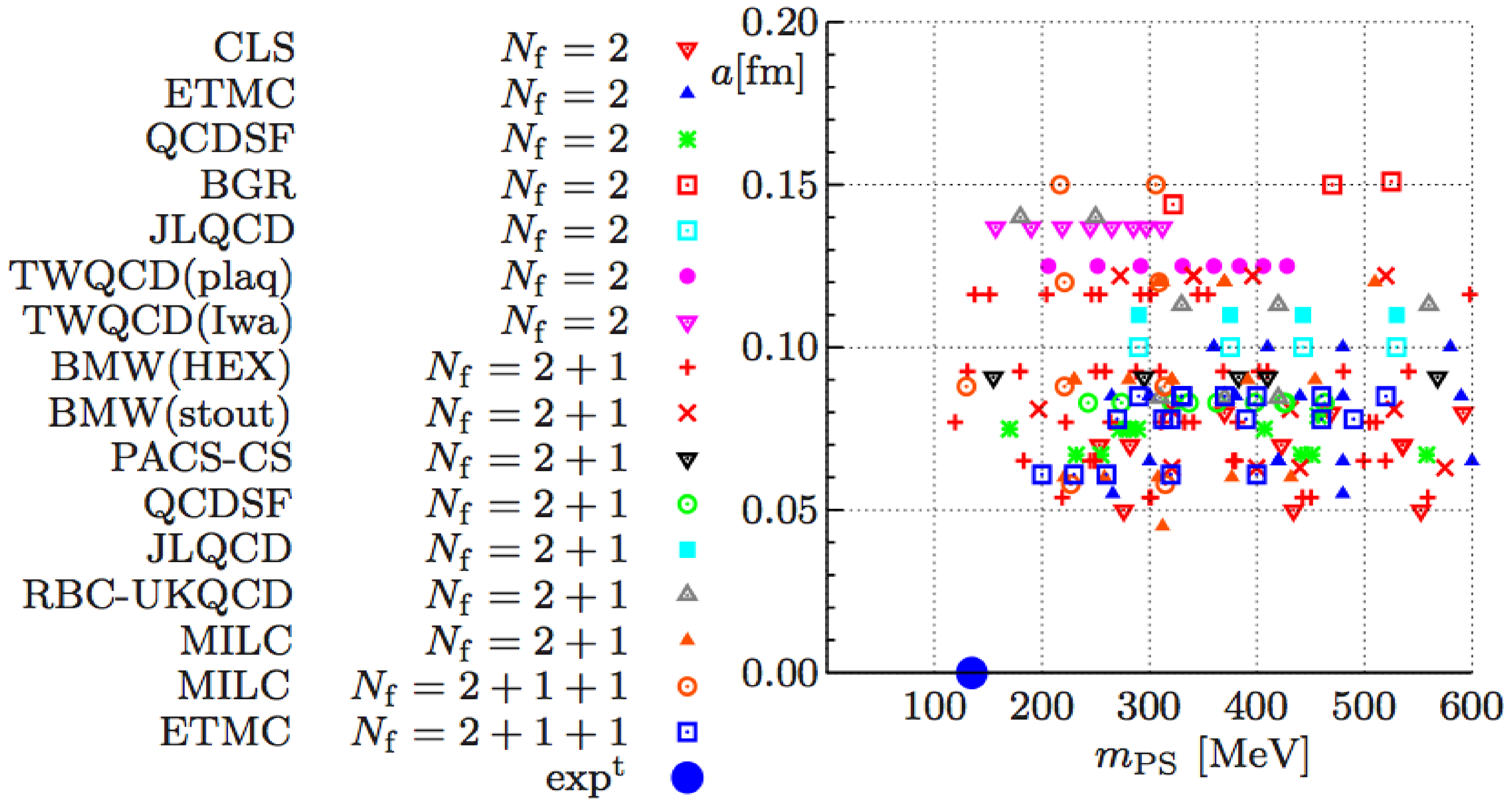
$$\langle O(U,\psi,\bar{\psi})\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\,O(U,\psi,\bar{\psi})e^{-S(U,\psi,\bar{\psi})}$$
$$\mathcal{Z} = \int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}e^{-S(U,\psi,\bar{\psi})}$$
$$S(U,\psi,\bar{\psi}) = S^{\mathrm{YM}}(U) + \bar{\psi}^i_x M^{ij}_{xy}(U)\psi^j_y$$
$$\mathcal{Z} = \int \mathcal{D}U\,\mathrm{Det}[\mathrm{M(U)}]e^{-S^{\mathrm{YM}}(U)} \equiv \int \mathcal{D}U e^{-S_{\mathrm{eff}}(U)}$$

Monte Carlo simulation: $\langle O\rangle \sim \frac{1}{\mathrm{N_{conf}}}\sum_i O(\{U\}_i)$: we have to build the statistical sample $\{U\}_i$ in function of the Boltzmann weight $e^{-S_{\mathrm{eff}}}$. Incorporating the quark loop effects hidden in $\mathrm{Det}[M(U)]$ is particularly expensive in computer time. Crucial in the extraction of α_s.

The weaponry:
Current simulation set up



In the past years tremendous progresses have been made by the lattice community to perform simulations that are closer to the physical point.



A task:
The running of ALPHA$_s$
… from the lattice!!!

Lattice: $\frac{1}{L^2} \ll p^2 \ll \frac{1}{a^2}$

Propagators in Landau gauge

$$\left(G^{(2)}\right)^{ab}_{\mu\nu}(p^2,\Lambda) = \frac{G(p^2,\Lambda)}{p^2}\,\delta_{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$\left(F^{(2)}\right)^{a,b}(p^2,\Lambda) = -\delta_{ab}\,\frac{F(p^2,\Lambda)}{p^2}$$

Renormalized in non-perturbative MOM-scheme:

$$G_R(p^2,\mu^2) = \lim_{\Lambda\to\infty} Z_3^{-1}(\mu^2,\Lambda)\,G(p^2,\Lambda)$$

$$F_R(p^2,\mu^2) = \lim_{\Lambda\to\infty} \widetilde{Z}_3^{-1}(\mu^2,\Lambda)\,F(p^2,\Lambda)$$

$$G_R(\mu^2,\mu^2) = F_R(\mu^2,\mu^2) = 1$$

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Ghost and gluon on the lattice

Landau gauge

$$F_U[g] = \mathrm{Re}\left[\sum_x \sum_\mu \mathrm{Tr}\left(1 - \frac{1}{N} g(x) U_\mu(x) g^\dagger(x+\mu)\right)\right]$$

Gluon:

$$A_\mu(x + \hat{\mu}/2) = \frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0} - \frac{1}{3}\mathrm{Tr}\left(\frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0}\right)$$

$$\left(G^{(2)}\right)^{a_1 a_2}_{\mu_1 \mu_2}(p) = \langle A^{a_1}_{\mu_1}(p) A^{a_2}_{\mu_2}(-p) \rangle$$

A task:

The running of ALPHA$_s$

… from the lattice!!!

Ghost and gluon on the lattice

Landau gauge

$$F_U[g] = \mathrm{Re}\left[\sum_x \sum_\mu \mathrm{Tr}\left(1 - \frac{1}{N} g(x) U_\mu(x) g^\dagger(x+\mu)\right)\right]$$

Ghost:

$$\left(F^{(2)}\right)^{ab}(x-y) \equiv \langle \left(M^{-1}\right)^{ab}_{xy} \rangle \,, \quad M(U) = -\frac{1}{N} \nabla \cdot \widetilde{D}(U)$$

$$\widetilde{D}(U)\eta(x) = \frac{1}{2}\left(U_\mu(x)\eta(x+\mu) - \eta(x)U_\mu(x) + \eta(x+\mu)U_\mu^\dagger - U_\mu^\dagger(x)\eta(x)\right)$$

A task:
The running of ALPHA$_s$
… from the lattice!!!

ETM
Collaboration:

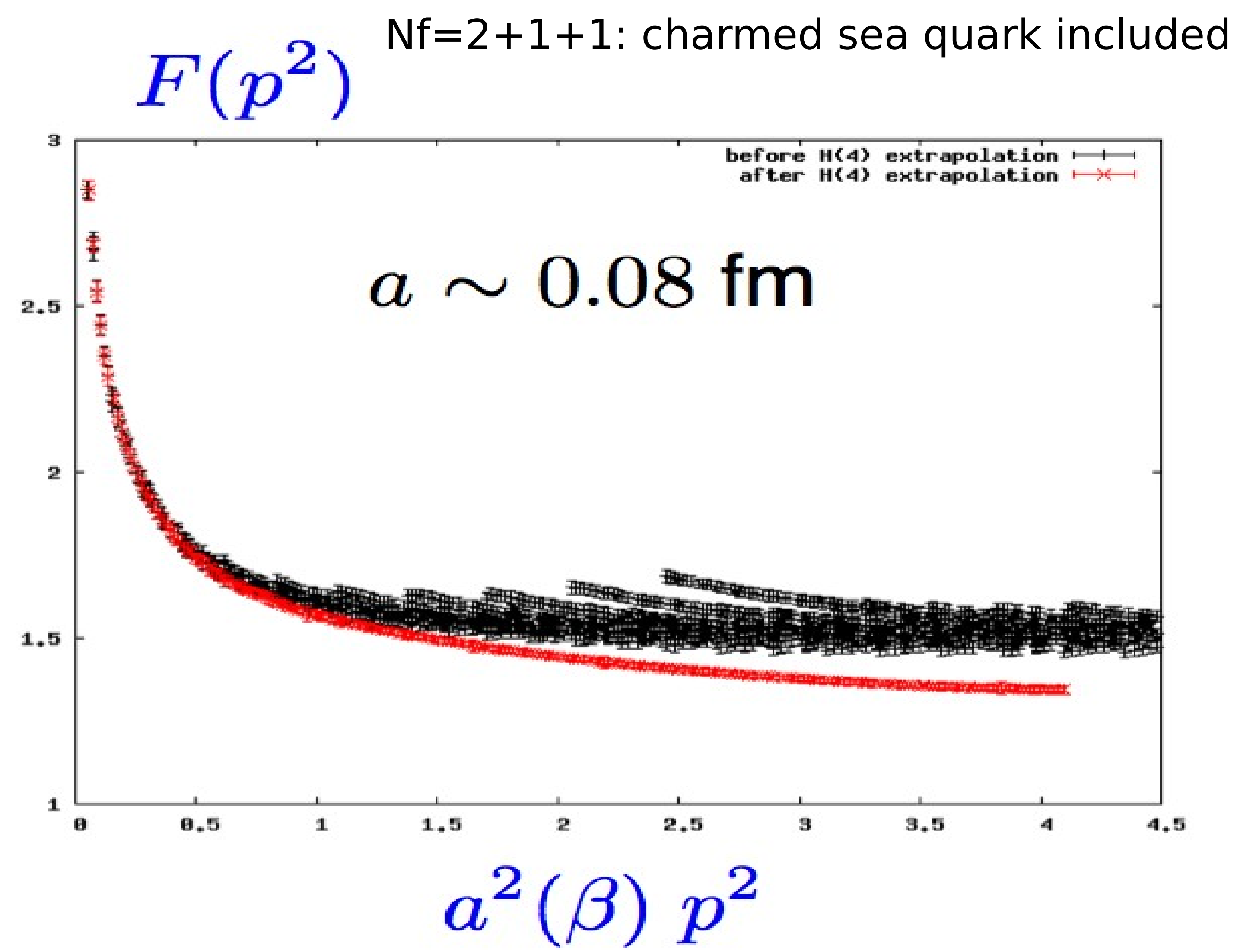
Set up parameters:

β	$\kappa_{\rm crit}$	$a\mu_l$	$a\mu_\sigma$	$a\mu_\delta$	$(L/a)^3 \times T/a$	confs.
1.90	0.1632700	0.0040	0.150	0.1900	$32^3 \times 64$	50
1.95	0.1612400	0.0035	0.135	0.1700	$32^3 \times 64$	50
2.10	0.1563570	0.0020	0.120	0.1385	$48^3 \times 96$	100

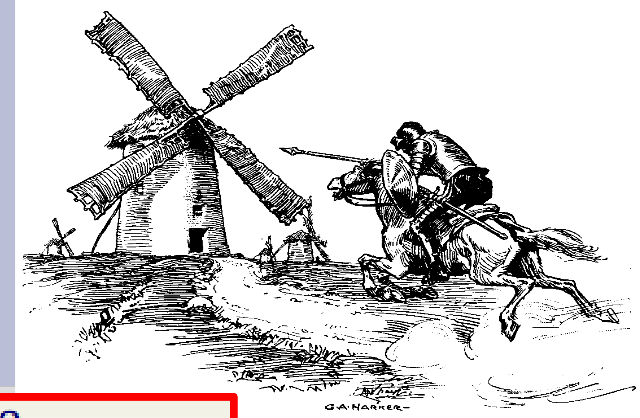
ETMC N_f=2+1+1 ensembles: $a^{\beta=2.1} \sim 0.06$ fm, $a^{\beta=1.95} \sim 0.08$ fm, $a^{\beta=1.9} \sim 0.09$ fm, $m_\pi \in$ [250-325] MeV

A task:
The running of ALPHA$_s$
… from the lattice!!!

Nf=2+1+1: charmed sea quark included!!!



A task:
The running of ALPHA$_s$
… from the lattice!!!



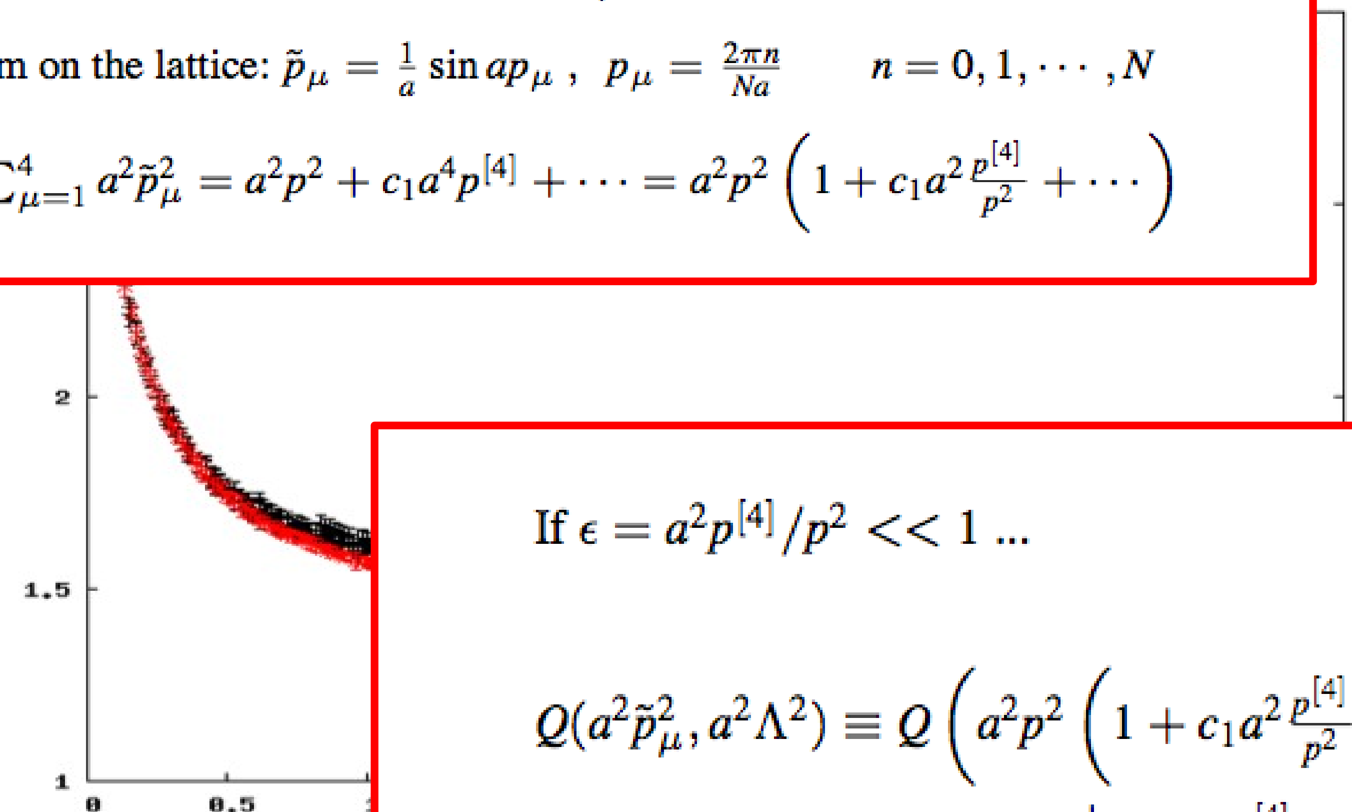
$O(4)$ breaking: $H(4)$ discretization artefacts[8]

uded!!!

Orbit labeled by $H(4)$-invariants: $p^{[2n]} = \sum_{\mu=1}^{4} p_\mu^{2n}, n = 1,2,3$

Momentum on the lattice: $\tilde{p}_\mu = \frac{1}{a}\sin a p_\mu \ , \ \ p_\mu = \frac{2\pi n}{Na} \qquad n = 0,1,\cdots,N$

$$a^2\tilde{p}^2 \equiv \sum_{\mu=1}^{4} a^2\tilde{p}_\mu^2 = a^2p^2 + c_1 a^4 p^{[4]} + \cdots = a^2p^2\left(1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \cdots\right)$$



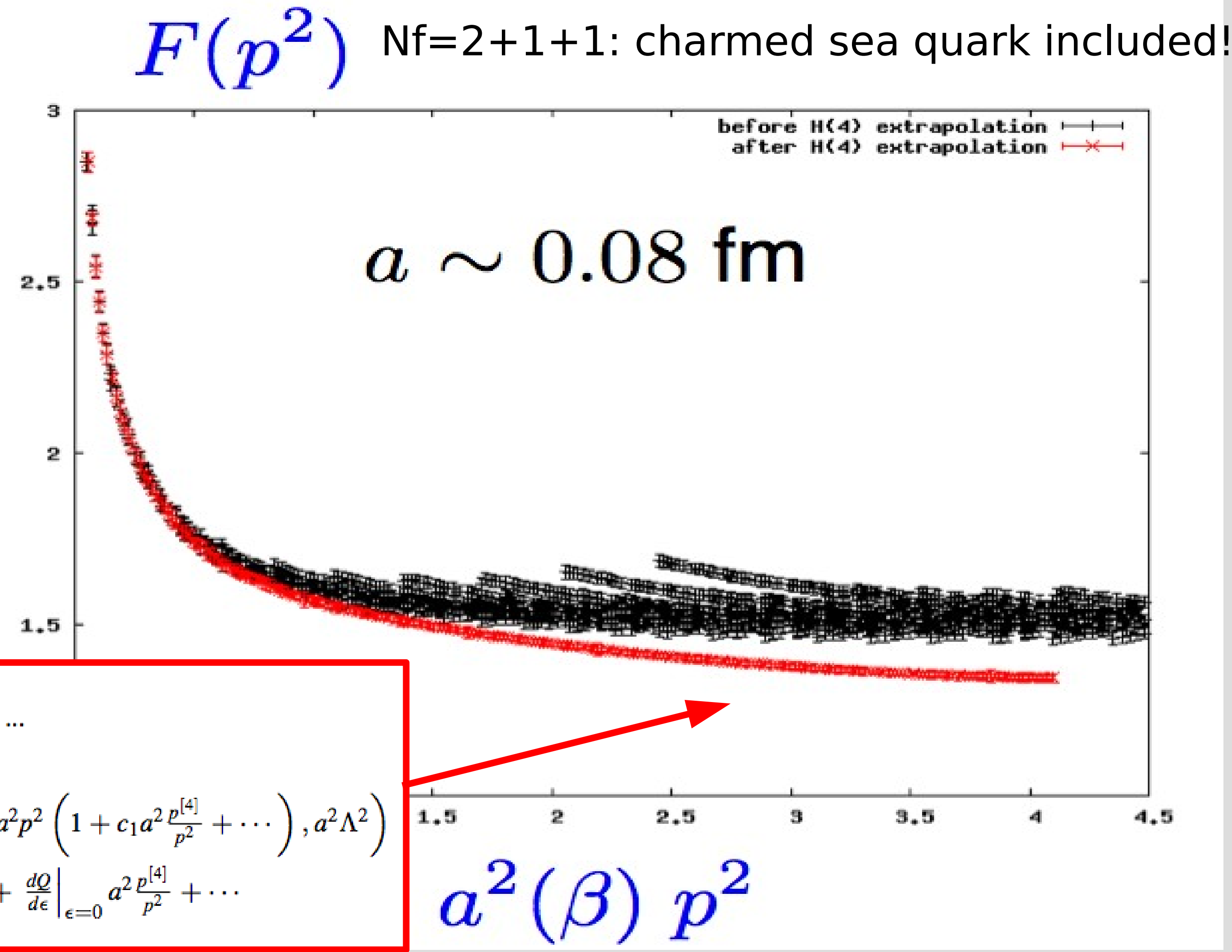
If $\epsilon = a^2p^{[4]}/p^2 << 1$ ...

$$Q(a^2\tilde{p}_\mu^2, a^2\Lambda^2) \equiv Q\left(a^2p^2\left(1 + c_1 a^2\frac{p^{[4]}}{p^2} + \cdots\right), a^2\Lambda^2\right)$$

$$= Q(a^2p^2, a^2\Lambda^2) + \left.\frac{dQ}{d\epsilon}\right|_{\epsilon=0} a^2\frac{p^{[4]}}{p^2} + \cdots$$

A task:
The running of ALPHA$_s$
… from the lattice!!!

Nf=2+1+1: charmed sea quark included!!!

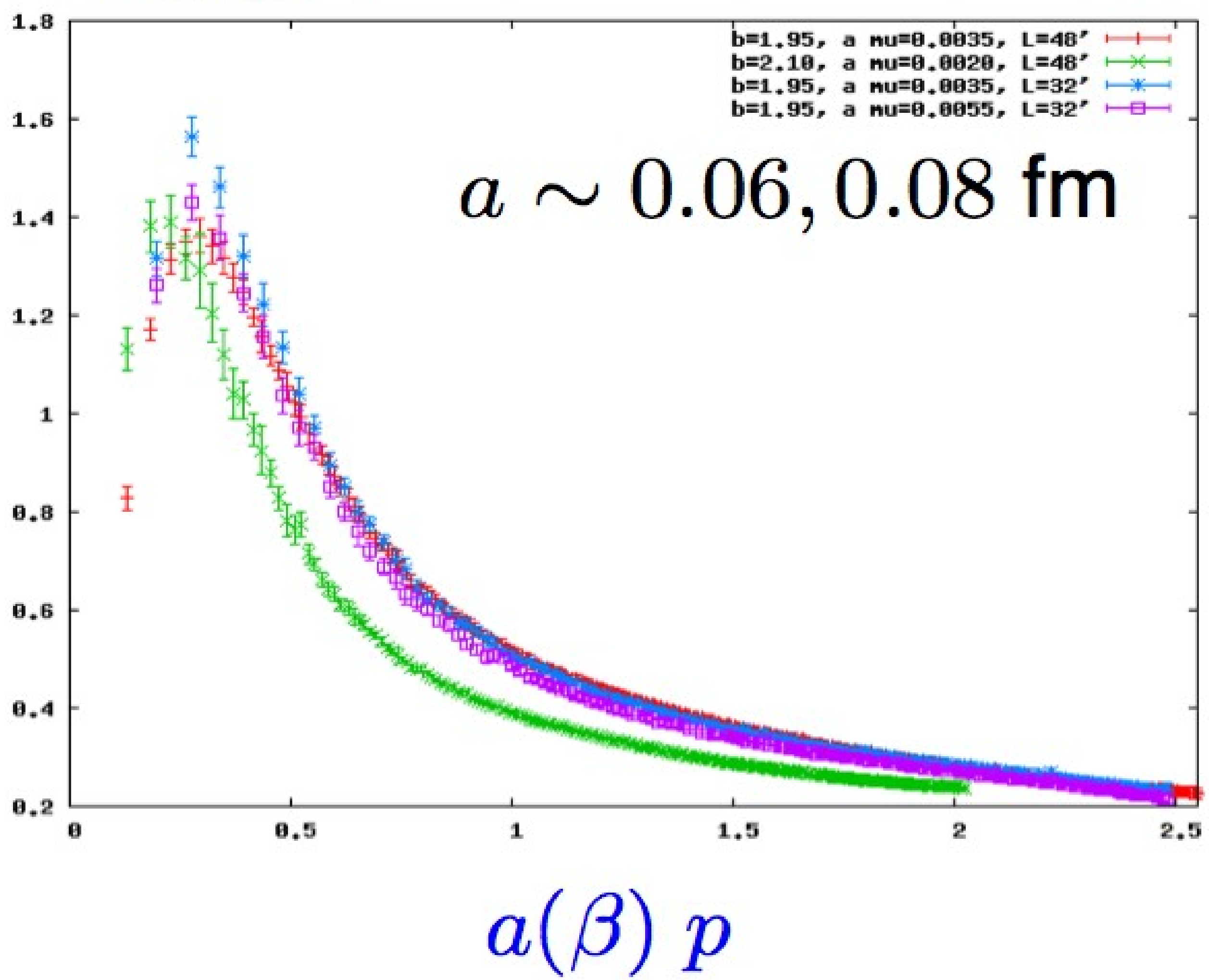


If $\epsilon = a^2 p^{[4]}/p^2 << 1$ ...

$$Q(a^2\tilde{p}^2_\mu, a^2\Lambda^2) \equiv Q\left(a^2p^2\left(1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \cdots\right), a^2\Lambda^2\right)$$
$$= Q(a^2p^2, a^2\Lambda^2) + \left.\frac{dQ}{d\epsilon}\right|_{\epsilon=0} a^2\frac{p^{[4]}}{p^2} + \cdots$$

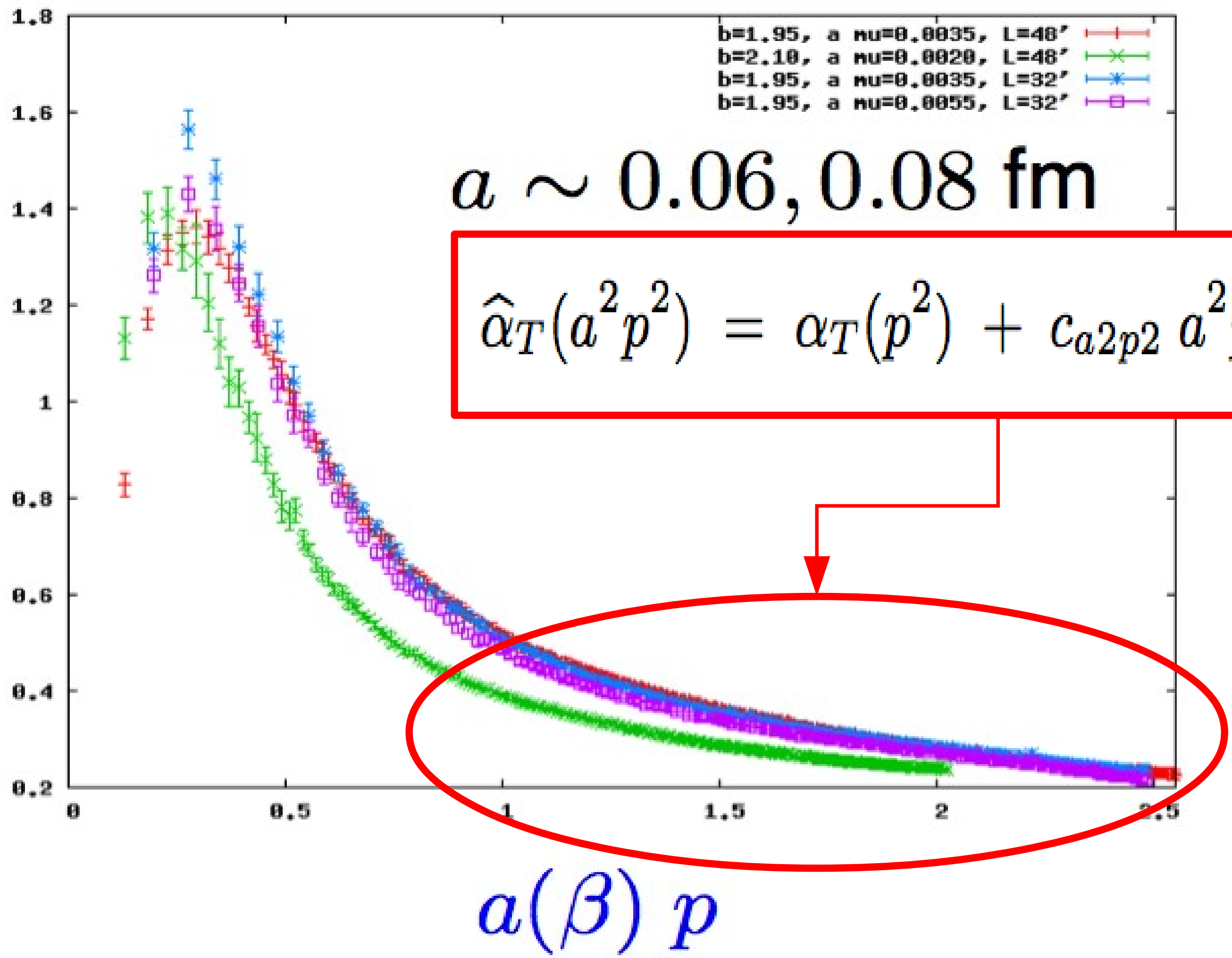
A task:
The running of ALPHA$_s$
… from the lattice!!!

$\hat{\alpha}_T(p^2)$ Nf=2+1+1: charmed sea quark included!!!



A task:
The running of ALPHA$_s$
… from the lattice!!!

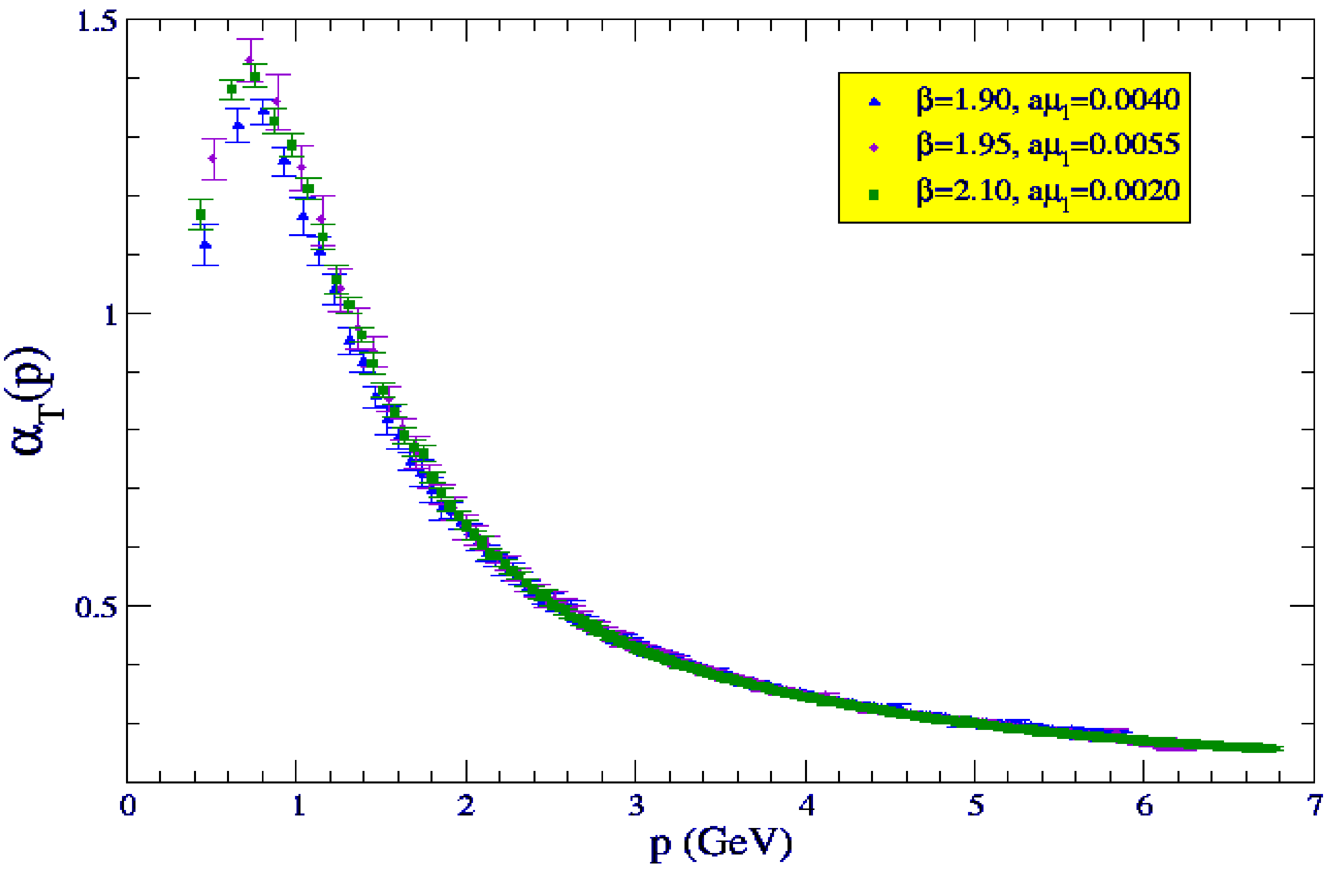
$\hat{\alpha}_T(p^2)$ Nf=2+1+1: charmed sea quark included!!!



$$\hat{\alpha}_T(a^2p^2) = \alpha_T(p^2) + c_{a2p2}\, a^2p^2 + \mathcal{O}(a^4)$$

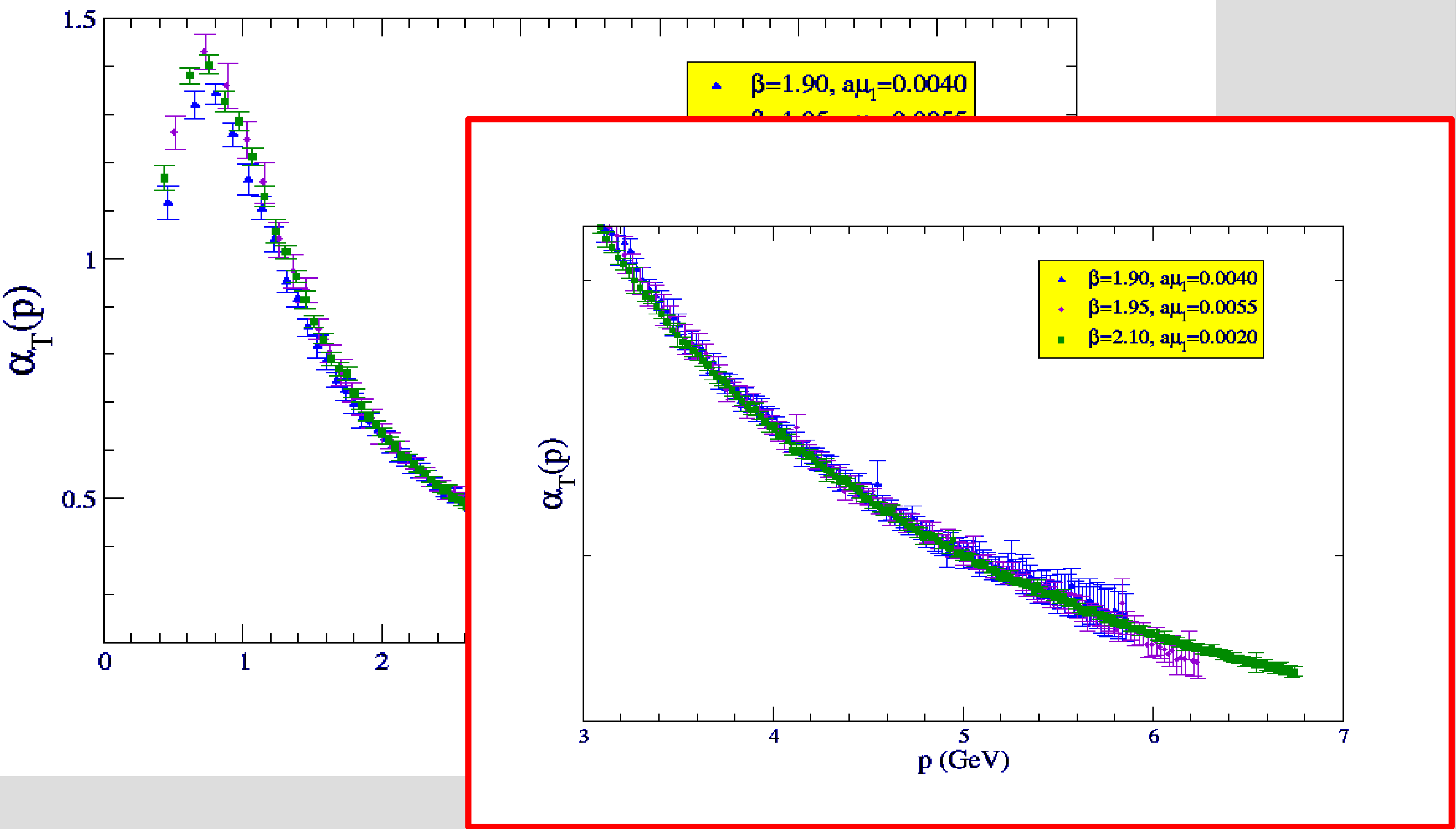
A task:
The running of ALPHA$_s$
… from the lattice!!!

Nf=2+1+1: charmed sea quark included!!!



A task:
The running of ALPHA$_s$
… from the lattice!!!

Nf=2+1+1: charmed sea quark included!!!



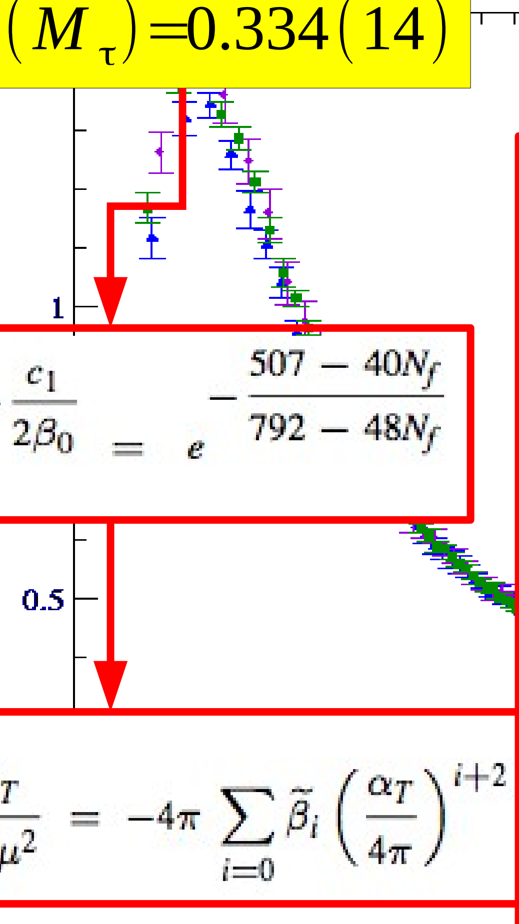
A task:

The running of ALPHA$_s$

… from the lattice!!!

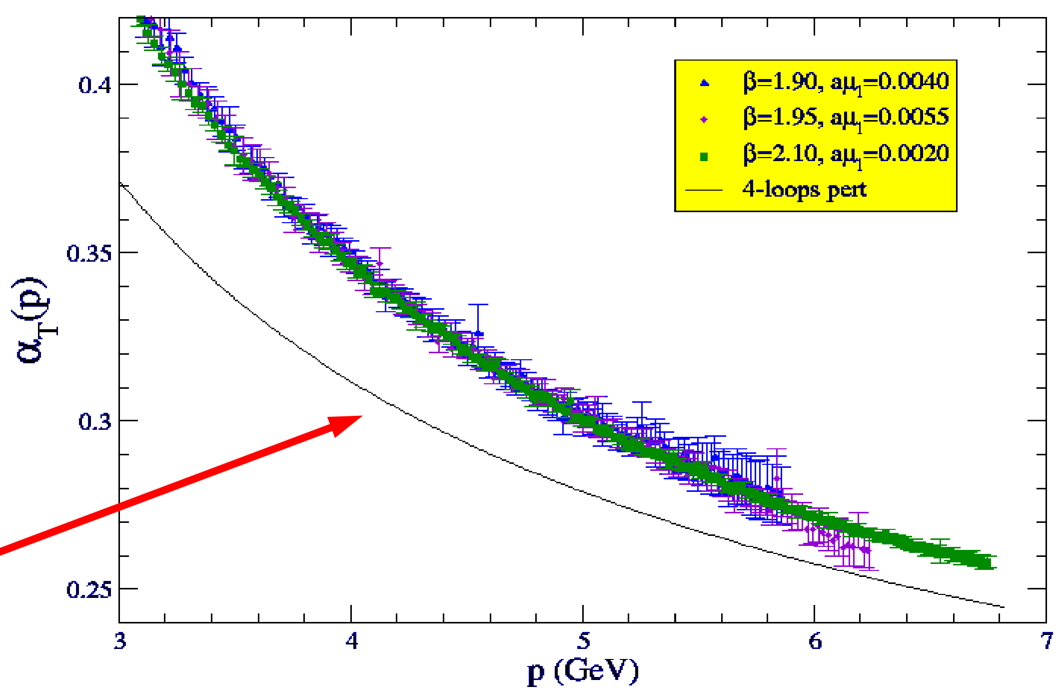
S. Bethke et al., arXiv:1110.0016

$$\alpha_{\overline{MS}}(M_\tau) = 0.334(14)$$

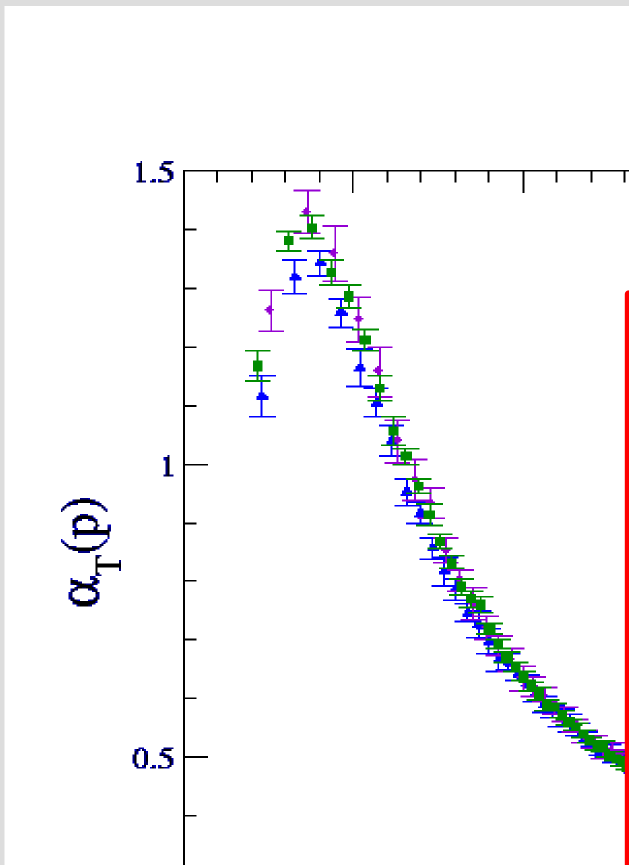


$$\frac{\Lambda_{\overline{\rm MS}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507-40N_f}{792-48N_f}}$$

$$\beta_T(\alpha_T) = \frac{d\alpha_T}{d\ln\mu^2} = -4\pi \sum_{i=0} \widetilde{\beta}_i \left(\frac{\alpha_T}{4\pi}\right)^{i+2}$$



A task:
The running of ALPHA$_s$
… from the lattice!!!



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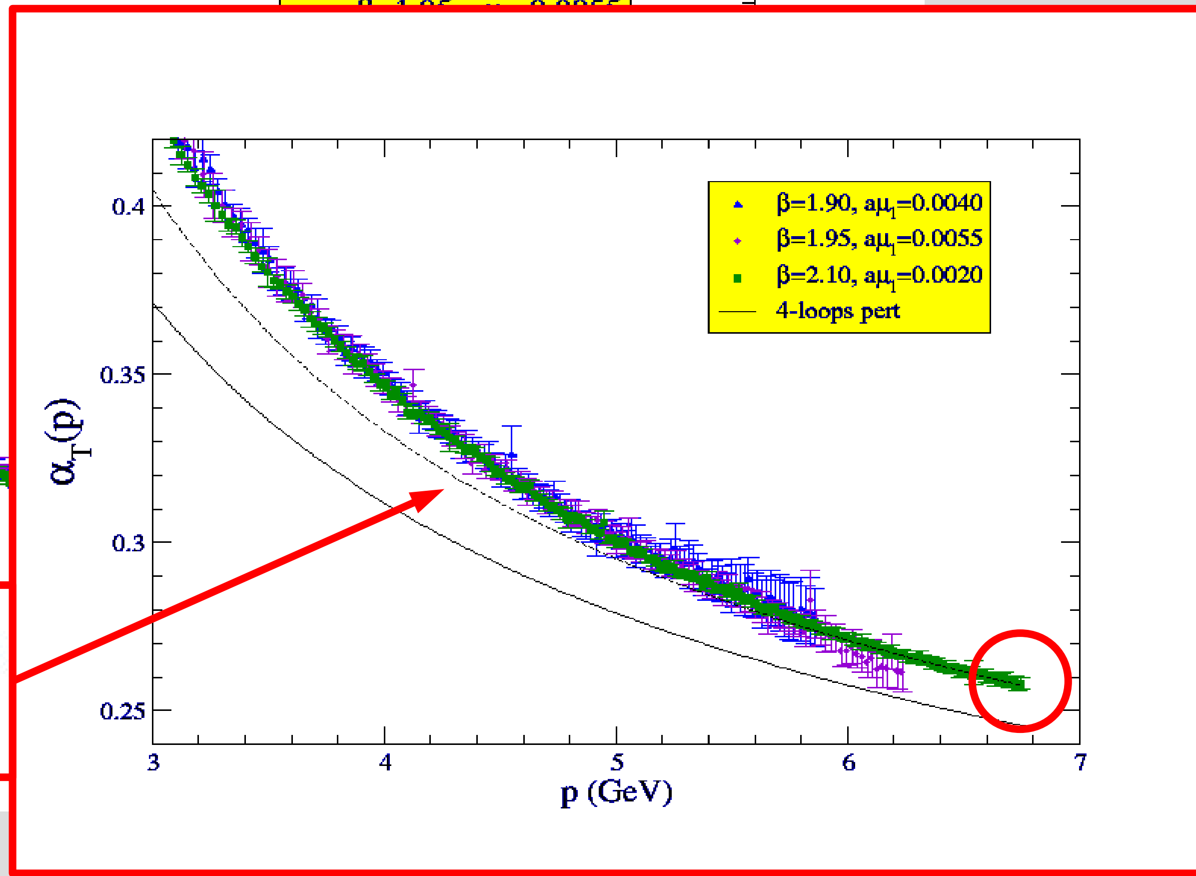

A task:

The running of ALPHA$_s$

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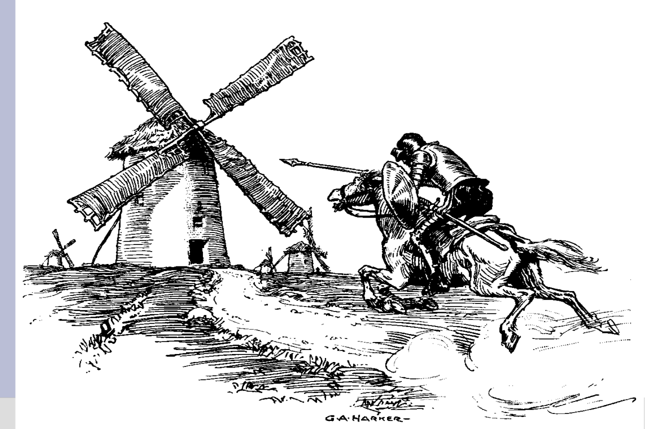
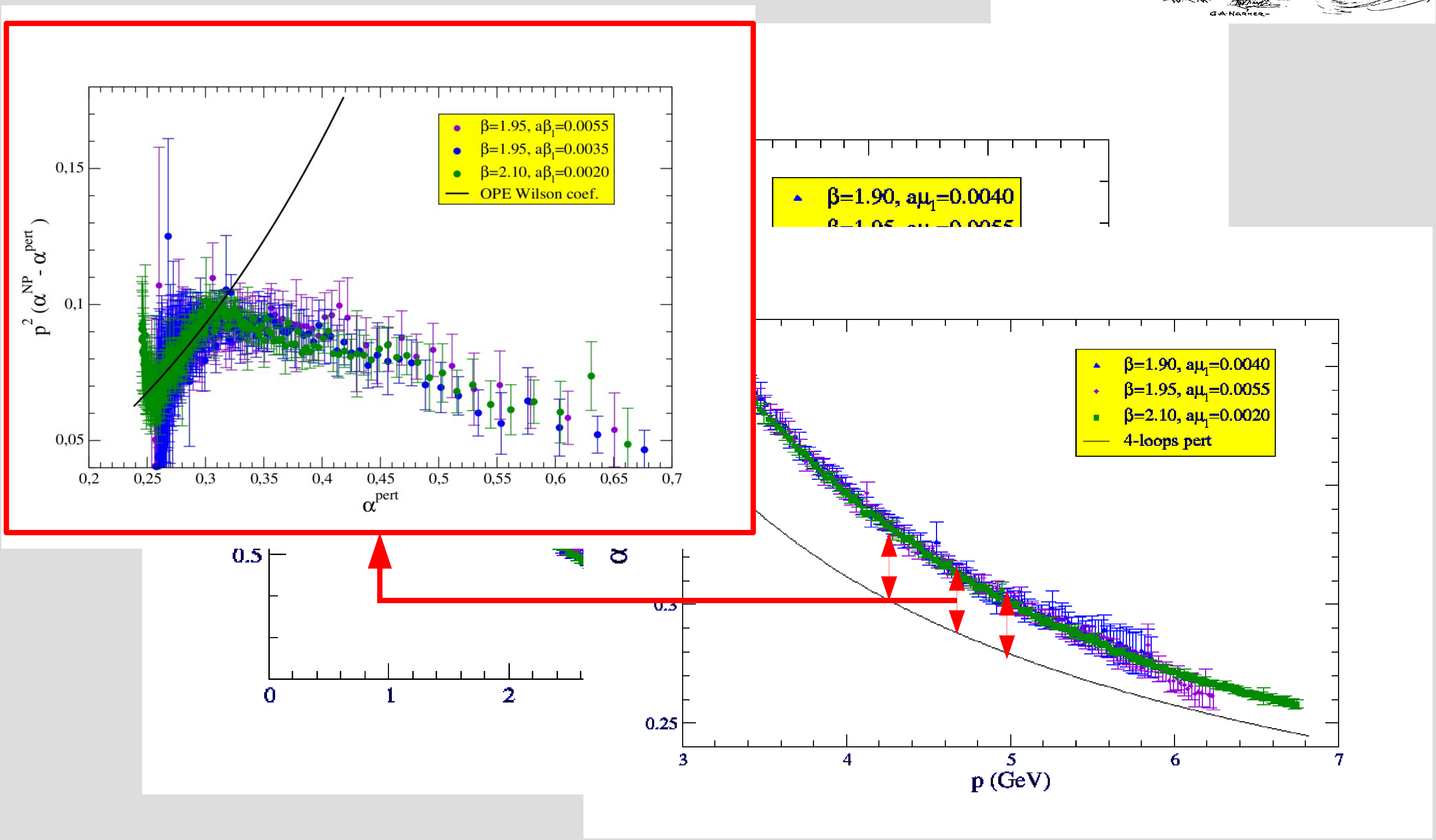


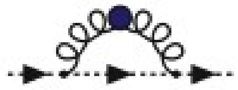


A task:

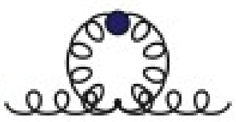
... then:

OPE power corrections

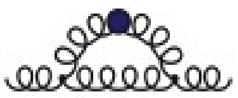
$$(F^{(2)})^{ab}(q^2) = (F^{(2)}_{\rm pert})^{ab}(q^2) + w^{ab}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \dots, \quad w^{ab} = 2\times$$



$$(G^{(2)})^{ab}_{\mu\nu}(q^2) = (G^{(2)}_{\rm pert})^{ab}_{\mu\nu}(q^2) + w^{ab}_{\mu\nu}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \dots, \quad w^{ab}_{\mu\nu} =$$



$$+\ 2\times$$



$$F_R(q^2,\mu^2) = F_{R,{\rm pert}}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right), \quad G_R(q^2,\mu^2) = G_{R,{\rm pert}}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)$$

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OPE power corrections

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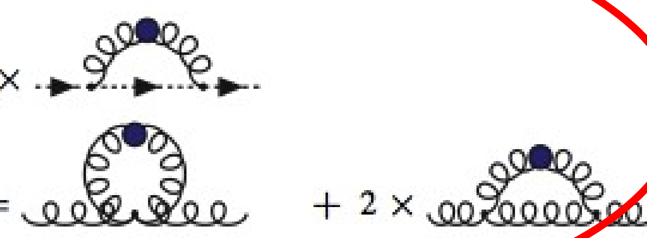

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A task:

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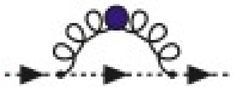
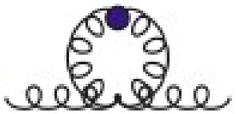
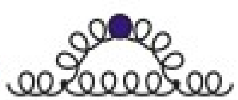
OPE power corrections

$$(F^{(2)})^{ab}(q^2) = (F^{(2)}_{\rm pert})^{ab}(q^2) + w^{ab}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots, \quad w^{ab} = 2\times$$ 

$$(G^{(2)})^{ab}_{\mu\nu}(q^2) = (G^{(2)}_{\rm pert})^{ab}_{\mu\nu}(q^2) + w^{ab}_{\mu\nu}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots, \quad w^{ab}_{\mu\nu} =$$  $+\ 2\times$ 

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Leading logarithm [4]: $\alpha_T(\mu^2) = \alpha_T^{\rm pert}(\mu^2)\left(1+\frac{9}{\mu^2}\left(\frac{\alpha_T^{\rm pert}(\mu^2)}{\alpha_T^{\rm pert}(q_0^2)}\right)^{1-\gamma_0^{A^2}/\beta_0}\frac{g_T^2(q_0^2)\langle A^2\rangle_{R,q_0^2}}{4(N_C^2-1)}\right)$

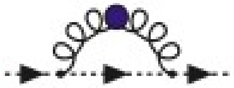
$$1-\gamma_0^{A^2}/\beta_0 = 1-\frac{105-8N_f}{132-8N_f} = \frac{9}{44-\frac{8}{3}N_f}$$

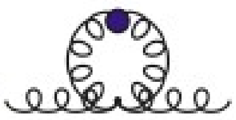
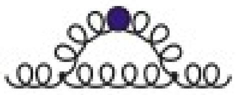
Ph. Boucaud et al, PRD79(2009)014508

A task:

… then:

OPE power corrections

$$(F^{(2)})^{ab}(q^2) = (F^{(2)}_{\rm pert})^{ab}(q^2) + w^{ab}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab} = 2\times$$ 

$$(G^{(2)})^{ab}_{\mu\nu}(q^2) = (G^{(2)}_{\rm pert})^{ab}_{\mu\nu}(q^2) + w^{ab}_{\mu\nu}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \ldots\,, \quad w^{ab}_{\mu\nu} =$$  $+\,2\times$ 

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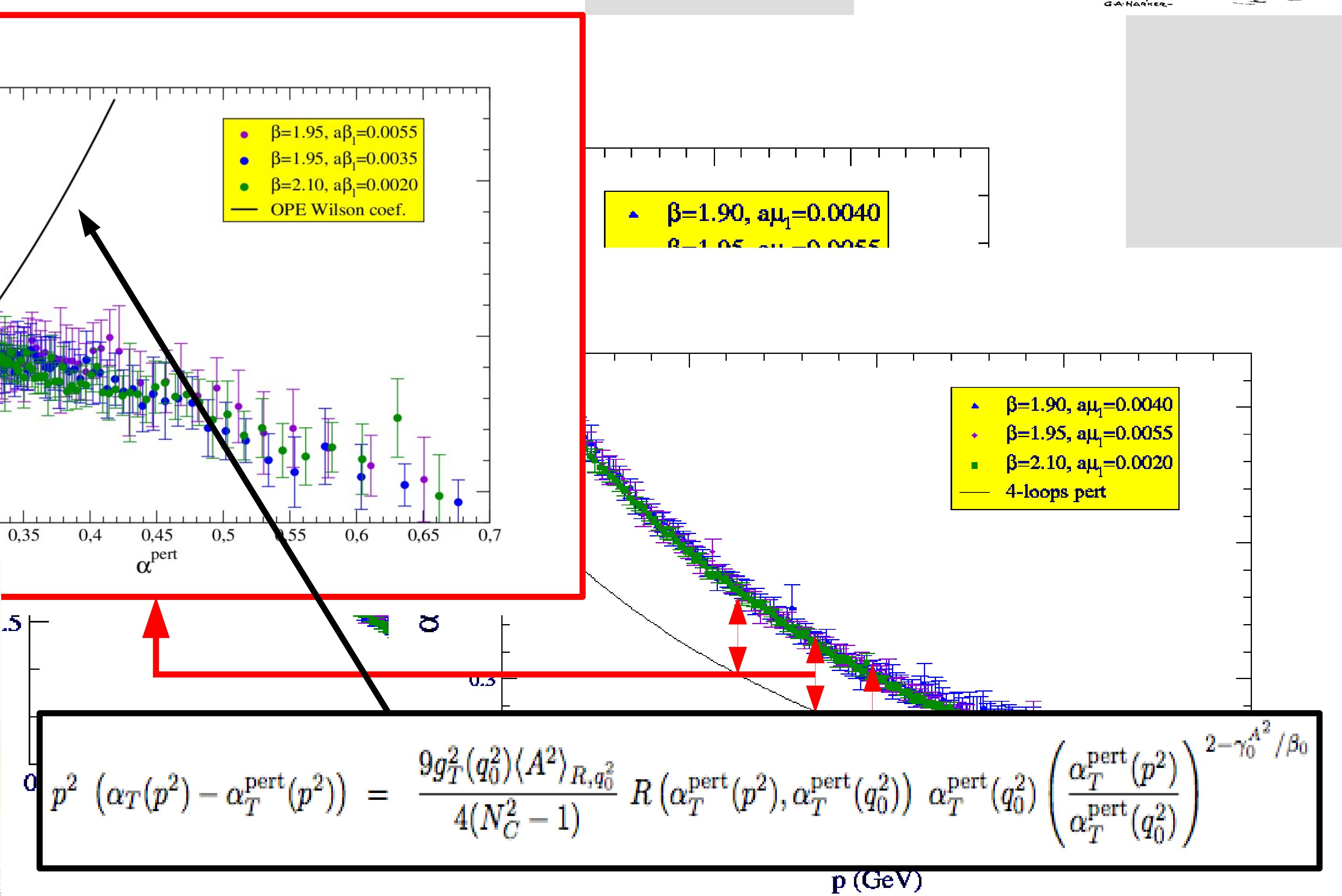
$$1-\gamma_0^{A^2}/\beta_0 = 1-\frac{105-8N_f}{132-8N_f} = \frac{9}{44-\frac{8}{3}N_f}$$

Chetyrkin & Maier, arXiv:0911.0594
J.A. Gracey, PLB552(2003)101 At the three-loop level !!!

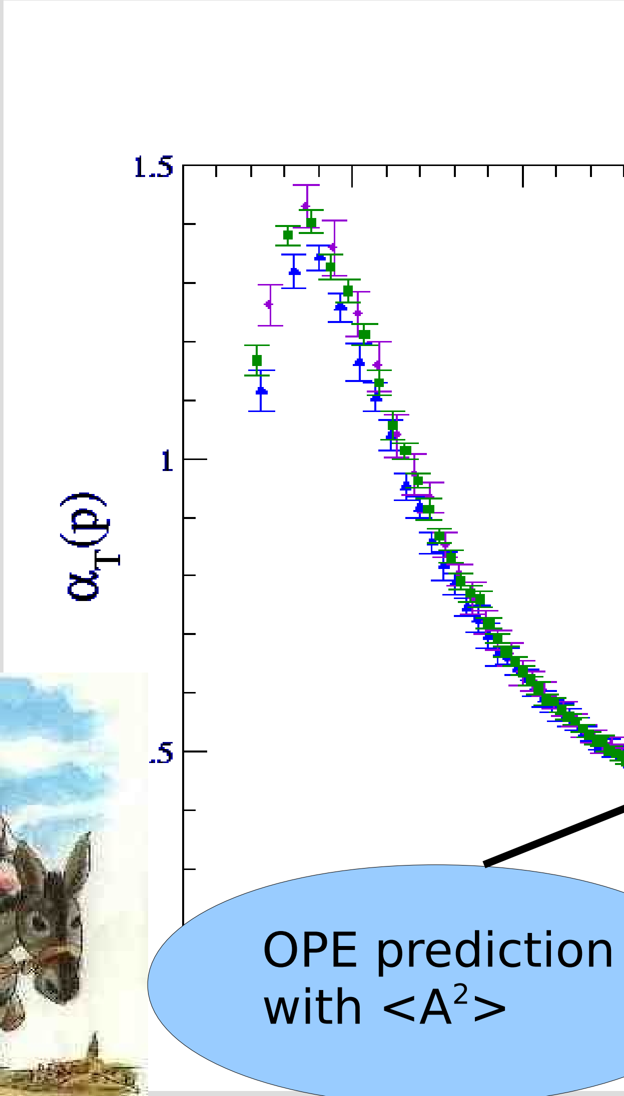
A task:

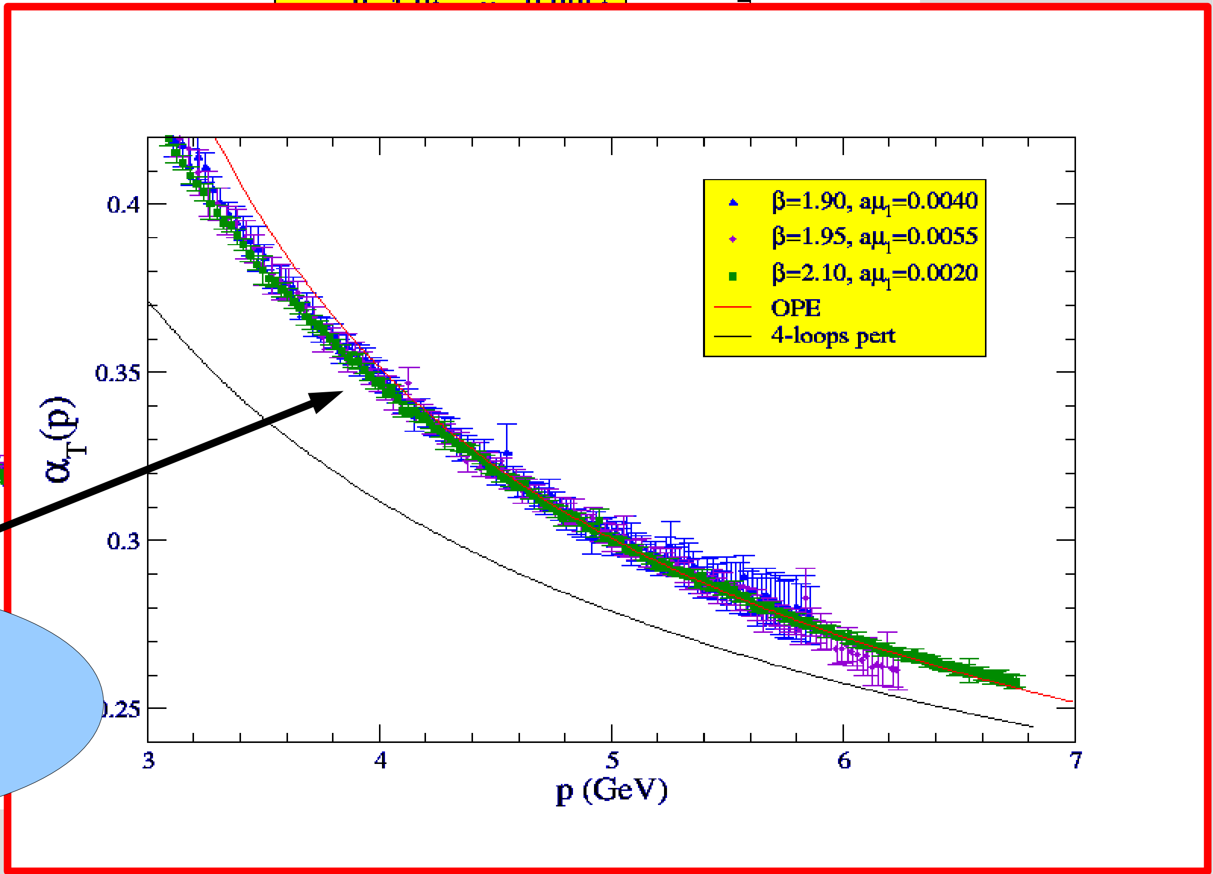
… a gluon condensate is needed!!!



$$p^2\left(\alpha_T(p^2)-\alpha_T^{pert}(p^2)\right) = \frac{9g_T^2(q_0^2)\langle A^2\rangle_{R,q_0^2}}{4(N_C^2-1)}\, R\left(\alpha_T^{pert}(p^2),\alpha_T^{pert}(q_0^2)\right)\,\alpha_T^{pert}(q_0^2)\left(\frac{\alpha_T^{pert}(p^2)}{\alpha_T^{pert}(q_0^2)}\right)^{2-\gamma_0^{A^2}/\beta_0}$$

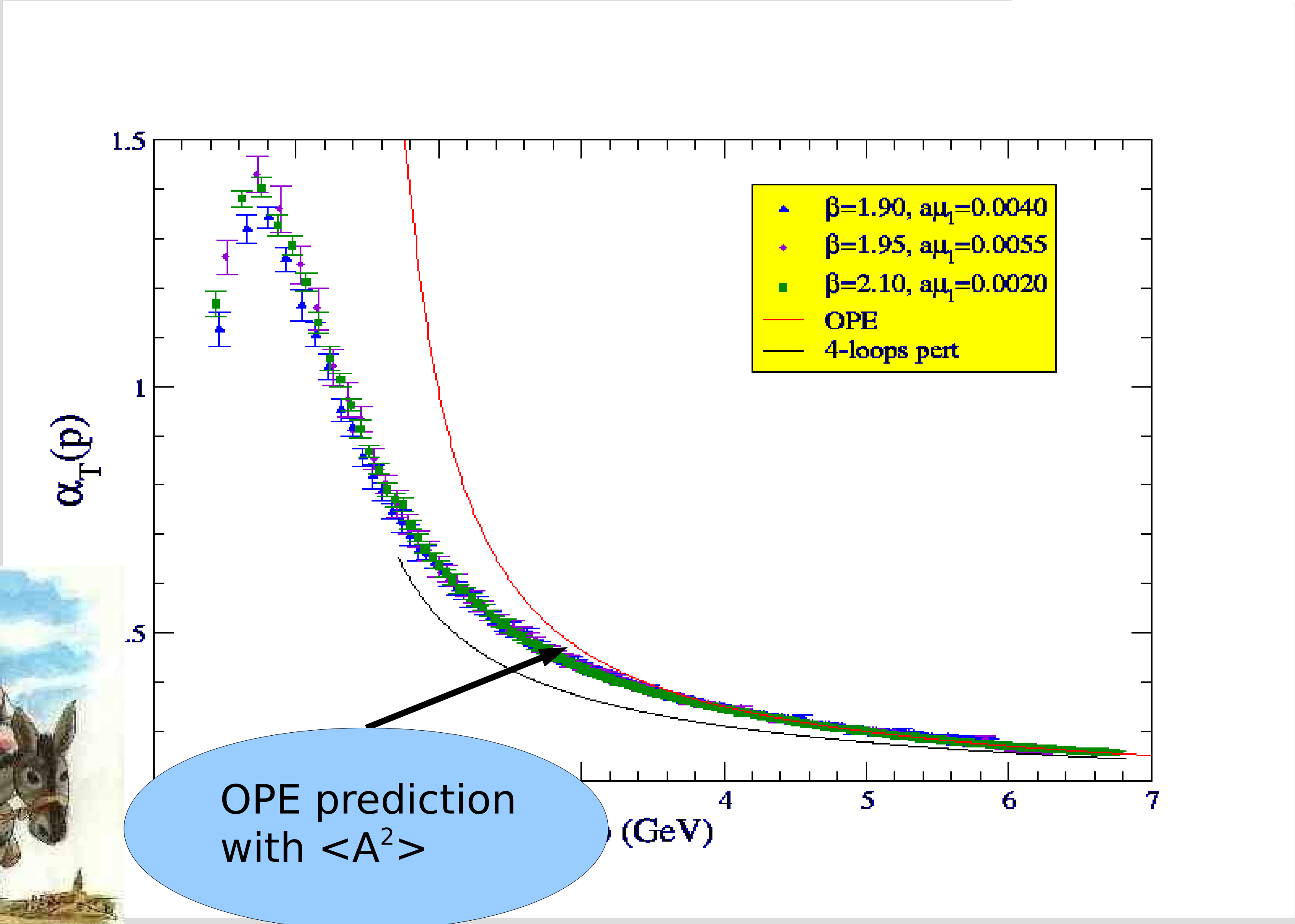
A task:
… a gluon condensate is needed!!!

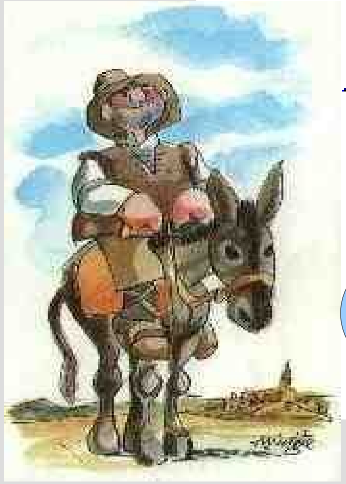




OPE prediction with <A^2>

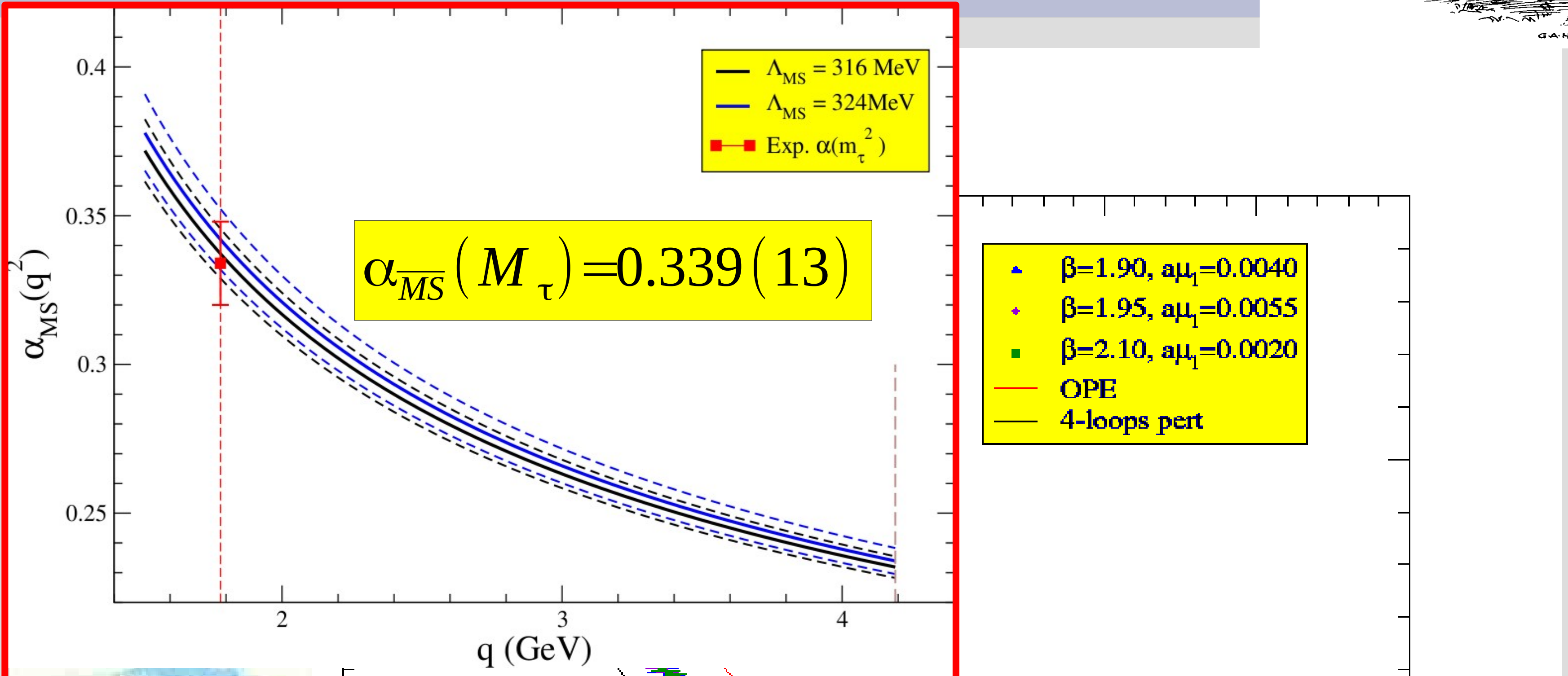
A task:
… a gluon condensate is needed!!!

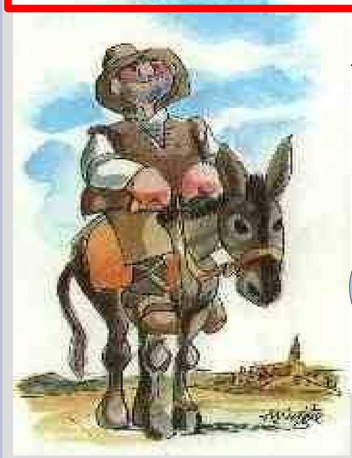




A task:

… a gluon condensate is needed!!!





A task:

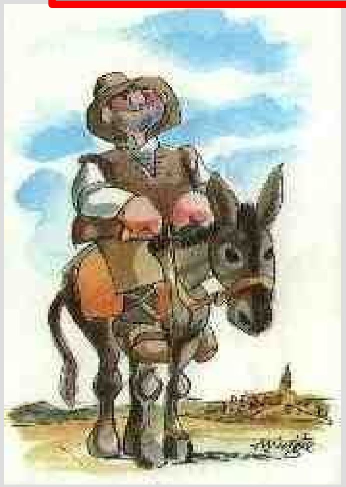
… a gluon condensate is needed!!!

A lattice regularized QCD action (including a charmed sea quark) with the physical scale fixed by: $f_\pi = 130.4(2)\,MeV$

$$\alpha_{\overline{MS}}(M_\tau) = 0.339(13)$$

$$\alpha_{\overline{\rm MS}}^{N_f=5}(m_b) = \alpha_{\overline{\rm MS}}^{N_f=4}(m_b)\left(1 + \sum_n c_{n0}\left(\alpha_{\overline{\rm MS}}^{N_f=4}(m_b)\right)^n\right)$$

$$\alpha_{\overline{MS}}(M_{Z0}) = 0.1200(14)$$



A task:
… a gluon condensate is needed!!!

A lattice regularized QCD action (including a charmed sea quark) with the physical scale fixed by: $f_\pi = 130.4(2)\,MeV$

$\alpha_{\overline{MS}}(M_\tau) = 0.339(13)$ ⟷ $\alpha_{\overline{MS}}(M_\tau) = 0.334(14)$

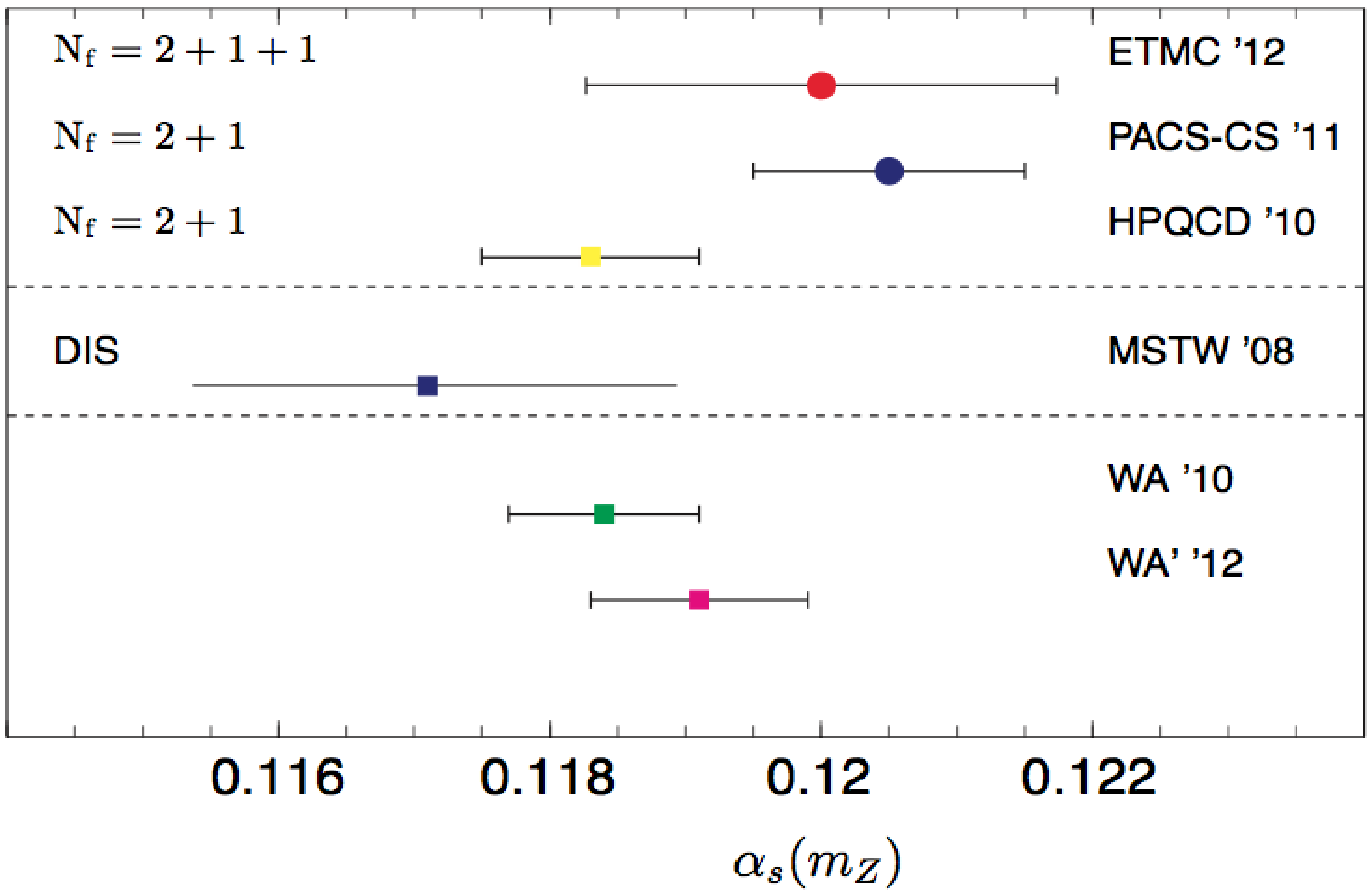
S. Bethke et al., arXiv:1110.0016 (tau decays)

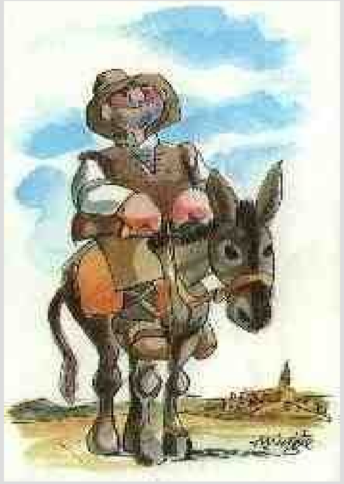
$$\alpha_{\overline{\rm MS}}^{N_f=5}(m_b) = \alpha_{\overline{\rm MS}}^{N_f=4}(m_b)\left(1 + \sum_n c_{n0}\left(\alpha_{\overline{\rm MS}}^{N_f=4}(m_b)\right)^n\right)$$

Eur. Phys. J. G 64:689(2009); "World average" (decays, scattering...)

$\alpha_{\overline{MS}}(M_{Z0}) = 0.1200(14)$ ⟷ $\alpha_{\overline{MS}}(M_{Z0}) = 0.1186(11)$

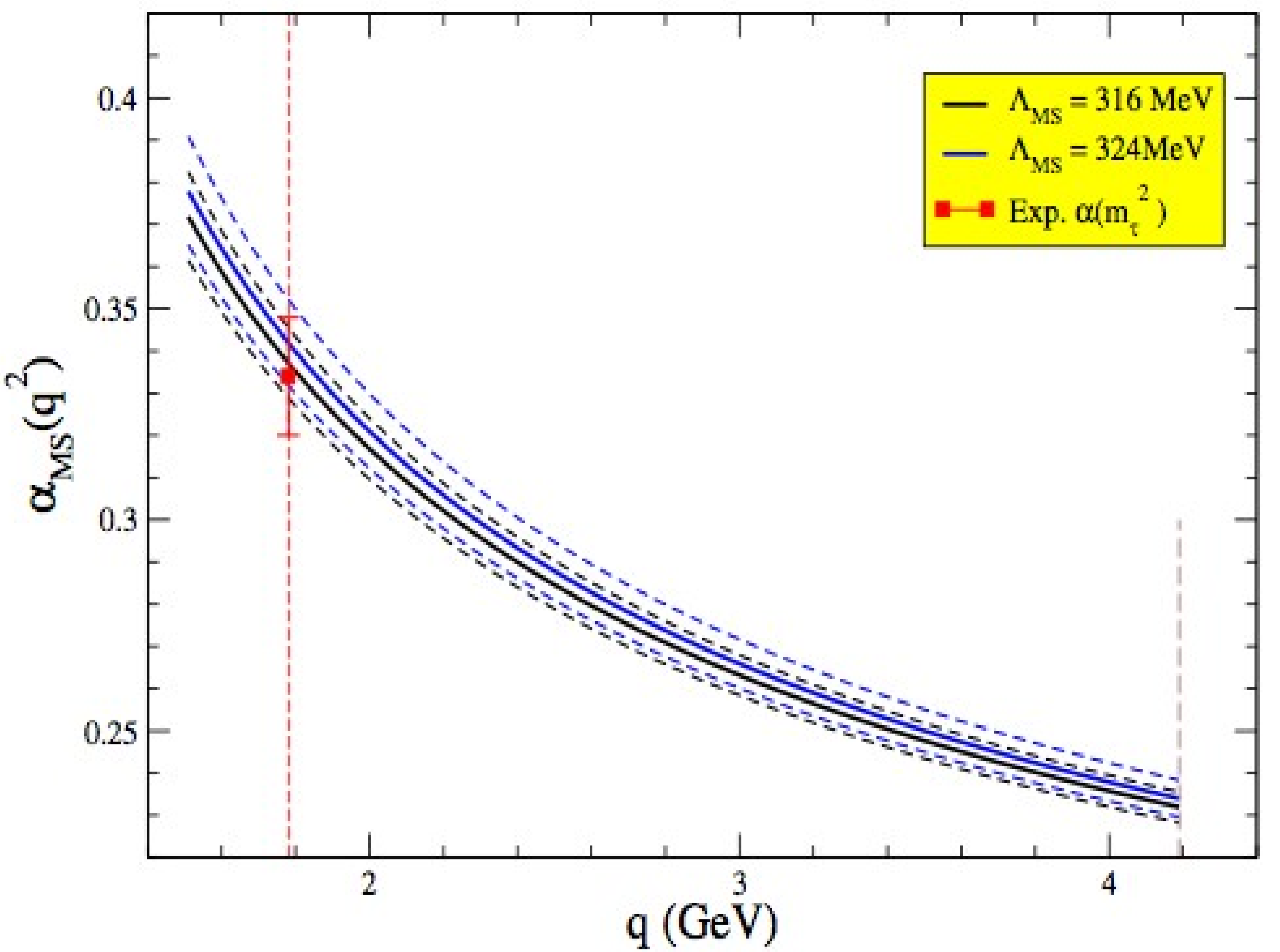
A task:
… a gluon condensate is needed!!!





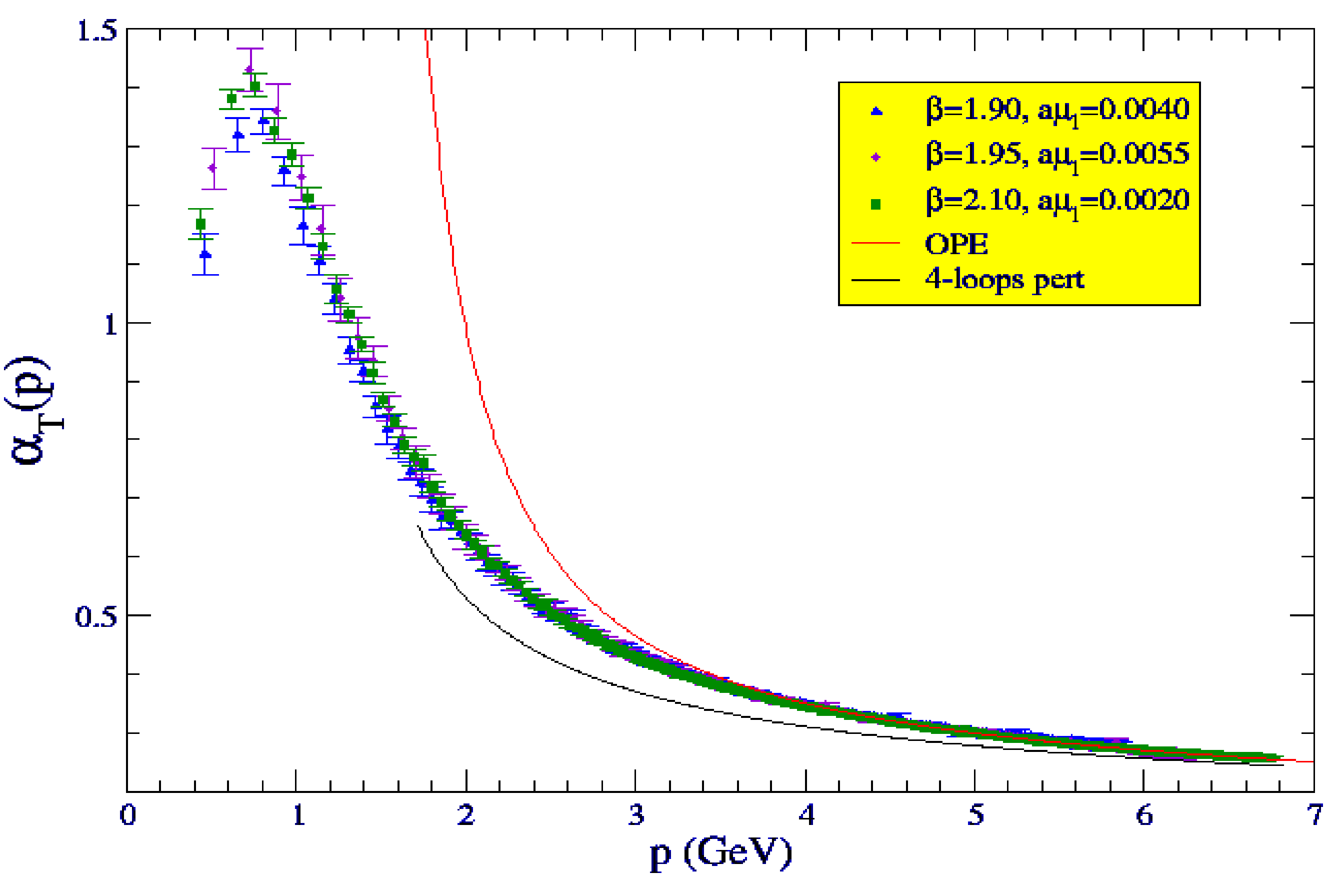
WA: PDG world average
WA': world average replacing $N_f = 2+1$ lattice results by $N_f = 2+1+1$

A task:
… a gluon condensate is needed!!!

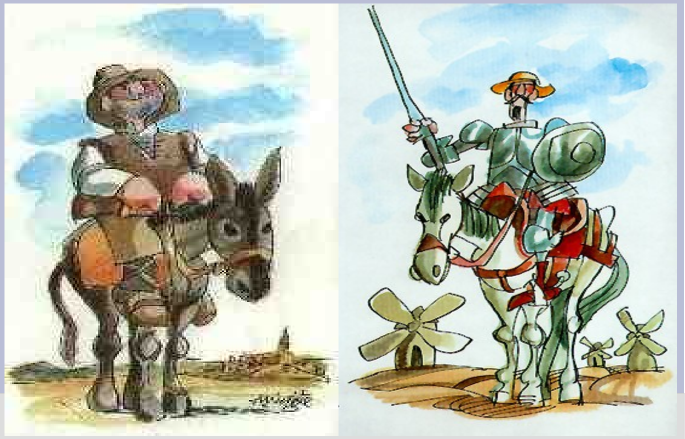


A second task:

How can we describe better the lower momenta region?



A second task:
The GPDSE analysis

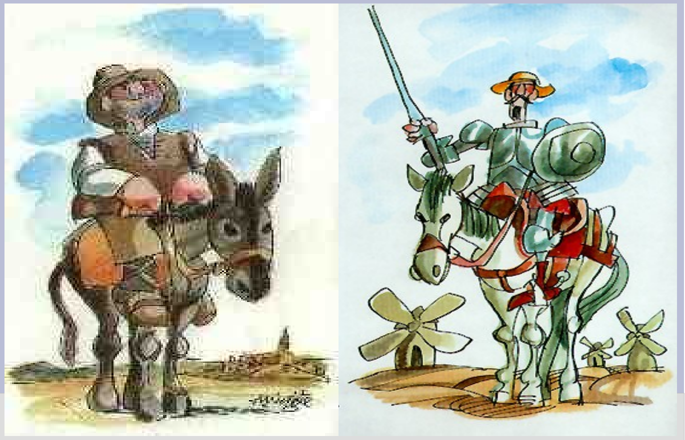


GPDSE:

$$\frac{1}{F(k^2)} = 1 + \alpha_T(\mu^2) \int d^4q\, K(k,q)\, H_1^{\text{bare}}(q,k) F(q^2)$$

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda \to \infty} \frac{g_0^2(\Lambda^2)}{4\pi} Z_3(\mu^2, \Lambda^2) \widetilde{Z}_3^2(\mu^2, \Lambda^2)$$

A second task:
The GPDSE analysis



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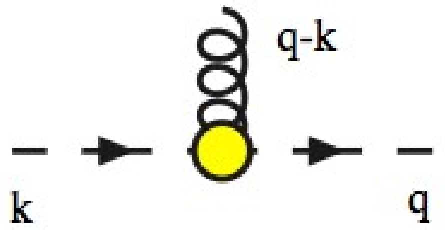
The usual approximation
Is to take this to be 1

Lattice inputs!!!

(Cfr. JHEP06(2008)012)

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

$$\widetilde{\Gamma}^{abc}_{\nu}(-q,k;q-k) =$$



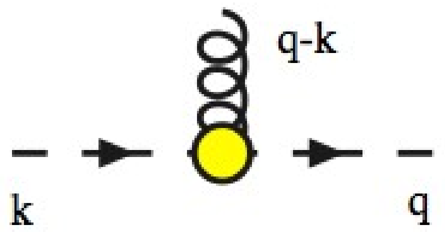
$$= i g_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

$$\widetilde{\Gamma}_R = \widetilde{Z}_1 \Gamma$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

The ghost-gluon vertex:

$\widetilde{\Gamma}^{abc}_{\nu}(-q,k;q-k) =$



$= i g_0 f^{abc} \left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k) \right)$

$$\widetilde{\Gamma}_R = \widetilde{Z}_1 \Gamma$$

Ghost Prop. DSE:

$$\frac{1}{F(k^2)} = 1 + \alpha_T(\mu^2) \int d^4q \; K(k,q) H_1^{\mathrm{bare}}(q,k) F(q^2)$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

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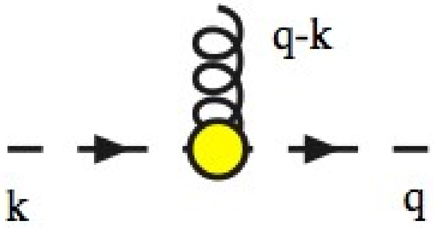

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OPE+SVZ analysis of the ghost-gluon vertex

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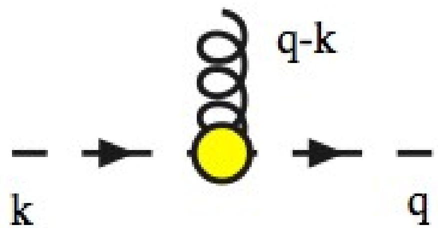

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A second task:
OPE+SVZ analysis of the ghost-gluon vertex

The ghost-antighost-gluon Green function:

$$
\begin{aligned}
V_\mu^{abc}(-q,k;q-k) &= \Gamma_{\mu'}^{a'b'c'}(-q,k;q-k)\, G_{\mu\mu'}^{bb'}(q-k)\, F^{aa'}(q)\, F^{cc'}(k) \\
&= \int d^4y\, d^4x\, e^{i(q-k)\cdot x}\, e^{ik\cdot y} \langle\, T\left(c^c(y) A_\mu^b(x) \overline{c^a}(0) \right) \rangle
\end{aligned}
$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

The ghost-antighost-gluon Green function:

$$V^{abc}_{\mu}(-q,k;q-k) = \Gamma^{a'b'c'}_{\mu'}(-q,k;q-k)\; G^{bb'}_{\mu\mu'}(q-k)\; F^{aa'}(q)\; F^{cc'}(k)$$
$$= \int d^4y\; d^4x\; e^{i(q-k)\cdot x}\; e^{ik\cdot y} \langle\, T\left(c^c(y) A^b_\mu(x) \overline{c^a}(0)\right)\,\rangle$$

OPE expansion:

$$V^{abc}_{\mu}(-q,k;q-k) = (d_0)^{abc}_{\mu}(q,k) + (d_2)^{abc\mu'\nu'}_{\mu a'b'}(q,k) \langle : A^{a'}_{\mu'}(0) A^{b'}_{\nu'}(0) : \rangle + \cdots$$

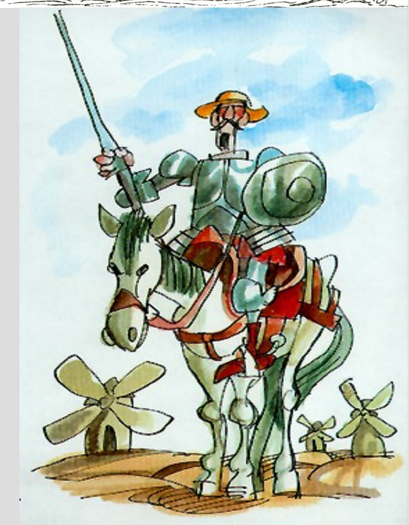
SVZ sum-rules:

$$w^{abc}_{\mu} = (d_2)^{abc\mu'\nu'}_{\mu a'b'}(q,k)\; \delta^{a'b'} g_{\mu'\nu'}$$
$$= 2I^{[1]} + 2I^{[1]}_s + 2I^{[2]} + 4I^{[3]} + I^{[4]} + 2I^{[5]}$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

SVZ sum-rules:

$$
\begin{aligned}
w_{\mu}^{abc} &= (d_2)_{\mu a'b'}^{abc\mu'\nu'}(q,k)\,\delta^{a'b'} g_{\mu'\nu'} \\
&= 2I^{[1]} + 2I_s^{[1]} + 2I^{[2]} + 4I^{[3]} + I^{[4]} + 2I^{[5]}
\end{aligned}
$$



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OPE+SVZ analysis of the ghost-gluon vertex

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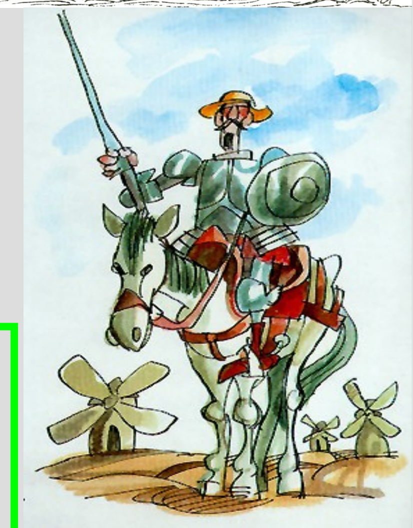

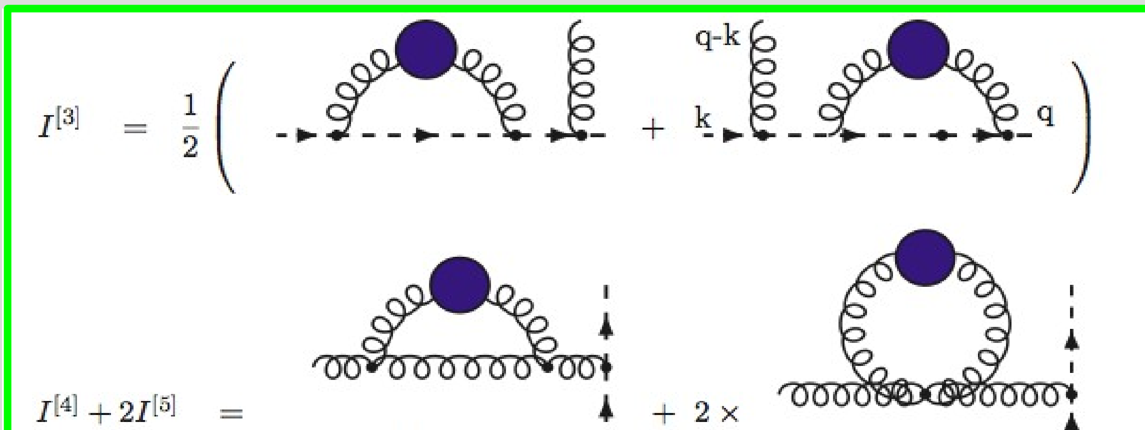

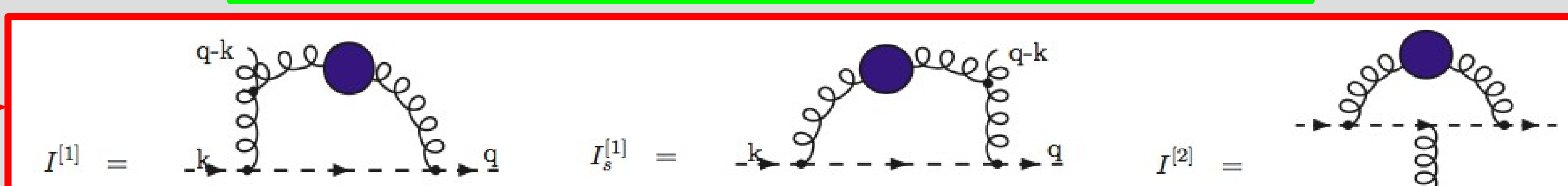

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

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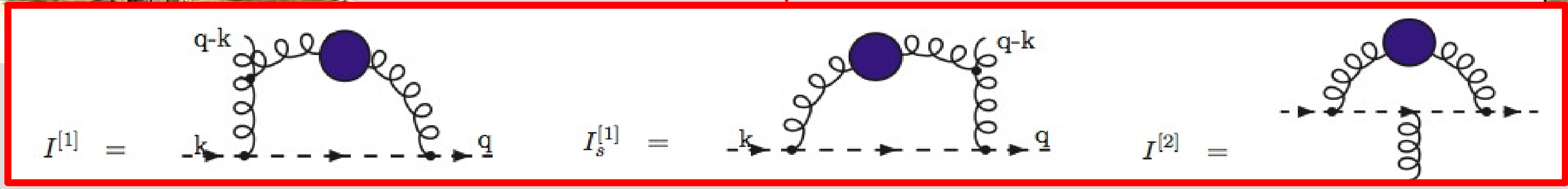
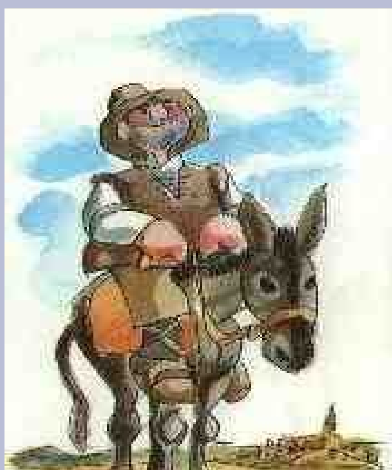
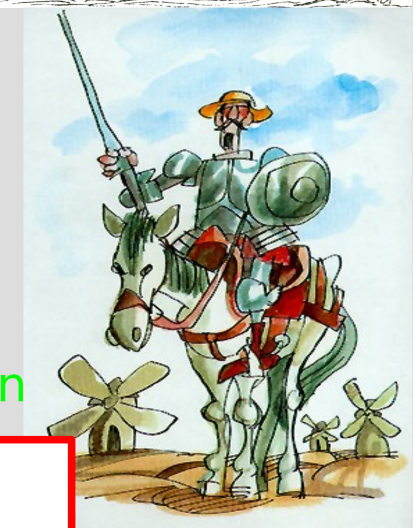
$$w_\mu^{abc} = (d_2)^{abc\mu'\nu'}_{\mu a'b'}(q,k)\,\delta^{a'b'}\,g_{\mu'\nu'} = 2I^{[1]} + 2I_s^{[1]} + 2I^{[2]} + 4I^{[3]} + I^{[4]} + 2I^{[5]}$$

External legs amputation



A second task:
OPE+SVZ analysis of the ghost-gluon vertex

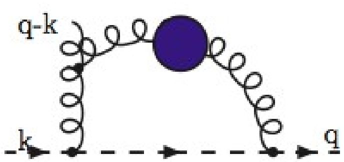


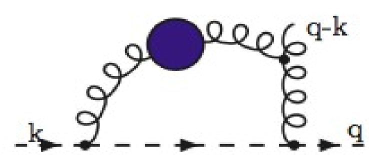


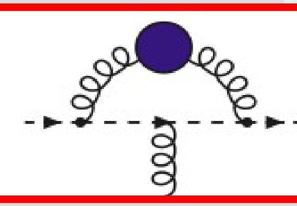
SVZ sum-rules:

$$
\begin{aligned}
w_\mu^{abc} &= (d_2)^{abc\mu'\nu'}_{\mu a'b'}(q,k)\, \delta^{a'b'} g_{\mu'\nu'} \\
&= 2I^{[1]} + 2I_s^{[1]} + 2I^{[2]} + 4I^{[3]} + I^{[4]} + 2I^{[5]}
\end{aligned}
$$

External legs amputation

$I^{[1]} =$ 

$I_s^{[1]} =$ 

$I^{[2]} =$ 

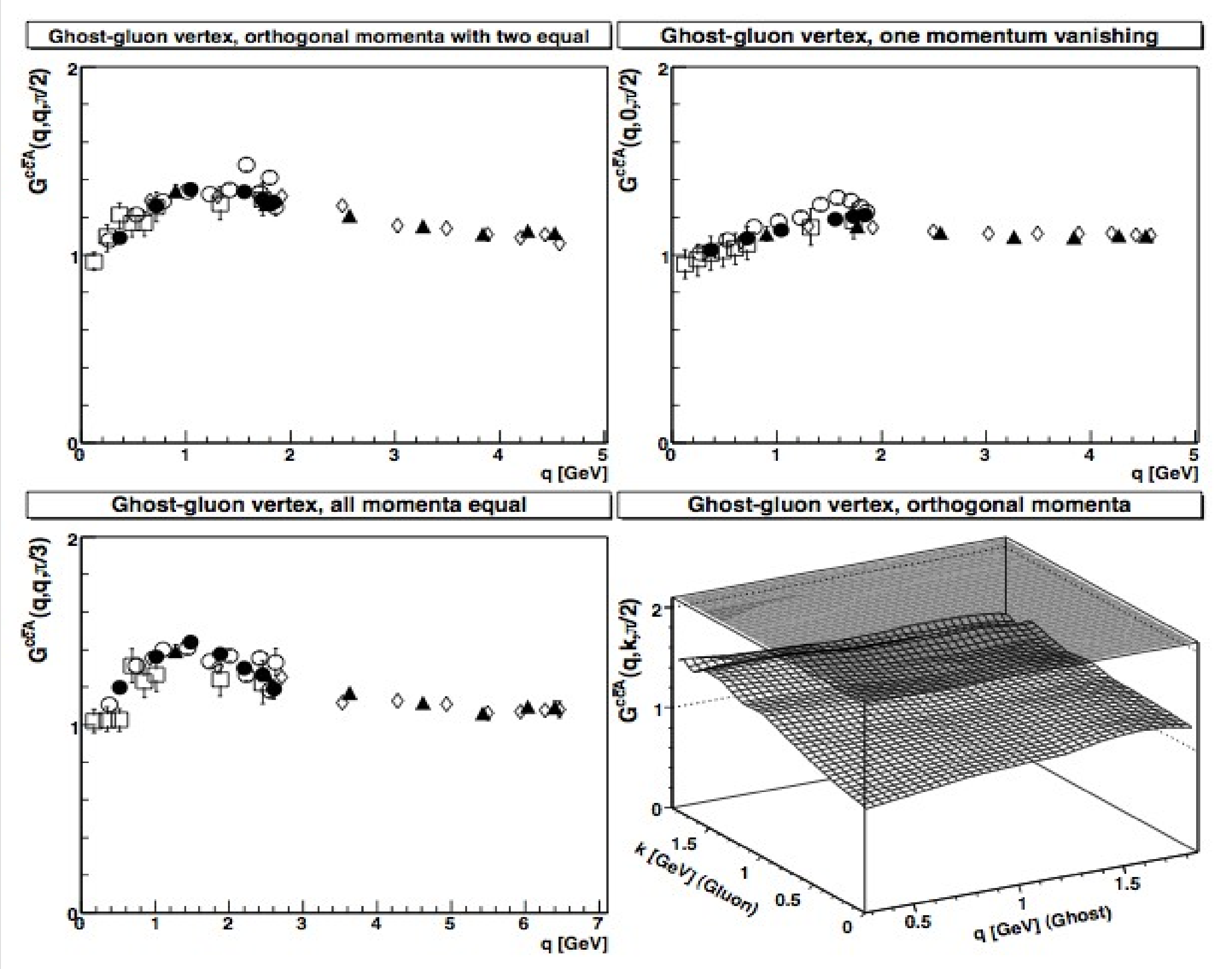
$$
H_1(q,k) = H_1^{\text{pert}}(q,k)\left(1 + s_V(q,k)\frac{\langle A^2\rangle}{4(N_C^2-1)} + \mathcal{O}(g^4, q^{-4}, k^{-4}, q^{-2}k^{-2})\right)
$$

$$
s_V(q,k) = \frac{N_C}{2}\left(2\,\frac{(q-k)\cdot q}{q^2(q-k)^2} + 2\,\frac{(k-q)\cdot k}{k^2(q-k)^2} + \frac{k\cdot q}{k^2 q^2}\right)
$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

SU(2) Lattice results to compare with:

(Phys.Rev.D77:094510,2008)



Three kinematical configurations:

- q-k=0
- $q^2=k^2$; ang(q-k,q)=pi/2
- $q^2=k^2$; ang(q-k,q)=pi/3

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

SU(2) Lattice results to compare with:

(Phys.Rev.D77:094510,2008)

$$H_1(q^2,k^2,\theta) = \widetilde{Z}_1^{-1}\left[1+\frac{N_C g^2\langle A^2\rangle}{8(N_C^2-1)}\right.$$
$$\times\left(\frac{\sqrt{k^2q^2}\cos\theta}{k^2q^2+m_{\rm IR}^4}+2\,\frac{q^2-\sqrt{k^2q^2}\cos\theta}{q^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right.$$
$$\left.\left.+2\,\frac{k^2-\sqrt{k^2q^2}\cos\theta}{k^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right)\right],$$

A second task:
OPE+SVZ analysis of the ghost-gluon vertex

SU(2) Lattice results to compare with:

(Phys.Rev.D77:094510,2008)

$$
\begin{aligned}
H_1(q^2,k^2,\theta) \;=\; & \widetilde{Z}_1^{-1}\left[1+\frac{N_C g^2\langle A^2\rangle}{8(N_C^2-1)}\right.\\
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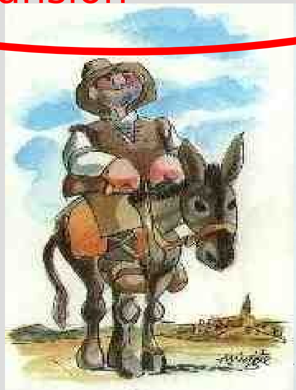
& \left.\left.+\,2\,\frac{k^2-\sqrt{k^2q^2}\cos\theta}{k^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right)\right],
\end{aligned}
$$

Infrared mass scale to “regulate” the spurious infinities from the OPE expansion

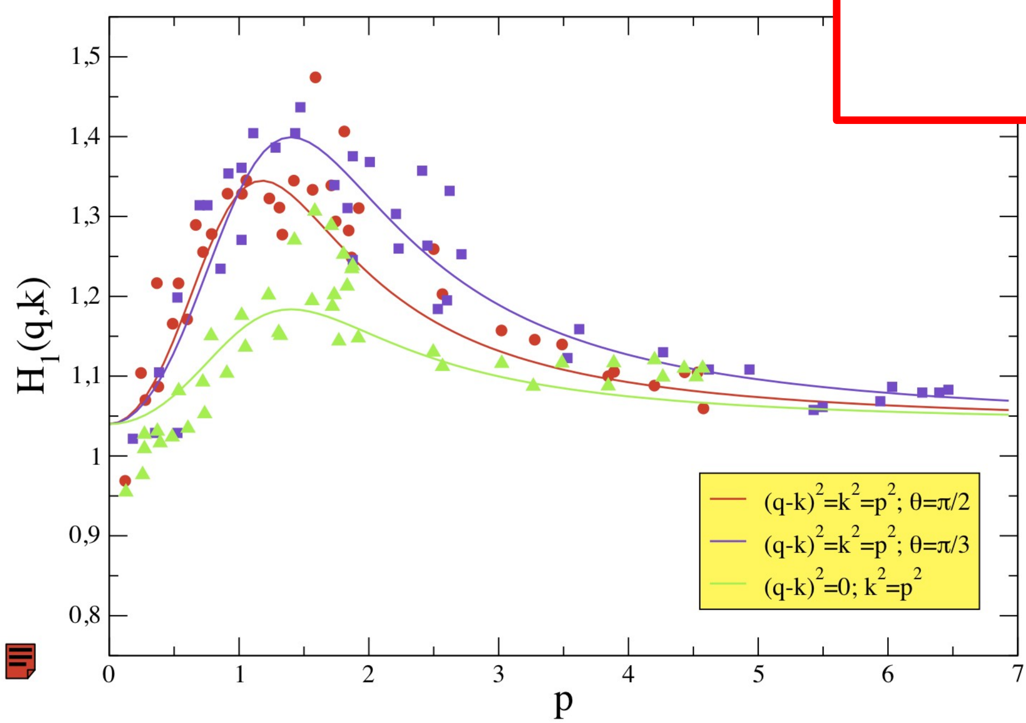


A second task:
OPE+SVZ analysis of the ghost-gluon vertex

SU(2) Lattice results to compare with:
(Phys.Rev.D77:094510,2008)

Very successful description!!!

$$
\begin{aligned}
H_1(q^2,k^2,\theta) &= \widetilde{Z}_1^{-1}\left[1 + \frac{N_C g^2\langle A^2\rangle}{8(N_C^2-1)}\right.\\
&\times\left(\frac{\sqrt{k^2q^2}\cos\theta}{k^2q^2+m_{\rm IR}^4} + 2\,\frac{q^2-\sqrt{k^2q^2}\cos\theta}{q^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right.\\
&\left.\left.+\,2\,\frac{k^2-\sqrt{k^2q^2}\cos\theta}{k^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right)\right],
\end{aligned}
$$



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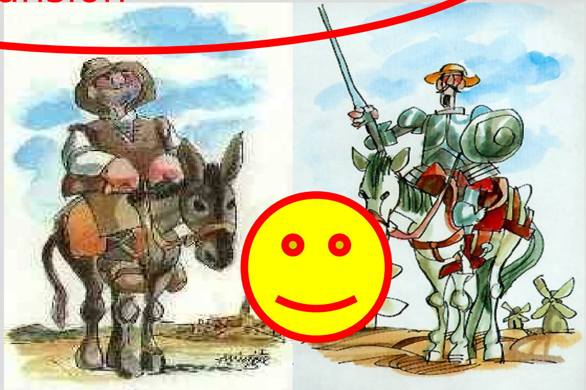

A second task:
… & the Taylor kinematics

$$H_1(q^2,k^2,\theta) = \widetilde{Z}_1^{-1}\left[1 + \frac{N_C g^2\langle A^2\rangle}{8(N_C^2-1)}\right.$$
$$\times \left(\frac{\sqrt{k^2q^2}\cos\theta}{k^2q^2+m_{\rm IR}^4} + 2\,\frac{q^2-\sqrt{k^2q^2}\cos\theta}{q^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right.$$
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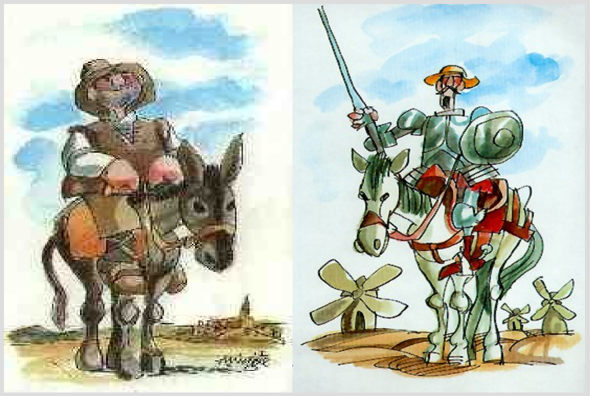

$k \to 0$

$$H_1(q,0) = \widetilde{Z}_1^{-1}\left(1 + N_C\frac{g^2\langle A^2\rangle}{4(N_C^2-1)}\,\frac{q^2}{q^4+m_{IR}^4}\right)$$

A second task:
… & the Taylor kinematics

$$H_1(q^2,k^2,\theta) = \widetilde{Z}_1^{-1}\left[1 + \frac{N_C g^2 \langle A^2\rangle}{8(N_C^2-1)}\right.$$
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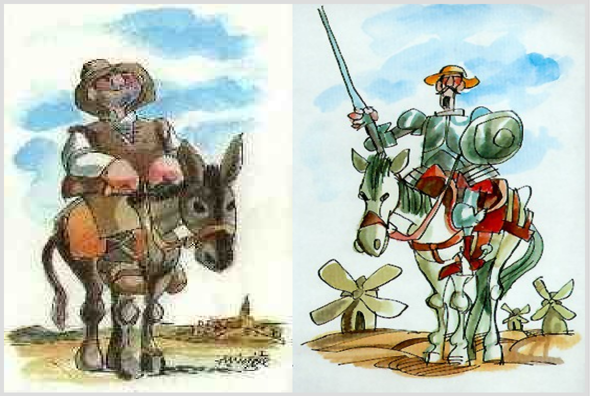

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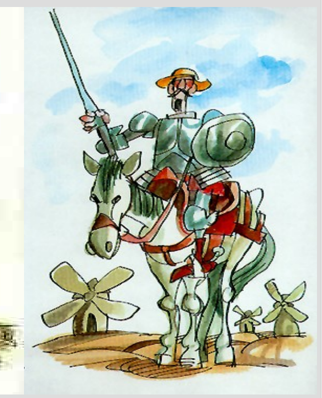
$$H_1(q,0) = \widetilde{Z}_1^{-1}\left(1 + N_C\frac{g^2\langle A^2\rangle}{4(N_C^2-1)}\,\frac{q^2}{q^4+m_{IR}^4}\right)$$

- Cfr. D. Dudal's talk !!!

A second task:
… & the Taylor kinematics

$$
\begin{aligned}
H_1(q^2,k^2,\theta) \;=\; & \widetilde{Z}_1^{-1}\left[1 + \frac{N_C g^2 \langle A^2\rangle}{8(N_C^2-1)}\right.\\
& \times \left(\frac{\sqrt{k^2q^2}\cos\theta}{k^2q^2+m_{\rm IR}^4} + 2\,\frac{q^2-\sqrt{k^2q^2}\cos\theta}{q^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right.\\
& \left.\left. + 2\,\frac{k^2-\sqrt{k^2q^2}\cos\theta}{k^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta)+m_{\rm IR}^4}\right)\right] ,
\end{aligned}
$$



$k \to 0$

$$
H_1(q,0) \;=\; \widetilde{Z}_1^{-1}\left(1 + N_C\frac{g^2\langle A^2\rangle}{4(N_C^2-1)}\,\frac{q^2}{q^4+m_{IR}^4}\right)
$$

- Cfr. D. Dudal's talk !!!
- What about the Taylor's theorem?

$$
\Gamma^{abc}_{{\rm bare},\mu}(-q,0;q) \;=\; -g f^{abc} q_\mu
$$

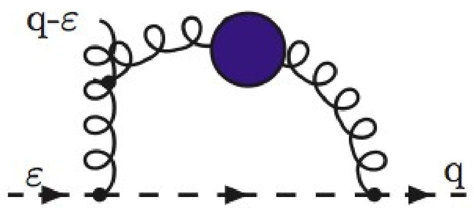
A second task:
… & the Taylor kinematics

OPE+SVZ for the asymmetric vertex:

$$\Gamma^{abc}_{\mu}(-q,\varepsilon;q-\varepsilon) \;=\; \Gamma^{abc}_{\text{pert},\mu}(-q,\varepsilon;q-\varepsilon) \;+\; \widetilde{v}^{abc}_{\mu}\,\frac{\langle A^2\rangle}{4(N_C^2-1)} \;+\;\cdots$$

$$\widetilde{v}^{abc}_{\mu} \;=\; 2I^{[1]}\;,$$

(According to PRD64(2001)114003)

$I^{[1]} \;=$ 

$$\Gamma^{abc}_{\text{bare},\mu}(-q,0;q) \;=\; -gf^{abc}q_{\mu}$$

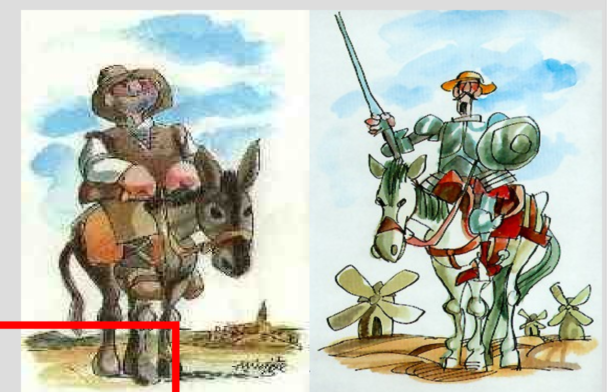
A second task:

… & the Taylor kinematics

OPE+SVZ for the asymmetric vertex:

$$\Gamma_\mu^{abc}(-q,\varepsilon;q-\varepsilon) = \Gamma_{\mathrm{pert},\mu}^{abc}(-q,\varepsilon;q-\varepsilon) + \tilde{v}_\mu^{abc}\,\frac{\langle A^2\rangle}{4(N_C^2-1)} + \cdots$$

$$\tilde{v}_\mu^{abc} = 2I^{[1]},$$

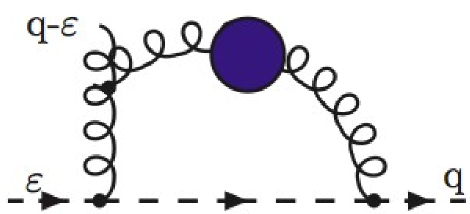


(According to PRD64(2001)114003)

$$H_1(q,\varepsilon) = H_1^{\mathrm{pert}}(q,\varepsilon) + N_C\, g^2 \frac{(q-\varepsilon)\cdot q}{q^2(q-\varepsilon)^2}\,\frac{\langle A^2\rangle}{4(N_C^2-1)},$$

$$H_2(q,\varepsilon) = H_2^{\mathrm{pert}}(q,\varepsilon) - N_C\, g^2 \frac{((q-\varepsilon)\cdot q)^2}{q^2(q-\varepsilon)^4}\,\frac{\langle A^2\rangle}{4(N_C^2-1)}.$$

$I^{[1]} =$



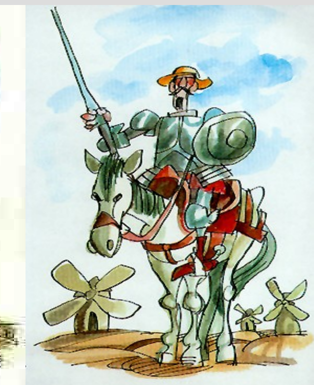
$$\Gamma_{\mathrm{bare},\mu}^{abc}(-q,0;q) = -g f^{abc} q_\mu$$

A second task:
… & the Taylor kinematics

OPE+SVZ for the asymmetric vertex:

$$\Gamma^{abc}_{\mu}(-q,\varepsilon;q-\varepsilon) \;=\; \Gamma^{abc}_{\text{pert},\mu}(-q,\varepsilon;q-\varepsilon) \;+\; \widetilde{v}^{abc}_{\mu}\,\frac{\langle A^2\rangle}{4(N_C^2-1)} \;+\;\cdots$$

$$\widetilde{v}^{abc}_{\mu} \;=\; 2I^{[1]}\,,$$

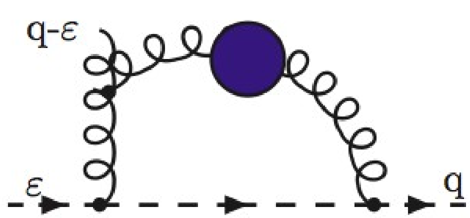


(According to PRD64(2001)114003)

$$H_1(q,\varepsilon) \;=\; H_1^{\text{pert}}(q,\varepsilon) \;+\; N_C\, g^2\,\frac{(q-\varepsilon)\cdot q}{q^2(q-\varepsilon)^2}\,\frac{\langle A^2\rangle}{4(N_C^2-1)}\,,$$

$$H_2(q,\varepsilon) \;=\; H_2^{\text{pert}}(q,\varepsilon) \;-\; N_C\, g^2\,\frac{((q-\varepsilon)\cdot q)^2}{q^2(q-\varepsilon)^4}\,\frac{\langle A^2\rangle}{4(N_C^2-1)}\,.$$

$I^{[1]} \;=$



$$\Gamma^{abc}_{\mu}(-q,0;q) \;=\; -g f^{abc}\, q_\mu \left(\, H_1^{\text{pert}}(q,0) \;+\; H_2^{\text{pert}}(q,0)\,\right)$$

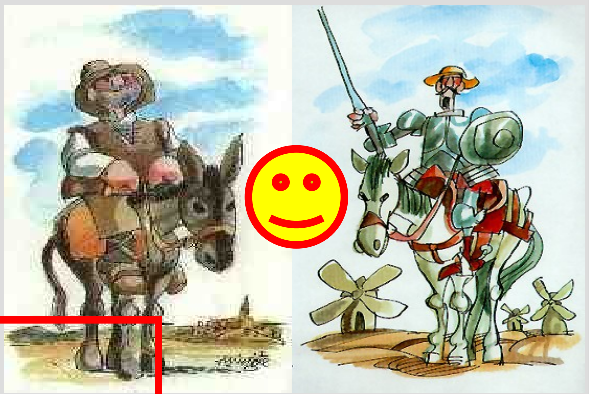
$$\Gamma^{abc}_{\text{bare},\mu}(-q,0;q) \;=\; -g f^{abc} q_\mu$$

A second task:
… & the Taylor kinematics

OPE+SVZ for the asymmetric vertex:

$$\Gamma^{abc}_{\mu}(-q,\varepsilon;q-\varepsilon) \;=\; \Gamma^{abc}_{\mathrm{pert},\mu}(-q,\varepsilon;q-\varepsilon) \;+\; \widetilde{v}^{abc}_{\mu}\,\frac{\langle A^2\rangle}{4(N_C^2-1)} \;+\;\cdots$$

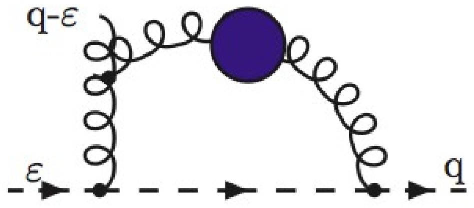
$$\widetilde{v}^{abc}_{\mu} \;=\; 2I^{[1]}\;,$$



(According to PRD64(2001)114003)

$$H_1(q,\varepsilon) \;=\; H_1^{\mathrm{pert}}(q,\varepsilon) \;+\; N_C\,g^2\,\frac{(q-\varepsilon)\cdot q}{q^2(q-\varepsilon)^2}\,\frac{\langle A^2\rangle}{4(N_C^2-1)}\;,$$

$$H_2(q,\varepsilon) \;=\; H_2^{\mathrm{pert}}(q,\varepsilon) \;-\; N_C\,g^2\,\frac{((q-\varepsilon)\cdot q)^2}{q^2(q-\varepsilon)^4}\,\frac{\langle A^2\rangle}{4(N_C^2-1)}\;.$$

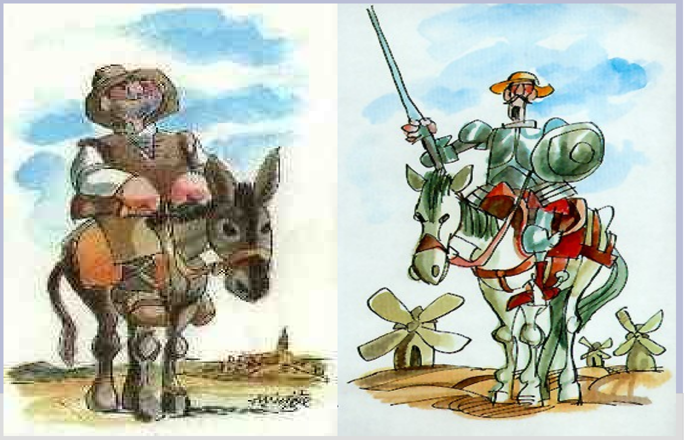
$I^{[1]} =$ 

$$\Gamma^{abc}_{\mu}(-q,0;q) \;=\; -gf^{abc}\,q_\mu\left(\;H_1^{\mathrm{pert}}(q,0) \;+\; H_2^{\mathrm{pert}}(q,0)\;\right)$$

Longitudinal and transverse corrections kill each other!!!

$$\Gamma^{abc}_{\mathrm{bare},\mu}(-q,0;q) \;=\; -gf^{abc}q_\mu$$

A final (preliminary) remark:
The GPDSE analysis



GPDSE:

$$\frac{1}{F(k^2)} = 1 + \alpha_T(\mu^2) \int d^4q \, K(k,q) \, H_1^{\text{bare}}(q,k) \, F(q^2)$$

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda \to \infty} \frac{g_0^2(\Lambda^2)}{4\pi} Z_3(\mu^2,\Lambda^2) \widetilde{Z}_3^2(\mu^2,\Lambda^2)$$

$$\begin{aligned} H_1(q^2,k^2,\theta) &= \widetilde{Z}_1^{-1} \left[1 + \frac{N_C g^2 \langle A^2 \rangle}{8(N_C^2-1)} \right. \\ &\times \left(\frac{\sqrt{k^2q^2}\cos\theta}{k^2q^2 + m_{\text{IR}}^4} + 2 \frac{q^2 - \sqrt{k^2q^2}\cos\theta}{q^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta) + m_{\text{IR}}^4} \right. \\ &\left. \left. + 2 \frac{k^2 - \sqrt{k^2q^2}\cos\theta}{k^2(q^2+k^2-2\sqrt{q^2k^2}\cos\theta) + m_{\text{IR}}^4} \right) \right] , \end{aligned}$$

Lattice inputs!!!

(Cfr. JHEP06(2008)012)

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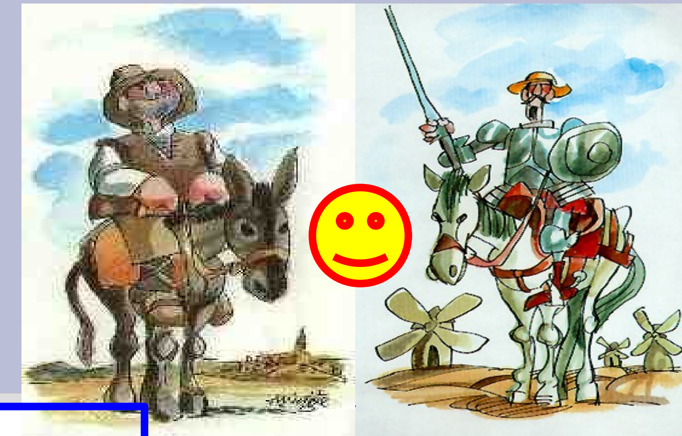
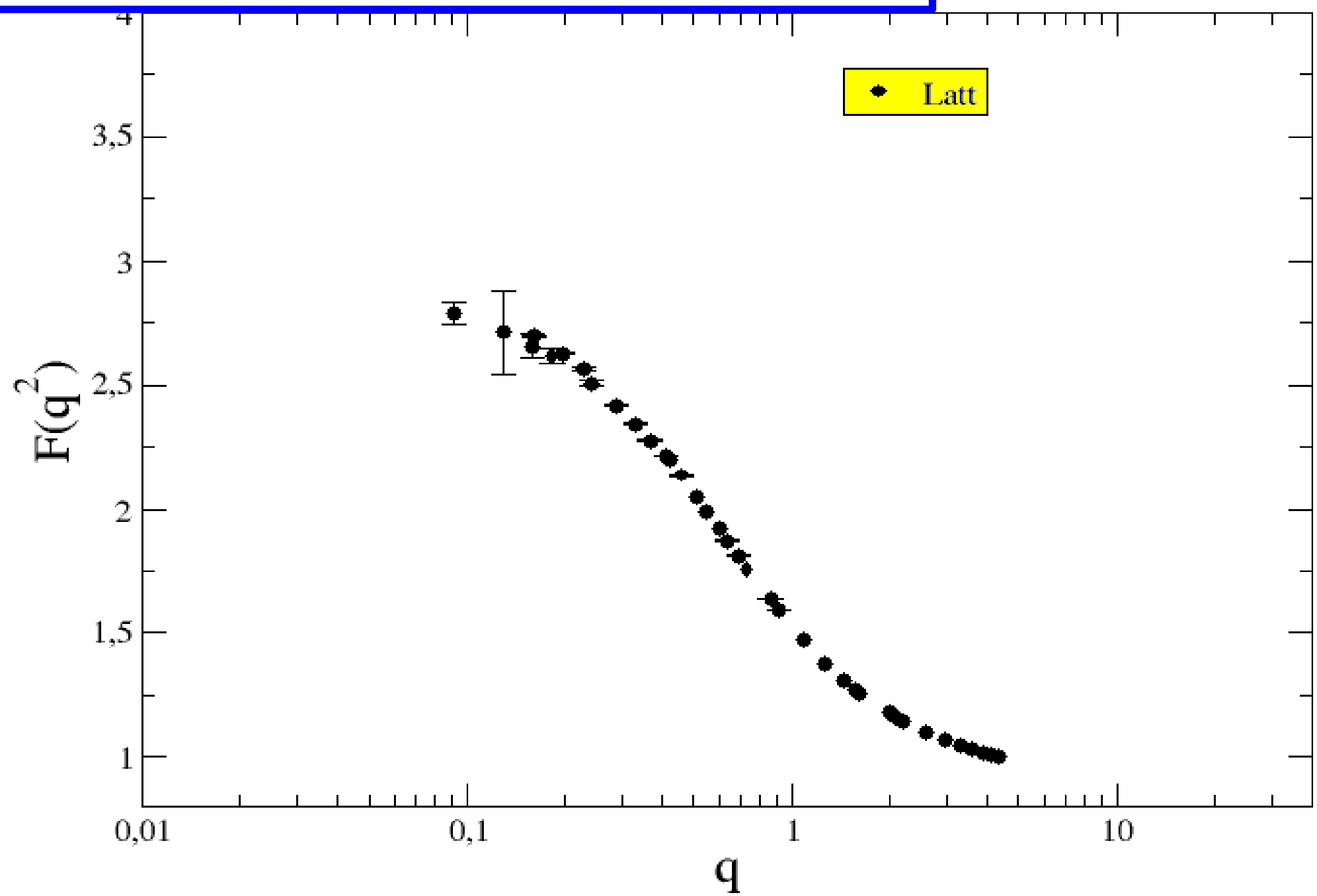
The GPDSE analysis



GPDSE:

$$\frac{1}{F(k^2)} = 1 + \alpha_T(\mu^2) \int d^4q \; K(k,q) H_1^{\rm bare}(q,k) F(q^2)$$



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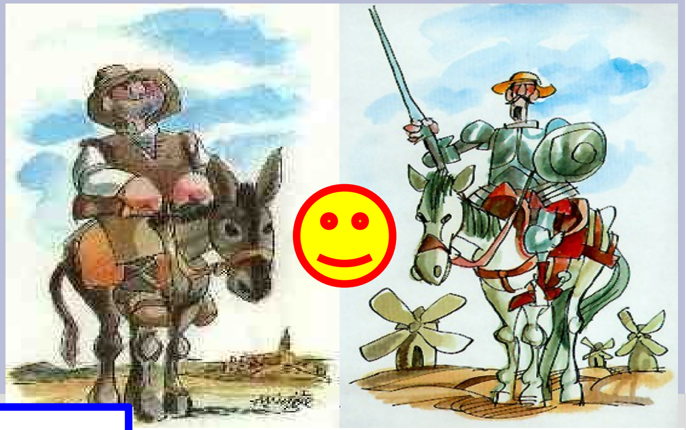

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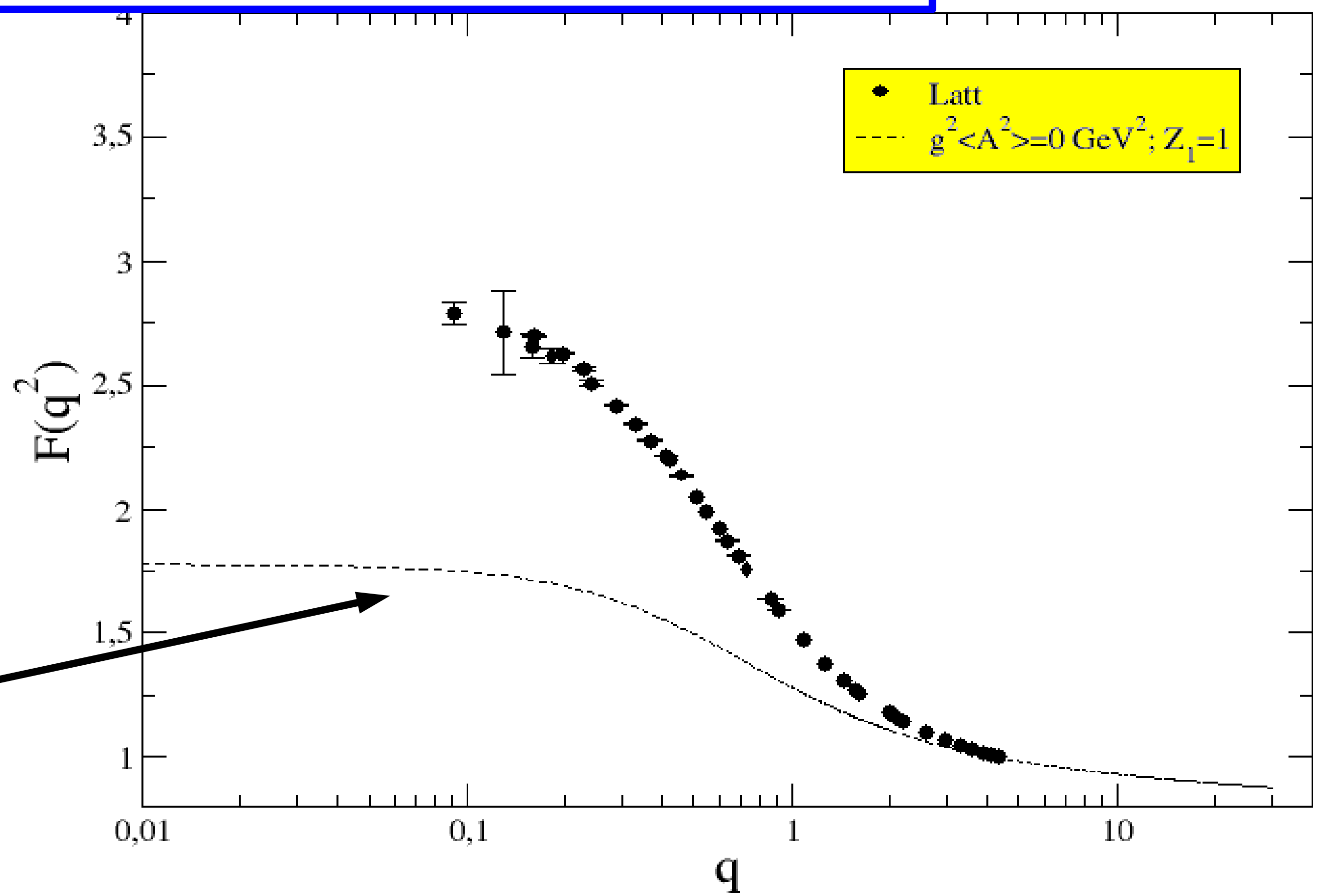
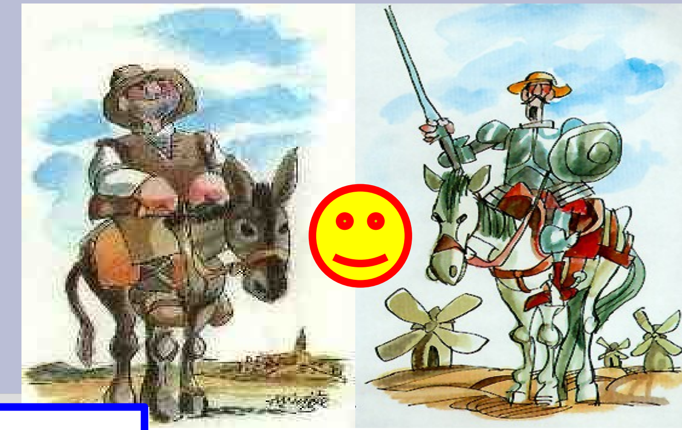


$H_1=1$

A final (preliminary) remark:

The GPDSE analysis

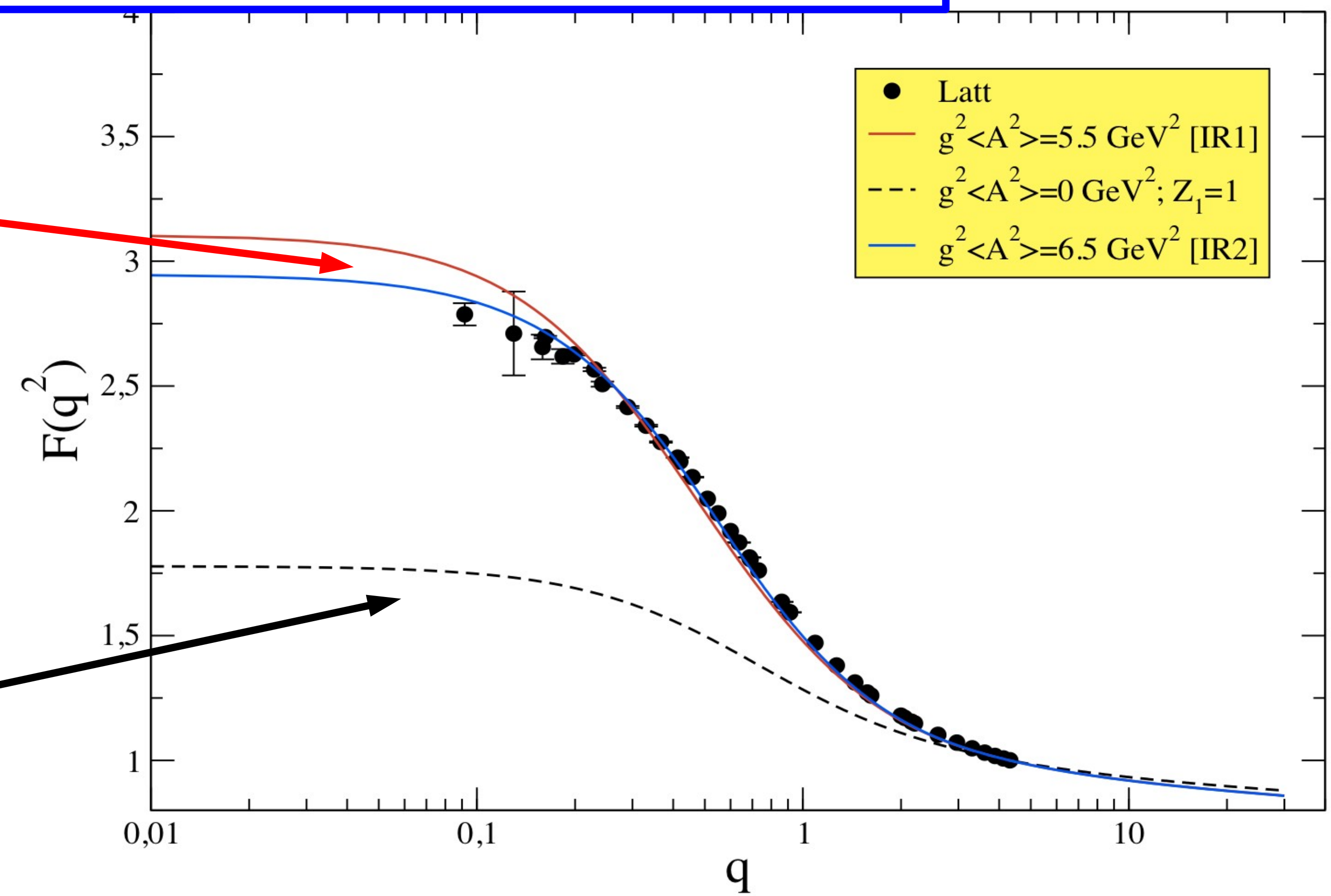


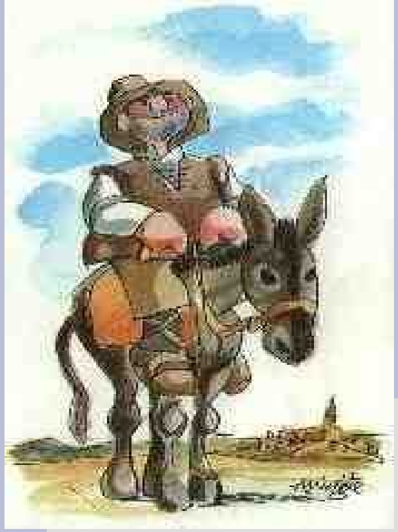
GPDSE:

$$\frac{1}{F(k^2)} = 1 + \alpha_T(\mu^2) \int d^4q \; K(k,q) H_1^{\rm bare}(q,k) F(q^2)$$

OPE-grounded H_1

$H_1=1$





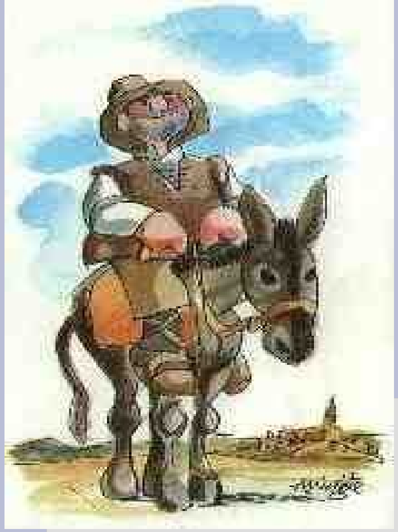
Epilogue:

- We concluded that an OPE contribution including a non-vanishing dimension-two gluon condensate is needed to describe the (experimental) running of alpha in T-scheme.

- The same OPE+SVZ approach is applied to compute non-perturbative corrections to the ghost-gluon vertex and this inspired a simple model describing its momentum behaviour in pretty good agreement with LQCD.

- We proved that, also in the OPE approach, longitudinal and transverse contributions cancel each other and the Taylor theorem still works (as it should be)

- We preliminary show that a quantitative description of the lattice ghost dressing function is possible from the GPDSE with lattice gluon inputs, only when OPE contributions for the ghost-gluon vertex are included.



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