In What Sense Is the Early Universe Fine-Tuned? (arXiv:1406.3057, Sean Carroll)

Jim Rich

SPP-IRFU
CEA-Saclay
91191 Gif-sur-Yvette

james.rich@cea.fr

November, 2014

Abstract

"It is commonplace in discussions of modern cosmology to assert that the early universe began in a special state. Conventionally, cosmologists characterize this fine-tuning in terms of the horizon and flatness problems. I argue that the fine-tuning is real, but these problems aren't the best way to think about it: causal disconnection of separated regions isn't the real problem, and flatness isn't a problem at all. Fine-tuning is better understood in terms of a measure on the space of trajectories: given reasonable conditions in the late universe, the fraction of cosmological histories that were smooth at early times is incredibly tiny. This discussion helps clarify what is required by a complete theory of cosmological initial conditions."

Horizon and Flatness problems

- Horizon problem: regions on the Last Scattering Surface separated by more than one degree were never in causal contact. So why is the temperature almost uniform?

  Conclusion of paper: this isn't the real problem

- Need $|\Omega_T - 1| < 10^{-55}$ in Early Universe to have $|\Omega_T - 1| < 10^{-2}$ today

  Conclusion of paper: this isn't a problem

Often claimed that inflation solves both "problems".

The “Past Hypothesis”

“With the caveat that the Past Hypothesis is necessary, let us assume that the universe we are trying to account for is one that was very nearly uniform (and spatially flat) at early times. What does it mean to day that such a state is fine-tuned?”

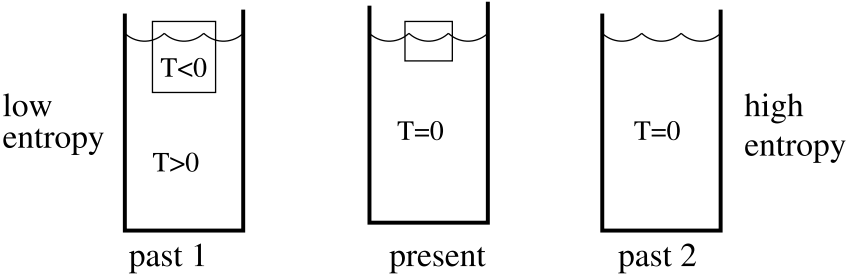
But in fact CMB observations don’t imply homogenity at early times!

“But the matter distribution at recombination could conceivably look very inhomogeneous on such slices; that would be compatible with our current observations as long as a direction-dependent cosmological redshift conspired to give an isotropic radiation field near the Earth today.”

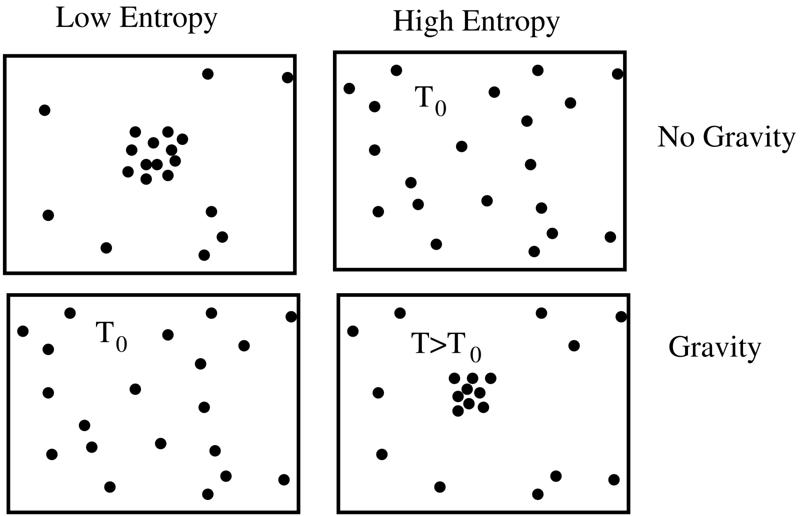
The “Past Hypothesis” rejects such conspiracies.

An interesting analogy

"Such conspiratorial conditions seem unlikely to us, but they are more numerous (in the measure to be discussed below) in the space of all possible initial conditions. Of course, we also know that most past conditions that lead to a half-melted ice cube in a glass of water look like a glass of liquid water at a uniform temperature, rather than the unmelted ice cube in warmer water we would generally expect. In both cases, our conventional reasoning assumes the kind of lower-entropy state postulated by the Past Hypothesis."



With gravity, high entropy ⇒ structures



The Entropy of our Hubble Volume

- Early universe: $S \sim 10^{88}$ (number of particles)
- Today: $S \sim 10^{103}$ (black holes at galactic centers)
- Maximum possible $S_{max} \sim 10^{122}$ (all matter in one black hole)

$$S_{BH} = \frac{A}{4G} = 4\pi GM^2 = 4\pi \frac{M^2}{M_{pl}^2}$$

The Early Universe was fine-tuned because its entropy was so small compared to the maximum possible

The Horizon Problem,, 1

> **Horizon problem (causal version).** *If different regions in the early universe have non-overlapping past light cones, no causal influence could have coordinated their conditions and evolution. There is therefore no reason for them to appear similar to us today.*

If that's as far as it goes, the horizon problem is perfectly well-formulated, if somewhat subjective. The causal formulation merely points out that there is no reason for a certain state of affairs (equal temperatures of causally disconnected regions) to obtain, but it doesn't give any reason for expecting otherwise. Characterizing this as a "problem," rather than merely an empirical fact to be noted and filed away, requires some positive expectation for what we think conditions near the Big Bang *should* be like: some reason to think that unequal temperatures would be more likely, or at least less surprising.

The Horizon Problem, 2

> **Horizon problem (equilibration version).** *If different regions in the early universe shared a causal past, they could have equilibrated and come to the same temperature. But when they do not, such as in a matter/radiation-dominated universe, equal temperatures are puzzling.*

"The equilibration formulation of the horizon problem seems stronger than the causal version; it attempts to provide some reason why equal temperatures across causally disconnected reasons should be surprising, rather than merely noting their existence."

But thermal equilibrium + gravity ⇒ inhomogenity!

"Indeed, the lack of time to equilibrate is seen to be a feature, rather than a bug: it would be even harder to understand the observed uniformity of the CMB if the plasma had had an arbitrarily long time to equilibrate."

So why is the temperature uniform?

"From this perspective, the thermal nature of the CMB radiation is especially puzzling. It cannot be attributed to "thermal equilibrium," since the early plasma is not in equilibrium in any sense."

Slow-roll inflation provides an answer:

"Central to the success of this model is the fact that the rolling field acts as a "clock," allowing regions that have been inflated to extreme distances to undergo reheating at compatible times [Anninos:1991ma]. It is thus crucially important that the universe during slow-roll inflation is *not* truly in equilibrium, even though its evolution is is approximately adiabatic; the evolving inflaton field allows for apparent coordination among widely-separated regions."

But fine-tuning required before inflation

Inflation, therefore, solves the puzzle raised by the horizon problem, in the following sense: given a sufficient amount of inflation, and a model that gracefully exits from the inflationary phase, we can obtain a homogeneous and isotropic universe in which distant points share a common causal past. The success of this picture can obscure an important point: the conditions required to get inflation started in the first place are extremely fine-tuned. This fine-tuning is often expressed in terms of entropy; a patch of spacetime ready to begin inflating has an enormously lower entropy than the (still quite low-entropy) homogeneous plasma into which it evolves [Penrose,Carroll:2004pn]. In this sense, inflation “explains” the fine-tuned nature of the early universe by positing an initial condition that is even more fine-tuned.

Does “eternal inflation” solve this problem?

Contemporary discussion of inflation often side-steps the problem of the required low-entropy initial condition by appealing to the phenomenon of eternal inflation: in many models, if inflation begins at all it continues without end in some regions of the universe, creating an infinite amount of spacetime volume [Guth:2000ka]. While plausible (although for some recent concerns see [Boddy:2014eba]), this scenario raises a new problem: rather than uniquely predicting a universe like the kind we see, inflation predicts an infinite variety of universes, making prediction a difficult problem. I won't discuss this issue here, but for recent commentary see [Ijjas:2013vea,Guth:2013sya,Linde:2014nna,Ijjas:2014nta].

Measure for flatness?

Entropy or “trajectories” tell us that the early universe was fine tuned:

“At heart, there is not much conceptual difference between studying the purported fine-tuning of the universe in terms of the measure on trajectories and quantifying the low entropy of the early state. There are relatively few initial conditions with low entropy, and the trajectories that begin from such conditions will have a small measure.”

How can we quantify fine-tuning for flatness?

Gibbons-Hawking-Stewart measure for curvature

In stat-mech, one uses the "Liouville measure" (all cells in 6N dimensional phase-space have equal probability), in part because the total phase-space available is time-independent.

"We are not arguing for some metaphysical principle to the effect that the universe *should* be chosen uniformly in phase space according to the Liouville measure; merely that, given this measure's unique status as being picked out by the dynamics, states that look natural in this measure tell us very little, while states that look unnatural might reveal useful information."

"In the case of general relativity, Gibbons, Hawking and Stewart (GHS, [Gibbons:1986xk]) showed that there is nevertheless a unique measure satisfying a small number of reasonable constraints: it is positive, independent of coordinate choices, respects the symmetries of the theory, and does not require the introduction of any additional structures."

GHS on the likelihood of extreme flatness

As we have argued, however, such a statement only has impact if the set of trajectories for which $\Omega_\kappa/\Omega_{\mathrm{matter/radiation}} < 10^{-55}$ in the very early universe is actually small. It *seems* small, since 10^{-55} is a small number. But that just means that it would be small if trajectories were chosen uniformly in the variable $\Omega_\kappa/\Omega_{\mathrm{matter/radiation}}$, for which we have given no independent justification. Clearly, this is a job for the GHS measure.

........The measure is then

$$\mu = 3\sqrt{\frac{3}{2}}H_*^{-2}\int_{H=H_*}\frac{1-\Omega_V-\frac{2}{3}\Omega_\kappa}{|\Omega_\kappa|^{5/2}\left(1-\Omega_V-\Omega_\kappa\right)^{1/2}}\,d\Omega_\kappa\,d\phi. \qquad (1)$$

This integral is divergent; it blows up as $\Omega_\kappa \to 0$

GHS $\Rightarrow$ extreme early flatness??

This divergence was noted in the original GHS paper [Gibbons:1986xk], where it was attributed to "universes with very large scale factors" due to a different choice of variables. That characterization isn't very useful, since "large scale factor" is a feature along the trajectory of any open universe, rather than picking out a particular type of trajectory. Later works [Hawking:1987bi,Coule:1994gd,Gibbons:2006pa] correctly described the divergence as arising from nearly-flat universes. Gibbons and Turok [Gibbons:2006pa] advocated dealing with the infinity by discarding all flat universes by fiat, and concentrating on the non-flat universes. Tam and I [Carroll:2010aj] took the opposite view: what (**??**) is telling us is that almost every Robertson-Walker cosmology is spatially flat. Rather than throwing such trajectories away, we should throw all of the others away and deal with flat universes.

GHS on Smoothness

The surprising result that almost all universes are spatially flat might raise the hope that a careful consideration of the measure might also explain the smoothness of the universe: perhaps almost all cosmological trajectories are extremely smooth at early times. Sadly, the opposite is true, as can be seen by extending the GHS measure to perturbed spacetimes [Carroll:2010aj]. ....A straightforward calculation shows that the measure evaluated on such a surface is

$$\mu = \int_{\eta=\eta_*} du\, dp_u. \tag{2}$$

In other words, the measure on a perturbation mode is completely uniform in the $\{u, p_u\}$ variables, much as we might have naïvely guessed, and in stark contrast to the flatness problem. All values for u and p_u are equally likely; there is nothing in the measure that would explain the small observed values of perturbations at early times. Hence, the observed homogeneity of our universe does imply considerable fine-tuning.

Amount for finetuning required for flatness on LSS

The total fraction of universes that were smooth at early times is just the product of the fractions corresponding to each mode. Choosing reasonable bounds for the largest and smallest modes considered, the total fraction of trajectories that are smooth at early times works out to be

$$f(\text{smooth at GUT scale}|\text{smooth at recomb.}) \approx 10^{-6.6\times10^7}. \quad (3)$$

This represents a very conservative estimate for the amount of fine-tuning involved in the standard cosmological model.

The reason why (3) is such a small number is that most trajectories that are smooth at last scattering contain modes that were large at earlier times but decayed. That is morally equivalent to trajectories that start with relatively high entropy, but that start with delicate correlations that cause the entropy to decrease as time passes.

Conclusion on fine-tuning

The real sense in which the early universe was fine-tuned is extremely simple: the overwhelming majority of cosmological trajectories, as quantified by the canonical measure, are highly nonuniform at early times, and we don't think the real universe was like that.

It's even worse than that

In the presence of a stable vacuum energy, the highest-entropy configuration for the universe to be in is empty de Sitter space [Carroll:2004pn]. The worry there is that vacuum fluctuations give rise to an ensemble of freak "Boltzmann brain" observers [Dyson:2002pf,Albrecht:2004ke,Bousso:2006xc].As argued in [Boddy:2014eba], however, quantum fluctuations in de Sitter space don't actually bring into existence decohered branches of the wave function containing such freak observers. Nevertheless, it seems reasonable to think that the space of trajectories containing one person or one galaxy in an otherwise empty background has a much greater measure than the kind of universe in which we live, with over a hundred billion galaxies; at least, such a situation has a much higher entropy. We are therefore still left with the fundamental cosmological question: "Why don't we live in a nearly-empty de Sitter space?"

Conclusion on inflation

Inflation, therefore, cannot solve this problem all by itself. Indeed, the measure reinforces the argument made by Penrose, that the initial conditions necessary for getting inflation to start are extremely fine-tuned, more so than those of the conventional Big Bang model it was meant to help fix. Inflation does, however, still have very attractive features. It posits an initial condition that, while very low-entropy, is also extremely simple, not to mention physically quite small. (With inflation, our observable universe could have been one Planck length across at the Planck density; without inflation, the same patch was of order one centimeter across at that time. That is an incredibly large volume, when considered in Planck units, over which to have initial homogeneity.) Therefore, while inflation does not remove the need for a theory of initial conditions, it gives those trying to construct such a theory a relatively reasonable target to shoot for.