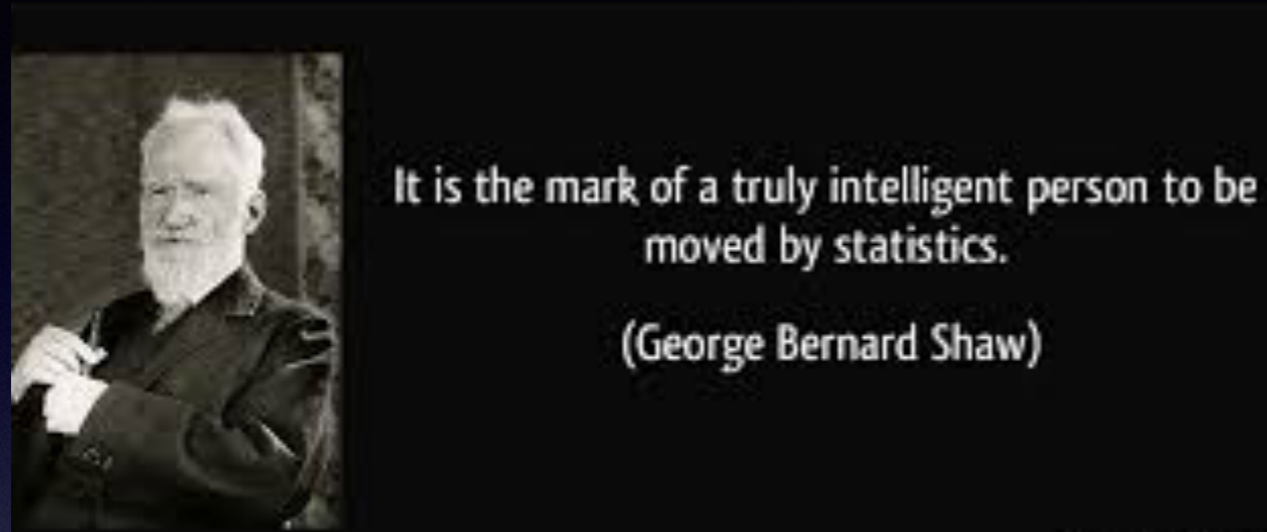


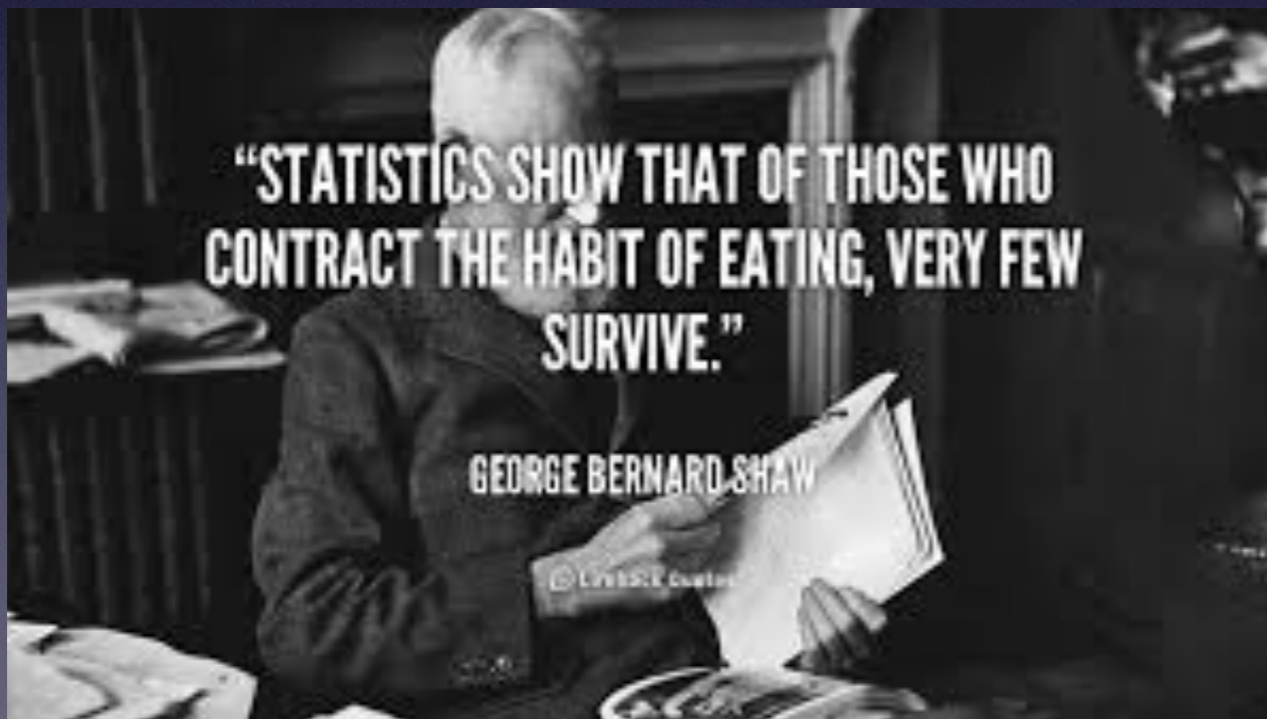
# Information and Statistics: A New Paradigm in the Study of Neutron Stars

Jorge Piekarewicz - Florida State University



It is the mark of a truly intelligent person to be moved by statistics.

(George Bernard Shaw)



“STATISTICS SHOW THAT OF THOSE WHO CONTRACT THE HABIT OF EATING, VERY FEW SURVIVE.”

GEORGE BERNARD SHAW

**ECT\***   
 EUROPEAN CENTRE FOR THEORETICAL STUDIES  
 IN NUCLEAR PHYSICS AND RELATED AREAS  
 TRENTO, ITALY  
 Institutional Member of the ESF Expert Committee NuPECC

**Information and Statistics in Nuclear Experiment and Theory ISNET-3**  
 Trento, November 16-20, 2015

**Main Topics**  
 Estimation of statistical uncertainties of calculated quantities,  
 assessment of systematic errors,  
 validation and verification of extrapolations,  
 information content of observables with respect to current theoretical models,  
 statistical tools of nuclear theory and planning of future experiments,  
 Bayesian methods and computational techniques,  
 novel methods of optimization

**Key Speakers**  
 Anatoli Afanasjev (Mississippi State University, USA), Enrique Ruiz Arriola (University of Granada, Spain), Julia Bliss (Technical University of Darmstadt, Germany), Rick Casten (Yale University, USA), Gianluca Colo (University of Milan and INFN, Italy), Andreas Ekstrom (University of Tennessee, USA), Christian Forsen (Chalmers University of Technology, Sweden), Dick Furnstahl (Ohio State University, USA), Krzysztof Graczyk (University of Wrocław, Poland), Titta Havrinen (University of Jyväskylä, Finland), Dave Ireland (University of Glasgow, UK), Yamen Jagannathan (Michigan State University, USA), Markus Kortelainen (University of Jyväskylä and Helsinki Institute of Physics, Finland), Amy Lovell (Michigan State University, USA), Rodrigo Navarro-Perez (Lawrence Livermore National Laboratory, USA), Witold Nazarewicz (Michigan State University, USA), Nils Paar (University of Basel, Switzerland), Alessandro Pastore (University of York, UK), Jorge Piekarewicz (Florida State University, USA), Scott Pratt (Michigan State University, USA), David Regnier (CEA Bruyères, France), Paul-Gerhard Reinhard (University of Erlangen, Germany), David Richards (Jefferson Laboratory, USA), Xavier Roca-Maza (University of Milan and INFN, Italy), Jan Ryeckebusch (Ghent University, Belgium), Nicolas Schunck (Lawrence Livermore National Laboratory, USA), Achim Schwenk (TU Darmstadt), Paul Stevenson (University of Surrey, UK), Rebecca Surman (University of Notre Dame, USA), Bartłomiej Szpak (Institute of Nuclear Physics PAN - Krakow), Sarah Wesolowski (Ohio State University, USA), Stefan Wild (Argonne National Laboratory, USA)

**Organizers**  
 David Ireland (University of Glasgow)  
 Witold Nazarewicz (FRIB/NSCL - Michigan State University)  
 Bartłomiej Szpak (Institute of Nuclear Physics PAN - Krakow)

**Director of the ECT\*:** Professor Wolfram Weise (ECT\*)

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For local organization please contact: Giammaria Ziglio - ECT\* Secretariat - Villa Tambosi - Strada delle Tabarelle 286 - 38123 Villazano (Trento) - Italy  
 Tel. +39-0461 314721 Fax(+39-0461) 314750, E-mail: [ect@ectstar.eu](mailto:ect@ectstar.eu) or visit <http://www.ectstar.eu>

Nuclear Physics Division  
CEA Saclay, France  
May 13, 2016

# My Collaborators

## My FSU Collaborators

- Genaro Toledo-Sanchez
- Karim Hasnaoui
- Bonnie Todd-Rutel
- Brad Futch
- Jutri Taruna
- **Farrukh Fattoyev**
- **Wei-Chia Chen**
- **Raditya Utama**



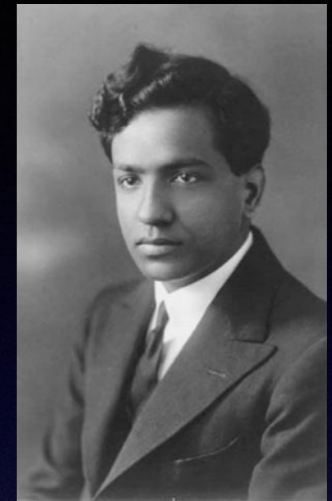
## My Outside Collaborators

- B. Agrawal (Saha Inst.)
- M. Centelles (U. Barcelona)
- G. Colò (U. Milano)
- C.J. Horowitz (Indiana U.)
- W. Nazarewicz (MSU)
- N. Paar (U. Zagreb)
- M.A. Pérez-Garcia (U. Salamanca)
- P.G.- Reinhard (U. Erlangen-Nürnberg)
- X. Roca-Maza (U. Milano)
- D. Vretenar (U. Zagreb)



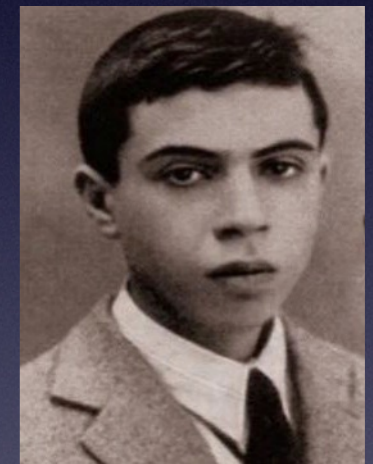
# Neutron Stars: Some Historical Facts

📌 Chandrasekhar shows that massive stars will collapse (1931)

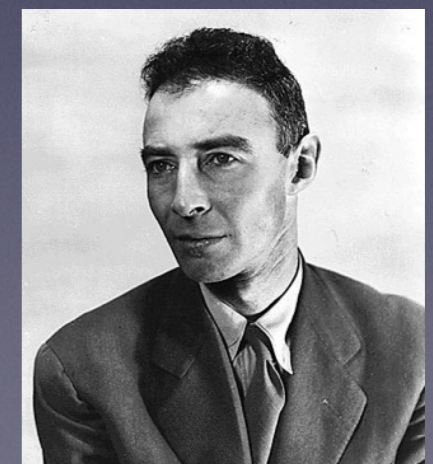


📌 Chadwick discovers the neutron (1932)  
(... predicted earlier by Majorana but never published)

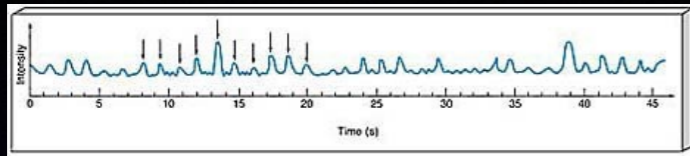
📌 Baade-Zwicky introduce the concept of a neutron star (1933)  
(... Landau mentions dense stars that look like giant nuclei)



📌 Oppenheimer-Volkoff use GR to compute the structure of neutron stars (1939)  
(... predict  $M_{\star} \simeq 0.7 M_{\odot}$  as maximum neutron star mass)



# Neutron Stars: Dame Jocelyn Bell Burnell



- Detected a bit of “scruff” (1967)
- Discovers amazing regularity in the signal (P=1.33730119 seconds)
- May the signal be from an alien civilization? (Little Green Man 1)
- Paper announcing first pulsar published [Observation of a Rapidly Pulsating Radio Source A Hewish, SJ Bell, et al., Nature 217, 709 (1968)]
- Nobel awarded to Hewish and Ryle (1974)
- “No-Bell” roundly condemned (Hoyle)

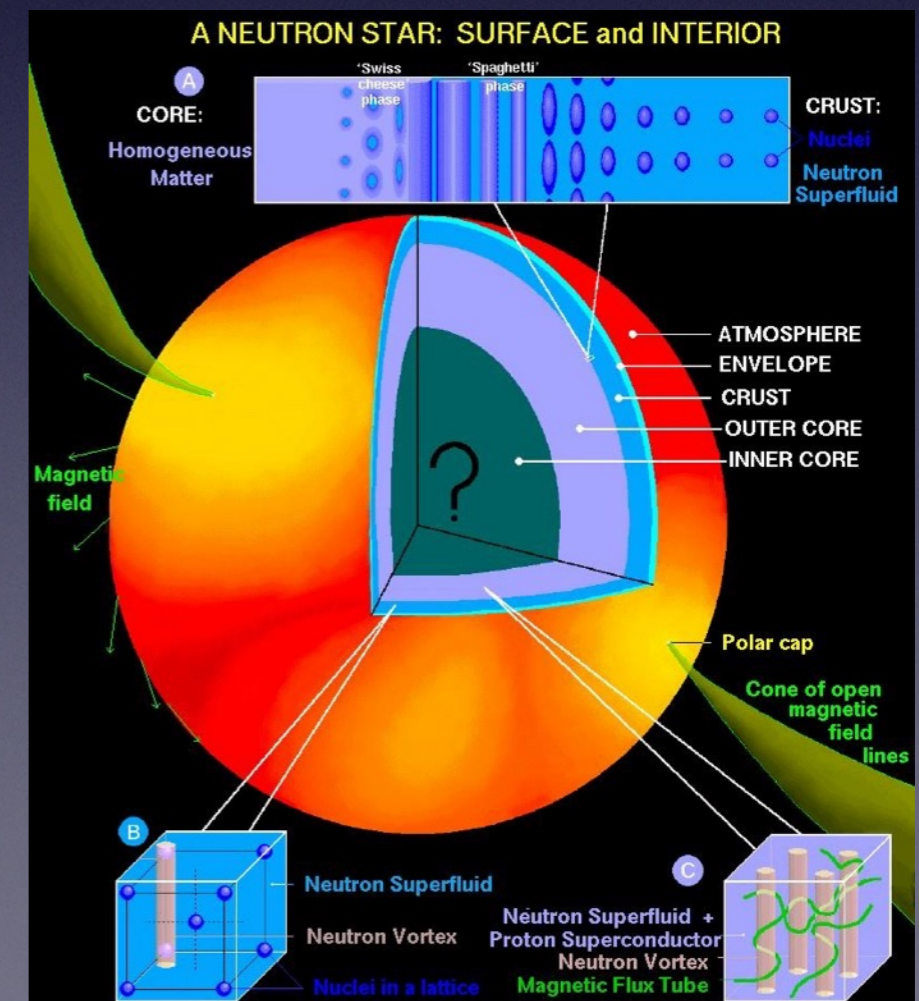
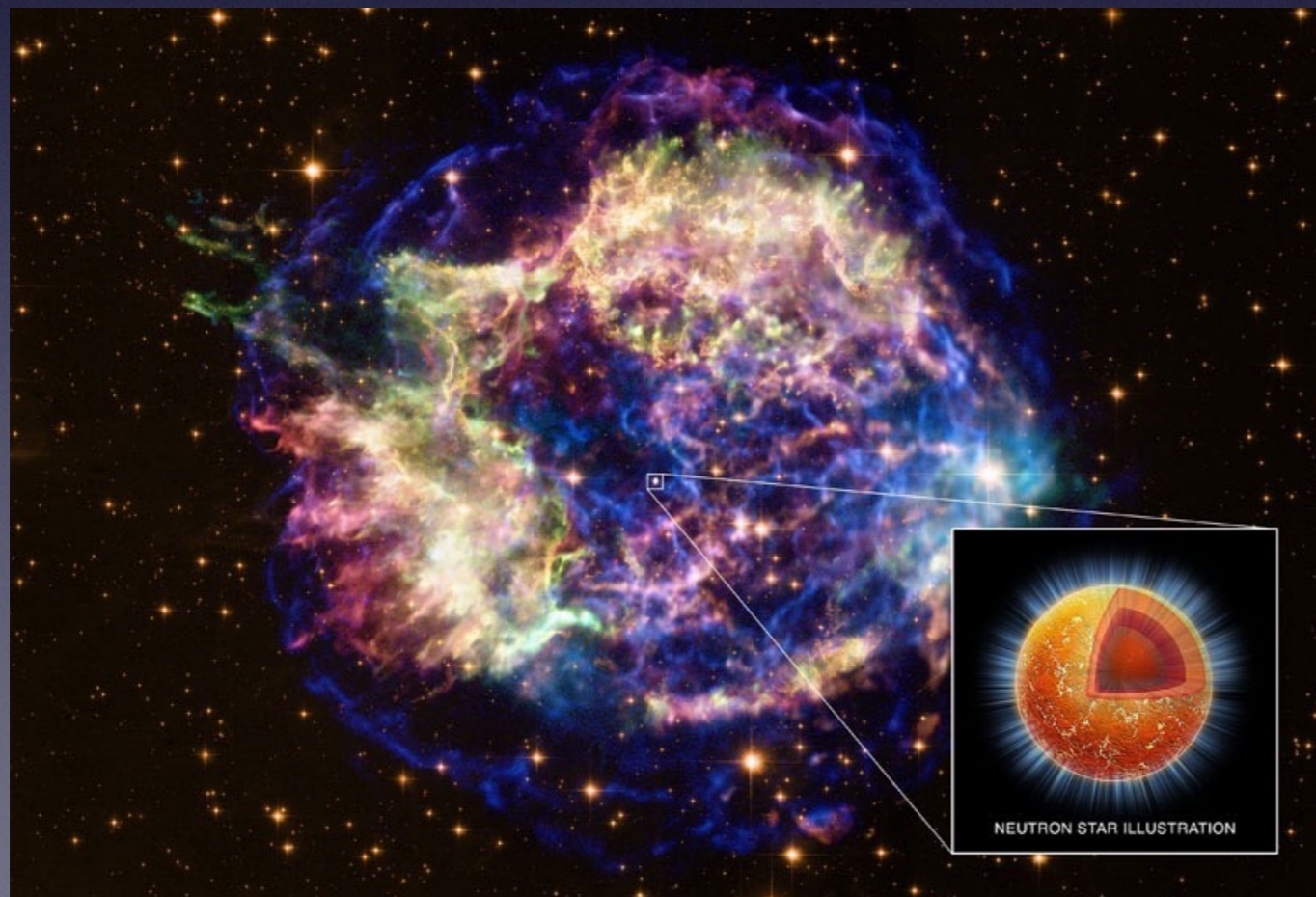


*“I believe it would demean Nobel Prizes if they were awarded to research students, except in very exceptional cases and I do not believe this is one of them”*



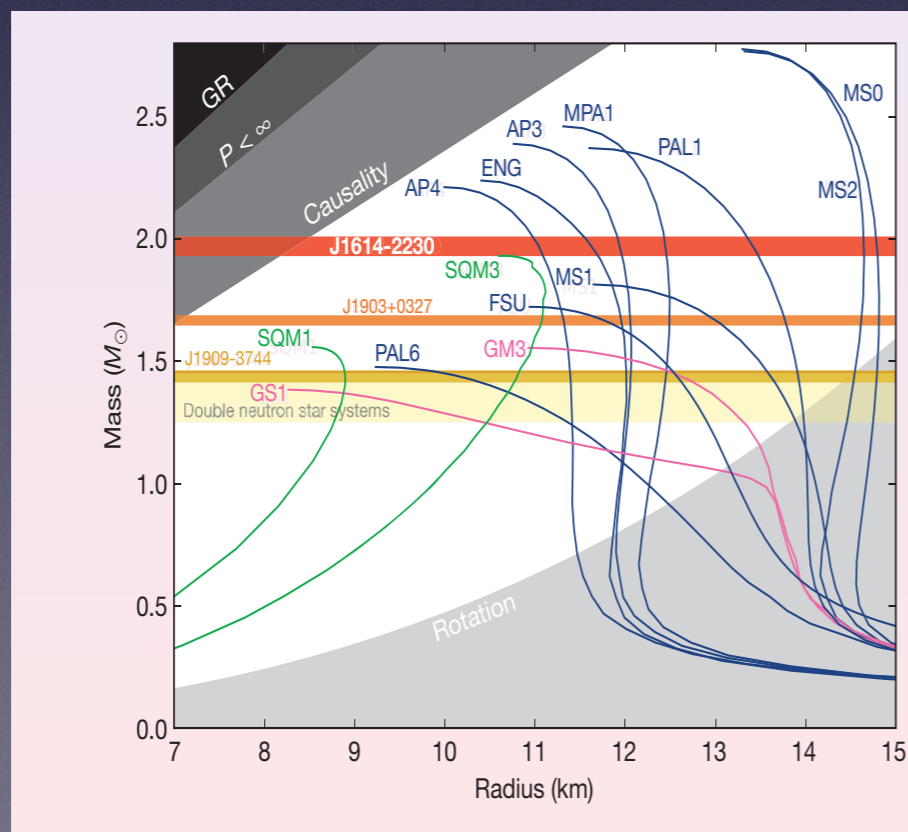
# The Anatomy of a Neutron Star

- Atmosphere (10 cm): Shapes Thermal Radiation ( $L=4\pi\sigma R^2T^4$ )
- Envelope (100 m): Huge Temperature Gradient ( $10^8\text{K} \leftrightarrow 10^6\text{K}$ )
- Outer Crust (400 m): Coulomb Crystal (Exotic neutron-rich nuclei)
- Inner Crust (1 km): Coulomb Frustration (“Nuclear Pasta”)
- Outer Core (10 km): Uniform Neutron-Rich Matter (n,p,e, $\mu$ )
- Inner Core (?): Exotic Matter (Hyperons, condensates, quark matter)



# Neutron Stars: Unique Cosmic Laboratories

- Neutron stars are the remnants of massive stellar explosions (CCSN)
  - Bound by gravity — NOT by the strong force
  - Catalyst for the formation of exotic state of matter
  - Satisfy the Tolman-Oppenheimer-Volkoff equation ( $v_{\text{esc}}/c \sim 1/2$ )
- Only Physics that the TOV equation is sensitive to: Equation of State
  - EOS must span about 11 orders of magnitude in baryon density
- Increase from  $0.7 \rightarrow 2$  Msun transfers ownership to Nuclear Physics!
- Predictions on stellar radii differ by several kilometers!



$$\frac{dM}{dr} = 4\pi r^2 \mathcal{E}(r)$$

$$\frac{dP}{dr} = -G \frac{\mathcal{E}(r)M(r)}{r^2} \left[ 1 + \frac{P(r)}{\mathcal{E}(r)} \right]$$

$$\left[ 1 + \frac{4\pi r^3 P(r)}{M(r)} \right] \left[ 1 - \frac{2GM(r)}{r} \right]^{-1}$$

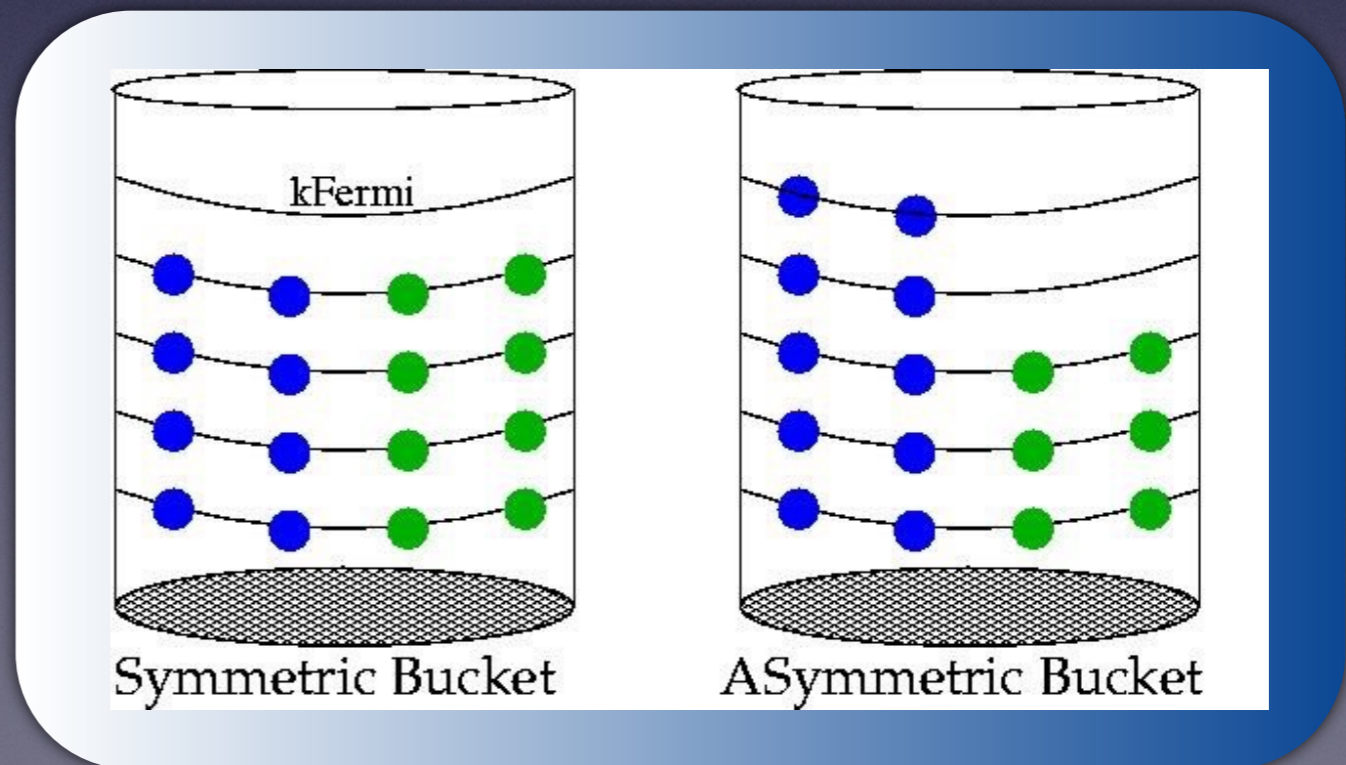
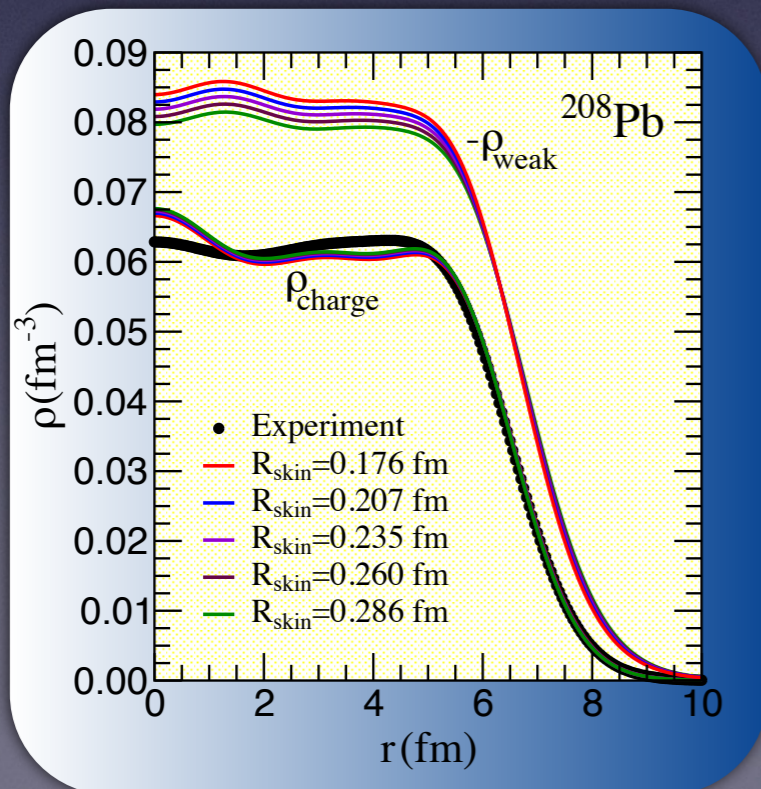
Need an EOS:  $P = P(\mathcal{E})$  relation

**Nuclear Physics Critical**



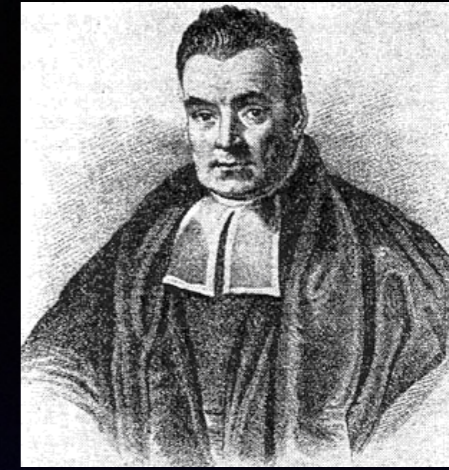
# The Equation of State of Neutron-Rich Matter

- **The EOS of asymmetric matter:**  $\alpha=(N-Z)/A$ ;  $x=(\rho-\rho_0)/3\rho_0$ ;  $T=0$
- $\rho_0 \simeq 0.15 \text{ fm}^{-3}$  — saturation density  $\leftrightarrow$  nuclear density
- $\mathcal{E}(\rho, \alpha) \simeq \mathcal{E}_0(\rho) + \alpha^2 \mathcal{S}(\rho) \simeq \left( \epsilon_0 + \frac{1}{2} K_0 x^2 \right) + \left( J + Lx + \frac{1}{2} K_{\text{sym}} x^2 \right) \alpha^2$
- **Symmetric nuclear matter saturates:**
  - $\epsilon_0 \simeq -16 \text{ MeV}$  — binding energy per nucleon  $\leftrightarrow$  nuclear masses
  - $K_0 \simeq 230 \text{ MeV}$  — nuclear incompressibility  $\leftrightarrow$  nuclear “breathing” mode
- **Density dependence of symmetry poorly constrained:**
  - $J \simeq 30 \text{ MeV}$  — symmetry energy  $\leftrightarrow$  masses of neutron-rich nuclei
  - $L \simeq ?$  — symmetry slope  $\leftrightarrow$  neutron skin ( $R_n - R_p$ ) of heavy nuclei ?



# Bayes' Theorem

Thomas Bayes (1701-1761)



$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

## A simple example: “False Positives”

- A: Individual is infected with the HIV virus
- B: Individual tests positive to HIV test

## The priors and the likelihood

- $P(A) = 1/200$  (“prior” knowledge; 0.5% of population is infected)
- $P(B|A) = 98/100$  (likelihood of the evidence; accuracy of test)
- $P(B) = (1/200) * (98/100) + (199/200) * (2/100) = 496 / (100 * 200)$

## The odds: the posterior probability

- $P(A|B) = 49/248 \approx 20\%$  (odds have increased from 0.5% but still very far away from 98%)



# Bayes' Theorem: Application to Model Building

PHYSICAL REVIEW C 90, 044305 (2014)



Building relativistic mean field models for finite nuclei and neutron stars

Wei-Chia Chen\* and J. Piekarewicz†

Department of Physics, Florida State University, Tallahassee, Florida 32306, USA

$$\text{Posterior} \leftarrow P(M|D) = \frac{P(D|M)P(M)}{P(D)} \rightarrow \text{Prior}$$

Likelihood
Marginal Likelihood

- QCD is the fundamental theory of the strong interactions!
- M: A theoretical MODEL with parameters and biases
- D: A collection of experimental and observational DATA

- The Prior  $P(M)$ : An insightful transformation in DFT  
 $(g_s, g_v, g_\rho, \kappa, \lambda, \Lambda_v) \iff (\rho_0, \epsilon_0, M^*, K, J, L)$

- The Likelihood  $P(D|M) \simeq \exp(-\chi^2/2)$   

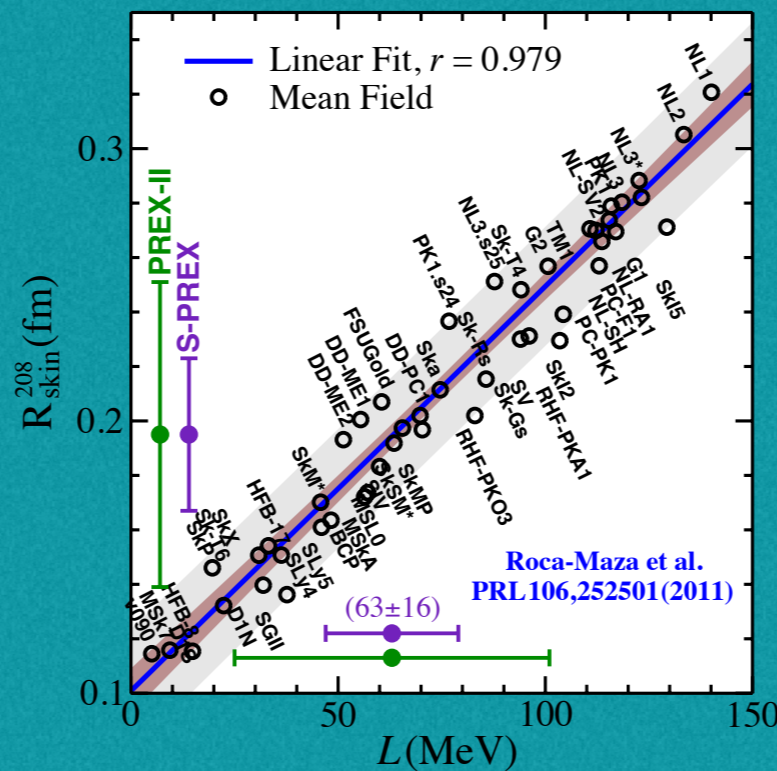
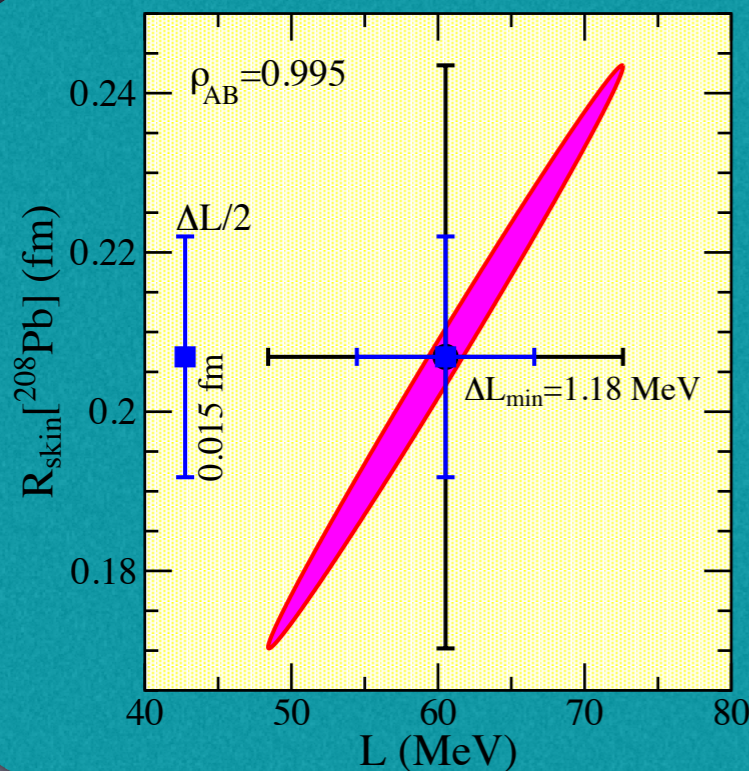
$$\chi^2(D, M) = \sum_{n=1}^N \frac{\left(O_n^{(\text{th})}(M) - O_n^{(\text{exp})}(D)\right)^2}{\Delta O_n^2}$$

- The Marginal Likelihood; overall normalization factor

# Searching for L: The Strategy

$P_{PNM} \simeq L\rho_0/3$  is not a physical observable

- Establish a powerful physical argument connecting  $L$  to  $R_{\text{skin}}$ 
  - Where do the extra 44 neutrons in  $^{208}\text{Pb}$  go? Competition between surface tension and the difference  $S(\rho_0) - S(\rho_{\text{surf}}) \simeq L$ .  
The larger the value of  $L$ , the thicker the neutron skin of  $^{208}\text{Pb}$
- Ensure that “your” accurately-calibrated DFT supports the correlation
  - Statistical Uncertainty: Theoretical error bars and correlation coefficients
  - What precision in  $R_{\text{skin}}$  is required to constrain  $L$  to the desired accuracy?
- Ensure that “all” accurately-calibrated DFT support the correlation
  - Systematic Uncertainty: As with all systematic errors, much harder to quantify (... “all models are equal but some models are more equal than others”)



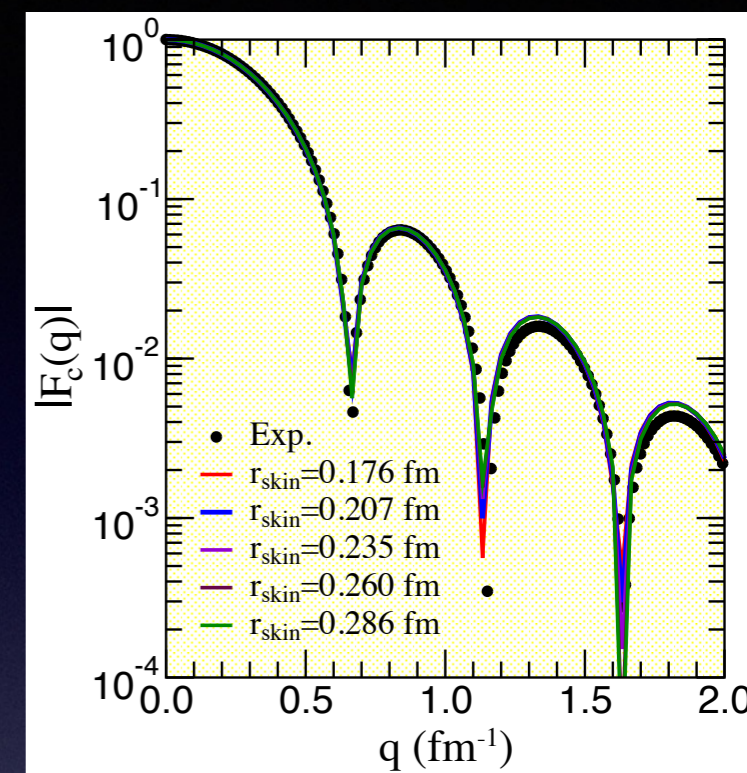
*New era in Nuclear Theory where predictability will be typical and uncertainty quantification will be demanded ...*

# Theory Informing Experiment

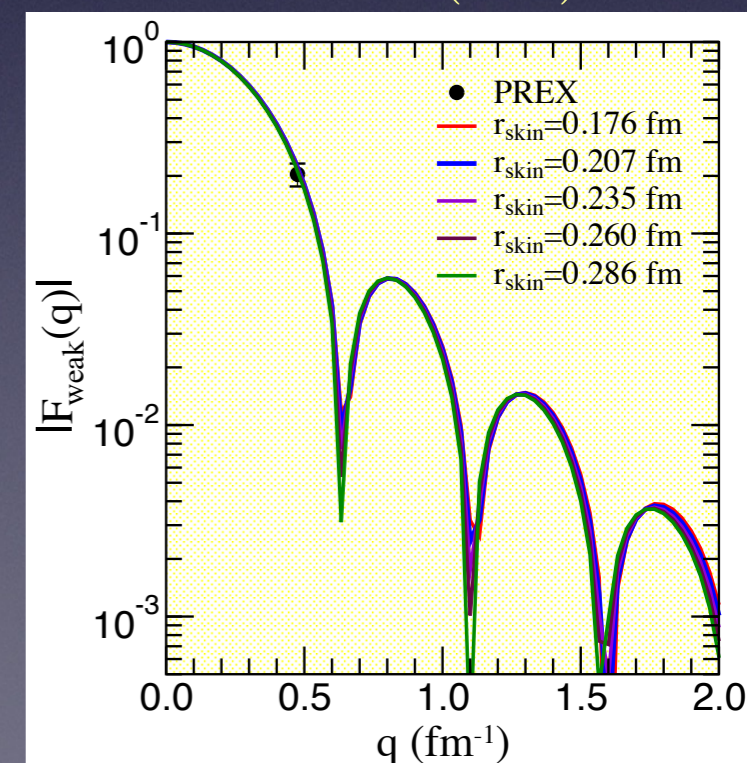
- PREX@JLAB: First electroweak evidence in favor of  $R_{\text{skin}}$  in Pb (error bars too large!)
- Precision required in the determination of the neutron radius/skin?
  - As precisely as “humanly possible” - fundamental nuclear structure property (*cf.* charge density)
  - To strongly impact Astrophysics?
- Is there a need for a systematic study over “many” nuclei?  
PREX, CREX, SREX, ZREX, ...
- Is there a need for more than one  $q$ -point?  
Radius and diffuseness ... or the whole form factor?

These questions will be addressed at the  
MITP Program “Neutron Skins of Nuclei”  
Mainz, May 17-27, 2016

$$R_{ch} = 5.5012(13) \text{ fm}$$



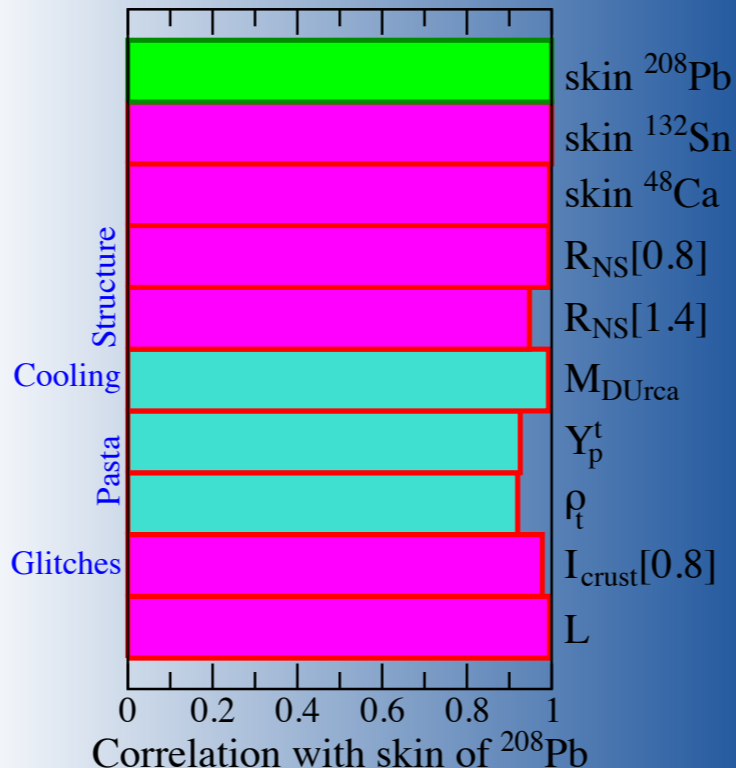
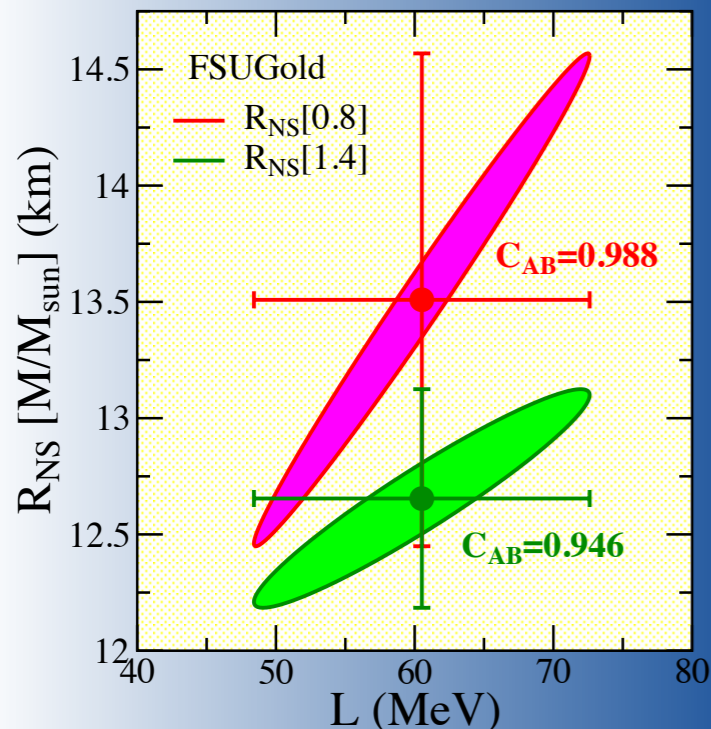
$$R_{wk} = 5.826(181) \text{ fm}$$



# Heaven and Earth

## The enormous reach of the neutron skin

- Neutron-star radii are sensitive to the EOS near  $2\rho_0$
- Neutron star masses sensitive to EOS at much higher density
- Neutron skin correlated to a host of neutron-star properties
- Stellar radii, proton fraction, enhanced cooling, moment of inertia
- We are at a dawn of a new era ... *the train has left the station*  
Predictability typical and uncertainty quantification demanded!

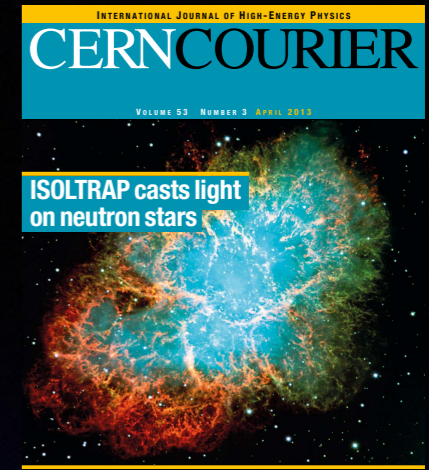


PHYSICAL REVIEW A 83,  
040001 (2011)  
Editorial: Uncertainty Estimates

*Papers presenting the results  
of theoretical  
calculations are expected to  
include  
uncertainty estimates for the  
calculations  
whenever practicable ...*

# The Composition of the Outer Crust

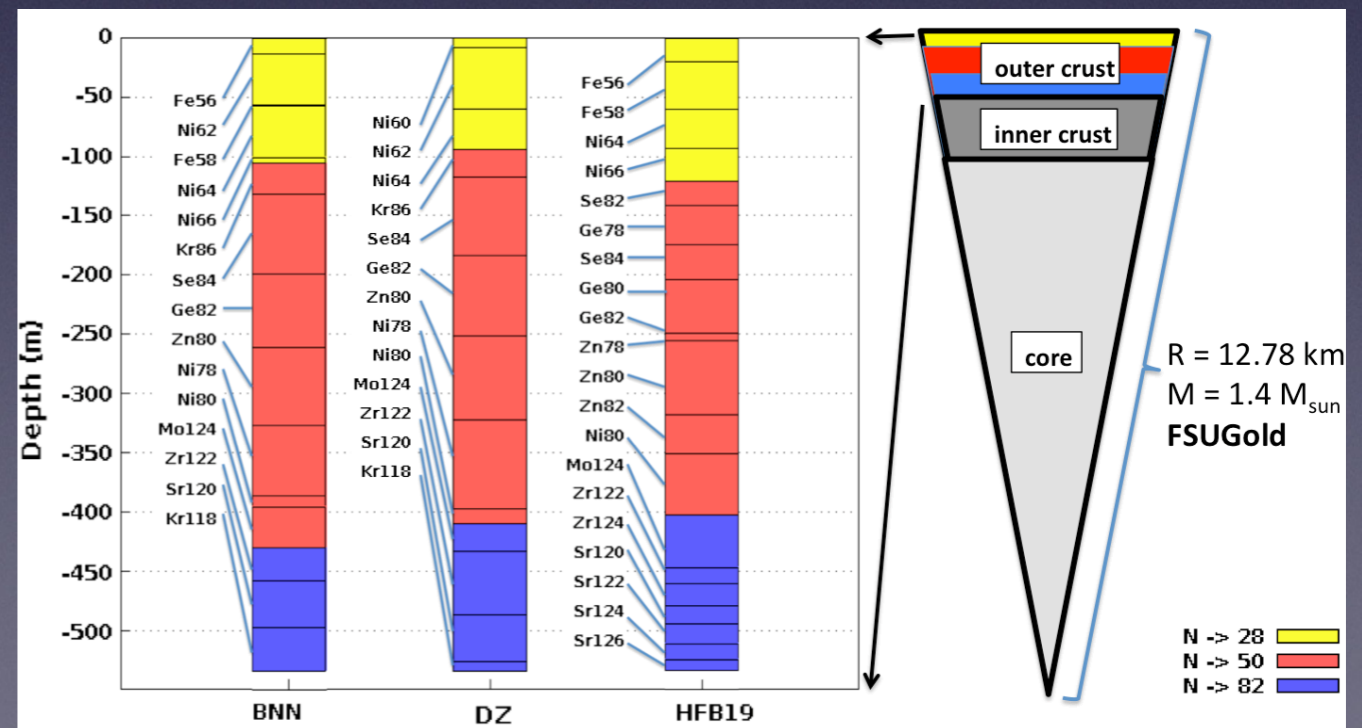
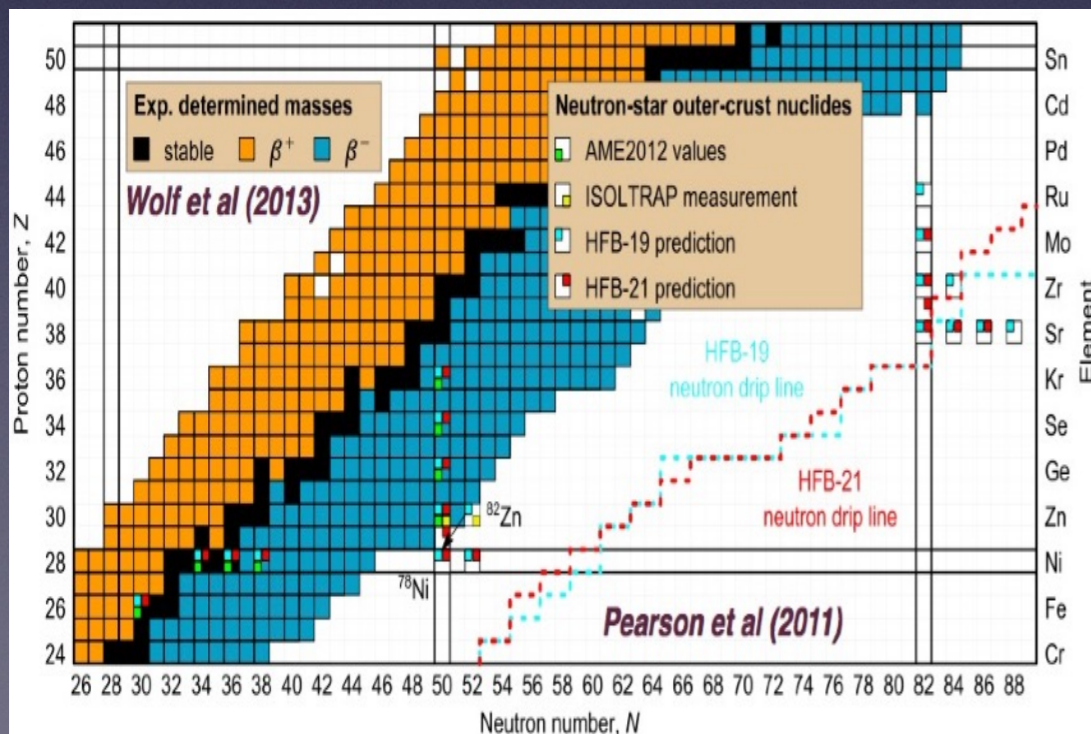
High sensitivity to nuclear masses



- System unstable to cluster formation
- BCC lattice of neutron-rich nuclei imbedded in e-gas
- Composition emerges from relatively simple dynamics
- Subtle composition between electronic and symmetry energy

$$E/A_{\text{tot}} = M(N, Z)/A + \frac{3}{4} Y_e^{4/3} k_F + \text{lattice}$$

- Precision mass measurements of exotic nuclei is essential
- Both for neutron-star crusts and r-process nucleosynthesis

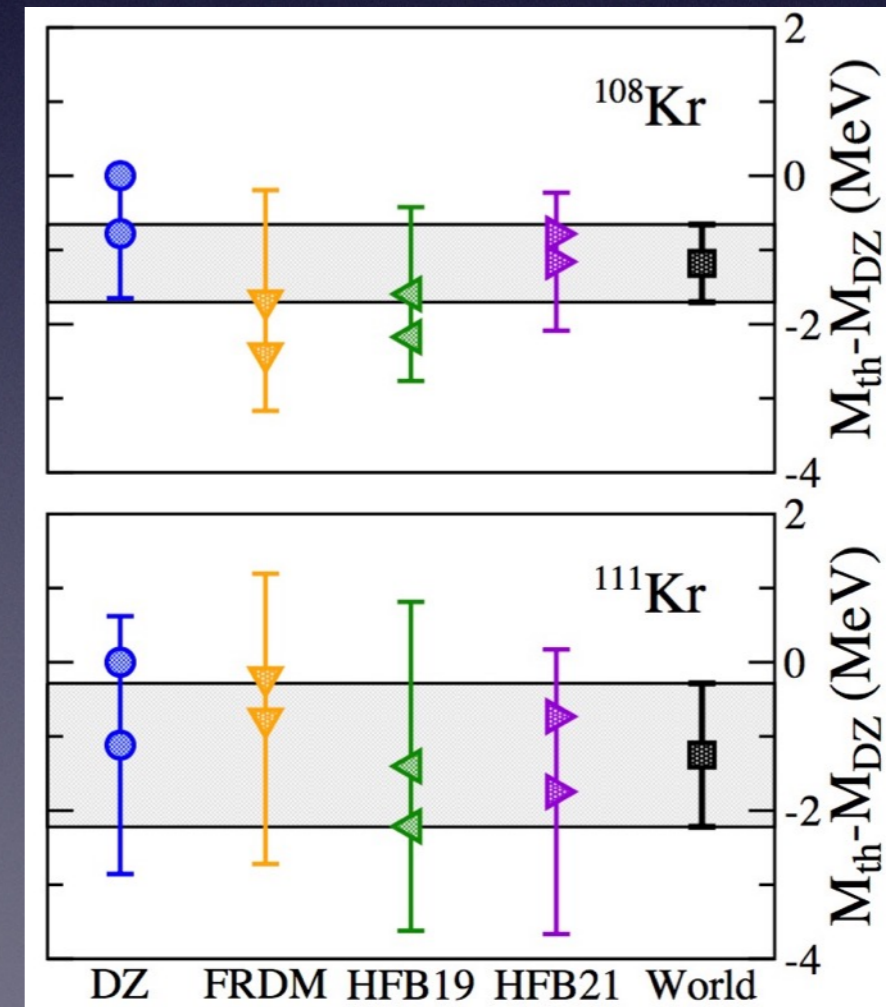
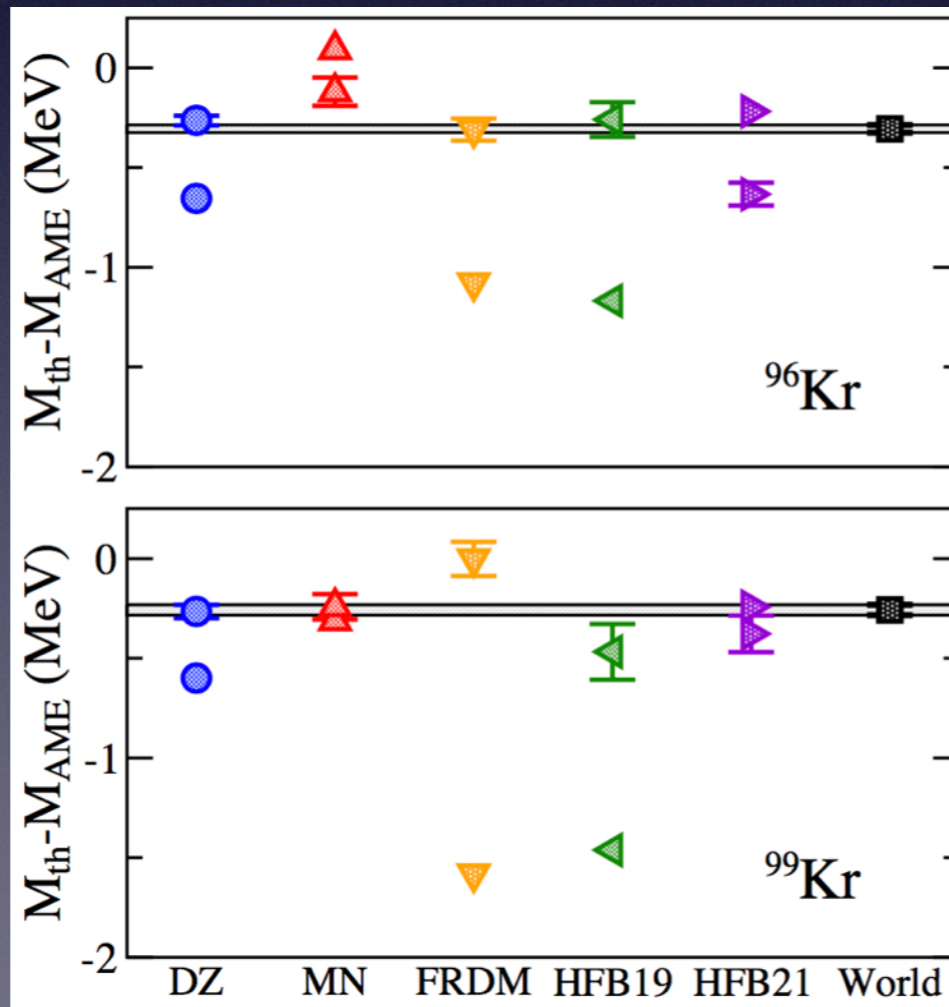
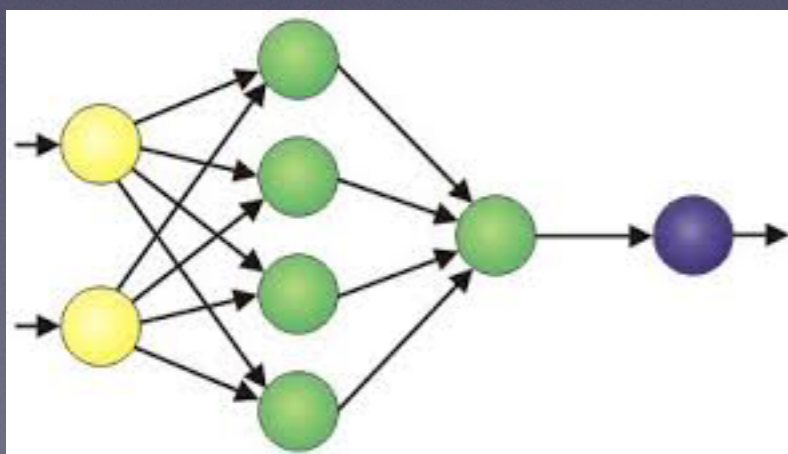
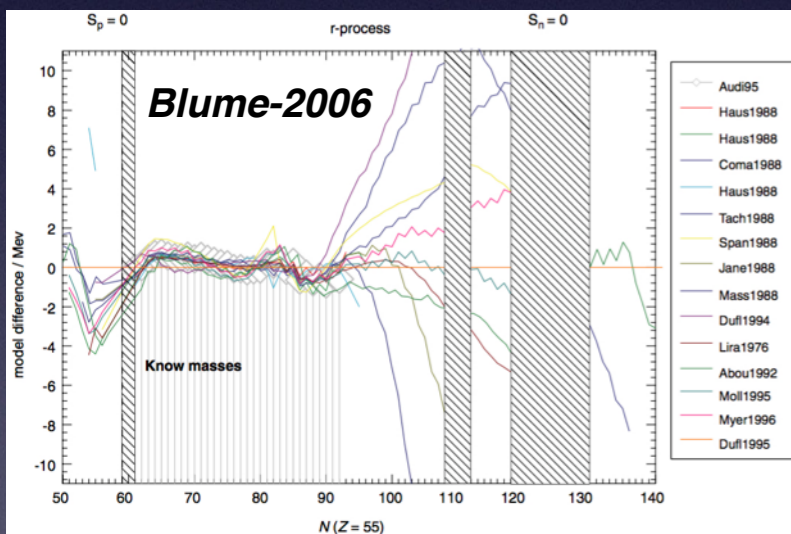


# DFT meets BNN

- Use DFT to predict nuclear masses
- Train BNN by focusing on residuals

$$M(N, Z) = M_{DFT}(N, Z) + \delta M_{BNN}(N, Z)$$

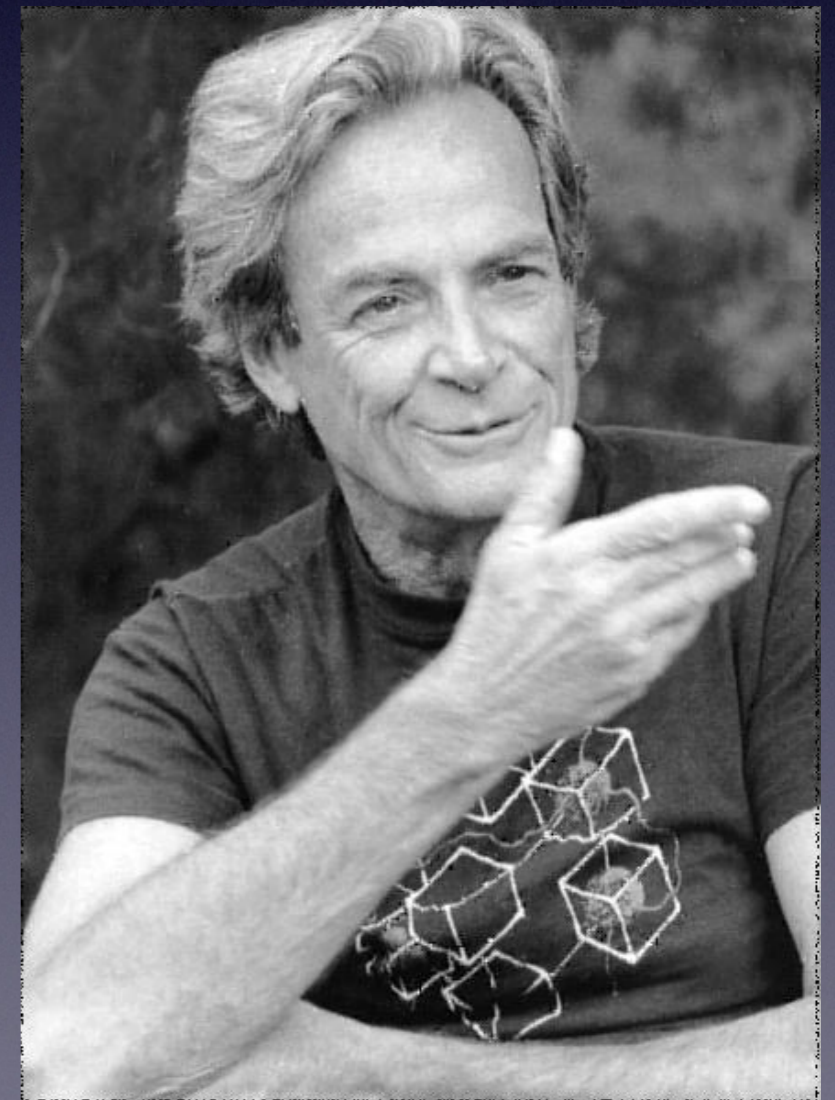
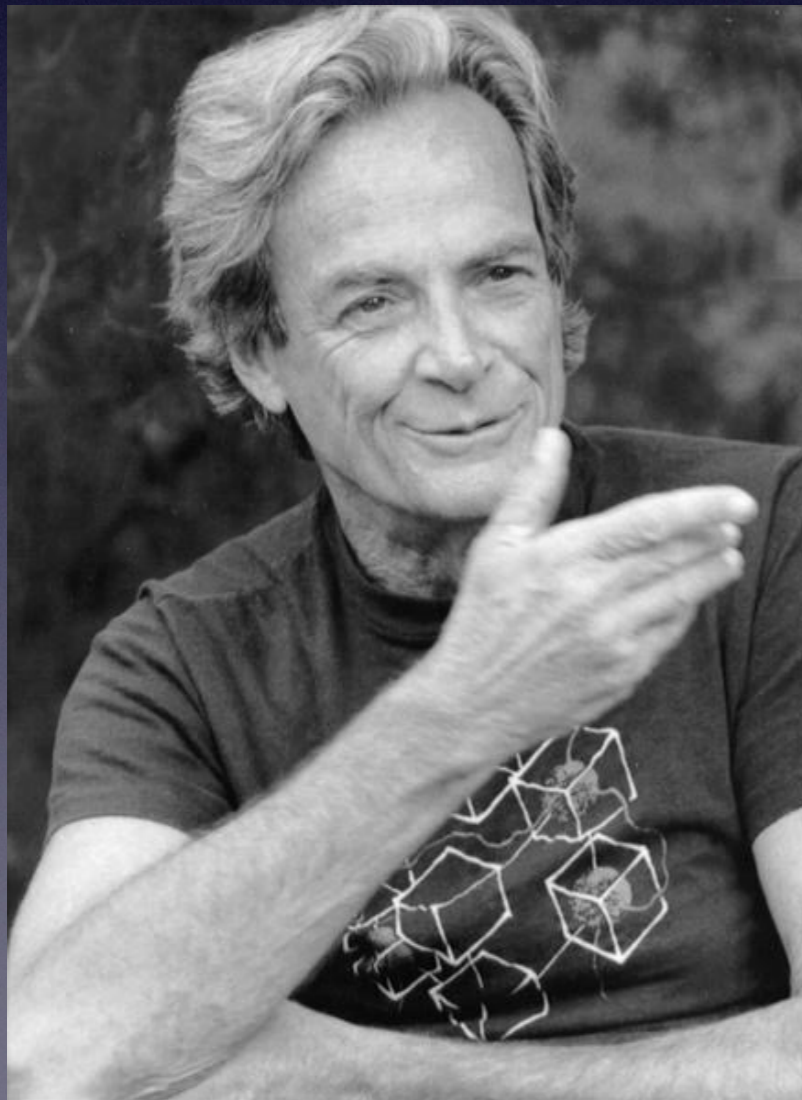
- Systematic scattering greatly reduced
- Predictions supplemented by theoretical errors



# Image Reconstructions meets BNN

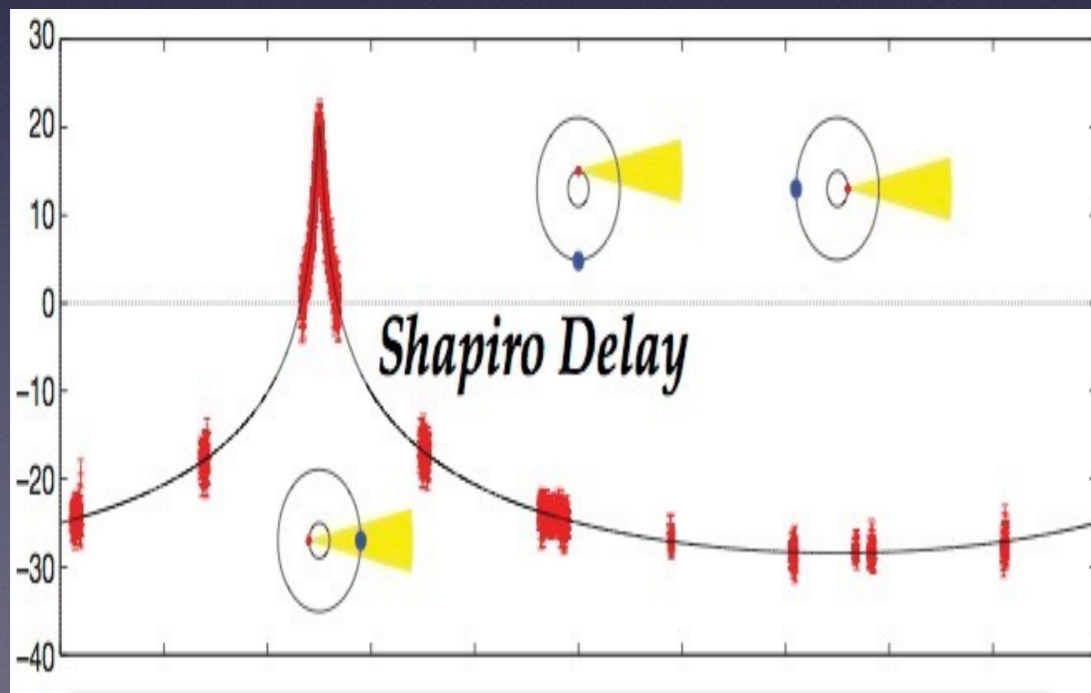
- Nature provides precise image of the world
- Models (DFT) aim to reproduce such image
- Image reconstruction (BNN) provides fine tuning

Image reconstructed using Garvey-Kelson “Mass Relations”

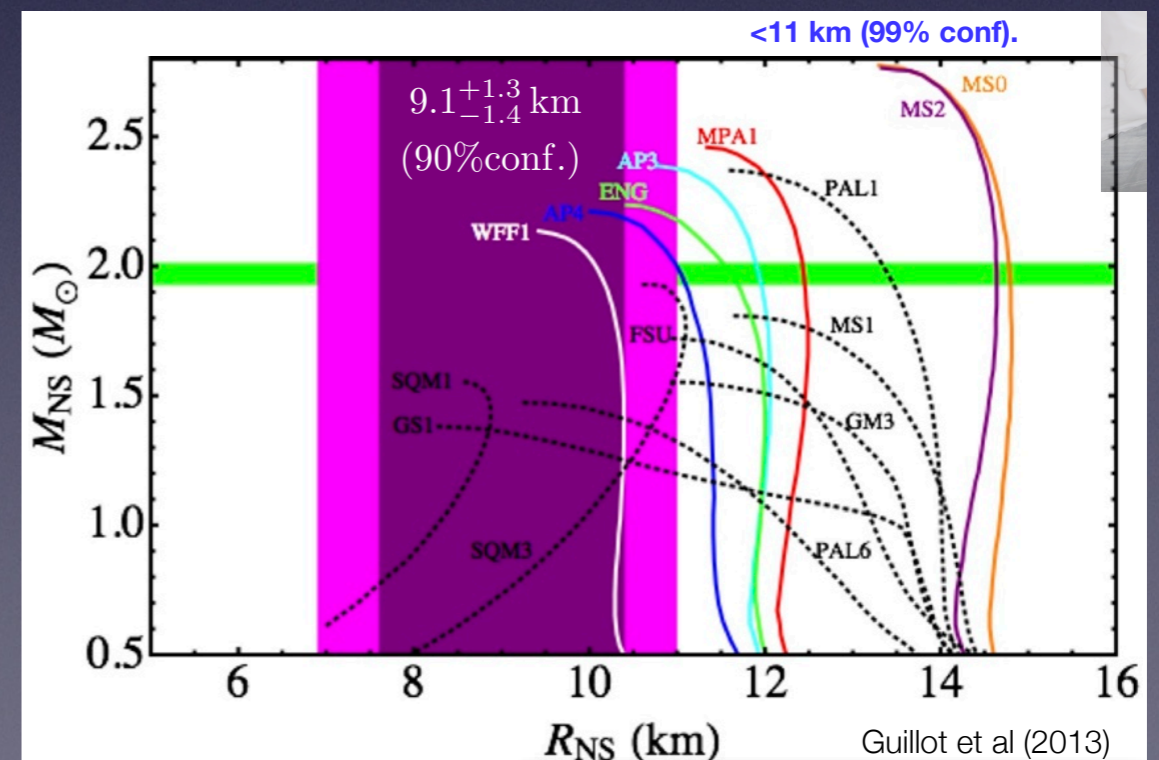


# Addressing Future Challenges

- Same dynamical origin to neutron skin and NS radius
  - Same pressure pushes against surface tension and gravity!
  - Correlation involves quantities differing by 18 orders of magnitude!
  - NS radius may be constrained in the laboratory (PREX-II, CREX, ...)
- 
- However, a significant tension has recently emerged!
  - Stunning observations have established the existence of massive NS
  - Recent observations has suggested that NS have small radii
  - Extremely difficult to reconcile both; perhaps evidence of a phase transition?



Time delay due to NS radiation dipping into gravitational well of WD!



WFF1 violates causality!

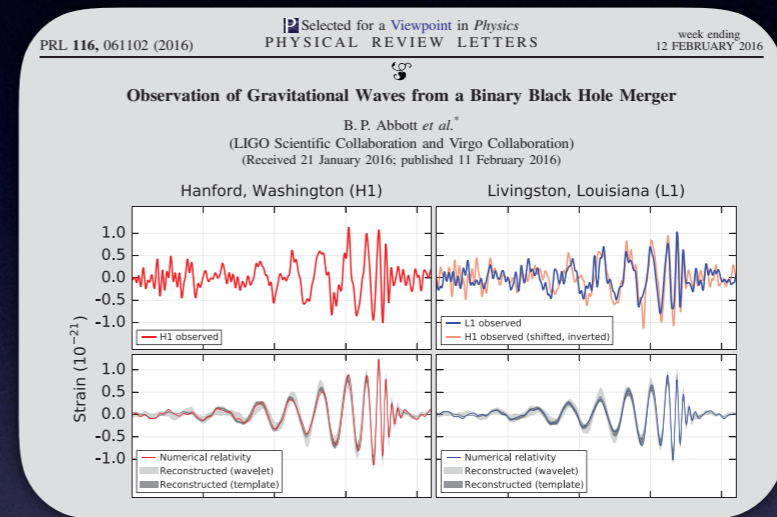


"We have detected gravitational waves. We did it"

David Reitze, February 11, 2016

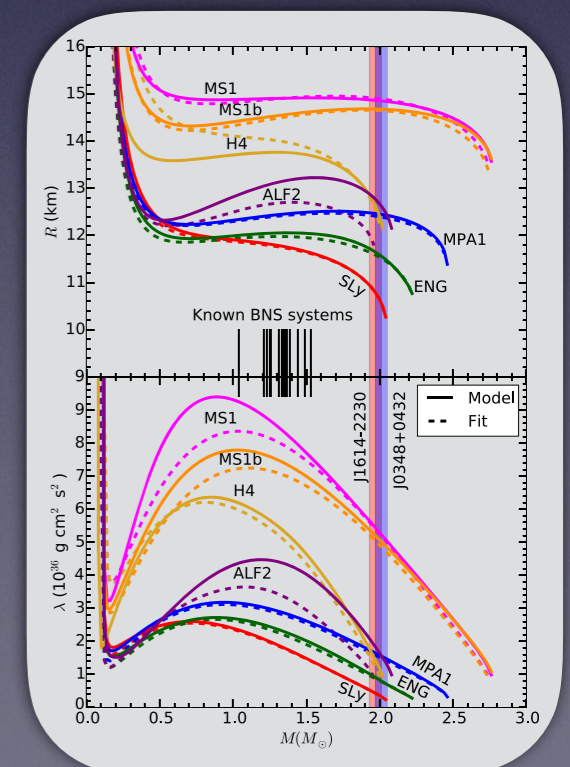
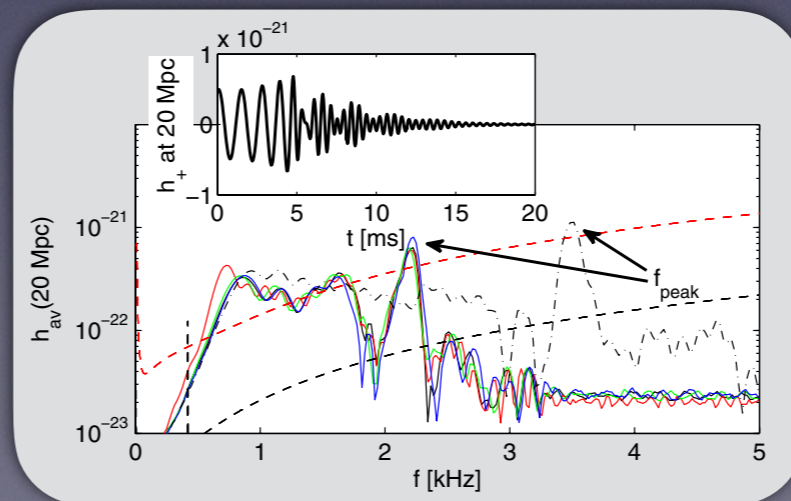
# The dawn of gravitational wave astronomy

- Initial black hole masses are 36 and 29 solar masses
- Final black hole mass is 62 solar masses, 3 solar masses radiated in GW



# What will we learn from NS mergers?

- Tidal polarizability scales as  $R^5$  ...
- NS radius can be measured to better than 1km!



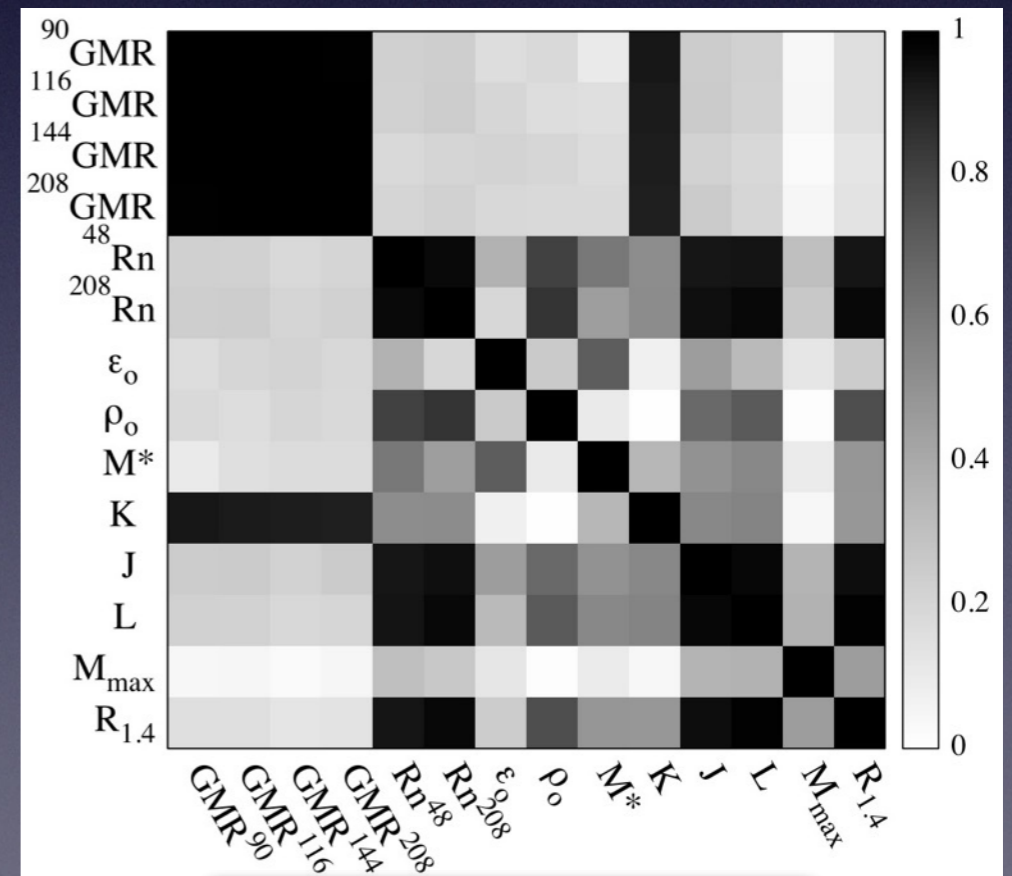
# Conclusions

PHYSICAL REVIEW A 83, 040001 (2011)  
Editorial: Uncertainty Estimates

*Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable ...*

*The train has left the station. The need for uncertainty estimates of theoretical models has been recognized in the nuclear physics community. So the question is not whether to do it or not, but how to do it best.*

(from a dedicated volume to uncertainty quantification in nuclear physics on JPG; 11/10/2015)



Wei-Chia Chen and JP  
PRC 90, 044305 (2014)