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Sterile Neutrinos as Dark Matter

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The simplest model that can accommodate a viable nonbaryonic dark matter candidate is the standard electroweak theory with the addition of right-handed (sterile) neutrinos. We consider a single generation of neutrinos with a Dirac mass μ and a Majorana mass M for the right-handed component. If $M \gg \mu$ (standard hot dark matter corresponds to $M=0$), then sterile neutrinos are produced via oscillations in the early Universe with energy density independent of M. However, M is crucial in determining the large scale structure of the Universe; for $M \sim 100$ eV, sterile neutrinos make an excellent warm dark matter candidate.

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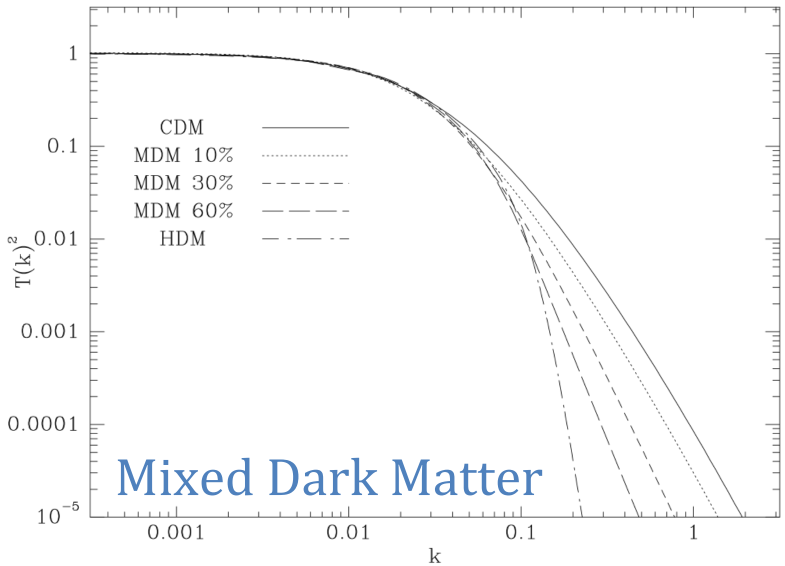
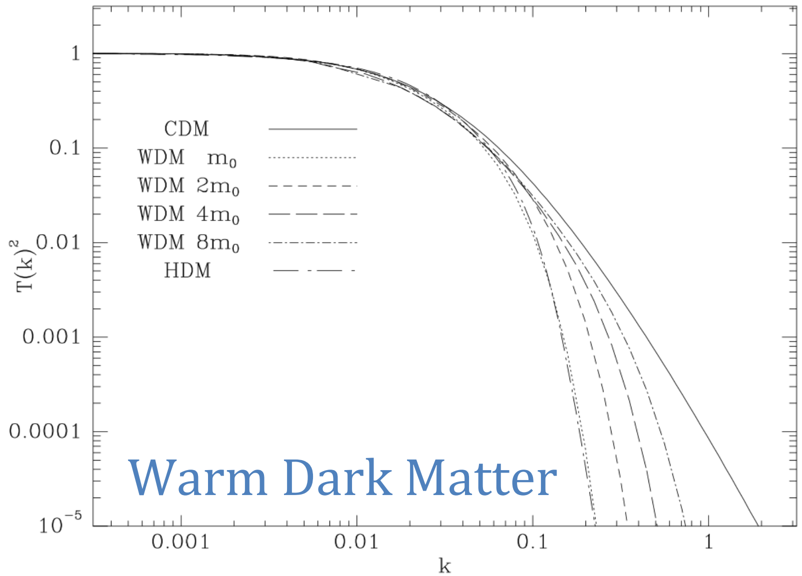
Limitations of CDM

- predicts far more low-mass halos than observed
  - deficit of power on $\leq 10h^{-1}Mpc$ scales
  - excess of power on $\geq 30h^{-1}Mpc$ scales

- simulated halo cores are more cuspy than those observed

Colombi, Dodelson & Widrow; **Astroph.J. 458**, 1996





Leptons

$$SU(2)\times U(1)$$

SM	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L$	$\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L$	$\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$	doublets
SM	e_R	μ_R	τ_R	singlets
νMSM	ν_{1R}	ν_{2R}	ν_{3R}	singlets

Lagrangian

$$L=\mu\left(\frac{\phi}{\upsilon}\right)\overline{\nu_L}\nu_R+M\nu_R\nu_R+\ldots$$

Dirac Majorana

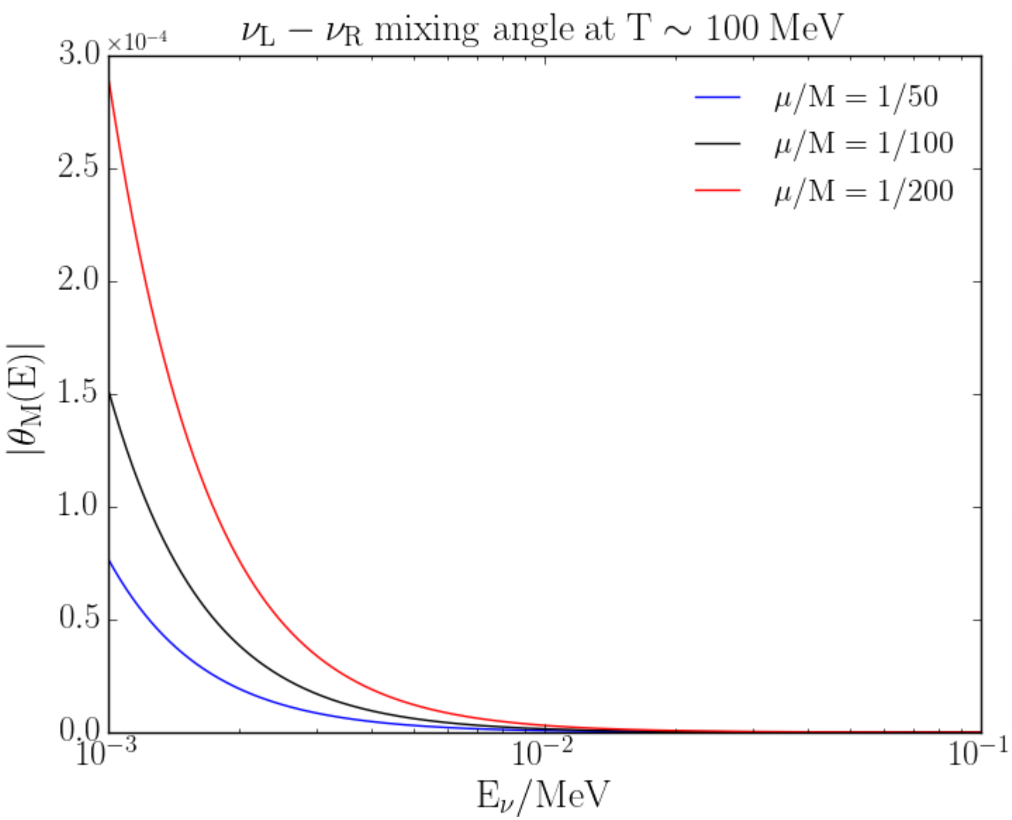
Seesaw

$$\begin{pmatrix}|\nu_A\rangle\\ |\nu_S\rangle\end{pmatrix}=\begin{pmatrix}\cos(\theta_M) & \sin(\theta_M)\\ -\sin(\theta_M) & \cos(\theta_M)\end{pmatrix}\begin{pmatrix}|\nu_1\rangle\\ |\nu_2\rangle\end{pmatrix}$$

The usual HDM case, wherein the active neutrinos constitute the dark matter, corresponds to $\{\mu=92h^2$ eV, $M\ll\mu\}$ or $\{\mu^2/M=92h^2$ eV, $M\gg\mu\}$. When sterile neutrinos are the dark matter, the relevant mass is M. At tree level, ν_R couples only to ν_L and therefore the most efficient way to produce sterile neutrinos [11–13] is via oscillations $\nu_L\to\nu_R$. The probability of observing a right-handed neutrino after a time t given that one starts with a pure monoenergetic left-handed neutrino is $\sin^2 2\theta_M \sin^2 vt/L$ where θ_M is the "mixing angle," L is the oscillation length, and v is the velocity of the neutrinos. In vacuum, and with $\mu\ll M$ (seesaw model) $\theta_M=\mu/M$ and $L=4E/(M^2-\mu^2)$ where E is the energy of the neutrinos. In the early Universe, the observation time t is replaced by the interaction time for the left-handed neutrinos. Recent work [14–16] has fine-tuned this picture taking into account the effect of finite density and temperature on the mixing angle.

Boltzmann Equation

$$\underbrace{\left(\partial_t \right.}_{\text{accumulation}} \underbrace{\left. - HE\partial_E\right)}_{\text{dilution}} f_S = \underbrace{\tfrac{1}{2}\sin^2(2\theta_M)\Gamma}_{\text{production}} f_A$$



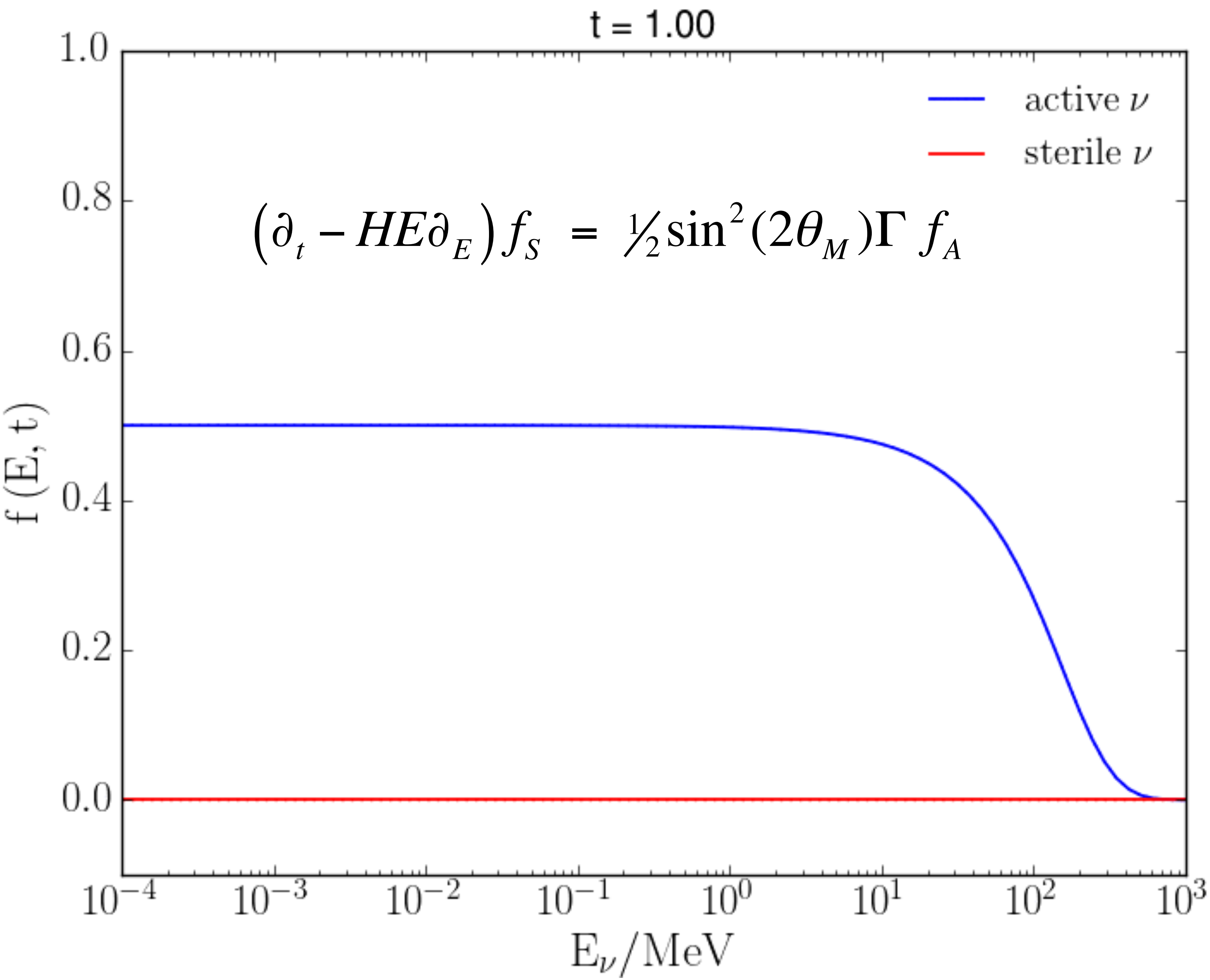
mixing angle

$$\sin^2(2\theta_M) = \frac{\mu^2}{\mu^2 + \left(\frac{c\Gamma E}{M} + \frac{M}{2}\right)^2}$$

collision rate

$$\Gamma \approx \frac{7\pi}{24} G_{Fermi}^2 T^4 E$$

$$\Gamma = \sigma n v$$



$$\left(\partial_t - HE\partial_E\right) f_S = \tfrac{1}{2}\sin^2(2\theta_M)\Gamma\, f_A$$

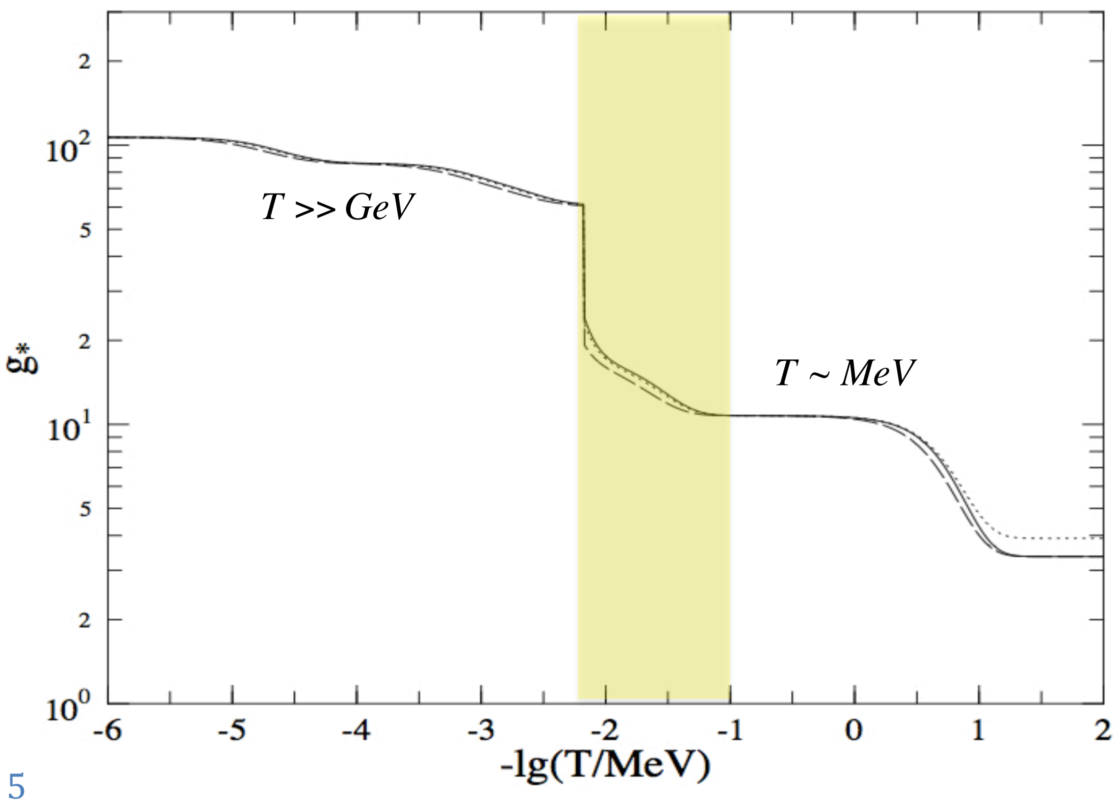
$$\frac{d\, n_S/n_A}{d\ln a} = \frac{\gamma}{H} + \frac{n_S}{n_A}\frac{d\ln g_*}{d\ln a}$$

number of ν_S in units of ν_A
produced in every dex of T

effective massless d.o.f.

$$\rho(T) = \frac{\pi^2}{30} g_*(T)\, T^4$$

$$g_* = g_{boson} + \frac{7}{8} g_{fermion}$$



Background entropy density
(adiabatic expansion)

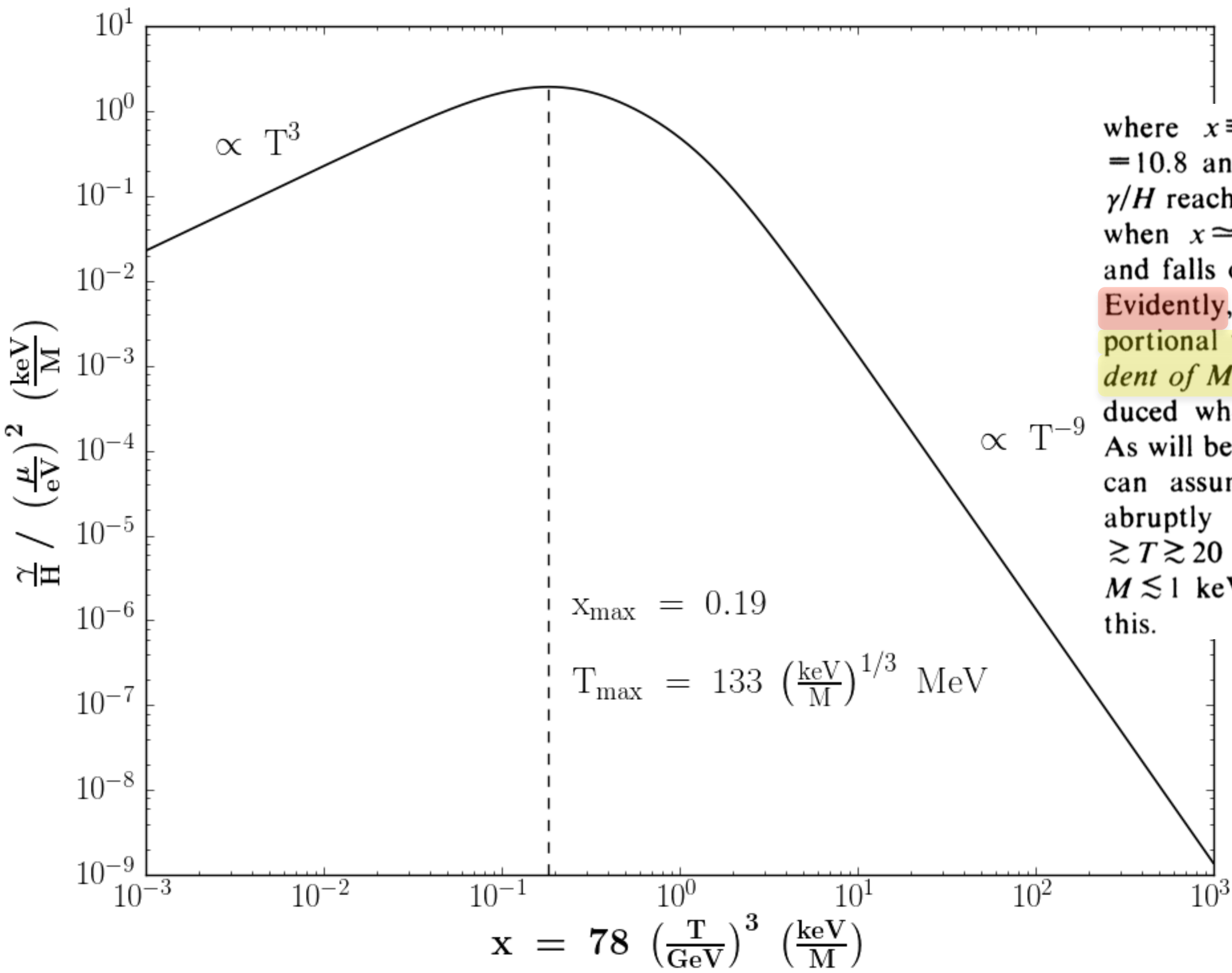
$$s = \frac{2\pi^2}{45} g_*\, T_\gamma^3 \propto a^{-3}$$

$$g_*\, T_\gamma^3 a^3 \qquad \text{cst}$$

$$\frac{d n_S/n_A}{d\ln a} = \frac{\gamma}{H} + \cancel{\frac{n_S}{n_A}\frac{d\ln g_*}{d\ln a}}$$

$$\frac{\gamma}{H}\left(\frac{M}{\mu^2}\right) \propto \int_0^\infty \frac{x\,y^3\,dy}{\left(e^y+1\right)\left(1+x^2y^2\right)^2}$$

$$x \propto T^3/M$$



where $x \equiv 78[T/(1\ \text{GeV})]^3[(1\ \text{keV})/M]$. Taking $g_* = 10.8$ and doing the integral numerically, we find that γ/H reaches a peak value of $1.9[\mu/(1\ \text{eV})]^2[(1\ \text{keV})/M]$ when $x \simeq 0.19$ or $T = T_{max} \simeq 133[M/(1\ \text{keV})]^{1/3}$ MeV and falls off as T^3 for $T \ll T_{max}$ and T^{-9} for $T \gg T_{max}$. Evidently, the number density in sterile neutrinos is proportional to M^{-1} so that the energy density is *independent of M*. Note also that most of the neutrinos are produced when the Universe has a temperature $T \simeq T_{max}$. As will be discussed below, our calculations simplify if we can assume that g_* is constant. Since g_* changes abruptly at $T \simeq 200$ MeV and varies slowly for $200 \gtrsim T \gtrsim 20$ MeV, this assumption will be pretty good for $M \lesssim 1$ keV but breakdown for masses much larger than this.

$$\cancel{\rho_S \propto M\, n_S}$$

$$\rho_{E,S} \propto Mc^2\, n_{E,S}$$

$$\frac{f_S}{f_A}\left(\frac{M}{\mu^2}\right) \propto \underbrace{\int_0^\infty \frac{y\,dx}{(1+x^2y^2)^2}}_{\text{independant of E and T}} \qquad y = E/T$$

$$\frac{f_S}{f_A} = \frac{6}{\sqrt{g_*}}\left(\frac{\mu}{eV}\right)^2\left(\frac{keV}{M}\right)$$

$$\Omega_S = \frac{f_S}{f_A}\left(\frac{M}{92h^2eV}\right)$$

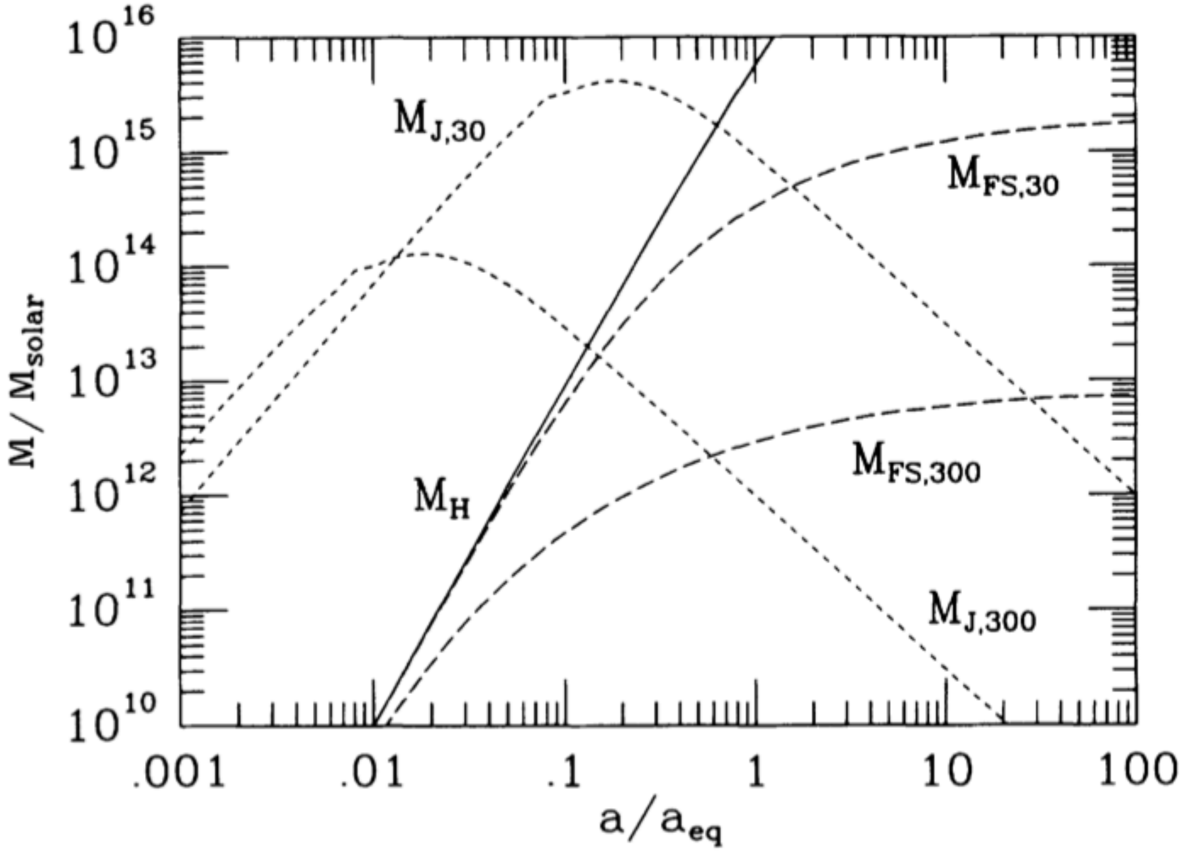
f_S has the same functional form as f_A and therefore $\Omega_S/\Omega_\nu = (M/m_\nu)(f_S/f_A)$. From the relation $m_\nu/\Omega_\nu \simeq 92h^2$ eV we find that $\Omega_S = 1$ for $\mu = 0.22h$ eV where we have again set $g_* = 10.8$. Finally, we note that the contribution of sterile neutrinos to the energy density of the Universe at the time of primordial nucleosynthesis [18] must be $\lesssim 0.5$ times the contribution of a light neutrino species if standard big bang nucleosynthesis [19] is to be believed. This in turn implies that $M \gtrsim 200h^2$ eV; that is, if sterile neutrinos are the dark matter then they are necessarily more massive than the standard HDM.

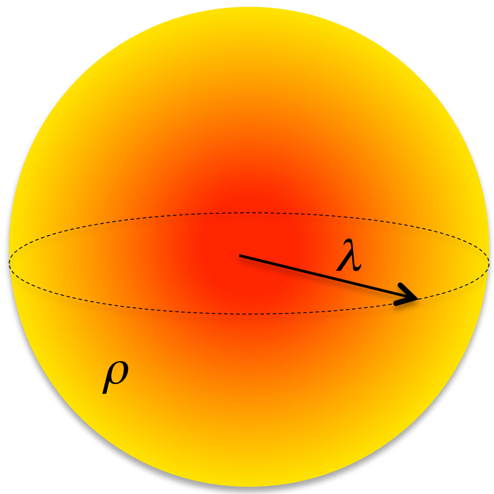
Structure Evolution

Hubble Horizon $\lambda_H(t) = a(t) \times \int_0^t \frac{dt}{a(t)} \propto t$

Free-Streaming Length Scale $\lambda_{FS}(t) = a(t) \times \int_0^t \frac{dt}{a(t)} \sqrt{\left\langle (p/E)^2 \right\rangle}$

Jeans Length Scale $\lambda_J(t) = \sqrt{\frac{\pi m_P^2 c_s^2}{\rho(t)}}$





$$M = \frac{4\pi}{3}\left(\frac{\lambda}{2}\right)^3 \rho$$

Jeans Length Scale

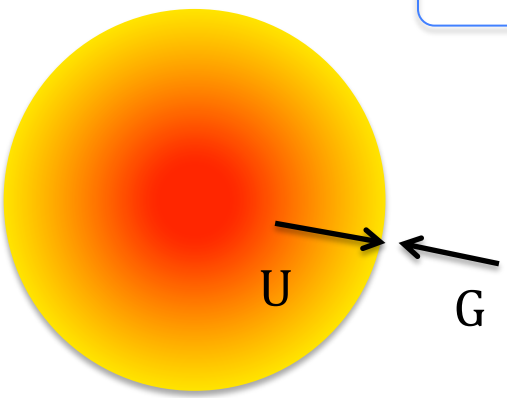
$$\lambda_J \propto \sqrt{\frac{c_s^2}{\rho}}$$

Vlasov–Poisson → moments
→ linearisation of perturbations
→ mode decomposition
→ dispersion relationship

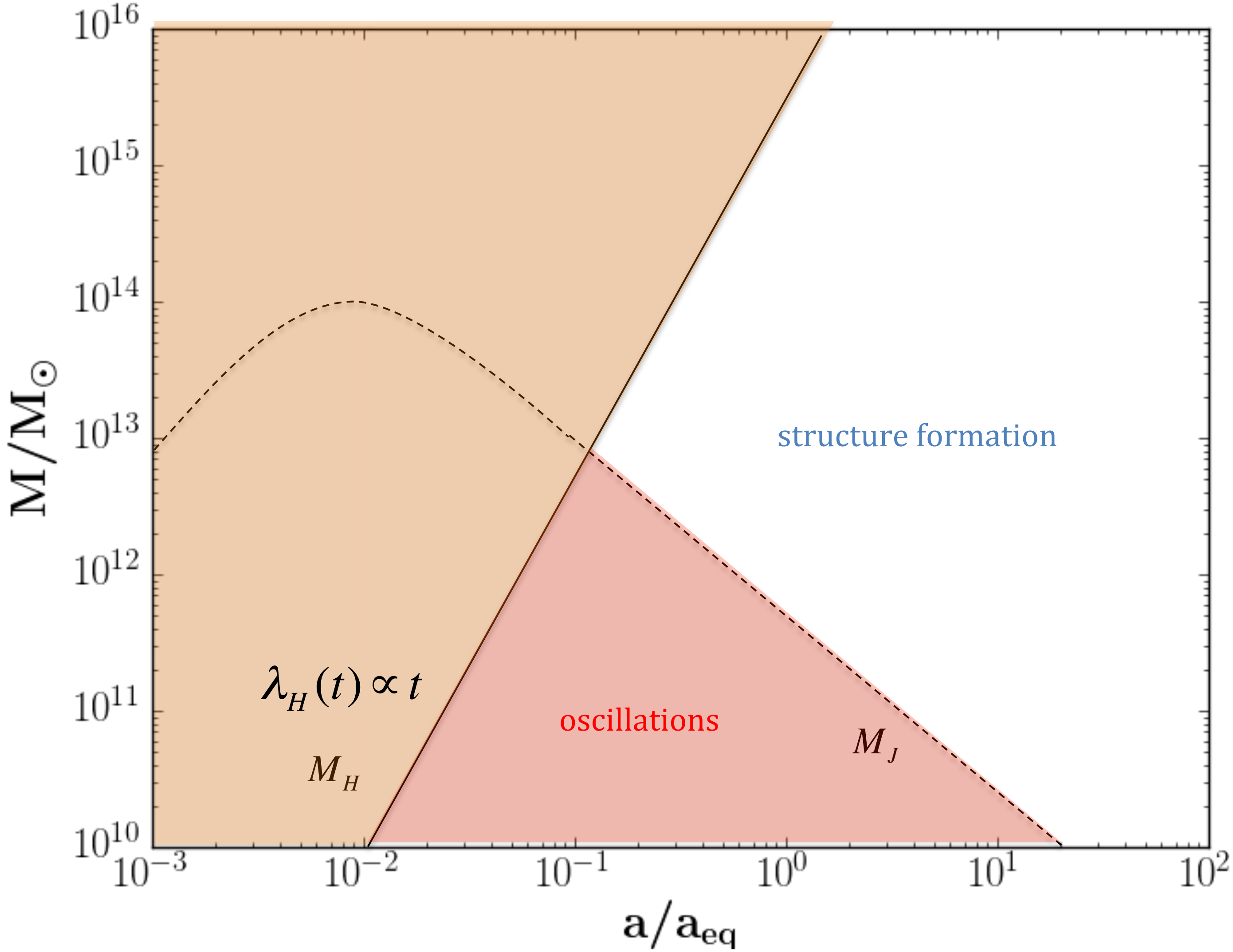
$$c_s^2 = \left.\frac{\partial P}{\partial \rho}\right|_S \Rightarrow c_s^2 \propto \rho^{\gamma-1} \Rightarrow \lambda_J \propto \rho^{\gamma-2/2}$$

$$P \propto \rho^{\gamma}$$

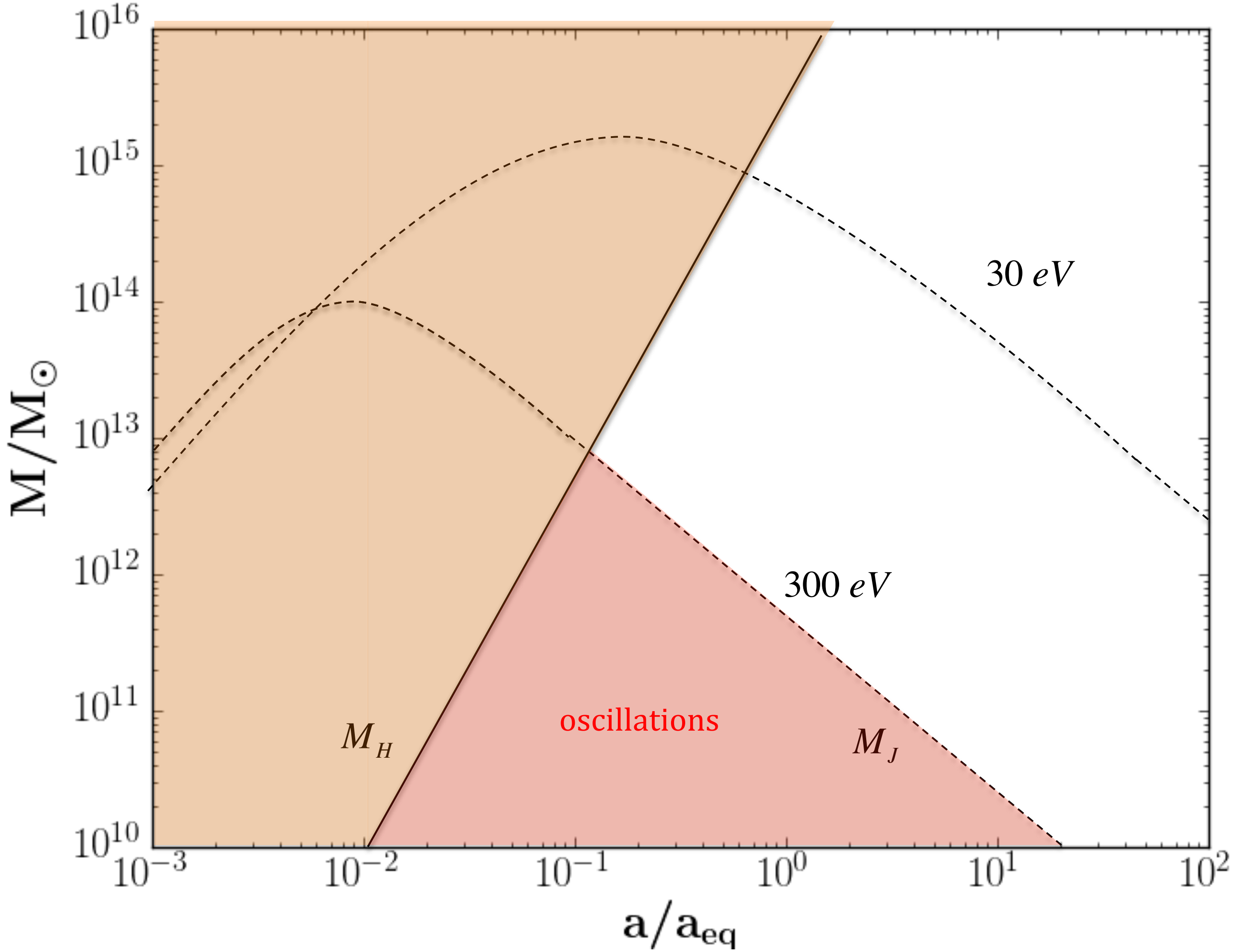
$$\begin{cases} \rho_{B,C} \propto a^{-3} \\ \rho_{\gamma,\nu} \propto a^{-4} \end{cases}$$



$$M_J \propto \lambda_J^3 \rho \propto \rho^{\frac{3}{2}\left(\gamma - 4/3\right)}$$



10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
$\mathrm{M/M_\odot}$
$\lambda_H(t) \propto t$
M_H
oscillations
M_J
structure formation
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
$\mathrm{a/a_{eq}}$



10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
$M/M_\odot$
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
a/a_{eq}
30 eV
300 eV
M_H
oscillations
M_J

Free-Streaming

$$\lambda_{FS}(t) = a(t) \times \int_0^t \frac{dt}{a(t)} \sqrt{\left\langle (p/E)^2 \right\rangle}$$

$$\left(\frac{p}{E}\right)^2 = \left(\frac{v}{c^2}\right)^2 \quad \Rightarrow \quad \sqrt{\left\langle (p/E)^2 \right\rangle} = \frac{1}{c^2}\sqrt{\frac{T}{m}} \longrightarrow T \propto a^{-1}$$

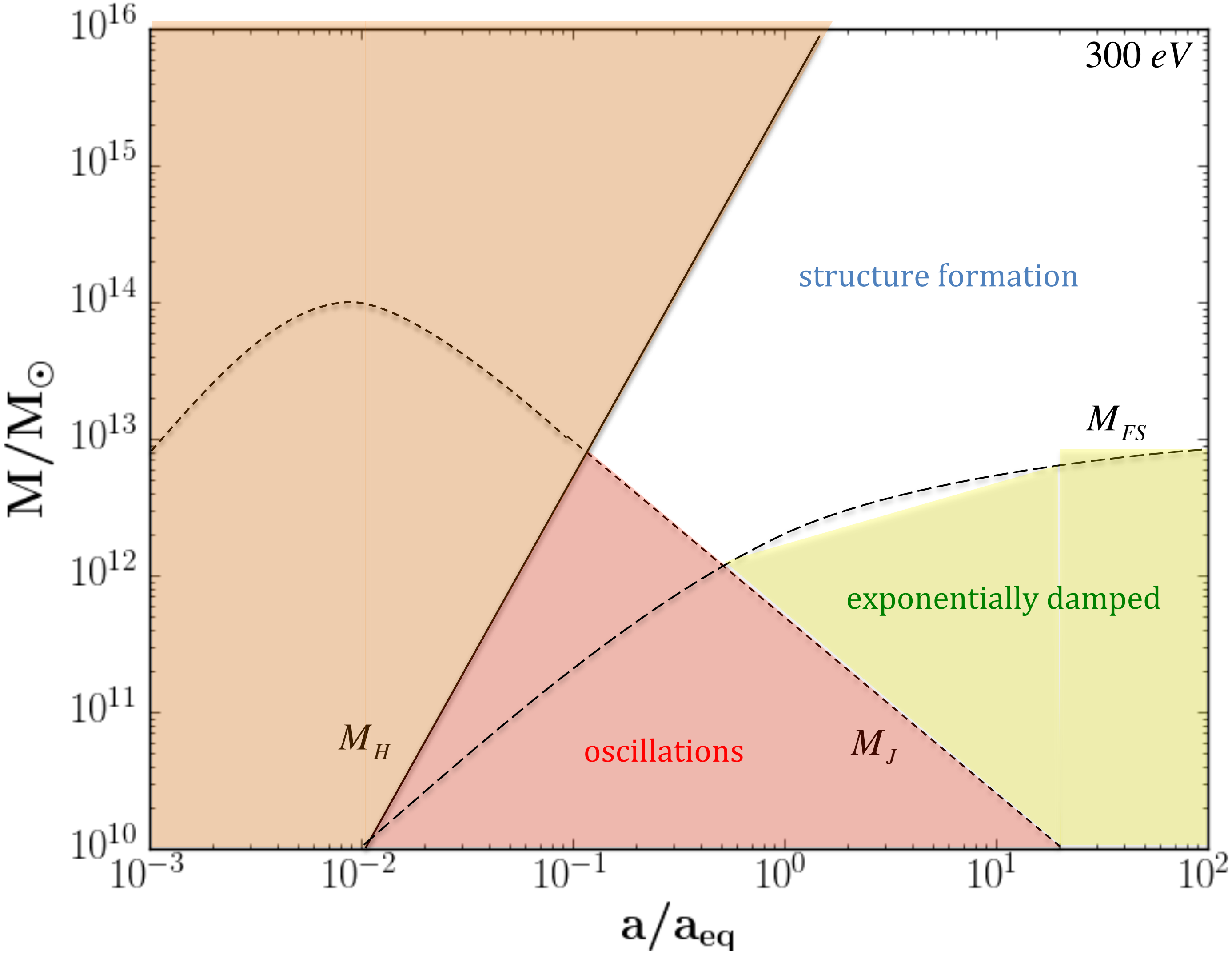
cosmological scale factor $a(t) \propto t^n$

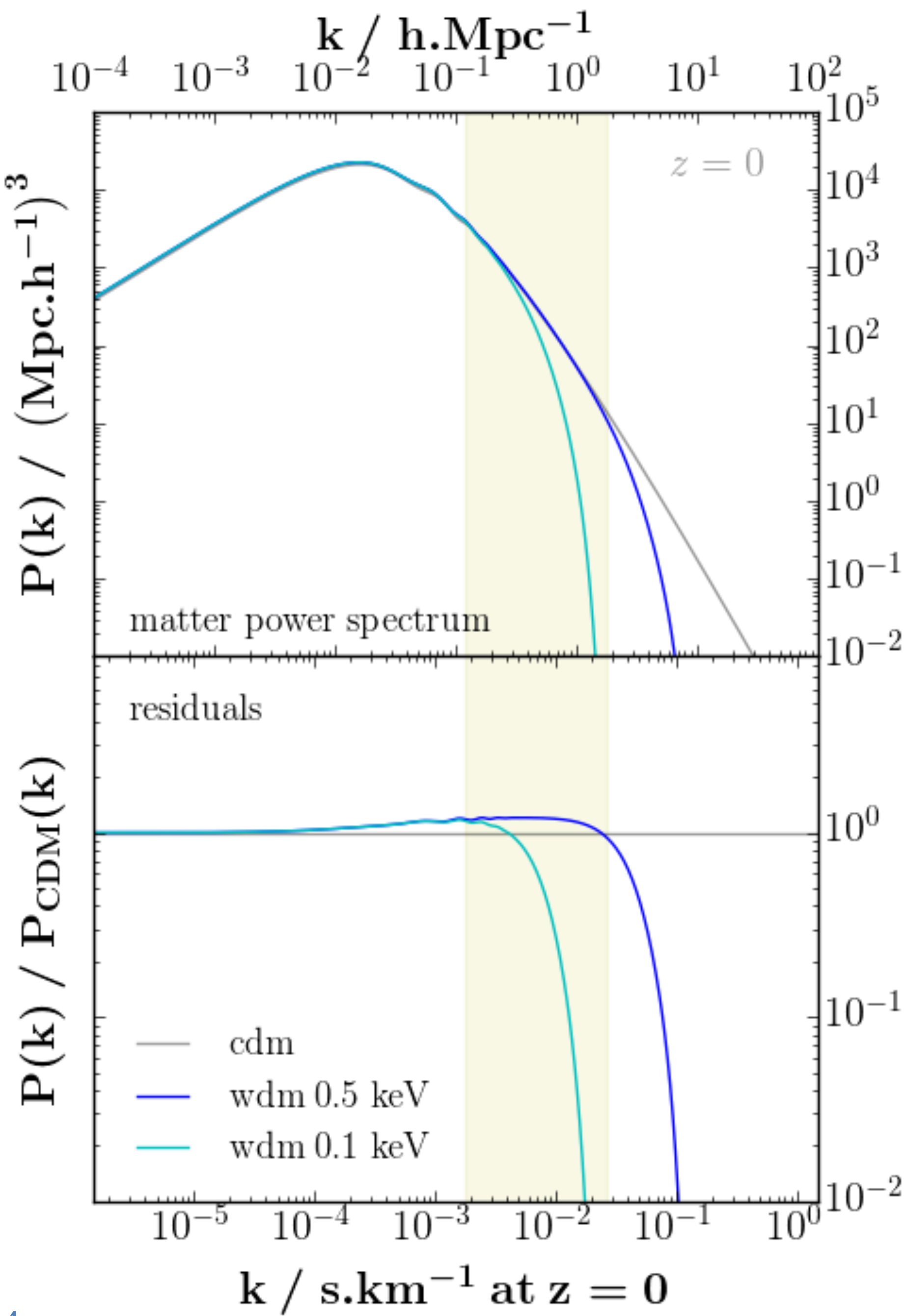
$$n = \begin{cases} 1/2 & \text{radiation – dominated} \\ 2/3 & \text{matter – dominated} \end{cases}$$

$$M_{FS} \propto \lambda_{FS}^3 \rho \propto a^{3(1/n - 1/2)} a^{-r}$$

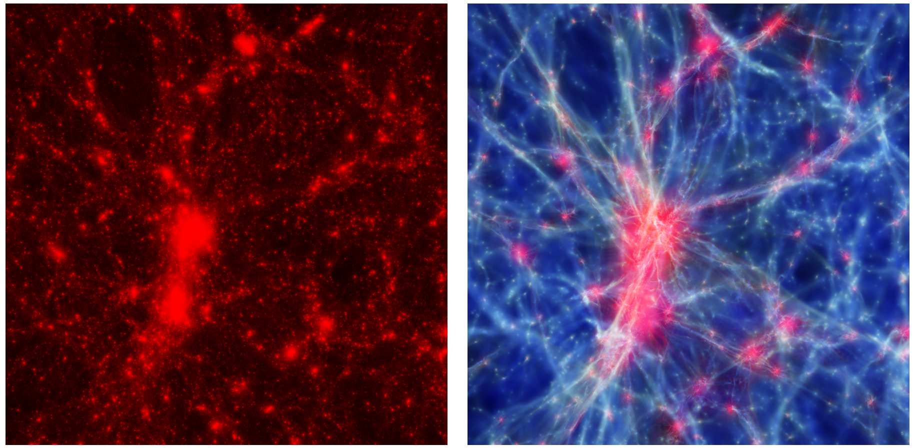
$$r = \begin{cases} 4 & \text{ultra – relativistic} \\ 3 & \text{non – relativistic} \end{cases}$$

	radiation–dominated	matter–dominated
non – relativistic	$M_{FS}^{NR} \propto a^{3/2}$	$M_{FS}^{NR} \propto cst$
ultra – relativistic	$M_{FS}^{UR} \propto a^{1/2}$	$M_{FS}^{UR} \propto a^{-1}$

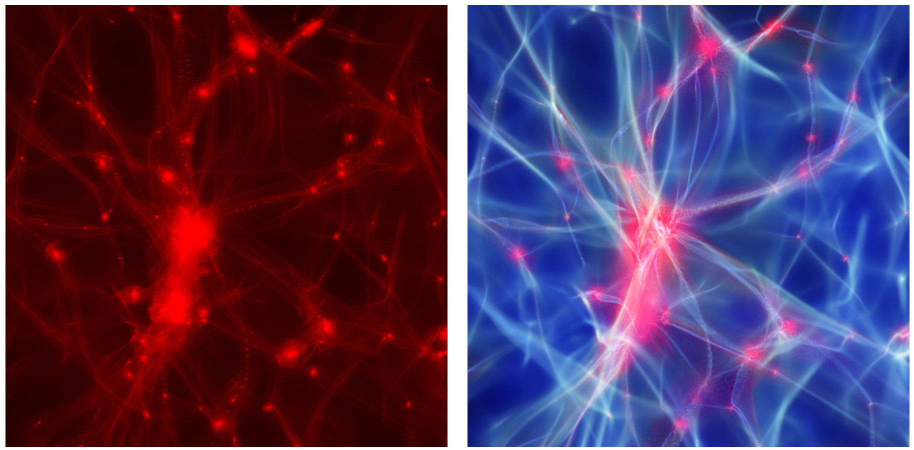

300 eV
structure formation
M_{FS}
exponentially damped
M_H
oscillations
M_J
$M/M_\odot$
a/a_{eq}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}



dark matter baryon gas

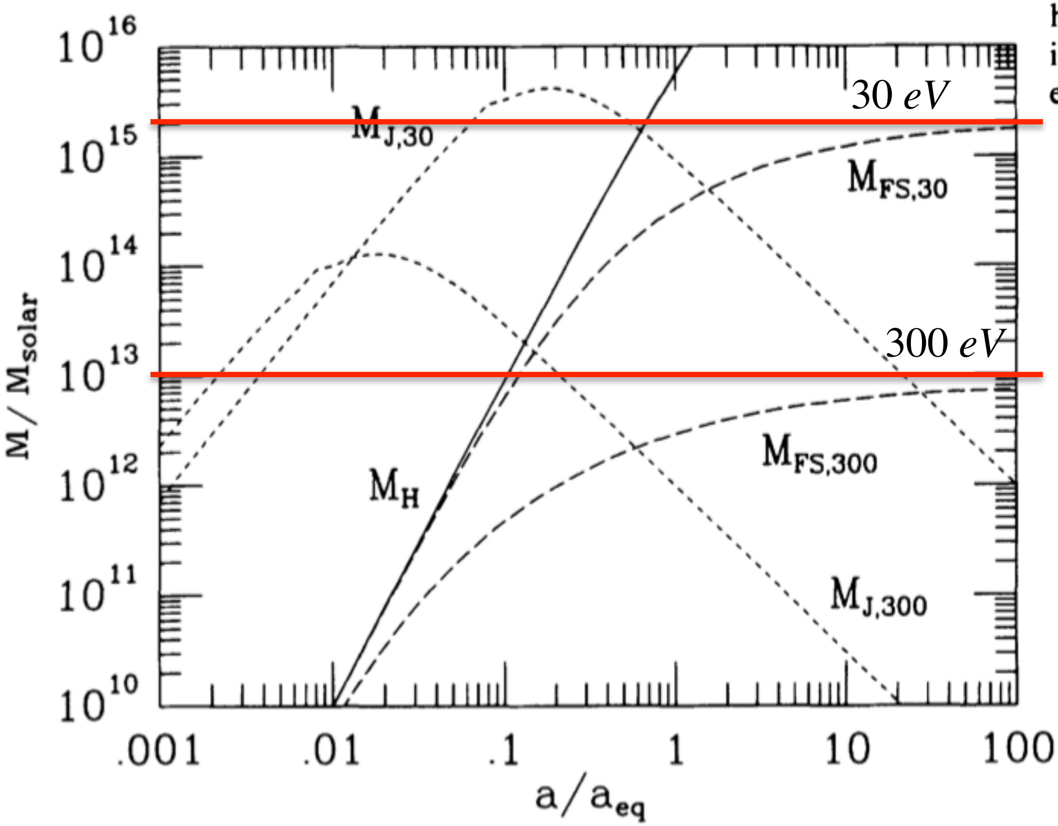


CDM

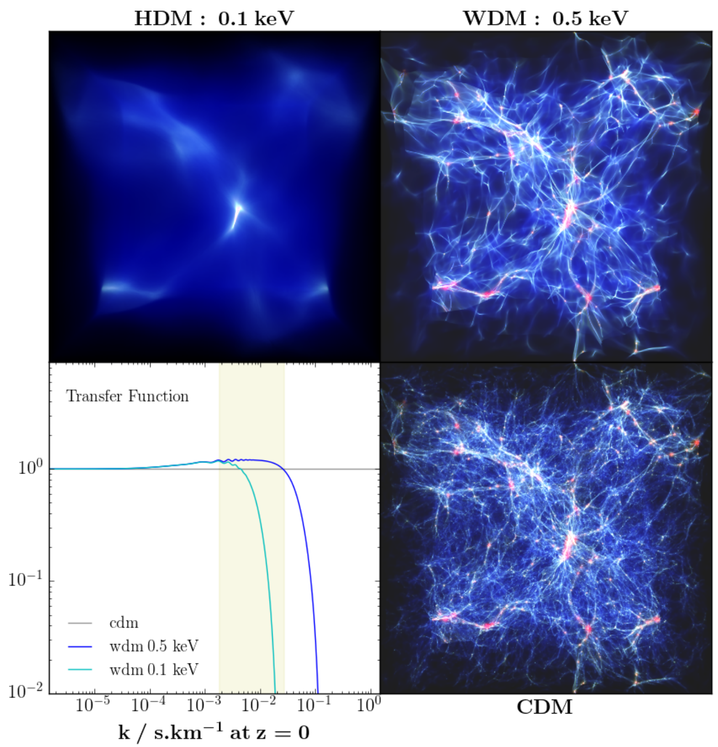


500 *eV*

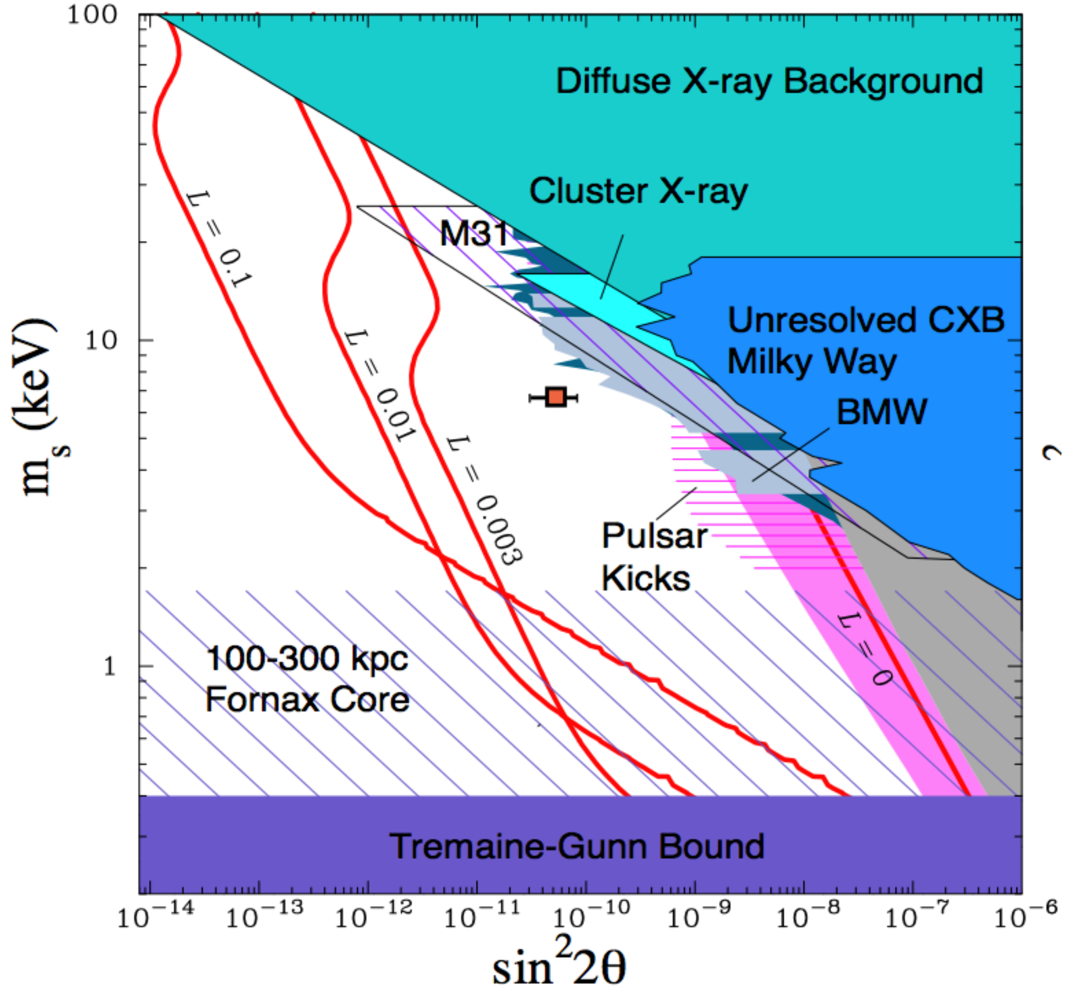
no dark matter candidate. For light neutrinos, the damping scale and the horizon scale at equality are roughly equal ($\sim 10^{15} M_\odot$), of order supercluster size. This scale is the first to go nonlinear. For sterile neutrinos, there is a large disparity between the two characteristic scales, so that perturbations with $10^{13} M_\odot \lesssim M \lesssim 10^{15} M_\odot$ are processed similarly; given an initial Harrison-Zel'dovich spectrum, they should all have the same final amplitude in linear theory. Power on scales smaller than this should be completely damped.



dark matter [20]. In particular, the pairwise velocity dispersions on scales of order 1–5 Mpc in a WDM universe are likely to be smaller than in CDM and hence more in accord with observations. There is more power in WDM than in HDM on these scales. This may not be enough: The largest challenge to warm dark matter is whether structure on galactic scales can form early enough to account for observations. On scales probed by the APM survey, WDM is a better fit either cold or hot dark matter (recall though that there is an extra degree of freedom, the mass). Another advantage WDM has over HDM is that since the neutrino mass is higher, it is possible to fit more neutrinos into a given galaxy, thus evading Tremaine-Gunn limits [21]. Finally we point out



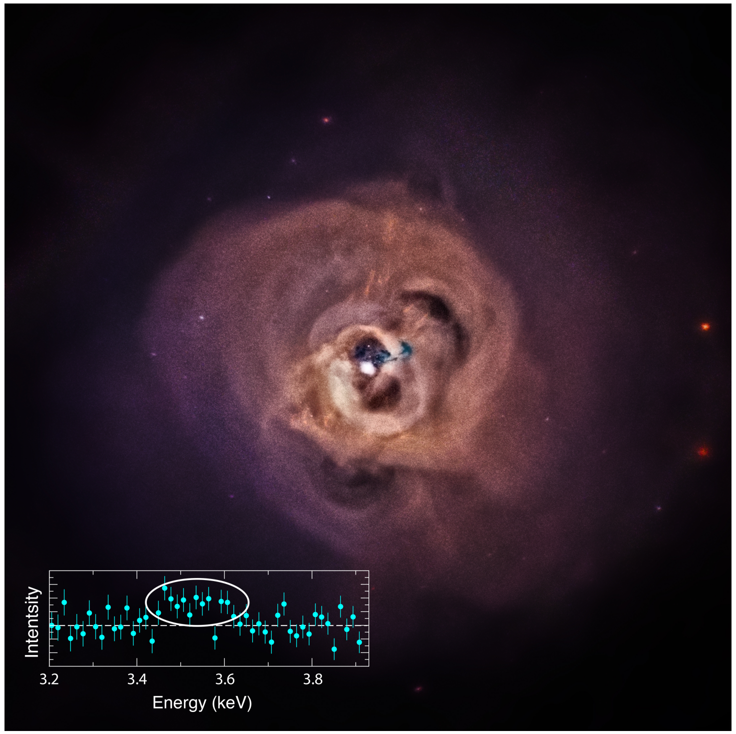
perspectives



E. Bulbul *et al.* 2014, **ApJ 789 13**
73 stacked XMM-N spectra

A. Boyarsky *et al.* 2014, **Phys. Rev. Lett. 113, 251301**
Andromeda & Perseus clusters

?



Perseus cluster
Chandra, XMM–Newton