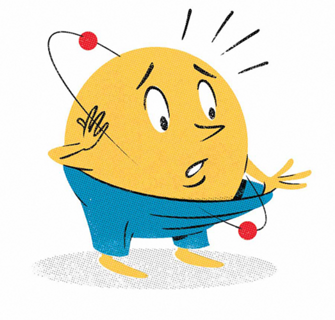


Critical Remarks on the Determination of The Proton Charge Radius

Egle Tomasi-Gustafsson

CEA, IRFU, DPhN and Université Paris-Saclay, France

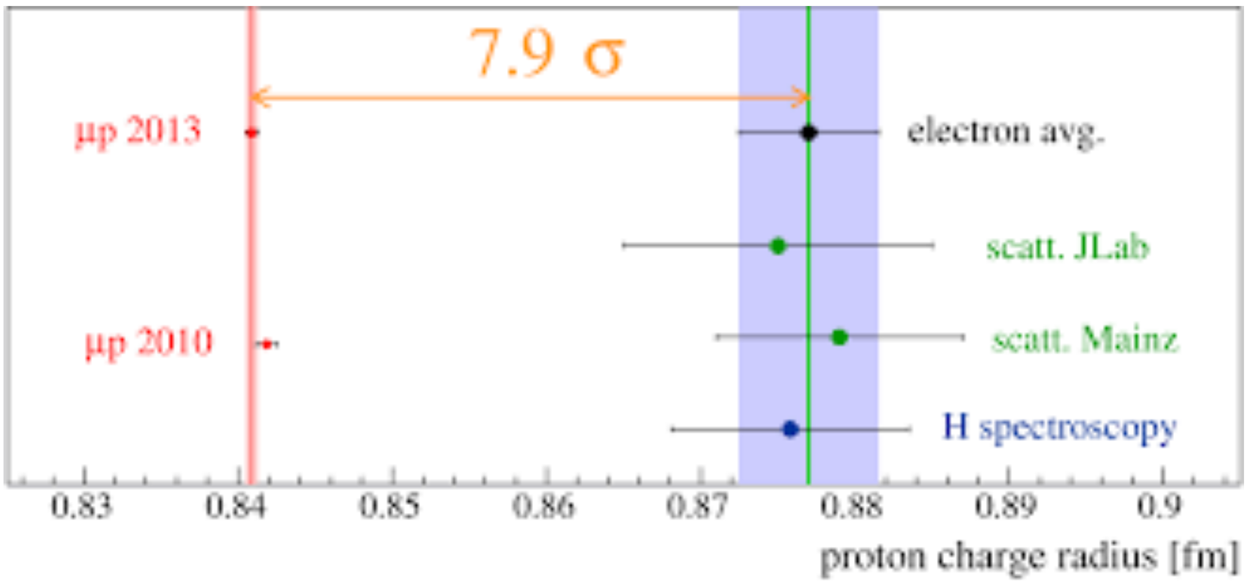
in collaboration with

G.I. Gakh, M.I. Konchatnji, N.P. Merenkov

NSC-KFTI Kharkov

Café DPhN, 15 Janvier 2018

The SIZE of the proton



Rp=0.897(18) fm

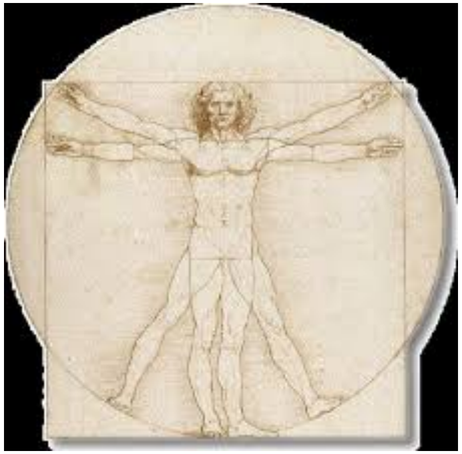
Rp=0.8768(69) fm

Rp=0.84184(67) fm (muonic H)

Rp=0.8335(95) fm (new H)

The New York Times

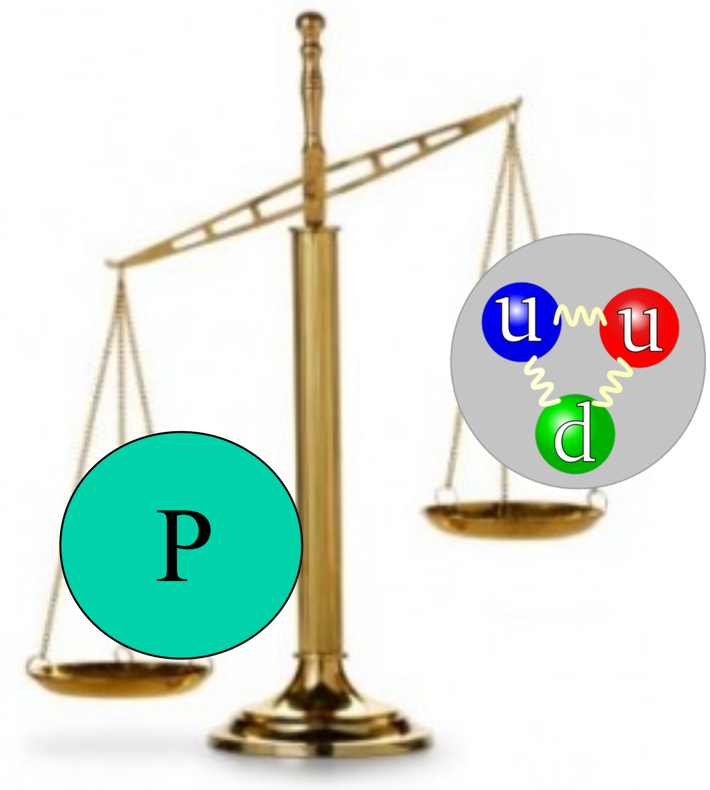
The proton

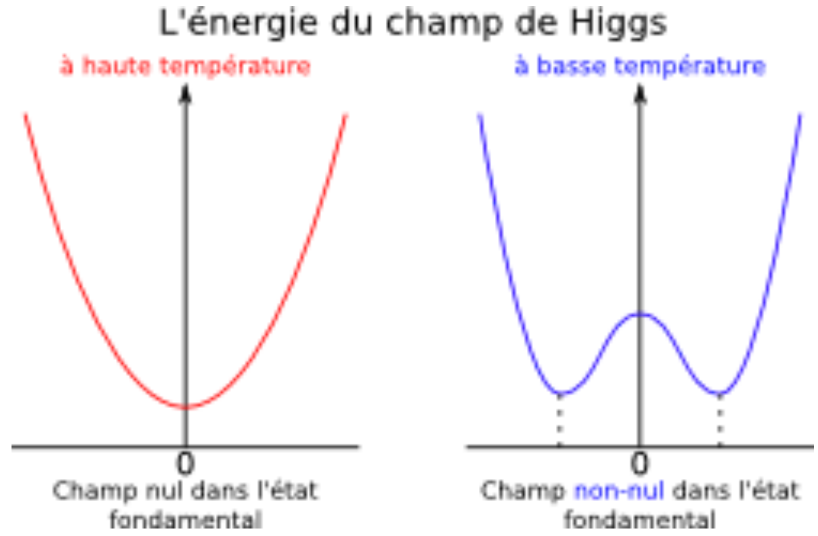


- Hadrons are 96% of visible matter
- Proton is the the most common particle in nature
- Its fundamental properties as
  - Mass
  - Spin
  - Size

  are still object of controversy

The MASS of the proton

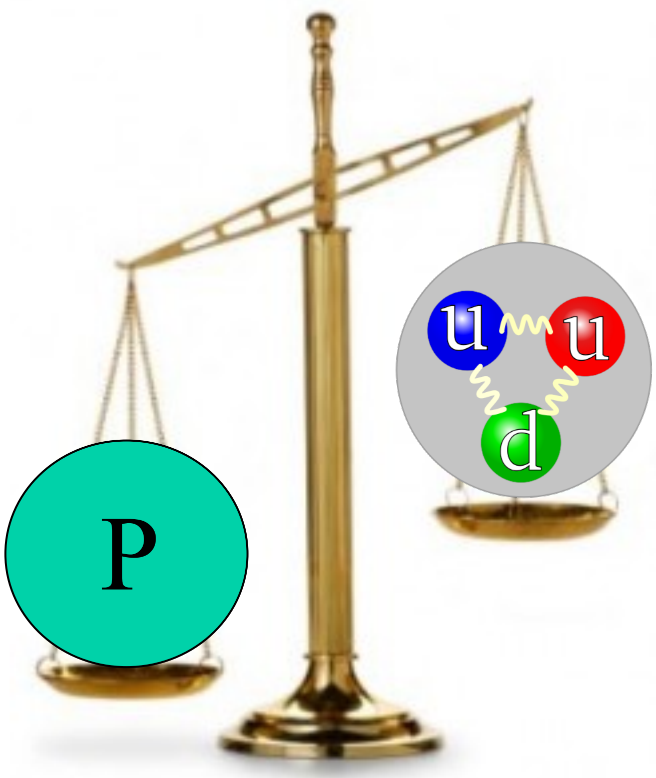




Mp=938,2720 MeV/c^2

Masses
u-quark=1.5-4 MeV/c^2
d-quark=4-8 MeV/c^2

The MASS of the proton



dynamically created by the strong interaction

Mp=938,2720 MeV/c^2

Antiproton-Proton collisions

$\bar{p}$anda

gluon-rich environment!

ATOMIC PHYSICS

International Nuclear Physics Conference 2010 (INPC2010) IOP Publishing
Journal of Physics: Conference Series **312** (2011) 032002 doi:10.1088/1742-6596/312/3/032002

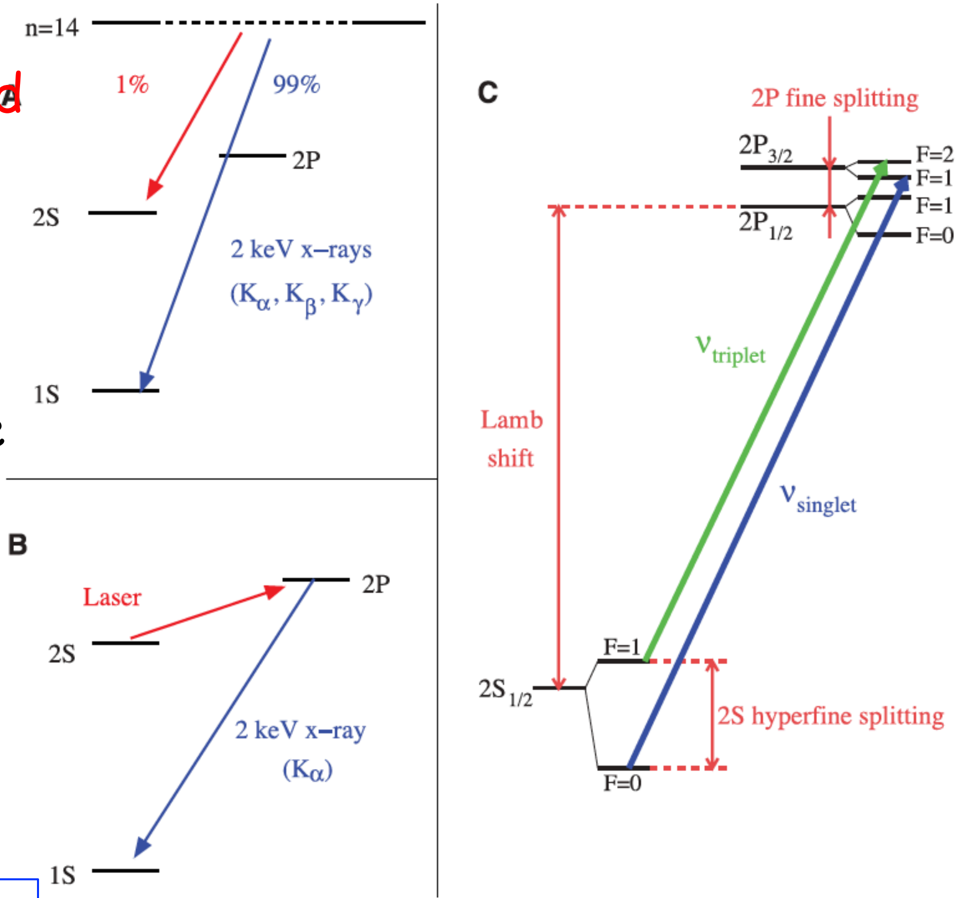
The proton radius puzzle

A Antognini[1,2], F D Amaro[3], F Biraben[4], J M R Cardoso[3], D S Covita[5], A Dax[6], S Dhawan[6], L M P Fernandes[3], A Giesen[7], T Graf[8], T W Hänsch[1,9], P Indelicato[4], L Julien[4], C-Y Kao[10], P Knowles[11], F Kottmann[2], E-O Le Bigot[4], Y-W Liu[10], J A M Lopes[3], L Ludhova[11], C M B Monteiro[3], F Mulhauser[11], T Nebel[1], F Nez[4], P Rabinowitz[12], J M F dos Santos[3], L A Schaller[11], K Schuhmann[7], C Schwob[4], D Taqqu[13], J F C A Veloso[5] and R Pohl[1]

Abstract. By means of pulsed laser spectroscopy applied to muonic hydrogen ($\mu^- p$) we have measured the $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ transition frequency to be 49881.88(76) GHz [1]. By comparing this measurement with its theoretical prediction [2, 3, 4, 5, 6, 7] based on bound-state QED we have determined a proton radius value of $r_p = 0.84184(67)$ fm. This new value differs by 5.0 standard deviations from the CODATA value of 0.8768(69) fm [8], and 3 standard deviation from the e-p scattering results of 0.897(18) fm [9]. The observed discrepancy may arise from a computational mistake of the energy levels in μp or H, or a fundamental problem in bound-state QED, an unknown effect related to the proton or the muon, or an experimental error.

Lamb shift and hyperfine splitting (1)

Negative μ beams at PSI are stopped in H_2 gas target at 1 hPa and 20°C



A) Formation of *μp atoms* in highly excited states. 1% populates the 2S state (τ=1 μs).

B) Laser excitation of 2S-2P transition

C) 2S and 2P energy levels.
ν_s and ν_p: measured transitions

$$\frac{1}{4}h\nu_s + \frac{3}{4}h\nu_t = \Delta E_L + 8.8123(2)\text{meV}$$

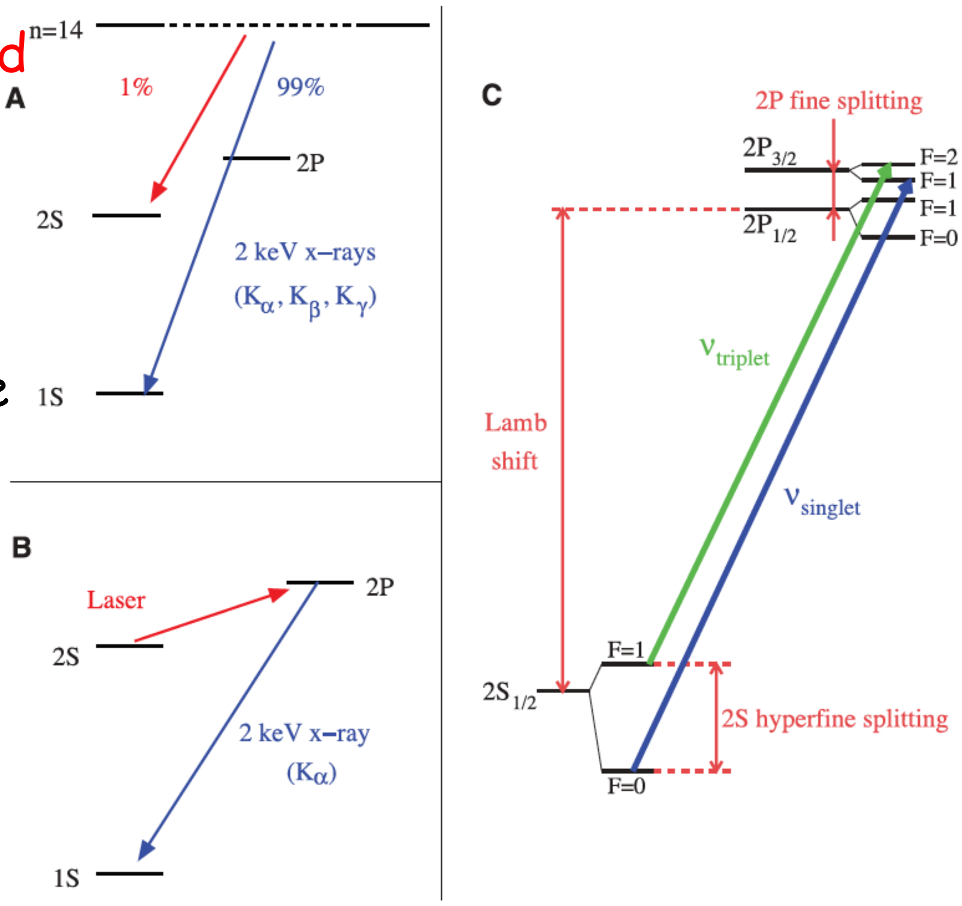
$$h\nu_s - h\nu_t = \Delta E_{HFS} - 3.2480(2)\text{meV}$$

$$\Delta E_L^{exp} = 202.3706(23)\ \text{meV}$$

$$\Delta E_{HFS}^{exp} = 22.8089(51)\ \text{meV}$$

Lamb shift and hyperfine splitting (1)

Negative μ beams at PSI are stopped in H_2 gas target at 1 hPa and 20°C



A) Formation of μp atoms in highly excited states. 1% populates the 2S state (τ=1 μs).

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C) 2S and 2P energy levels.

ν_s and ν_p: measured transitions

An electron in S state has some probability to be inside the proton.
The electric field (charge distribution) is modified by the proton size.
The ν_s and ν_p transitions are affected by the proton size (few %)

Lamb shift and hyperfine splitting

$$\Delta E_{\text{finite size}} = \frac{2\pi Z\alpha}{3} r_E^2 |\Psi(0)|^2$$

Atomic wave function at the origin

$|\Psi(0)|^2 \approx m_r^3$, $m_r(\mu p \text{ system}) \cong 186\, m_e$

H radius : 60000 x p radius

μH Bohr radius is ≈ 200 times smaller: larger sensitivity!

$$\frac{1}{4} h\nu_s + \frac{3}{4} h\nu_t = \Delta E_L + 8.8123(2)\text{meV}$$

$$h\nu_s - h\nu_t = \Delta E_{HFS} - 3.2480(2)\text{meV}$$

$$\Delta E_L^{exp} = 202.3706(23)\text{ meV}$$

$$\Delta E_{HFS}^{exp} = 22.8089(51)\text{ meV}$$

$$\Delta E_L^{th} = 206.0336(15) - 5.2275(10) r_E^2 + \Delta E_{TPE}$$

$$\Delta E_{TPE} = 0.0332(20)\ \text{meV}$$

$$r_E = 0.84087(26)^{exp}(29)^{th}\ \text{fm} = 0.84087(39)\ \text{fm}$$

International Nuclear Physics Conference 2010 (INPC2010) IOP Publishing
Journal of Physics: Conference Series **312** (2011) 032002 doi:10.1088/1742-6596/312/3/032002

The proton radius puzzle

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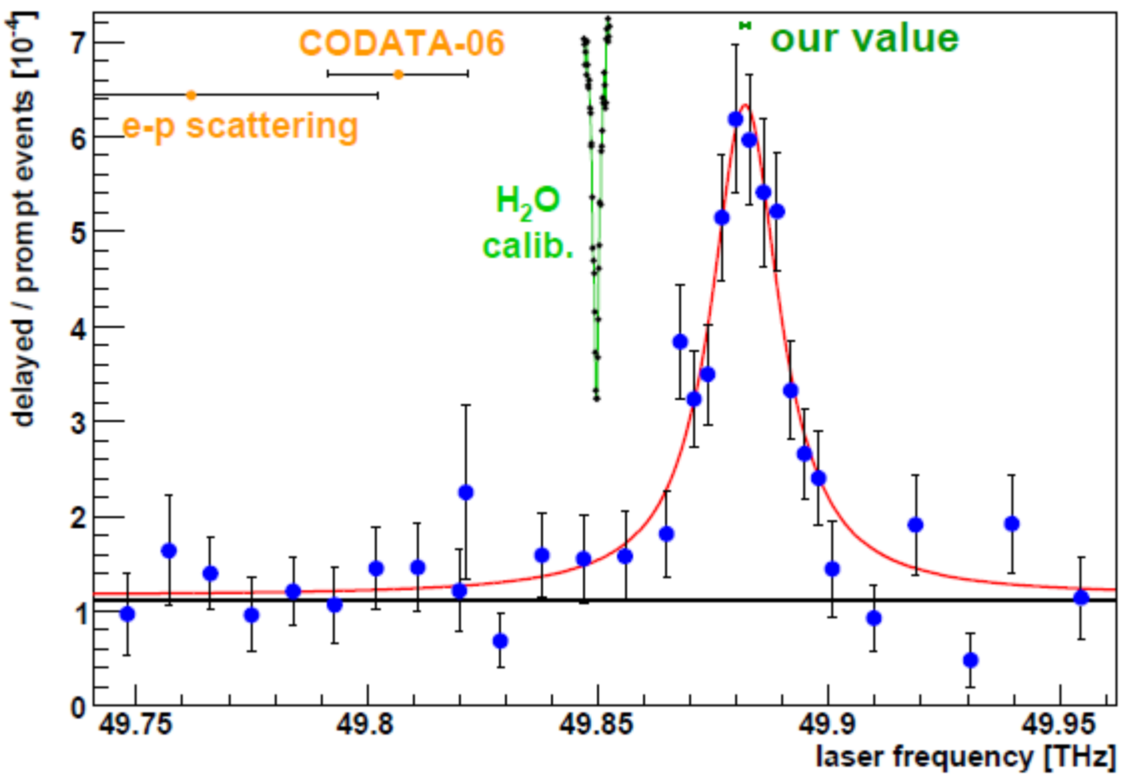
ulhauser[11],
s[3],
13,



Abstract. B rogen ($\mu^- p$) we have
measured the Iz [1]. By comparing
this measurem n bound-state QED
we have detern v value differs by 5.0
standard devia 3 standard deviation
from the e-p sc ncy may arise from a
computational oblem in bound-state
QED, an unkn nental error.

PHYSICS

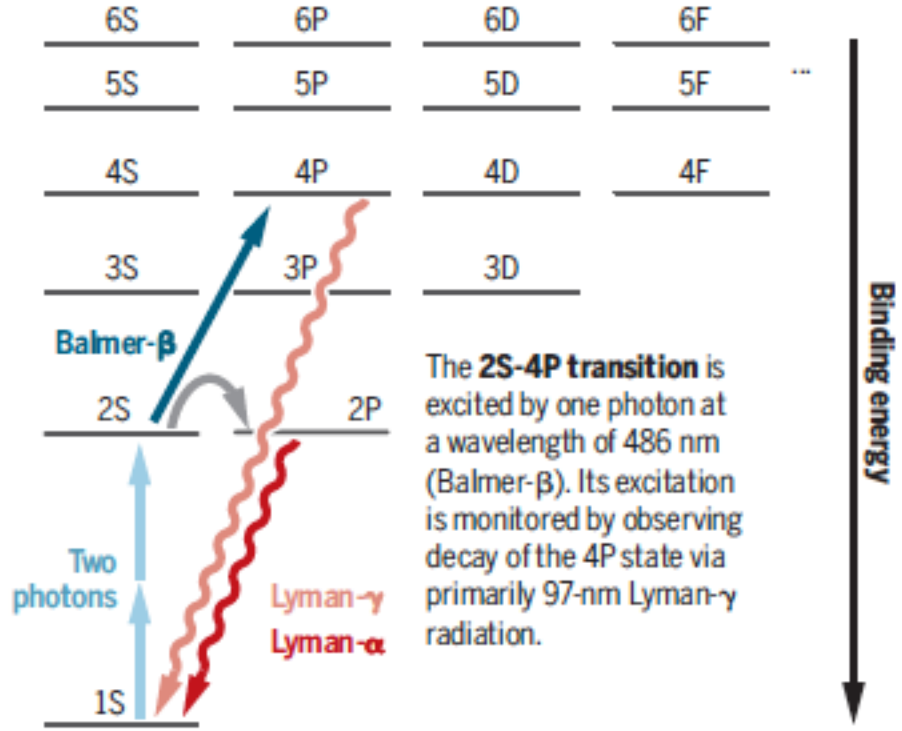
The proton radius revisited

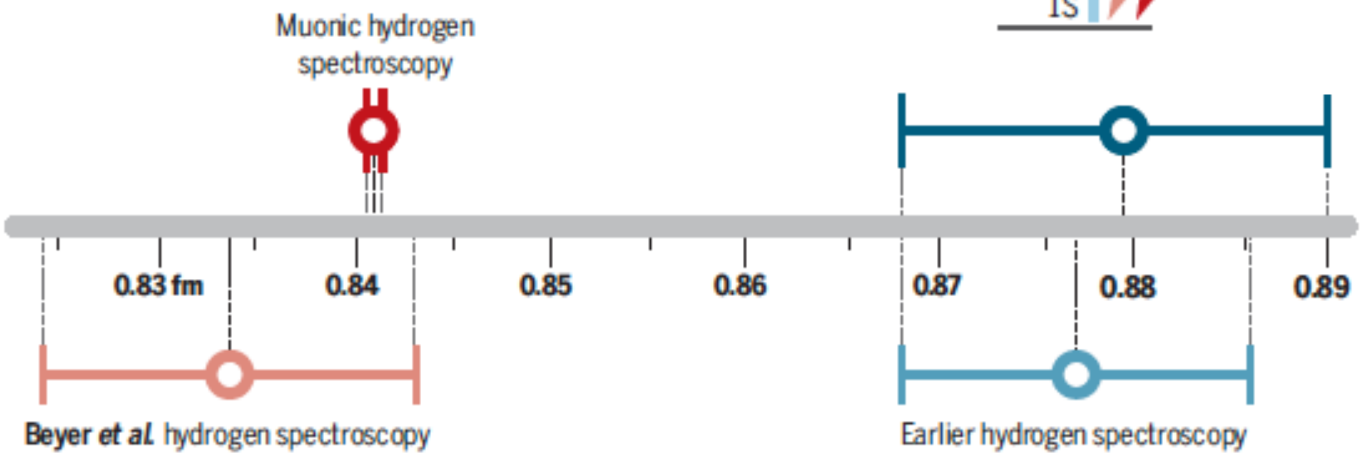
Hydrogen spectroscopy brings a surprise in the search for a solution to a long-standing puzzle

Science 06 Oct 2017: Vol. 358, 6359, pp. 39 DOI: 10.1126/ science.aao3969

New !

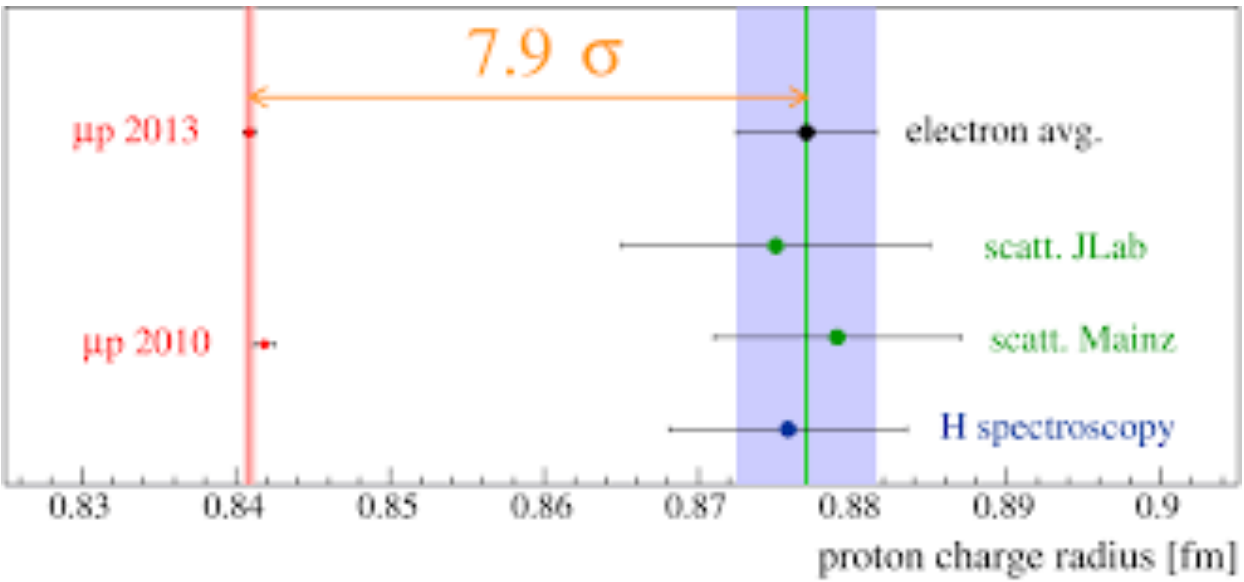
R_p=0.8335(95) fm





new Rydberg constant, deuterium…

The SIZE of the proton



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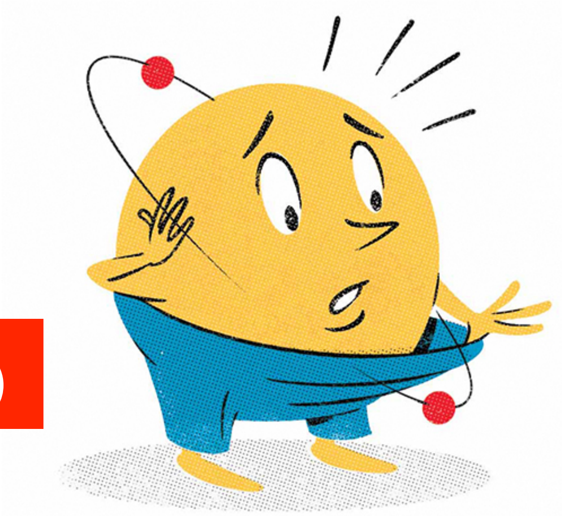
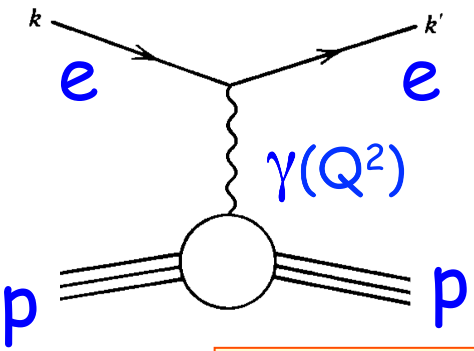


The New York Times

Hadron physics: e-p scattering

ep-elastic scattering : Rosenbluth separation

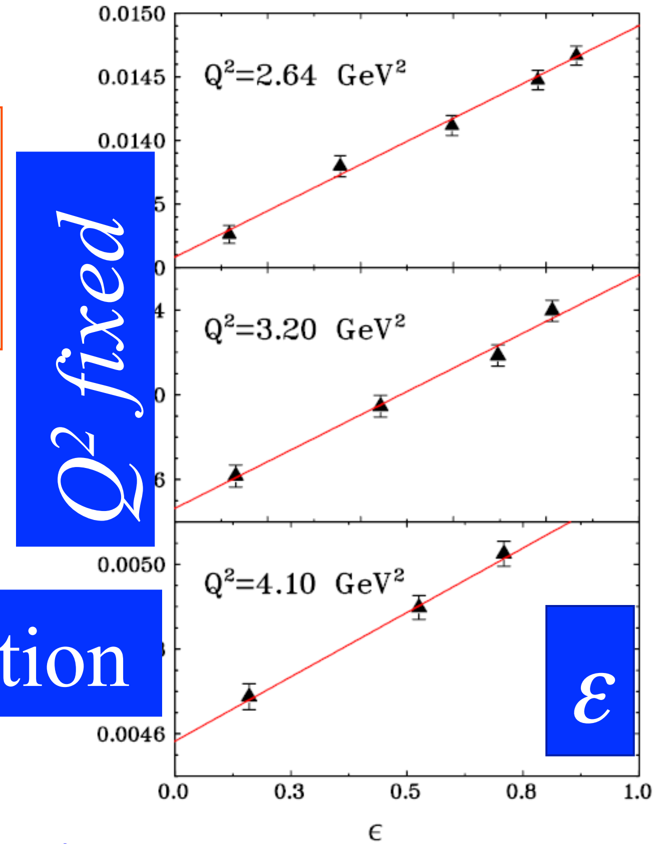


$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{Mott}\frac{1}{(1+\tau)}\left(G_E^2(Q^2)+\frac{\tau}{\varepsilon}G_M^2(Q^2)\right)$$

1950

$$\varepsilon=\left(1+2(1+\tau)\tan^2\left(\frac{\theta_e}{2}\right)\right)^{-1},\tau=\frac{Q^2}{4M^2}$$

$$\sigma_R=\varepsilon G_E^2+\tau G_M^2$$



Linearity of the reduced cross section

→ $\tan^2\theta_e$ dependence

→ Holds for 1γ exchange only

PRL 94, 142301 (2005)

Root mean square radius

$$F(q) = \frac{\int_\Omega d^3\vec{x} e^{i\vec{q}\cdot\vec{x}} \rho(\vec{x})}{\int_\Omega d^3\vec{x} \rho(\vec{x})}.$$

In *non-relativistic approach*
(and also in relativistic but in *Breit frame*)
FFs are Fourier transform of the density

density $\rho(r)$	Form factor $F(q^2)$	r.m.s. $< r_c^2 >$	comments
δ	1	0	pointlike
e^{-ar}	$\frac{a^4}{(q^2+a^2)^2}$	$\frac{12}{a^2}$	dipole
$\frac{e^{-ar}}{r}$	$\frac{a^2}{q^2+a^2}$	$\frac{6}{a^2}$	monopole
$\frac{e^{-ar^2}}{r^2}$	$e^{-q^2/(4a^2)}$	$\frac{1}{2a}$	gaussian
ρ_0 for $x \leq R$ 0 for $r \geq R$	$\frac{3(\sin X - X\cos X)}{X^3}$ $X = qR$	$\frac{3}{5}R^2$	square well

Root mean square radius

$$F(q)=\frac{\int_\Omega d^3\vec{x}e^{i\vec{q}\cdot\vec{x}}\rho(\vec{x})}{\int_\Omega d^3\vec{x}\rho(\vec{x})}.$$

$$<r_c^2>=\frac{\int_0^\infty x^4\rho(x)dx}{\int_0^\infty x^2\rho(x)dx}.$$

Expanding in Taylor series:

$$F(q)\sim 1-\frac{1}{6}q^2<r_c^2>+O(q^2),$$

$$\langle r^2_{E/M}\rangle=-\frac{6\hbar^2}{G_{E/M}(0)}\frac{dG_{E/M}(Q^2)}{dQ^2}\bigg|_{Q^2=0}.$$

RMS *is the limit of the* ***form factor derivative*** *for Q^2->0*

PRL 105, 242001 (2010) PHYSICAL REVIEW LETTERS week ending 10 DECEMBER 2010

High-Precision Determination of the Electric and Magnetic Form Factors of the Proton

J. C. Bernauer,[1,*] P. Achenbach,[1] C. Ayerbe Gayoso,[1] R. Böhm,[1] D. Bosnar,[2] L. Debenjak,[3] M. O. Distler,[1,†] L. Doria,[1] A. Esser,[1] H. Fonvieille,[4] J. M. Friedrich,[5] J. Friedrich,[1] M. Gómez Rodríguez de la Paz,[1] M. Makek,[2] H. Merkel,[1] D. G. Middleton,[1] U. Müller,[1] L. Nungesser,[1] J. Pochodzalla,[1] M. Potokar,[3] S. Sánchez Majos,[1] B. S. Schlimme,[1] S. Širca,[6,3] Th. Walcher,[1] and M. Weinriefer[1]

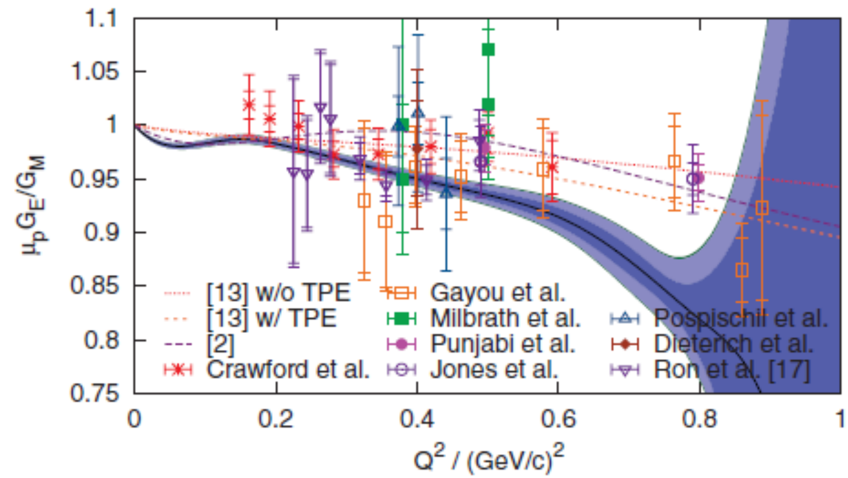
Mainz, A1 collaboration (1400 points)

$Q^2 > 0.004$ GeV^2

- Radiative corrections
- Two photon exchange
- Coulomb corrections

$$\langle r_E^2\rangle^{1/2} = 0.879(5)_{\rm stat}(4)_{\rm syst}(2)_{\rm model}(4)_{\rm group}\ {\rm fm},$$

$$\langle r_M^2\rangle^{1/2} = 0.777(13)_{\rm stat}(9)_{\rm syst}(5)_{\rm model}(2)_{\rm group}\ {\rm fm}.$$



What about extrapolation to $Q^2 \to 0$?

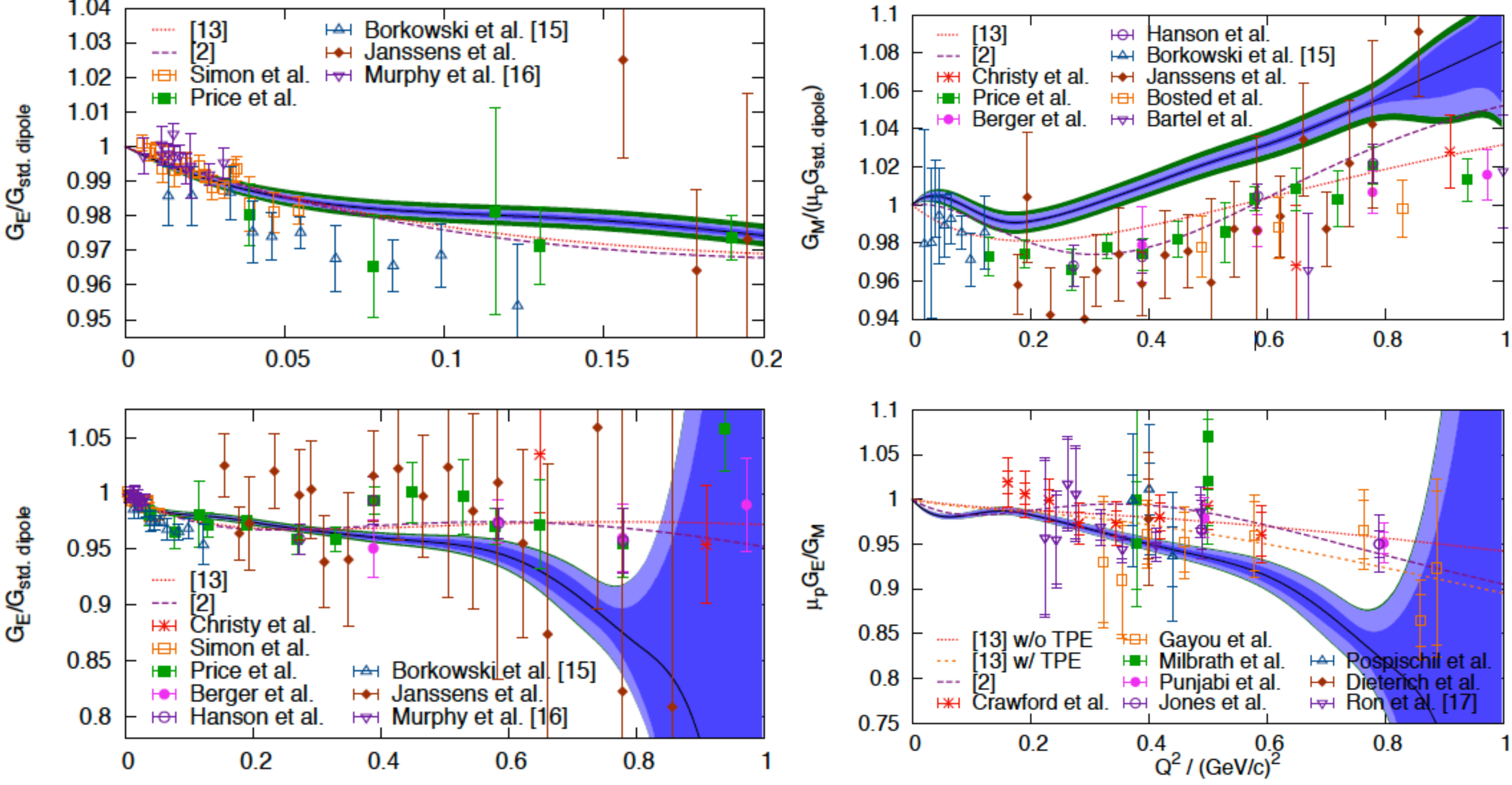
G.I. Gakh, A. Dbeyssi, E.T-G, D. Marchand, V.V. Bytev, Phys.Part.Nucl.Lett. 10 (2013) 393, Phys.Rev. C84 (2011) 015212

Mainz ep elastic scattering

GEp

$$\left\langle r_{E/M}^2 \right\rangle = -\frac{6\hbar^2}{G_{E/M}(0)} \frac{dG_{E/M}(Q^2)}{dQ^2}\Bigg|_{Q^2=0}$$

GMp



Mainz ep elastic scattering

$$\left\langle r^2_{E/M} \right\rangle = -\frac{6\hbar^2}{G_{E/M}(0)} \frac{\mathrm{d}G_{E/M}(Q^2)}{\mathrm{d}Q^2}\Big|_{Q^2=0}$$

1) Rosenbluth extraction

2) Direct extraction
(assuming a function for FFs)

Spline

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.875(5)_{\mathrm{stat.}}(4)_{\mathrm{syst.}}(2)_{\mathrm{model}}\,\mathrm{fm},$$

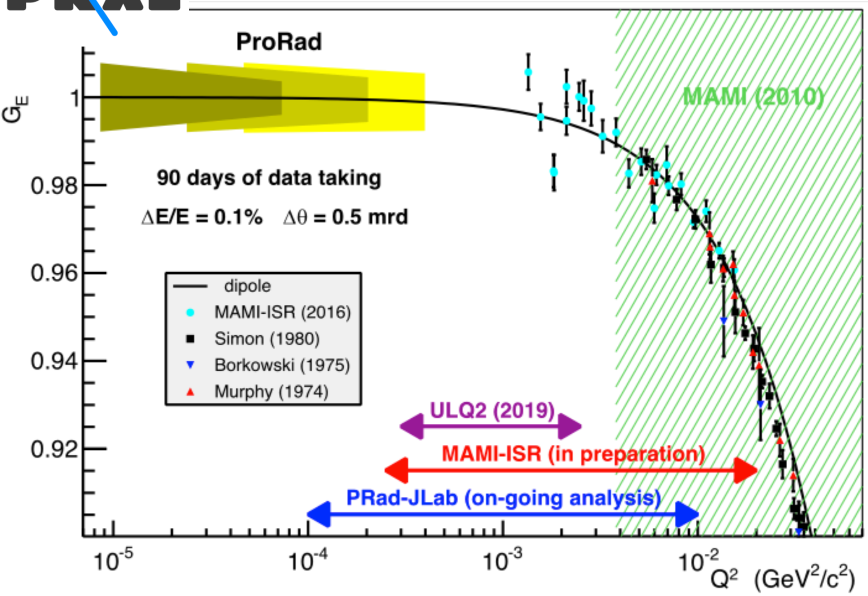
$$\langle r_M^2 \rangle^{\frac{1}{2}} = 0.775(12)_{\mathrm{stat.}}(9)_{\mathrm{syst.}}(4)_{\mathrm{model}}\,\mathrm{fm}$$

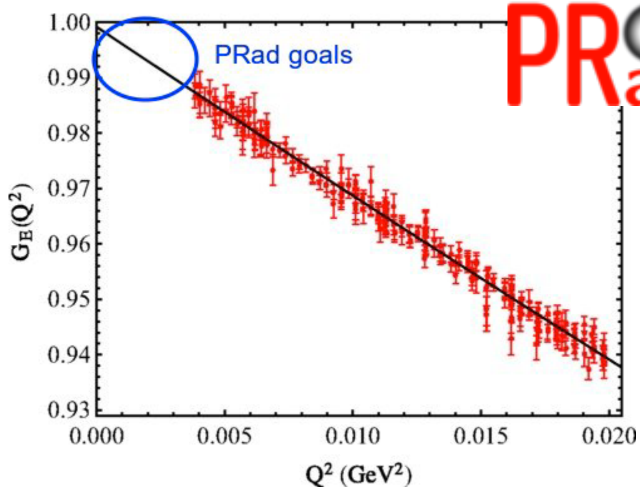
Polynomial

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.883(5)_{\mathrm{stat.}}(5)_{\mathrm{syst.}}(3)_{\mathrm{model}}\,\mathrm{fm},$$

$$\langle r_M^2 \rangle^{\frac{1}{2}} = 0.778(^{+14}_{-15})_{\mathrm{stat.}}(10)_{\mathrm{syst.}}(6)_{\mathrm{model}}\,\mathrm{fm}.$$

Planned ep experiments



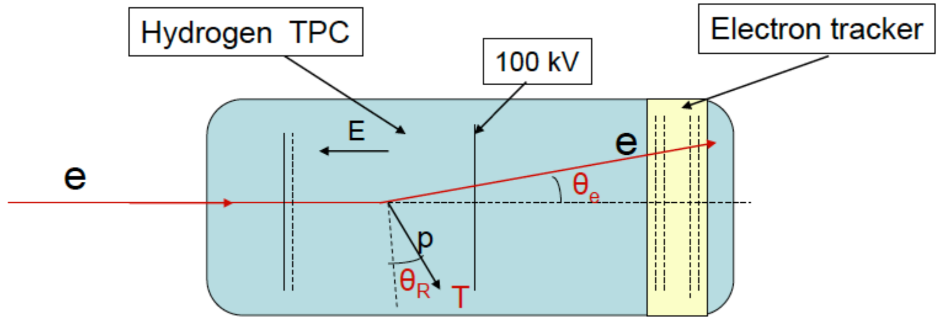


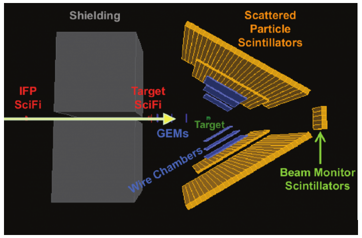
Systematics at sub%
Abs Norm by Moller

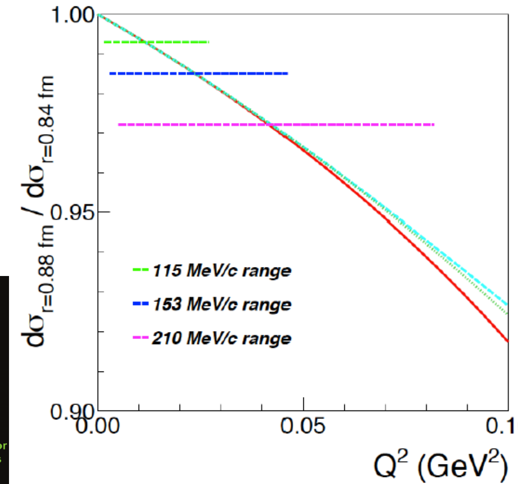
MUSE@PSI: muon beam

PNPI@MAMI: e and p detection

Combined recoiled proton@forward tracker detector







Proton-Electron Elastic Scattering

Polarization effects in elastic proton-electron scattering

G. I. Gakh, A. Dbeyssi, D. Marchand, E. Tomasi-Gustafsson, and V. V. Bytev
Phys. Rev. C **84**, 015212 – Published 28 July 2011

Письма в ЭЧАЯ. 2013. Т. 10, № 5(182). С. 642–649

PROTON–ELECTRON ELASTIC SCATTERING AND THE PROTON CHARGE RADIUS

G.I. Gakh, A. Dbeyssi,E. Tomasi-Gustafsson, D. Marchand, V.V. Bytev

Radiative corrections to elastic proton-electron scattering measured in coincidence

G. I. Gakh, M. I. Konchatnij, N. P. Merenkov, and E. Tomasi-Gustafsson
Phys. Rev. C **95**, 055207 – Published 30 May 2017

Proton-Electron Elastic Scattering

Inverse kinematics

Three possible applications:

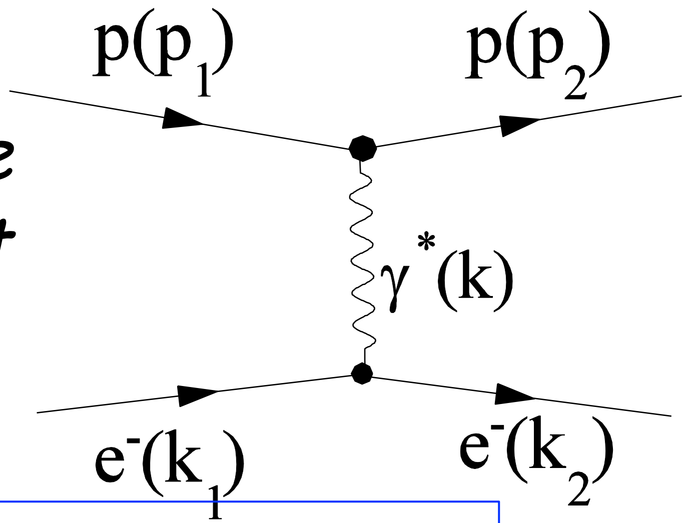
1. *Beam polarimeters for high energy polarized proton beams, Novosibirsk (1997)*

2. *Polarized (anti)protons (ASSIA, PAX at FAIR)*
   F. Rathman (1993), C. J. Horowitz and H. O. Meyer (1994),
   A.I.~Milstein, S. G. Salnikov and V. M. Strakhovenko(2008),
   T. Walcher, H. Arenhoevel (2006-2009) erratum;
   S. O'Brien, N. H. Buttimore (2006)...

3. ***Proton Radius***

Proton-Electron Elastic Scattering

- *Inverse kinematics : projectile heavier than the target → take into account the electron mass*



- *Specific kinematics:*
  - *very small scattering angles*
  - *very small transferred momenta*

- *'Equivalent total energy s'*

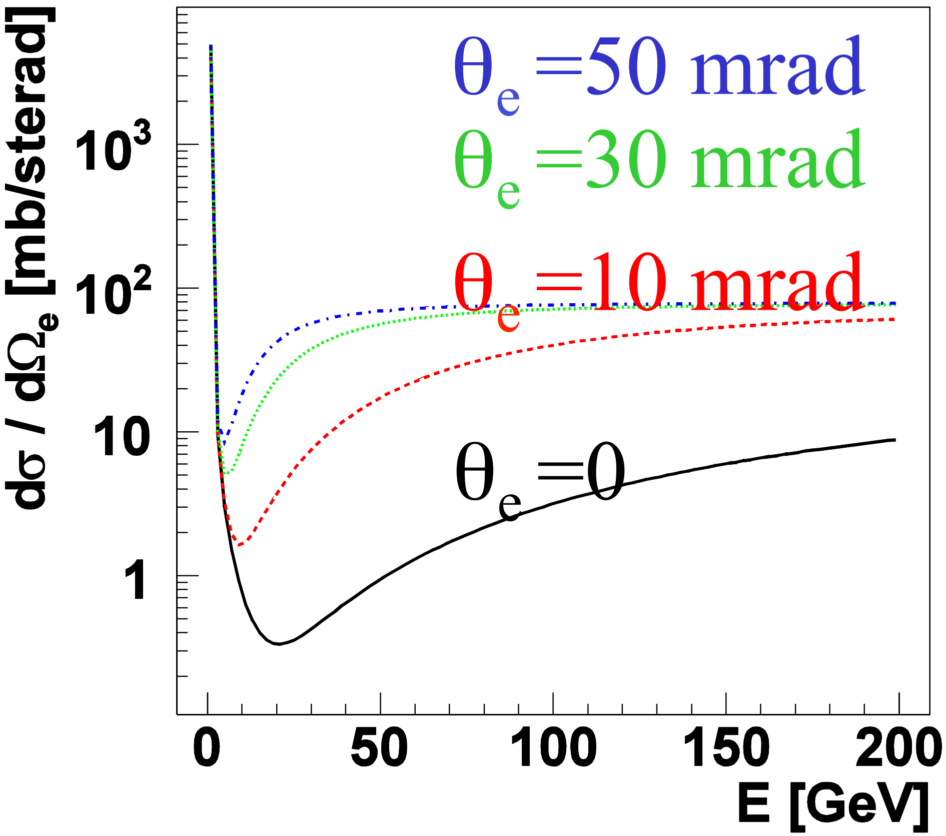
$$E = \frac{M}{m}\epsilon \sim 2000\ \epsilon.$$

A.I Akhiezer and M.P. Rekalo,
Hadron Electrodynamics, Naukova Dumka, Kiev (1977)

Proton-electron elastic scattering: The differential cross section

$$\frac{d\sigma}{d\Omega_e} = \frac{1}{32\pi^2}\frac{1}{mp}\frac{\vec{k}_2^3}{-k^2}\frac{\overline{|\mathcal{M}|^2}}{E+m},$$

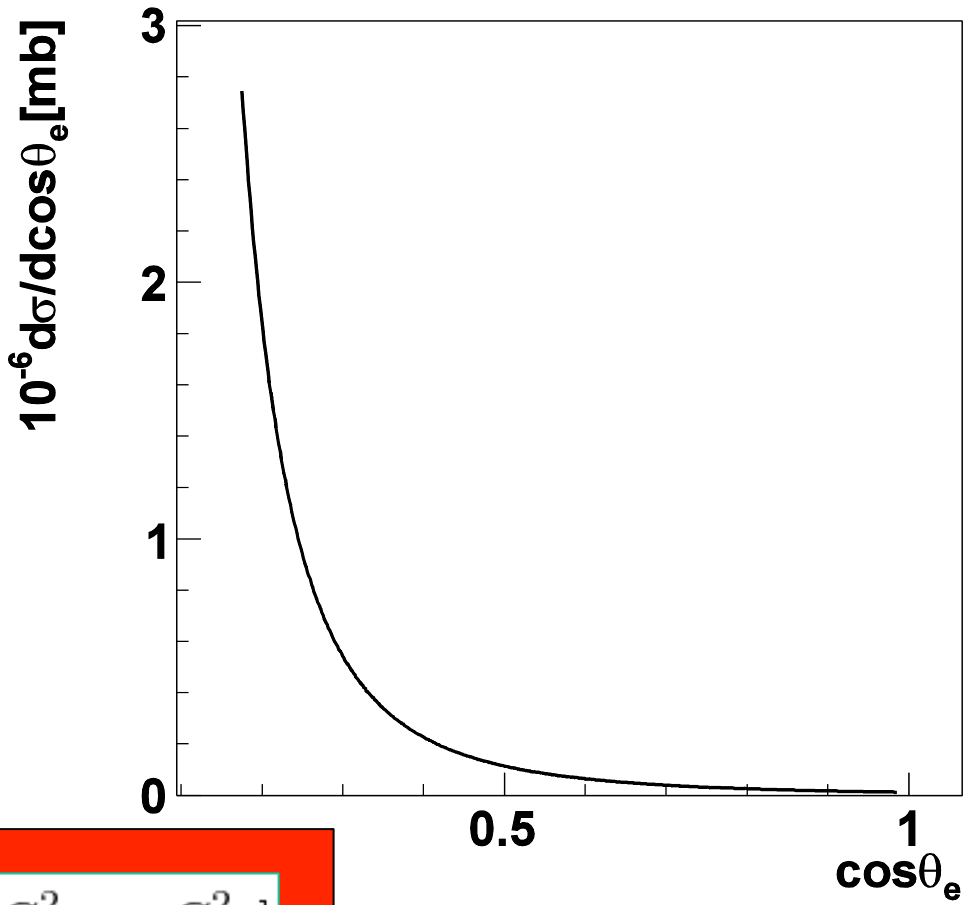
- The electron mass can not be neglected
- Interesting structure in the GeV region



Steep rise at small energy

The cross section at E=100 MeV

- Cross section is huge
- Only Electric FF contributes !

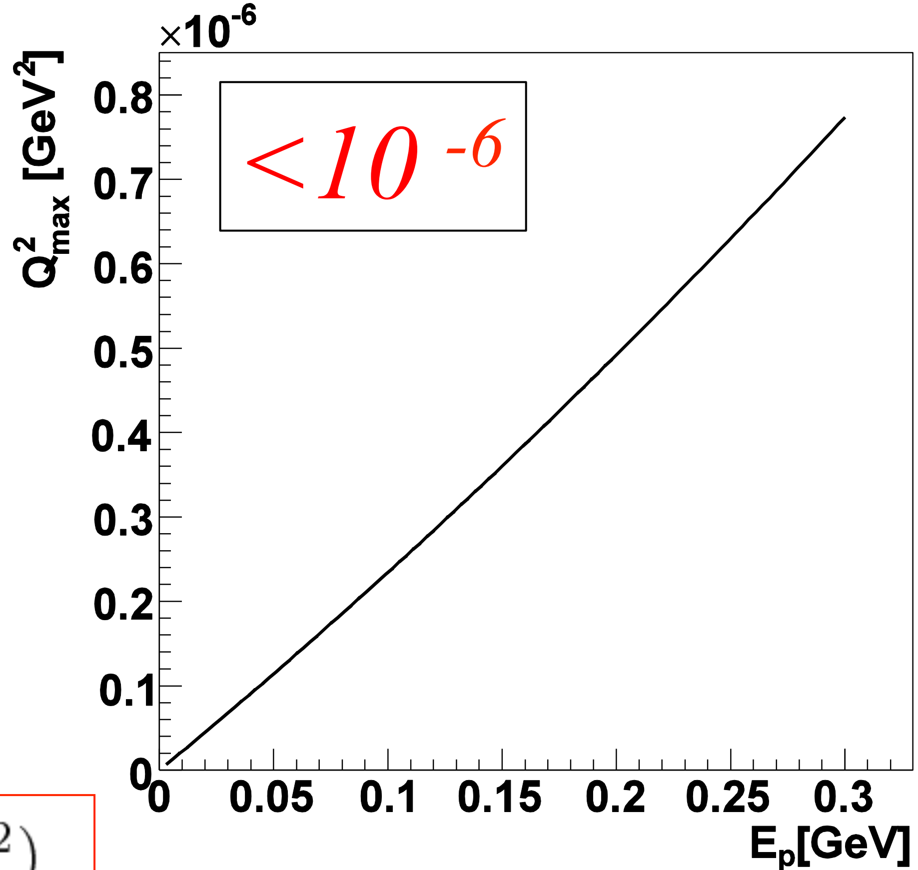


$$\frac{d\sigma}{dQ^2} = \frac{\pi\alpha^2}{2m^2\vec{p}^2}\frac{\mathcal{D}}{Q^4},$$

$$\mathcal{D} = -Q^2(-Q^2+2m^2)G_M^2 + 2[G_E^2+\tau G_M^2]\left[-Q^2M^2+\frac{1}{1+\tau}\left(2mE-\frac{Q^2}{2}\right)^2\right],$$

$$\tau = \frac{Q^2}{4M^2}$$

Proton-Electron Kinematics (E=100 MeV)



$$(-k^2)_{max} = \frac{4m^2(E^2 - M^2)}{M^2 + 2mE + m^2}$$

k^2 proportional to m^2 !!

Low Q^2 Form Factor Parametrizations

Radial expansion

$$\frac{G_{E,M}(q^2)}{G_{E,M}(0)} = 1 + \frac{1}{6} q^2 r^2_{E,M} + O(q^4),$$

$$G_E = 1 + 3.496\, q^2, \quad G_M = 2.793 + 8.65\, q^2.$$

$<r^2>=0.814$

Expansion to 4th order:

Dipole fit

$$G_E(q^2) = G,\ G_M(q^2) = \mu_p G,\ G = (1 - 1.41 q^2)^{-2},$$

$$G_E = 1 + 2.82 q^2 + 5.96 q^4,\ G_M = 2.793 + 7.88 q^2 + 16.65 q^4.$$

Low Q^2

$$G_E(q^2) = (1 - 1.517\, q^2)^{-2},\ G_M(q^2) = \mu_p (1 - 1.37 q^2)^{-2}$$

$$G_E = 1 + 3.034 q^2 + 6.91 q^4,\ G_M = 2.793 + 7.65 q^2 + 15.72 q^4.$$

Sum of monopoles

$$F_1(q^2) = \sum_1^3 \frac{n_i}{d_i - q^2},\ F_2(q^2) = \sum_1^3 \frac{m_i}{g_i - q^2},$$

$$G_E = 1 + 3.017\, q^2 + 7.22\, q^4, \quad G_M = 2.793 + 8.239\, q^2 + 20.31\, q^4$$

$<r^2>=0.657$

$<r^2>=0.706$

$<r^2>=0.702$

PHYSICAL REVIEW C **75**, 035202 (2007)

Dispersion analysis of the nucleon form factors including meson continua

M. A. Belushkin* and H.-W. Hammer†

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Ulf-G. Meißner‡

Helmholtz-Institut für Strahlen- und Ke

Institut für Kernphysi

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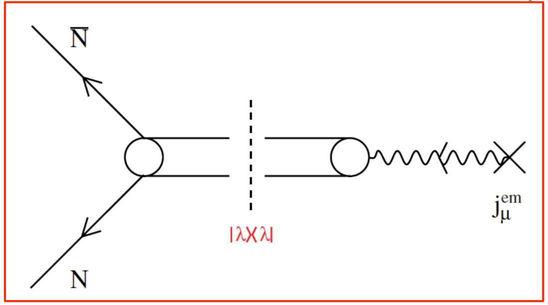
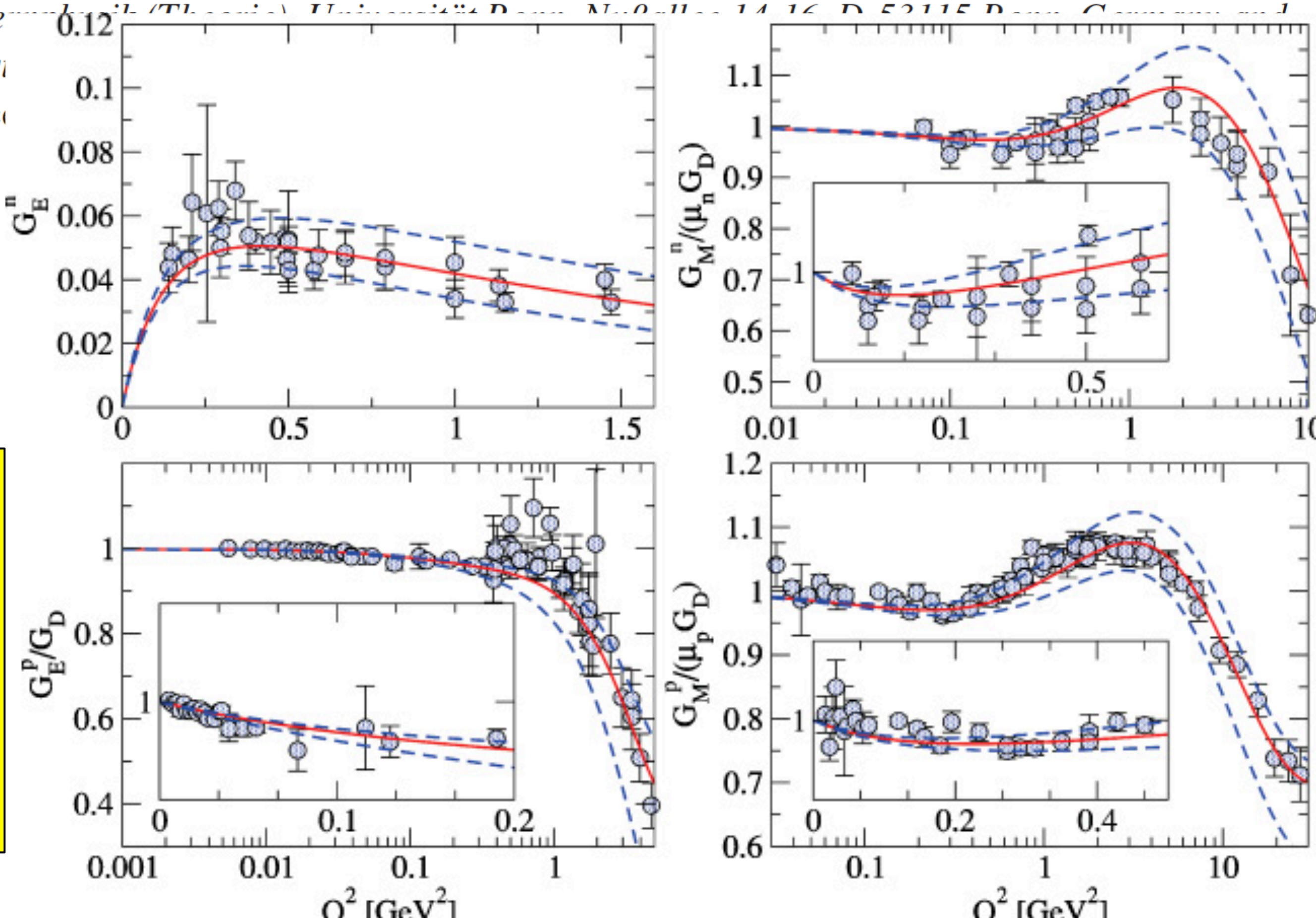


Superconvergent relations
pQCD asymptotics
Broad resonance
2π,KK,ρπ continuum



PHYSICAL REVIEW C **75**, 035202 (2007)

Dispersion analysis of the nucleon form factors including meson continua

M. A. Belushkin* and H.-W. Hammer†
Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), Universität Bonn, Nußallee 14-16, D-53115 Bonn, Germany

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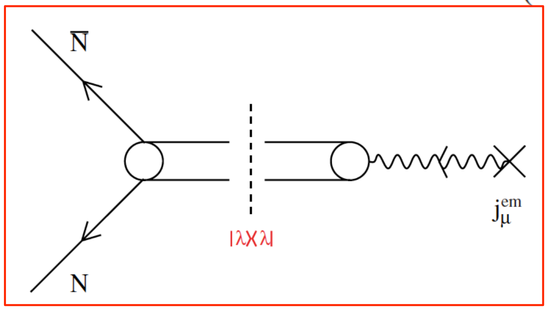
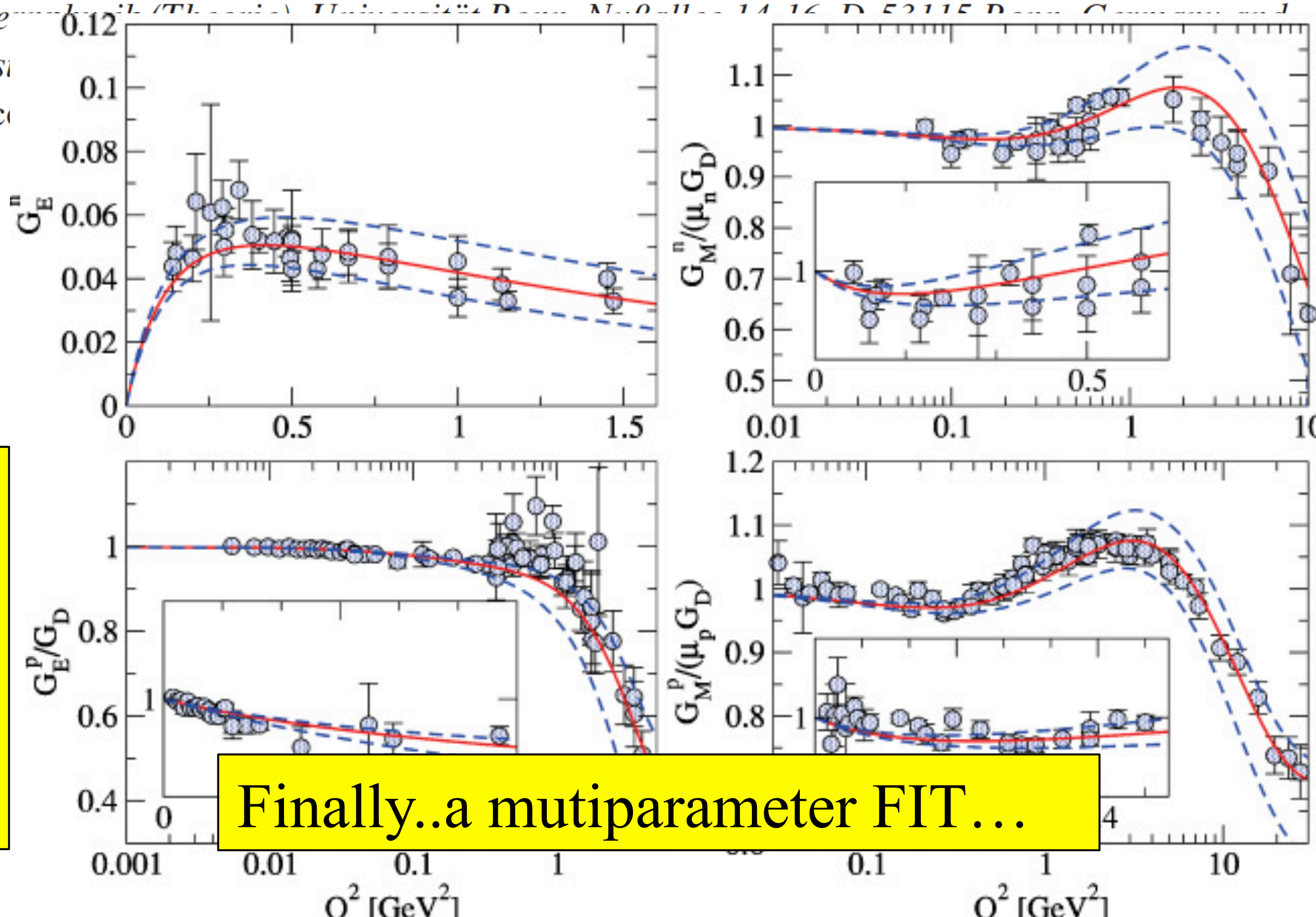
Institut für Kernphysi
(Rec





Superconvergent relations
pQCD asymptotics
Broad resonance
2π,KK,$\rho\pi$ continuum

Finally..a mutiparameter FIT…

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(Received 4 September 2006; published 6 March 2007)

	SC approach	Explicit pQCD app.	Ref. [23]	Recent determ.
r_E^p (fm)	0.844 (0.840 . . . 0.852)	0.830 (0.822 . . . 0.835)	0.848	0.886(15) [72–74]
r_M^p (fm)	0.854 (0.849 . . . 0.859)	0.850 (0.843 . . . 0.852)	0.857	0.855(35) [73,75]
$(r_E^n)^2$ (fm^2)	−0.117 (−0.11 . . . −0.128)	−0.119 (−0.108 . . . − 0.13)	−0.12	−0.115(4) [52]
r_M^n (fm)	0.862 (0.854 . . . 0.871)	0.863 (0.859 . . . 0.871)	0.879	0.873(11) [76]

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ArXiv 1406.2962v2[Hep-ph]

Reduction of the proton radius discrepancy by 3 σ

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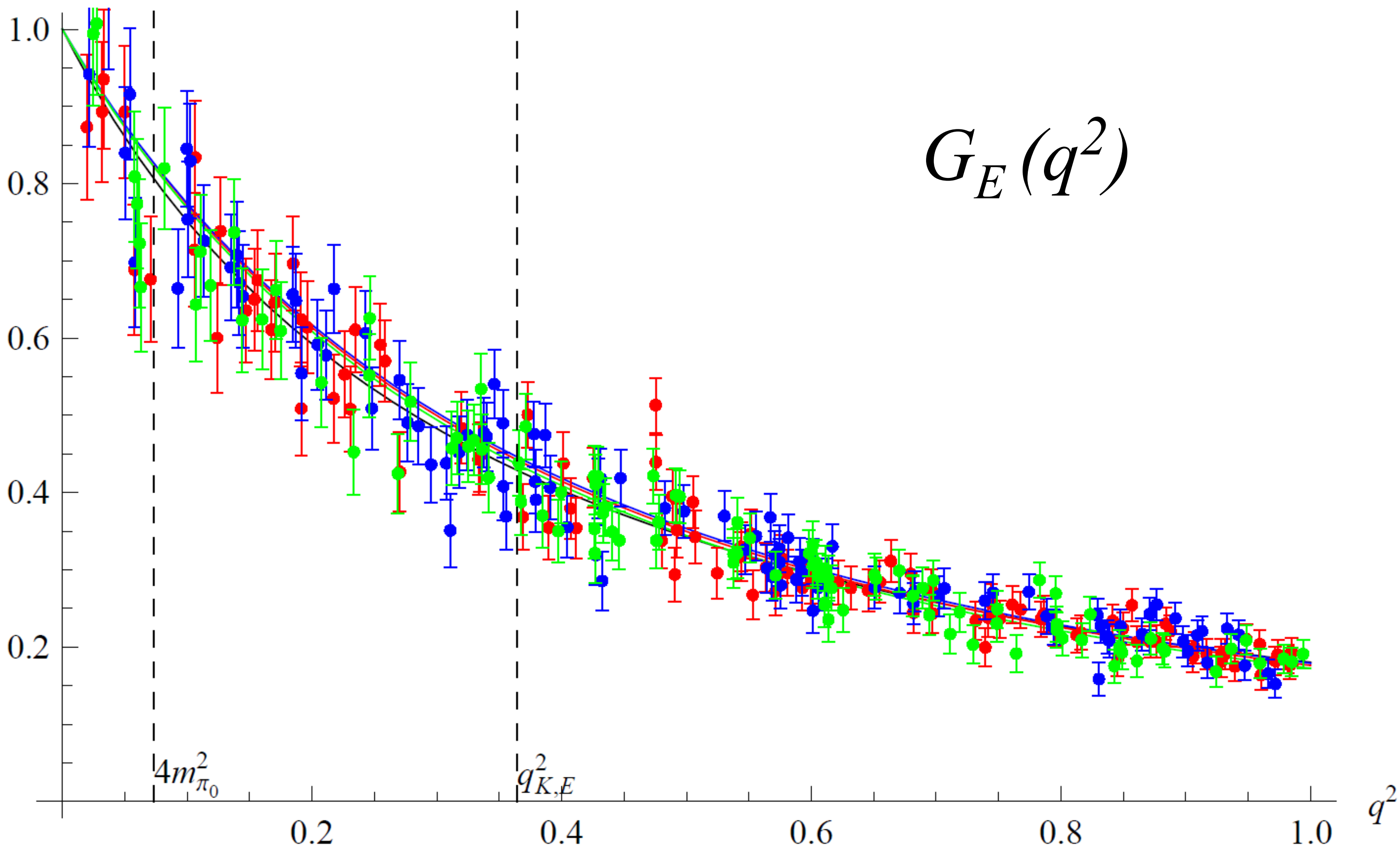
We show that in previous analyses of electron-proton scattering, the uncertainties in the statistical procedure to extract the proton charge radius are underestimated. Using a fit function based on a conformal mapping, we can describe the scattering data with high precision and extract a radius value in agreement with the one obtained from muonic hydrogen.

Why I do not trust the fits

Slide from Savely Karshenboim



Why I do not trust the fits

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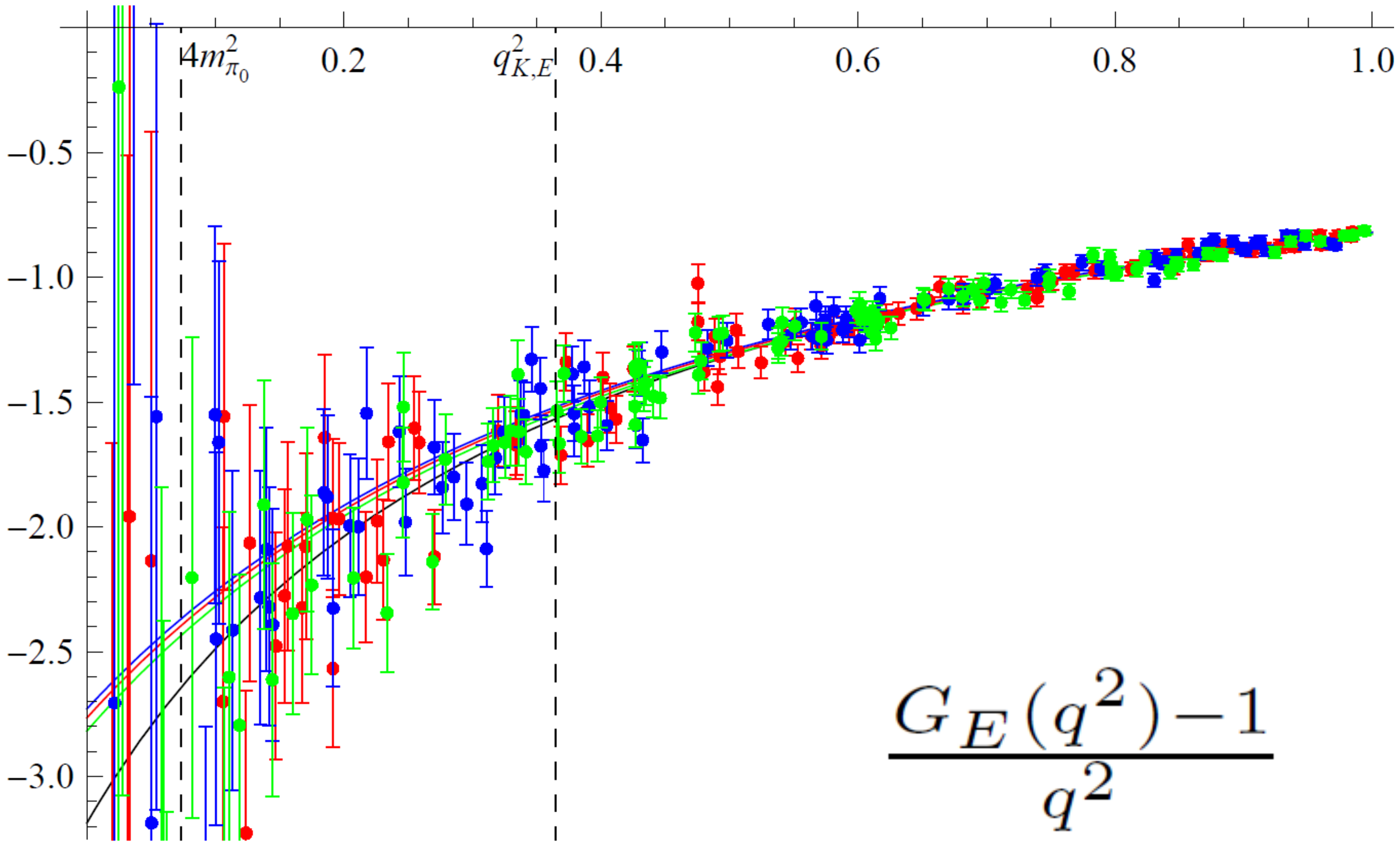


Conclusions

- Discrepancy between the determination of the proton radius:
  - CODATA (ep scattering & H) and muonic hydrogen
  - ep elastic scattering and μH
  - Recent and previous Hydrogen Lamb shift experiments
  - Tension between analysis of ep-scattering: extrapolation to $Q^2=0$!!!

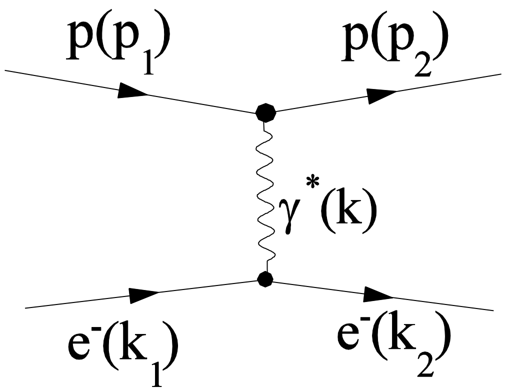
The problem is on derivatives, not on observables !

- Our contribution:
  - Very low transferred momenta can be reached by proton-electron elastic scattering (inverse kinematics)
  - Fully relativistic description of proton-electron scattering : kinematics, differential cross section, polarization phenomena and radiative corrections

The unpolarized cross section (I)

- *The matrix element*

$$\mathcal{M} = \frac{e^2}{k^2} j_\mu J_\mu,$$



- *The leptonic tensor* $j_\mu = \bar{u}(k_2)\gamma_\mu u(k_1),$

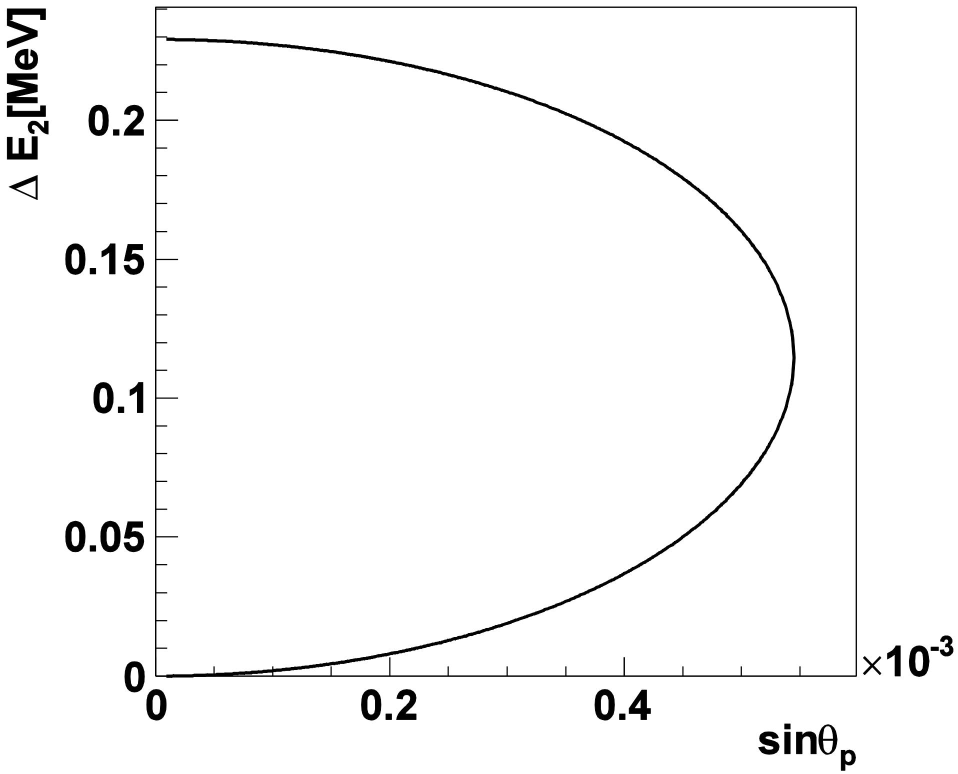
- *The hadronic tensor*

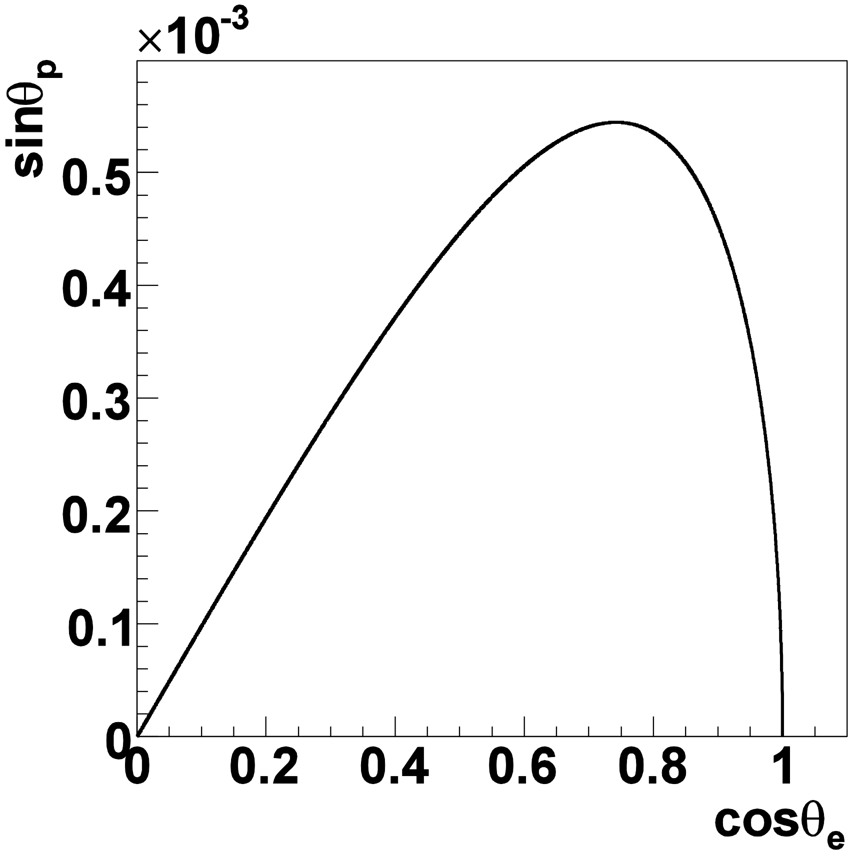
$$\begin{aligned} J_\mu &= \bar{u}(p_2)\left[F_1(k^2)\gamma_\mu - \frac{1}{2M}F_2(k^2)\sigma_{\mu\nu}k_\nu\right]u(p_1) \\ &= \bar{u}(p_2)\left[G_M(k^2)\gamma_\mu - F_2(k^2)P_\mu\right]u(p_1). \end{aligned}$$

$P_\mu = (p_1 + p_2)_\mu/(2M)$.

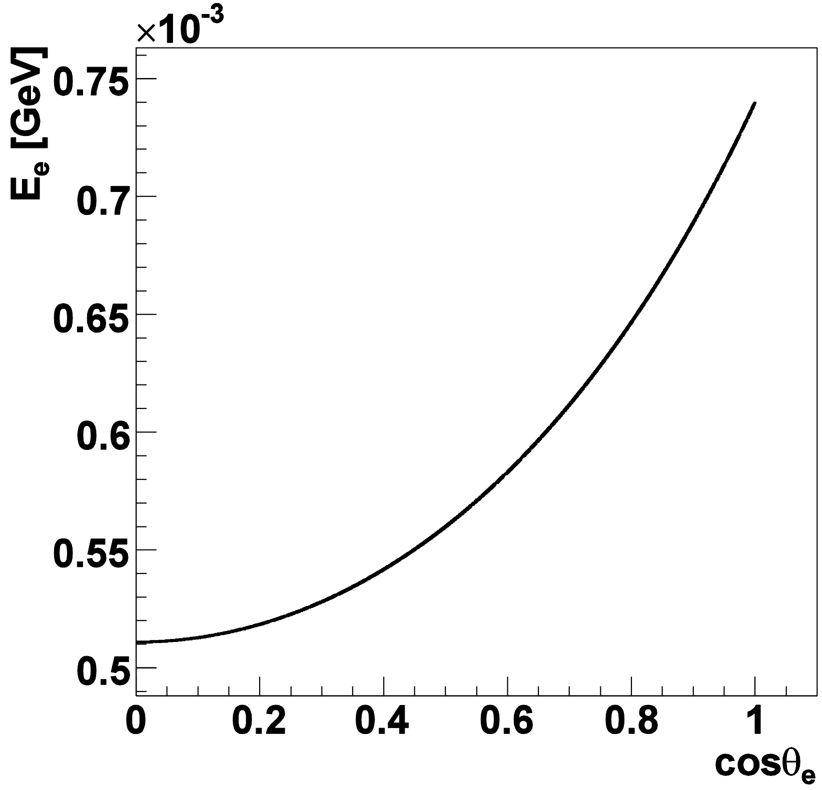
$$G_M(k^2) = F_1(k^2) + F_2(k^2)$$
$$G_E(k^2) = F_1(k^2) - \tau F_2(k^2)$$

The proton kinematics (E=100 MeV)



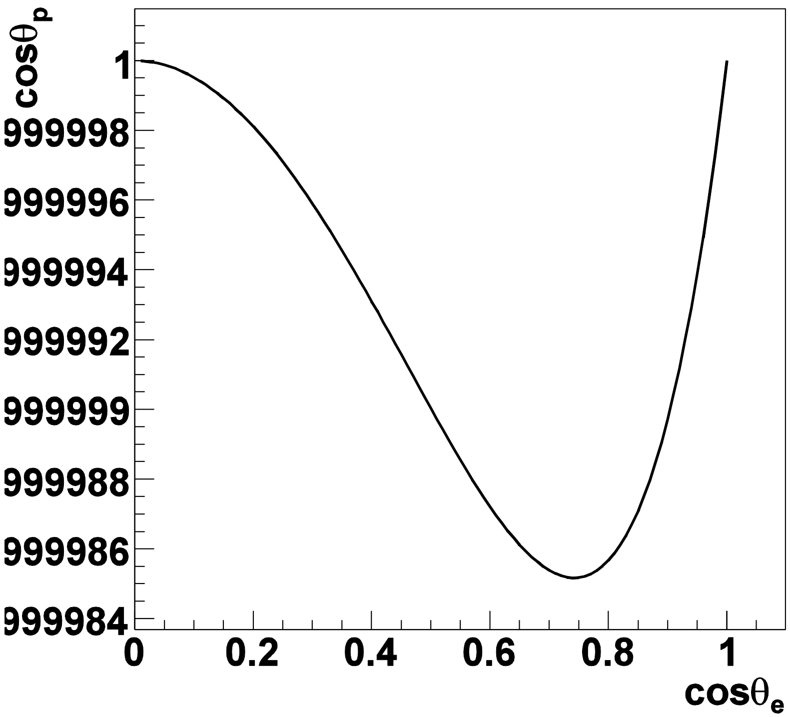


Proton-Electron Kinematics

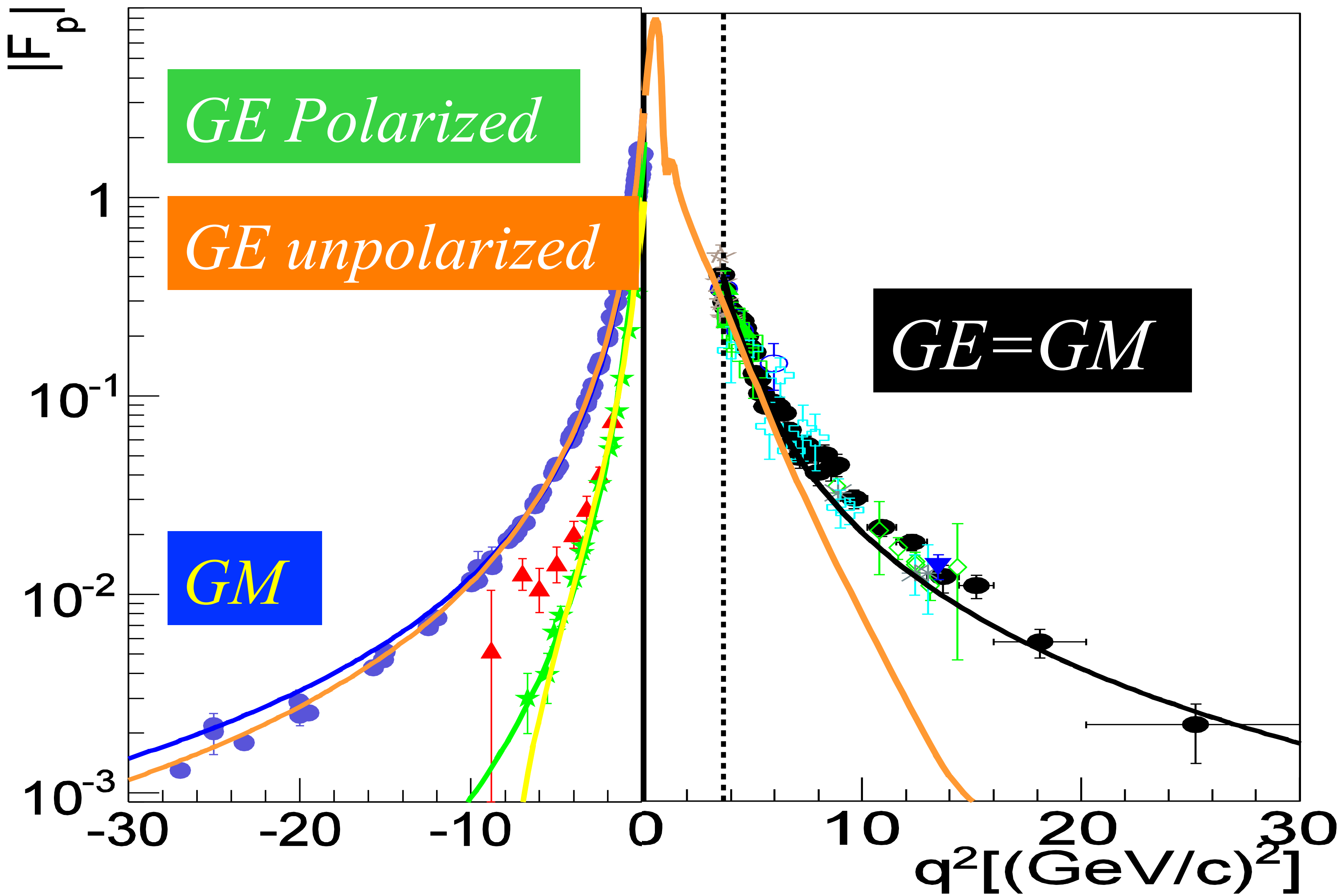


E(e)- cos θ(e)

cosθ(p)- cos θ(e)

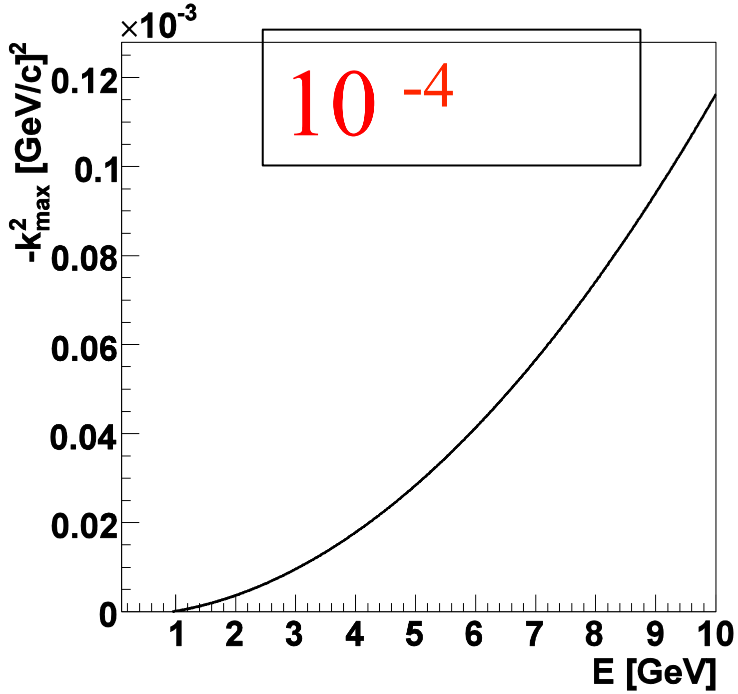


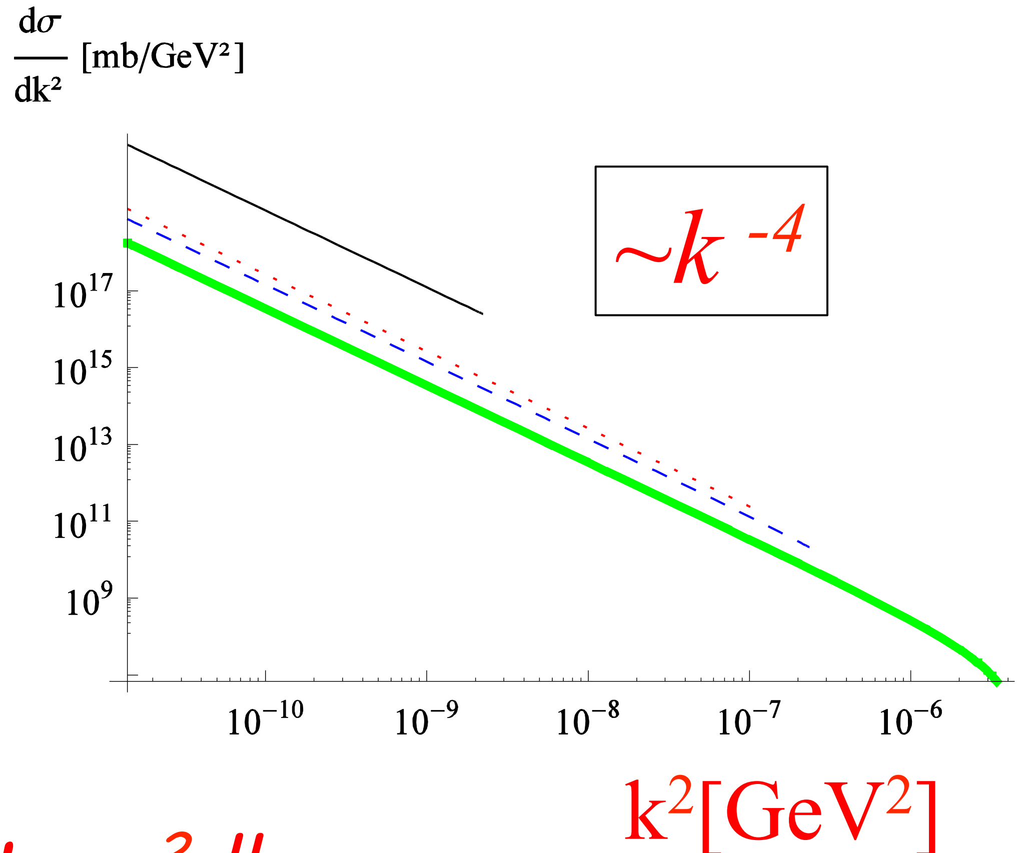
Hadron Electromagnetic Form Factors



Proton-Electron elastic scattering

$$(-k^2)_{max} = \frac{4m^2(E^2 - M^2)}{M^2 + 2mE + m^2}$$





k^2 proportional to m^2 !!

Extraction of electromagnetic form factors for $k^2 \to 0$

Applications I

Polarimetry of high energy (anti)proton beams

Polarization phenomena

1) Polarization transfer coefficients

$$p + \vec{e} \longrightarrow \vec{p} + e$$

2) Spin correlation coefficients

$$\vec{p} + \vec{e} \longrightarrow p + e$$

3) Depolarization coefficients

$$\vec{p} + e \longrightarrow \vec{p} + e$$

Depolarization coefficients

- *Initial and final proton spins*

- *The polarized cross section*

$$\frac{d\sigma}{dk^2}(\eta_1,\eta_2)=\left(\frac{d\sigma}{dk^2}\right)_{un}\left[1+D_{tt}S_{1t}S_{2t}+D_{nn}S_{1n}S_{2n}+D_{\ell\ell}S_{1\ell}S_{2\ell}+D_{t\ell}S_{1t}S_{2\ell}+D_{\ell t}S_{1\ell}S_{2t}\right]$$

- *The coefficients*

$$\begin{aligned}\mathcal{D}D(\eta_1,\eta_2) \;=\; & 2(1+\tau)^{-1}\Big\{k\cdot\eta_1 k\cdot\eta_2 G_M(k^2)\left[k^2\left(G_M(k^2)-G_E(k^2)\right)+2m^2(1+\tau)G_M(k^2)\right]\\ & +k^2(1+\tau)G_M^2(k^2)(2k_1\cdot\eta_2 k_2\cdot\eta_1-m^2\eta_1\cdot\eta_2)\\ & +4G_M(k^2)(k\cdot\eta_1 k_1\cdot\eta_2-k\cdot\eta_2 k_1\cdot\eta_1)\left[M^2\tau\left(G_E(k^2)-G_M(k^2)\right)\right.\\ & \left.+mE\left(G_E(k^2)+\tau G_M(k^2)\right)\right]\\ & -\eta_1\cdot\eta_2\left(G_E^2(k^2)+\tau G_M^2(k^2)\right)\left[k^2(M^2-2mE)+4m^2E^2\right]\Big\}.\end{aligned}$$

Polarization

- *Polarized lepton tensor*

$$L_{\mu\nu}^{(p)} = 2im\epsilon_{\mu\nu\alpha\beta}k_\alpha S_\beta,$$

- *Polarized hadronic tensor*

$$W_{\mu\nu}(\eta_j) = -2iG_M(k^2)\left[MG_M(k^2)\epsilon_{\mu\nu\alpha\beta}k_\alpha\eta_{j\beta} + + F_2(k^2)(P_\mu\epsilon_{\nu\alpha\beta\gamma} - P_\nu\epsilon_{\mu\alpha\beta\gamma})p_{1\alpha}p_{2\beta}\eta_{j\gamma}\right]$$

The transverse beam polarization induces effects smaller by M/E

Polarization transfer coefficients

- *Initial electron and final proton spin*

$$S \equiv (0, \vec{\xi}), \;\; \eta_2 \equiv \left(\frac{1}{M} \vec{p}_2 \cdot \vec{S}_2, \vec{S}_2 + \frac{\vec{p}_2 (\vec{p}_2 \cdot \vec{S}_2)}{M(E_2 + M)} \right)$$

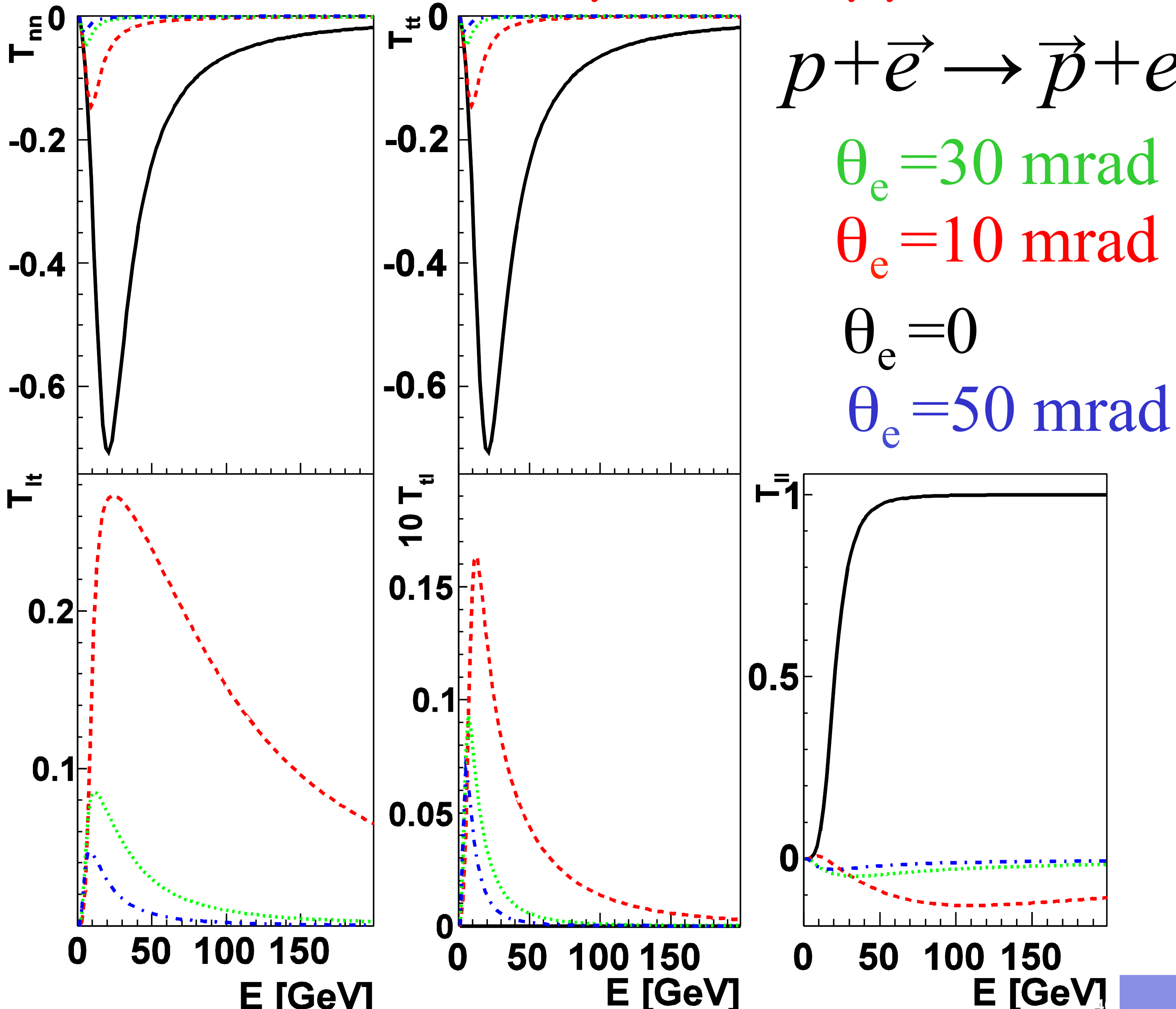
- *The polarized cross section*

$$\frac{d\sigma}{dk^2}(\vec{\xi}, \vec{S}_2) = \left(\frac{d\sigma}{dk^2} \right)_{un} [1 + T_{\ell\ell}\xi_\ell S_{2\ell} + T_{nn}\xi_n S_{2n} + T_{tt}\xi_t S_{2t} + T_{\ell t}\xi_\ell S_{2t} + T_{t\ell}\xi_t S_{2\ell}] \, ,$$

- *The coefficients*

$$\mathcal{D}T(S, \eta_2) = 4mMG_M(k^2) \left[G_E(k^2)(k \cdot Sk \cdot \eta_2 - k^2 S \cdot \eta_2) - k^2 F_2(k^2) P \cdot SP \cdot \eta_2 \right]$$

Polarization transfer coefficients



Polarization correlation coefficients

- *Initial electron and proton spins*

$$S \equiv (0, \vec{\xi}),\ \eta_1 = \left(\frac{\vec{p}\cdot\vec{S}_1}{M}, \vec{S}_1 + \frac{\vec{p}(\vec{p}\cdot\vec{S}_1)}{M(E+M)} \right)$$

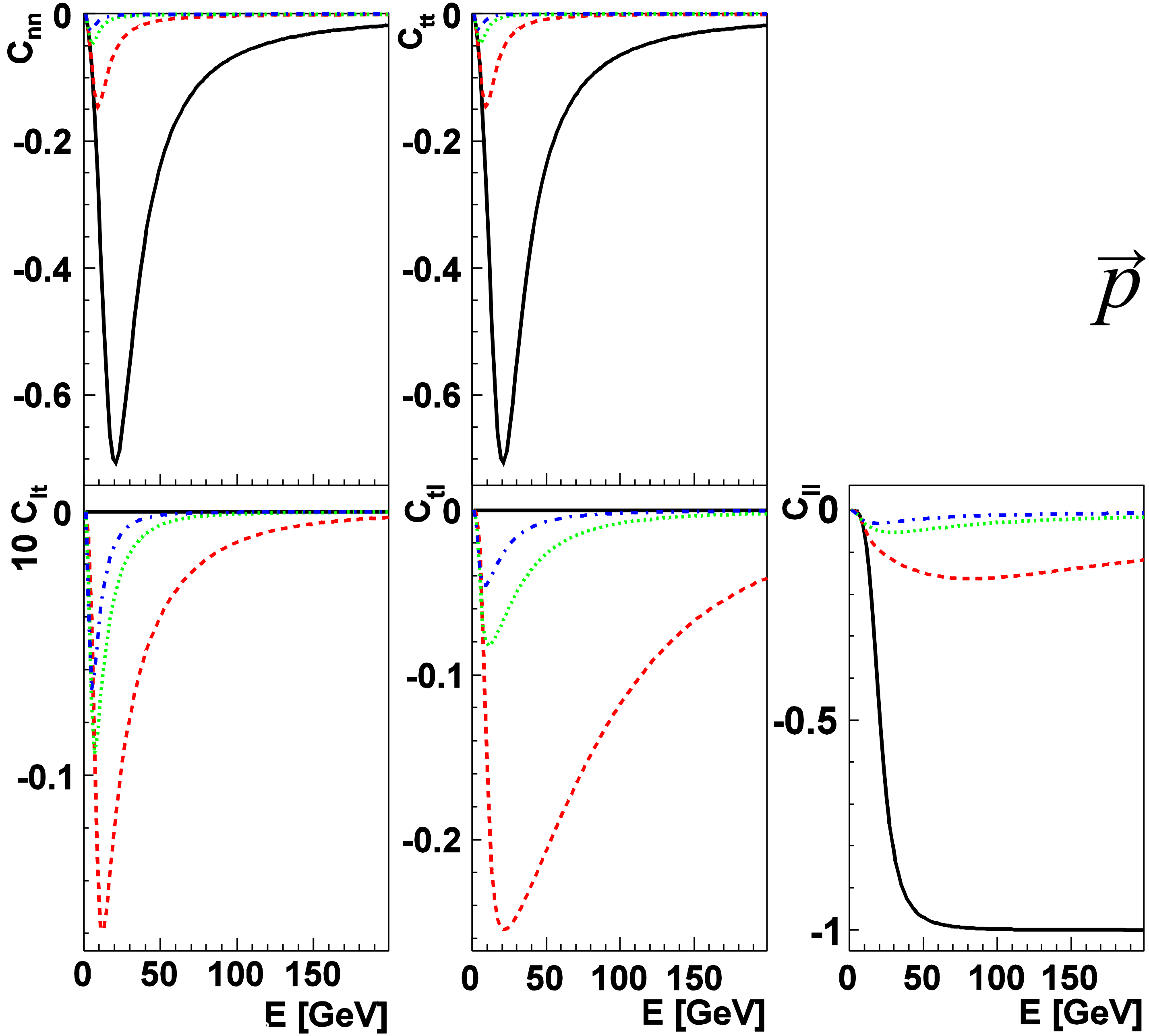
- *The polarized cross section*

$$\frac{d\sigma}{dk^2}(\vec{\xi},\vec{S}_1) = \left(\frac{d\sigma}{dk^2}\right)_{un} \left[1 + C_{\ell\ell}\xi_\ell S_{1\ell} + C_{tt}\xi_t S_{1t} + C_{nn}\xi_n S_{1n} + C_{\ell t}\xi_\ell S_{1t} + C_{t\ell}\xi_t S_{1\ell}\right],$$

- *The coefficients*

$$\mathcal{D}C(S,\eta_1) = 8mMG_M(k^2)\left[(k\cdot Sk\cdot\eta_1 - k^2 S\cdot\eta_1)G_E(k^2) + \tau k\cdot\eta_1(k\cdot S + 2p_1\cdot S)F_2(k^2)\right].$$

Spin correlation coefficients



$$\vec{p}+\vec{e} \longrightarrow p+e$$

$\theta_e = 30$ mrad

$\theta_e = 10$ mrad

$\theta_e = 0$

$\theta_e = 50$ mrad

Polarization by Spin Flip?

Ongoing experiments:

Spin Filtering with polarized targets

Spin Filtering with antiprotons at AD (CERN)

Our contribution to this problem:
large polarization effects appear at large energies.

$$p+\vec{e}\rightarrow \vec{p}+e$$

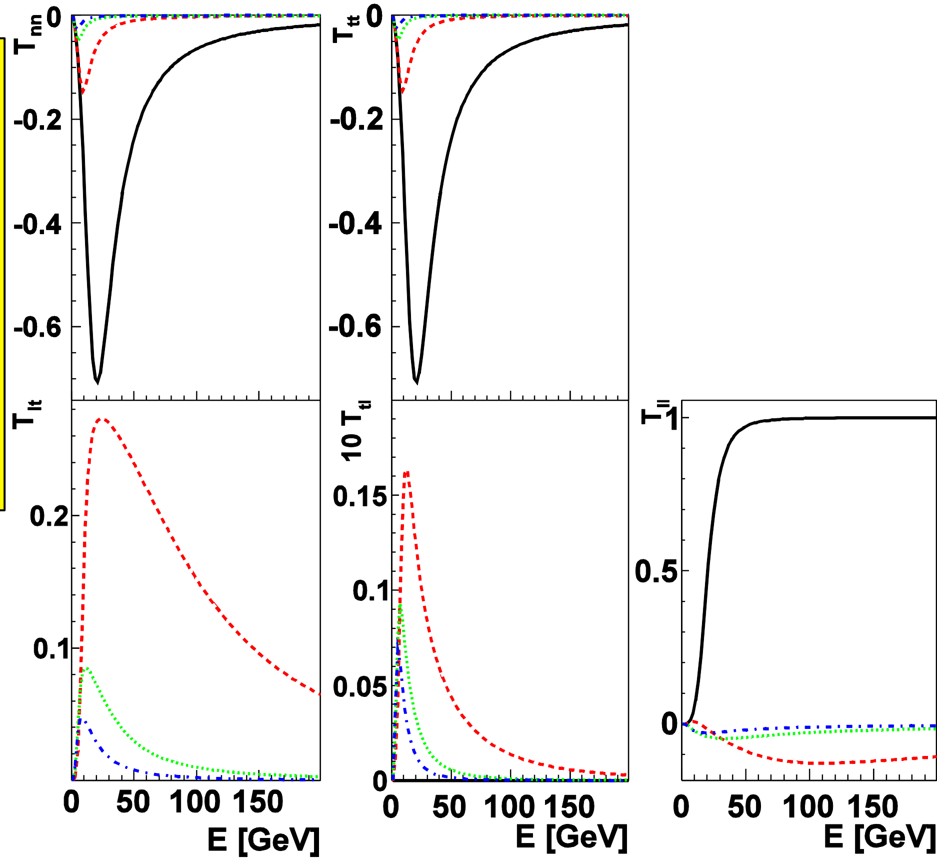
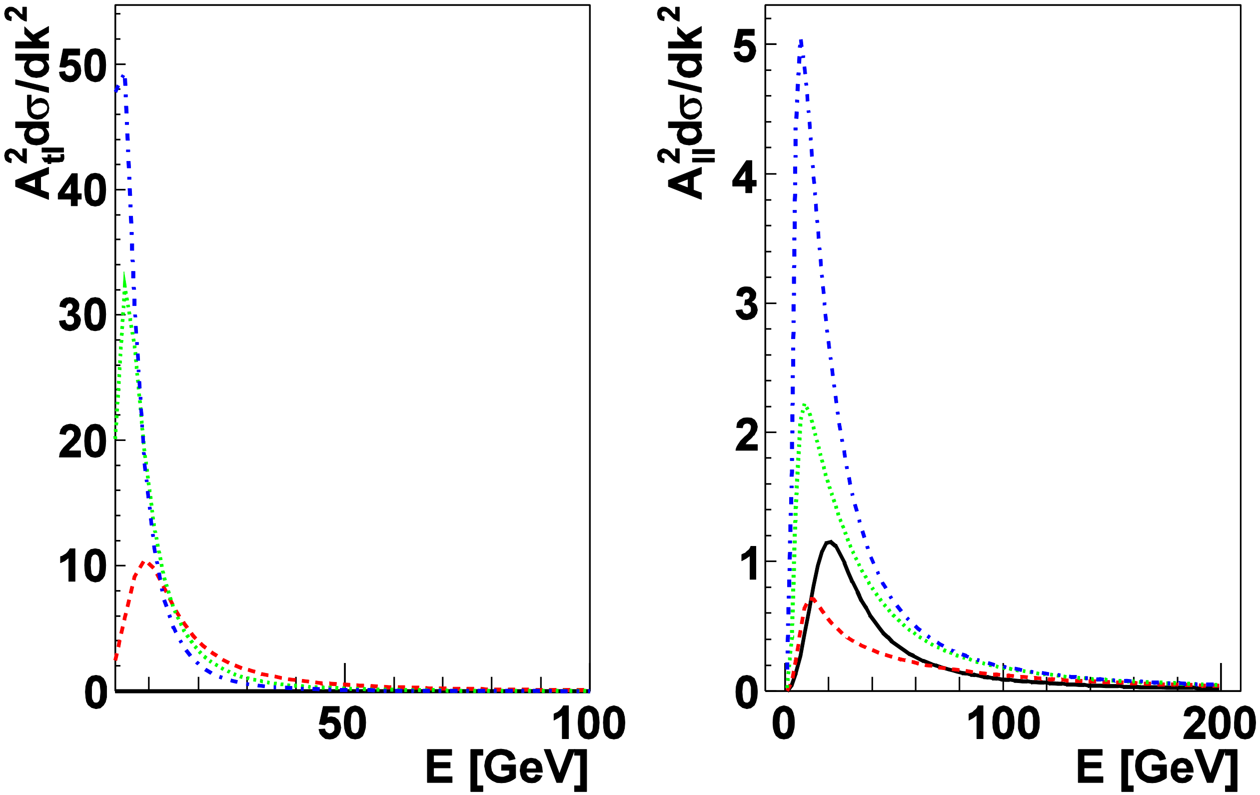


Figure of Merit

$$\mathcal{F}^2(\theta_p) = \epsilon(\theta_p) A^2_{ij}(\theta_p), \quad \epsilon(\theta_p) = N_f(\theta_p)/N_i$$

$$\left(\frac{\Delta P(\theta_p)}{P}\right)^2 = \frac{2}{N_i(\theta_p)\mathcal{F}^2(\theta_p)P^2} = \frac{2}{Lt_m(d\sigma/d\Omega)d\Omega A^2_{ij}(\theta_p)P^2},$$



$\vec{p} + \vec{e} \to p + e$

θ_e =30 mrad

θ_e =10 mrad

θ_e =0

θ_e =50 mrad

Figure of Merit

$$\mathcal{F}^2(\theta_p) = \epsilon(\theta_p) A^2_{ij}(\theta_p), \quad \epsilon(\theta_p) = N_f(\theta_p)/N_i$$

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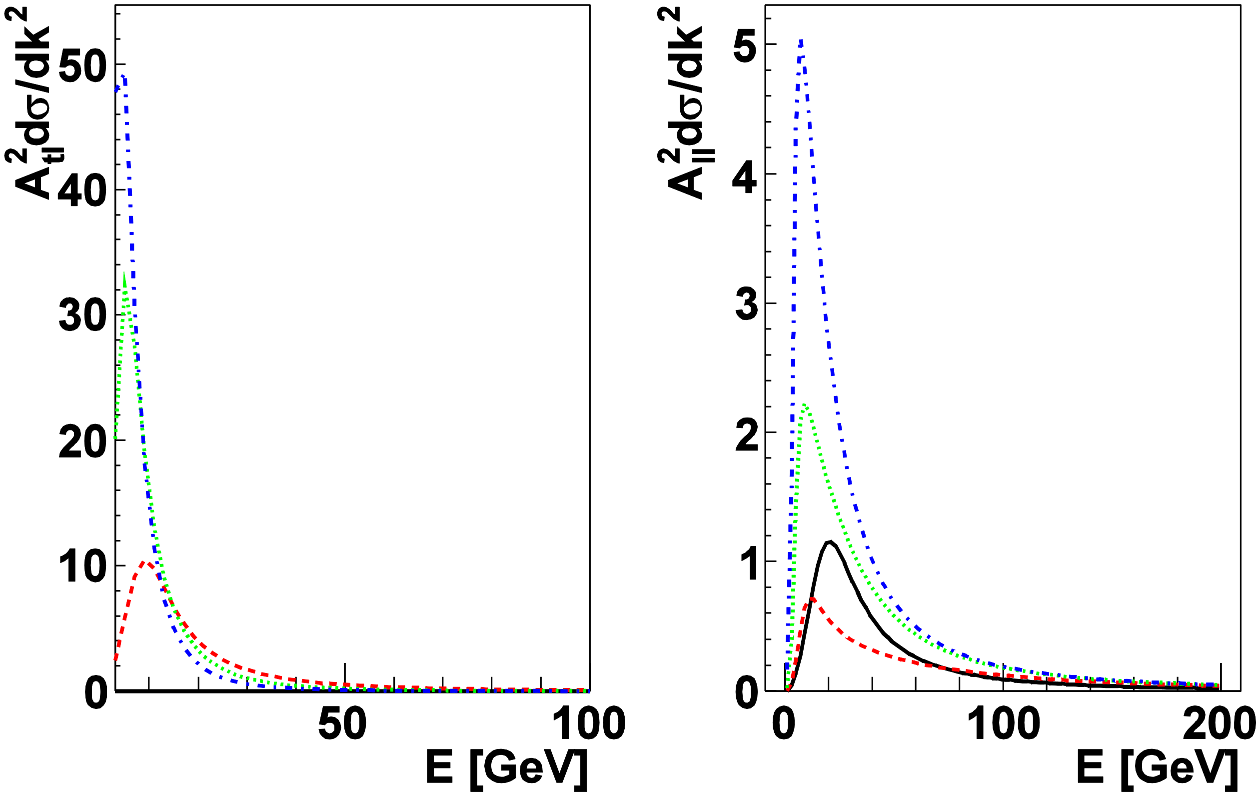


$$\vec{p} + \vec{e} \to p + e$$

$\theta_e = 30$ mrad

$\theta_e = 10$ mrad

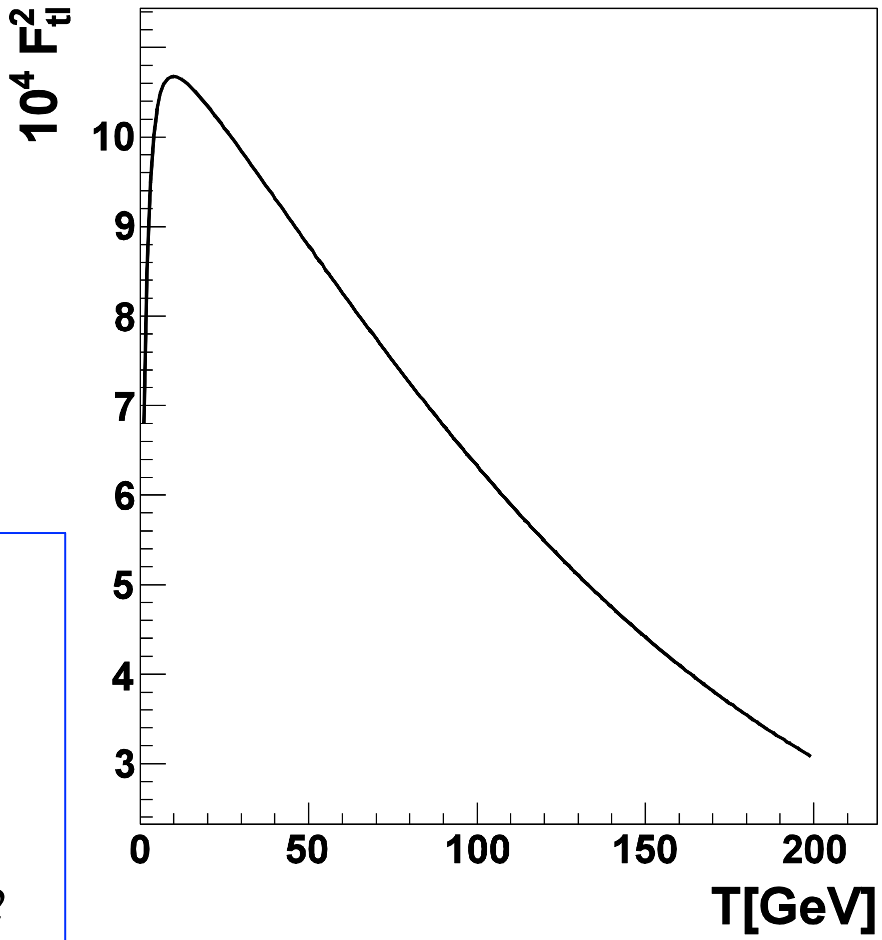
$\theta_e = 0$

$\theta_e = 50$ mrad

Polarimetry

Polarized beam
on polarized target

$$F^2 = \int \frac{d\sigma}{dk^2} A_{ij}^2(k^2)\mathrm{d}k^2$$



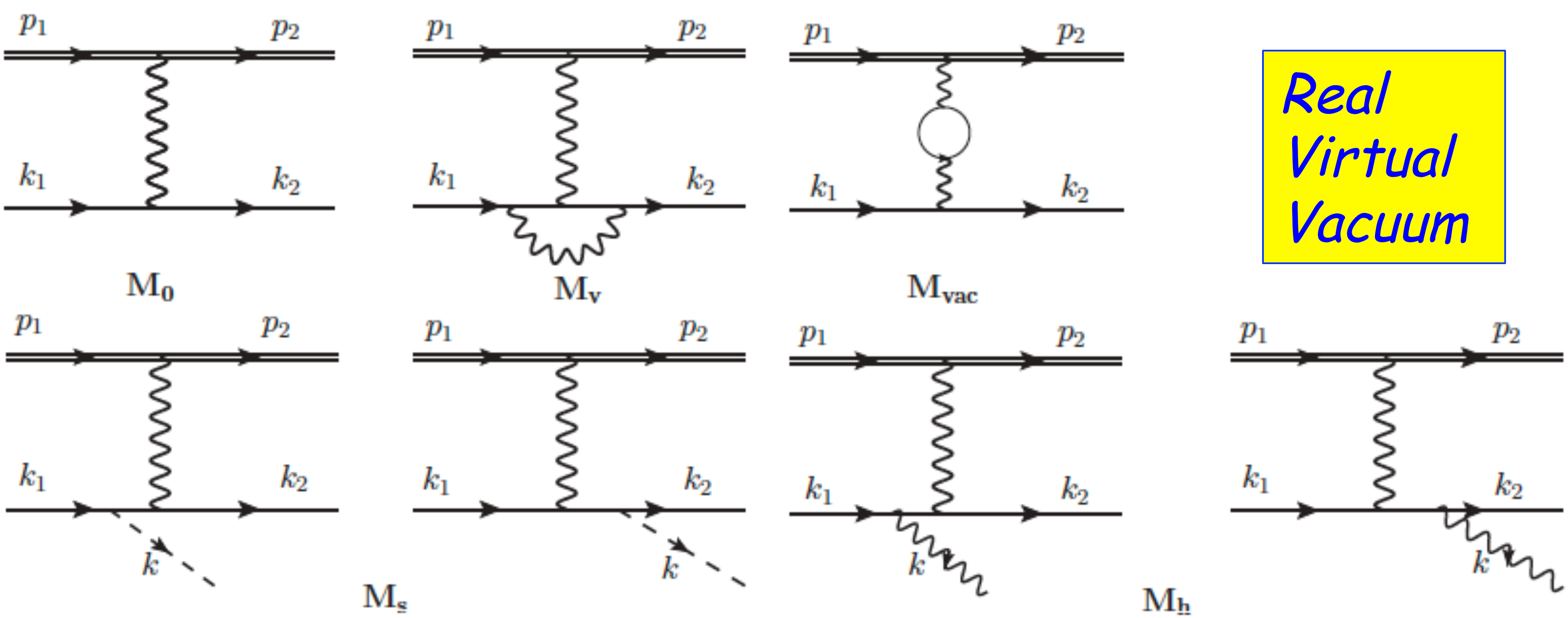
F^2 Max at E~10 GeV
$L=10^{32}\ cm^{-2}s^{-1}$

N beam=6 x10^{17} p s^{-1}
N target =2 x10^{14} atomes/cm^2
ΔP=1% in t=3m

Radiative corrections to elastic proton-electron scattering measured in coincidence

G. I. Gakh, M. I. Konchatnij, N. P. Merenkov, and E. Tomasi-Gustafsson

Phys. Rev. C **95**, 055207 – Published 30 May 2017



Soft Radiative Corrections (α^3)

Hard Radiative Corrections

Soft Radiative Corrections (α^3)

$$d\sigma^{(RC)} = (1+\delta_1+\delta_2+\delta^{(s)}+\delta^{(\mathrm{vac})})d\sigma^{(B)} = (1+\delta_0+\bar\delta+\delta^{(\mathrm{vac})})d\sigma^{(B)},$$

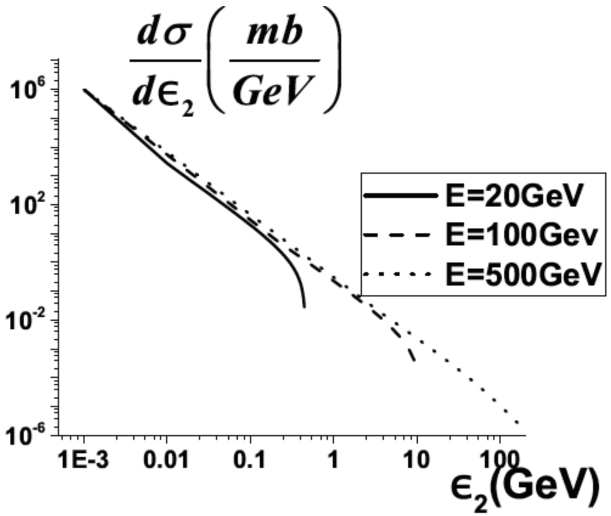
$$\delta_0 = \frac{2\alpha}{\pi}\ln\frac{\bar\omega}{m}\left[\frac{\epsilon_2}{k_2}\ln\left(\frac{\epsilon_2+k_2}{m}\right)-1\right],$$

$$\bar\delta = \frac{\alpha}{\pi}\left\{-1-2\ln 2+\frac{\epsilon_2}{k_2}\left[\ln\left(\frac{\epsilon_2+k_2}{m}\right)\left(1+\ln\left(\frac{\epsilon_2+k_2}{m}\right)+2\ln\left(\frac{m}{k_2}\right)+\frac{m+3\epsilon_2}{2\epsilon_2}-\right.\right.\right.$$
$$\left.-\ln\left(\frac{\epsilon_2+m}{k_2}\right)-\frac{1}{2}\ln\left(\frac{Q^2}{m^2}\right)\right)+4m\frac{M^2q^2}{\epsilon_2\mathcal{D}}\ln\left(\frac{\epsilon_2+k_2}{m}\right)(G_E^2-2\tau G_M^2)-$$
$$\left.\left.-\frac{\pi^2}{6}+Li_2\left(\frac{\epsilon_2-k_2}{\epsilon_2+k_2}\right)+Li_2\left(\frac{\epsilon_2+k_2+m}{2(\epsilon_2+m)}\right)-Li_2\left(\frac{\epsilon_2-k_2+m}{2(\epsilon_2+m)}\right)\right]\right\}.$$

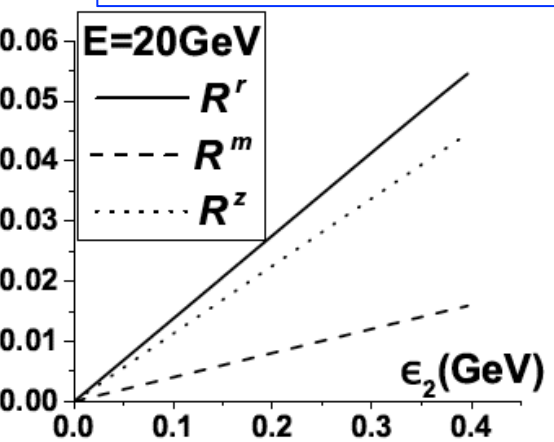
$$\delta^{(\mathrm{vac})} = \frac{2\alpha}{3\pi}\left\{-\frac{5}{3}+4\frac{m^2}{Q^2}+\left(1-2\frac{m^2}{Q^2}\right)\sqrt{1+4\frac{m^2}{Q^2}}\ln\frac{\sqrt{1+4\frac{m^2}{Q^2}}+1}{\sqrt{1+4\frac{m^2}{Q^2}}-1}\right\}.$$

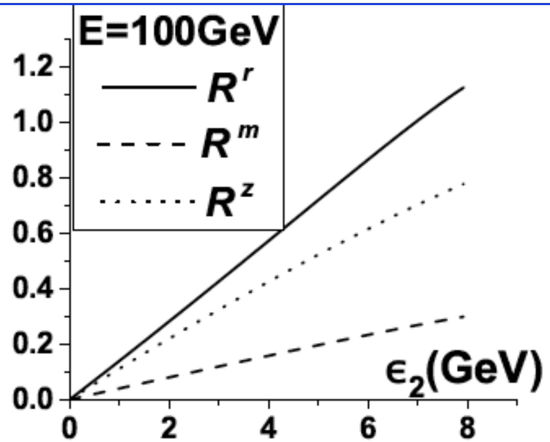
Cross section and FFs

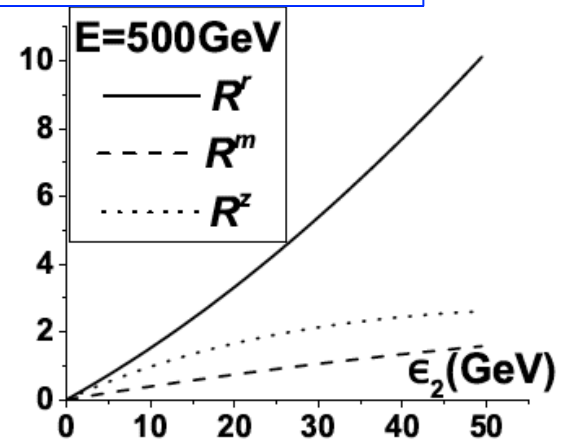
Born cross section with dipole FFs



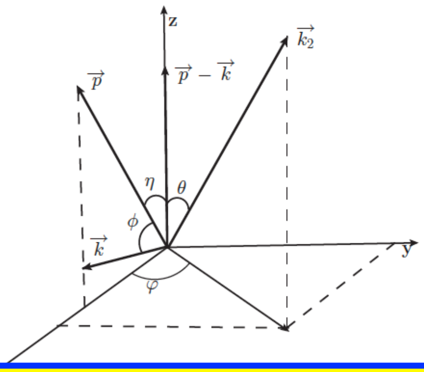
$$R^r = 1 - \frac{d\,\sigma^r}{d\,\sigma^{sd}}\,, \quad R^m = 1 - \frac{d\,\sigma^m}{d\,\sigma^{sd}}\,, \quad R^z = 1 - \frac{d\,\sigma^z}{d\,\sigma^{sd}}\,,$$



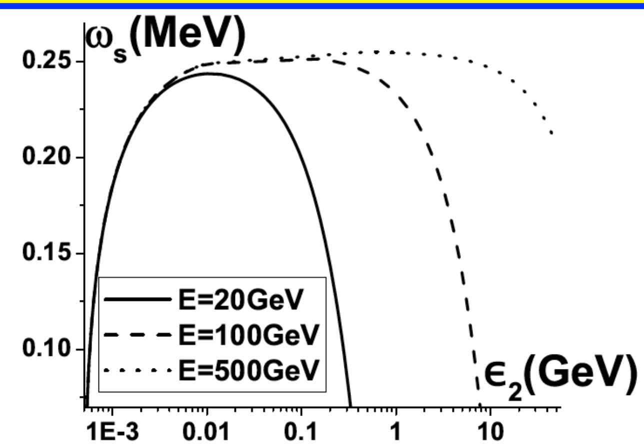




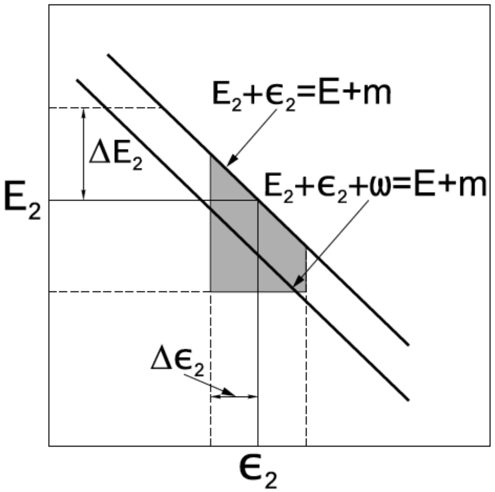
Hard Radiative Corrections (α^3)

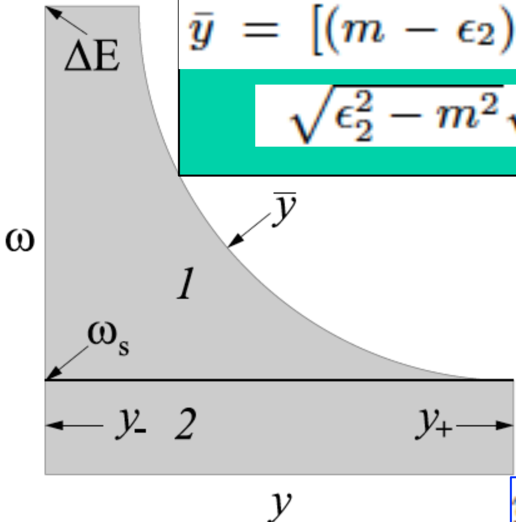


Maximum energy of the hard photon
Emitted in the whole solid angle



Kinematically allowed region
-for proton (E_2) and
- electron (ε_2) energy

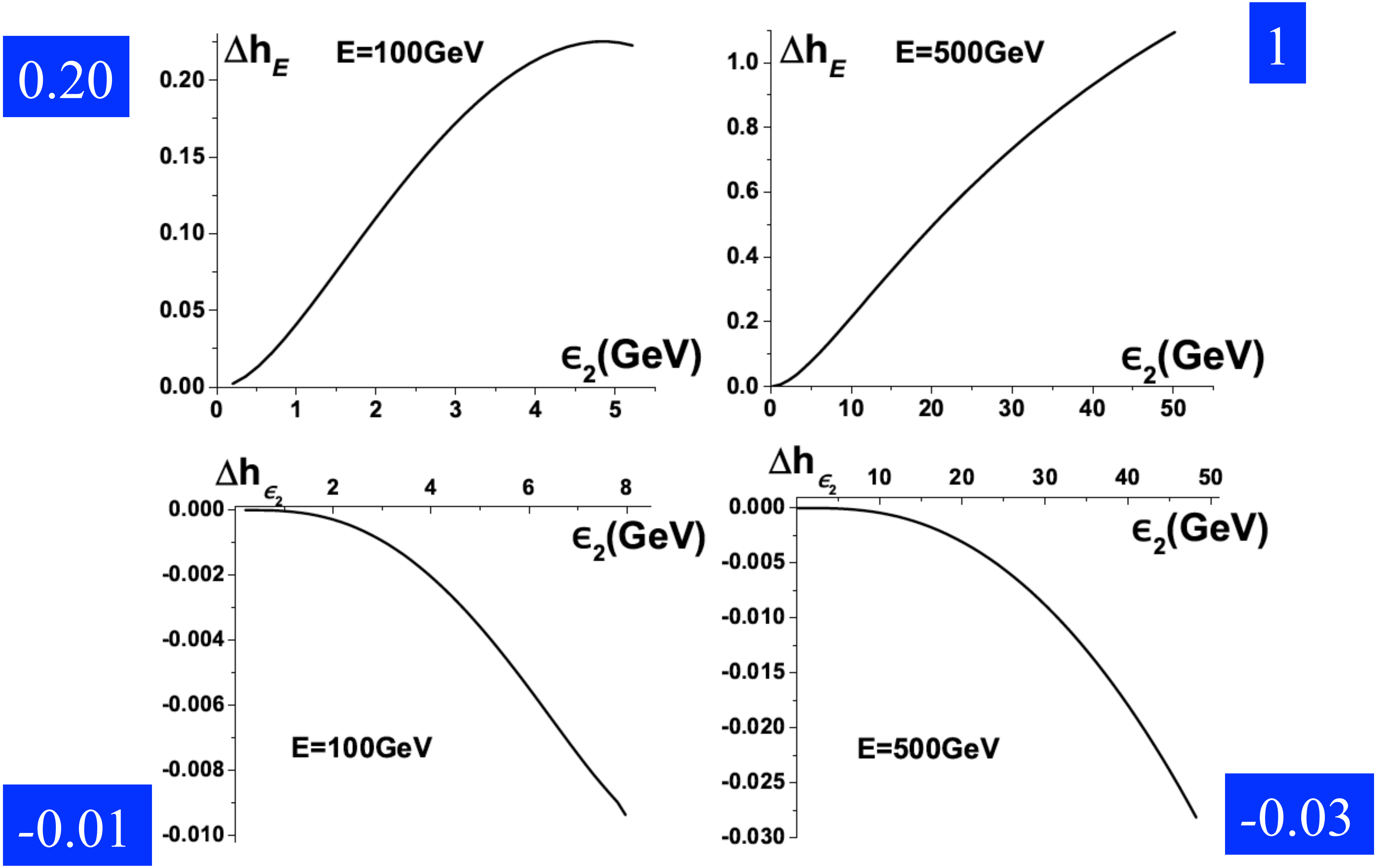




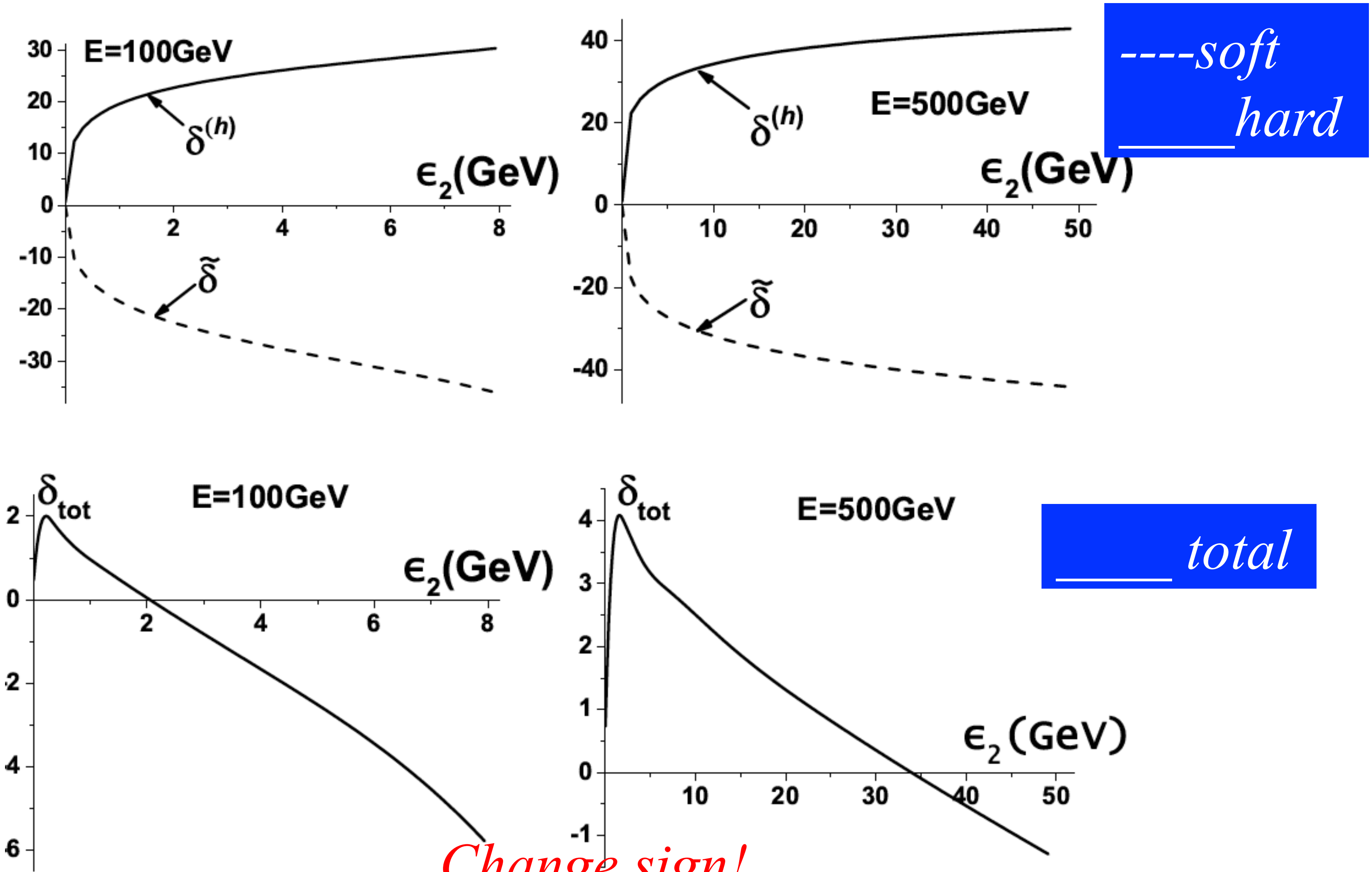
$$\bar{y} = [(m-\epsilon_2)(E-\epsilon_2-\omega) + \sqrt{\epsilon_2^2-m^2}\sqrt{(E+m-\epsilon_2-\omega)^2-M^2}]/\omega.$$

$$y_\pm = E \pm p,$$

Results for Hard Photon Corrections (α^3)



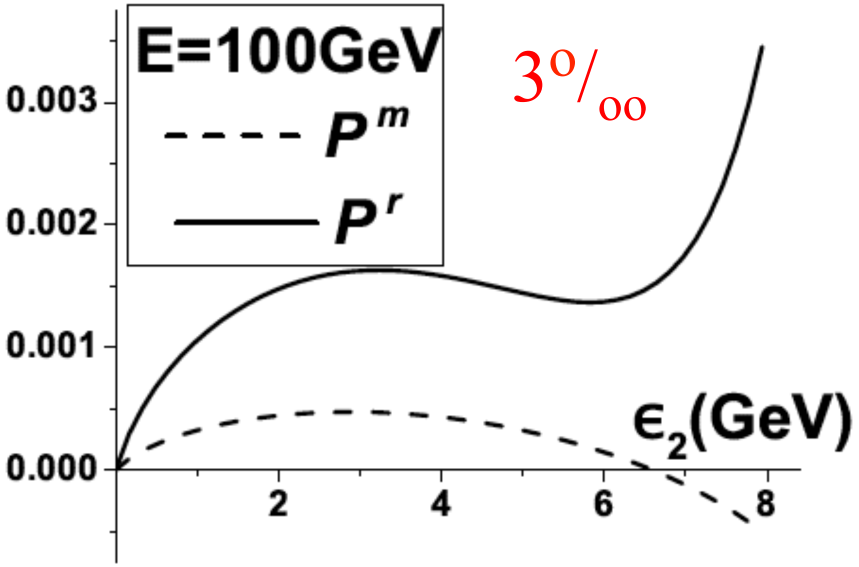
Results for Radiative Corrections (α^3)

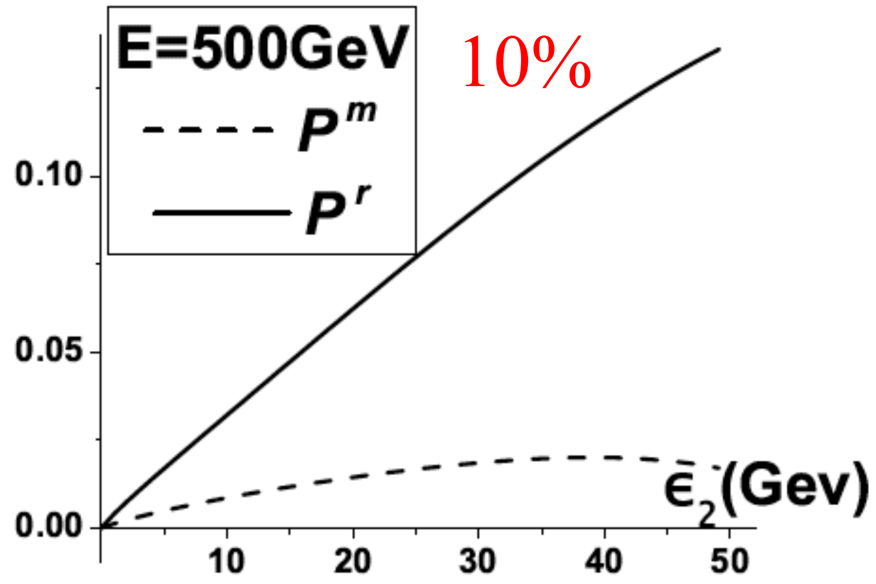


Change sign!

Sensitivity of RC to FFs

$$P^i = \frac{1+\delta^i_{tot}}{1+\delta_{tot}} - 1\,, \quad i = r, m\,,$$



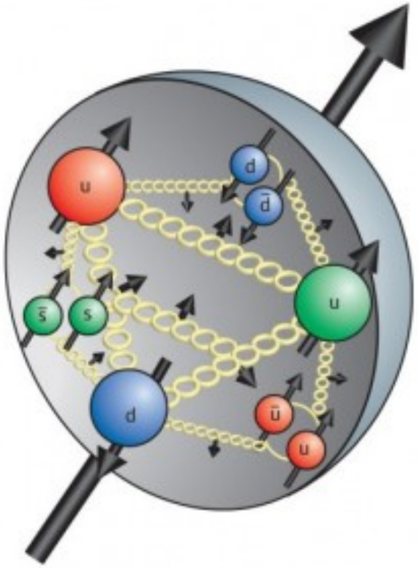


δ_{tot} : *RC for dipole parametrization*

Application

Precise measurement of the proton radius

The SPIN of the proton



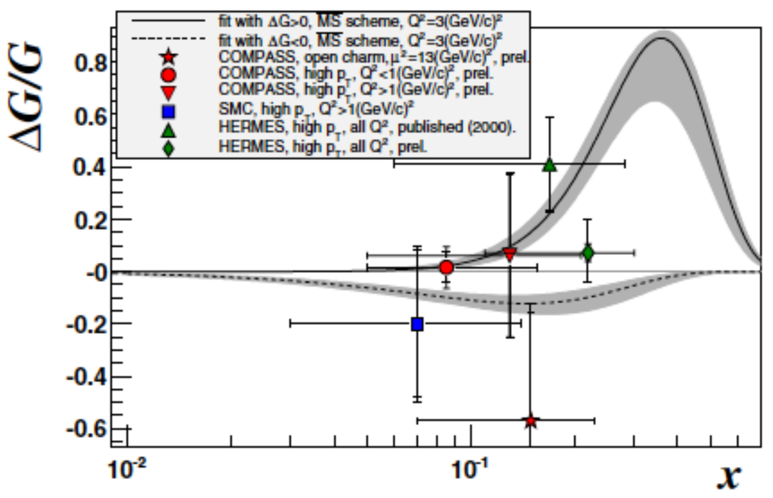
$S = 1/2$

$\Delta\Sigma \quad + \quad \Delta G \quad + \quad L$

Quarks

gluons

orbital momentum



Measured: ~ 1/4

And if …
a proof of quark substructure?

RC : what we learned

- The *sensitivity* of the cross section to FFs *grows with proton beam energy*
- The *hard photon correction* depends on the uncertainty in the energy of the scattered particles
- *Strong cancellation* between the *positive hard correction* and the *negative virtual and soft*: at E=100 GeV $\delta s \sim \delta h \sim 20$ %, but the sum $\delta \sim 6\%$
- Taking into account the proton structure does not change essentially the estimation at so small Q^2
- *Two photon exchange is ~0.1%*
- **Model independent radiative corrections for *pe* elastic scattering have been calculated** *for a cross section measured at permille accuracy.*
- Model dependent corrections are small and can not affect the cross section