Do Dark Matter Axions Form a Condensate with Long-Range Correlation? (Guth, Hertzberg, Prescod-Weinstein, arXiv:1412.5930)

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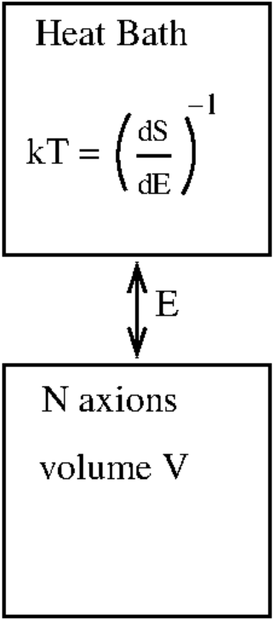
The seminal paper

Cosmic axion thermalization
O. Erken, P. Sikivie, H. Tam and Q. Yang, arXiv:1111.1157

Axions differ from the other cold dark matter candidates in that they form a degenerate Bose gas. It is shown that their huge quantum degeneracy and large correlation length cause cold dark matter axions to thermalize through gravitational self-interactions when the photon temperature reaches approximately 500 eV. When they thermalize, the axions form a Bose-Einstein condensate. ....
The rethermalization of axions that are about to fall in a galactic potential well causes them to acquire net overall rotation as they go to the lowest energy state consistent with the total angular momentum they acquired by tidal torquing. This phenomenon explains the occurrence of caustic rings of dark matter in galactic halos. We find that photons may reach thermal contact with axions.....



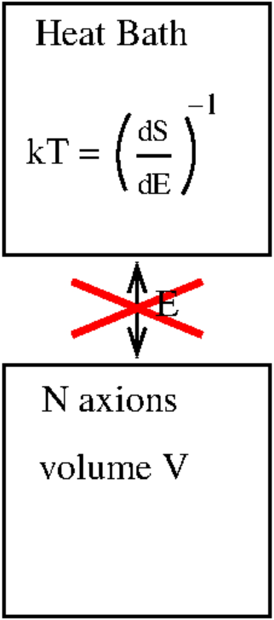
Reminder on B-E condensates.

At low temperature:

$$kT < kT_c \propto \frac{\hbar^2 (N/V)^{2/3}}{m_a}$$

the total entropy (bath + axions) is maximized when N_0 axions are in the ground state

$$N_0 = N(1 - T/T_c)$$



Insulate the Axions from bath.

At low mean energy per axion:

$$\frac{\langle p^2 \rangle}{2m_a} < kT_c \propto \frac{\hbar^2 (N/V)^{2/3}}{m_a}$$

the axion entropy is maximized when N_0 axions are in the g.s.

$$N_0 = N\left(1 - \frac{T_a}{T_c}\right) \qquad T_a^2 = \frac{\langle p^2 \rangle}{2m_a} T_c$$

Collisions between axions necessary for thermalization.

Do Dark Matter Axions Form a Condensate with Long-Range Correlation?

Guth, Hertzberg, and Prescod-Weinstein, arXiv:1412.5930

......We point out that there is an essential difference between the thermalization and formation of a condensate due to repulsive interactions, which can indeed drive long-range order, and that due to attractive interactions, which can lead to localized Bose clumps (stars or solitons) that only exhibit short range correlation. While the difference between repulsion and attraction is not present in the standard collisional Boltzmann equation, we argue that it is essential to the field theory dynamics, and we explain why the latter analysis is appropriate for a condensate. Since the axion is primarily governed by attractive interactions gravitation and scalar-scalar contact interactions we conclude that while a Bose-Einstein condensate is formed, the claim of long-range correlation is unjustified.

Outline

- Classical or Quantum Field?
- The initial axion state.
- Evolution toward thermal equilibrium?
- Consequences of Thermalization to BE condensate?
- Further developments

Classical vs. quantum fields

Classical field:

- Field, e.g. $\phi(x,t)$, is an "element of (classical) reality"
- Solution of a field equation (Maxwell, Klein-Gordan....)
- Modes ($\vec{k}$) have arbitrary amplitudes
- In thermal equilibrium, each mode takes energy kT.

Quantum fields

- Field is an operator on a space of states.
- Only $\langle\phi(x,t)\rangle$ satisfies classical field eqn. (Ehrenfest)
- Modes have discreet squared amplitudes
- In thermal equilibrium, modes with $E_{\vec{k}} \gg kT$ not excited.
- Higher order statistics differ from classical theory: counter coincidences correctly given.

Guth et al treat the axion as a classical field $\Rightarrow$ 3 problems

- In contact with a heat bath (temperature T), the high-$\vec{k}$ modes suck an infinite energy from the bath.

But this problem is manageable for "misalignment" axions since they are not coupled to a heat bath.

- "quantum fluctuations" of ϕ are ignored

But for misalignment axions the phase-space density is enormous (many axions per mode) so the fluctuations should be unimportant.

- While classical fields can correctly give counting rates they give incorrect correlations between detectors.

But this is unimportant for axions because of large density.

Photoelectric effect for classical EM field

Initial state: electron bound by energy E_b.

Perturbation: oscillating classical electric field of frequency ω.

Fermi Golden Rule $\Rightarrow$ Amplitude to eject the electron to a state with energy $E_f > 0$:

$$A(t) \propto \int^t dt \exp i[\omega - (E_b + E_f)/\hbar)]t$$

$A(t)$ increases with time only if $\omega \sim (E_b + E_f)/\hbar$.

Note: particle physicists prefer to replace ω with $\hbar\omega/\hbar = E_\gamma/\hbar$. This leads to the incorrect belief that the photoelectric effect requires photons.

Particles $\rightarrow$ classical field

Particles $\rightarrow$ modes

$$E_a^2 = p_a^2 + m_a^2 \quad \rightarrow \quad \omega_k^2 = c^2 k^2 + w_o^2 \qquad \omega_0 = \frac{m_a c^2}{\hbar}$$

particle velocity $\rightarrow$ mode group velocity

$$v_g(k) = \frac{d\omega}{dk} = \frac{kc}{(k^2 + \omega_0^2/c^2)^{1/2}} \;\rightarrow \frac{kc^2}{\omega_0} = \frac{k\hbar}{m_a}$$

k dispersion $\rightarrow$ coherence length,time

$$\langle \phi(x,t)\phi(x+L,t)\rangle \sim 0 \quad \text{for} \quad L \gg L_{coh} \sim 1/\Delta k$$

(Fourier uncertainty principle)

CMB as a classical EM field

Modes have random phases and random squared amplitudes drawn from Planck distribution with $T = 2.7$K.

$$1/\Delta k \sim \frac{\hbar c}{KT} \quad \Rightarrow \langle \vec{E}(x,t)\vec{E}(x+L,t)\rangle \sim 0 \;\; \text{for} \;\; L >> 1\text{mm}$$

$$\Rightarrow \langle \vec{E}(x,t)\vec{E}(x,t+\Delta t)\rangle \sim 0 \;\; \text{for} \;\; \Delta t >> 1\text{ps}$$

.

Note: this has nothing to do with $\Delta T/T \sim 10^{-5}$, which describes the statistics of $\vec{E} \cdot \vec{E}$ rather than $\vec{E}$.

Galactic axions as a classical scalar field

$$m_a = 10^{-5}\text{eV} \qquad \Rightarrow \omega_0/2\pi = m_a c^2/2\pi\hbar = 2.39\text{Ghz}$$

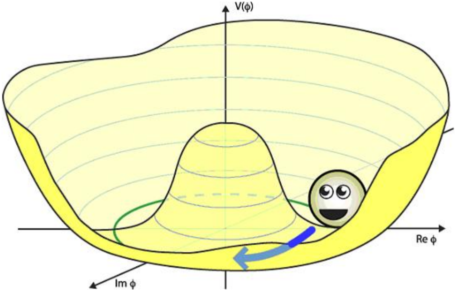
$$\Delta v \sim 10^{-3}c, \qquad \hbar\Delta k = m_a \Delta v$$

$$\Rightarrow L_{coh} \sim 2\pi/\Delta k \sim 2\pi 10^3 \hbar/m_a c \sim 200\text{m}$$

$$\hbar\omega(k) \sim m_a c^2 + \hbar^2 k^2/2m_a$$

$$\Rightarrow t_{coh} \sim 2\pi/\Delta\omega \sim 0.6\text{ms}$$

Misalignment production of axions



$V(\phi)$ initially symmetric: high frequency and zero frequency modes

ϕ_a is zero frequency (Goldstone) mode, starts at arbitrary position at minimum.

At $T \sim \Lambda_{qcd}$, $V(\phi)$ "tilts" and ϕ_a starts to roll and oscillate about minimum.

$m_a = V''(\phi_a)$ at minimum.

Misalignment axions as a classical scalar field

Axion field is coherent over horizon at $T \sim \Lambda_{qcd}$:

$$L_{coh} \sim \frac{c}{\sqrt{G\Lambda_{qxd}^4}} \sim \frac{m_{pl}}{\Lambda_{qcd}^2} \qquad \Rightarrow \Delta k \sim \frac{\Lambda_{qcd}^2}{m_{pl}} \sim 10^{-12}\,\mathrm{eV}$$

All important modes have $v_g \ll c$ if $m_a \gg 10^{-12}\,\text{eV} \Rightarrow$ CDM.

$\Rightarrow$ QCD axions are CDM because they are produced with a large L_{coh}.

Expansion of universe

$$L_{coh}(today) = \frac{m_{pl}}{\Lambda_{qcd}^2} \times \frac{\Lambda_{qcd}}{T_0} \sim \mathrm{pc}$$

(in absence of structure formation and/or thermalization)

Initial axions: orders of magnitude

wave number dispersion: $\Delta k \sim \Lambda^2_{\rm qcd}/m_{\rm Pl}$

velocity dispersion: $\Delta k/m_a \sim \Lambda^2_{\rm qcd}/m_a m_{\rm Pl}$

axion-photon ratio:

$$\eta_a \sim \eta_a(today) \sim (\Omega_a/m_a)(\Omega_\gamma/T_0) \sim 4000\,T_0/m_a$$

axion number density: $n_a \sim \eta_a \Lambda^3_{\rm qcd}$

axion phase-space density: $f_a = n_a/(\Delta k)^3 \sim 10^{64}$

$f_a \gg 1$ is equivalent to $\langle p^2/2m_a \rangle \ll T_c$
Therefore, if the axions thermalize they will form a BEC.

System evolution: Boltzmann eqn. or wave eqn.?

Boltzmann:

- governs density in phase space of particles.
- Uses squared amplitudes for scattering (cross sections)
- By including stimulated emission and Fermi blocking, evolution to Planck, Bose-Einstein or Fermi-Dirac distribution is automatic.

Boltzmann term for $(1,2) \leftrightarrow (1'2')$:

$$\frac{D}{Dt}\mathcal{N}_{p_1} = \int \frac{d^3p_2}{(2\pi)^3}\, d\sigma\, v_{\rm rel}\Big[\mathcal{N}_{p_1'}\mathcal{N}_{p_2'}(1+\mathcal{N}_{p_1})(1+\mathcal{N}_{p_2}) - \mathcal{N}_{p_1}\mathcal{N}_{p_2}(1+\mathcal{N}_{p_1'})(1+\mathcal{N}_{p_2'})\Big] \quad (80)$$

System evolution: Boltzmann eqn. or wave eqn.?

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- governs density in phase space of particles.
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Wave:

- governs a wave amplitude but ignores quantum fluctuations
- non-linear terms couple modes
- approach to thermal equilibrium not clear (to me).

In both cases we expect thermal equilibrium to obtain if the characteristic time for system evolution is shorter than the Hubble time.

Guth et al.'s procedure (1) the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \quad (4)$$

In order to take the non-relativistic limit, let's re-write the real field ϕ in terms of a complex field ψ as follows

$$\phi(\mathbf{x}, t) = \frac{1}{\sqrt{2m}}\left(e^{-imt}\psi(\mathbf{x}, t) + e^{imt}\psi^*(\mathbf{x}, t)\right) \quad (5)$$

We substitute this into eq. (1) and dispense with terms that go as powers of e^{-imt} and e^{imt}, as they are rapidly varying and average out to approximately zero. We then obtain the following non-relativistic Lagrangian for ψ

$$\mathcal{L} = \frac{i}{2}(\dot{\psi}\psi^* - \psi\dot{\psi}^*) - \frac{1}{2m}\nabla\psi^*\cdot\nabla\psi - \frac{\lambda}{16m^2}(\psi^*\psi)^2 \quad (6)$$

Guth et al.'s procedure (2) The field eqn.

Then using the Hamilton-Jacobi equations, we obtain the following approximate equation of motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|} \quad (23)$$

This is rather more complicated than the standard one-particle Schrödinger equation; this equation is non-linear and non-local.

......

For the case of self-interaction, the equation governing the evolution of modes is

$$i\,\dot{\psi}_k = \frac{k^2}{2m}\psi_k + \frac{\lambda}{8m^2}\int\frac{d^3k'}{(2\pi)^3}\int\frac{d^3k''}{(2\pi)^3}\,\psi_{k'}\psi^*_{k''}\psi_{k+k''-k'} \quad (74)$$

Evolution from nearly homogeneous field

$$i\,\dot{\psi}_k = \frac{k^2}{2m}\psi_k + \frac{\lambda}{8m^2}\sum_{k'}\sum_{k''}\psi_{k'}\psi^*_{k''}\psi_{k+k''-k'}$$

(Replaced integrals by sums to emphasize that one has to be careful with the discrete low-k modes.)
For nearly homogeneous initial conditions, the sum is dominated by by (k', k'') such that two of $(k', k'', k + k'' - k')$ are zero.

At small $k^2 < |\lambda| n_a/2m_a$ the interaction term dominates.
For $\lambda < 0$ (attractive axion-axion interaction) this leads to growth of the perturbation causing the system to collapse. Because of this, no large-scale correlation can be established. (according to Guth et al.).

(Attractive) Gravity causes low-k modes collapse

So again we find instability for a condensate of long-range correlation. Modes that satisfy

$$k < k_J = (16\pi G m^3 n_0)^{1/4} \qquad (57)$$

are unstable. Here k_J is a type of Jeans wavenumber, as it separates the regime where gravity dominates, leading to collapse, and the regime where pressure dominates, leading to oscillations. This pressure is, from the particle point of view, a type of "quantum pressure," arising from the uncertainty principle: even though the background particles are at rest, a perturbation of wavelength $2\pi/k$ implies that at least some of the particles are localized on this distance scale, requiring an increase in the energy, with the accompanying restoring force.

The interaction rate

Guth et al.'s estimate of the effective interaction rate:

$$\Gamma_k \sim \frac{8\pi G\, m^2\, n_{\rm ave}}{k^2} \tag{1}$$

It is linear in G (as first noted by Sikivie et al) and becomes greater than the expansion rate at late time. However, Guth et al claim the thermalization is to "localized clumps" rather than to a system with large correlation length.

Possible Consquences of a large correlation length

Galactic caustics:

Evidence for Ring Caustics in the Milky Way: Sikivie, 0109296.

Effects of a caustic ring of dark matter on the distribution of stars and interstellar gas, Chakrabarty & Sikivie, 1808.00027

Testing the Dark Matter Caustic Theory Against Observations in the Milky Way, Dumas et al , 1508.04494

Baryon cooling:

Axion dark matter and the 21-cm signal, Sikivie, 1805.05577

Natural explanation for 21cm absorption signals via axion-induced cooling, Yang et al, 1805.04426

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