

# Weak Binding Effects in Nuclear Structure: $^{40}\text{Mg}$ and The Kerman Problem in the Continuum

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*Recent advances on proton-neutron pairing, session II  
2-6 September 2019*



Work supported under contract number DE-AC02-05CH11231.

## Part 1

Short introduction

Spectroscopy of  $^{40}\text{Mg}$  at RIKEN

Some technical details and results

Qualitative interpretation of the spectra

Weak binding effects

## Part 2

What is the Kerman's Problem anyway?

What we did and why

Some (preliminary) results



## **Part 1**

Short introduction

Spectroscopy of  $^{40}\text{Mg}$  at RIKEN

Some technical details and results

Qualitative interpretation of the spectra

Weak binding effects

# Elusive magic numbers

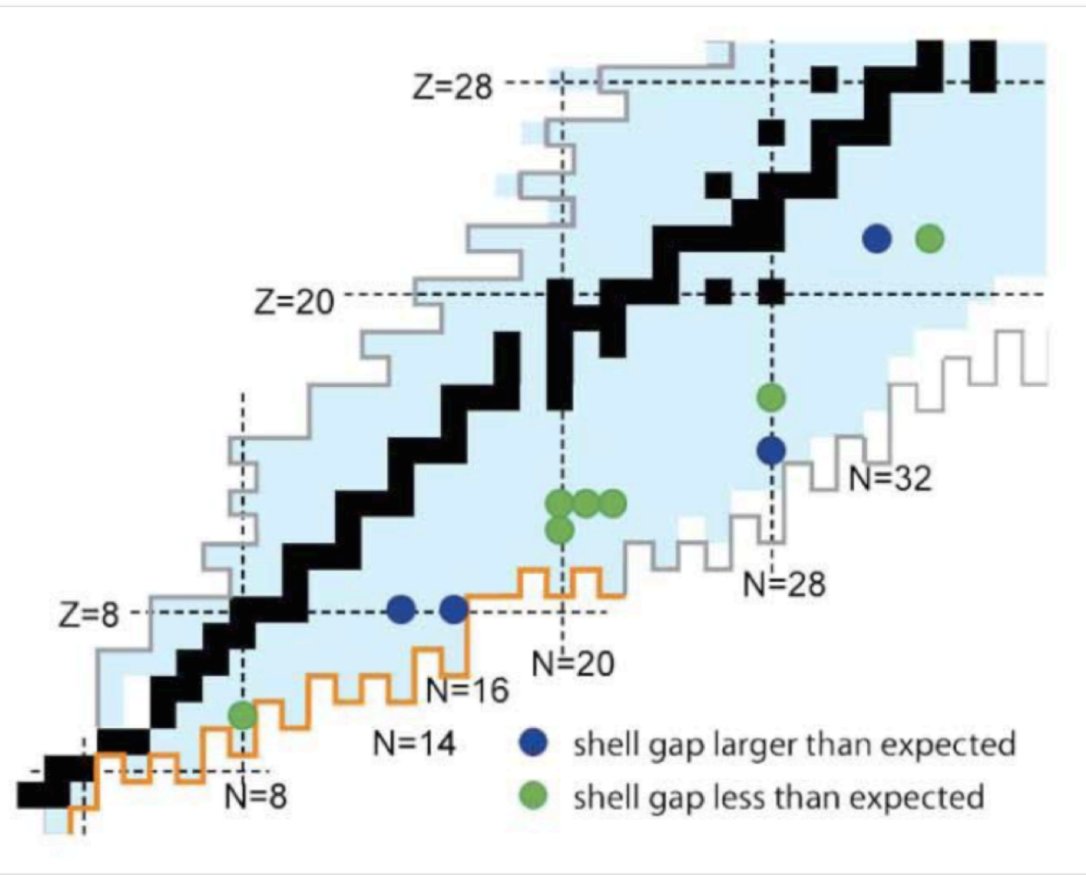
Robert V. F. Janssens

Standard magic numbers are generally correct only for stable and near stable isotopes

Experimental studies of new isotopes has given insight into the role of tensor and 3-body forces in nuclei

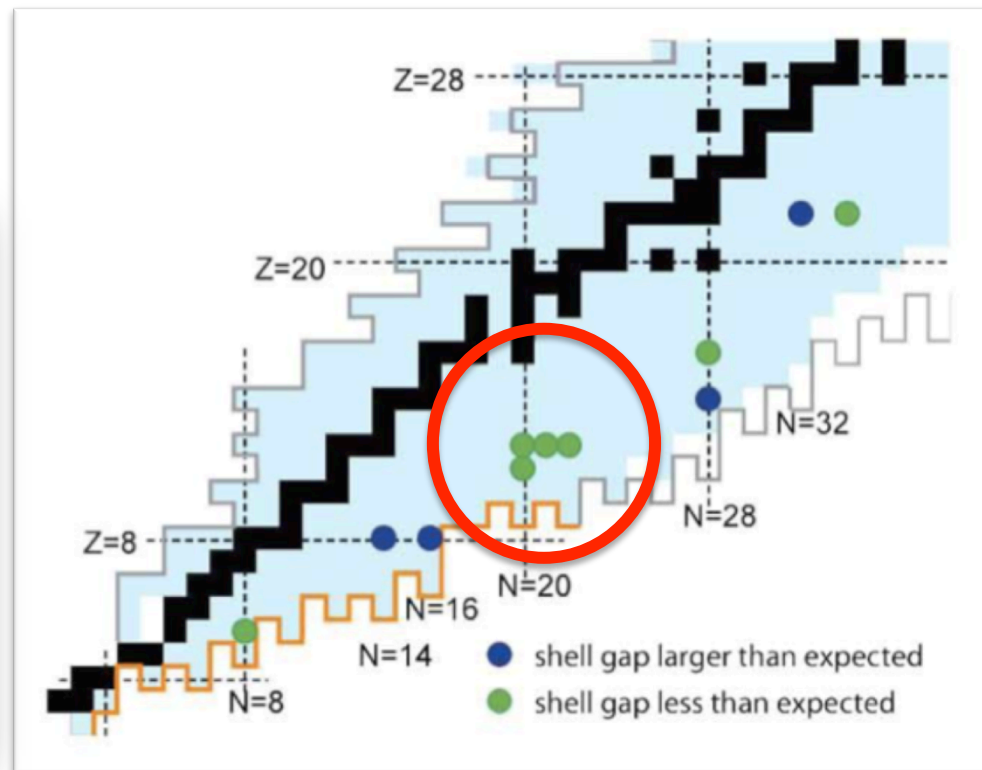
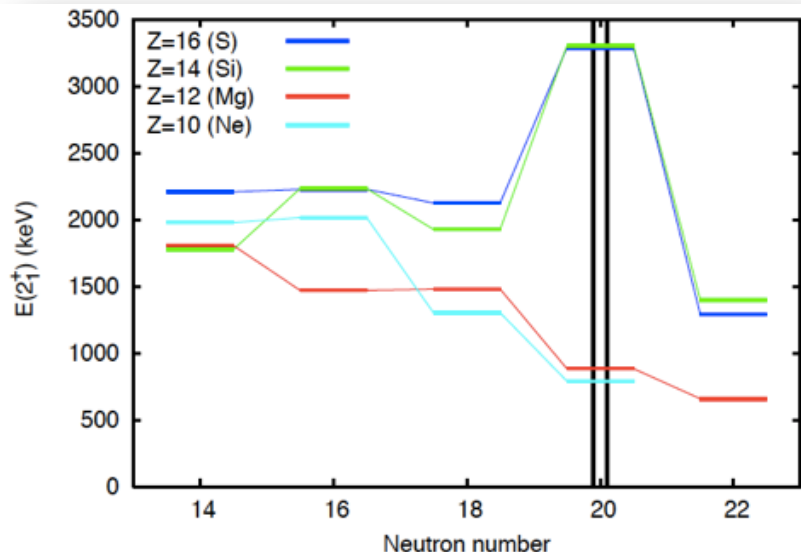
N/Z (Isospin) dependence

Role of weak binding and coupling to the continuum



# Evolution of Shell Structure and Collectivity

## N=20 shell gap



A. Poves and J. Retamosa, Phys. Lett. B 184, 311 (1987).

E.K. Warburton, J.A. Becker and B.A. Brown, Phys. Rev. C 41, 1147 (1990).

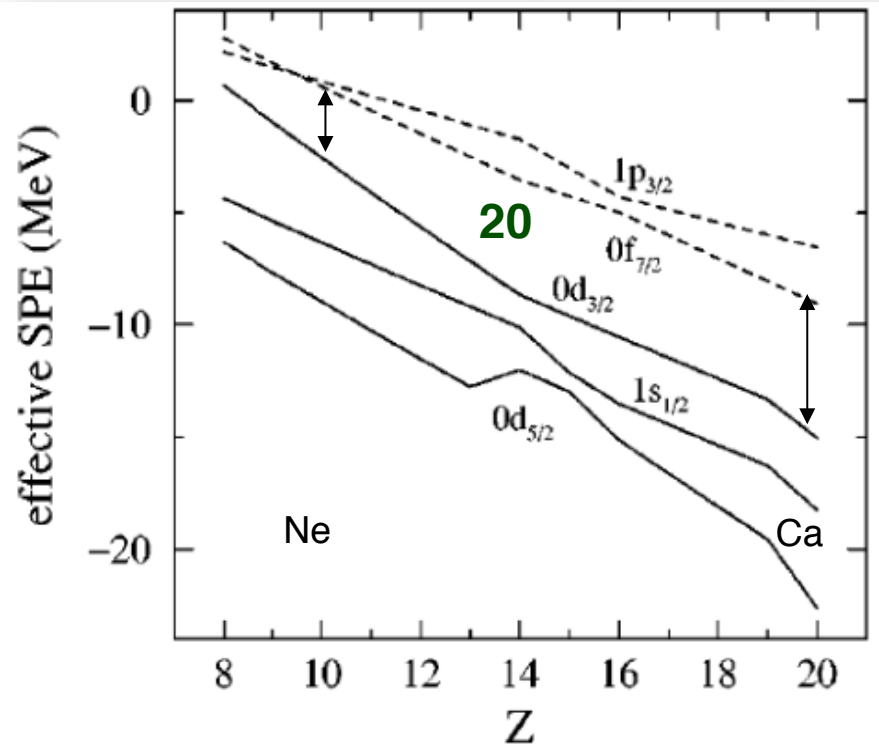
T. Otsuka *et al.*, Phys. Rev. Lett. **87**, 082502 (2001).

O. Sorlin and M. Porquet, Prog. Part.Nucl. Phys. **61**, 602 (2008).

K. Heyde and J. L. Wood, Rev. Mod. Phys. **83**, 1467 (2011).

# Evolution of Shell Structure and Collectivity

Y.Utsuno et al PRC 60 (1999) 011301(R)



**N=20 shell gap**

Role of the  $\pi d_{5/2} - \nu d_{3/2}$  interaction



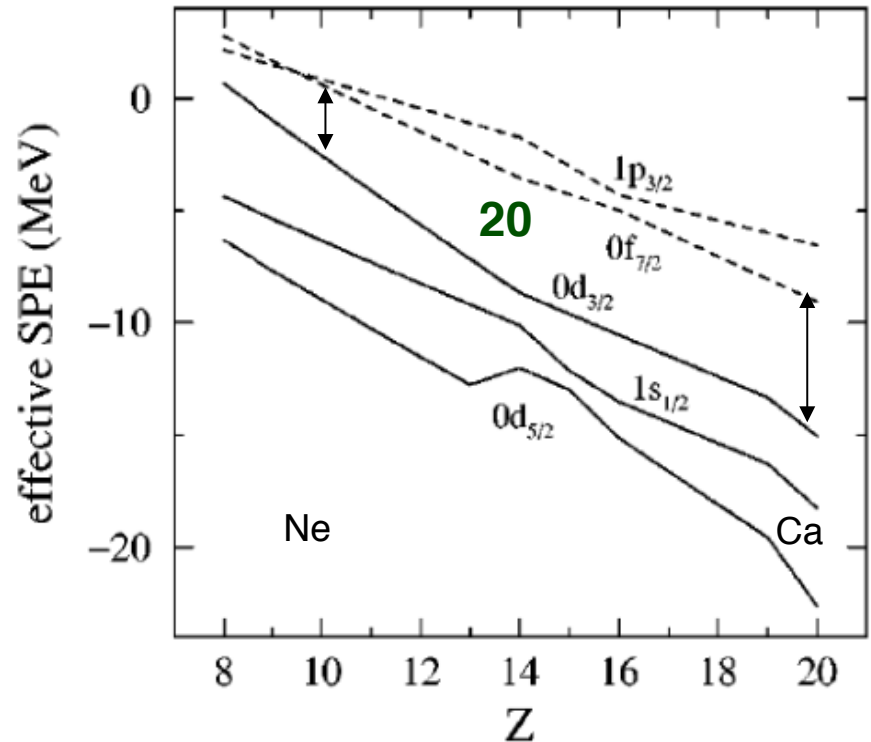
$\Delta l = \Delta j = 2$   
 $\rightarrow$  Quadrupole Correlations

A delicate balance between the monopole field and correlations.

$$H = E_{sp} + GP^{\dagger}P + xQ.Q$$

# Evolution of Shell Structure and Collectivity

Y.Utsuno et al PRC 60 (1999) 011301(R)



N=20 shell gap

Role of the  $\pi d_{5/2} - \nu d_{3/2}$  interaction

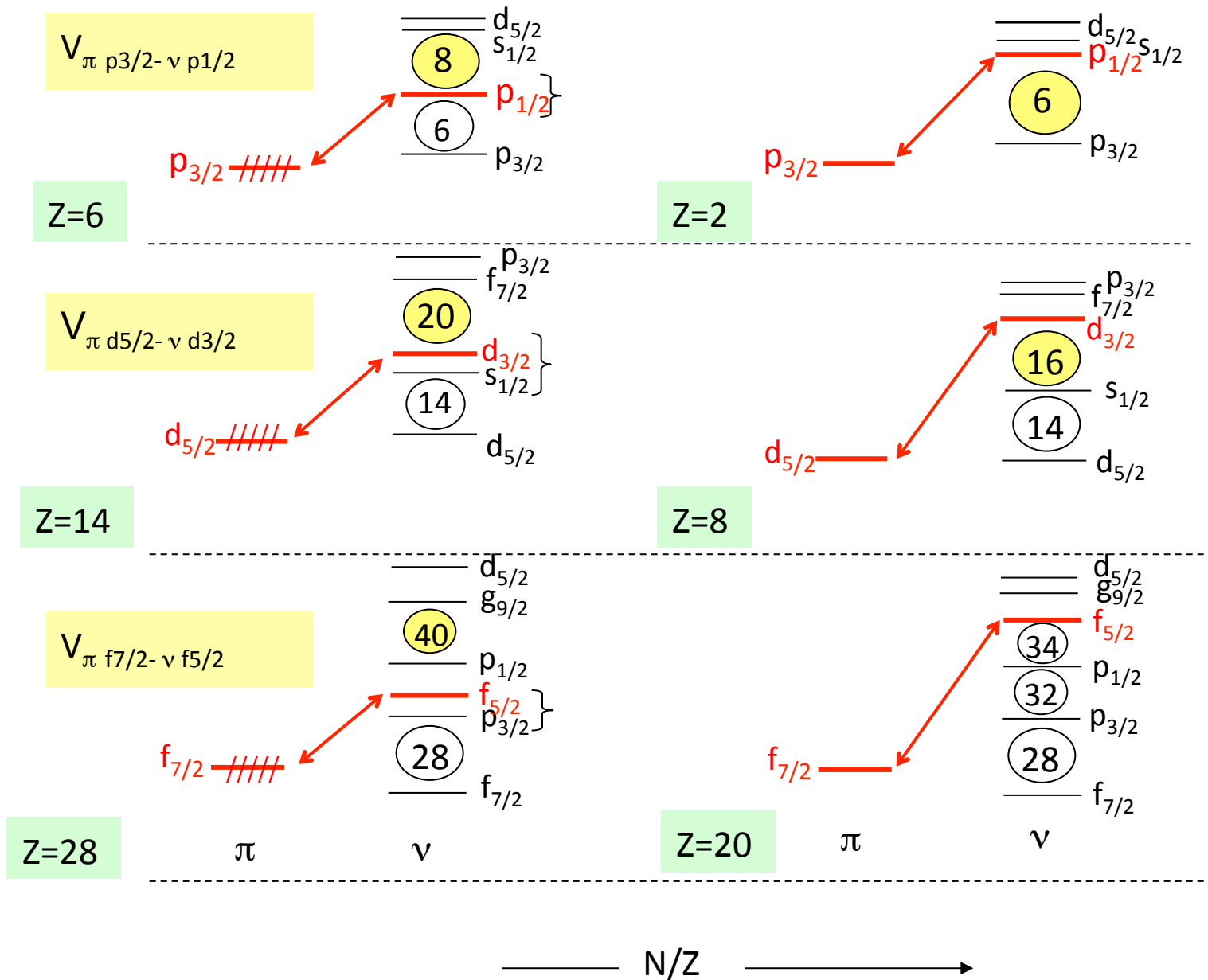
$$\Delta l = \Delta j = 2$$

→ Quadrupole  
Correlations

Or in the words of Andres Zuker:

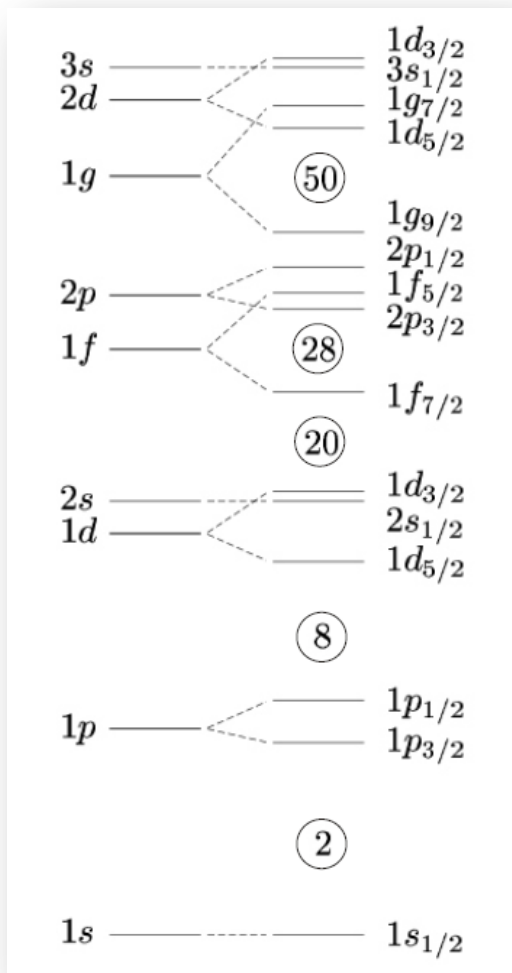
*"Pairing plus Quadrupole propose and Monopole disposes"*

Coherent and Random Hamiltonians, CRN Preprint 1994

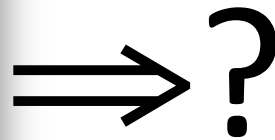




# “Exotic” Shell Structure and Collectivity

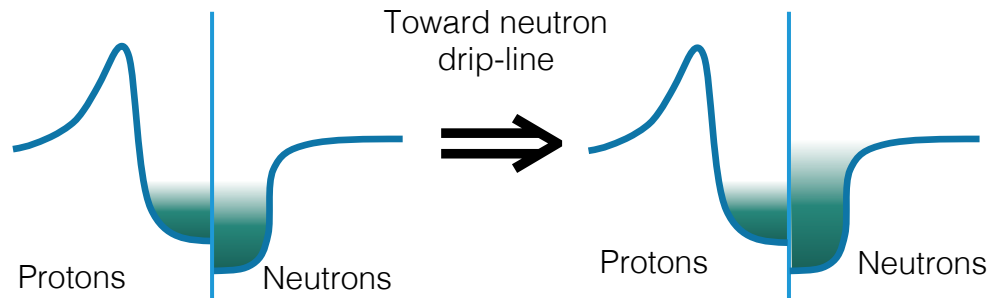


Near the valley of  $\beta$  stability



A driving question in nuclear science:

Is the shell-model description static across the entire chart of nuclides?



Approaching the  
drip-lines

## Weakly bound systems

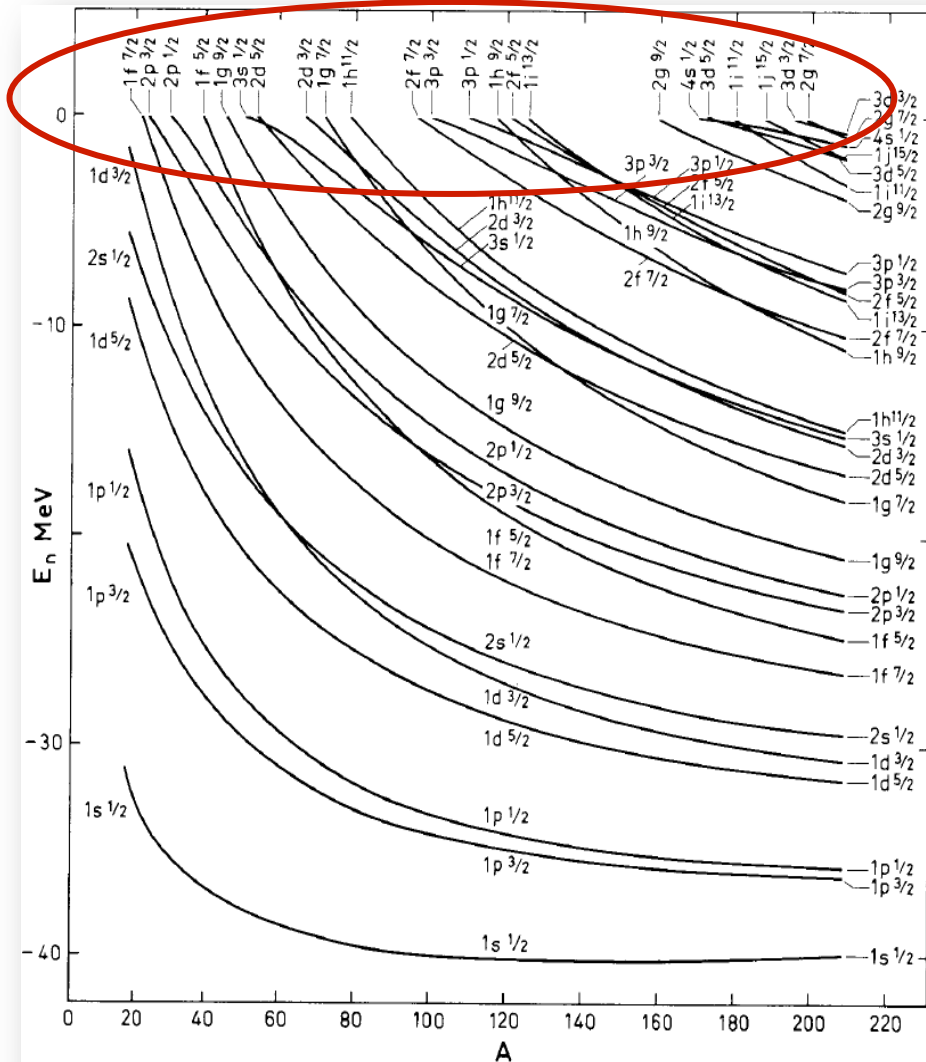
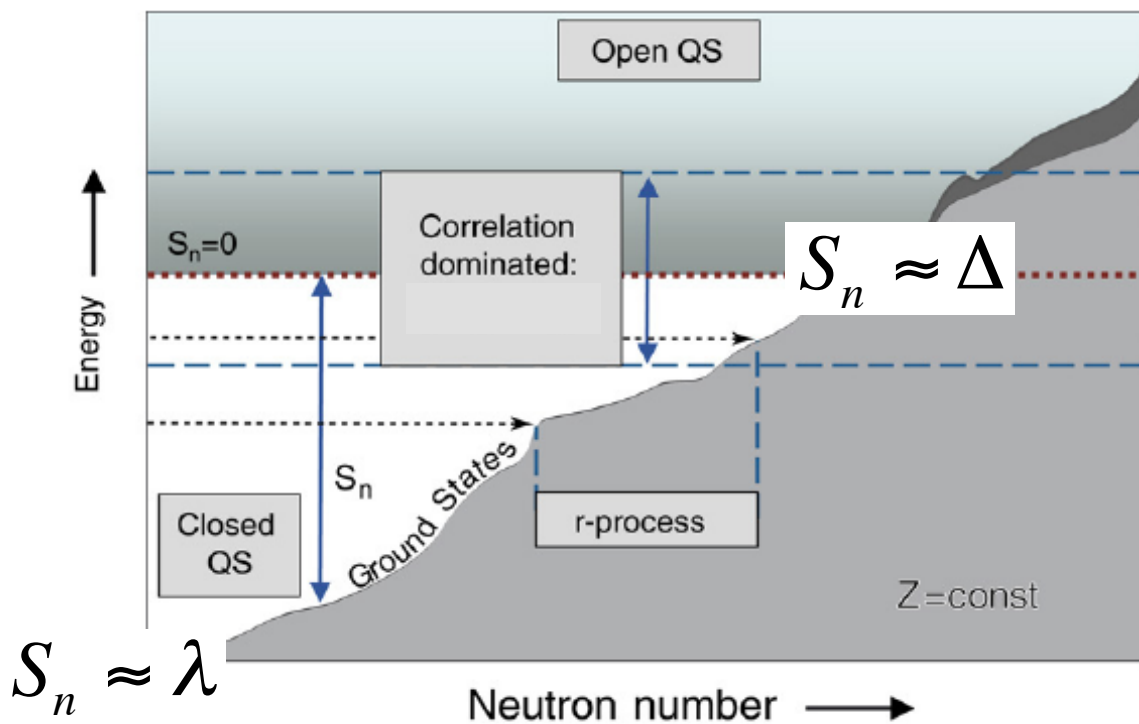


Figure 2-30 Energies of neutron orbits calculated by C. J. Veje (private communication).

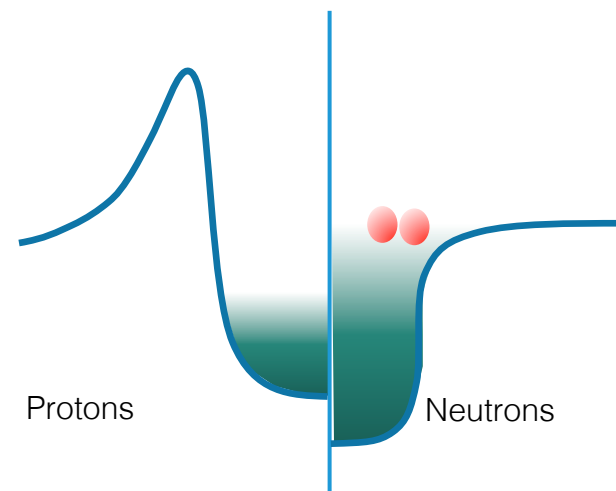
- low  $l$  levels ( $s, p$ )  $\rightarrow$  extended wavefunctions (“halos”)
- Valence nucleons can become decoupled from the core
- Coupling to continuum states

# Weakly bound systems

*J. Dobaczewski et al. / Progress in Particle and Nuclear Physics 59 (2007) 432–445*



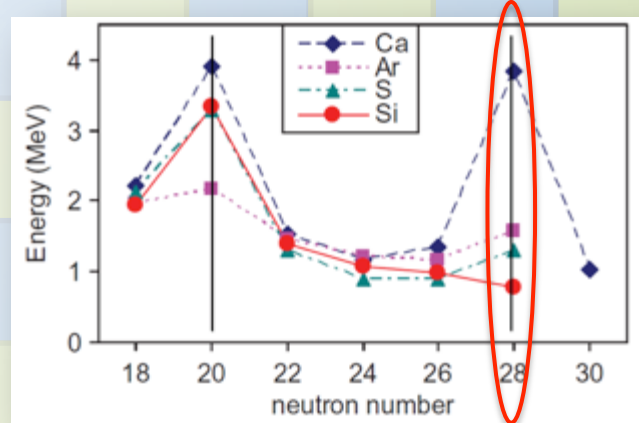
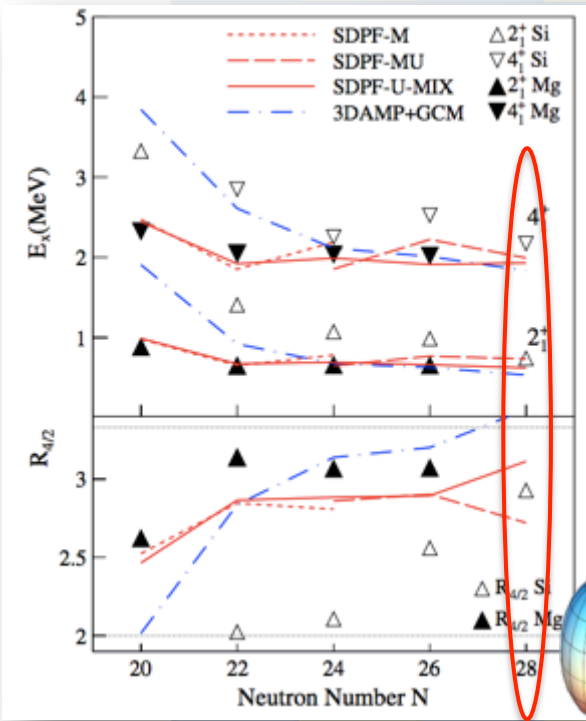
$$S_n \approx \Delta + \lambda$$



$$\lambda / \Delta \approx 1$$

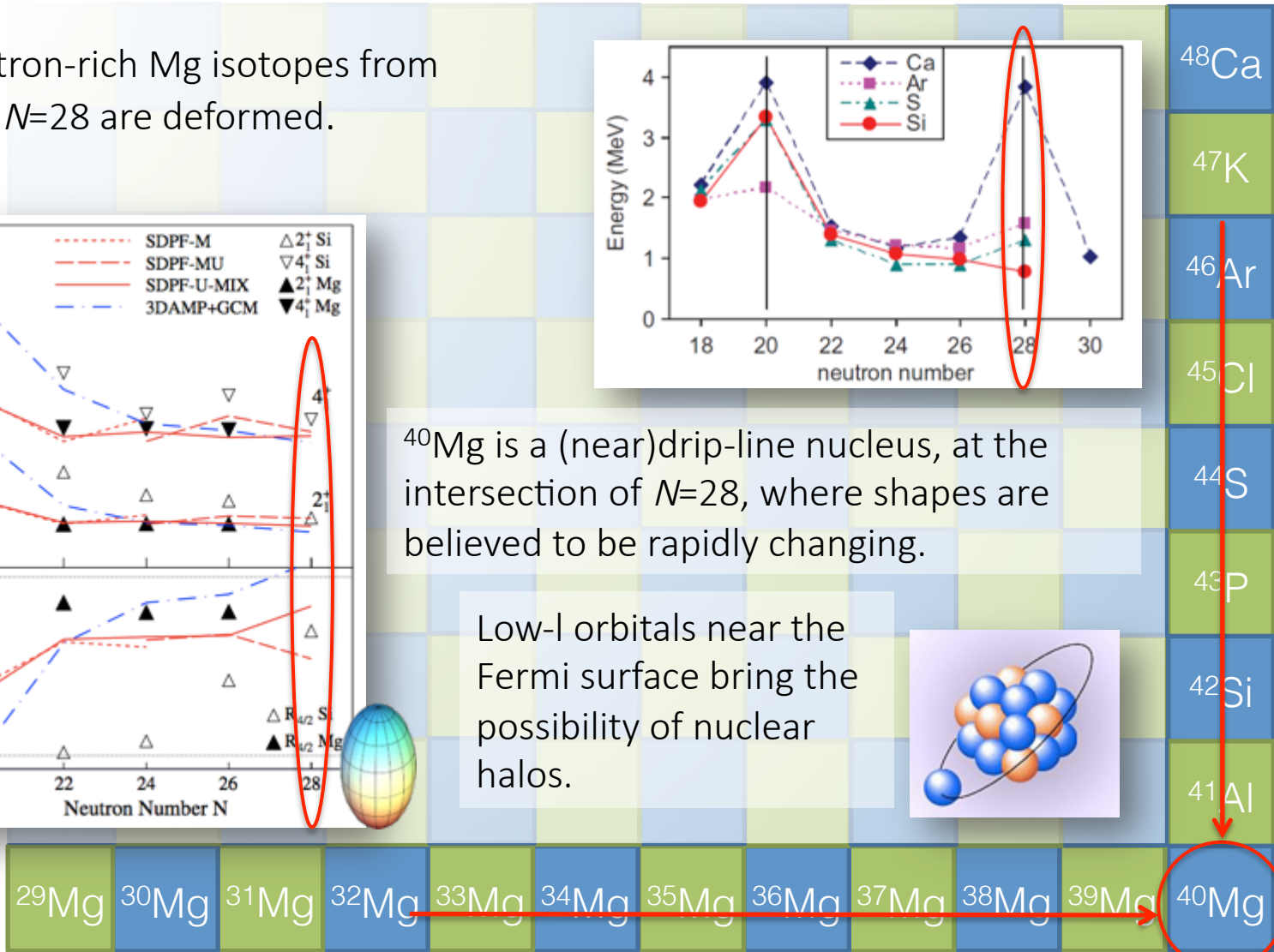
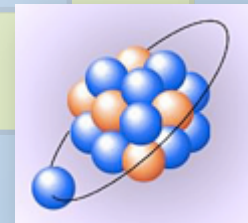
# What to expect in $^{40}\text{Mg}$

The neutron-rich Mg isotopes from  $N=20$  to  $N=28$  are deformed.



$^{40}\text{Mg}$  is a (near)drip-line nucleus, at the intersection of  $N=28$ , where shapes are believed to be rapidly changing.

Low- $l$  orbitals near the Fermi surface bring the possibility of nuclear halos.



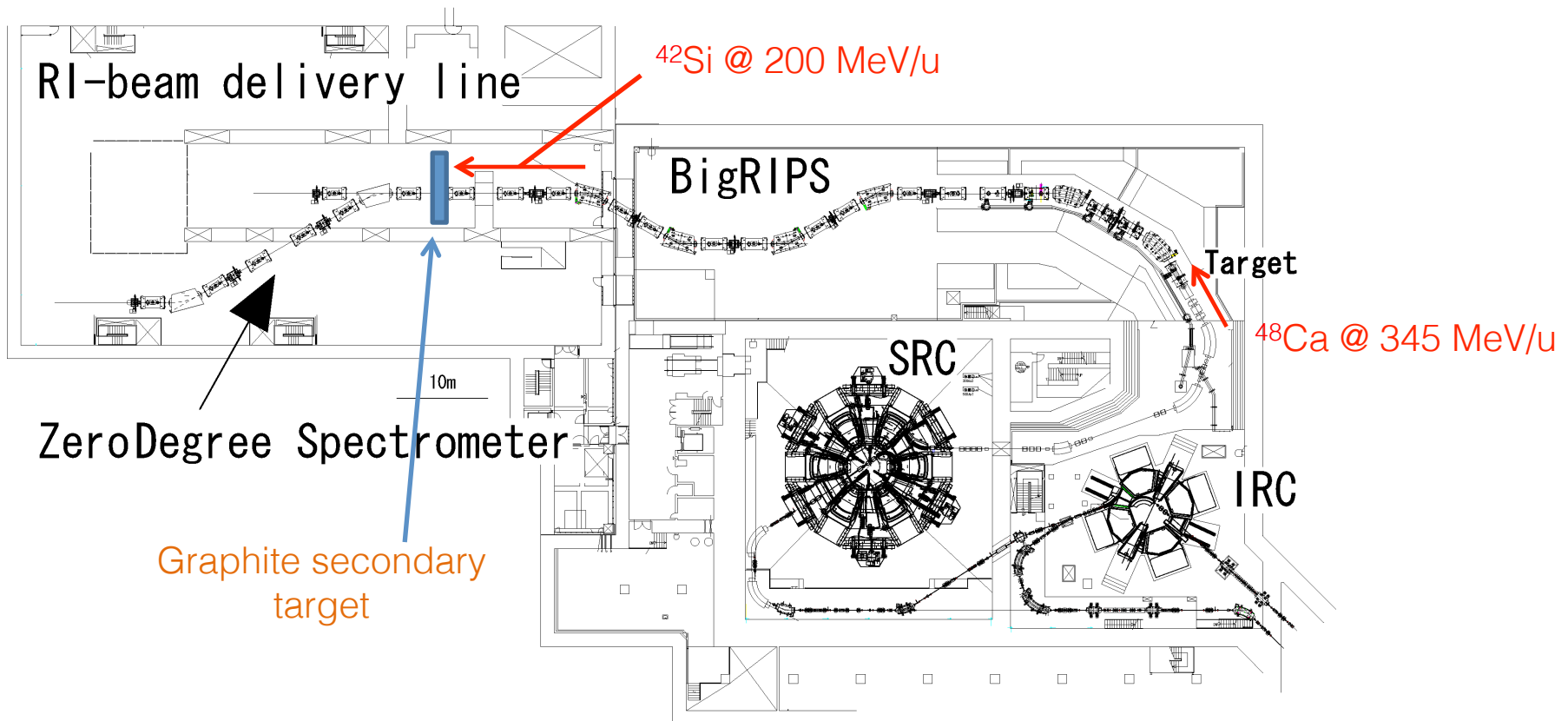
**$N=28$**

Gaodefroy and Grevy, Nucl. Phys. News **20**, 13 (2010); Li *et al.*, PRC **84**, 054304 (2011).  
Nowacki and Poves, PRC **79**, 014310 (2009); Doornenbal *et al.*, PRL **111**, 212502 (2013)

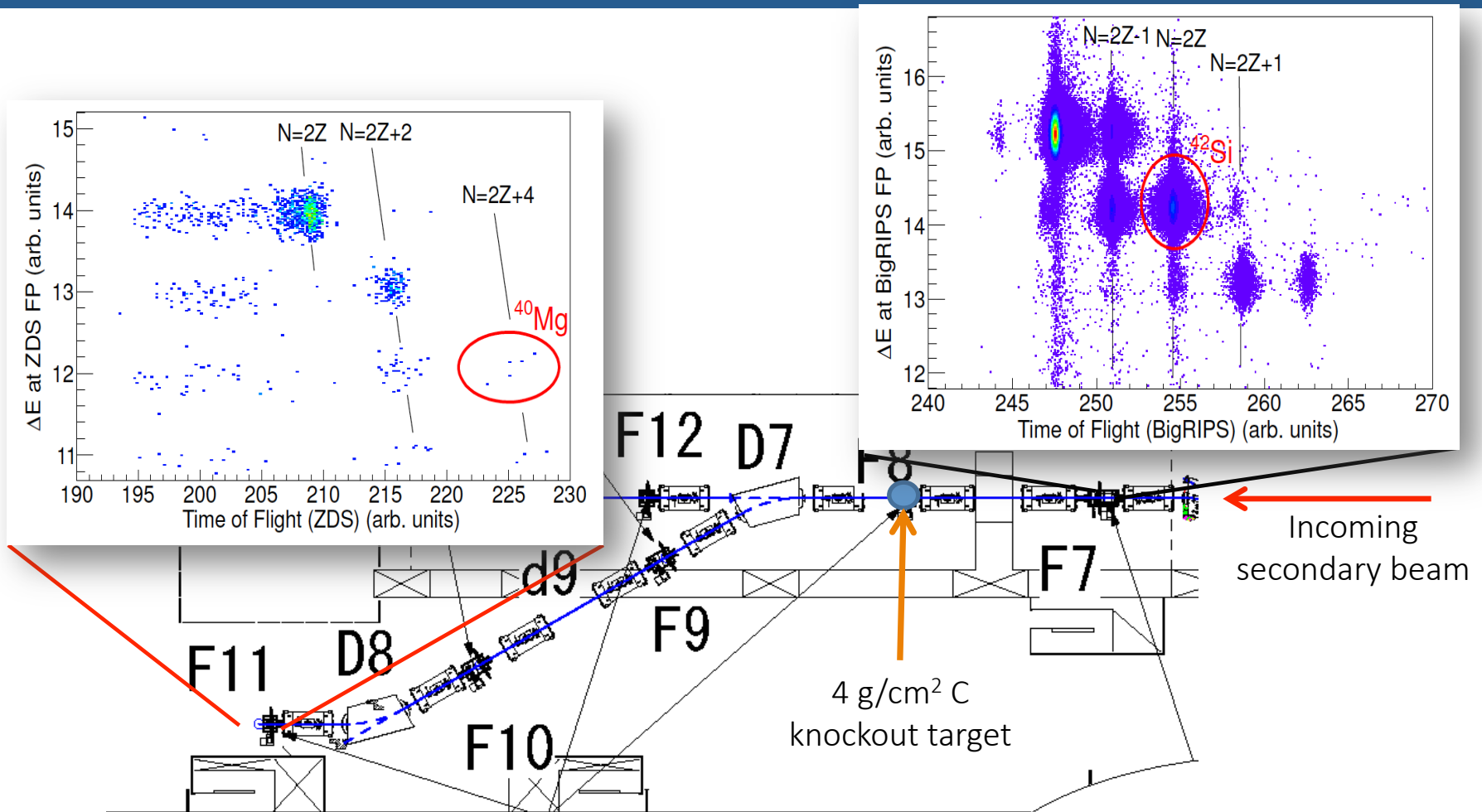
# A brief history

December 2010 – Sunday Campaign (NP1312-RIBF03)

$^{42}\text{Si}$  produced at a rate of 25 pps/100 pA following fragmentation of a high-intensity  $^{48}\text{Ca}$  primary beam at RIBF in RIKEN



# 2p Knockout: $^{42}\text{Si} \Rightarrow ^{40}\text{Mg}$



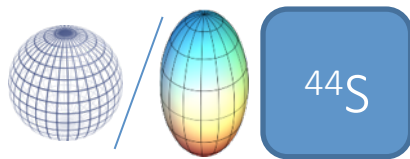
- Approximately 10 hours of beam-on-target
- 5 events of  $^{40}\text{Mg}$  observed -- measured inclusive  $\sigma_{(-2p)}$  of  $40(18) \mu\text{b}$

# N = 28 Coexisting Shapes

Calculations and data indicate that the low-energy structure in  $^{44}\text{S}$ ,  $^{42}\text{Si}$ , and  $^{40}\text{Mg}$  is dominated by two major, co-existing configurations:

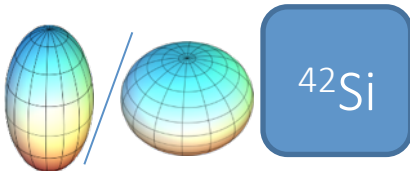
Spherical and Prolate in  $^{44}\text{S}$ , Oblate and Prolate in  $^{42}\text{Si}$  and  $^{40}\text{Mg}$ .

This suggests that a two-state(shape) mixing model can provide a description of their structure:



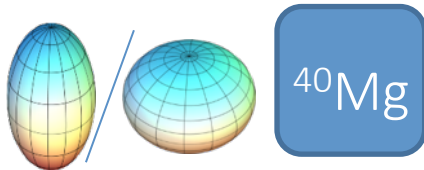
$$|^{44}\text{S}, 0_1^+\rangle = 0.35 |0^+; \text{S}\rangle + 0.94 |0^+; \text{P}\rangle$$

Force et al., Phys. Rev. Lett. 105, 102501 (2010).



$$|^{42}\text{Si}, 0_1^+\rangle = +\alpha |0^+; \text{O}\rangle + \beta |0^+; \text{P}\rangle$$

$$|^{42}\text{Si}, 0_2^+\rangle = -\beta |0^+; \text{O}\rangle + \alpha |0^+; \text{P}\rangle$$



$$|^{40}\text{Mg}, 0_1^+\rangle = +\gamma |0^+; \text{O}\rangle + \delta |0^+; \text{P}\rangle$$

$$\mathcal{R} = \frac{\overbrace{[\sigma_{42}(0_1^+) + \sigma_{42}(0_2^+)]}^{^{44}\text{S}-2p}}{\underbrace{\sigma_{40}(0_1^+)}^{^{42}\text{Si}-2p}}$$

# $\sigma$ Ratios to Constrain “Shape” Amplitudes

Cross-section ratio  $\mathcal{R}$  plotted as a function of the prolate component (probability) in the  $^{42}\text{Si}$  ( $\alpha^2$ ) and  $^{40}\text{Mg}$  ( $\beta^2$ ) ground-state wave functions.

$$|^{42}\text{Si}, 0_1^+\rangle = +\alpha |0^+; \text{O}\rangle + \beta |0^+; \text{P}\rangle$$

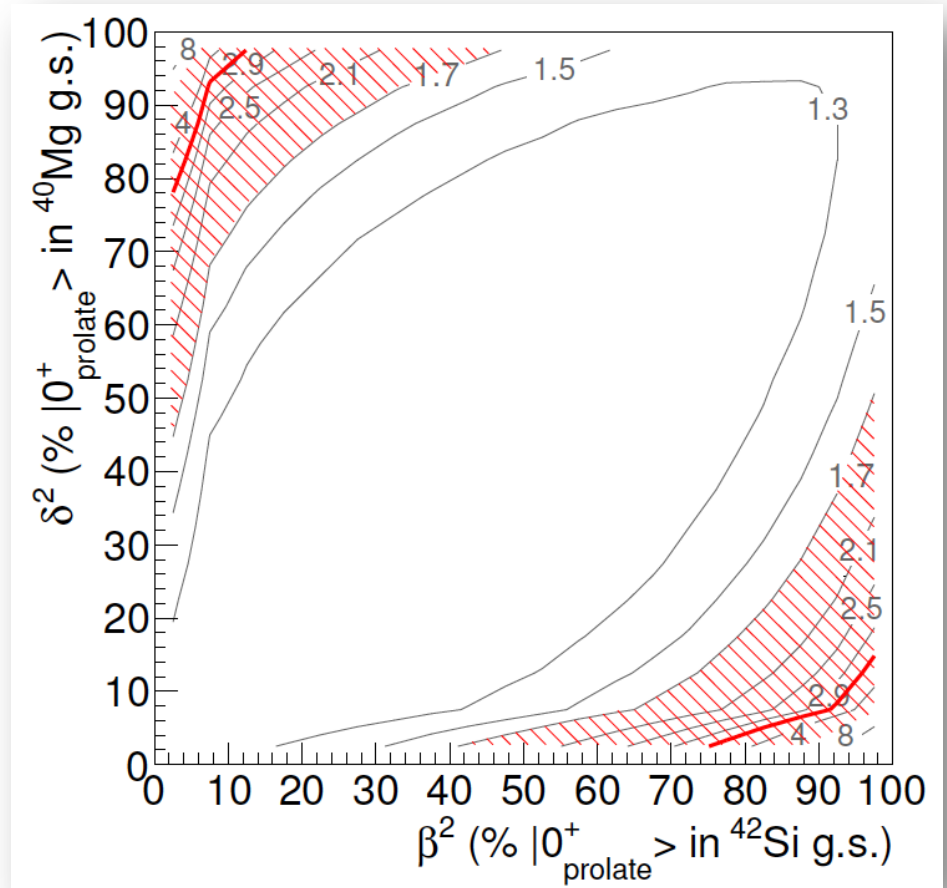
$$|^{42}\text{Si}, 0_2^+\rangle = -\beta |0^+; \text{O}\rangle + \alpha |0^+; \text{P}\rangle$$

$$|^{40}\text{Mg}, 0_1^+\rangle = +\gamma |0^+; \text{O}\rangle + \delta |0^+; \text{P}\rangle$$

Experimental value

$$\mathcal{R} = 3.3_{-1.6}^{+2.4}$$

Dominant deformations in the  $^{42}\text{Si}$  and  $^{40}\text{Mg}$  ground states are consistently opposite.





# $^{40}\text{Mg}$ : Where we left it in 2014

Based on the inclusive cross-section from  $^{42}\text{Si}(-2p)$ :

- $^{40}\text{Mg}$  likely only has one bound  $0^+$  state (the ground state)
- The ground state deformation is likely opposite in sign to that of  $^{42}\text{Si}$

Open questions:

- Are there *any* bound excited states in  $^{40}\text{Mg}$ ?
- Is  $E(2^+)$  in line with expectations from shell-model?
- Is the ground state consistent with prolate deformation?
- Is there evidence for weak-binding effects in the spectrum of  $^{40}\text{Mg}$ ?

# $^{40}\text{Mg}$ in December 2016

## What did we learn in NP0906-RIBF03 for spectroscopy in $^{40}\text{Mg}$ ?

- Cross-section for  $^{42}\text{Si}(-2p)$  is **low** – with 150 pA of  $^{48}\text{Ca}$  primary beam, we expect only 25  $^{40}\text{Mg}$  / day using a  $4\text{g}/\text{cm}^2$  C target
- With branch to  $2^+$  as predicted by shell-model calculations\* would expect only 10 counts in a photopeak
- Measured production rate for  $^{41}\text{Al}$  (0.6pps/100pA  $^{48}\text{Ca}$ )
- Measured background with thick carbon target in -1p channel

## Proposed for NP0906-RIBF03R2

- Use  $^{41}\text{Al}$  reactions:
  - $^{41}\text{Al}(-1p)$  – C target
  - $^{41}\text{Al}(-1p)$  and  $^{41}\text{Al}(p,2p)$  –  $\text{CH}_2$  target

⇒ Increased (total)  $^{40}\text{Mg}$  production  
⇒ Greater  $2^+$  population  
⇒ Improved peak/background (more  $2^+ / ^{40}\text{Mg}$ )

## $^{41}\text{Al}(-1p)^{40}\text{Mg}$ on Carbon

State	Energy (keV)	$\sigma$ (mb) SDPF-MU
$0^+$	0.0	1.35
$2^+$	733.0	2.10
$0^+$	1683.0	0.58

\* Tostevin and Brown, private communication.

## First Spectroscopy of the Near Drip-line Nucleus $^{40}\text{Mg}$

H. L. Crawford,<sup>1,\*</sup> P. Fallon,<sup>1</sup> A. O. Macchiavelli,<sup>1</sup> P. Doornenbal,<sup>2</sup> N. Aoi,<sup>3</sup> F. Browne,<sup>2</sup> C. M. Campbell,<sup>1</sup> S. Chen,<sup>2</sup> R. M. Clark,<sup>1</sup> M. L. Cortés,<sup>2</sup> M. Cromaz,<sup>1</sup> E. Ideguchi,<sup>3</sup> M. D. Jones,<sup>1,†</sup> R. Kanungo,<sup>4,5</sup> M. MacCormick,<sup>6</sup> S. Momiyama,<sup>7</sup> I. Murray,<sup>6</sup> M. Niikura,<sup>7</sup> S. Paschalis,<sup>8</sup> M. Petri,<sup>8</sup> H. Sakurai,<sup>2,7</sup> M. Salathe,<sup>1</sup> P. Schrock,<sup>9</sup> D. Steppenbeck,<sup>9</sup> S. Takeuchi,<sup>2,10</sup> Y. K. Tanaka,<sup>11</sup> R. Taniuchi,<sup>7</sup> H. Wang,<sup>2</sup> and K. Wimmer<sup>7</sup>

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Editors' Suggestion

### First Spectroscopy of the Near Dr

H. L. Crawford,<sup>1,\*</sup> P. Fallon,<sup>1</sup> A. O. Macchiavelli,<sup>1</sup> D. ...  
S. Chen,<sup>2</sup> R. M. Clark,<sup>1</sup> M. L. ...  
M. MacCormick,<sup>6</sup> S. ...

# PHYSICS TODAY

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DOI:10.1063/PT.6.1.20190225a

25 Feb 2019 in *Research & Technology*

## Neutron-rich magnesium has unexpected transitions

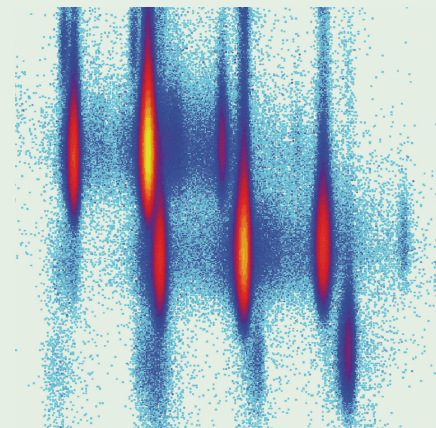
An exotic isotope near the edge of stability bucks trends established by nuclei with lower mass n

Christine Middleton

# PHYSICAL REVIEW LETTERS

Articles published week ending 8 FEBRUARY 2019

PRL 122 (5), 050401–059901, 8 February 2019 (294 total pages)



5

Published by American Physical Society



Volume 122, Number 5

# Production of $^{40}\text{Mg}$ by fast-beam fragmentation

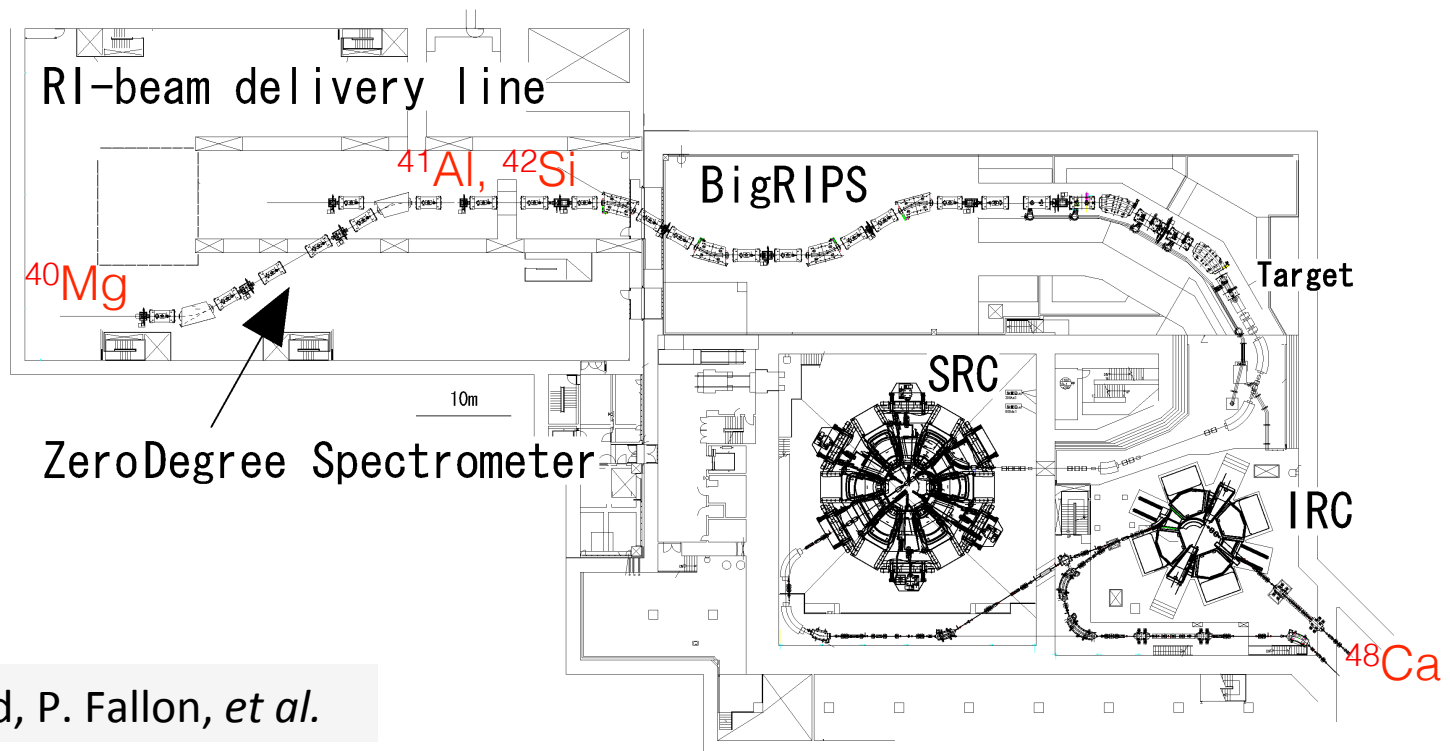
Two Measurements at RIKEN/RIBF - high energy  $^{48}\text{Ca}$  beam 345 MeV/u

December 2010

$^{48}\text{Ca} \rightarrow ^{42}\text{Si}$  (200 MeV/u), 2p Knockout:  $^{42}\text{Si} - 2p \rightarrow ^{40}\text{Mg}$  ( $v/c \sim 60\%$ )

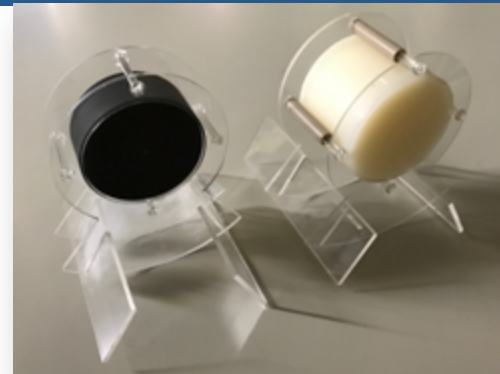
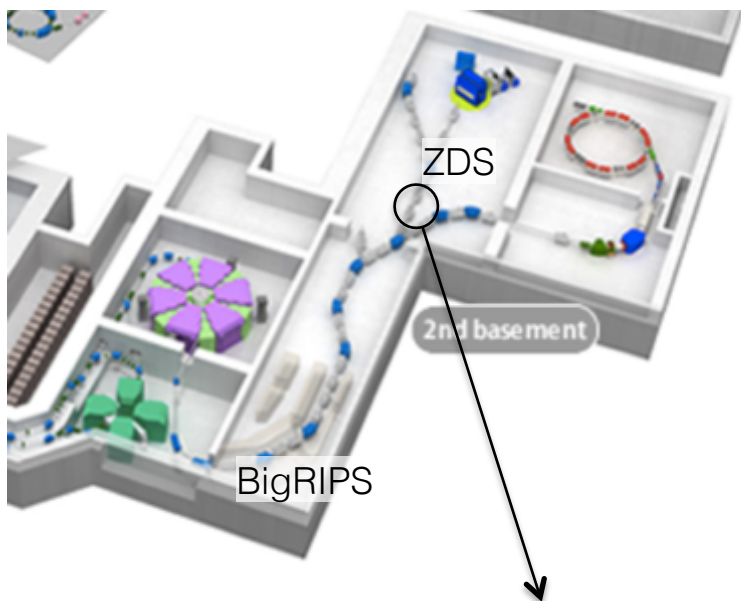
December 2016

$^{48}\text{Ca} \rightarrow ^{41}\text{Al}$  (240 MeV/u), 1p Knockout:  $^{41}\text{Al} - 1p \rightarrow ^{40}\text{Mg}$  ( $v/c \sim 60\%$ )

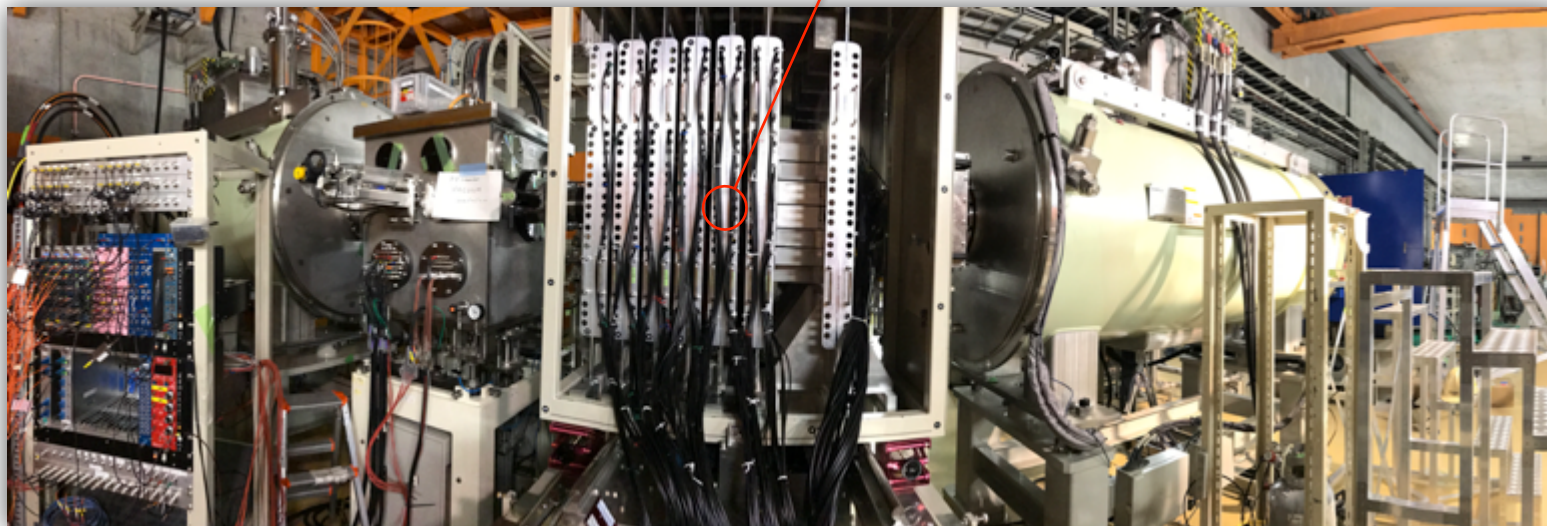


H. L. Crawford, P. Fallon, *et al.*

# $^{40}\text{Mg}$ : 2016 setup

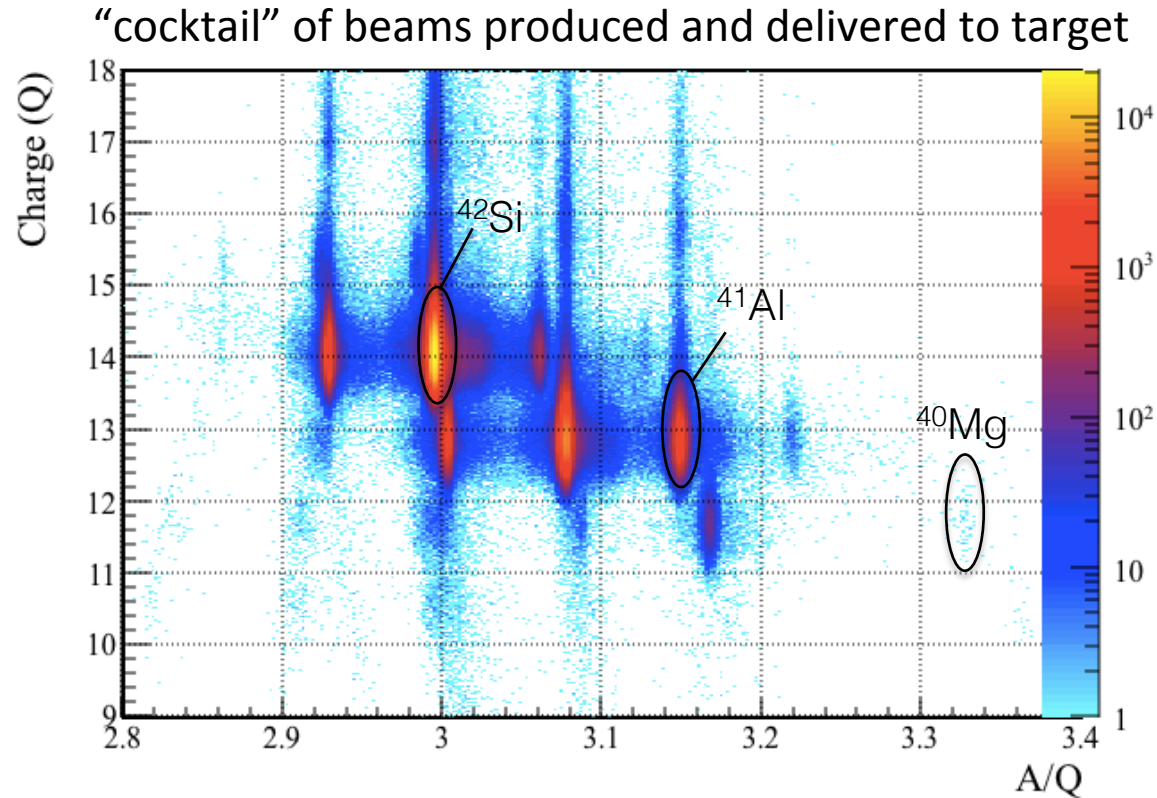


Self-supporting Carbon (graphite) and  $\text{CH}_2$  targets  
 $\text{CH}_2 \Rightarrow 3.82 \text{ g/cm}^2$ ; Carbon  $\Rightarrow 3.80 \text{ g/cm}^2$



DALI2  $\gamma$  detector 186  $\text{NaI}(\text{Tl})$  scintillators covering  $4\pi$

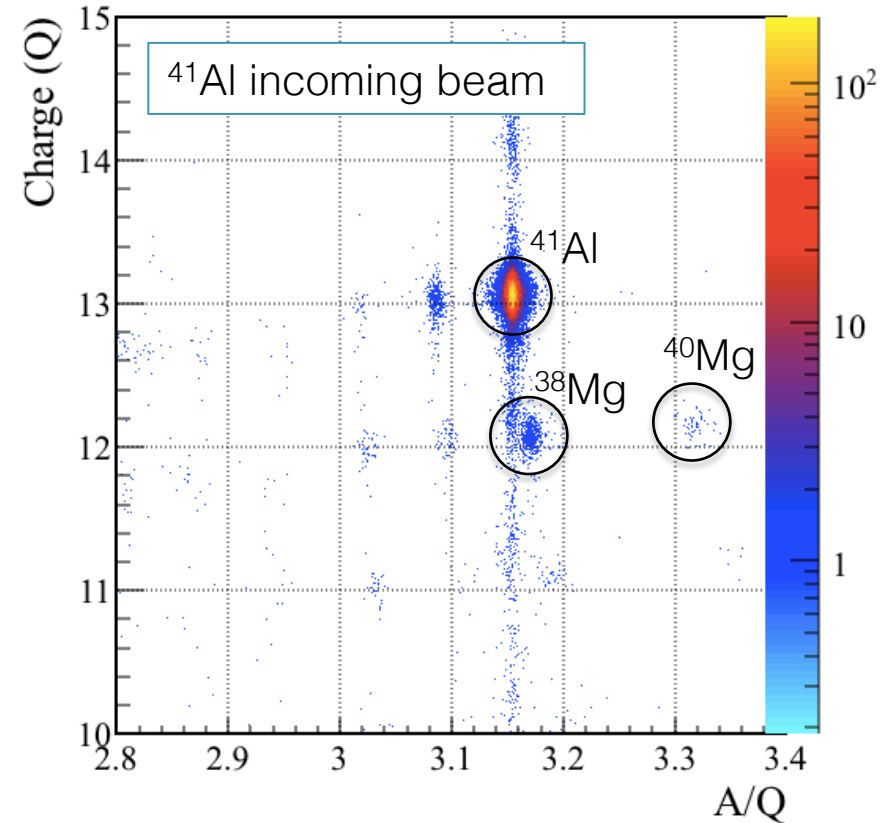
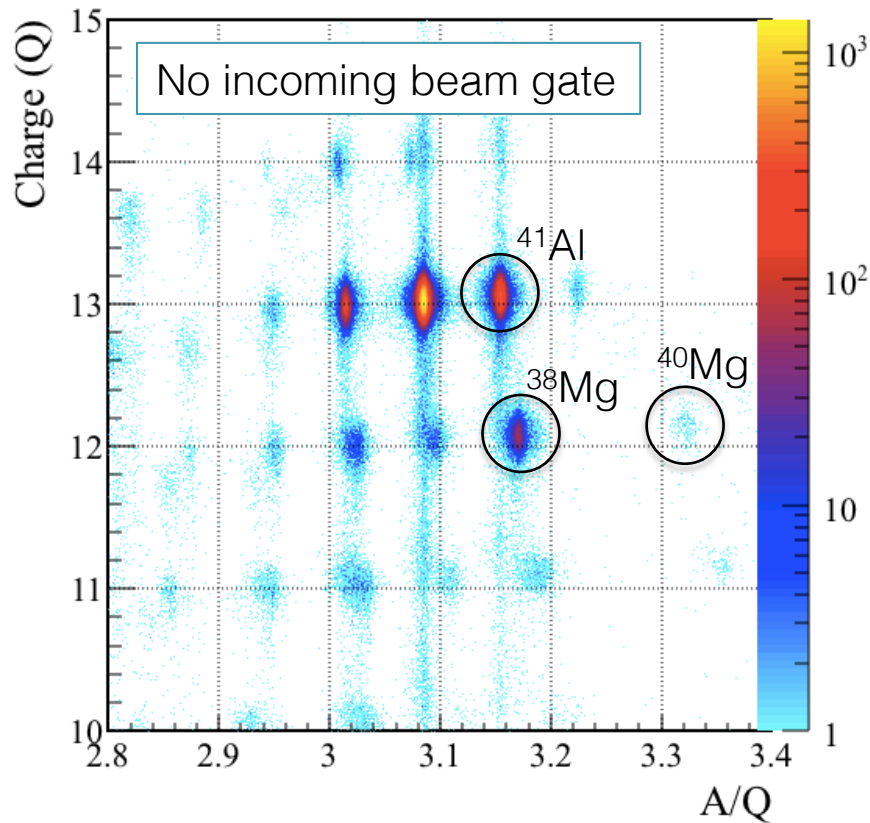
# Event-by-event identification of incoming beam



- BigRIPS fragment separator was centered on  $^{41}\text{Al}$
- ~3% of incoming beam was  $^{41}\text{Al}$ ;  $^{42}\text{Si}$  and  $^{40}\text{Mg}$  were both in acceptance of BigRIPS
- Average  $^{48}\text{Ca}$  primary beam intensity of order 400 pA for ~6 days !!

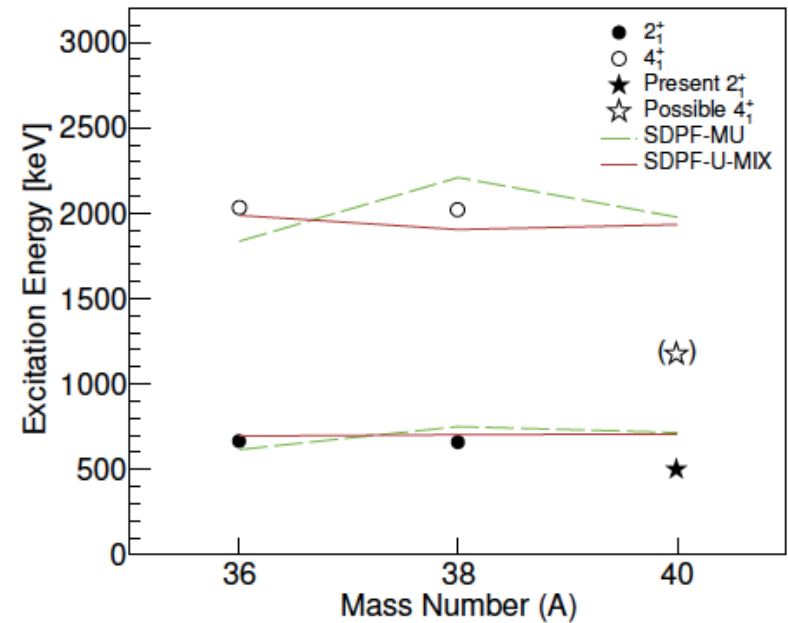
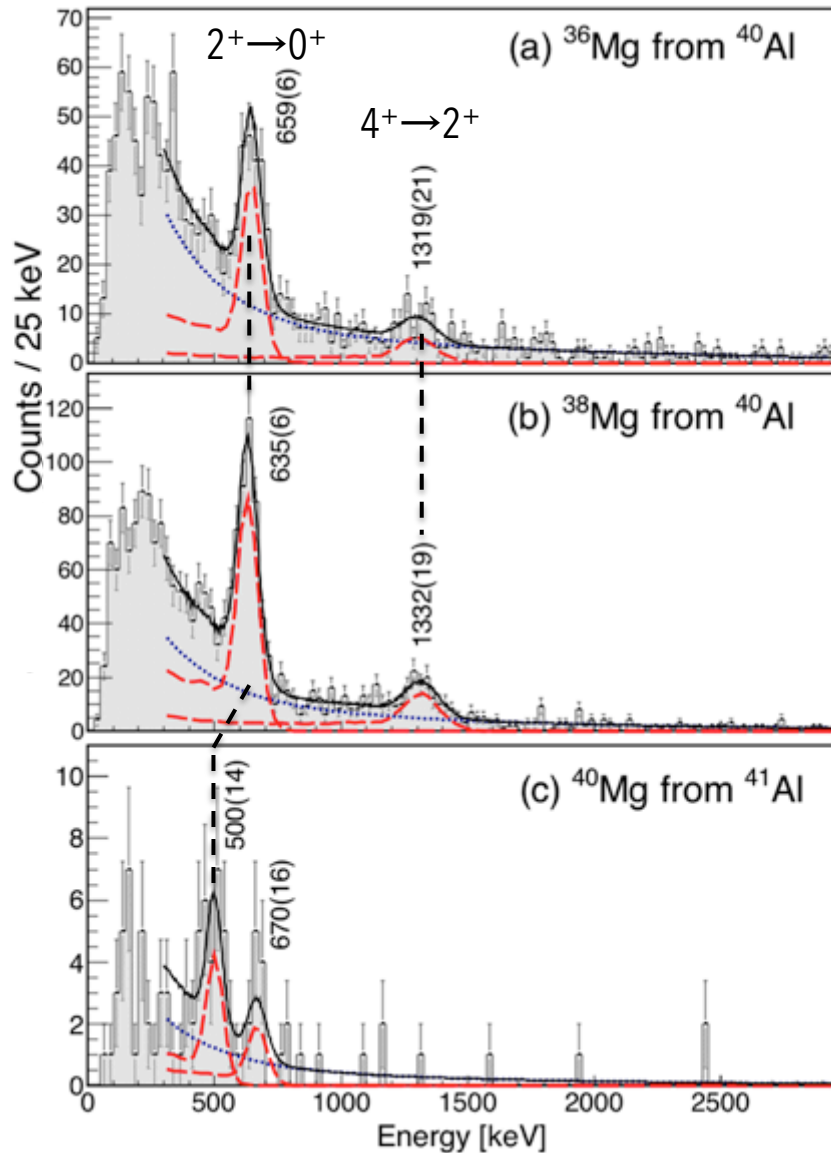
# Event-by event identification of reaction products

Secondary reaction products identified at the focal plane



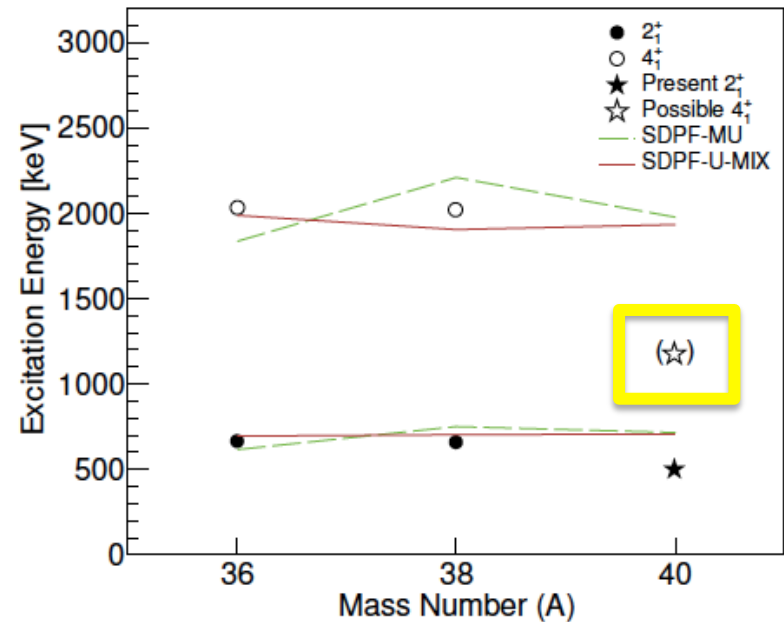
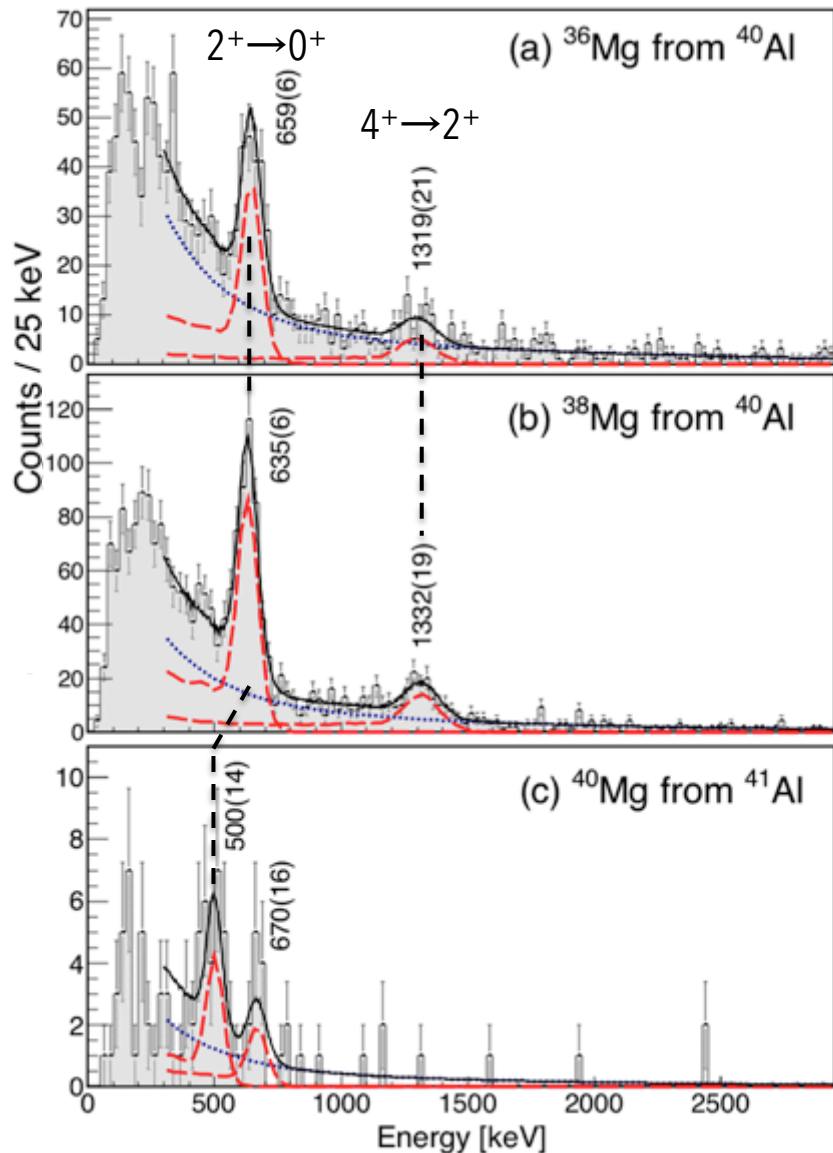


# Results



- **500 keV transition assigned to  $2^+ \rightarrow 0^+$**
- Observe a 20% decrease in  $^{40}\text{Mg}$   $2^+$  energy relative to  $^{38}\text{Mg}$ .
- Relative change in  $2^+$  (more robust prediction than absolute value) is not captured in calculations

# Results: $^{40}\text{Mg}$ (670 keV transition)



## • 670 keV transition ?

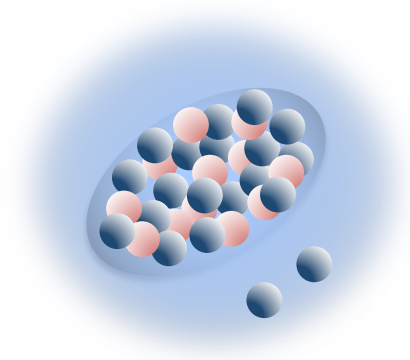
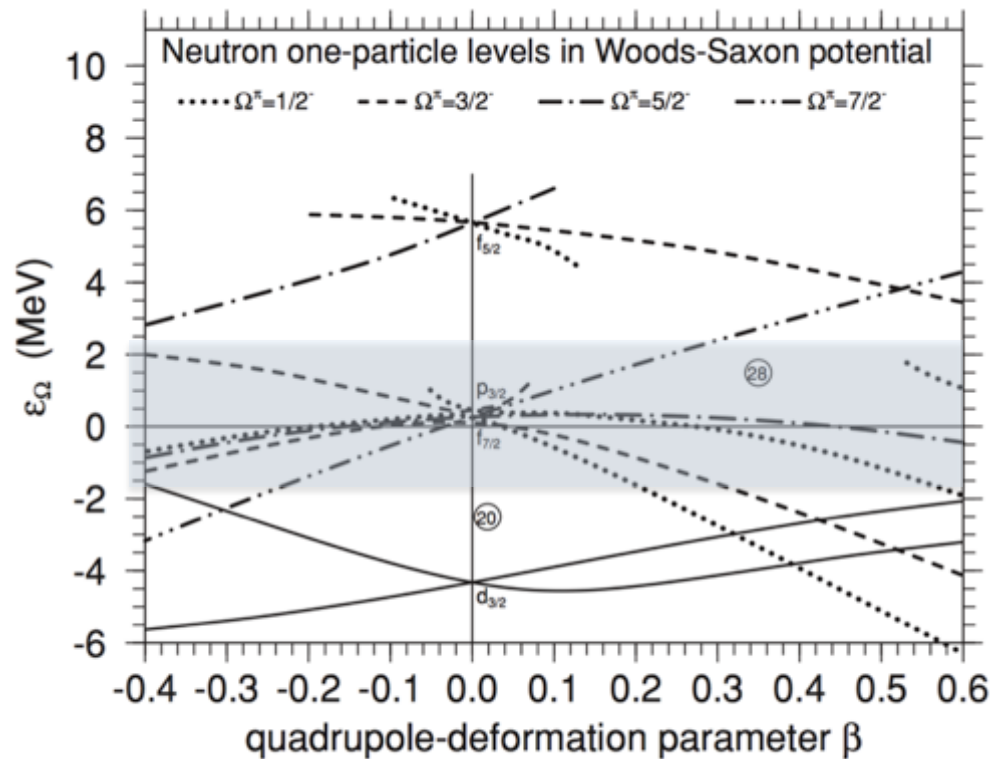
$$4^+_1 \rightarrow 2^+_1 \quad 2^+_2 \rightarrow 0^+_1 \quad 2^+_2 \rightarrow 2^+_1 \quad 0^+_2 \rightarrow 2^+_1 \quad + \dots$$

- No scenario fits with existing expectations (systematics) nor predictions from calculation
- Breakdown of systematics and theory predictions may suggest something is happening at the dripline ??

\* preferred

# Weakly bound neutrons in $^{40}\text{Mg}$

- 2-body NN interaction works to reduce the N=28 shell gap when removing protons from  $^{48}\text{Ca}$
- Occupation of low  $l$  levels ( $p_{3/2}$ ) may lead to extended wavefunctions (“halos”)

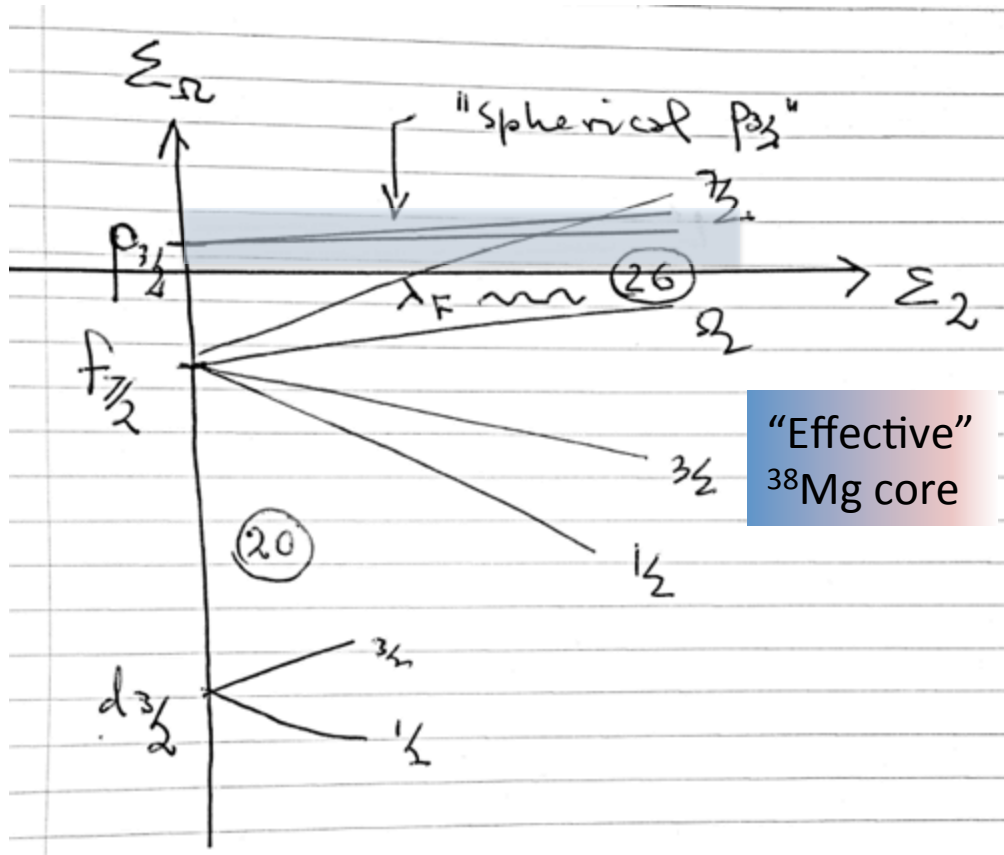


- Could we consider  $^{40}\text{Mg}$  as a deformed  $^{38}\text{Mg}$  core and a 2-neutron  $p$ -wave halo ?

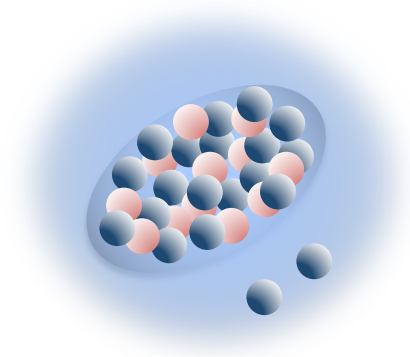
I. Hamamoto PRC 79, 014307 (2009)

# Weakly bound neutrons in $^{40}\text{Mg}$

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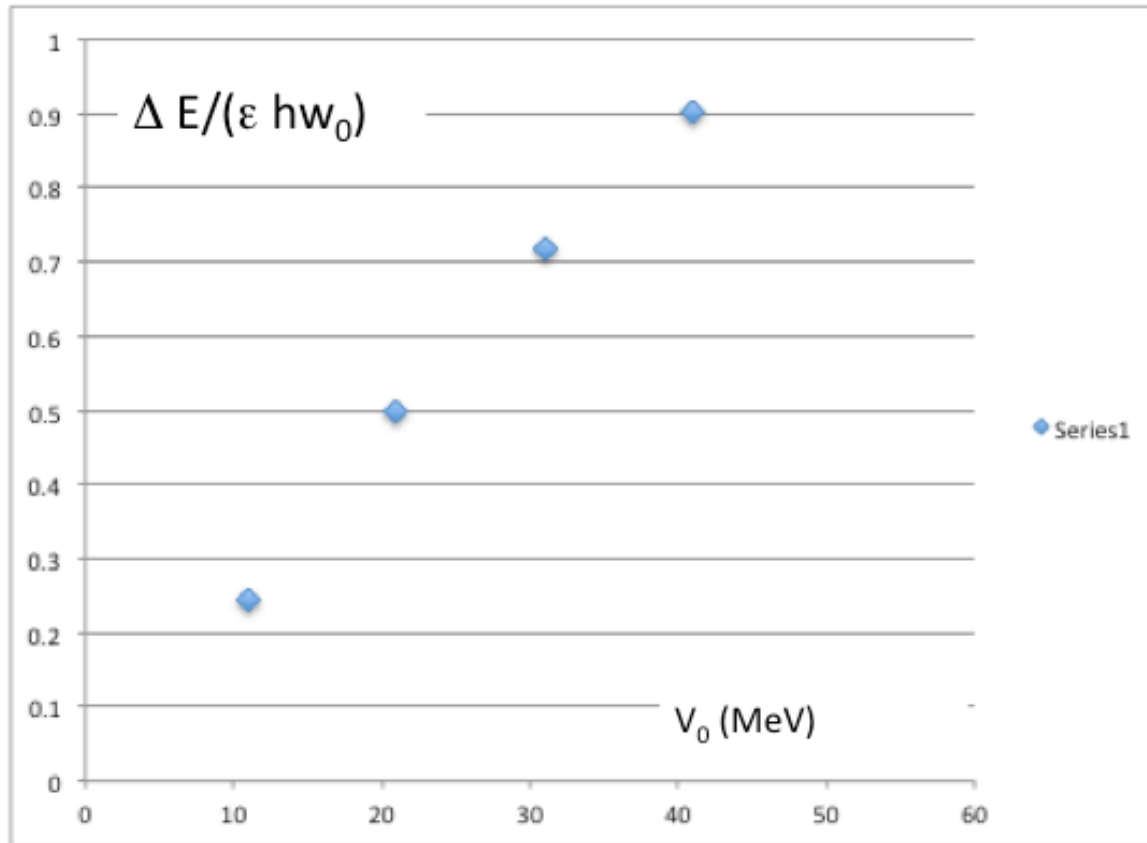


Schematic single-particle scenario



- Could we consider  $^{40}\text{Mg}$  as a deformed  $^{38}\text{Mg}$  core and a 2-neutron  $p$ -wave halo ?

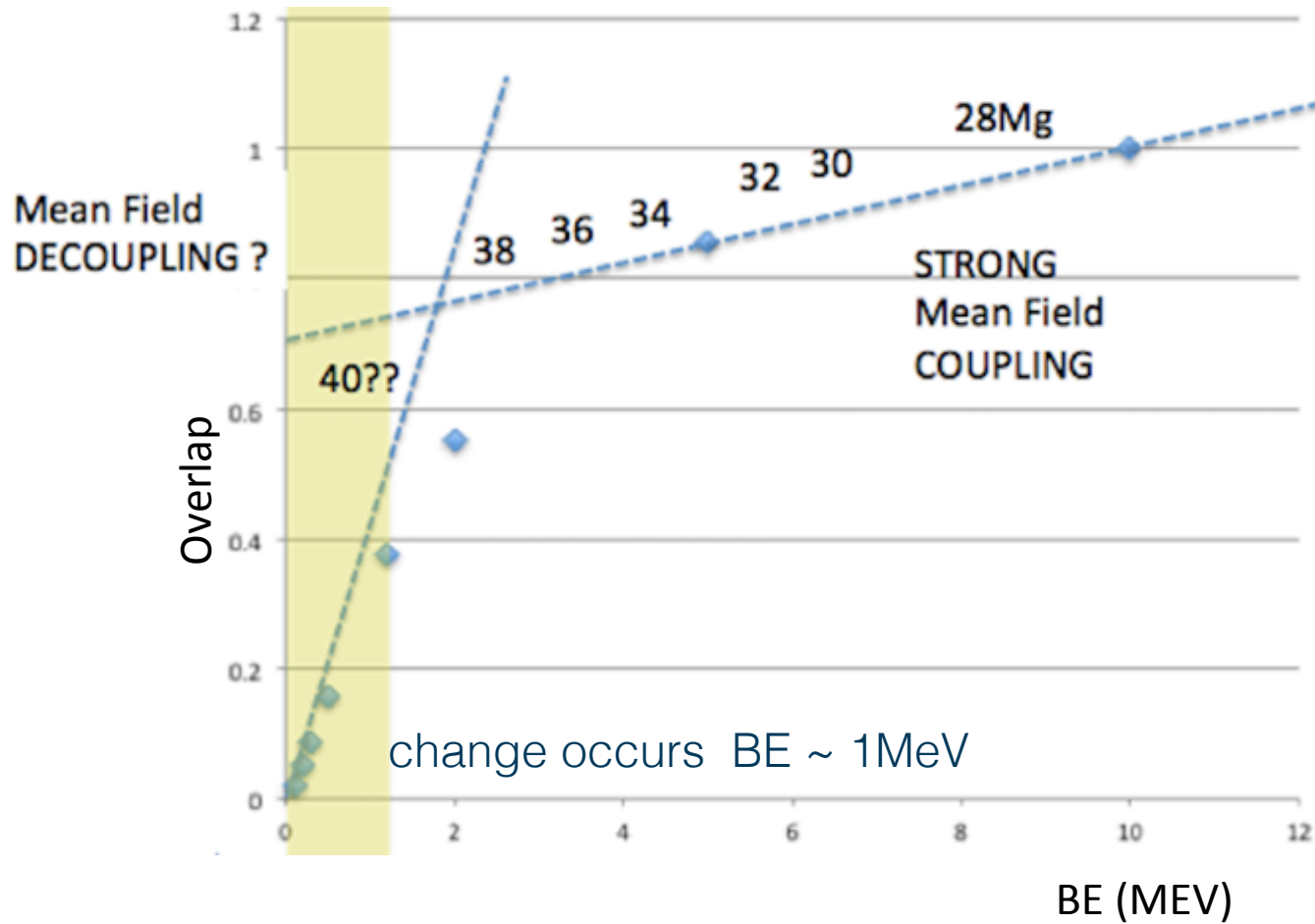
# Weakly bound neutrons in $^{40}\text{Mg}$



- Energy separation between the  $\Omega= 1/2$  and  $3/2$  Nilsson levels as a function of the WS depth ( $V_0$ ), showing the approach to the spherical limit for weak binding

# Indications of weak binding - geometric overlap

Volume  $p_{3/2}$  overlap as a function of BE calculated in a Woods-Saxon\*



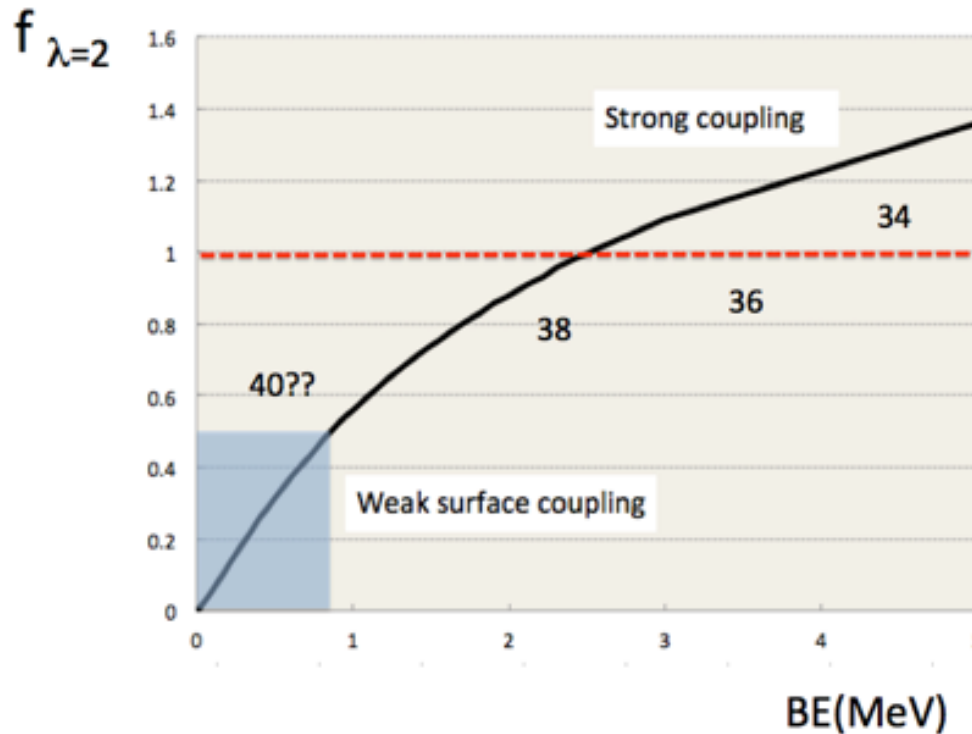
\*<https://www.volya.net/>

# Indications of weak binding – PV coupling

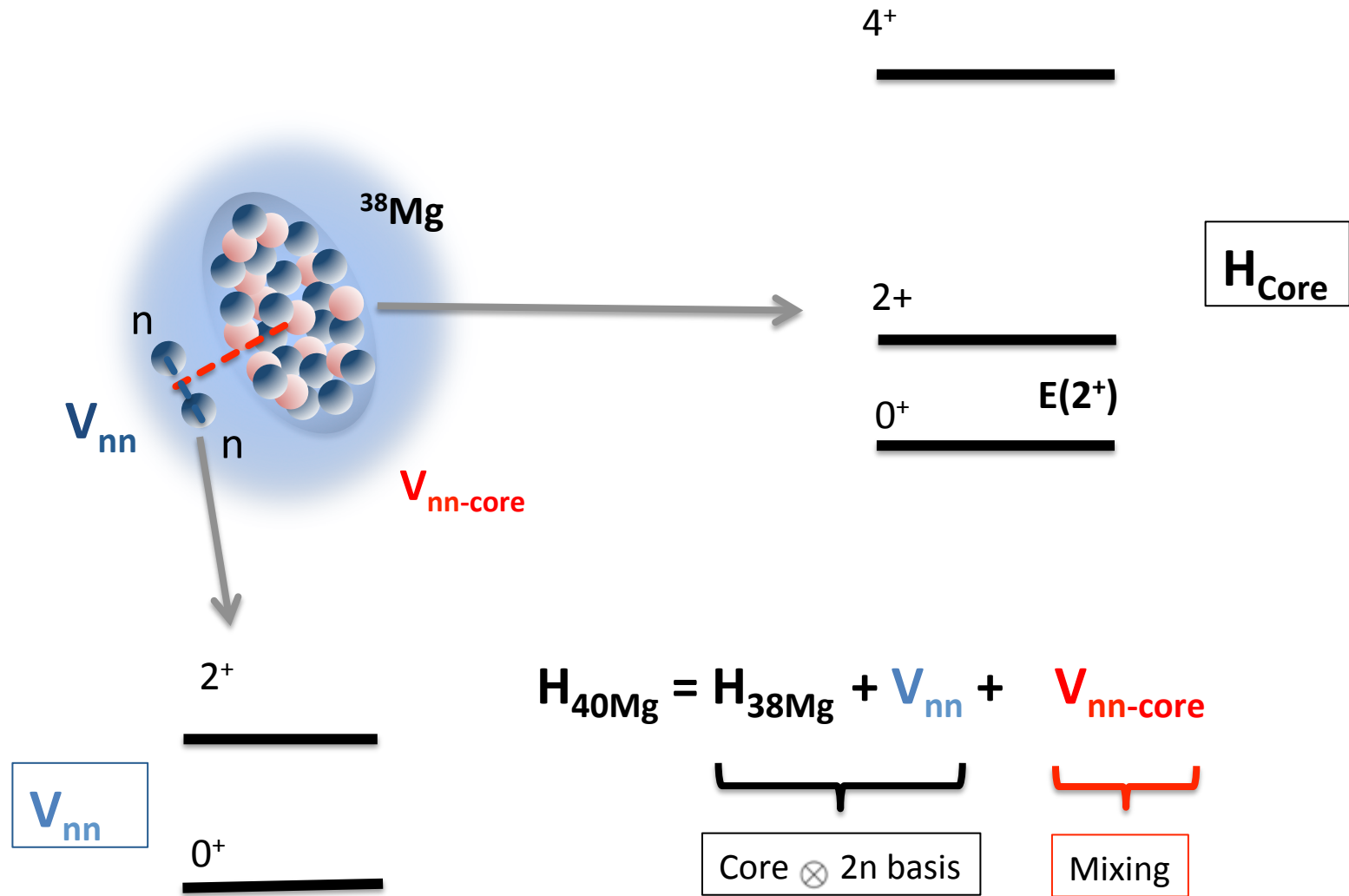
A dimensionless parameter that is often useful in characterizing the strength of the particle-vibration coupling is obtained by dividing a standard coupling matrix element by  $\hbar\omega_\lambda$ . Thus, for a shape vibration, we may employ the parameter (see Eq. (6-209))

B&M, Vol II pag.419

$$f_\lambda = \left( \frac{2\lambda + 1}{16\pi} \right)^{1/2} \left( \frac{\hbar\omega_\lambda}{2C_\lambda} \right)^{1/2} \frac{\langle k_\lambda \rangle}{\hbar\omega_\lambda} \quad (6-212)$$

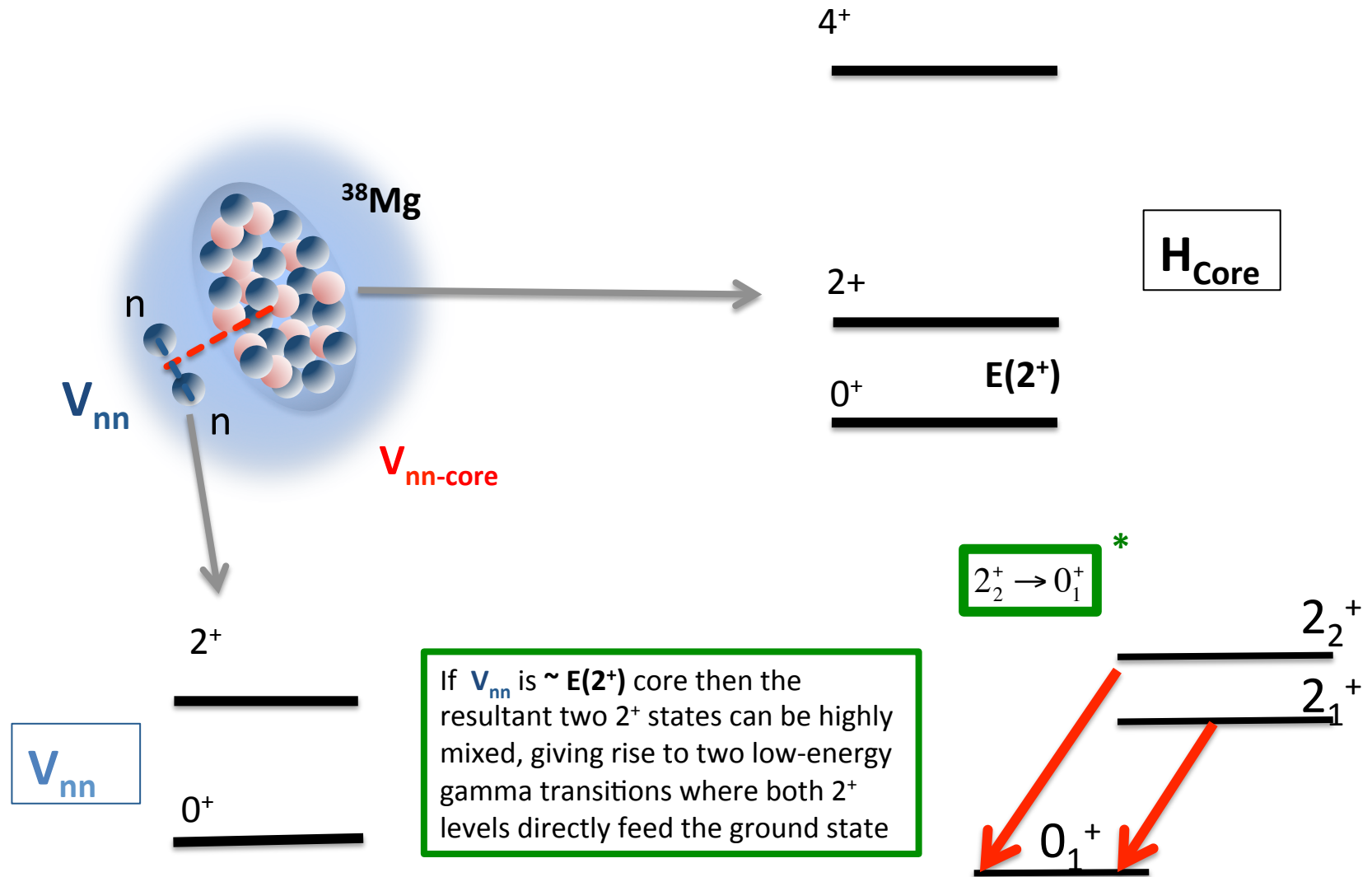


# Weak coupling of two degrees of Freedom

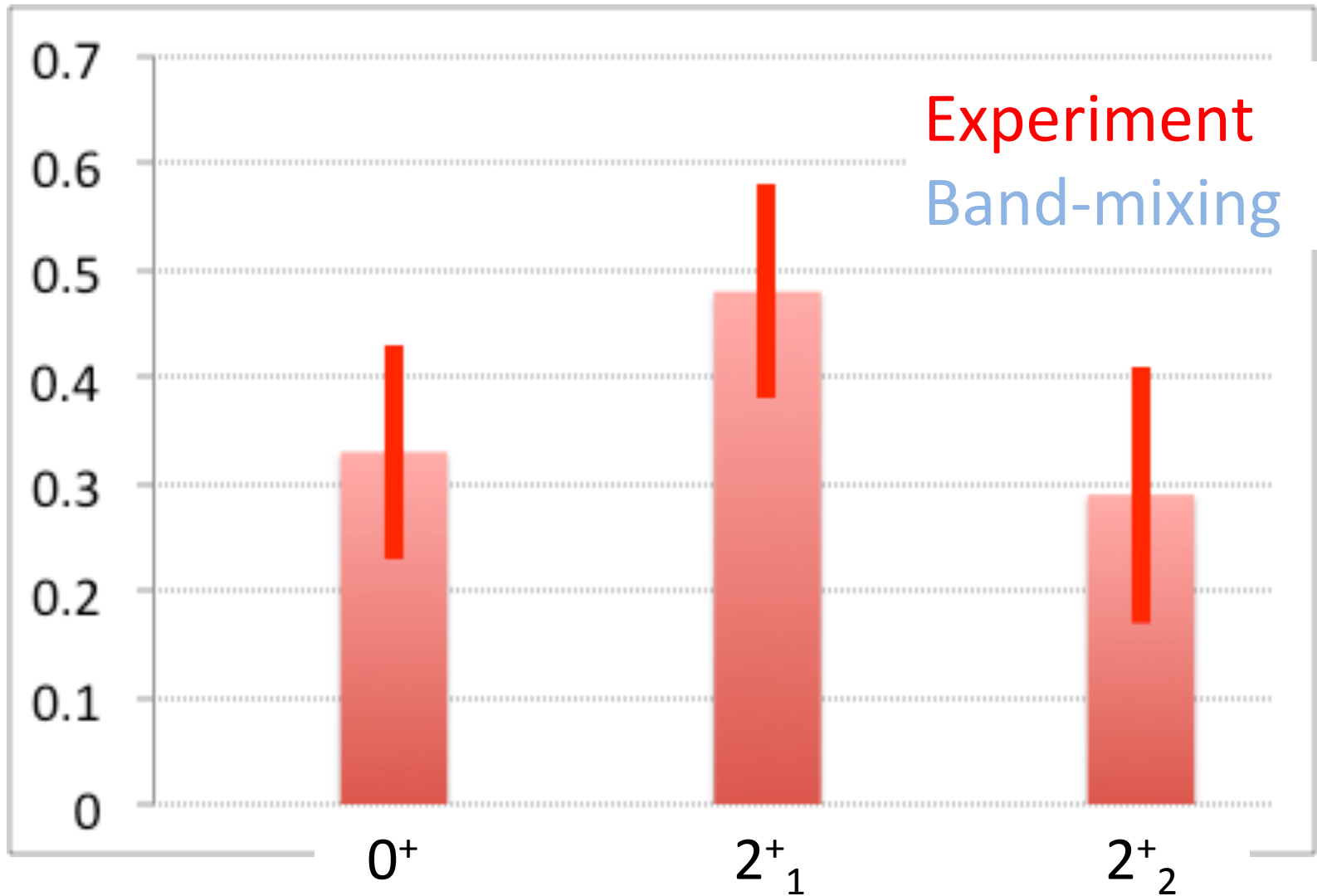




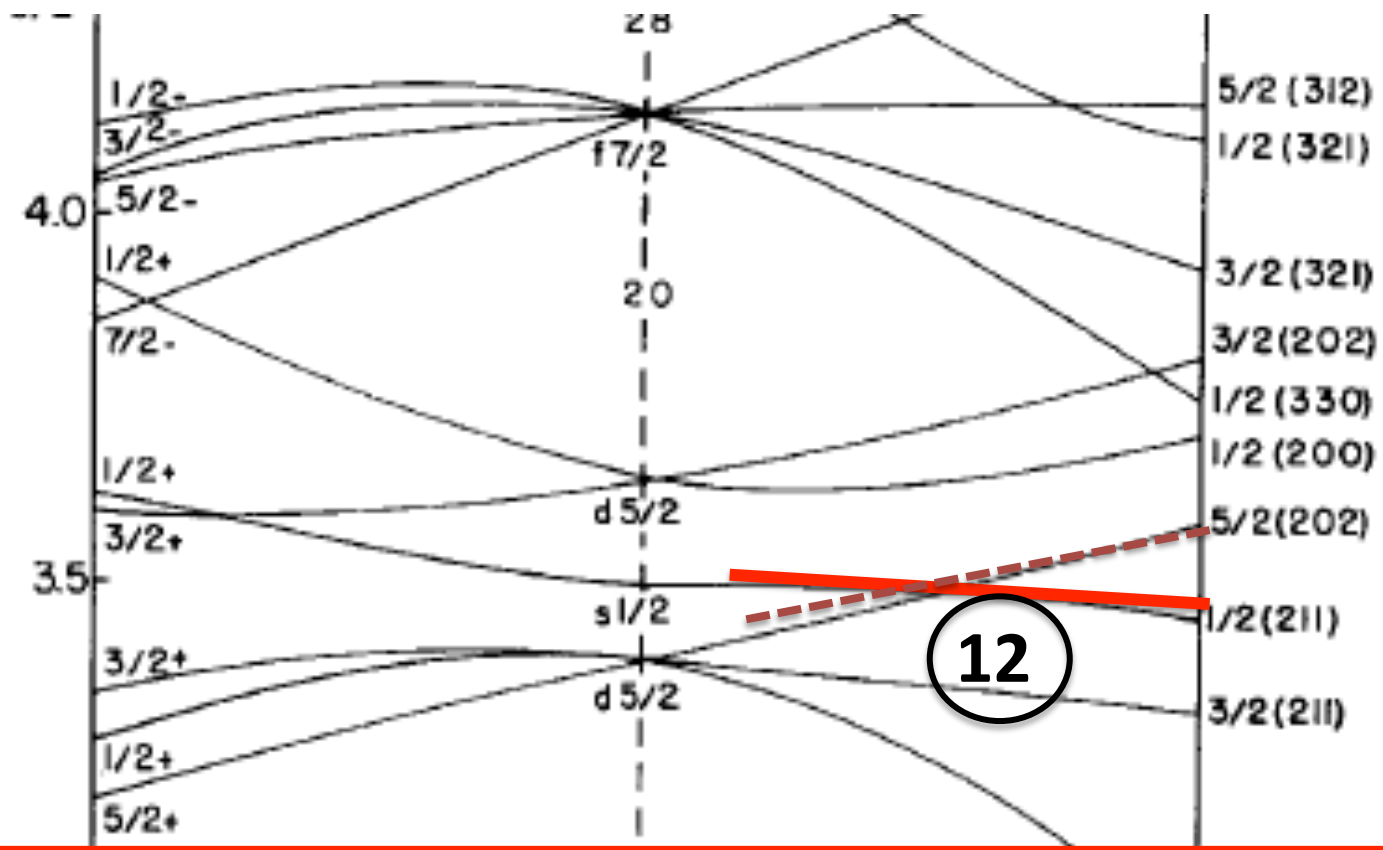
# Weak coupling of two degrees of Freedom



# Relative populations from gamma-ray and particle singles intensities

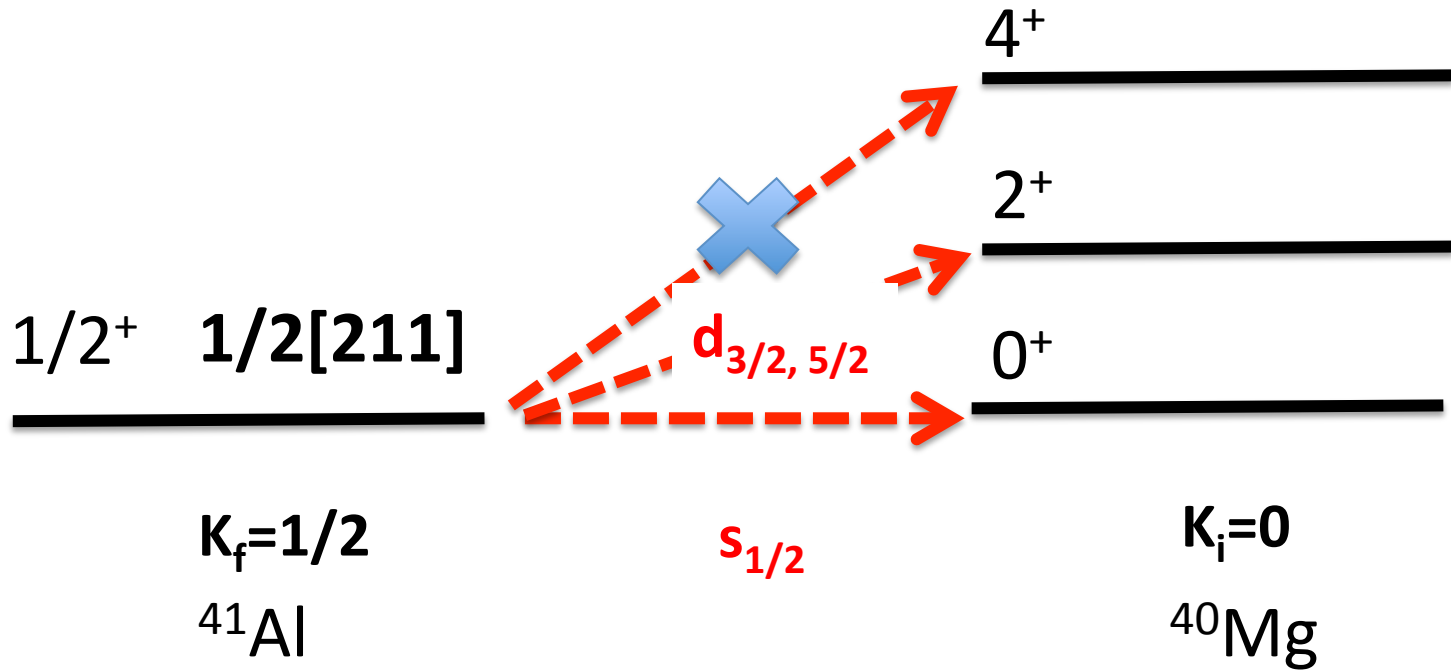


# Nilsson proton levels



$K = 1/2, (211)$								
$E$		3.628406	3.584660	3.533333	3.500000	3.506007	3.488141	3.457374
$a$		.884746	1.199178	1.400000	1.000000	.800406	.142953	-.113242
$l$	$j$							
0	1/2	-.357242	-.490690	.730296	1.000000	.805854	.529234	.370421
2	3/2	.609920	.513667	-.326599	.000000	.424450	.677771	.753501
2	5/2	.707373	.703825	-.600001	.000000	.412844	.510429	.543162

# Rotational part

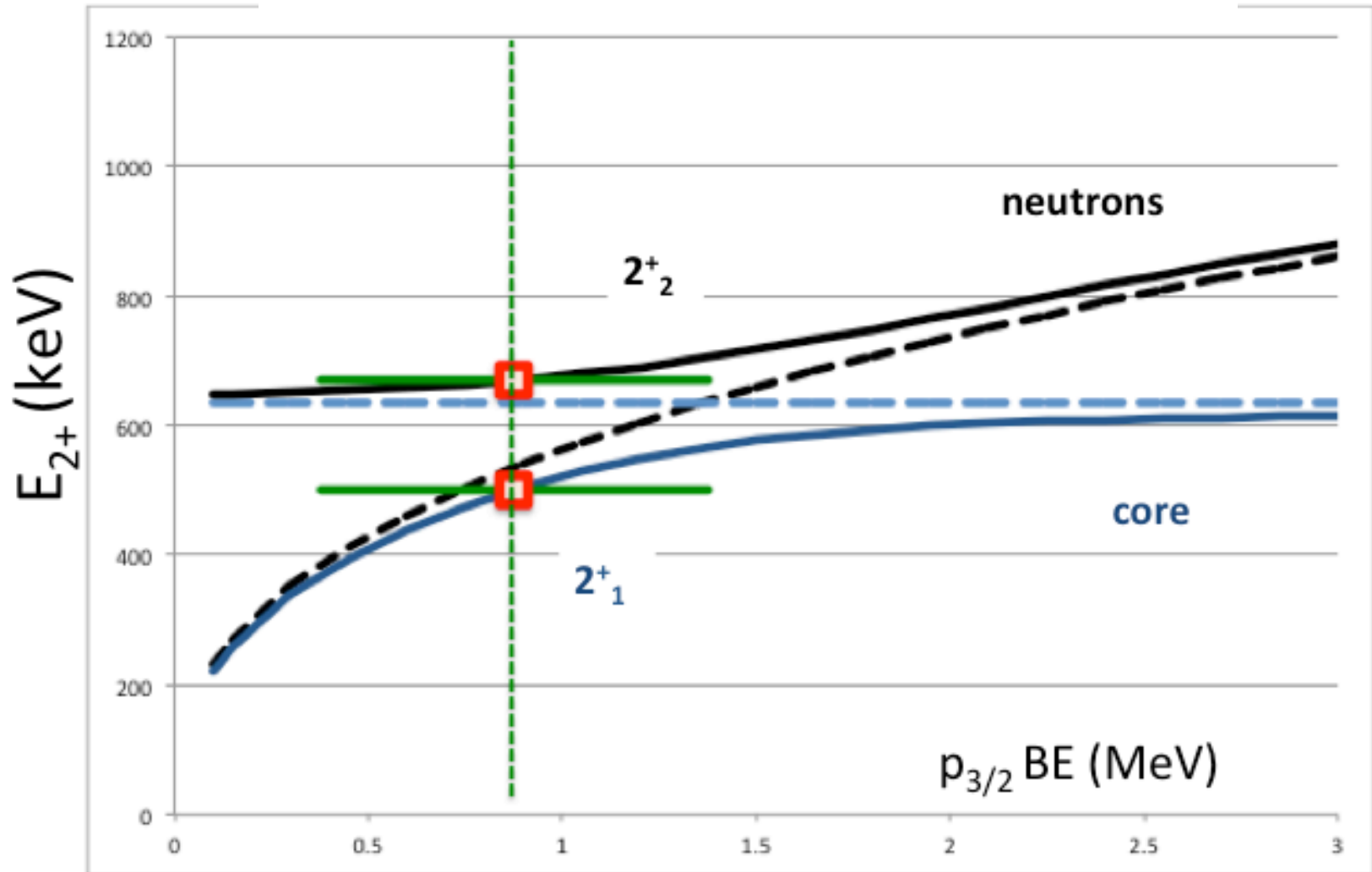


$$S_{i,f} = 2 \langle I_i j K_i \Delta K | I_f K_f \rangle^2 C_{j,\ell}^2 \langle \phi_f | \phi_i \rangle^2$$

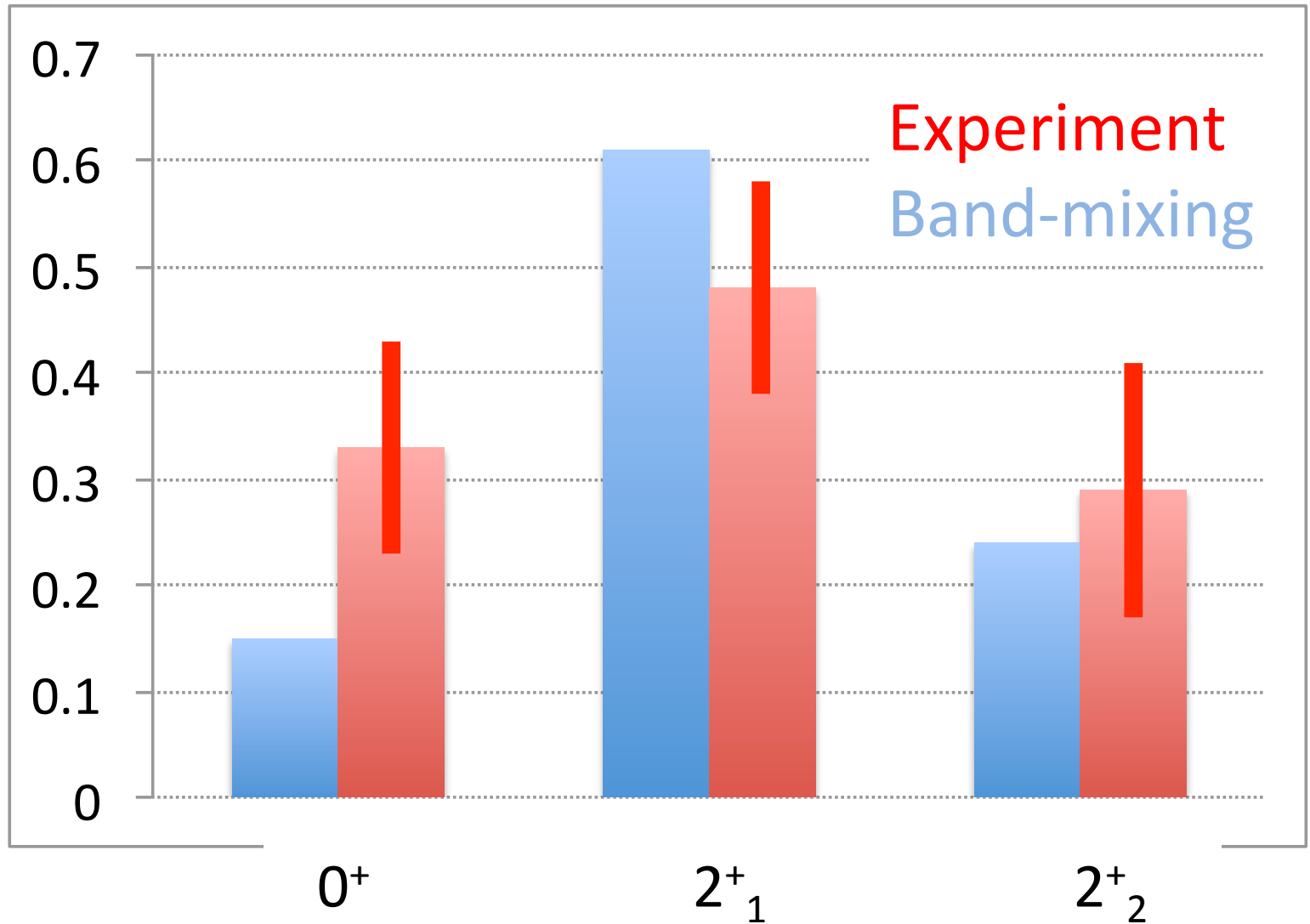
Single particle part ( $1/2 \rightarrow 2^+_{2n}$ ) treated as a parameter  $S_{sp}$

# Minimization results

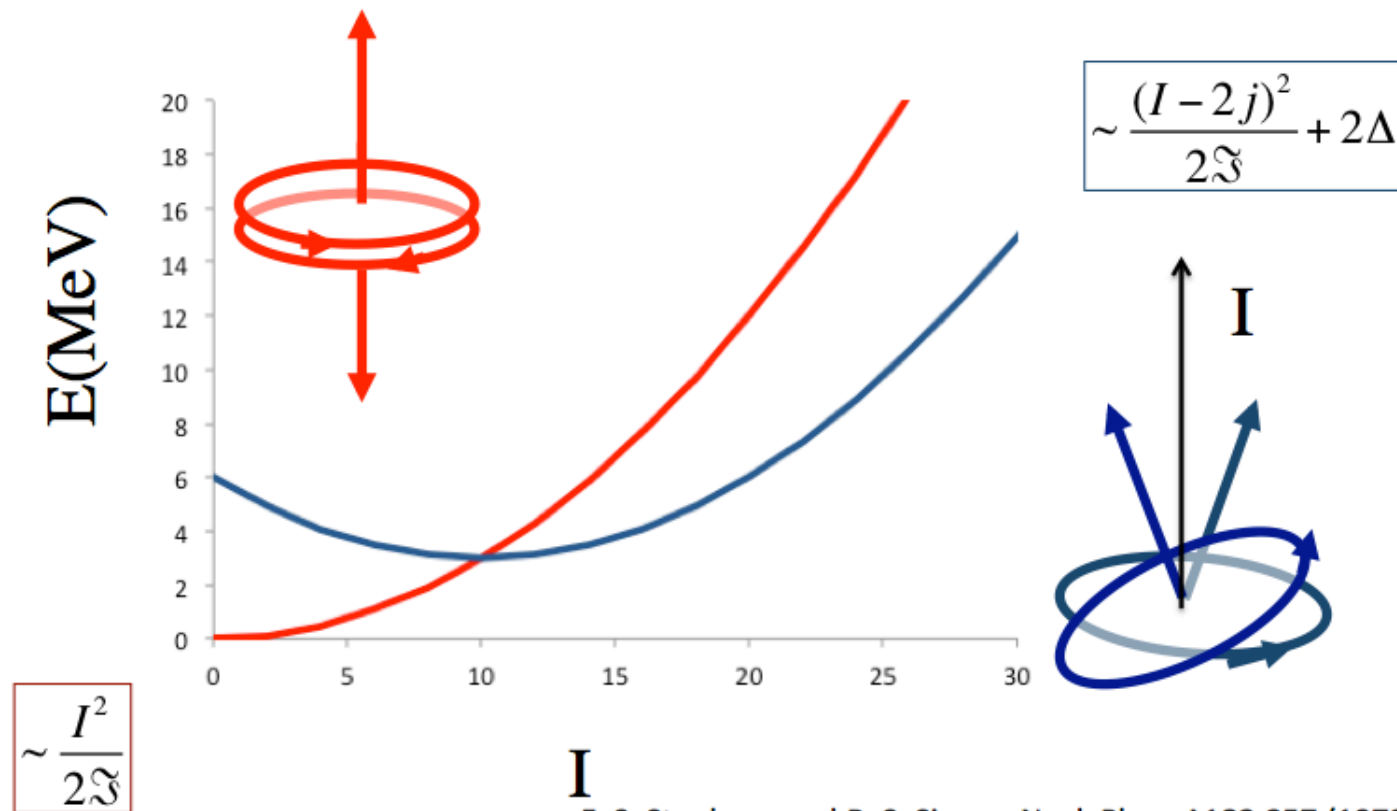
$$BE = 877 \text{ keV} \quad V_{nn\text{-Core}} = 69 \text{ keV} \quad S_{sp} = 0.13$$



# Relative populations from gamma-ray and particle singles intensities



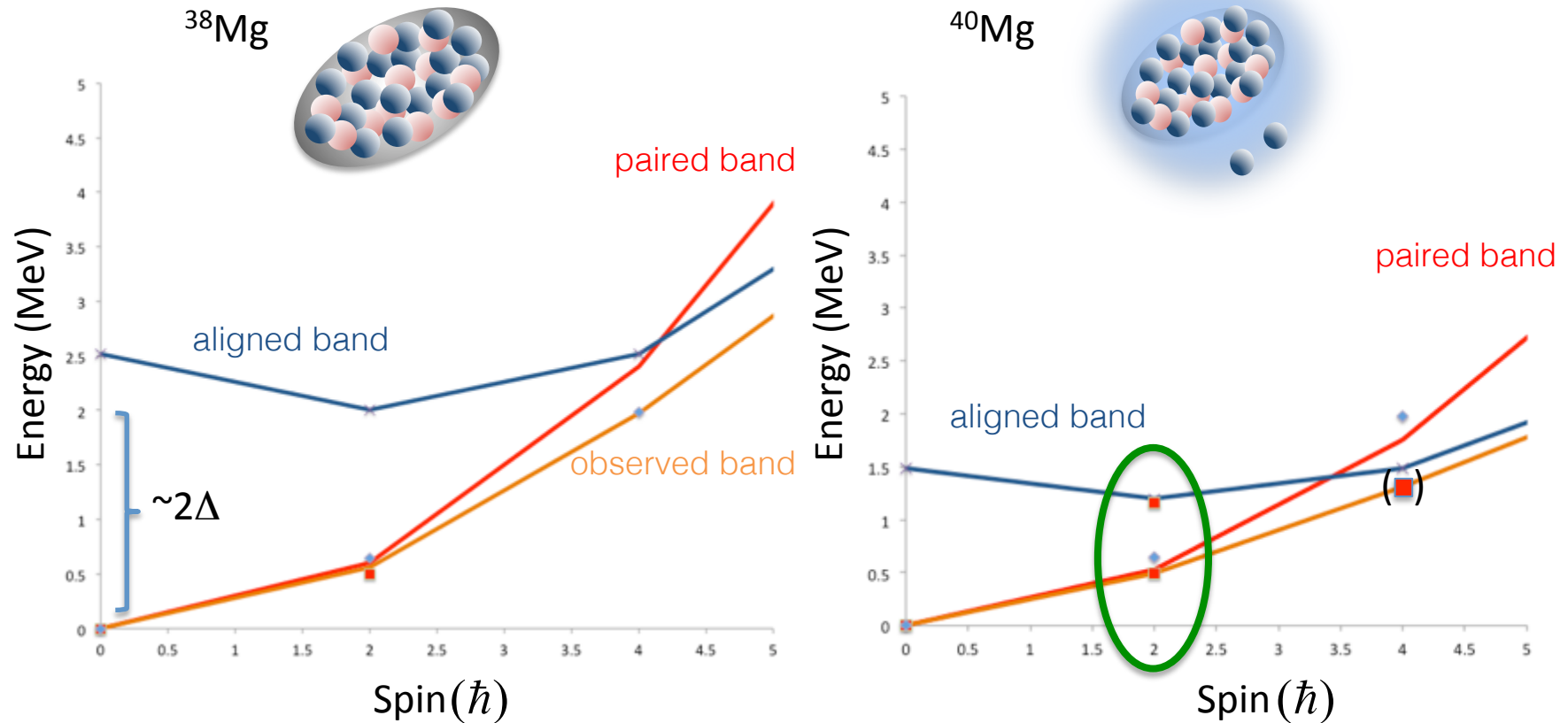
# An alternative scenario: Rotation and Alignment



$I$

F. S. Stephens and R. S. Simon, *Nucl. Phys.* A183,257 (1972).

# Rotation in $^{40}\text{Mg}$ – aligned-band crossing



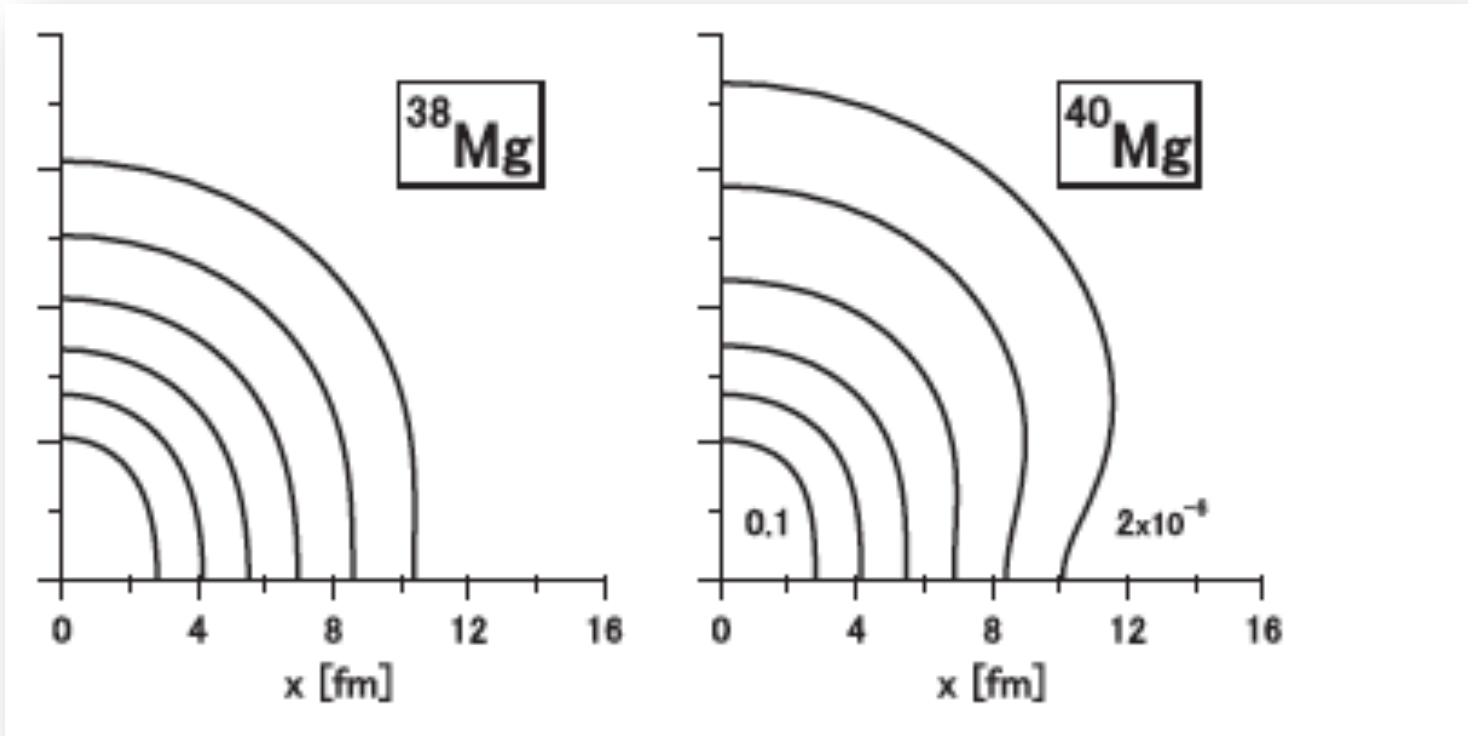
In  $^{40}\text{Mg}$ , the energy to break a neutron  $p_{3/2}$  pair needs to be reduced by 1/2  
 Quenched pairing due to reduced overlap?



# HFB Calculation of Mg deformed ground states

H. NAKADA AND K. TAKAYAMA

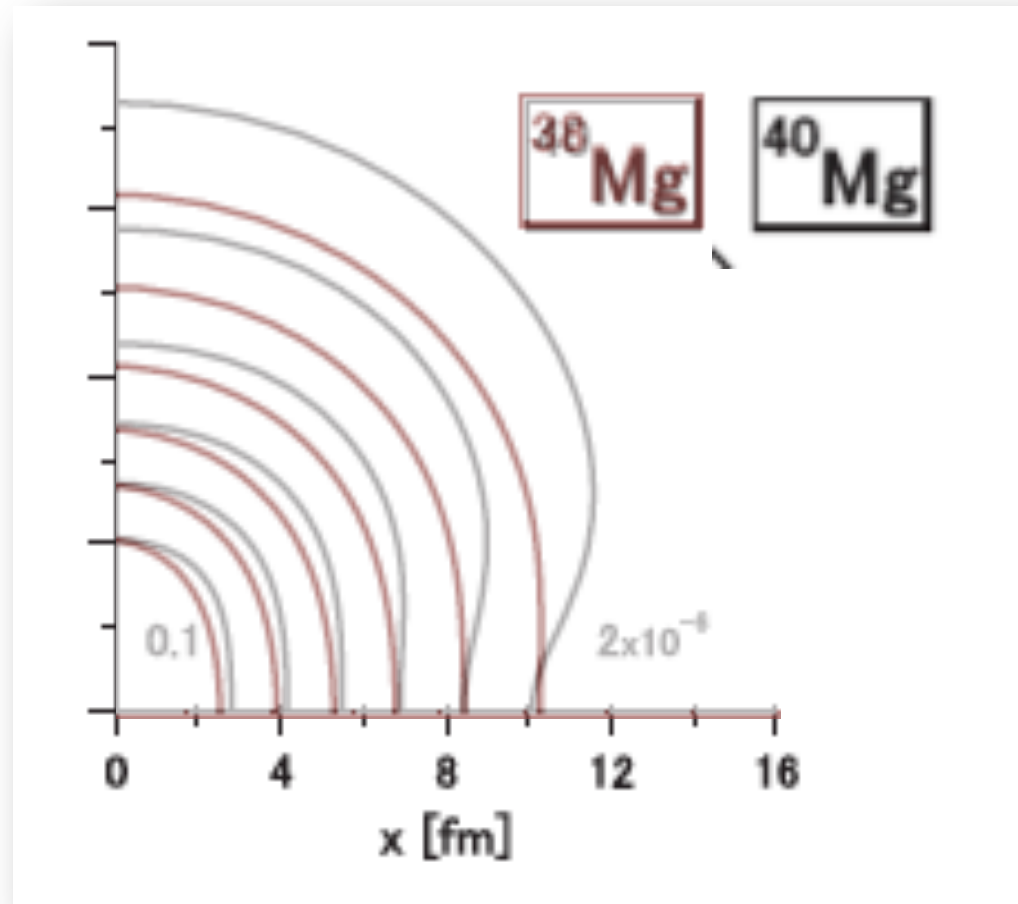
PHYSICAL REVIEW C 98, 011301(R) (2018)



# HFB Calculation of Mg deformed ground states

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$p_{3/2}$  deformed halo and quenched pairing

# Summary

- First data on excitation modes in a heavy weakly bound nucleus -  $^{40}\text{Mg}$ 
  - Observed spectrum does not fit with existing expectations and existing calculations
  - Breakdown of experimental systematics and theory may suggest something new is happening at the neutron dripline
  - **Qualitative arguments indicate that weak binding effects could reproduce the spectrum seen in  $^{40}\text{Mg}$** 
    - The observation of two low lying states maybe a consequence of “weakly coupled” (deformed?) 2 neutron-halo
    - Next → Implications for population pattern → SF’s
  - Microscopic models taking into account extended wavefunctions and coupling to the continuum would be needed to provide a quantitative description

# Summary

- First data on excitation modes in a heavy weakly bound nucleus -  $^{40}\text{Mg}$ 
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  - Breakdown of experimental data may suggest something new is happening
  - Qualitative analysis of binding effects could reproduce observed states
    - The observed states maybe a consequence of “weakly coupled” 2 neutron-halo
    - Next  $\rightarrow$  Improved population pattern  $\rightarrow$  SF’s
  - Microscopic models taking into account extended wavefunctions and coupling to the continuum would be needed to provide a quantitative description

LAUGH TEST

FAIL

?

Many Thanks to  
George Bertsch, Rolo Id Betan, Osvaldo Civitarese,  
Roberto Liotta and Nicu Sandulescu !

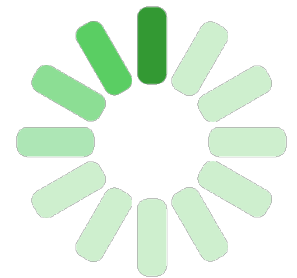
## Part 2

What is the Kerman's Problem anyway?

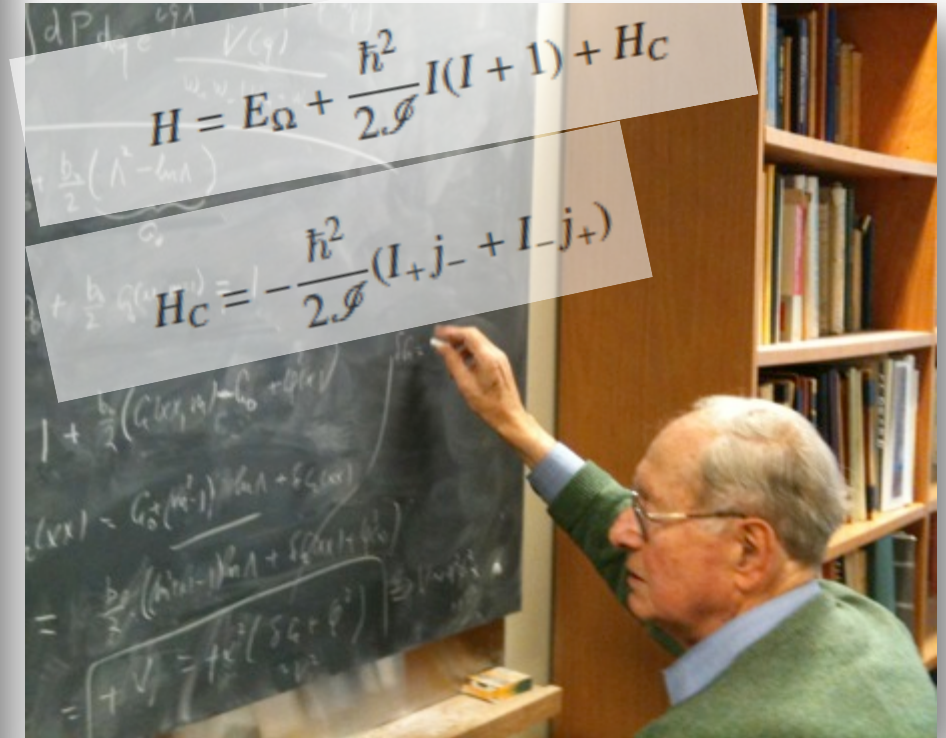
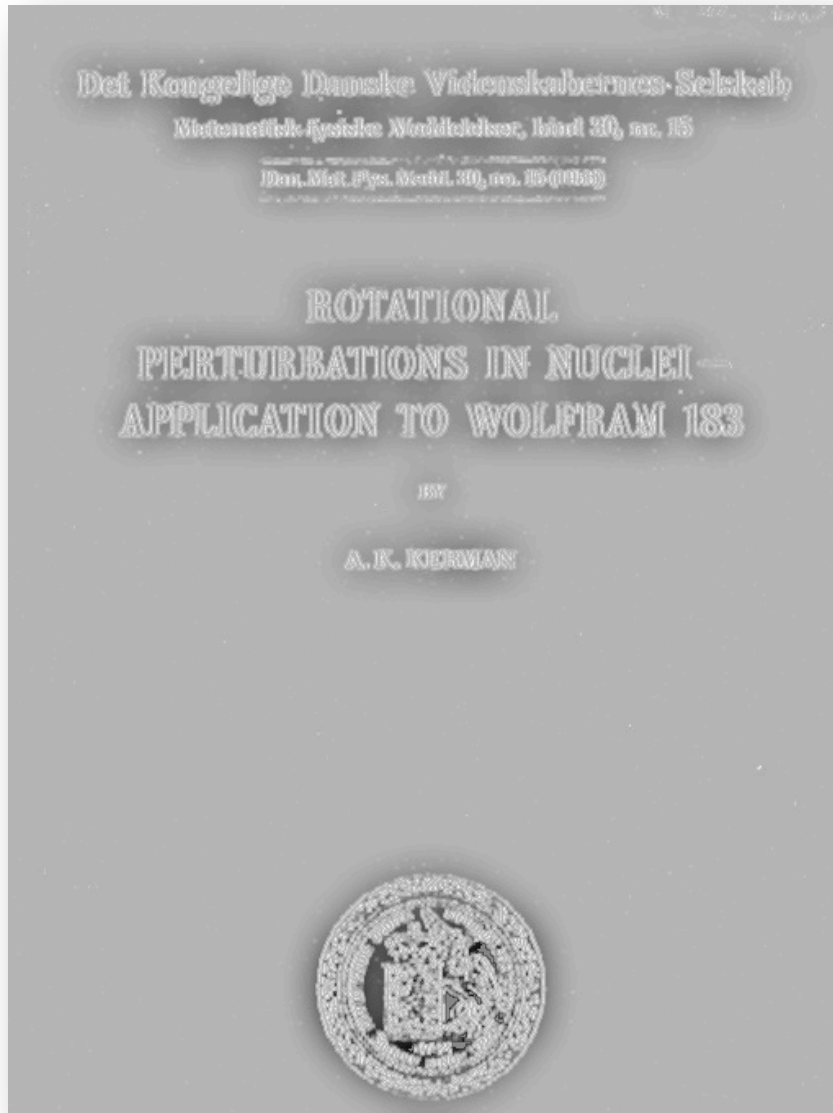
What we did and why

Some (preliminary) results

**In Progress**



# Arthur Kerman 1929-2017



# The Particle plus Rotor Model 101

$$\begin{aligned}
 H &= H_p + H_{\text{rot}} = H_p + (\hbar^2/2\mathcal{I})\mathbf{R}^2 \\
 &= H_p + (\hbar^2/2\mathcal{I})(R_x^2 + R_y^2),
 \end{aligned}$$

$$\begin{aligned}
 H &= H_p + (\hbar^2/2\mathcal{I})[I(I+1) - K^2] + H_c \\
 &+ (\hbar^2/2\mathcal{I})[\langle \mathbf{j}^2 \rangle - \Omega^2],
 \end{aligned}$$



$$\begin{aligned}
 H_c &= -2(\hbar^2/2\mathcal{I})[I_x j_x + I_y j_y] \\
 &= -(\hbar^2/2\mathcal{I})[I_+ j_- + I_- j_+].
 \end{aligned}$$

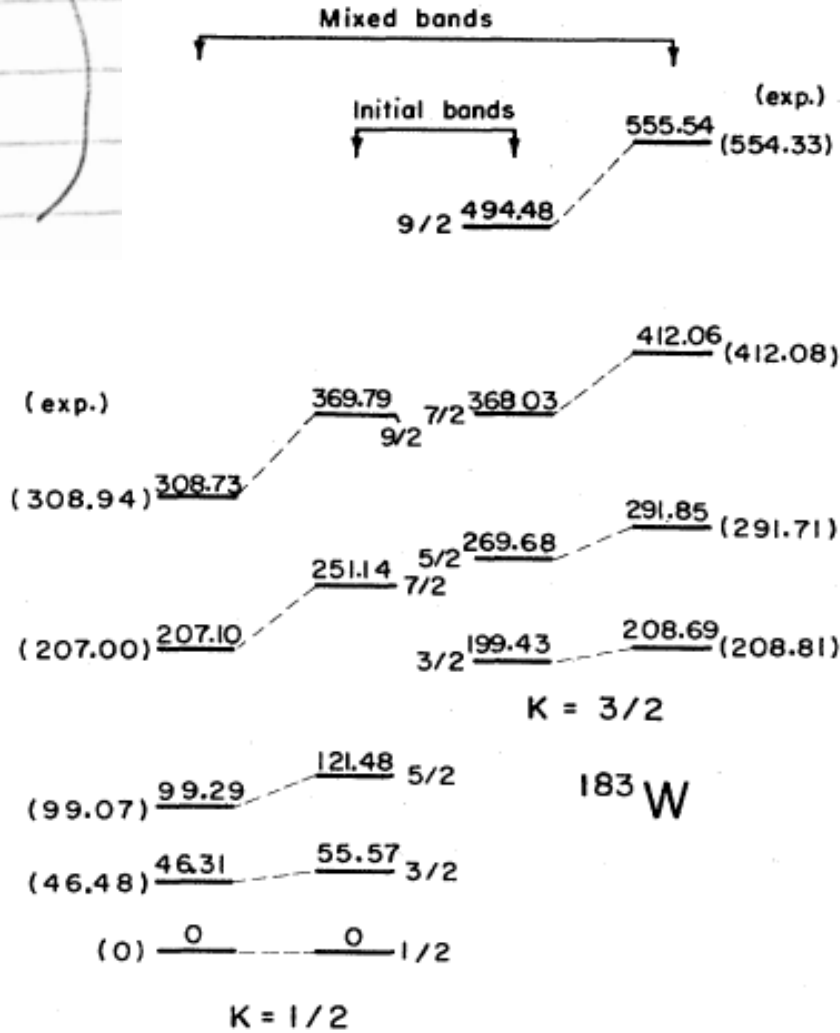
$$\begin{aligned}
 \langle I, \Omega \pm I | H_c | I, \Omega \rangle \\
 = -(\hbar^2/2\mathcal{I})[(I \mp K)(I \pm K + 1)]^{1/2} \langle \Omega \pm 1 | j_{\pm} | \Omega \rangle,
 \end{aligned}$$

$$\langle j, \Omega \pm 1 | j_{\pm} | j, \Omega \rangle = [(j \mp \Omega)(j \pm \Omega + 1)]^{1/2}.$$

# 2x2 matrix

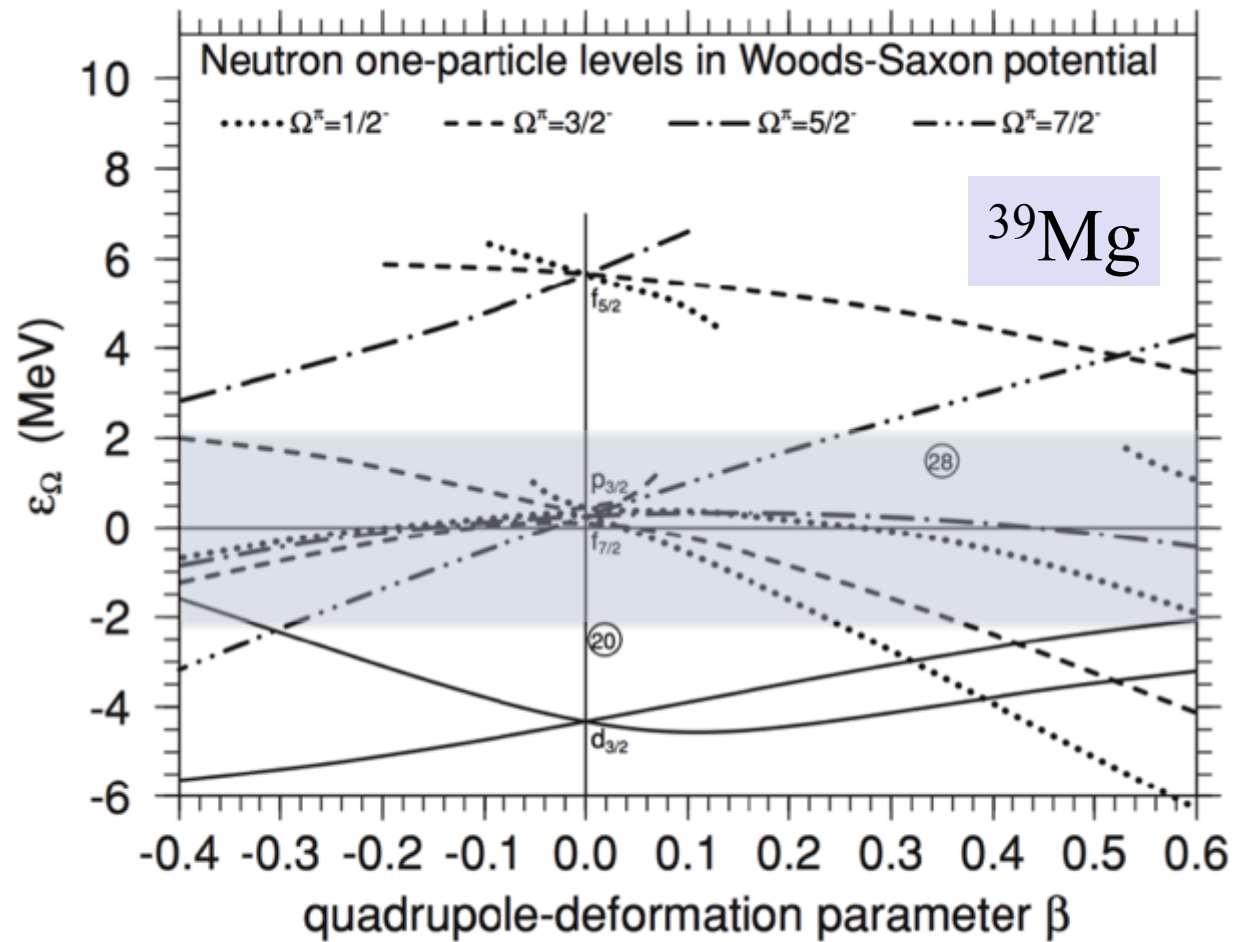
$$\begin{pmatrix} E_{3/2} & V_C \\ V_C & E_{1/2} \end{pmatrix}$$

# The first study





# Weakly Bound Systems



I. Hamamoto, Phys. Rev. C 79, 014307 (2009)

K. Fosse, J. Rotureau, N. Michel, Quan Liu, and W. Nazarewicz, Phys. Rev. C 94, 054302(2016)

Thus :

→ Kerman's Problem in the Continuum

Thus :

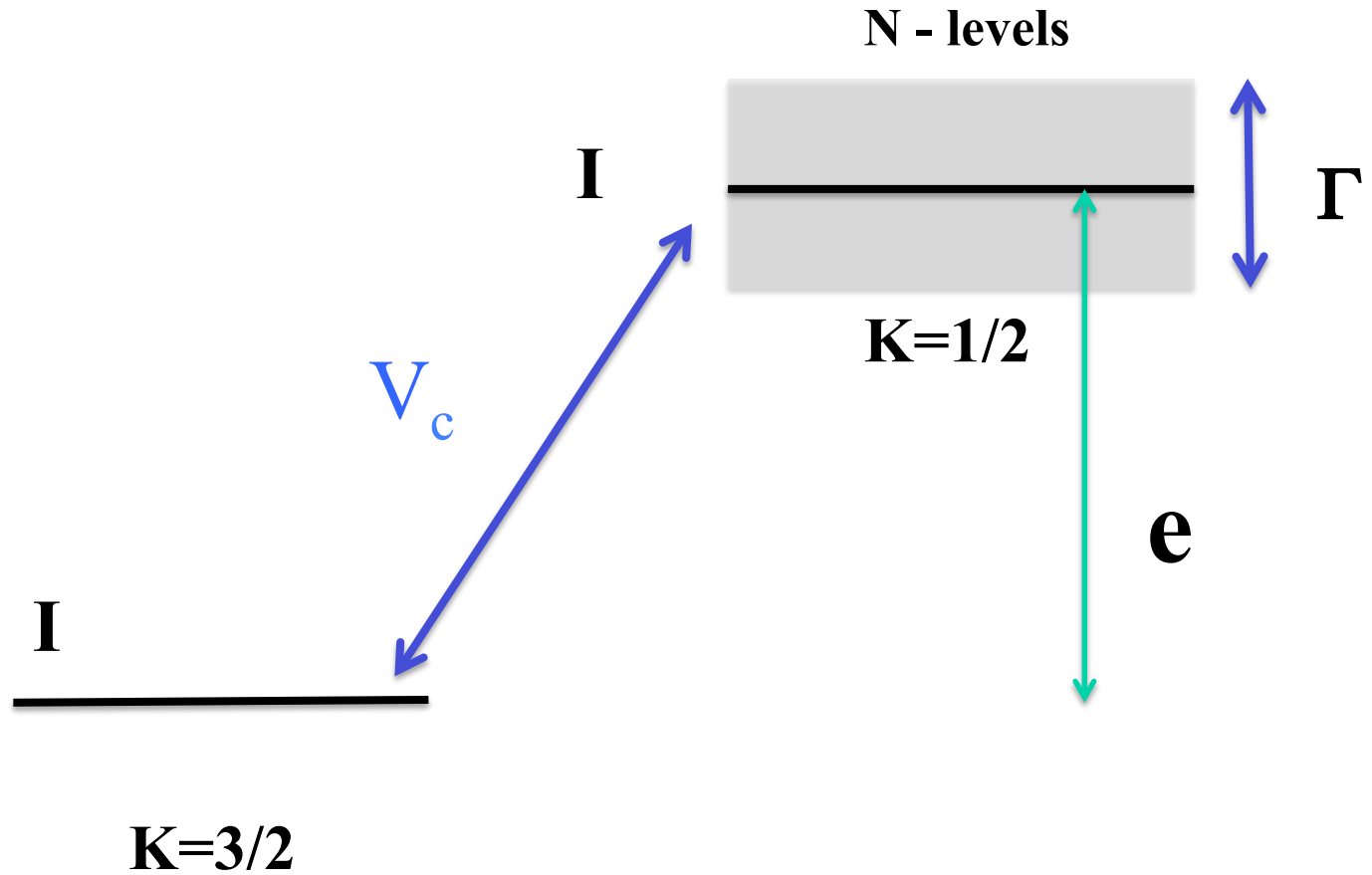
→ Kerman's Problem in the Continuum



Arthur Kerman to Rick Casten, ca. 1980

*“Experimentalists should not dabble in thought ...”*

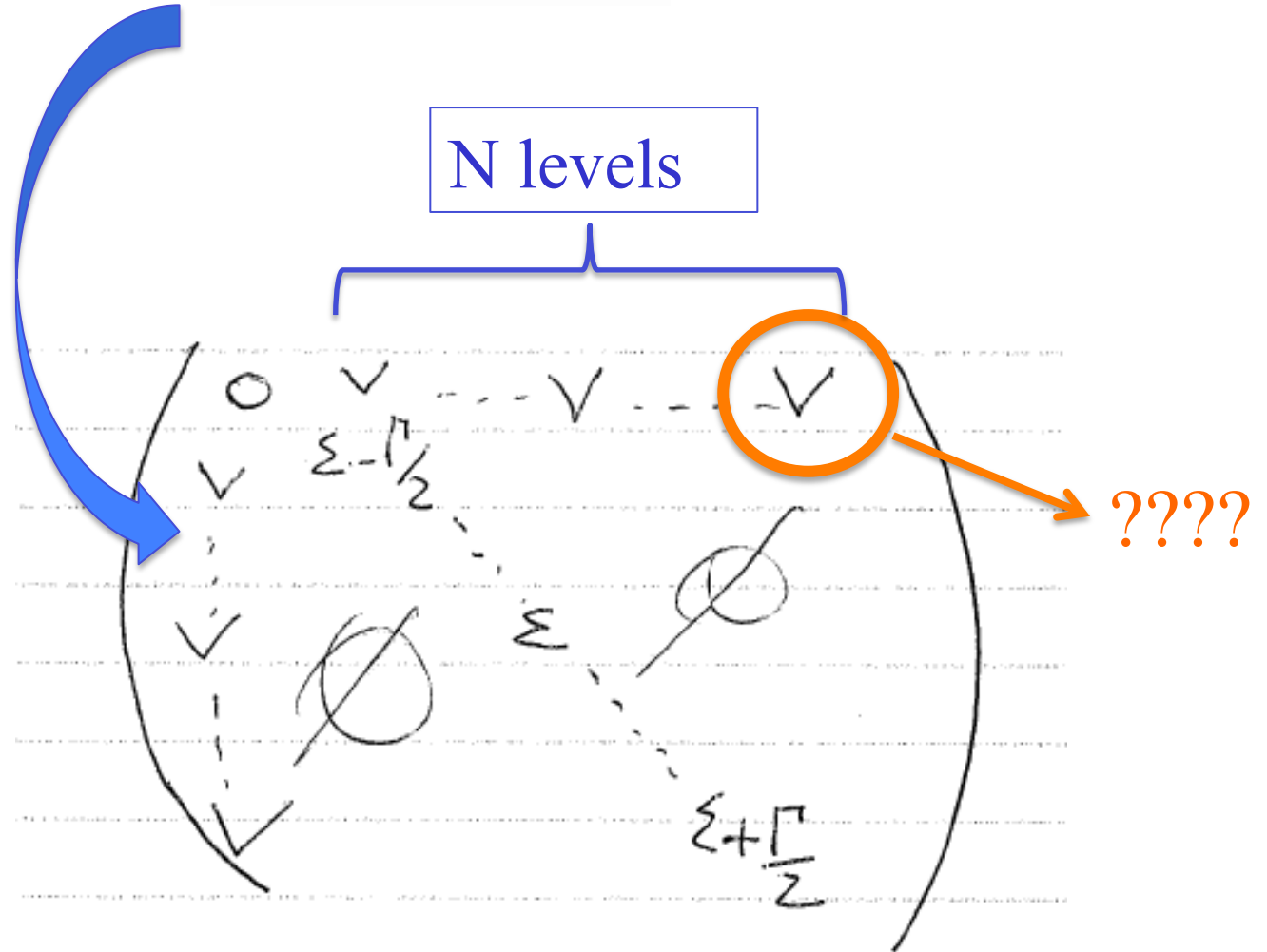
# Two-level $\rightarrow$ N levels



The 2x2 matrix

$$\begin{pmatrix} E_{3/2} & V_C \\ V_C & E_{1/2} \end{pmatrix}$$

Goes into an  
(N+1)x(N+1)  
matrix



In the limit  $\Gamma \rightarrow 0$

$$\begin{pmatrix} 0 & V & \dots & V & \dots & V \\ V & \dots & \varepsilon & \text{X} & \dots & \\ V & \text{X} & \dots & \varepsilon & \dots & \\ V & \dots & \dots & \dots & \dots & \varepsilon \end{pmatrix}$$

for which Mathematica tells me that the lowest eigenvalue is:

$$\lambda = \frac{1}{2} \varepsilon - \sqrt{\left(\frac{\varepsilon}{2}\right)^2 + NV^2}$$

with  $V = \frac{V_0}{\sqrt{N}}$  then  $\lambda = \frac{\varepsilon}{2} - \sqrt{\left(\frac{\varepsilon}{2}\right)^2 + V_0^2}$   
the 2x2 solution!

Consider now

$$E_{3/2} = e_1 + A I(I+1)$$
$$E_{1/2} = e_2 + A I(I+1) + \underbrace{(-)^{I+1/2} A_1(I+1/2)}_{a(I)}$$

Decoupling term

Take  $e_2 - e_1 = e$

And energies in units of the rotational constant  $A$  !

First order perturbation solution ( $V_c \ll e$ ):

$$E_{3/2} = I(I+1) + \frac{V_c}{1+\tilde{a}} \left[ 1 + \frac{\tilde{\Gamma}}{2(1+\tilde{a})} + \dots \right]$$

2x2 Solution

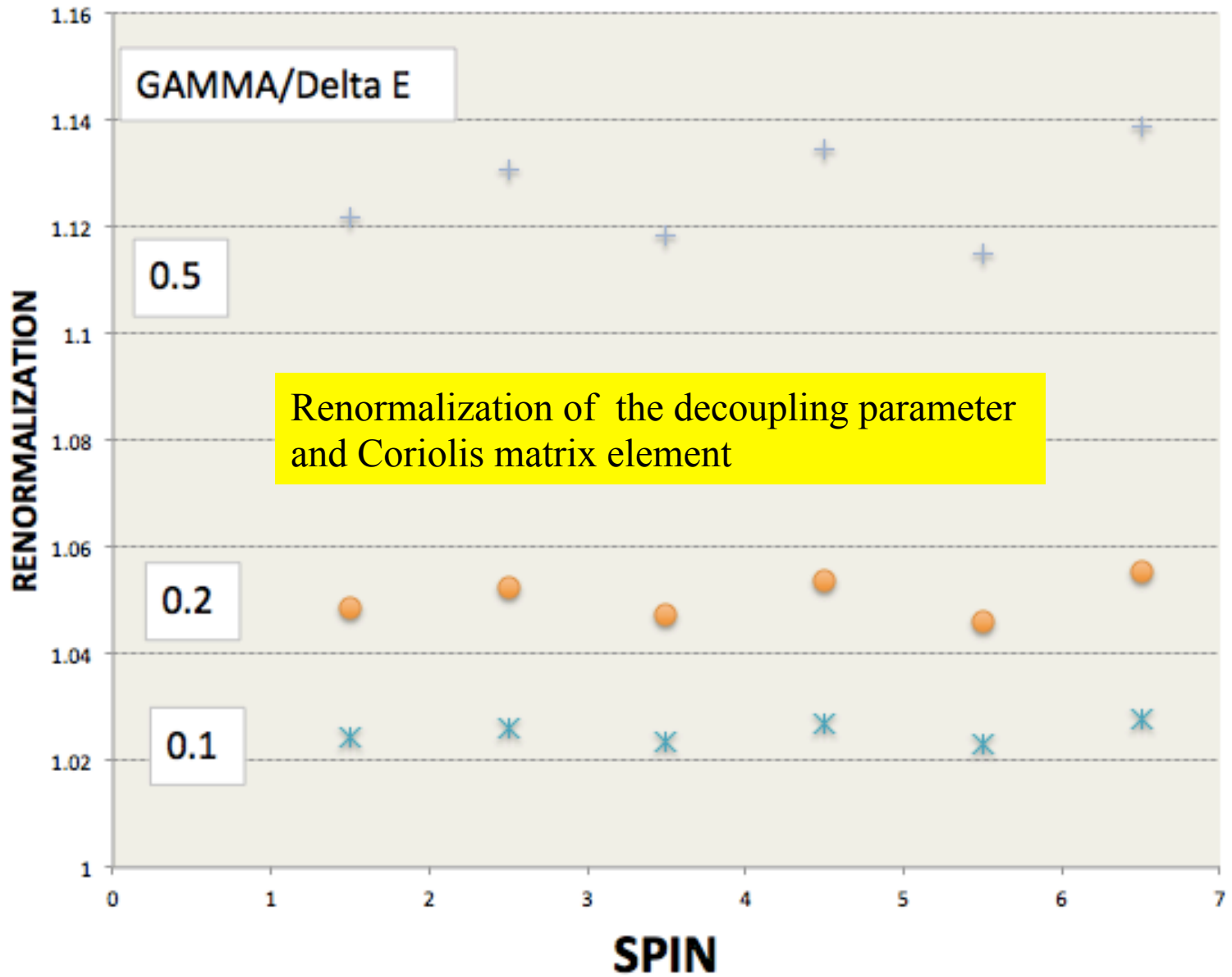
Renormalization factor

with  $\tilde{a} = \frac{a}{\epsilon}$  and  $\tilde{\Gamma} = \frac{\Gamma}{2\epsilon}$  and  $\tilde{V}_c^2 = \frac{V_c^2}{A \cdot \epsilon}$

In the original Kerman paper :

$a = 0.2$     $e = 210 \text{ keV}$ ,  $A = 15 \text{ keV}$ , and  $V_c = 20 \text{ keV}$





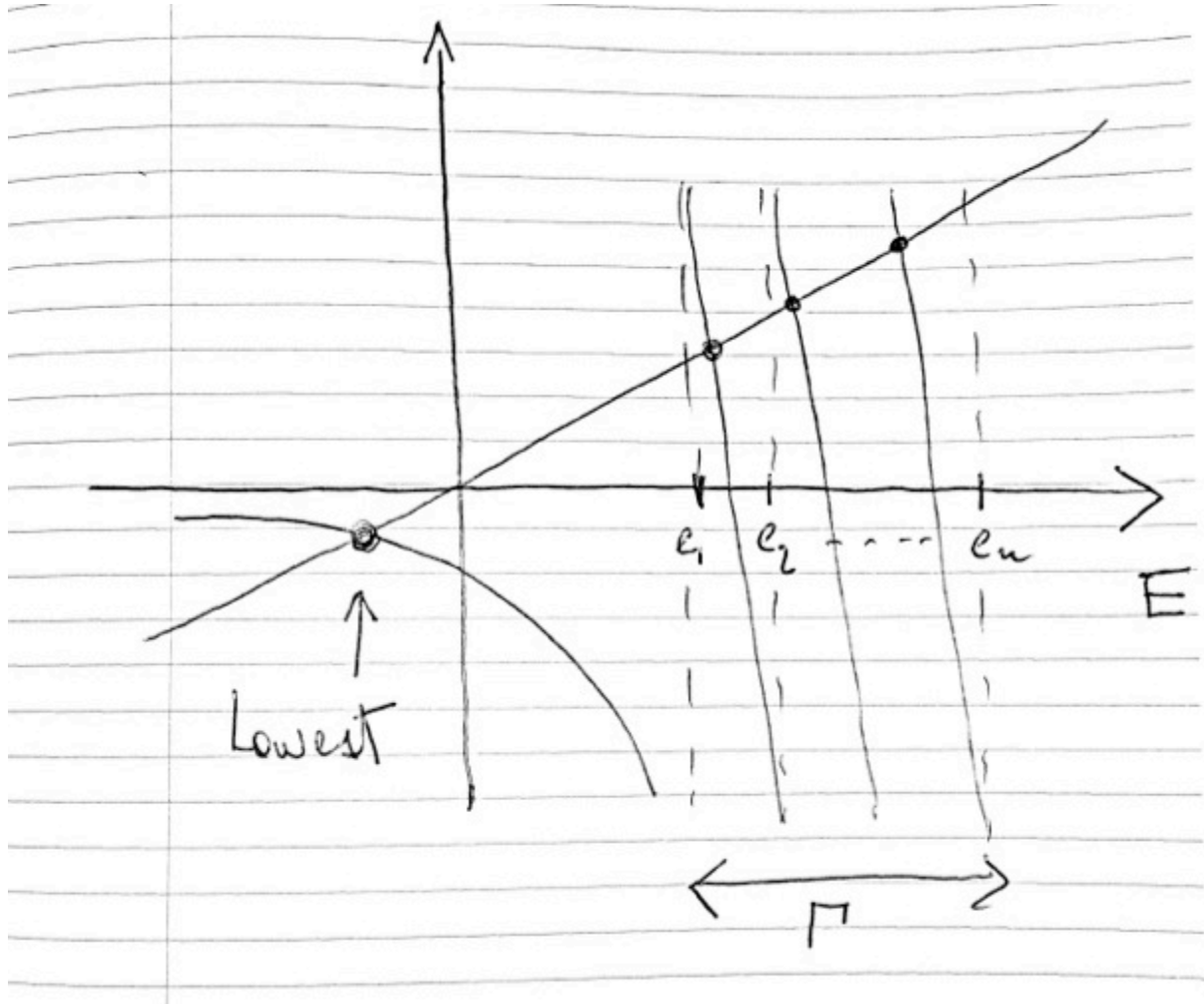
Full solution:

$$\begin{bmatrix} E_0 & v_1 & v_2 & \dots \\ v_1 & \epsilon_1 & 0 & \dots \\ v_2 & 0 & \epsilon_2 & \dots \\ \vdots & & & \ddots \end{bmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix} = E \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix}$$



$$E_0 + \sum \frac{v_i^2}{E - \epsilon_i} = E$$

$$E_0 + \sum \frac{V_i^2}{E - \epsilon_i} = E$$



# Summary

The evolution of Shell Structure and Collective motion in weakly bound nuclei is a topic of much interest in nuclear structure

A “2x2” Kerman-model calculation, including Coriolis mixing with an unbound state, was used to explore possible (general) consequences on rotational properties of an odd-A system

Qualitative effects seem to appear when the width becomes comparable to the intrinsic level separation energy.

Next steps:

- 1) Full solution
- 2) Extension to a single-j Nilsson multiplet

# Summary

The evolution of Shell Structure and Collective motion in weakly bound nuclei is a topic of much interest in nuclear structure

A “2x2” Kerman-model calculation (including Coriolis mixing with an unbound state, was used to study the (general) consequences on rotational spectra of a system

Qualitative effects on the width becomes comparable to the separation energy.

Next steps:

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LAUGH TEST  
FAIL

**Merçi!**