Four-Body Scale in Universal Few-Boson Systems.

בצלאל בזק יוהנס קירשר סבאסטיאן מלך מנואל פאבון

B. Bazak [1] *J. Kirscher* [2] S. König [3] M. Pavón Valderrama [4]

N. Barnea [1] U. van Kolck [5,6]

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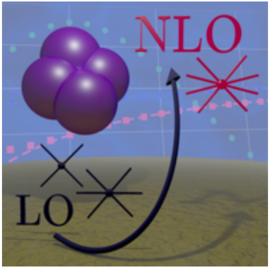
[4] Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University

[5] Institut de Physique Nucléaire, Université Paris-Saclay

[6] Department of Physics, University of Arizona



Phys. Rev. Lett. **122**, *143001*

$A = 2$

The Problem

Identification of correlations between 2-, 3-, ... A-body characteristics and phenomena in $(A + 1 \to \infty)$*-body systems.*

"Can we understand complexity (nuclear chart, molecules) from few-body dynamics?"

bosonic bosonic

$A = d =$ flavour-space dimension

fermionic cinoimref

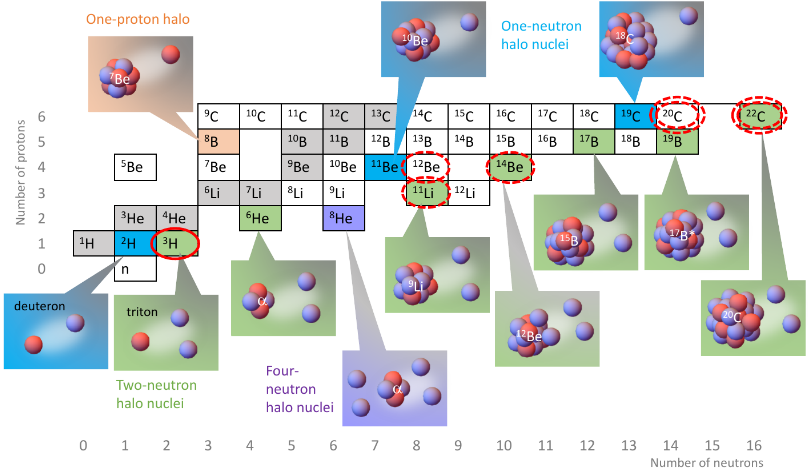
$A = 2$

interaction range $\ll$ scattering (correlation) length

$$k \cot \delta_0(k) = -\frac{1}{a_0} \qquad\qquad + \mathcal{O}(k^2)$$

bosonic bosonic

fermionic cinoimref



P. Naidon, S. Endo, Rep. Prog. Phys. **80** *056001*

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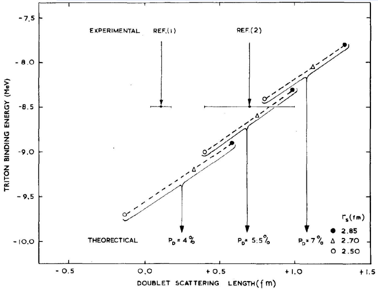
bosonic bosonic

$A = 3$

Phillips line*, Efimov phenomena†



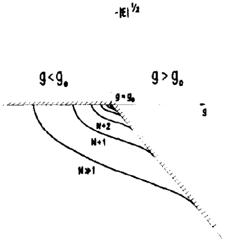


Fig. 1. The level spectrum of three neutral spinless particles. The scale is not indicative.

* A.C. Phillips (1967)

† V. Efimov (1970)

$A = 2$

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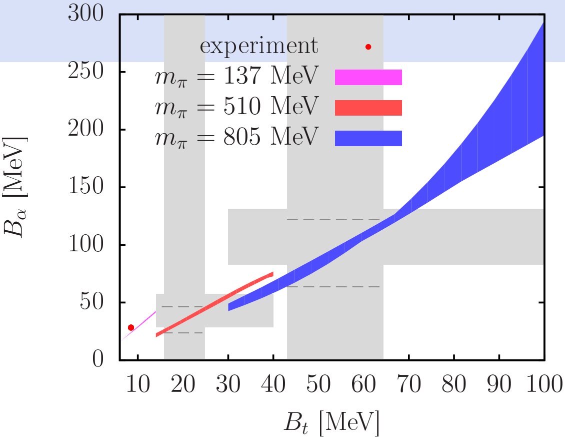
bosonic bosonic

$A = 4$

Tjon line*



* J.A. Tjon (1974)

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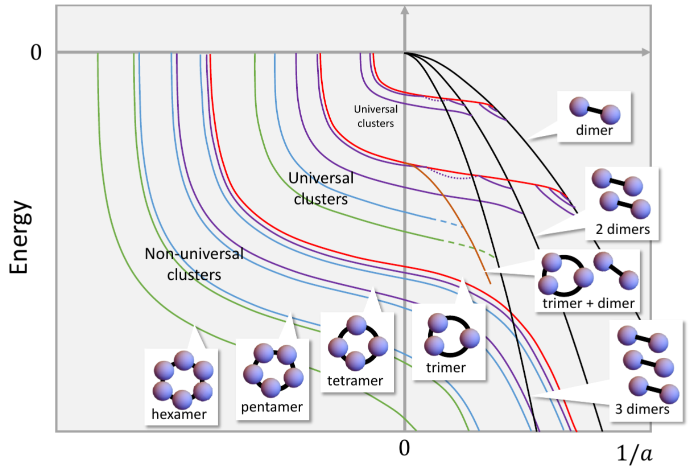


bosonic

P. Naidon, S. Endo, Rep. Prog. Phys. **80** *056001*

$A = d$ =flavour-space dimension

$A = 2$

interaction range $\ll$ scattering (correlation) length

$$k \cot \delta_0(k) = -\frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4)$$

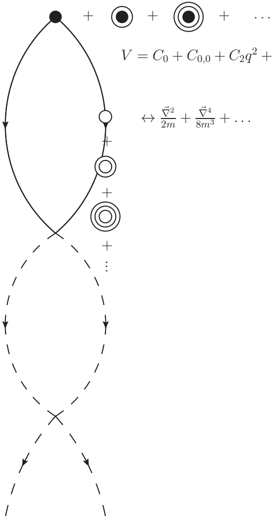
bosonic bosonic

Effect of a range-setting short-distance 2-body perturbation on universal spectra

?

$A = d =$flavour-space dimension

Minimal field-theoretical formulation.*



$$V = C_0 + C_{0,0} + C_2 q^2 + \ldots$$

$$\leftrightarrow \frac{\vec{\nabla}^2}{2m} + \frac{\vec{\nabla}^4}{8m^3} + \ldots$$

* P. F. Bedaque, J.-W. Chen, H. W. Hammer, D. B. Kaplan, U. van Kolck, G. Rupak, M. J. Savage (199x-201y)

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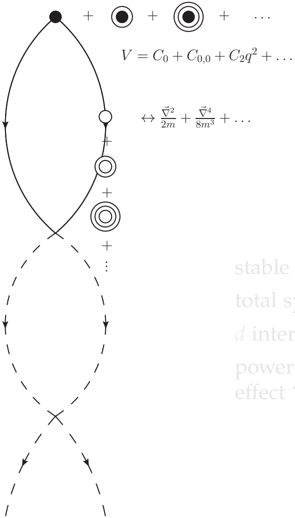

$$\mathcal{L}_{LO} = N^T \left[i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_N} \right] N + C\,(N^T N)^2 + D\,(N^T N)^3$$

stable particles;

total system energy < particle-production thresholds;

d internal flavour states;

power counting ("structural and diagrammatic")
effect "if" they collide vs. effect how often they collide

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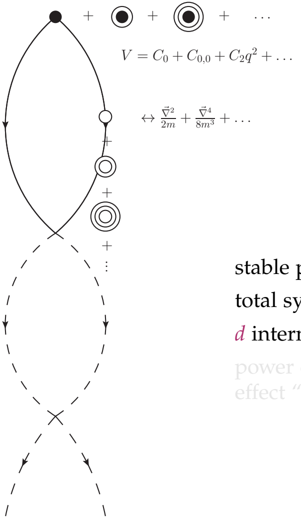

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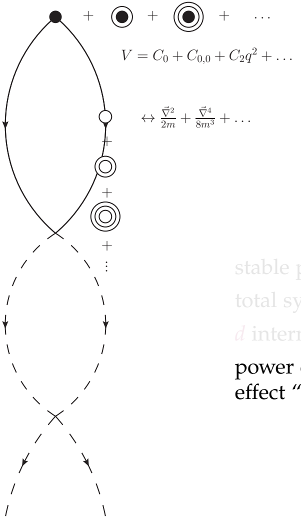

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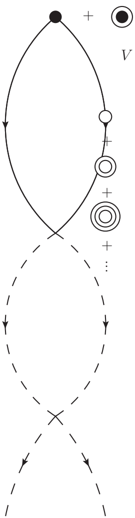

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$$\mathcal{L}_{LO} = N^T \left[i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_N} \right] N + C\,(N^T N)^2 + D\,(N^T N)^3$$

(א) encode the ignorance about substructure in C, D (renormalize).

(ב) “group transform” within the unobservable.

(ג) trace this transformation in an observable.

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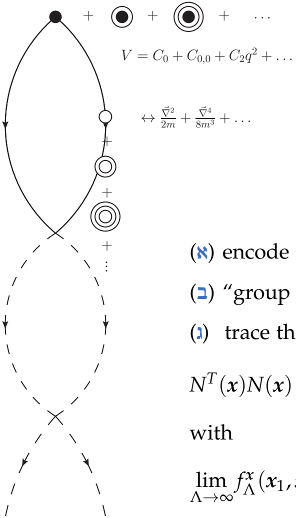

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(ג) trace this transformation in an observable.

$$N^T(\boldsymbol{x})N(\boldsymbol{x})\; N^T(\boldsymbol{x})N(\boldsymbol{x}) \rightarrow f^{\boldsymbol{x}}_{\Lambda}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4)\; N^T(\boldsymbol{x_1})N(\boldsymbol{x_2})N^T(\boldsymbol{x_3})N(\boldsymbol{x_4})$$

with

$$\lim_{\Lambda\rightarrow\infty} f^{\boldsymbol{x}}_{\Lambda}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4) = \prod_{i=1}^{4} \delta(\boldsymbol{x} - \boldsymbol{x_i})$$

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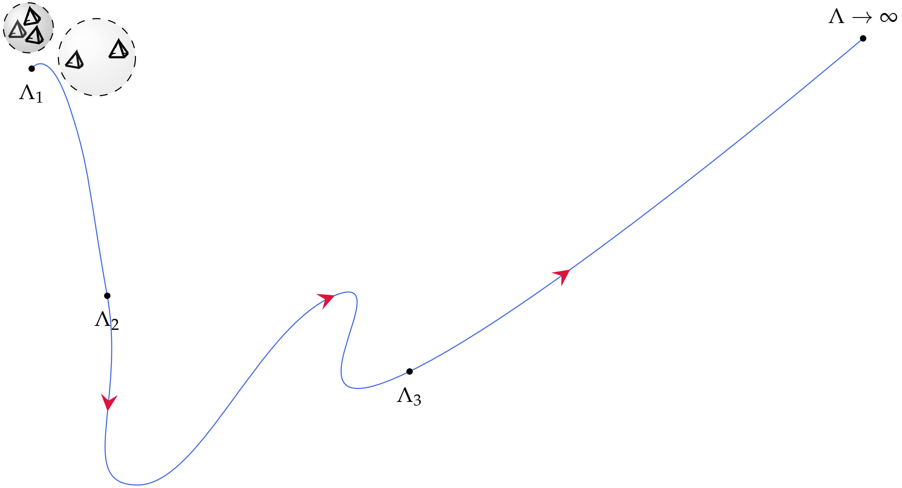
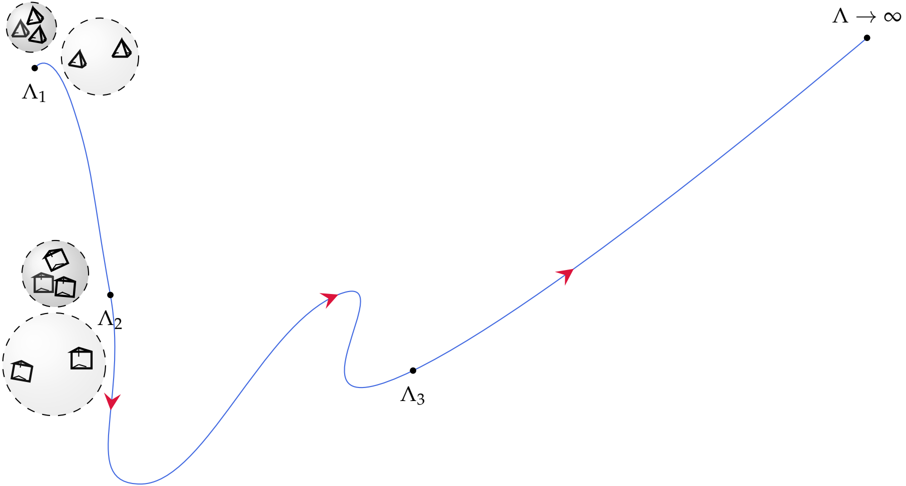
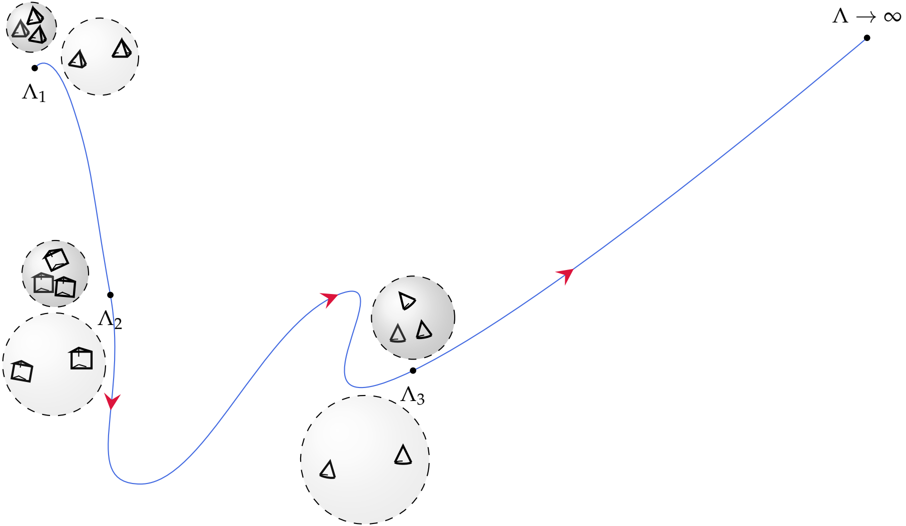


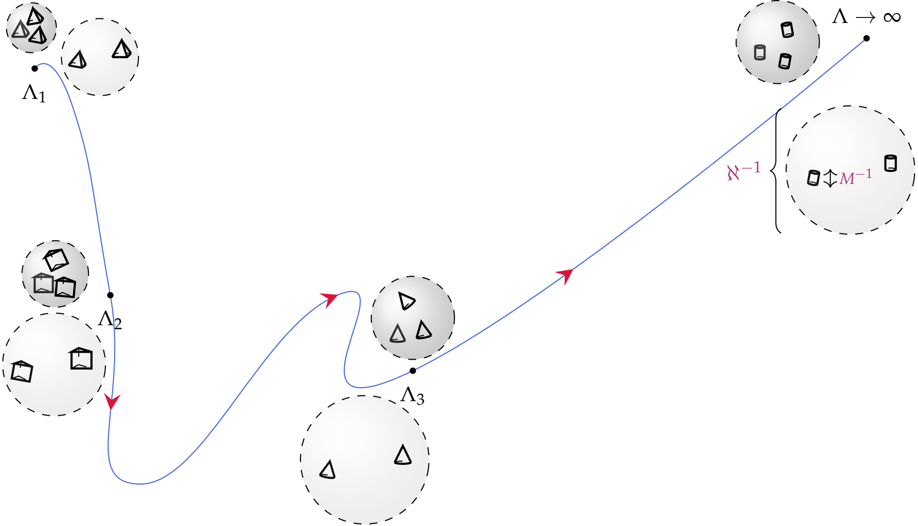
$\Lambda \to \infty$
Λ_1
Λ_2
Λ_3



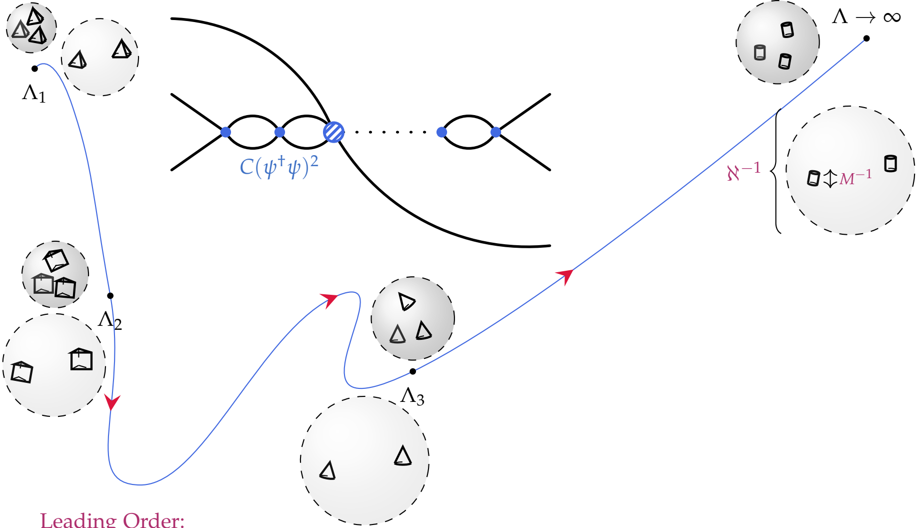
Λ → ∞
Λ_1
Λ_2
Λ_3



$\Lambda \to \infty$
Λ_1
Λ_2
Λ_3



Λ_1
Λ_2
Λ_3
$\Lambda \to \infty$
$\aleph^{-1}$
M^{-1}

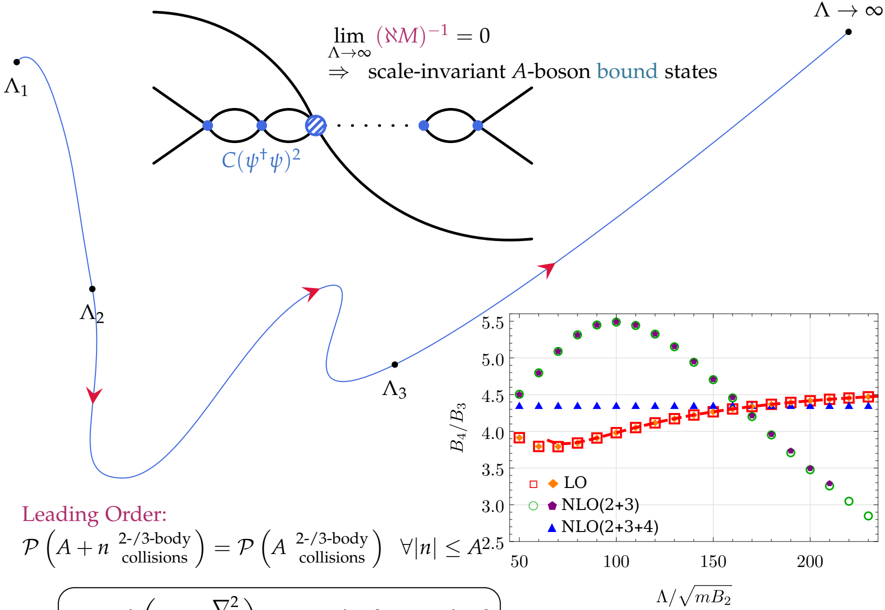


Leading Order:

$$\mathcal{P}\left(A + n \begin{array}{c}\text{2-/3-body}\\ \text{collisions}\end{array}\right) = \mathcal{P}\left(A \begin{array}{c}\text{2-/3-body}\\ \text{collisions}\end{array}\right) \quad \forall |n| \leq A$$

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3$$

P. F. Bedaque, H.-W. Hammer, and U. van Kolck, NPA ***646****, 444 (1999).*



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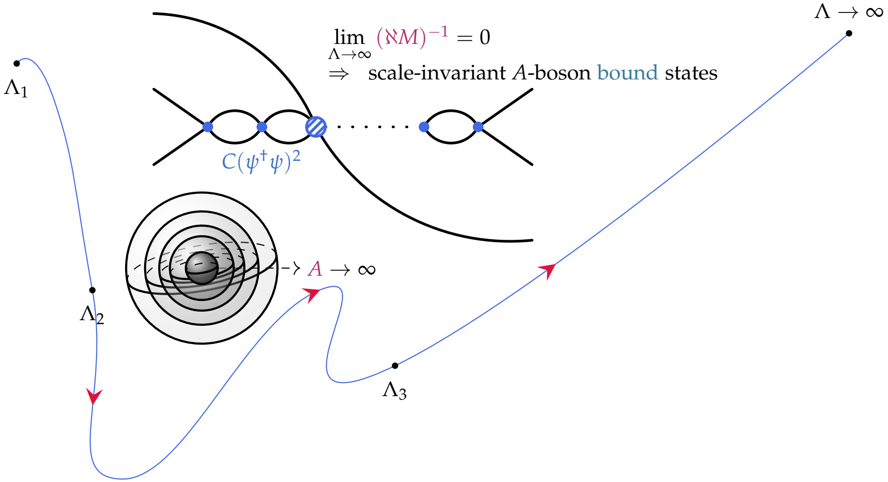
$\Psi = 12 \cdot \mathbf{Y} + 6 \cdot \mathbf{H} \overset{\text{id.}}{\to} \mathbf{Y} + \mathbf{H}$ (S. König, accurate calibration); $\Psi = \sum_n c_n \cdot e^{-\boldsymbol{\eta}^\dagger \mathbf{A}_n \boldsymbol{\eta}}$ (B. Bazak, $A > 4$).



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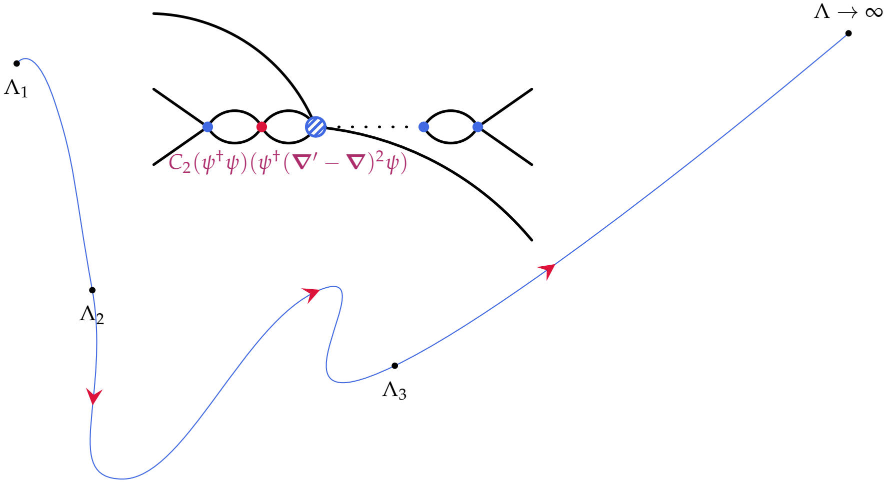
J. von Stecher, PRL **107***, 200402 (2011); M. Gattobigio, A. Kievsky, and M. Viviani, PRA* **84***, 052503 (2011);*
B. Bazak, M. Eliyahu, and U. van Kolck, PRA **94***, 052502 (2016).*



Next-to-leading Order (3 bodies, 3 constraints):

Two long-range constraints, one short-range constraint.

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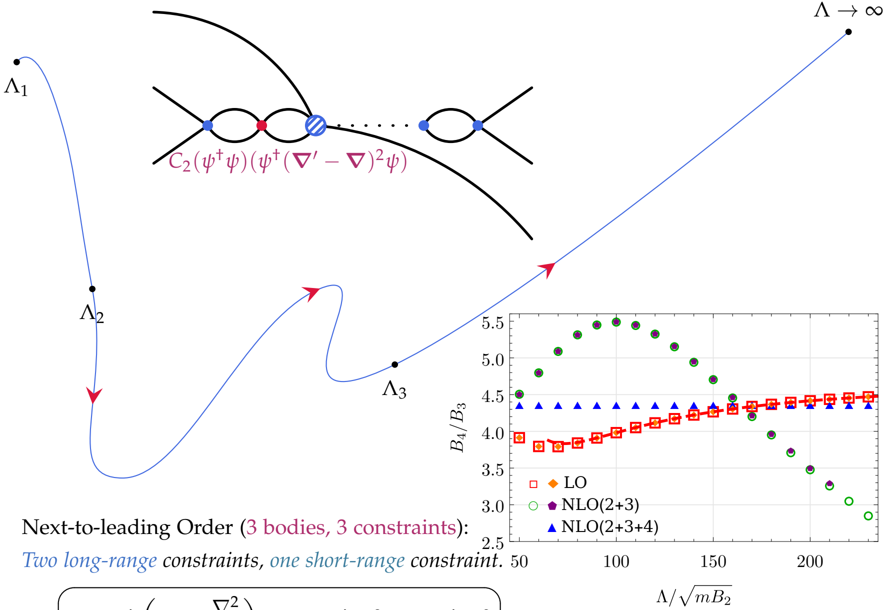

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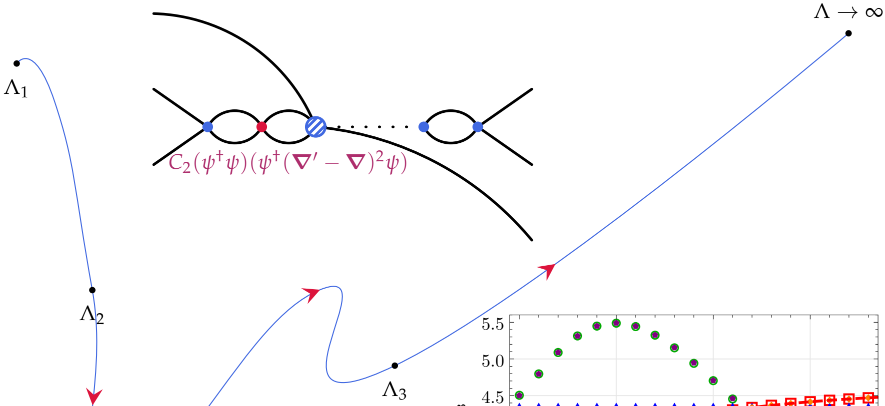
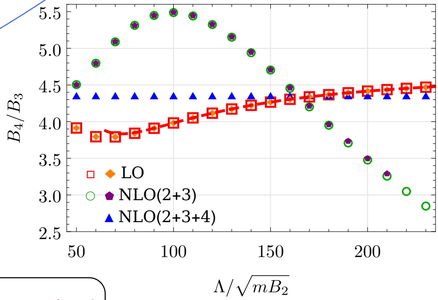
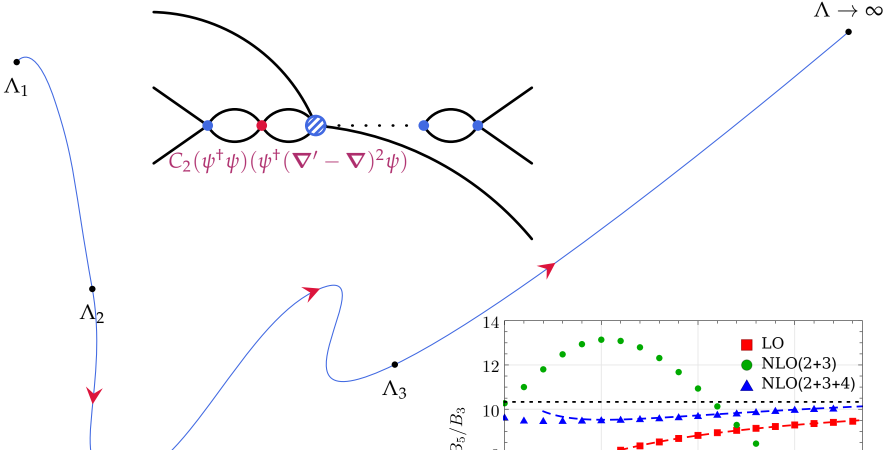


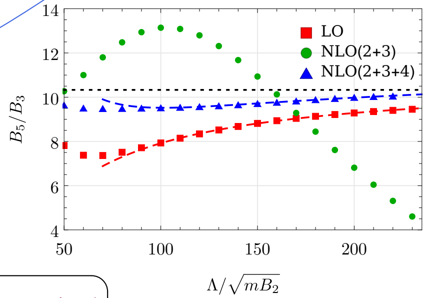


(3 bodies, 3 constraints):

⇒ four-body details are resolved!

$$\mathcal{L} = \psi^\dagger\left(\mathrm{i}\partial_0 + \frac{\nabla^2}{2m}\right)\psi - C(\psi^\dagger\psi)^2 - D(\psi^\dagger\psi)^3 - F(\psi^\dagger\psi)^4$$





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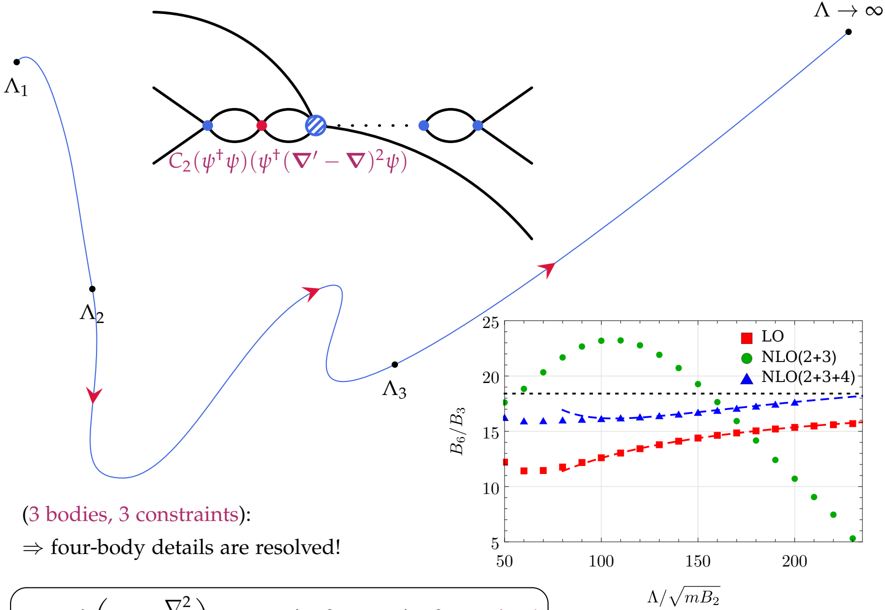

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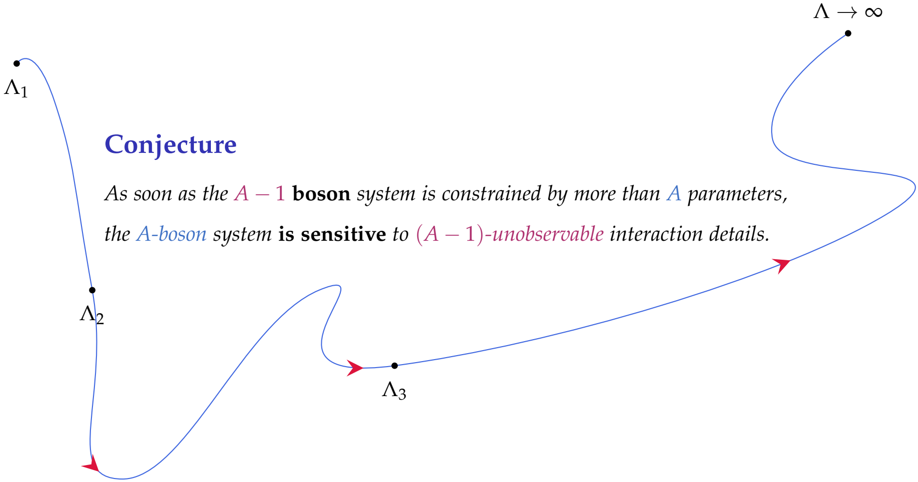
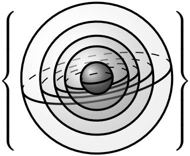


Conjecture

As soon as the $A-1$ **boson** *system is constrained by more than* A *parameters, the* A*-boson system* **is sensitive** *to* $(A-1)$*-unobservable interaction details.*

$\lim_{|\boldsymbol{r}_{nm}|\to 0}$

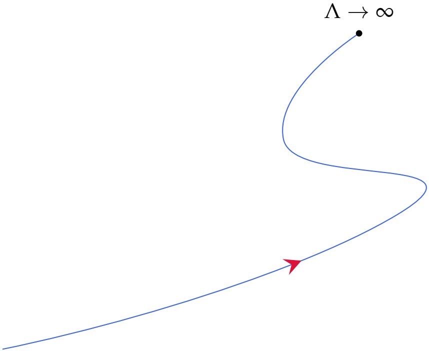


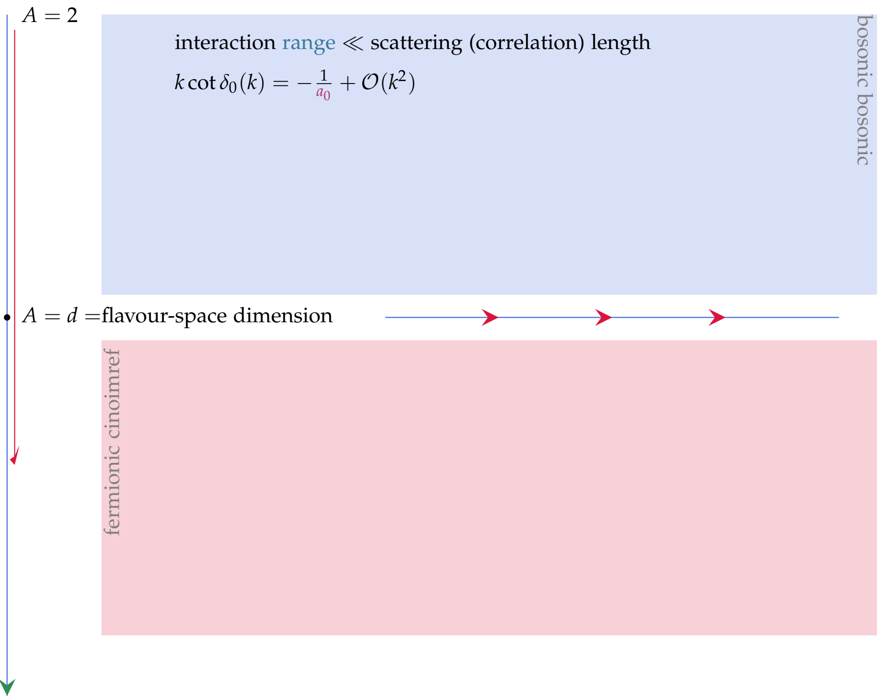
$$= \frac{u(\boldsymbol{r}_{12}\ ,\ \ldots\ ,\ \boldsymbol{r}_{A-1,A})}{\prod_{i<j} |\boldsymbol{r}_{ij}|} \quad \forall\ n,\ m$$

and with $0 < |u| < \infty$ for any $|r_{nm}| \to 0$ (no finite polynomial)

Little Fugue in G Minor

Johann Sebastian Bach 1685 - 1750





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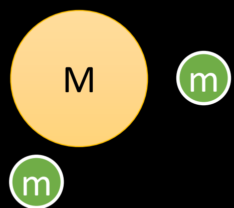
Background

O. I. Kartavtsev et al. (Sov. Phys. JETP, 108(3) 365–373)
M. Gattobigio et al. (Phys. Rev. C **100**, 034004)
L. Contessi et al. (Phys. Lett. B **772** 839-848)



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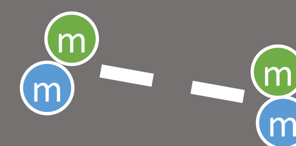


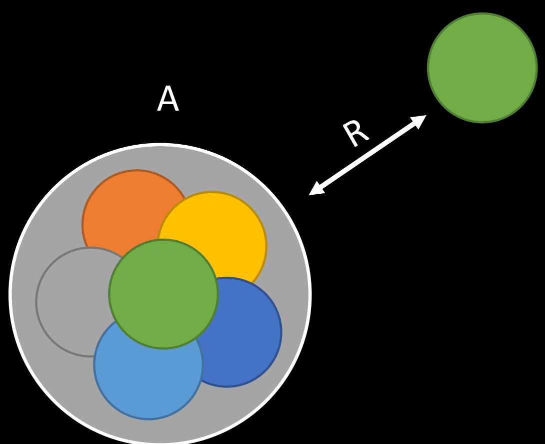
D. S. Petrov et al. (Phys. Rev. A **71**, 01270

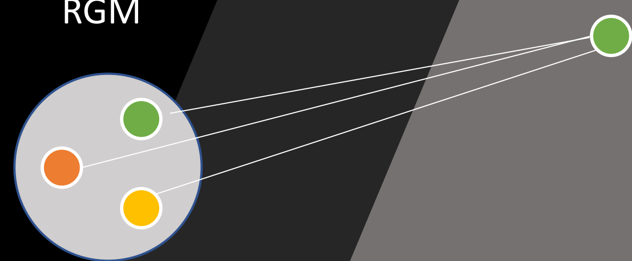
M. Gattobigio et al. (Phys. Rev. C **100**, 034004)

Is this just a case or a common feature for all the systems with more particles than fermionic degrees of freedom?

Semi - Analytic calculation

RGM for core interaction
and then an extra fermion is added.





Pro:
- Many-body problem -> two-body

Contro:
- Interaction is energy dependent
- ‘Core’ Wave function is approximated

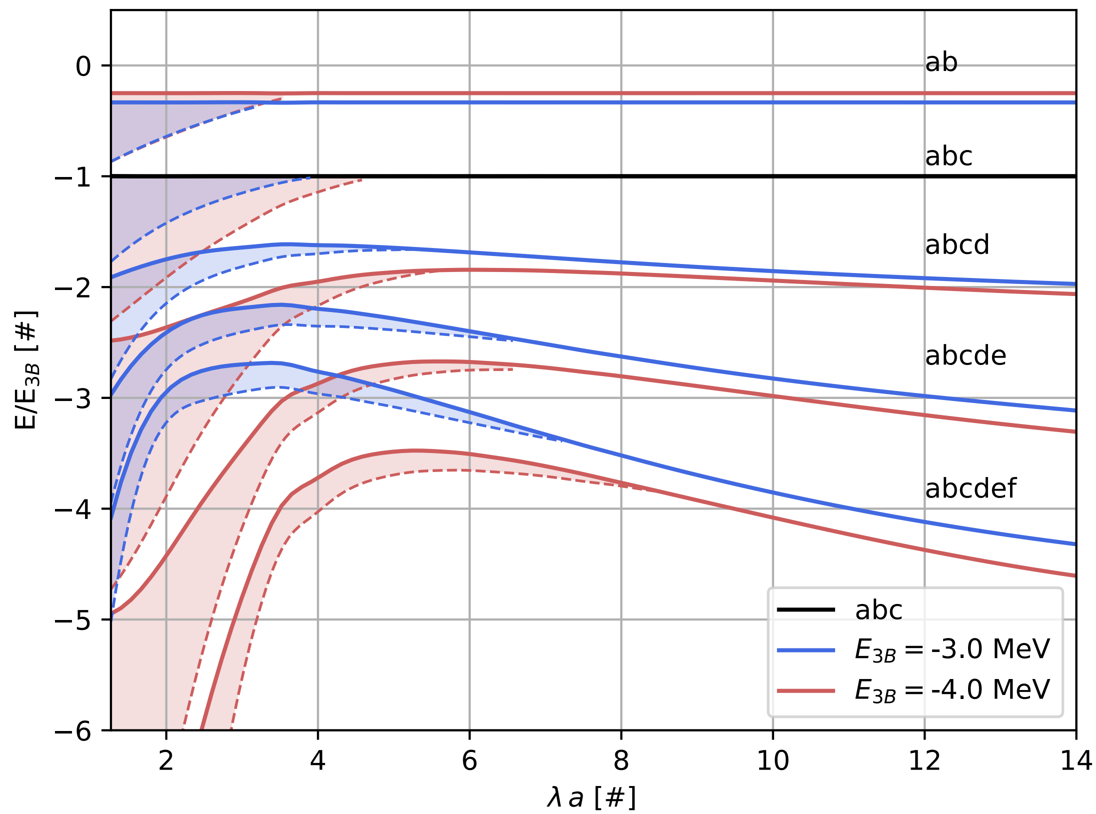
S. Endo et al. Few-Body Systems vol 51 2-4 207-217 (2011)

Close to unitarity AB..Z + A

Each P-wave state breaks in
N bosons + 1 fermion (ABC..Z + A)
for **sufficient large cut-off λ_c**.

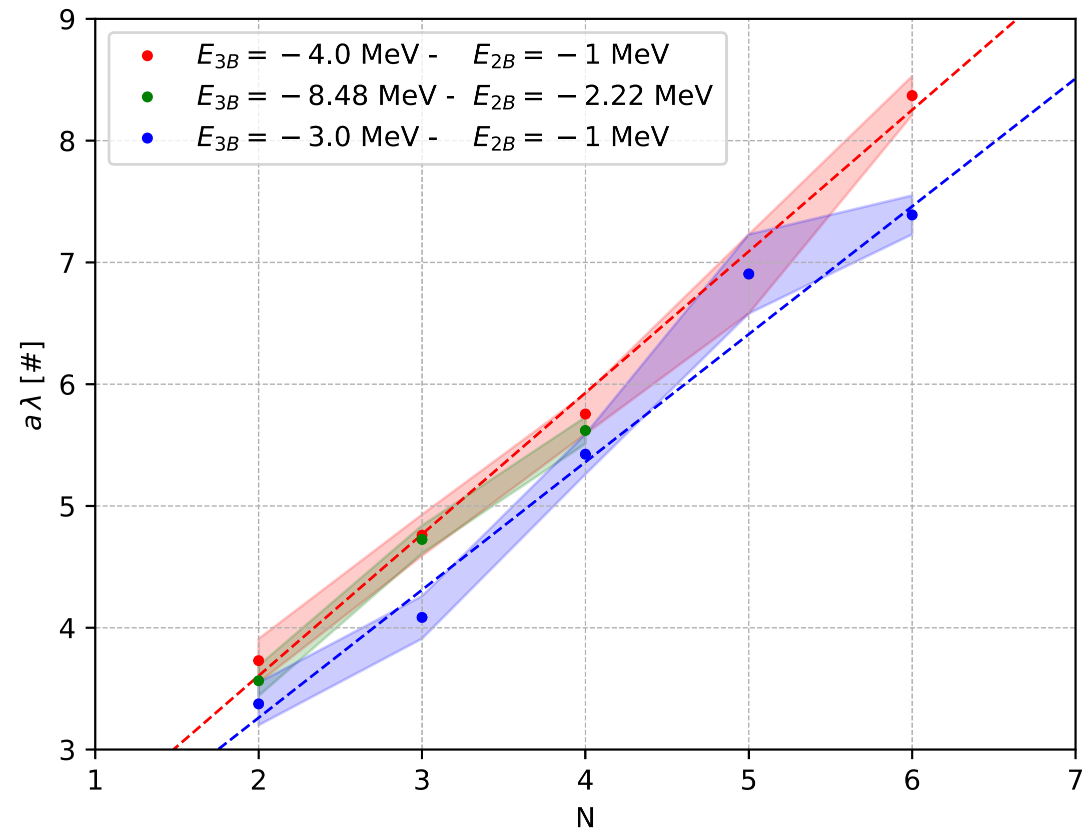
λ_c depends on:

- The ratio E_{3b}/E_{2b}.
  (Larger ration → stable for lower r_0)
  the limit is the unitary-limit
- the **number of particles**.
  (more particles → stable for lower r_0)
  what happen for infinite bosons?



This is also equivalent to the effective range!

Critical Lambda vs. number of bosons



λ_c increases w/ $\boldsymbol{E_{3b}/E_{2b}}$

λ_c increases linearly w/ the number of bosons **N**.

↓

There is no N for which $\lambda_c \to \infty$

↓

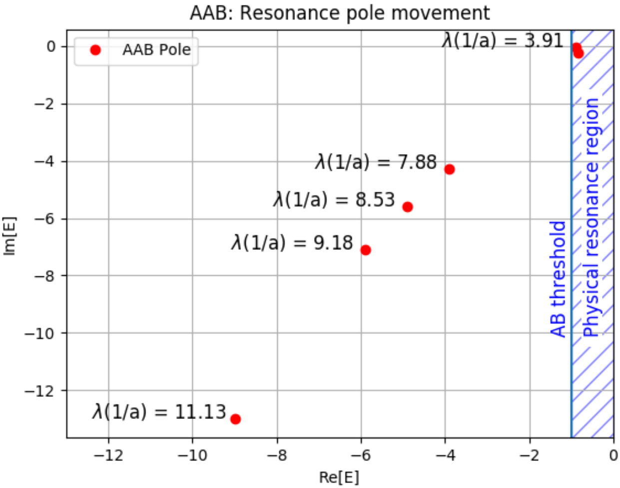
P-wave systems sooner or later will break for any number of particles.

P-wave pole motion for $\Lambda \to \infty$.

M. Schäfer (Prague)

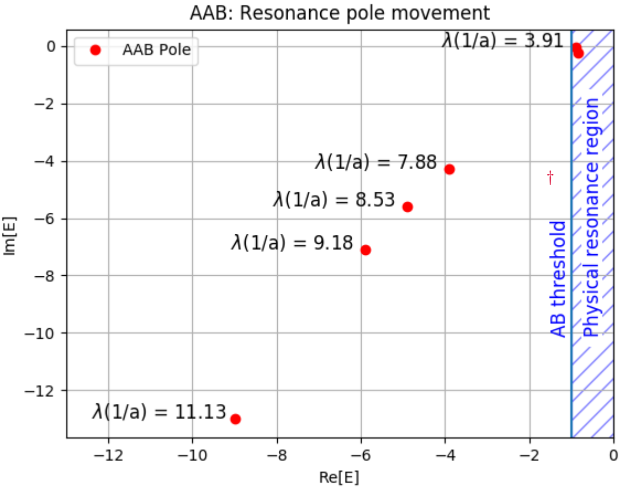
P-wave pole motion for $\Lambda \to \infty$.

M. Schäfer (Prague)



P-wave pole motion for $\Lambda \to \infty$.

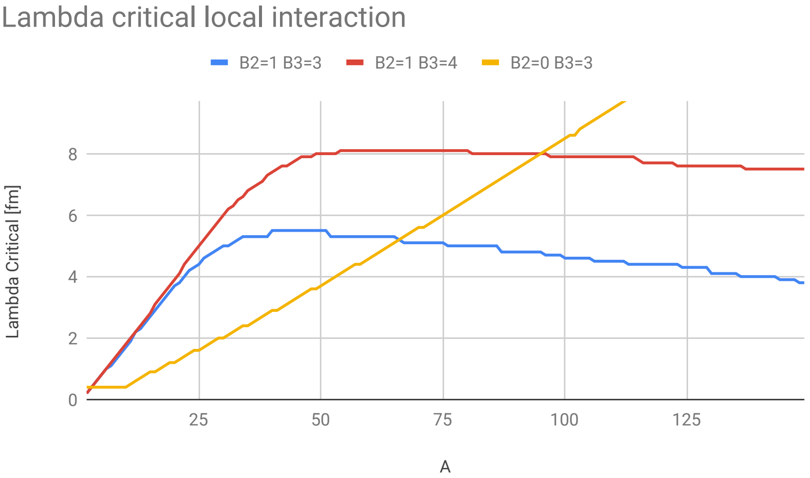
M. Schäfer (Prague)



†Neither real nor imaginary part of pole location show convergent behaviour for $\Lambda \to \infty$.

Critical stability range for $A \to \infty$.

M. Schäfer, L. Contessi, J.Kirscher


Lambda critical local interaction
B2=1 B3=3
B2=1 B3=4
B2=0 B3=3
Lambda Critical [fm]
0
2
4
6
8
25
50
75
100
125
A

Critical stability range for $A \to \infty$.

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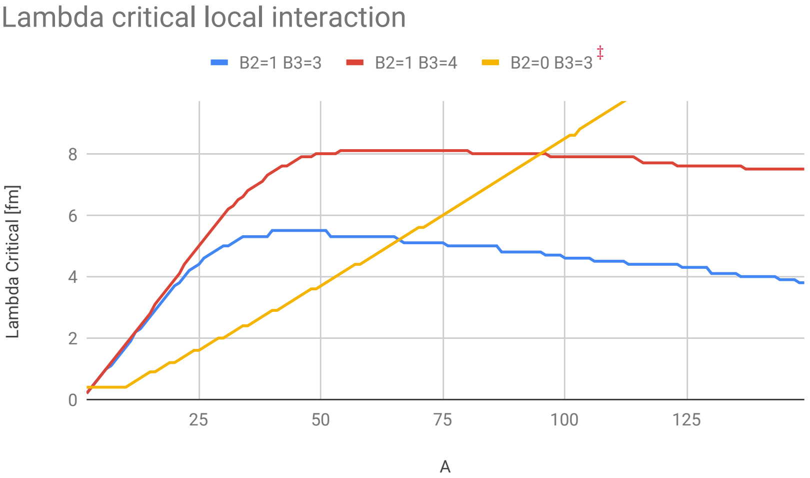

‡ $\exists A^* : \Lambda_c(A^*) > \Lambda_c(A) \; \forall A \neq A^*$.

CRITICAL STABILITY RANGE FOR $A \to \infty$.

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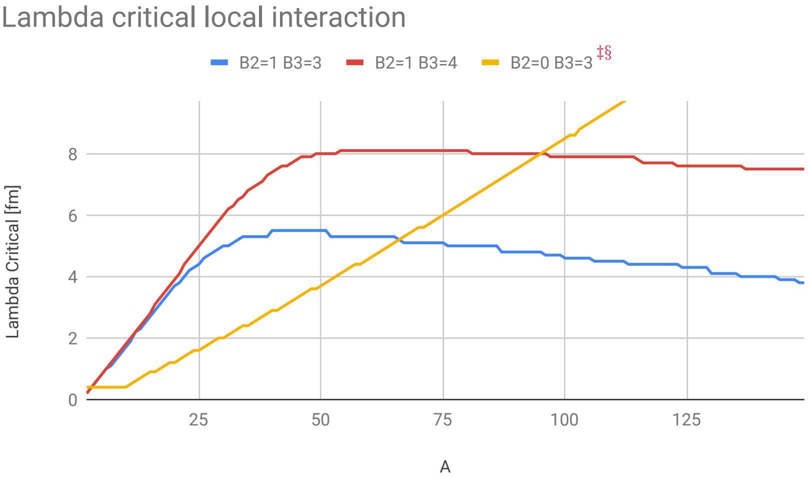

‡ $\exists A^* : \Lambda_c(A^*) > \Lambda_c(A)\ \forall A \neq A^*$.

§ Any scale related to the prediction of a disintegration at $\Lambda_c < \Lambda_{\text{breakdown}}$ vanishes in the unitarity limit.