MODEL OF DARK MATTER AND DARK ENERGY BASED ON GRAVITATIONAL POLARIZATION

Luc Blanchet

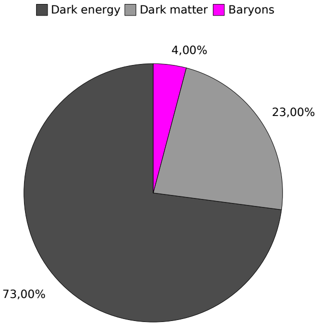
Gravitation et Cosmologie ($\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$)
Institut d'Astrophysique de Paris

15 septembre 2008

Outline of the talk

PHENOMENOLOGY OF DARK MATTER AND MOND

Concordance model in cosmology

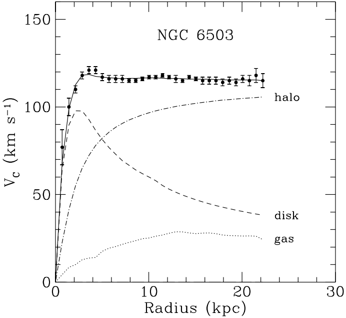


- Einstein field equations with a cosmological constant Λ are assumed to be correct and $\Omega_\Lambda = 73\%$ is measured from the Hubble diagram of supernovas
- Density of baryons $\Omega_\mathrm{b} = 4\%$ is known from Big Bang nucleosynthesis and from CMB anisotropies
- Dark matter is non-baryonic and accounts for the observed dynamical mass of bounded astrophysical systems [Oort 1932, Zwicky 1933]

The dark matter paradigm [Bertone, Hooper & Silk 2004]

1. Dark matter is made of unknown non-baryonic particles, *e.g.*
   - neutrinos (but $\Omega_\nu \lesssim 7\%$)
   - neutralinos (predicted by super-symmetric extensions of the standard model)
   - light scalar [Boehm, Cassé & Fayet 2003]
   - axions
   - $\cdots$
2. It accounts for the observed mass discrepancy between the dynamical and luminous masses of clusters of galaxies
3. It triggers the formation of large-scale structures by gravitational collapse and predicts the scale-dependence of density fluctuations
4. It suggests some universal dark matter density profile around ordinary masses [Navarro, Frenk & White 1995]
5. It has difficulties at explaining naturally the flat rotation curves of galaxies and the Tully-Fisher relation [McGaugh & de Blok 1998, Sanders & McGaugh 2002]

Rotation curves of galaxies are flat

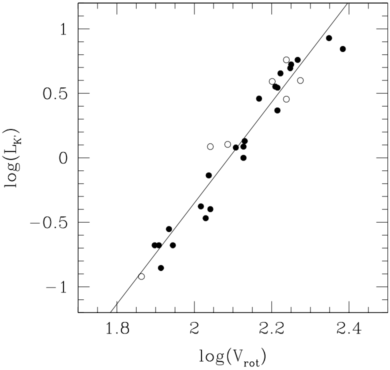


For a circular orbit $v(r) = \sqrt{\dfrac{G\,M(r)}{r}}$

The fact that $v(r)$ is approximately constant implies that beyond the optical disk

$$M_{\text{halo}}(r) \approx r \qquad \rho_{\text{halo}}(r) \approx \frac{1}{r^2}$$

The Tully-Fisher empirical relation [Tully & Fisher 1977]



The relation between the asymptotic flat velocity and the luminosity of spirals is

$$v_{\rm flat} \propto L^{1/4}$$

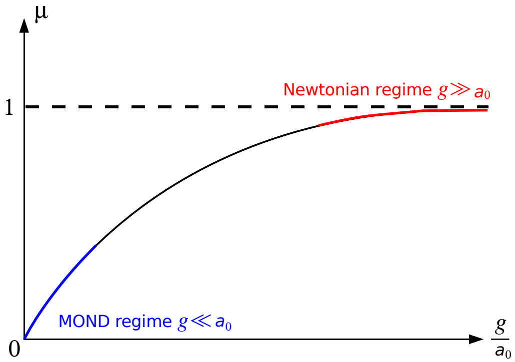
Modified Newtonian dynamics (MOND) [Milgrom 1983]

1. MOND was proposed as an alternative to the dark matter paradigm
2. It states that there is no dark matter and we witness a violation of the fundamental law of gravity
3. MOND is designed to account for the basic phenomenology of the flat rotation curves of galaxies and the Tully-Fisher relation
4. The basic fact about MOND is that the apparent need for dark matter arises in regions of weak gravitational field $g \ll a_0$

The Newtonian gravitational field is modified in an *ad hoc* way

$$\mu\left(\frac{g}{a_0}\right) \boldsymbol{g} = \boldsymbol{g}_{\mathrm{N}}$$

The MOND function



In the MOND regime (when $g \ll a_0$) we have [Milgrom 1983]

$$\mu\left(\frac{g}{a_0}\right) \approx \frac{g}{a_0}$$

Recovering flat rotation curves and Tully-Fisher

1. For a spherical mass $g_{\mathrm{N}} = \frac{G\,M}{r^2}$ hence $g \approx \frac{\sqrt{G\,M\,a_0}}{r}$
2. For circular motion $\frac{v^2}{r} = g$ thus v is constant and we get

$$\boxed{v_{\text{flat}} \approx \left(G\,M\,a_0\right)^{1/4}}$$

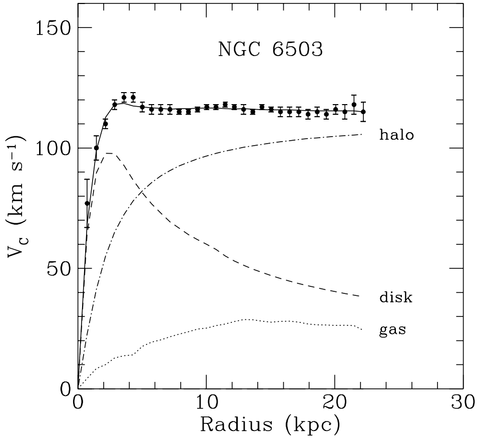
3. Assuming $L/M = \text{const}$ one naturally explains the Tully-Fisher relation
4. The numerical value of the critical acceleration is measured as

$$\boxed{a_0 \approx 1.2\,10^{-10}\,\mathrm{m/s^2}}$$

This value of a_0 is mysteriously close to the acceleration scale associated with the cosmological constant Λ

$$a_0 \approx 1.3\,a_\Lambda \quad \text{where} \quad a_\Lambda = \frac{1}{2\pi}\left(\frac{\Lambda}{3}\right)^{1/2}$$

The MOND fit of galactic rotation curves



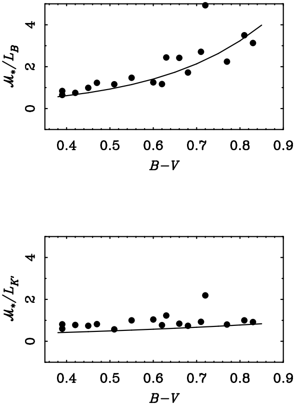
$\Longleftarrow$ the solid line is the MOND fit

Many galaxies are accurately fitted with MOND [Sanders 1996, McGaugh & de Blok 1998]

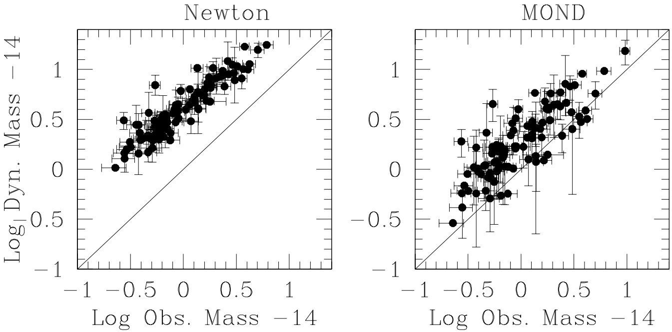
Fit of the mass-to-luminosity ratio

The fit of rotation curves is actually a one-parameter fit. The mass-to-luminosity ratio M/L of each galaxy is adjusted (and is therefore measured by MOND)

M/L shows the same trend with colour as is implied by models of population synthesis [Sanders & Verheijen 1998]



Problem with galaxy clusters [Gerbal, Durret *et al* 1992, Sanders 1999]



The mass discrepancy is $\approx 4-5$ with Newton and ≈ 2 with MOND

RELATIVISTIC MOND THEORIES

Non-relativistic theory for MOND

- The MOND equation can be rewritten as the modified Poisson equation

$$\nabla \cdot \Big[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \Big] = -4\pi\, G\, \rho$$

where $\boldsymbol{g} = \nabla U$ is the gravitational field and ρ the density of ordinary matter

- This equation is derivable from the Lagrangian [Bekenstein & Milgrom 1984]

$$L = \frac{a_0^2}{8\pi}\, k \left(\frac{\nabla U^2}{a_0^2} \right) + \rho\, U$$

where k is related to the MOND function by $k'(x) = \mu(\sqrt{x})$.

Relativistic scalar-tensor theory for MOND [Bekenstein & Sanders 1994]

1. The tensor part is the usual Einstein-Hilbert action

$$L_g = \frac{1}{16\pi} R[g, \partial g, \partial^2 g]$$

2. The scalar field is given by an aquadratic kinetic term

$$L_\phi = \frac{a_0^2}{8\pi}\, h \left(\frac{g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi}{a_0^2} \right)$$

3. The matter fields are universally coupled to gravity

$$L_\mathrm{m} = L_\mathrm{m}\Big[\Psi,\, \underbrace{\tilde{g}_{\mu\nu} \equiv e^{2\phi} g_{\mu\nu}}_{\text{physical metric}}\Big]$$

- Light signals do not feel the presence of the scalar field ϕ because the physical metric $\tilde{g}_{\mu\nu}$ is conformally related to the Einstein frame metric $g_{\mu\nu}$
- Since we observe huge amounts of dark matter by gravitational lensing (weak and strong) this theory is not viable

Tensor-vector-scalar theory [Bekenstein 2004, Sanders 2005]

1. The tensor part is the Einstein-Hilbert action L_g
2. The vector field part for $W_{\mu\rho} = \partial_\mu V_\nu - \partial_\nu V_\mu$ is

$$L_{\rm V} = -\frac{1}{32\pi}\Big[K \underbrace{g^{\mu\nu}g^{\rho\sigma}W_{\mu\rho}W_{\nu\sigma}}_{\text{standard spin-1 action}} -2\lambda \underbrace{\left(g^{\mu\nu}V_\mu V_\nu + 1\right)}_{\substack{\text{tells that } V^\mu \text{ is} \\ \text{time-like and unitary}}}\Big]$$

3. The scalar action reads

$$L_\phi = -\overbrace{\frac{1}{2}\left(g^{\mu\nu} - V^\mu V^\nu\right)\partial_\mu\phi\partial_\nu\phi}^{\text{dynamical scalar field } \phi} - \overbrace{\frac{\sigma^4}{4\ell^2}F\left(k\,\sigma^2\right)}^{\substack{\text{non-dynamical} \\ \text{scalar field } \sigma}}$$

4. Matter fields are coupled to the non-conformally related physical metric

$$\tilde{g}_{\mu\nu} = e^{-2\phi}(g_{\mu\nu} + V_\mu V_\nu) - e^{2\phi}V_\mu V_\nu$$

This theory evolved recently toward Einstein-æther like theories [Zlosnik, Ferreira & Starkman 2007; Li, Mota & Barrow 2007; Halle, Zhao & Li 2007]

Particle dark matter versus MOND

The alternative seems to be

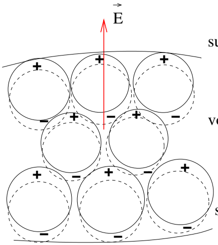
1. Either accept the existence of cold dark matter but
   - made of unknown non-baryonic particles yet to be discovered
   - which fails to reproduce in a natural way the flat rotation curves of galaxies
2. Or postulate a modification of the fundamental law of gravity (MOND and its relativistic extensions) but
   - on an *ad hoc* and not yet physically justified basis

Here we shall adopt a different approach

1. Keep the standard law of gravity namely general relativity and its Newtonian limit when $c \to \infty$
2. Use the phenomenology of MOND to guess what could be the (probably unorthodox) nature of dark matter
3. Propose a natural physical mechanism for MOND
4. Develop a relativistic matter model and apply it to cosmology

GRAVITATIONAL POLARIZATION AND MOND

The electric field in a dielectric medium



The atoms in a dielectric are modelled by electric dipole moments

$$\boldsymbol{\pi}_e = q\,\boldsymbol{\xi}$$

The polarization vector is

$$\boldsymbol{\Pi}_e = n\,\boldsymbol{\pi}_e$$

Density of polarization charges $\rho_e^{\rm polar} = -\boldsymbol{\nabla}\cdot\boldsymbol{\Pi}_e$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = \frac{\rho_e + \rho_e^{\rm polar}}{\varepsilon_0} \quad\Longleftrightarrow\quad \boldsymbol{\nabla}\cdot\left(\boldsymbol{E} + \frac{\boldsymbol{\Pi}_e}{\varepsilon_0}\right) = \frac{\rho_e}{\varepsilon_0}$$

The dipoles and polarization vector are aligned with the electric field

$$\frac{\boldsymbol{\Pi}_e}{\varepsilon_0} = \underbrace{\chi_e(E)}_{\text{electric susceptibility}}\;\boldsymbol{E}$$

Interpretation of the MOND equation [Blanchet 2007]

The MOND equation in the form of a modified Poisson equation

$$\nabla \cdot \left[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \right] = -4\pi\, G\, \rho$$

is formally analogous to the equation of electrostatics inside a dielectric. We pose

$$\mu = 1 + \underbrace{\chi(g)}_{\substack{\text{gravitational}\\ \text{susceptibility}}} \quad \text{and} \quad \underbrace{\boldsymbol{\Pi}}_{\substack{\text{gravitational}\\ \text{polarization}}} = -\frac{\chi}{4\pi\, G}\, \boldsymbol{g}$$

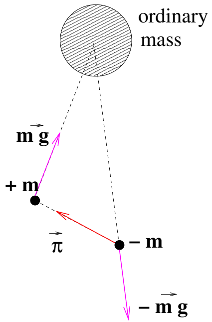
The MOND equation is equivalent to

$$\Delta U = -4\pi\, G\, (\rho + \rho_{\text{polar}})$$

In this interpretation the Newtonian law of gravity is not violated but we are postulating a new form of dark matter consisting of "polarization masses" with density

$$\boxed{\rho_{\text{polar}} = -\nabla \cdot \boldsymbol{\Pi}}$$

Sign of the gravitational susceptibility



The "digravitational" medium is modelled by individual dipole moments $\boldsymbol{\pi}$

$$\boldsymbol{\pi} = m\,\boldsymbol{\xi}$$
$$\boldsymbol{\Pi} = n\,\boldsymbol{\pi}$$

We suppose that the dipoles consist of particles doublets with

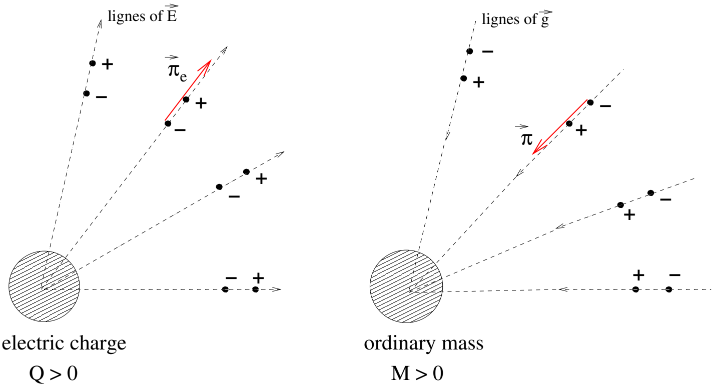
- opposite gravitational masses $m_{\rm g} = \pm m$
- positive inertial masses $m_{\rm i} = m$

1. The gravitational force is governed by a negative Coulomb law
2. The dipoles tend to align in the same direction as the gravitational field thus

$$\boxed{\chi < 0}$$

which is nicely compatible with MOND since $0 < \mu < 1 \Longrightarrow -1 < \chi < 0$

Electric screening versus gravitational anti-screening



Screening by polarization charges

$$\boxed{\chi_e > 0}$$

Anti-screening by polarization masses

$$\boxed{\chi < 0}$$

The dipolar internal force

1. The dipole moments cannot be stable in a gravitational field and we must invoke some non-gravitational internal force which is attractive between unlike masses
2. Consider a gravitational plasma, *i.e.* a medium composed of particles

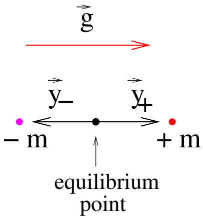
$$(m_{\rm i}, m_{\rm g}) = (m, \pm m)$$

   in equal numbers (so the plasma is globally neutral) and with local number densities $n_\pm$
3. We find that to obtain MOND we must postulate the Gauss law

$$\boldsymbol{\nabla}\cdot\boldsymbol{f}_{\rm int} = -\frac{4\pi\, G\, m}{\chi}\left(n_+ - n_-\right)$$

where $\chi < 0$ is the susceptibility of the dipolar medium

Gravitational plasma oscillations



Let $\boldsymbol{y}_\pm$ be the displacements from the equilibrium position at which the plasma is locally neutral, *i.e.* $n_+ = n_- = n$. The equation of motion is

$$m\,\frac{d^2\boldsymbol{y}_\pm}{dt^2} = \pm\, m\left(\boldsymbol{f}_{\text{int}} + \boldsymbol{g}\right)$$

Consider a small departure from equilibrium. The density perturbation reads $n_\pm = n\left(1 - \boldsymbol{\nabla}\cdot\boldsymbol{y}_\pm\right)$ to first order in $\boldsymbol{y}_\pm$

We obtain the harmonic oscillator

$$\frac{d^2\boldsymbol{y}_\pm}{dt^2} + \omega^2\,\boldsymbol{y}_\pm = \pm\,\boldsymbol{g}$$

in which ω is the usual plasma frequency given here by

$$\omega = \sqrt{-\frac{8\pi\,G\,m\,n}{\chi}}$$

Non viability of the quasi-Newtonian model

The quasi-Newtonian model

- Suggests that the gravitational analogue of the electric polarization is possible
- Yields a simple and natural explanation of the MOND equation
- Requires the existence of a new non-gravitational force

BUT THIS MODEL IS NOT VIABLE

- Is not relativistic
- Involves negative gravitational masses so violates the equivalence principle
- Does not allow to answer questions related to cosmology

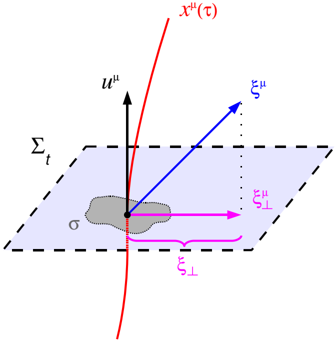
RELATIVISTIC MODEL OF DARK MATTER AND DARK ENERGY

Action principle for dipolar fluid [Blanchet 2007]

We propose a matter action in standard general relativity of the type

$$S = \int d^4x \sqrt{-g}\, L\left[J^\mu, \xi^\mu, \dot{\xi}^\mu, g_{\mu\nu}\right]$$

where the current density J^μ and the dipole moment ξ^μ are two independent dynamical variables



- The current density $J^\mu = \sigma u^\mu$ is conserved

$$\nabla_\mu J^\mu = 0$$

- The covariant time derivative is denoted

$$\dot{\xi}^\mu \equiv \frac{\mathrm{D}\xi^\mu}{\mathrm{d}\tau} = u^\nu \nabla_\nu \xi^\mu$$

- Projection perpendicular to the velocity

$$\xi^\mu_\perp \equiv \perp_{\mu\nu} \xi^\nu$$

Dipolar dark matter Lagrangian [Blanchet & Le Tiec 2008]

$$L = \sigma \left[-1 - \sqrt{\left(u_\mu - \dot{\xi}_\mu\right)\left(u^\mu - \dot{\xi}^\mu\right)} + \frac{1}{2}\dot{\xi}_\mu \, \dot{\xi}^\mu \right] - \mathcal{W}(\Pi_\perp)$$

1. The mass term σ represents the inertial mass of the dipole moments (say $\sigma = 2m\,n$)
2. The second term is a potential term inspired by the action of particles with spins in general relativity [Papapetrou 1951, Bailey & Israel 1980]
3. The third term is a kinetic-like term for the dipole moment and will tell how its evolution differs from parallel transport
4. The potential function $\mathcal{W}$ depends on the polarization field

$$\boxed{\Pi_\perp = \sigma\, \xi_\perp}$$

and describes a non-gravitational force $\mathcal{F}^\mu$ internal to the dipole moment

Equations of motion and evolution

Varying the action with respect to the dynamical variables J^μ and ξ^μ we find that one equation can be solved with $\left[\left(u_\mu - \dot{\xi}_\mu\right)\left(u^\mu - \dot{\xi}^\mu\right)\right]^{1/2} = 1$

Equation of motion of dipolar fluid

$$\underbrace{\dot{u}^\mu = -\mathcal{F}^\mu}_{\text{non-geodesic motion}} \qquad \text{where} \qquad \underbrace{\mathcal{F}^\mu \equiv \hat{\xi}^\mu_\perp \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\Pi_\perp}}_{\text{dipolar internal force}}$$

Equation of evolution of dipole moment

$$\dot{\Omega}^\mu = \frac{1}{\sigma}\nabla^\mu\left(\mathcal{W} - \Pi_\perp \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\Pi_\perp}\right) - \underbrace{\xi^\nu_\perp R^\mu_{\ \rho\nu\sigma} u^\rho u^\sigma}_{\text{coupling to Riemann curvature}}$$

$$\text{where} \quad \Omega^\mu \equiv \perp^\mu_\nu \dot{\xi}^\nu_\perp + u^\mu\left(1 + \xi_\perp \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\Pi_\perp}\right)$$

The equations depend only on the (space-like) perpendicular projection $\xi^\mu_\perp = \perp^\mu_\nu \xi^\nu$ which represents the physical dipole moment variable

Stress-energy tensor

Varying the action with respect to the metric we obtain

$$T^{\mu\nu} = \underbrace{r}_{\substack{\text{energy}\\\text{density}}} u^\mu u^\nu + \underbrace{\mathcal{P}}_{\text{pressure}} \perp^{\mu\nu} + 2 \underbrace{Q^{(\mu}}_{\text{heat flux}} u^{\nu)} + \underbrace{\Sigma^{\mu\nu}}_{\substack{\text{anisotropic}\\\text{stresses}}}$$

where

$$\begin{aligned} r &= \rho + \mathcal{W} - \Pi_\perp \mathcal{W}' \\ \mathcal{P} &= -\mathcal{W} + \frac{2}{3}\Pi_\perp \mathcal{W}' \\ Q^\mu &= \sigma\,\dot{\xi}^\mu_\perp + \Pi_\perp \mathcal{W}' u^\mu - \Pi^\lambda_\perp \nabla_\lambda u^\mu \\ \Sigma^{\mu\nu} &= \left(\frac{1}{3}\perp^{\mu\nu} - \hat{\xi}^\mu_\perp \hat{\xi}^\nu_\perp\right)\Pi_\perp \mathcal{W}' \end{aligned}$$

The mass density ρ contains the rest mass σ and a dipolar term $-\nabla_\lambda \Pi^\lambda_\perp$ which appears as a relativistic generalisation of the polarization mass density

$$\boxed{\rho = \sigma - \nabla_\lambda \Pi^\lambda_\perp}$$

Cosmological perturbations at large scales

1. Apply first-order perturbation theory around FLRW background

$$\begin{aligned} g_{\mu\nu} &= \overline{g}_{\mu\nu} + \delta g_{\mu\nu} \\ u^\mu &= \overline{u}^\mu + \delta u^\mu \\ \xi_\perp^\mu &= 0 + \delta\xi_\perp^\mu \qquad \Longleftarrow \quad \text{dipole moment is perturbative} \\ \Pi_\perp &= 0 + \delta\Pi_\perp \qquad \Longleftarrow \quad \text{polarization field is perturbative} \end{aligned}$$

2. Suppose that the potential takes the form

$$\mathcal{W}(\Pi_\perp) = \mathcal{W}_0 + \frac{1}{2}\mathcal{W}_2\,\Pi_\perp^2 + \mathcal{O}\left(\Pi_\perp^3\right)$$

3. Apply standard SVT gauge-invariant formalism with $\delta\xi_\perp^\mu = \left(0, D^i y + y^i\right)$
   - equations of motion

$$\begin{aligned} V' + \mathcal{H}\,V + \Phi &= -\mathcal{W}_2\,\overline{\sigma}\,a^2\,y \\ V_i' + \mathcal{H}\,V_i &= -\mathcal{W}_2\,\overline{\sigma}\,a^2\,y_i \end{aligned}$$

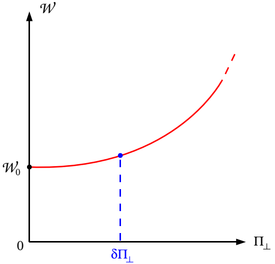
   - evolution equations

$$\begin{aligned} y'' + \mathcal{H}\,y' - \mathcal{W}_2\,\overline{\sigma}\,a^2\,y &= 0 \\ y_i'' + \mathcal{H}\,y_i' - \mathcal{W}_2\,\overline{\sigma}\,a^2\,y_i &= 0 \end{aligned}$$

Cosmological expansion of the fundamental potential

We assumed that the potential $\mathcal{W}$ reaches a minimum for the FLRW background

$$\begin{aligned} \mathcal{W}(\Pi_\perp) &= \mathcal{W}_0 + \frac{1}{2}\mathcal{W}_2\,\Pi_\perp^2 + \mathcal{O}\left(\Pi_\perp^3\right) \\ \mathcal{F}^\mu &= \underbrace{\mathcal{W}_2\,\Pi_\perp^\mu}_{\text{contributes at first-order cosmological perturbation}} + \mathcal{O}\left(\Pi_\perp^2\right) \end{aligned}$$



The minimum of the "fundamental" potential is the cosmological constant

$$\mathcal{W}_0 = \frac{\Lambda}{8\pi}$$

Stress-energy tensor in linear cosmological perturbation

1. To first order in the perturbation, $T^{\mu\nu} = T^{\mu\nu}_{\text{de}} + T^{\mu\nu}_{\text{dm}}$ where

$$\begin{aligned} T^{\mu\nu}_{\text{de}} &= -\mathcal{W}_0\, g^{\mu\nu} \\ T^{\mu\nu}_{\text{dm}} &= \rho\, u^\mu\, u^\nu + \underbrace{2Q^{(\mu}\, u^{\nu)}}_{\text{heat flux term}} \end{aligned}$$

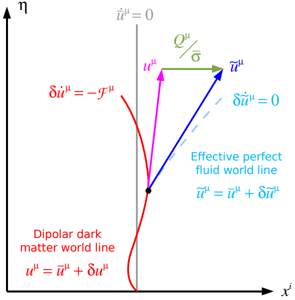
2. Posing $\widetilde{u}^\mu = u^\mu + Q^\mu/\overline{\rho}$ we can recast the dark matter fluid into a perturbed pressureless perfect fluid at linear perturbation order

$$T^{\mu\nu}_{\text{dm}} = \rho\, \widetilde{u}^\mu\, \widetilde{u}^\nu$$

3. The energy density of the dipolar dark matter fluid is

$$\rho = \underbrace{\sigma}_{\substack{\text{rest mass}\\ \text{energy density}}} - \underbrace{D_i \Pi^i_\perp}_{\substack{\text{dipolar polarization}\\ \text{energy density}}}$$

Agreement with the Λ-CDM scenario



The dipolar fluid is undistinguishable from

- standard DE (a cosmological constant)
- standard CDM (a pressureless perfect fluid)

at the level of first-order cosmological perturbations

Adjusting Λ so that $\Omega_{\text{de}} \approx 0.73$ and $\overline{\sigma}$ so that $\Omega_{\text{dm}} \approx 0.23$ the model is consistent with CMB fluctuations

Non-relativistic limit of the model ($c \to +\infty$)

1. Equation of motion of the dipolar particle

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \underbrace{\boldsymbol{g}}_{\substack{\text{local}\\\text{gravitational field}}} - \underbrace{\boldsymbol{\mathcal{F}}}_{\substack{\text{internal}\\\text{force}}}$$

2. Evolution equation of the dipole moment

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} = \boldsymbol{\mathcal{F}} + \frac{1}{\sigma}\boldsymbol{\nabla}\left(\mathcal{W} - \Pi\,\mathcal{W}'\right) + \underbrace{(\boldsymbol{\xi}\cdot\boldsymbol{\nabla})\,\boldsymbol{g}}_{\text{tidal effect}}$$

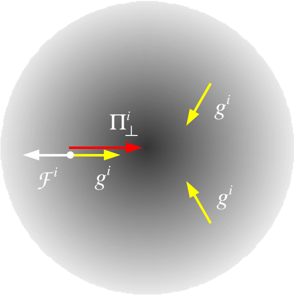
3. Conservation of the number of particles

$$\partial_t \sigma = -\boldsymbol{\nabla}\cdot(\sigma\boldsymbol{v})$$

4. Poisson equation for the gravitational potential

$$\boldsymbol{\nabla}\cdot\boldsymbol{g} = -4\pi\Big(\underbrace{\rho_{\mathsf{b}}}_{\substack{\text{baryonic}\\\text{matter}}} + \sigma - \boldsymbol{\nabla}\cdot\boldsymbol{\Pi}\Big)$$

Dipolar dark matter in a central gravitational field



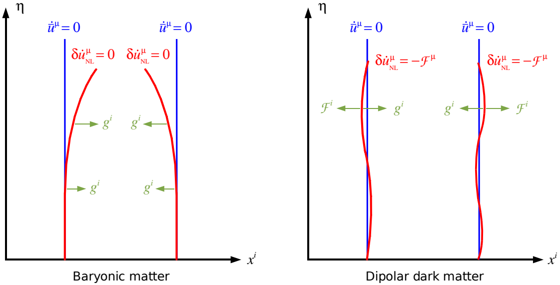
In spherical symmetry we find that there is a solution for which the dipolar fluid is static (i.e. at rest), the dipole moment is aligned with the gravitational field, and varies on a very long time-scale so that it is practically in equilibrium

$$\begin{aligned} \boldsymbol{v} &= \mathbf{0} \\ \mathbf{\Pi} &= -\Pi(r)\,\boldsymbol{e}_r \end{aligned}$$

In this solution the internal force balances the gravitational field

$$\boldsymbol{\mathcal{F}} = \boldsymbol{g}$$

Weak clustering of dipolar dark matter



- Baryonic matter follows the geodesic equation $\dot{u}^\mu = 0$, therefore collapses in regions of overdensity
- Dipolar dark matter obeys $\dot{u}^\mu = -\mathcal{F}^\mu$, with the internal force $\mathcal{F}^i$ balancing the gravitational field g^i created by an overdensity

We expect that the mass density of dipolar dark matter in a galaxy at low redshift remains close to its mean cosmological value

$$\sigma \approx \overline{\sigma} \ll \rho_\mathsf{b} \quad \text{and} \quad \boldsymbol{v} \approx \mathbf{0}$$

Recovering MOND in a galaxy at low red-shift

Using $\boldsymbol{v} \approx \mathbf{0}$ in the equation of motion

$$\boldsymbol{g} = \hat{\mathbf{\Pi}} \frac{d\mathcal{W}}{d\Pi} \quad \Longleftarrow \quad \text{polarization of the dipolar medium}$$

Using $\sigma \approx \overline{\sigma} \ll \rho_\mathsf{b}$ in the gravitational field equation

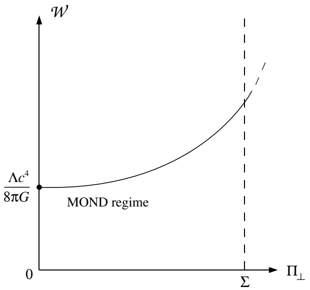
$$\boldsymbol{\nabla} \cdot \Big[\boldsymbol{g} - 4\pi\,\mathbf{\Pi}\Big] = -4\pi\,\rho_\mathsf{b} \quad \Longleftarrow \quad \text{modification of the Poisson equation}$$

1. Since the polarization $\mathbf{\Pi}$ is aligned with the gravitational field $\boldsymbol{g}$ we recover the MOND equation with MOND function $\mu = 1 + \chi$ where

$$\mathbf{\Pi} = -\frac{\chi}{4\pi}\,\boldsymbol{g}$$

2. There is one-to-one correspondence between μ and the potential $\mathcal{W}$

The fundamental potential $\mathcal{W}$



The potential $\mathcal{W}$ is determined through third order

$$\mathcal{W} = \frac{\Lambda}{8\pi} + 2\pi\,\Pi_\perp^2 + \underbrace{\frac{16\pi^2}{3a_0}\,\Pi_\perp^3}_{\text{MOND acceleration appears here}} + \mathcal{O}\left(\Pi_\perp^4\right)$$

Though the model is purely *classical* it is tempting to interpret the cosmological constant Λ as a "vacuum polarization", i.e. the residual polarization that remains when the "classical" part of the polarization $\Pi_\perp \to 0$

Order of magnitude of the cosmological constant

1. Introduce a purely numerical coefficient α such that

$$a_0 = \frac{1}{2\pi\,\alpha}\left(\frac{\Lambda}{3}\right)^{1/2} \qquad \left(\text{i.e.}\quad a_0 = \frac{a_\Lambda}{\alpha}\right)$$

2. Write the fundamental potential as $\mathcal{W} = \frac{3\pi\,a_0^2}{2}\,f\left(\frac{\Pi_\perp}{a_0}\right)$ with

$$f(x) = \underbrace{\alpha^2 + \frac{4}{3}\,x^2 + \frac{32\pi}{9}\,x^3 + \mathcal{O}\left(x^4\right)}_{\text{some ``universal'' function of } x\,\equiv\,\Pi_\perp/a_0}$$

3. The numerical coefficients in $f(x)$ are expected to be of the order of one, hence the cosmological constant should be of the order of

$$\boxed{\Lambda \sim a_0^2}$$

in good agreement with the observations (which give $\alpha \approx 0.8$)

Conclusion

1. We usually face two alternatives to the issue of dark matter:
   - Either accept the existence of CDM particles but which fail to reproduce in a natural way the rotation curves of galaxies
   - Or postulate an *ad hoc* alteration of the fundamental theory of gravity (MOND and its relativistic extensions like TeVeS)
2. Here we proposed a third alternative:
   - Keep the standard law of gravity but add to the ordinary matter some non-standard dark matter in order to explain MOND
3. More precisely we show:

- The phenomenology of MOND can be naturally interpreted as resulting from a mechanism of gravitational polarization
- A relativistic model of gravitational polarization reproducing MOND at galactic scales can be built in standard GR
- The model is in agreement with the standard Λ-CDM scenario at the level of first-order cosmological perturbations
- In this model the cosmological constant is interpreted as an effect of vacuum polarization and scales with the MOND acceleration as $\Lambda \sim a_0^2$