

Interactions between nuclei: phenomenology and theory

Raymond Mackintosh
Department of Physics and Astronomy,
The Open University,
Milton Keynes MK7 6AA, UK.
E-mail: r.mackintosh@open.ac.uk

Nucleon-nucleus potentials: 'impossible' but true

Back in the nuclear stone age....

Fermi's resonances, Bohr's liquid drop --- everyone 'knew' that a nuclear shell model (or the extension to the unbound case) is 'impossible'. Single particle model phenomenology was *abandoned* in the 1930s.

But the shell model IS possible, and a single-particle potential model of neutron scattering is possible IF (at low energies) you study *average* cross sections. The relative motion of two nucleons in a nucleus is essentially that of free particles --- until they are much closer than the average separation of nucleons in nuclear matter.

How is this possible? The key (Brueckner, Bethe, Weisskopf) is the Pauli exclusion principle... this is behind the shell-model, nuclear matter and ...the complex nuclear 'optical model' potential.

By 1960, there was consistent picture involving the same non-local (for exchange) potential for the shell model (bound nucleons) and the optical model (nucleons in the continuum). This conceptually important encouraged a 'nuclear matter' approach to the optical model potential.

.Now in the present age: what are the limits to a single particle potential?

.

Non-locality of the nucleon-nucleus potential.

Knock-on exchange (just the Fock term of Hartree Fock) makes the nucleon OMP non-local.

Nonlocal potential requires integro-differential equation with

$$V(r)\Psi(\mathbf{r}) \Rightarrow \int V(\mathbf{r}, \mathbf{r}')\Psi(\mathbf{r}')dV.$$

Note that the non-local potential has a local equivalent (LEP).... But it is strictly an exact equivalent *for elastic scattering only*... The Perey effect implies that uncorrected local equivalent potentials are not strictly appropriate for applications.

Most of the energy dependence of the (local) proton OMP arises from the fact that the local equivalent of a non-local potential (which is mostly due to knock-on exchange) is energy dependent.

There are various formal ways of calculating the LEP, but inversion is a practical one (see later).

Folding models (local version!)

Behind calculations of the OMP lies the folding model (FM) in its various variants and developments. The key equation depends on the nuclear density $\rho(\mathbf{r}')$ which is the nuclear density at point \mathbf{r}' in the nucleus. (We ignore spin, isospin etc here). The (local) potential at r is then

$$V(r) = \int \rho(\mathbf{r}') v_{nn}(|\mathbf{r} - \mathbf{r}'|) d\mathbf{r}'$$

More modern forms include a specific density dependence based on nuclear matter considerations:

$$v_{nn}(|\mathbf{r} - \mathbf{r}'|) \Rightarrow v_{nn}(|\mathbf{r} - \mathbf{r}'|, \rho) = v_{nn}(|\mathbf{r} - \mathbf{r}'|)F(\rho)$$

A local representation of exchange is usually included, but depends on local nucleon energy: don't renormalize.

Optical potentials for composite projectiles

In the 'iron age', a potential model for composite nuclei seemed even more implausible than for nucleons, but by the later 1950s successful phenomenological OMPs for ${}^2\text{H}$, ${}^4\text{He}$, modelled on local nucleon OMPs, had been found. The ambiguities were more serious, related to lesser penetration.

Later, nucleon folding models (FMs) were generalized either by:

1. Folding the nucleon-target (or cluster-target) interactions over the density of the projectile (e.g. Watanabe), or,
2. Folding a nucleon-nucleon (N-N) interaction over both densities.

Almost invariably, FMs involve approximations: local density LDA, local representation of exchange. (The range of the exchange non-locality goes like $1/A_{\text{proj}}$. But exchange non-locality is not the only kind of non-locality).

The density dependence of the N-N interaction has been included more recently, often based on nuclear matter theory.

Limitations of folding model; (nucleons or composite)

1. Almost always, nothing characteristic of a nucleus except the nuclear density (or proton and neutron densities) is included. Usually, not included are:
2. the density gradients (LDA),
3. the nuclear collectivity,
4. the possibility of `virtual' coupling processes, including projectile excitations.
5. the nature of the specific outer shell orbitals...

The collectivity effect was recognized a long time ago!

[Note: I am concerned with energies below those for which impulse approximation, KMT etc are appropriate.]

A bit more history

- 1962-1963, Buck notes effect of channel coupling on elastic scattering; Perey finds term depending upon β^2 in real and imaginary parts; Perey-Buck find departures from OMP systematics at the deformed rare earth transition near $A=150$.
- 1967, Glendenning *et al* find single potential valid across the shape transition as long as rotational coupling is included; Feshbach invoked, but new part has not yet a name.
- 1971, Study of 'collective contributions to OMP': refit elastic scattering angular distribution with OM code. Surface V .
- 1973, CRC calculations to determine contributions of pickup (deuteron) channels on proton scattering.

Is there a better way to find the contributions due to specific coupled channels?

The dynamic polarization potential, DPP

Feshbach theory (Ann Phys **5**, 357 (1958), also 1962) suggests a separation of the OMP into two terms,

$$V_{\text{OMP}} = V_{\text{FM}} + V_{\text{DPP}}$$

and the question is: how can we determine the second term? This, the DPP, contains the effect of ALL coupled channels.

In Feshbach formalism, the second term involves a very complicated Green's function involving all possible couplings between open (and closed) channels.

But do we want the effect of *all* coupled channels? We set aside serious conceptual problem and consider contribution of specific channels.

One approach: Green's functions

Evaluate the DPP due to specific channels (or even OMP) using Feshbach formalism, e.g. Coulter and Satchler in 1977.

Problems:

1. It is easy --- *if* you neglect channel-channel coupling (i.e. include 'up-down' coupling only) but *much* harder if you include all couplings, or exchange.
2. You get a non-local and L -dependent potential. Note that this is independent of exchange non-locality.
3. Do the inelastic channels fit the data?

There is a way to get the local equivalent potential directly:- apply S-matrix-to-potential inversion to the elastic channel S-matrix from the full coupled (reaction) channel calculation. But, why bother?

Why bother?

There are now well-developed folding models (FMs)... but:

1. FMs generally involve local density approximation (LDA) and do not fully account for surface gradients.... They cannot accommodate L -dependence, for example.
2. FMs only give average properties. For example, FMs can not explain Perey's deformation dependence of the OMP, nor can they describe effects of specific unfilled shells.
3. We expect (and find) coupling effects to be large in the surface and NOT represented by renormalizing the FM. (Surely if a consistent FM potential requires renormalization, it has failed?)
4. Very interesting effects are found with exotic nuclei (e.g. ${}^6\text{He}$) that are outside the scope of FMs.

In short: other properties of nucleus determine the OMP besides the nuclear density.

Conceptual problem with calculated DPP

- In the background, we have:

$$V = V_{\text{FM}} + V_{\text{DPP}}$$

- Problem: V_{FM} is not based on bare NN interaction – the effective interaction involves many excitations.... And so does (much of) V_{DPP} and they are not orthogonal (Breakup and maybe stripping DPPs are OK ?)
- Possible solution for future? develop a potential model that is analogous to the Strutinsky model --- smooth trend described by FM, plus `shell corrections' representing the features that depend on departures from smoothly varying properties, and also from the validity of LDA.
- Glendenning *et al* in 1968 conceptually separated specific channels that depend on particular nuclei from the high-lying channels that can be assumed to vary smoothly. See also PR C67, 034607 (2003) for formal expression of DPP.

Three important aspects of DPPs

1. Describing the departure from average FM properties of nucleon and OMPs and heavy ion OMPs ('shell corrections'); extrapolation from beta stability; differences between global potentials and potentials that accurately fit elastic scattering for specific nuclei.
2. Breakup and transfer DPPs for loosely bound nuclei and cluster nuclei.
3. Threshold effects (related by dispersion relations).

This list is not exhaustive and the categories have overlaps. There have been particularly successful calculations for cases 2 and 3.

The (psychological) importance of threshold effects.

Threshold effects are helpful since coupled reaction channel, CRC, effects have inescapable consequences.

Coupled reaction channels have substantial effects on elastic scattering well above threshold, but, since they are not easily reconciled with conventional models, those who do not wish to be convinced of their importance have not been convinced.

But the CRC theory is the same, and has been verified by the threshold phenomena!

How to calculate the local DPP.

1. Perform as complicated and realistic a reaction calculation (e.g. CRC, CDCC or better) as you can manage. It will involve a 'bare' potential corresponding to FM.
2. 'Invert' the elastic channel S-matrix from the elaborate (e.g. CRC) calculation to get the local potential V that gives the same S-matrix.
3. Then the DPP due to (e.g. CRC) is:- $V_{\text{DPP}} = V - V_{\text{bare}}$

This is much better than finding V by fitting the elastic cross section from a CRC calculation.

Paradigm case: breakup of ${}^6\text{Li}$

1. Case of ${}^6\text{Li}$ was exception to the successful M3Y folding model applied by Satchler and Love.... The real part had to be renormalized by factor of ~ 0.6
2. One can get a good fit to elastic scattering data by including projectile breakup (using adiabatic model, later CDCC) .
3. Story completed by `inverting' the elastic S-matrix from calculations that included coupling ... yields potential showing appropriate repulsion in surface.
4. But inversion revealed more: a generic pattern of surface repulsion and interior attraction that applied also to breakup of deuterons.
5. There are also many other applications of inversion besides determining DPPs.
6. So what does it mean -- `inverting' the elastic S-matrix-- ?

Inversion... What is it?

- Case 1
forward $V(r) \rightarrow S_l$ EASY
inversion $S_l \rightarrow V(r)$ MUCH HARDER

- Case 2
forward $S_l \rightarrow \sigma(\theta)$ TRIVIAL
inversion $\sigma(\theta) \rightarrow S_l$ OFTEN VERY HARD

- Case 3
forward $V(r) \rightarrow \sigma(\theta)$ EASY
inversion $\sigma(\theta) \rightarrow V(r)$ VERY HARD

- Case 1 inversion is now routinely possible,
- Case 2 and Case 3 inversion can present formidable difficulties, although R -matrix fits solve Case 2 in restricted cases, and optical model searches apply to Case 3 for many years.
- We mostly discuss Case 1 inversion, but also Case 3.

Review: V.I. Kukulin and R.S. Mackintosh, *J. Phys. G: Nucl. Part .Phys.* 30, R1 (2004)

Classes of $S_l \rightarrow V(r)$ inversion

Formal inversion methods apply to two classes:

- **Fixed- l inversion.**

a single l for all energies $S_l \rightarrow V(r)$, local potential.

- **Fixed energy inversion.**

for all l at a single energy $S_l \rightarrow V(r)$, local potential.

The problem in each case is the all.

Formal inversion methods

- Fixed l :- Gel'fand-Levitan, Marchenko
- Fixed E :- Newton Sabatier, NS, and related methods due to Lipperheide, Fiedeldey etc

Disadvantages of traditional methods (e.g. NS):

1. Require highly precise S_l , tendency to instability (NS method).
2. Fixed- l requires large energy ranges, but nuclear potentials are energy-dependent (has been used for N-N).
3. Not adaptable to cases with small ranges of l (NS).
4. Mostly applied to spin 0; NS can handle spin 1/2.
5. Cannot easily be generalized.

Practical versions of NS can handle finite range of l and a number of applications have yielded real physical insights.

Inversion based on wave function.

Various kinds of 'Trivial Equivalent Local Potential', TELP, have been found useful. These are based on the wave function rather than S-matrix. Some weighting procedure is required.

In some, but not all, cases they give similar results to S-matrix equivalent potentials. Full and detailed comparisons over a range of cases have not yet been made. Not guaranteed to give same fit to cross section.

Raises interesting question concerning the meaning of potential models.

Obvious limitation: In many cases, there are no wave functions available.

Other methods of S-to-V inversion

1. Semi-classical methods based on WKB and Glauber (eikonal) methods. (Not an exhaustive list.)
2. The iterative-perturbative, IP, method.

All inversions described here are applications of IP inversion.

IP inversion: underlying principles, 1.

Key idea: the response of the elastic scattering S -matrix to small changes is assumed to be linear (often surprisingly accurate):

$$\Delta S_l = -\frac{i m}{\hbar^2 k} \int_0^{\infty} (u_l(r))^2 \Delta V(r) dr.$$

where $u_l(r)$ is normalized with $u_l(r) \rightarrow (I_l(r) - S_l(r)O_l(r))$, where $I_l(r)$ and $O_l(r)$, are incoming and outgoing Coulomb wave functions.

Take a known 'starting potential' $V(r)$ giving S_l . With added term:

$$V(r) \rightarrow \hat{V}(r) = V(r) + \sum c_i v_i(r)$$

it gives $S_l + \Delta S_l$. Functions $v_i(r)$ belong to a suitable 'inversion basis'.

IP inversion: underlying principles, 2.

Solve linear equations with $\Delta S_l = S_l^{\text{target}} - S_l$ and $\Delta V = \sum c_i v_i(r)$ to find amplitudes c_i such that \hat{V} gives S_l closer to S_l^{target} . Use SVD to solve these over-determined linear equations. By iterating, $S_l + \Delta S_l$ converges to S_l^{target} .

In many applications there is a natural starting potential, SRP; it can often be a zero potential. (Apply diverse bases and SRPs to establish uniqueness of results.)

The quality of the inversion is quantified using σ_{inv} defined by

$$\sigma_{\text{inv}}^2 = \sum_L |S_L^{\text{tar}} - S_L^{\text{inv}}|^2$$

IP inversion can be generalized indefinitely

- i. **Fixed-energy inversion.** S_l , 'all l ', one E . (Problem: at low energies the potential is under-determined - too few partial waves to define potential.)
- ii. **Mixed case (energy bite) inversion.** Problem of under-determination can often be solved given $S_l(E)$ over a range of energies ('energy bite').
Some l , some E , $S_l(E) \rightarrow V(r)$
 For a narrow energy bite, this effectively includes dS_l/dE as input information.
- iii. **Energy dependent inversion.** The potential, particularly imaginary parts, varies with energy. IP can be extended to determine $V(r, E)$ directly.
Some l , some E , $S_l(E) \rightarrow V(r, E)$
 Published cases are limited to the factored form, $\sum_i f_i(E) V_i(r)$, where i indicates real-central, imaginary-central, etc.
- iv. **Inversion to fit bound state and resonance energies.** Energies of bound states can be included with S_l as input data.
- v. **Direct data to potential inversion.** Example: $\vec{d}+^4\text{He}$ many energies, many observables, tensor interaction included.

IP: Identical bosons, Majorana potential

Identical bosons. One can determine $V(r)$ given S_l for even- l only, e.g. $^{12}\text{C} + ^{12}\text{C}$.

Majorana terms. We often require Wigner and Majorana terms for each component:

$$V_W(r) + (-1)^l V_M(r)$$

- Theory and experiment often make Majorana terms obligatory for light nuclei; they are even required for $p + ^{16}\text{O}$. Odd-parity and even-parity potentials always have different radial forms.
- Inversion of both empirical and theoretical S_l will often present a choice:
either oscillatory pure Wigner potentials
or relatively smooth potentials with Majorana components.
- We have established highly oscillatory l -independent potentials having the same set of S_l as explicitly parity-dependent phenomenological potentials.
- But, wavy l -independent potentials also result from alternative sources of l -dependence (Feshbach).

IP: Spin cases handled by IP inversion

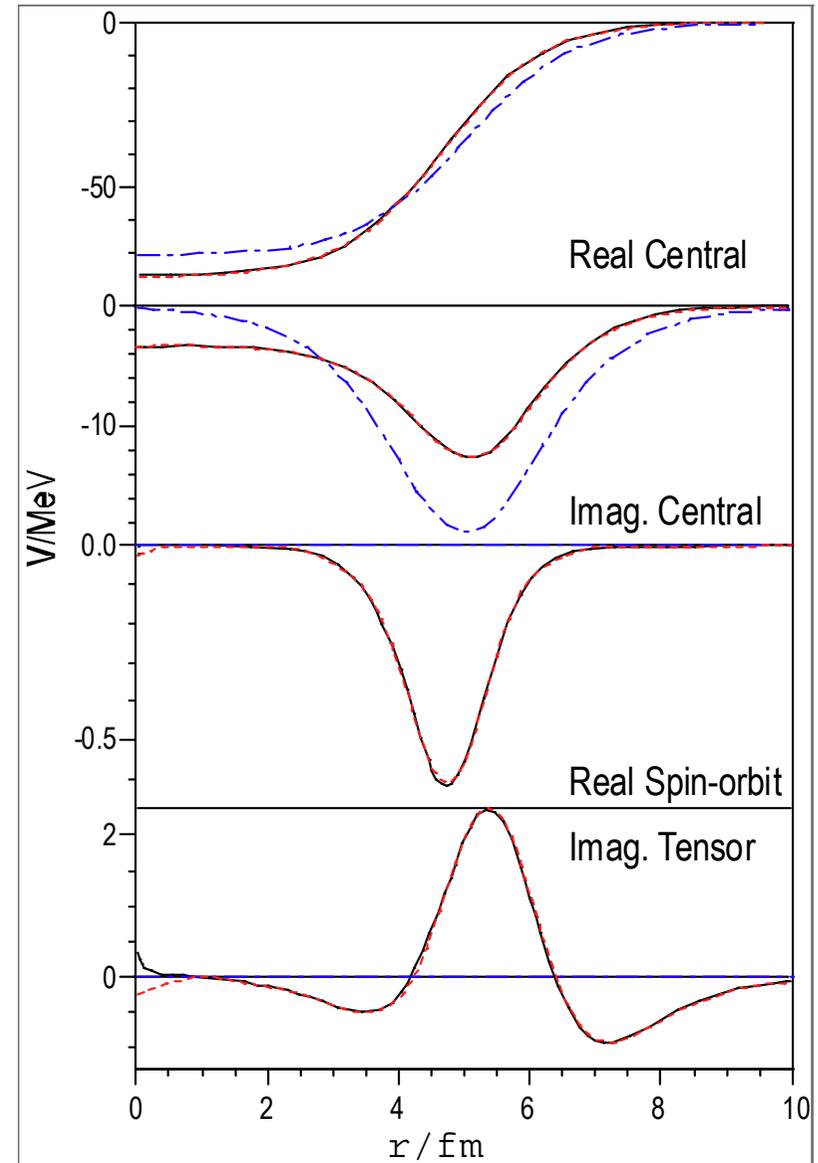
- **Spinless projectiles.** $S_l \rightarrow V(r)$
- **Spin $\frac{1}{2}$ projectiles.** $S_{lj} \rightarrow V(r) + \ell \cdot \sigma V_{ls}$
- **Spin one projectiles.** Vector spin-orbit and $T_R = ((s \cdot \hat{r})^2 - 2/3)V_R(r)$ tensor potentials can be determined from non-diagonal S_{ll}^j . This is *coupled channel inversion*.
- **High channel spin.** For cases like $d + {}^3\text{He}$, or $p + {}^6\text{Li}$, independent potentials for each possible channel spin have been determined.

In general, **all** spin-dependent components can have real and imaginary, Wigner and Majorana terms. These different components of the potentials can all be expanded in different bases if required.

How well does it work?

- **Test case.** Deuterons on ^{58}Ni at 56 MeV.
 Solid lines: target (known) potential,
Dash-dot: inversion SRP (starting potentials), which were zero for the real spin-orbit and tensor terms.
Dots: V found by inversion.

- **Noisy S -matrix.** IP inversion can be applied to noisy data and produce meaningful $V(r)$... the iterative process is under control.



Information on nuclear interactions from inversion

I. Inversion of S_p , S_{lj} or S_{ll}^j , obtained from theory

1. DPPs have been found for inelastic, breakup and reaction channels
2. $V(r)$ from RGM and similar S -matrix.
3. Local $V(r)$ equivalent to non-local potentials.
4. $V(r)$ for impulse approximation S_L , or $S(b)$ from Glauber and other eikonal models

II. Inversion of S_p , S_{lj} or S_{ll}^j , from analysis of experiment

1. (few partial waves) Parameterized R -matrix or effective range fits at low energies.
2. (many partial waves) High energy two-step phenomenology. (E.g. ^{11}Li , $^{12}\text{C} + ^{12}\text{C}$ from 140 to 2400 MeV)

III. Direct observable $\rightarrow V(r)$ inversion.

S -matrix search convoluted with IP inversion; S -matrix is byproduct. Can handle many energies simultaneously to give multi-component $V(E)$.

Pickup coupling and proton scattering.

It has long been known that **pickup coupling** ($p \rightarrow d \rightarrow p$ for proton scattering) makes a significant contribution to the proton optical potential.

It explains why really precise fits for nucleon elastic scattering below 50 MeV for closed shell target nuclei have not been found with conventional OM.

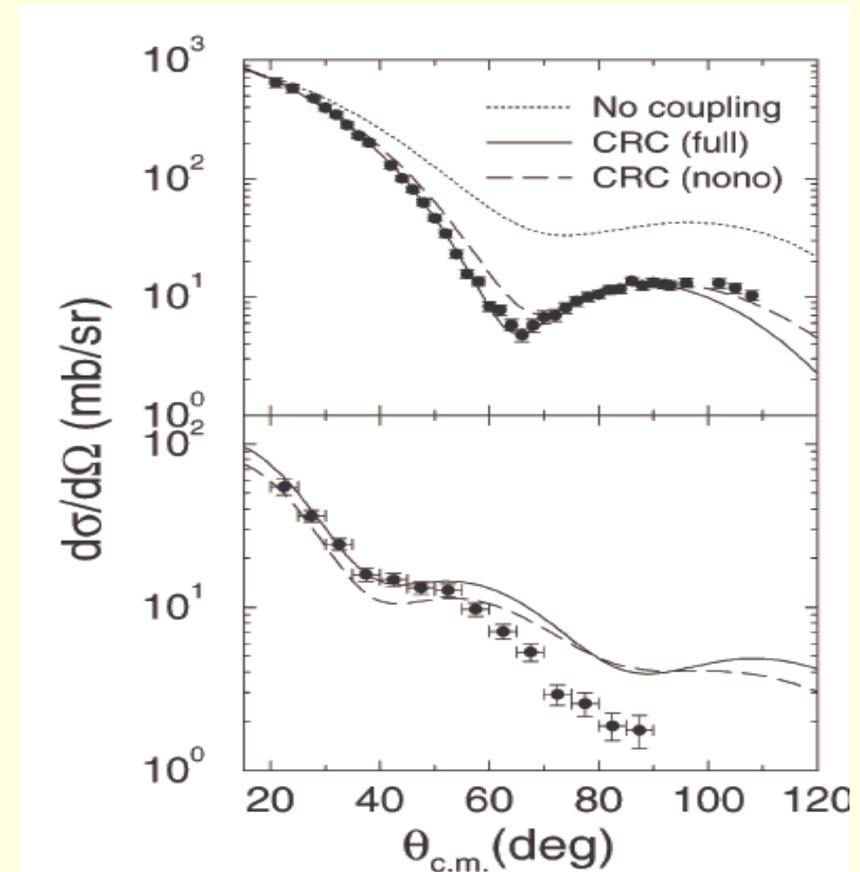
What contribution does this process make to the OMP?

The contribution to the proton OMP can be quantified in terms of the changes in the volume integrals of the real and imaginary terms J_R and J_I , and the change in the rms radius of the imaginary part, R_I .

Effect on ${}^8\text{He}(p,p)$ elastic scattering cross sections

The coupling of deuteron channels has a large influence on the elastic scattering of protons on ${}^8\text{He}$ at 15.6 MeV. The figure also shows the influence of 'non-orthogonality' corrections (they are omitted in 'nono' curves.)

The lower part shows the (p,d) cross section.



Contribution to OMP for $^8\text{He} + p$

We can carry out a full finite-range pickup calculations including non-orthogonality terms, for protons on ^8He , Phys. Lett. **B 619** (2005) 82. The pickup coupling greatly improved the fit to the elastic scattering.

	J_R	$\langle r^2 \rangle_R^{1/2}$	J_I	$\langle r^2 \rangle_I^{1/2}$	J_{SOR}	J_{SOI}
OM	704.14	3.092	55.37	3.336	26.60	0.005
CRC	653.94	2.938	307.47	4.138	40.27	1.250
NONO	571.28	2.840	252.62	4.360	33.15	6.55

OM=Bare, CRC=CRC, NONO=CRC with non-orthogonality term omitted

The table quantifies the pickup contribution for 15.6 MeV protons in terms of volume integrals and RMS radii. The repulsive real DPP is large at the nuclear centre although the effect on J_R is modest. The DPP could **not** be represented by renormalizing a folding model potential.

Example: Pickup coupling effects in $p + {}^{10}\text{Be}$; 1

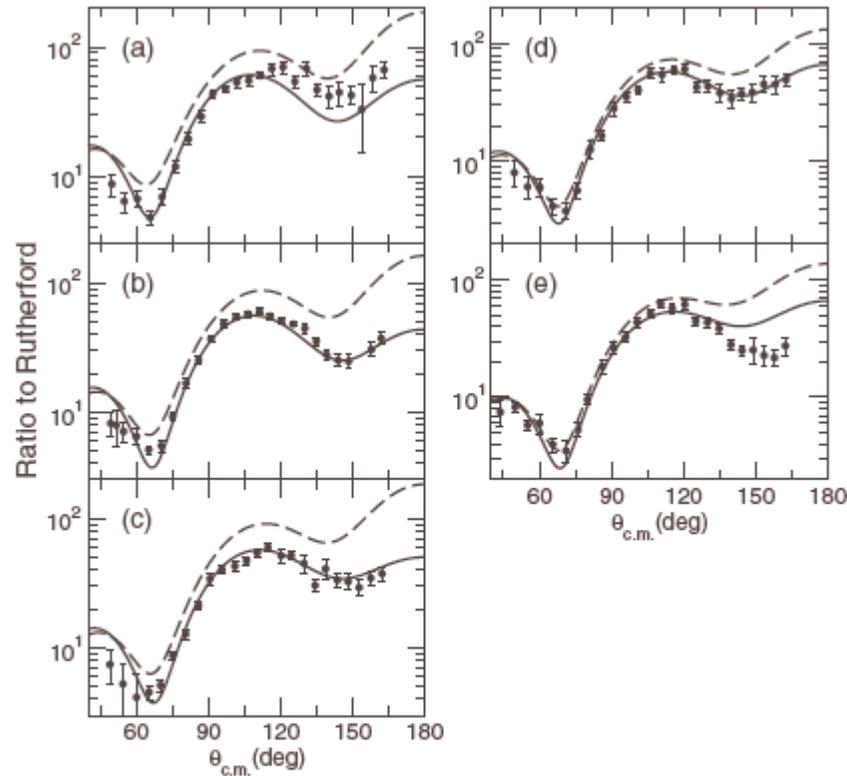


FIG. 1. Data for ${}^{10}\text{Be}(p, p)$ elastic scattering [15] compared with the full CRC calculations (full curves) and the no-coupling calculations (dashed curves). (a) 16 MeV, (b) 15 MeV, (c) 14 MeV, (d) 13 MeV, (e) 12 MeV; plotted as ratio to Rutherford in each case.

What contribution does the coupling of pickup (deuteron) channels make to proton scattering for a more normal nucleus similar in mass to ${}^8\text{He}$?

<= See the effect on elastic scattering angular distributions... Note the smaller effect at 12 MeV; the fit is not so good there.... Resonance? Exchange requiring Majorana? The effect is similar to that for ${}^8\text{He}$ despite lesser neutron skin – clearly it is a general effect.

But what is the contribution to the OMP?

Example: Pickup coupling effects in $p + {}^{10}\text{Be}$; 2

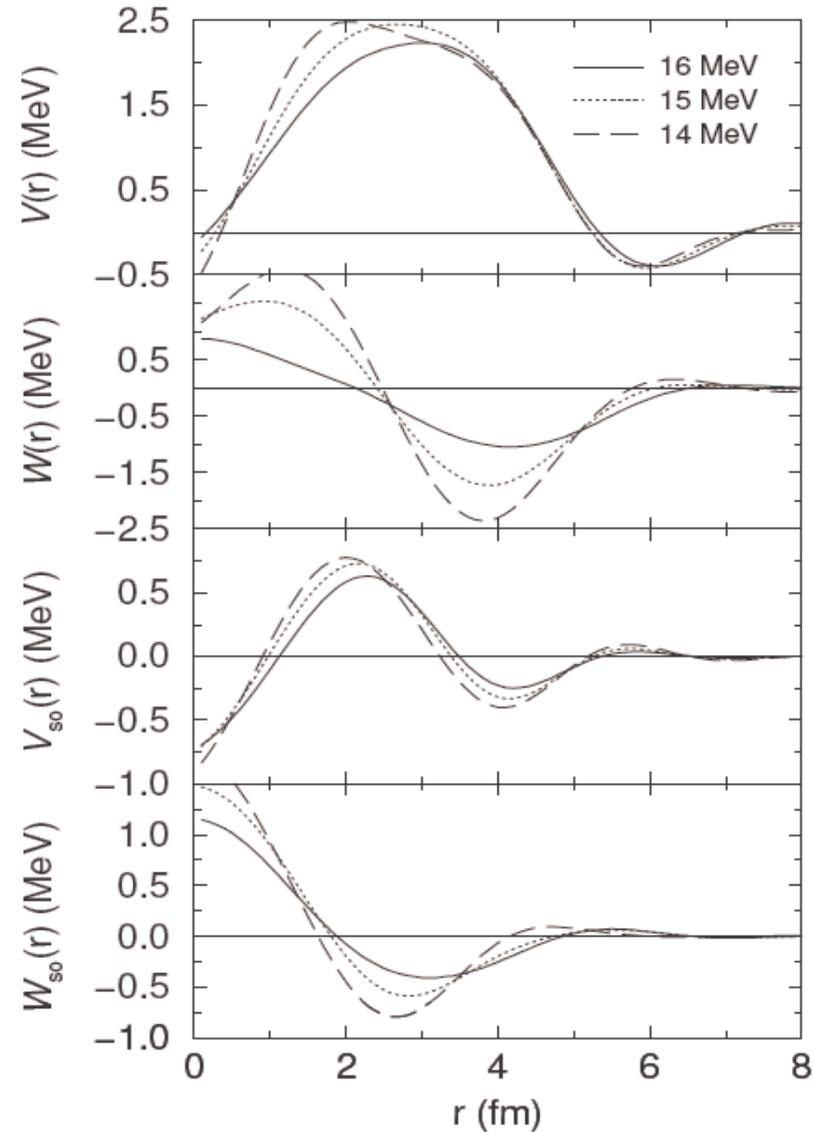
Are the effects of pickup coupling specific to the halo target nucleus ${}^8\text{He}$ or are they more general?

Here are the DPPs for protons on ${}^{10}\text{Be}$ at 14, 15 and 16 MeV.

Net repulsion (at 16 MeV, decreasing the real volume integral J_R by 66.9 MeV fm^3), and absorption (at 16 MeV increasing J_W by 57.3 MeV fm^3). DPP emissive near nuclear centre; effect is to move absorption outwards.

These are similar to general effects found previously, but with smaller reduction in J_R due to inclusion of non-orthogonality effects.

Shapes for the central terms of the DPP are like those for $p + {}^8\text{He}$, but shifted outwards; S-O shapes different.



Example: Pickup coupling effects in $p + {}^{10}\text{Be}$; 3

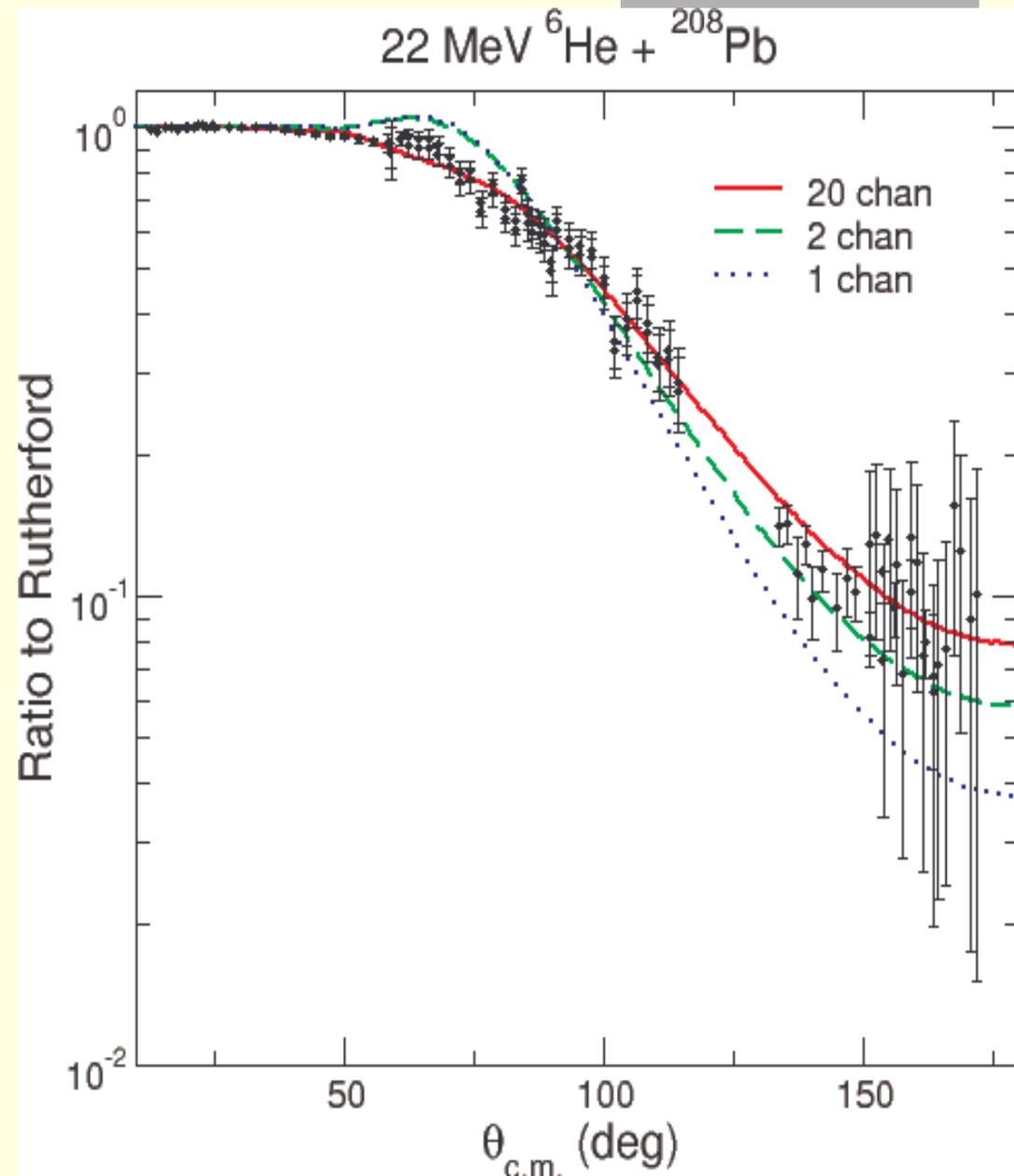
The contribution to the proton OMP can be quantified in terms of the changes in the volume integrals of the real and imaginary terms J_R and J_I , and the change in the rms radius of the imaginary part R_I :

E_{lab} (MeV)	ΔJ_R (MeV fm ³)	ΔJ_I (MeV fm ³)	ΔR_I (fm)
16	-66.9	57.3	0.623
15	-68.8	67.9	0.626
14	-67.9	77.2	0.694
13	-44.9	47.2	0.699
12	-24.4	50.1	0.613

${}^6\text{He}$ scattering from ${}^{208}\text{Pb}$ at 22 MeV

Coupling to $2+$ resonance and continuum breakup states of the ${}^6\text{He}$ projectile at 22 MeV.

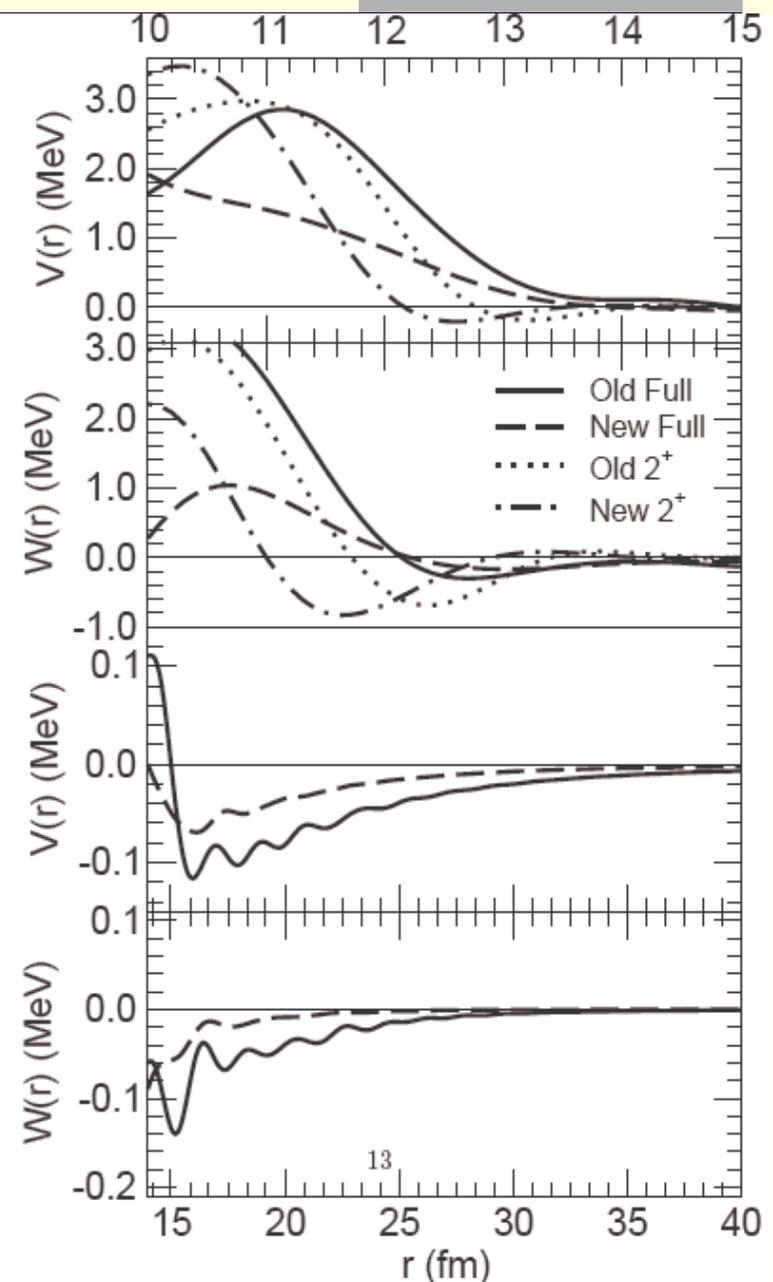
Calculated using CDCC formalism.



DPP due to breakup for ${}^6\text{He}$ scattering from ${}^{208}\text{Pb}$

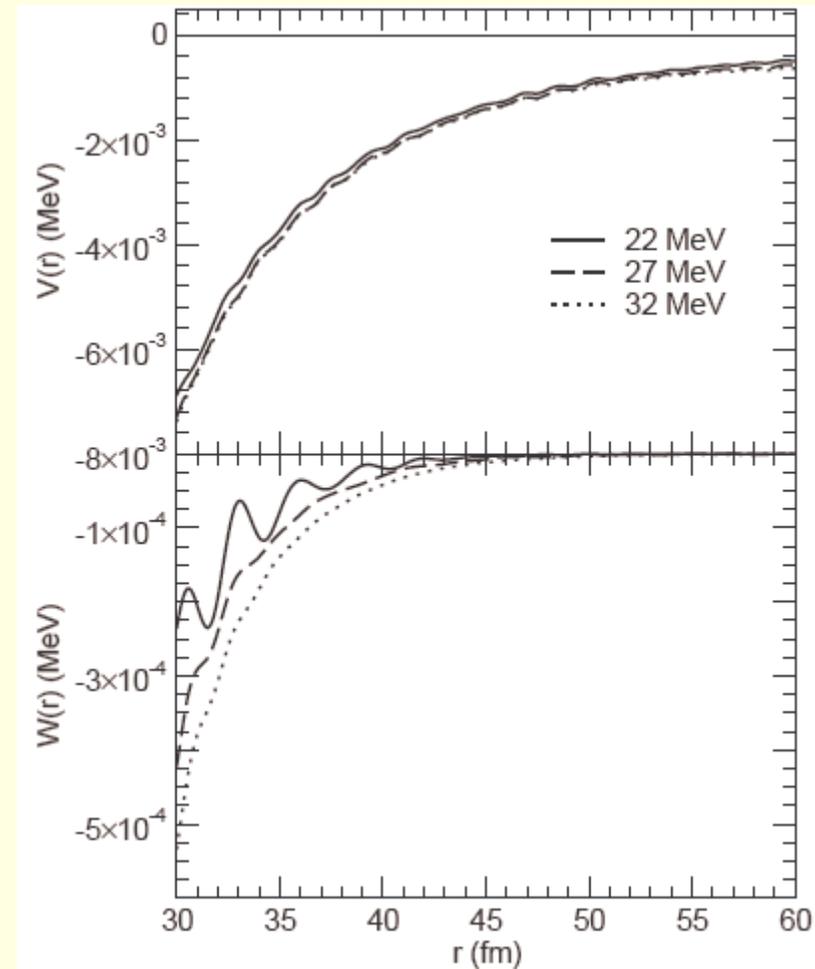
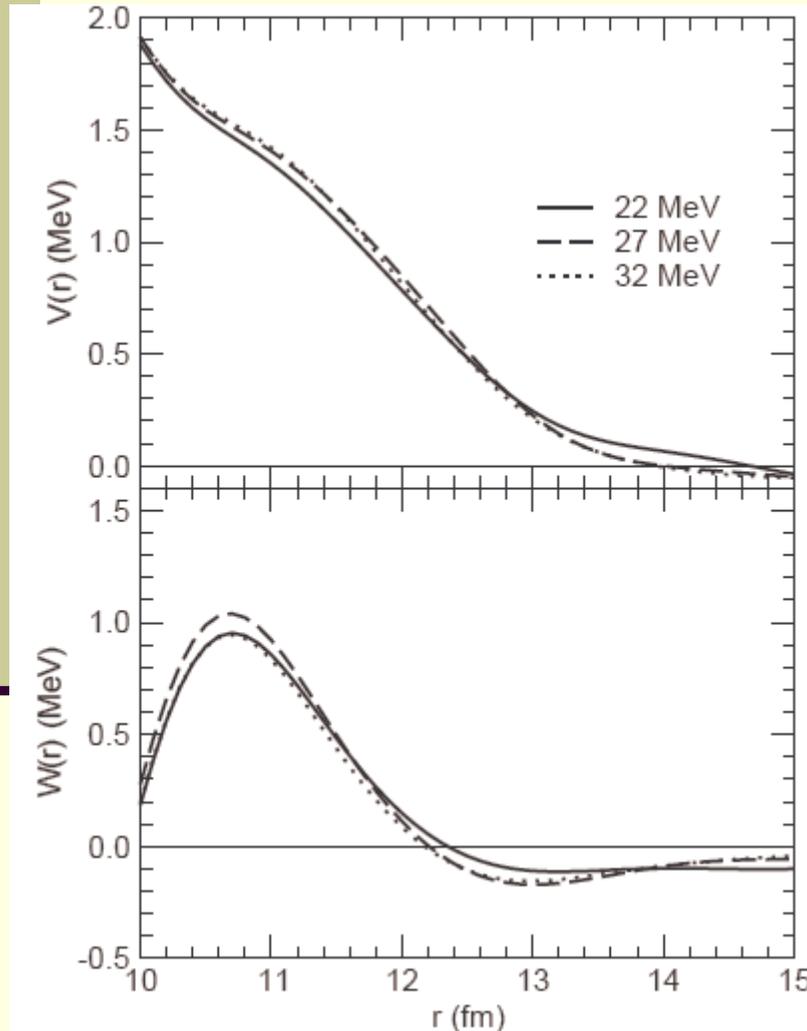
1. DPP due to $2+$ and dipole excitation

- ${}^6\text{He}$ incident on ${}^{208}\text{Pb}$, 22, 27, 32 MeV -- these DPPs are at 27 MeV.
- The breakup was calculated using model due to Moro et al. The DPP has a long range attractive tail generated by the Coulomb dipole interaction.
- There is short range repulsive and emissive DPP around 11 fm. Regions of emissiveness often appear in local potentials that represent non-local and L -dependent DPP; unitarity is not broken.
- The DPP is not well-defined for $r \leq 10$ fm.



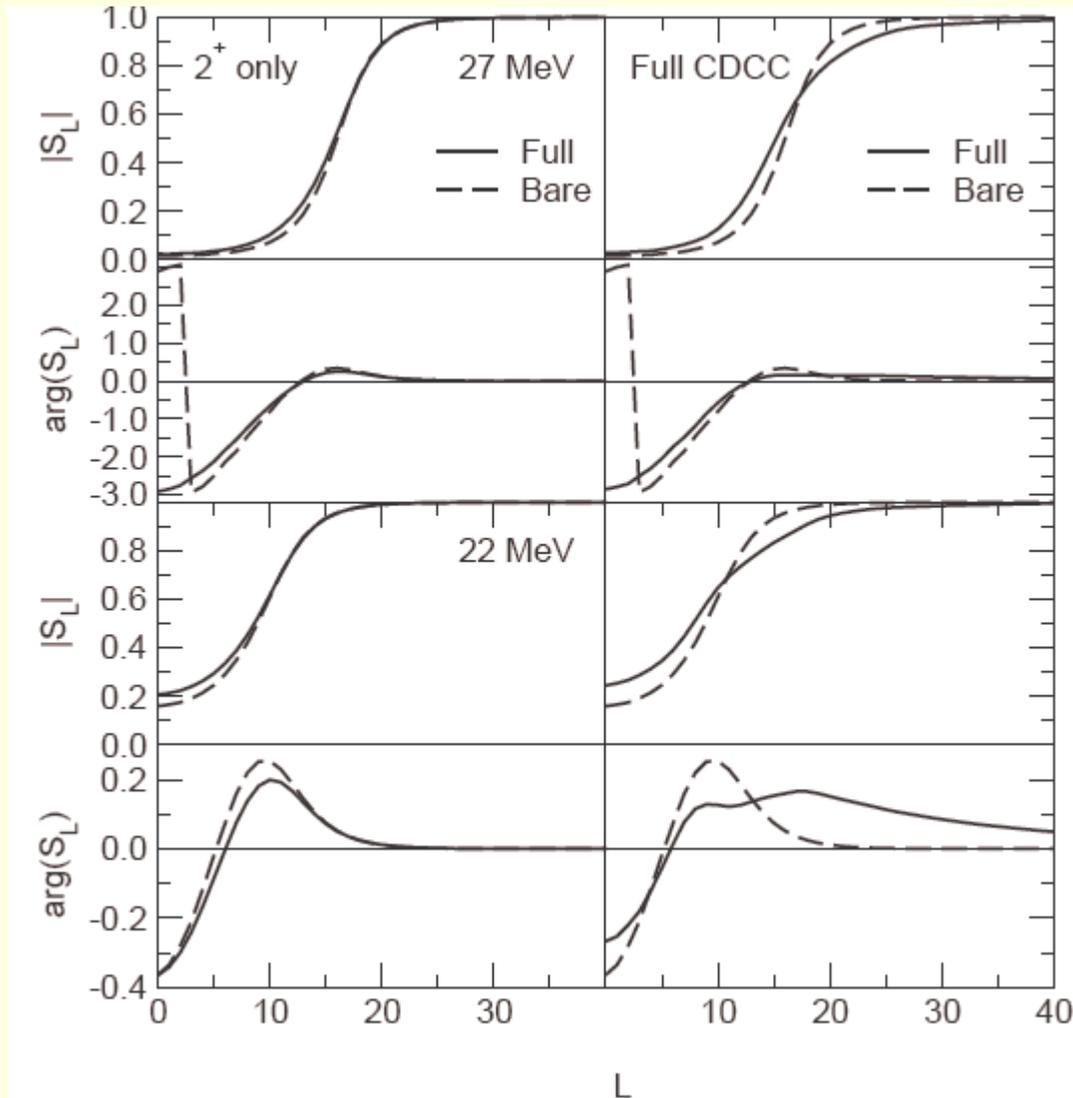
The DPP due to breakup for ${}^6\text{He}$ scattering from ${}^{208}\text{Pb}$

2. The DPP depends little on energy ...



The DPP due to breakup for ${}^6\text{He}$ scattering from ${}^{208}\text{Pb}$

3. ... although S_L does vary as the Coulomb barrier is approached:



For low L $|S_L|$ increases as coupling switched on.

$p + {}^6\text{He}$: DPP due to breakup.

With K. Rusek, uses 'old' di-neutron cluster model, at 21.57 MeV CM. Breakup into continuum states with $L=0, 1, 2, 3$ plus $L=2$ resonance at 1.8 MeV.

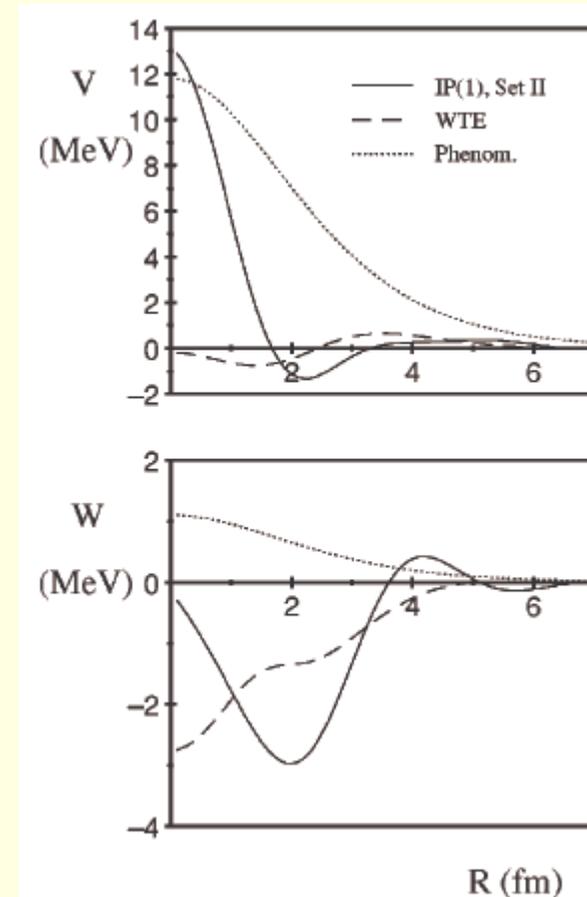
Note emissive region in imaginary part.

Very similar results at 32.8 MeV CM.

The WTE results are in poor agreement for real part.

The DPP does not look like a multiple of the bare potential.

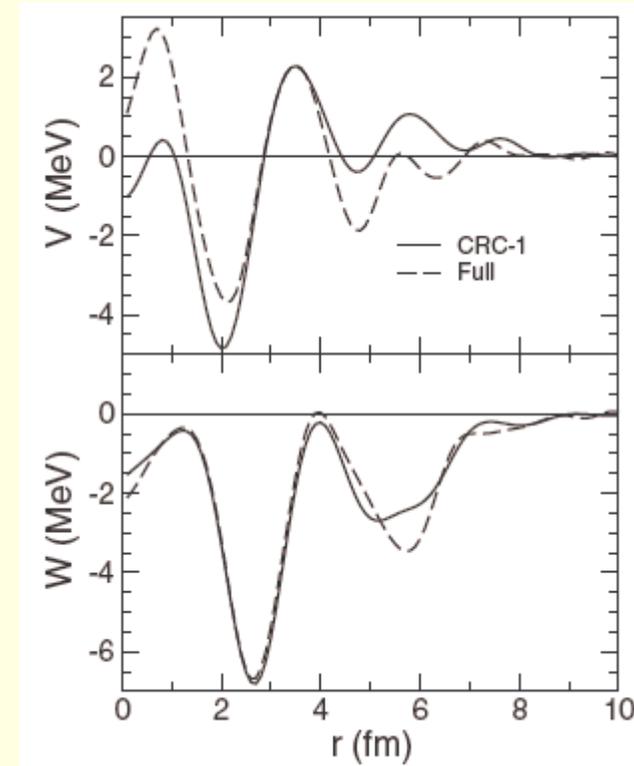
Effect of breakup is to make the nucleus seem smaller, r_{rms} for real part is reduced, so nuclei are larger than FM might suggest.



DPP at $E(\text{lab}) = 21.57$ MeV, except for the phenomenological, 32.8 MeV. WTE is weighted trivially equivalent. Substantial effect on spin-orbit potential is not shown.

Pickup and breakup contributions to $d + {}^{40}\text{Ca}$ OMP

- 52 MeV deuterons; substantial DPP due to pickup and breakup, especially wavy imaginary term.
- A significant T_R tensor force arises mostly from reorientation in deuteron breakup channels.
- Such couplings cannot be omitted from an account of deuteron scattering.
- Phenomenological wavy potentials now become plausible.
- The DPP, especially imaginary part, cannot be represented by a local density model, or by renormalizing FM.
- N Keeley and RSM, Phys Rev C 77, 0546003 (2008)



The d-⁴He interaction derived from multi-energy data

- 'Direct data → V inversion' is an alternative to optical model fitting with parameterized forms for determining potentials from data, especially when there are many data and many parameters.
- There are many data for d-⁴He scattering: a full set of polarization observables (including all 3 tensor analyzing powers) for many angles, all for many energies in the range of 4 - 13 MeV.
- 1000 data points were fitted, Nucl. Phys. **A723** (2003) 45, (five observables, a wide angular range and many energies) to produce a multi-component (Wigner and Majorana, central, spin-orbit and tensor) multi-energy potential (components were functions of energy) giving a reasonable representation of shape resonances.
- This would have been a formidable job for standard optical model codes since they would have had to include the coupled channel calculation for the tensor interaction, for each energy, within the search.
- The simultaneous fit to all energies reduces chance of noisy or spurious data introducing spurious features in potential.
- The facility of the IP method to control the degree of fitting is essential; it is possible to demand smooth potentials.

Understanding nucleus-nucleus interactions....

.... With a little help from inversion

- The systematic properties of DPPs and exchange effects need to be established before we understand nucleus-nucleus interactions. To get a deeper understanding of nucleus-nucleus interactions we must explore where folding models fail... e.g. get clues to limits of LDA, and need for L -dependence.
- We should be able to understand OMP for every target nucleus, not just trends, but we also wish to predict with confidence the OMP for target nuclei far from stability.
- As reaction theories progress and produce better S-matrices, IP inversion will be there to provide local potentials, linking to precision phenomenology.
- In particular, Glauber and other eikonal models, RGM, impulse and KMT models etc do not produce potentials (how else to interpret the inexact fits?).
- The IP algorithm is indefinitely (?) generalizable, and has found wider applications than originally envisaged. Are there any suggestions for new applications?

Thanks to

1. Andrzej Kobos, Andy Ioannides and specially Shirley Cooper for help in developing IP inversion.
2. Nick Keeley, expert on Fresco and much else.
3. Valérie Lapoux for the invitation.

p + ${}^6\text{He}$: exchange processes.

- Inversion of RGM S-matrix, real potential only (no inelastic channels). The presence or absence of a Majorana term (required by inversion) changes the differential cross section by a factor of two over a range of angles in the backward hemisphere. No effect in forward hemisphere as expected for 'heavy particle stripping' contribution. The Majorana term is smaller than for p+ ${}^4\text{He}$, as expected.
- Volume integrals:
- Wigner term: $J_W = -709(1 - 0.0064 E_{\text{lab}}) \text{ MeV fm}^3$
- Majorana term:
- $J_M = 66.5(1 - 0.024 E_{\text{lab}} + 0.036 \times 10^{-4} E_{\text{lab}}^2) \text{ MeV fm}^3$