

Spin-dependent parton distributions of the nucleon

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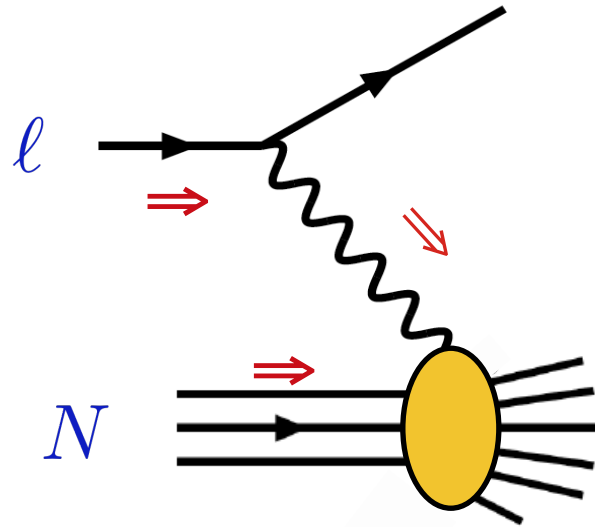
Saclay, 18.02.2011

Outline:

- Introduction: Nucleon helicity structure
- Polarized high-energy collisions in QCD
- Global analysis of pol. parton distributions
- Applications of QCD resummation
- Conclusions & Outlook

**Introduction:
Nucleon Helicity Structure**

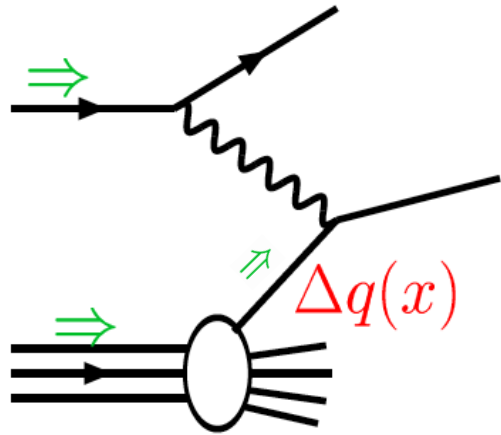
Mid '70s: "Polarized DIS"



$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \sim \frac{g_1(x, Q^2)}{F_1(x, Q^2)}$$

First at **SLAC**, later **CERN**, **DESY**, **Jefferson Lab**

Parton Model:

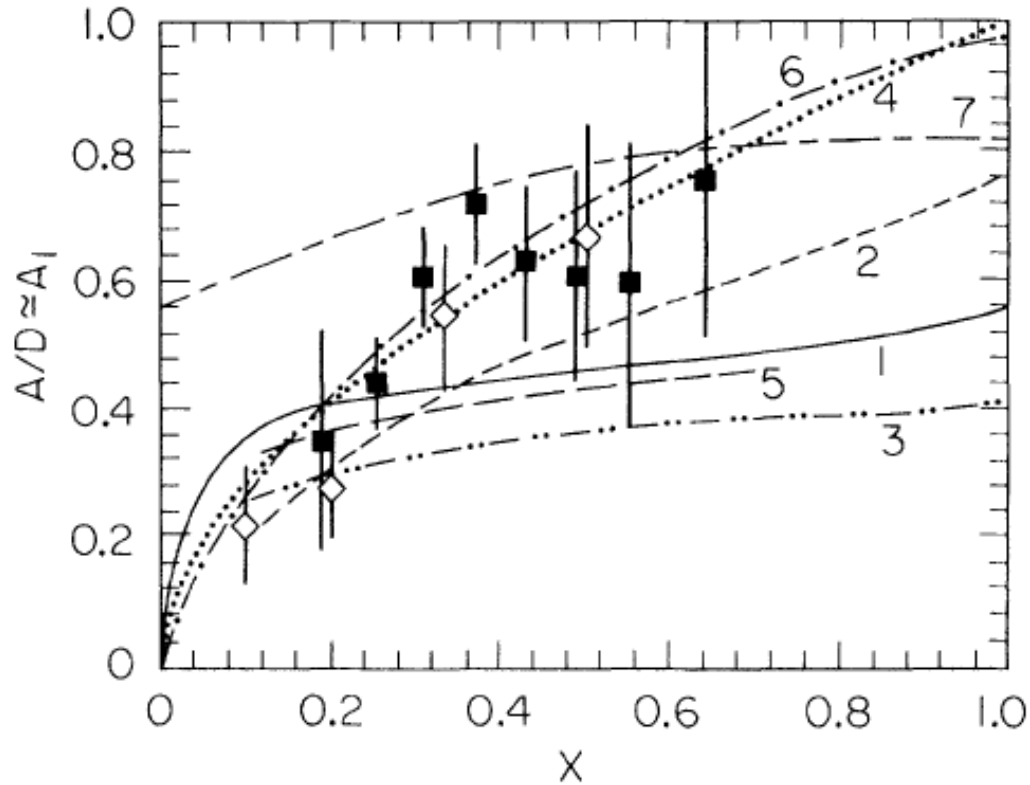


$$A_1 = \frac{\sum_q e_q^2 [\Delta q(x) + \Delta \bar{q}(x)]}{\sum_q e_q^2 [q(x) + \bar{q}(x)]}$$

$$\Delta q(x) = \left| \begin{array}{c} P, + \\ \Rightarrow \\ \text{Oval} \\ \text{partons} \end{array} \left. \begin{array}{c} xP \\ \nearrow \\ \text{partons} \end{array} \right\} X \right|^2 - \left| \begin{array}{c} P, + \\ \Rightarrow \\ \text{Oval} \\ \text{partons} \end{array} \left. \begin{array}{c} xP \\ \nearrow \\ \text{partons} \end{array} \right\} X \right|^2$$

$$\Delta q(x) = \frac{1}{4\pi} \int dy^- e^{-iy^- xP^+} \langle P, S | \bar{\psi}(0, y^-, \mathbf{0}_\perp) \gamma^+ \gamma_5 \psi(0) | P, S \rangle$$

All was well ... initially...

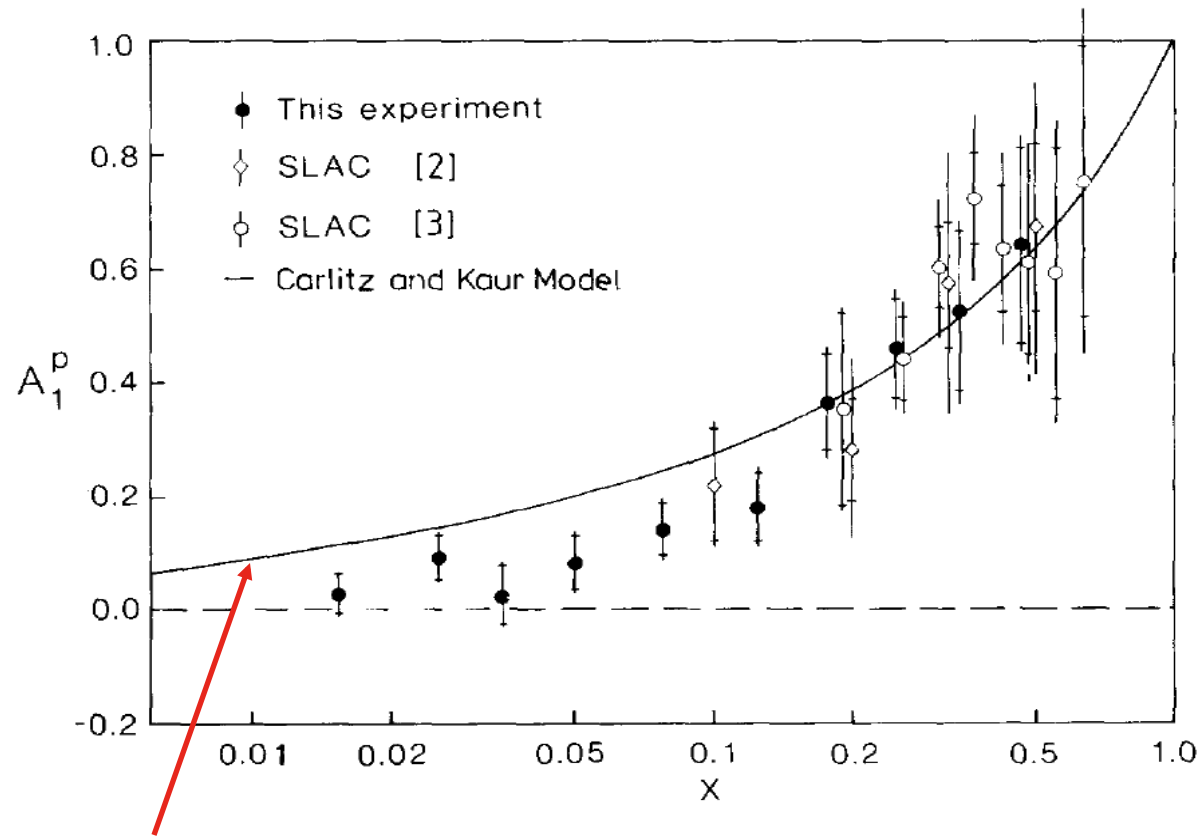


SLAC-E80, E130

(1976-83)

But then:

EMC (1987)



If quarks and anti-quarks carried
~70% of the proton spin...

$$2 \int_0^1 dx g_1(x, Q^2) = \frac{4}{9} \Delta\Sigma_u + \frac{1}{9} \Delta\Sigma_d + \frac{1}{9} \Delta\Sigma_s$$

$$\Delta\Sigma_q \equiv \int_0^1 dx (\Delta q + \Delta\bar{q})(x, Q^2) \propto \langle P, s | \bar{\psi}_q \gamma^\mu \gamma_5 \psi_q | P, s \rangle$$

(axial charges)

use SU(3) to obtain *non-singlet* combinations from baryon decays:

Bjorken;
Ellis, Jaffe;
Sehgal;
Karliner, Lipkin;
Ratcliffe;...

$$\Delta\Sigma_u - \Delta\Sigma_d = g_A = 1.257 \pm \dots$$

$$\Delta\Sigma_u + \Delta\Sigma_d - 2\Delta\Sigma_s = 3F - D = 0.58 \pm 0.03$$

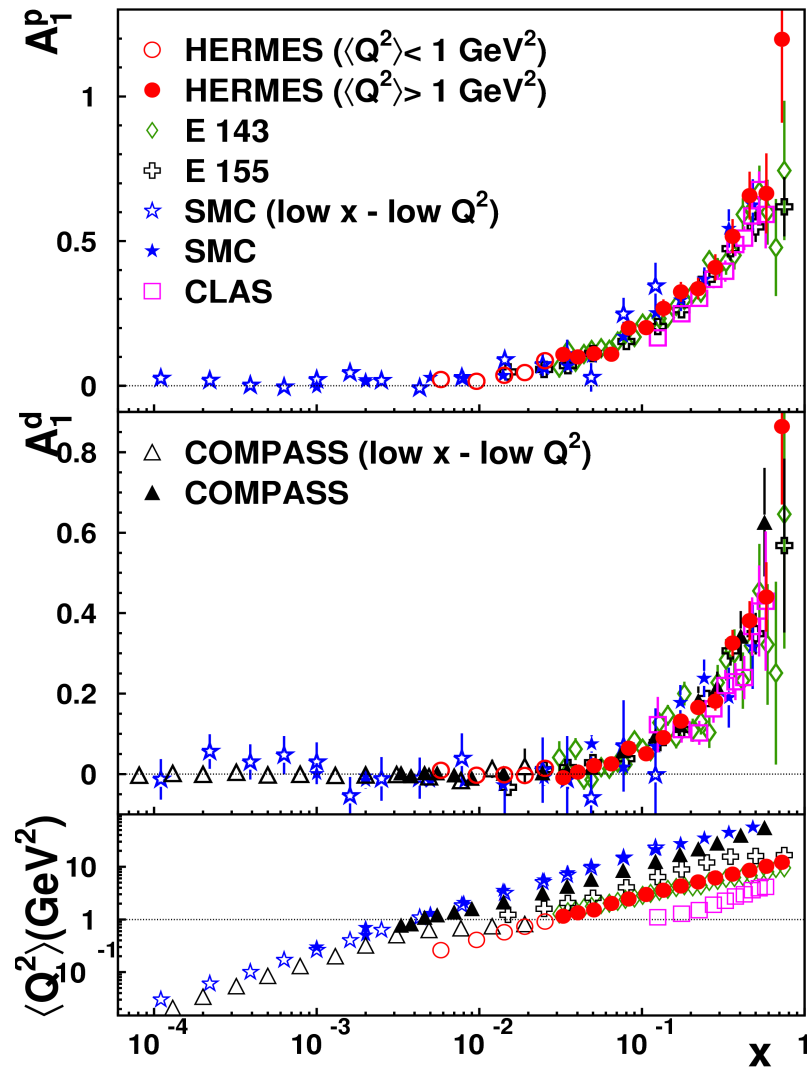
$$\Delta\Sigma = \Delta\Sigma_u + \Delta\Sigma_d + \Delta\Sigma_s = 0.12 \pm 0.17$$

EMC '89

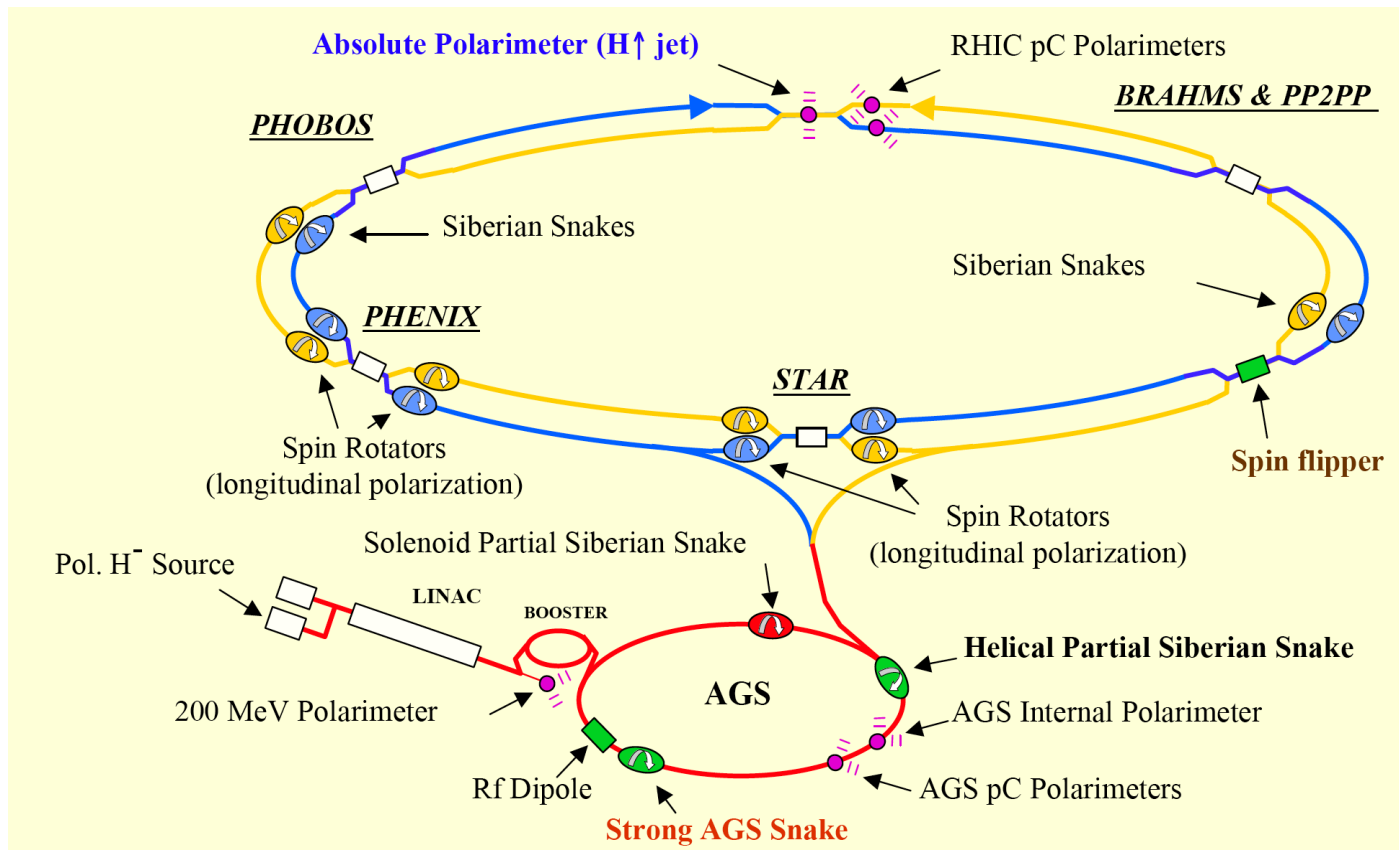
• Note,

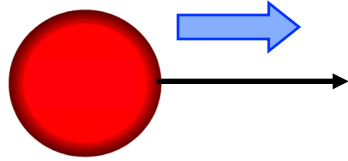
$$\Delta\Sigma = \Delta\Sigma_u + \Delta\Sigma_d + \Delta\Sigma_s = 3F - D + 3\Delta\Sigma_s$$

Today:



Plus: polarized pp at RHIC

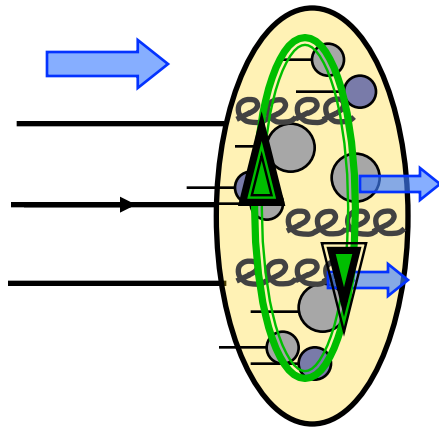




$$\frac{1}{2} = \langle P, \frac{1}{2} | \hat{J}_z | P, \frac{1}{2} \rangle$$

$$\hat{J}_z = \int d^3x \left[\frac{1}{2} \bar{\psi} \gamma_z \gamma_5 \psi - i \psi^\dagger (\vec{x} \times \vec{\nabla})_z \psi + (\vec{E} \times \vec{A})_z + E_i (\vec{x} \times \vec{\nabla})_z A_i \right]$$

- Gives rise to proton spin sum rule:

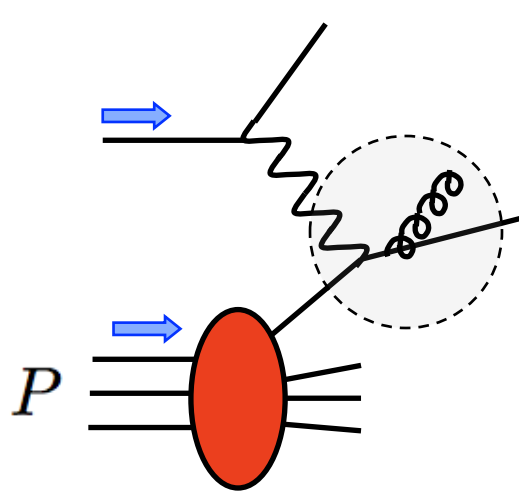


$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + \Delta G + L_g$$

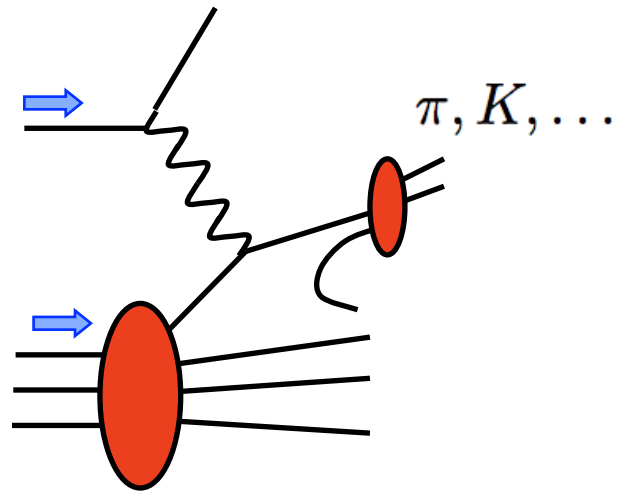
Jaffe, Manohar; Jaffe, Bashinsky;
Brodsky; Chen et al.; Wakamatsu

Polarized high-energy collisions in QCD

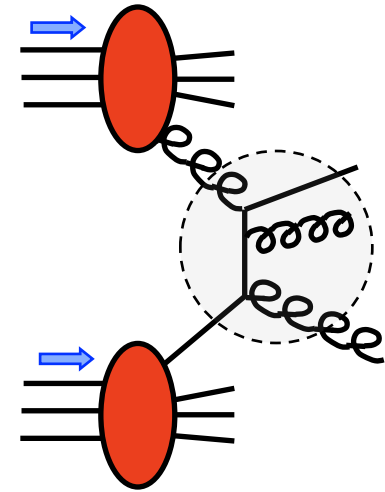
The probes of nucleon helicity structure :



DIS



SIDIS



pp (RHIC)

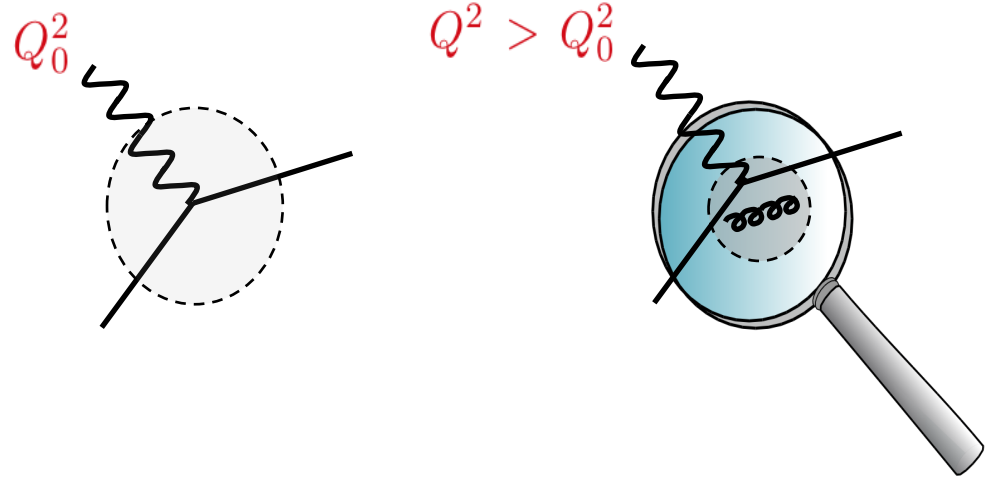
DIS
$$\Delta\sigma = \sum_{f=q,\bar{q},g} \int dx \Delta f(x, Q^2) \Delta\hat{\sigma}^f(xP, \alpha_s(Q^2)) + \dots$$

pp
$$\Delta\sigma = \sum_{a,b=q,\bar{q},g} \int dx_a \Delta f_a(x_a, p_{\perp}^2) \int dx_b \Delta f_b(x_b, p_{\perp}^2) \Delta\hat{\sigma}^{ab}(x_a P, x_b P', \alpha_s(p_{\perp}^2)) + \dots$$

$$\Delta\hat{\sigma} = \Delta\hat{\sigma}_{\text{LO}} + \alpha_s \Delta\hat{\sigma}_{\text{NLO}} + \dots$$

A lot of theory work:

- **DGLAP** evolution:



$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} \Delta q(x, \mu^2) \\ \Delta g(x, \mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta \mathcal{P}_{qq} & \Delta \mathcal{P}_{qg} \\ \Delta \mathcal{P}_{gq} & \Delta \mathcal{P}_{gg} \end{pmatrix} \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} \left(\frac{x}{z}, \mu^2 \right)$$

$$\Delta \mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} \Delta \mathcal{P}_{ij}^{\text{LO}} + \left(\frac{\alpha_s}{2\pi} \right)^2 \Delta \mathcal{P}_{ij}^{\text{NLO}} + \left(\frac{\alpha_s}{2\pi} \right)^3 \Delta \mathcal{P}_{ij}^{\text{NNLO}} + \dots$$

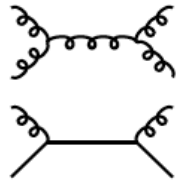
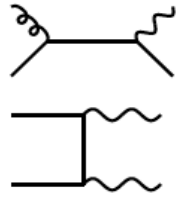
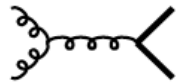
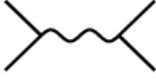
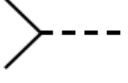
↑
Ahmed, Ross
Altarelli, Parisi, ...

↑
Mertig, van Neerven
WV

↑
Moch, Rogal,
Vermaseren, Vogt
(ij = qq, qg)

Polarized pp scattering (RHIC) :

NLO:

Reaction	Dom. partonic process	probes	LO Feynman diagram
$\vec{p}\vec{p} \rightarrow \pi + X$	$\vec{g}\vec{g} \rightarrow gg$ $\vec{q}\vec{g} \rightarrow qg$	Δg	
$\vec{p}\vec{p} \rightarrow \text{jet(s)} + X$	$\vec{g}\vec{g} \rightarrow gg$ $\vec{q}\vec{g} \rightarrow qg$	Δg	(as above)
$\vec{p}\vec{p} \rightarrow \gamma + X$ $\vec{p}\vec{p} \rightarrow \gamma + \text{jet} + X$ $\vec{p}\vec{p} \rightarrow \gamma\gamma + X$	$\vec{q}\vec{g} \rightarrow \gamma q$ $\vec{q}\vec{g} \rightarrow \gamma q$ $\vec{q}\vec{q} \rightarrow \gamma\gamma$	Δg Δg $\Delta q, \Delta \bar{q}$	
$\vec{p}\vec{p} \rightarrow DX, BX$	$\vec{g}\vec{g} \rightarrow c\bar{c}, b\bar{b}$	Δg	
$\vec{p}\vec{p} \rightarrow \mu^+\mu^- X$ (Drell-Yan)	$\vec{q}\vec{q} \rightarrow \gamma^* \rightarrow \mu^+\mu^-$	$\Delta q, \Delta \bar{q}$	
$\vec{p}\vec{p} \rightarrow (Z^0, W^\pm)X$ $p\vec{p} \rightarrow (Z^0, W^\pm)X$	$\vec{q}\vec{q} \rightarrow Z^0, \vec{q}'\vec{q} \rightarrow W^\pm$ $\vec{q}'\vec{q} \rightarrow W^\pm, q'\vec{q} \rightarrow W^\pm$	$\Delta q, \Delta \bar{q}$	

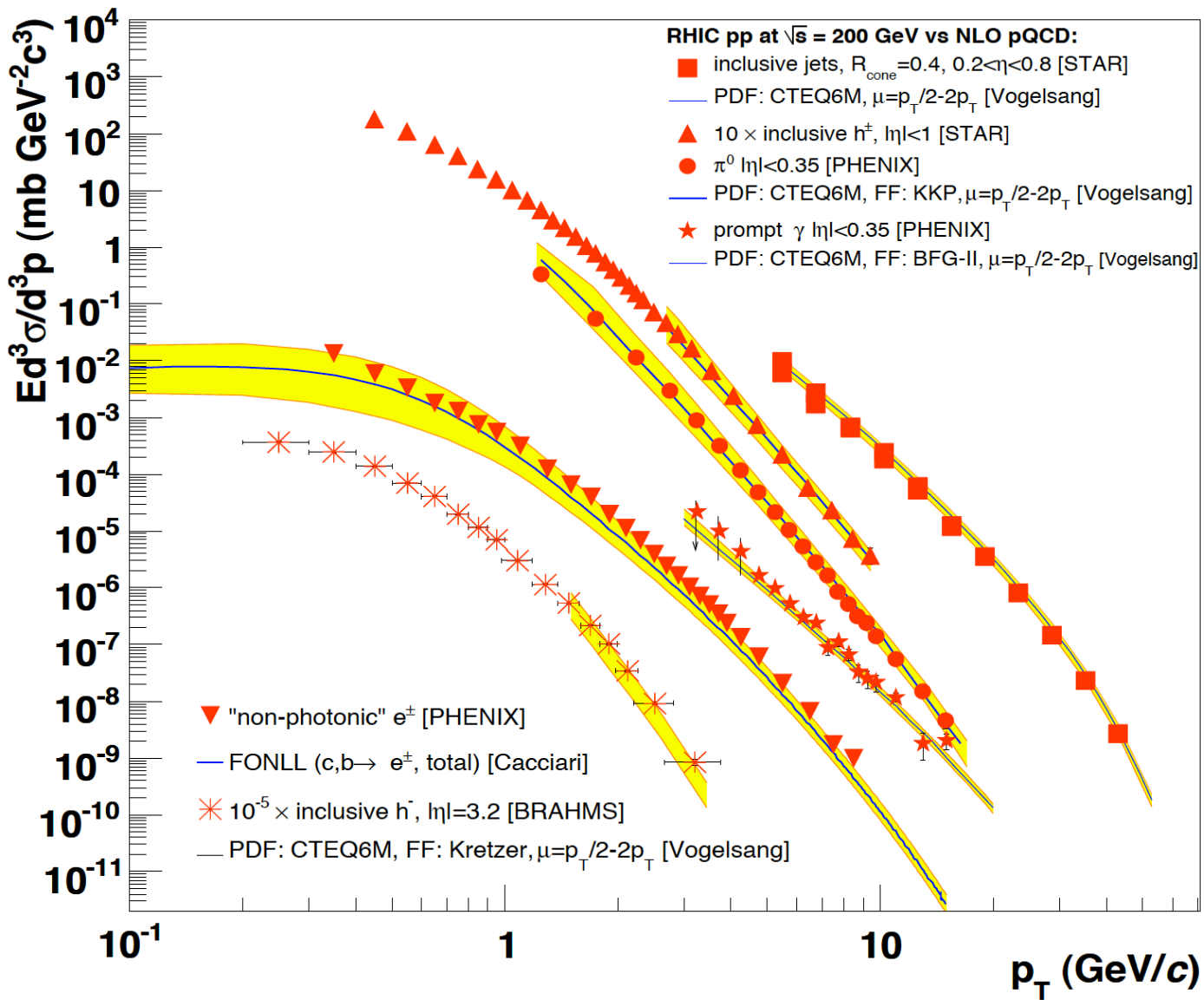
Jäger, Schäfer,
Stratmann, WV

Jäger, Stratmann,
WV; Signer et al.

Gordon, WV;
Contogouris et al.;
Gordon, Coriano;
Frixione. WV

Stratmann, Bojak

Weber; Gehrmann;
Kamal; Smith,
van Neerven,
Ravindran;
Nadolsky, Yuan;
de Florian, WV



**Global analysis of polarized
parton distributions: technique**

Long history of NLO QCD analyses of helicity parton distributions in DIS:

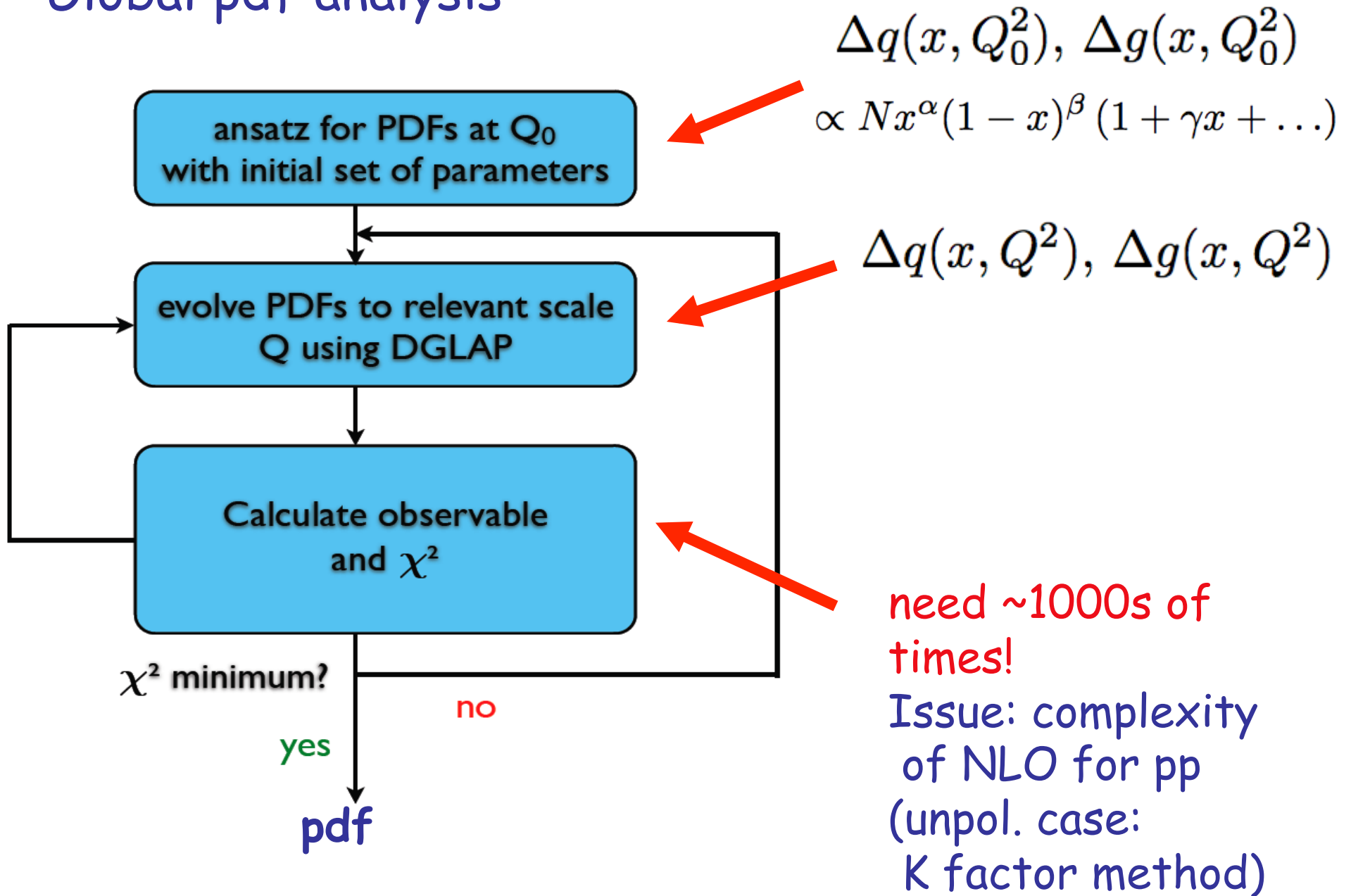
GRSV Glück, Reya, Stratmann, WV
GS Gehrman, Stirling
ABFR Altarelli, Ball, Forte, Ridolfi
BB Blümlein, Böttcher
BBS Bourely, Buccella, Soffer
LSS Leader, Sidorov, Stamenov
AAC Hirai, Kumano, Saito
DNS de Florian, Navarro, Sassot

...

First NLO (\overline{MS}) "global analysis" of all DIS, SIDIS, RHIC data sets:

DSSV de Florian, Sassot, Stratmann, WV

"Global pdf analysis"



Typical example $pp \rightarrow \pi X$:

$$\Delta\sigma = \sum_{abc} \int dx_a \int dx_b \int dz_c \Delta f_a(x_a) \Delta f_b(x_b) \Delta \hat{\sigma}_{ab \rightarrow cX} D_c(z_c)$$

One evaluation @ NLO: $\sim O(10 \text{ sec.})$

Assume ~ 10 data points and 5,000 calls during fit:

\rightarrow Time = $O(10^6 \text{ sec.})$

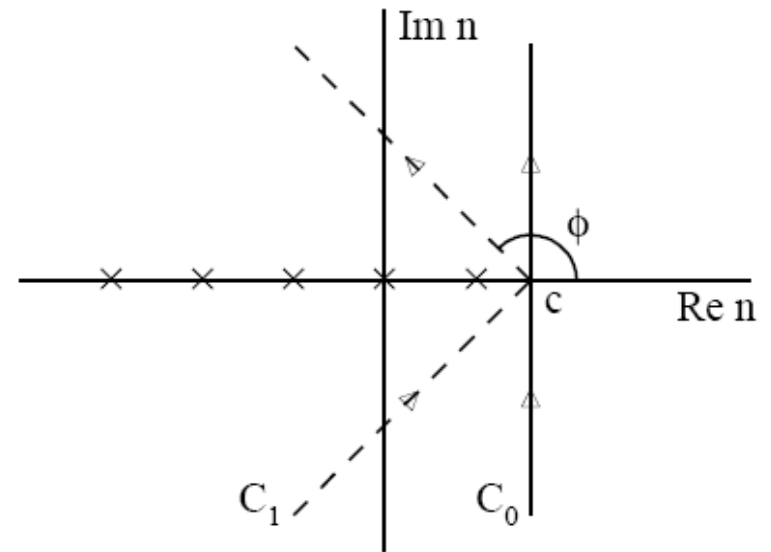
(typically need 100s of fits)

Mellin moments of a function $f(x)$:

$$f^n \equiv \int_0^1 dx x^{n-1} f(x)$$

Inverse transformation :


$$f(x) = \frac{1}{2\pi i} \int_C dn x^{-n} f^n$$




Mellin method for pp scattering : example $pp \rightarrow \pi X$

Stratmann, WV; Berger, Graudenz, Hampel, Vogt; Kosover

$$\Delta\sigma = \sum_{abc} \int dx_a \int dx_b \int dz_c \Delta f_a(x_a) \Delta f_b(x_b) \Delta \hat{\sigma}_{ab \rightarrow cX} D_c(z_c)$$



$$\frac{1}{2\pi i} \int_{\mathcal{C}} dn x_a^{-n} \Delta f_a^n$$



$$\frac{1}{2\pi i} \int_{\mathcal{C}_m} dm x_b^{-m} \Delta f_b^m$$

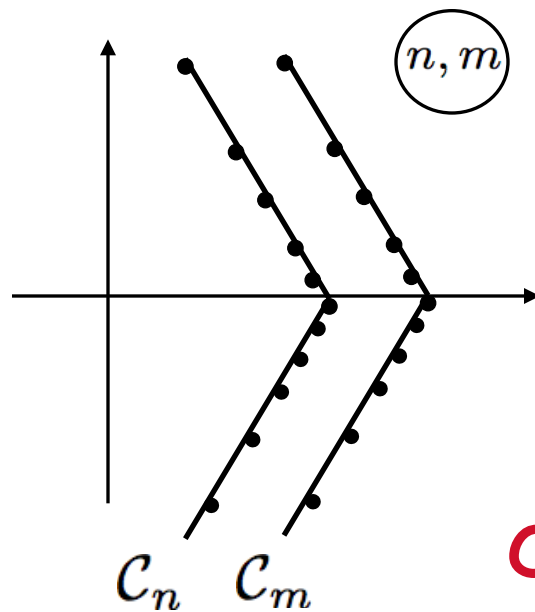
$$= \frac{1}{(2\pi i)^2} \sum_{abc} \int_{\mathcal{C}_n} dn \int_{\mathcal{C}_m} dm \Delta f_a^n \Delta f_b^m \int dx_a \int dx_b \int dz_c x_a^{-n} x_b^{-m} \Delta \hat{\sigma}_{ab \rightarrow cX} D_c(z_c)$$

$$\frac{1}{(2\pi i)^2} \sum_{abc} \int_{C_n} dn \int_{C_m} dm \Delta f_a^n \Delta f_b^m \int dx_a \int dx_b \int dz_c x_a^{-n} x_b^{-m} \Delta \hat{\sigma}_{ab \rightarrow cX} D_c(z_c)$$

Standard Mellin inverses

Contains all dependence on fit parameters

Completely independent of pdfs. Can be "pre-calculated" prior to fit



Discretize for (Gaussian) integration:
64 × 64 positions

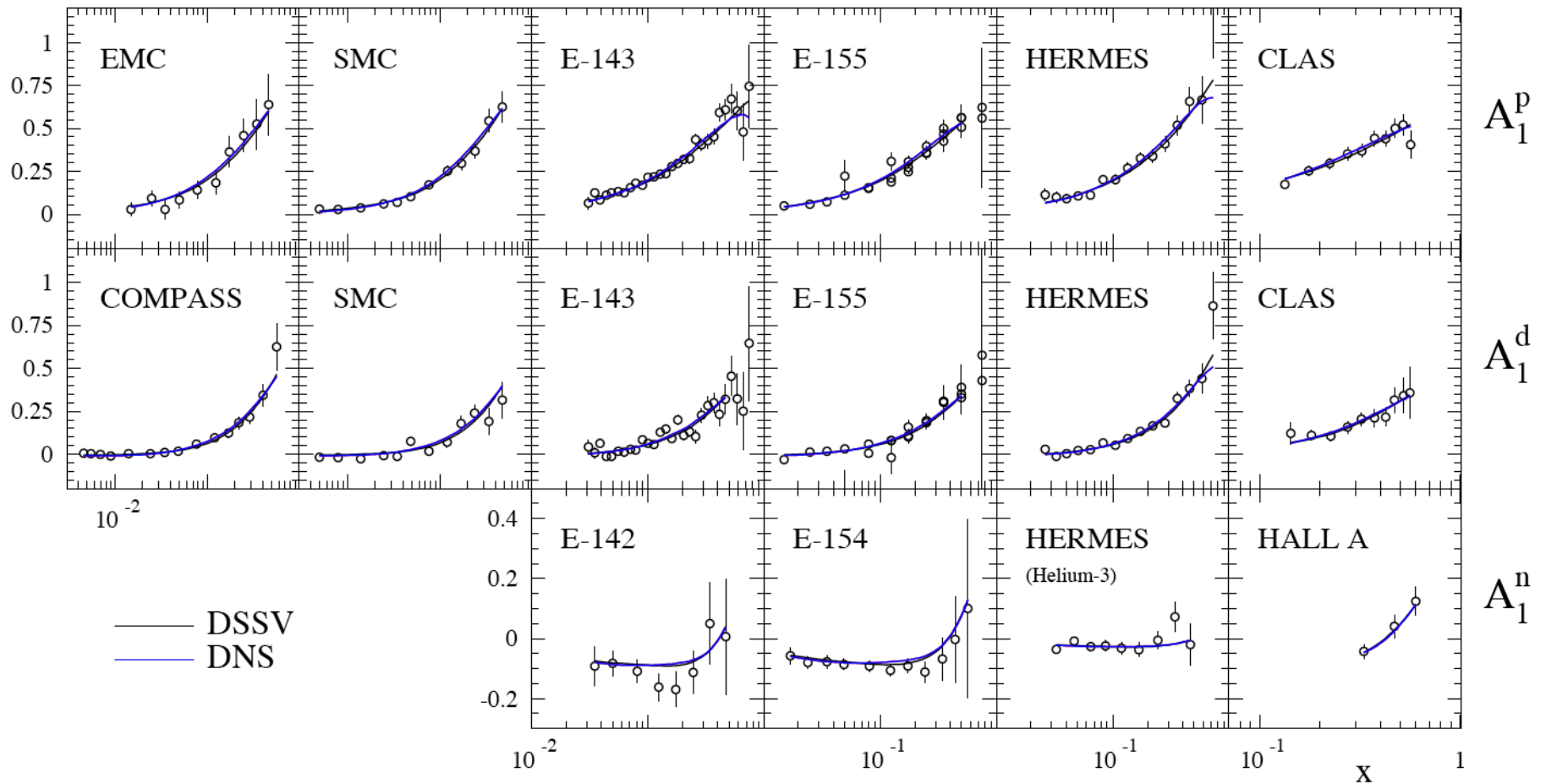
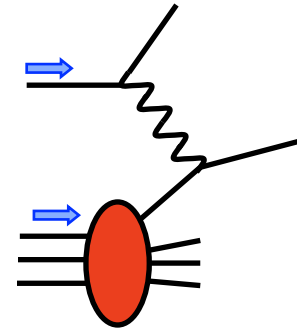
Evaluate "matrix"

$$(\Delta \tilde{\sigma}_{ab \rightarrow cX})_{n_i m_j}$$

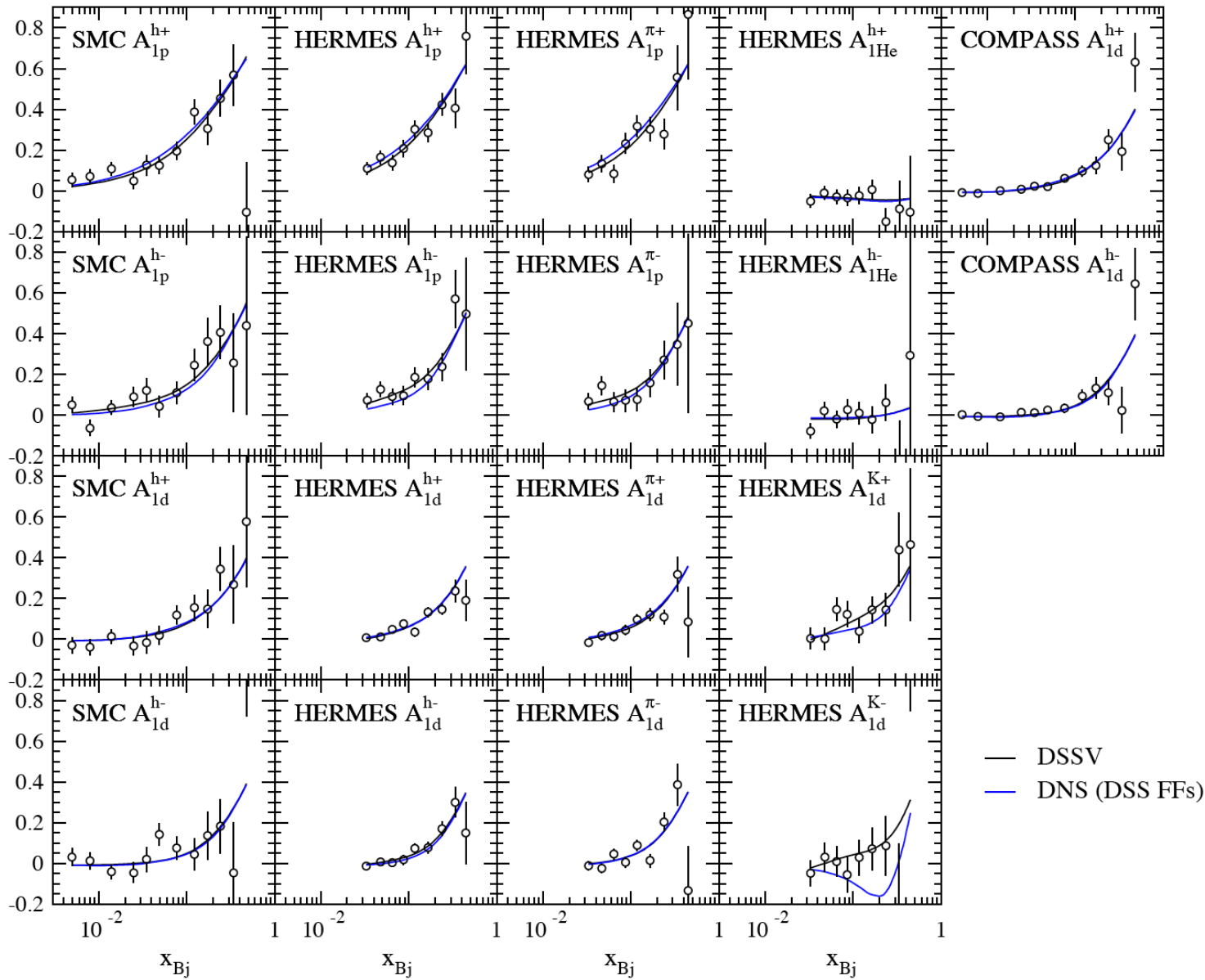
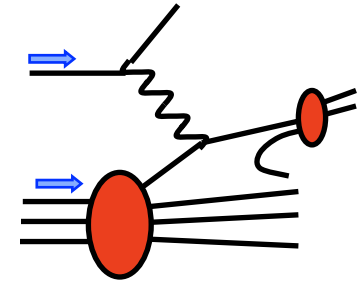
$O(10 \text{ sec.}) / \text{point} \rightarrow O(1 \text{ msec.}) / \text{point}$

Global analysis: results

Spin asymmetries in inclusive DIS:

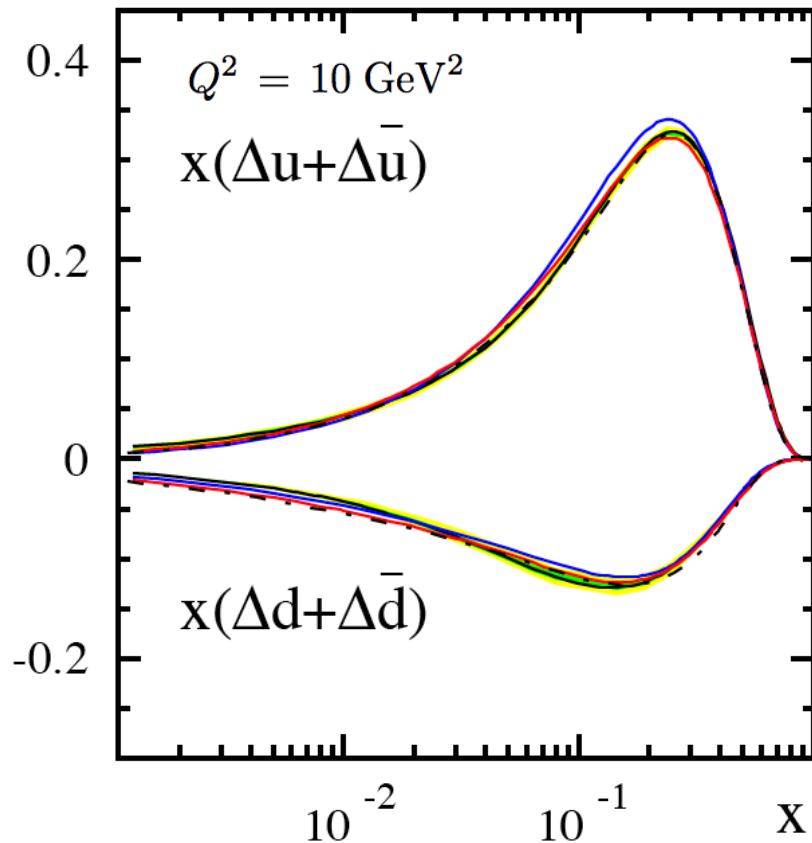


Spin asymmetries in semi-inclusive DIS:



What's the emerging picture ?

- best determined: $\Delta u + \Delta \bar{u}$, $\Delta d + \Delta \bar{d}$



Comparison with:

DNS de Florian, Navarro, Sassot

GRSV Glück, Reya, Stratmann, WV

Similar results:

Leader, Stamenov, Sidorov

Blümlein, Böttcher; & HERMES

Hirai, Kumano, Saito (AAC)

COMPASS

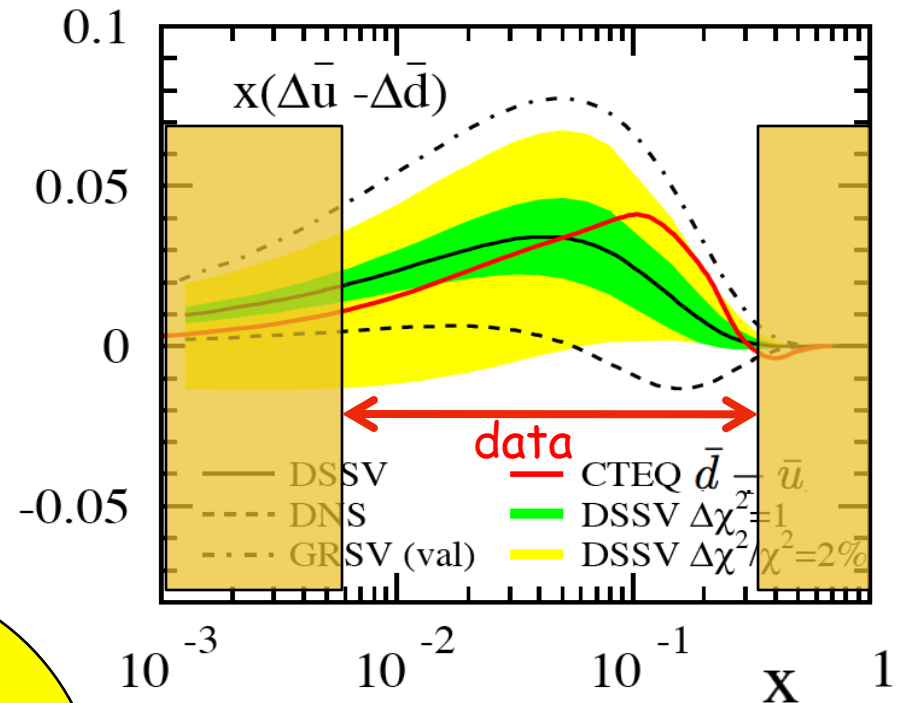
- first moments agree with lattice:
gives confidence in small- x extrapolations (?)

- light flavor sea :

driven by
SIDIS

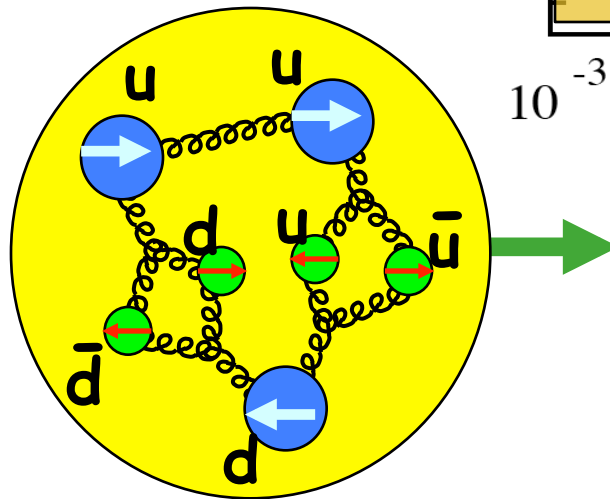
$$\Delta \bar{u} > 0$$

$$\Delta \bar{d} < 0$$



- qualitatively:

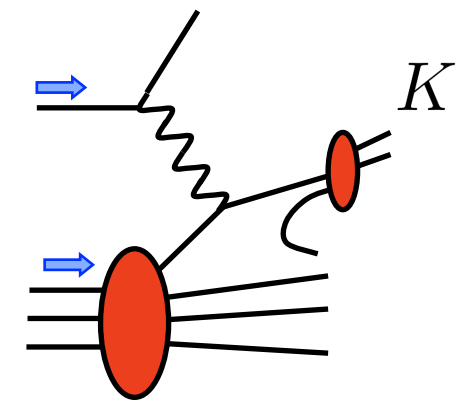
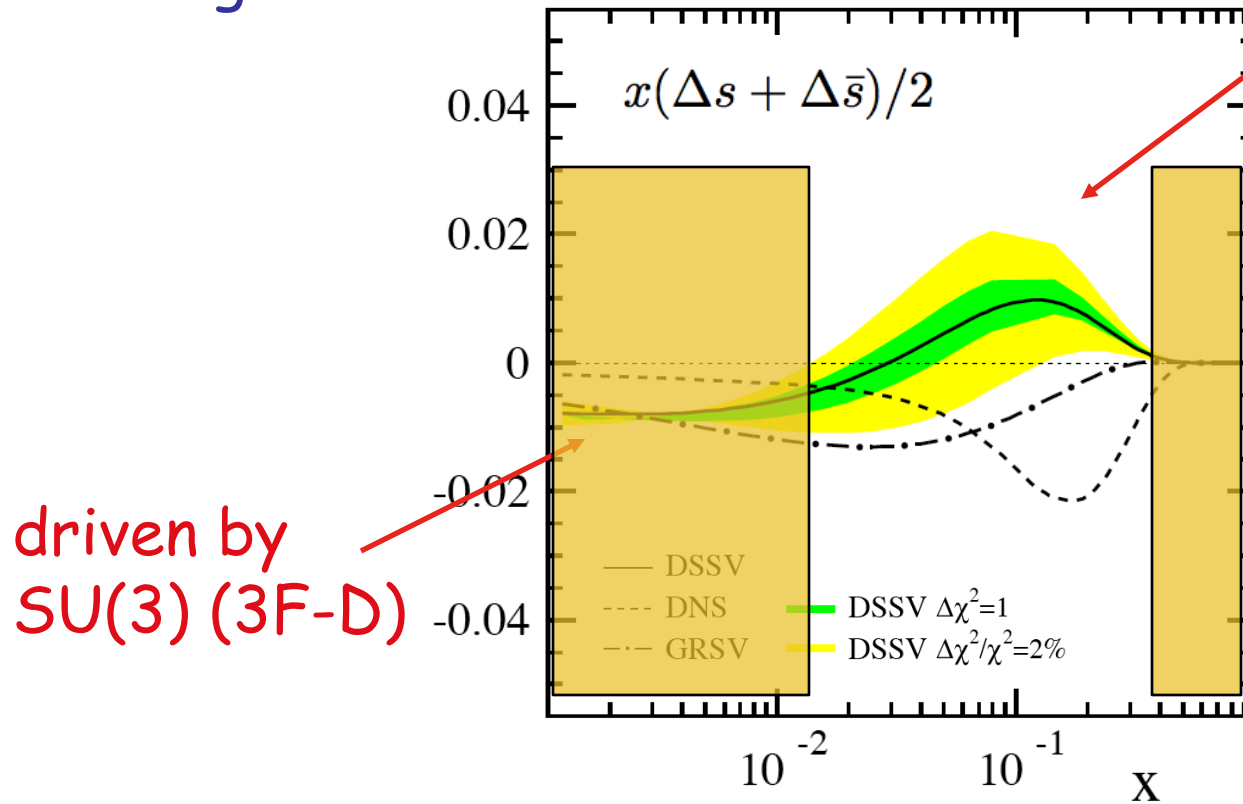
Glück,Reya;...



- large- N_c , chiral quark models, meson cloud

Thomas,Signal,Cao; Holtmann,Speth,Fässler; Diakonov,Polyakov,Weiss;
Schäfer,Fries; Kumano; Wakamatsu; Bourrely, Soffer ...

- strangeness :



$$\int_{0.001}^1 dx \Delta s(x) = -0.006 \pm 0.01 \quad (\Delta\chi^2 = 1)$$

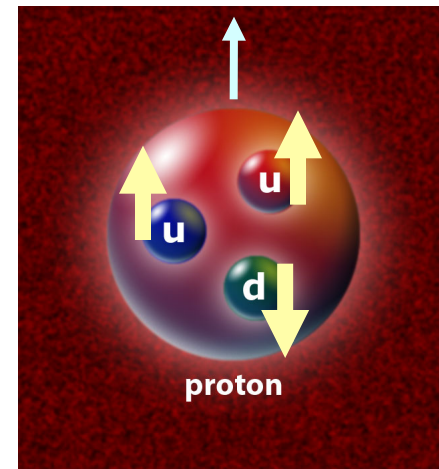
$$\int_0^1 dx \Delta s(x) = -0.057 \pm ? \quad \text{using F,D and SU(3)}$$

- total quark and anti-quark spin contribution :

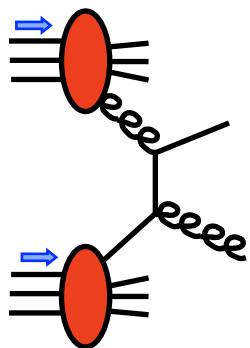
$$\int_{0.001}^1 dx \Delta\Sigma = 0.366 \pm 0.016 \quad (\Delta\chi^2 = 1)$$

$$\int_0^1 dx \Delta\Sigma = 0.242 \pm ?$$

- in any case, $\Delta\Sigma \ll 1$



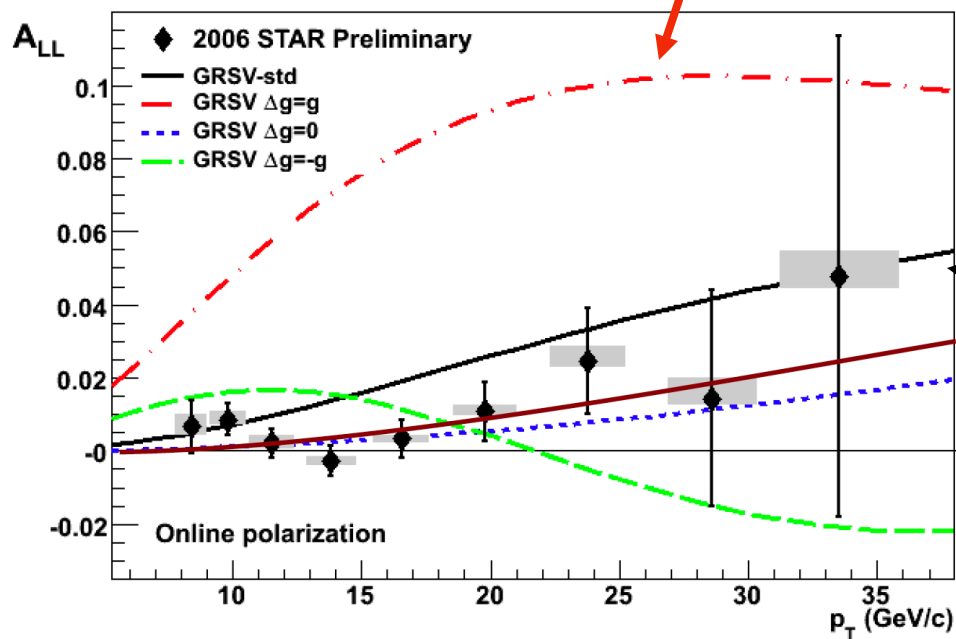
- polarized glue ?



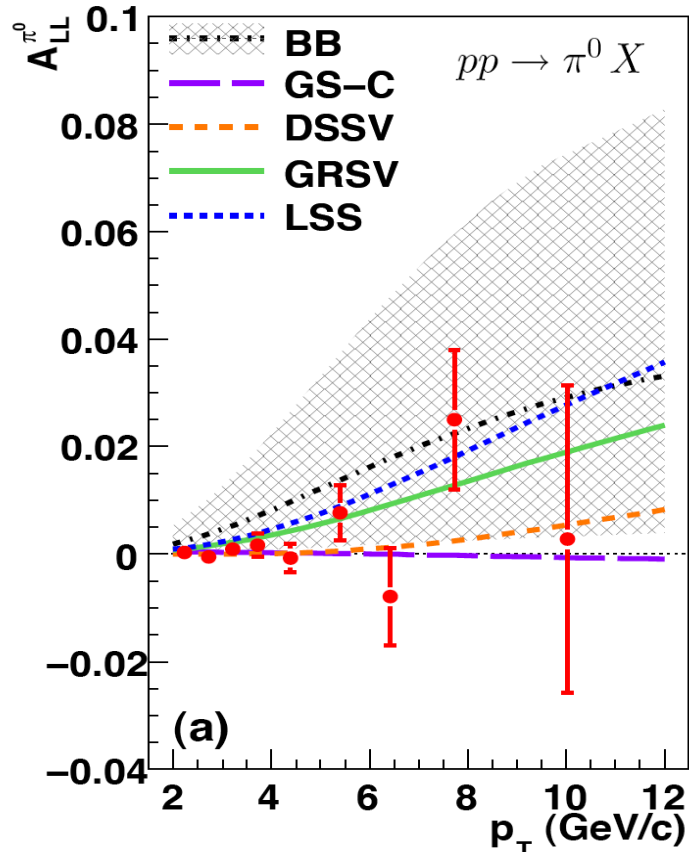
(Altarelli et al.)

$$\Delta G \approx 1.8$$

$pp \rightarrow \text{jet } X$



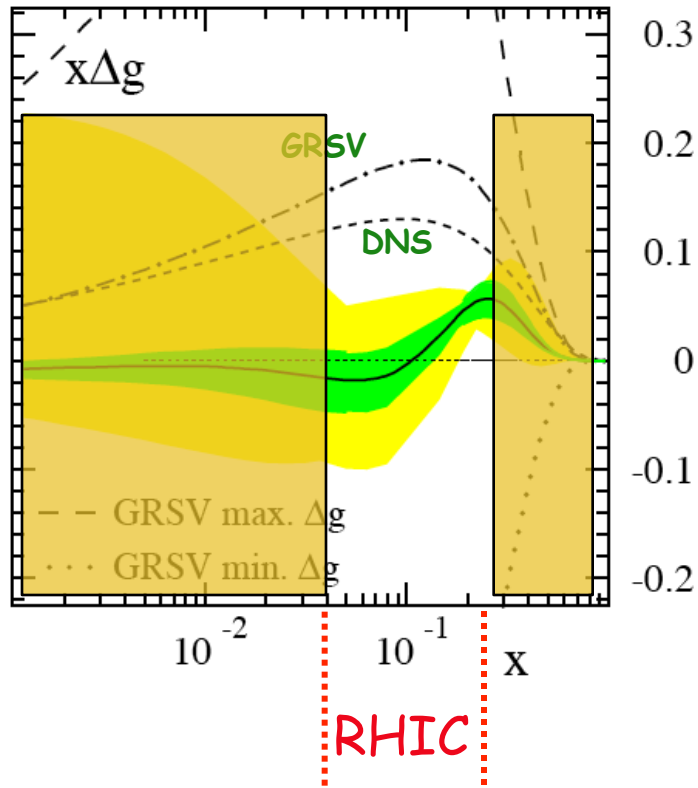
STAR



$$\Delta G \approx 0.4 \quad (Q^2 = 1 \text{ GeV}^2)$$

DSSV

$$\Delta G = 0$$

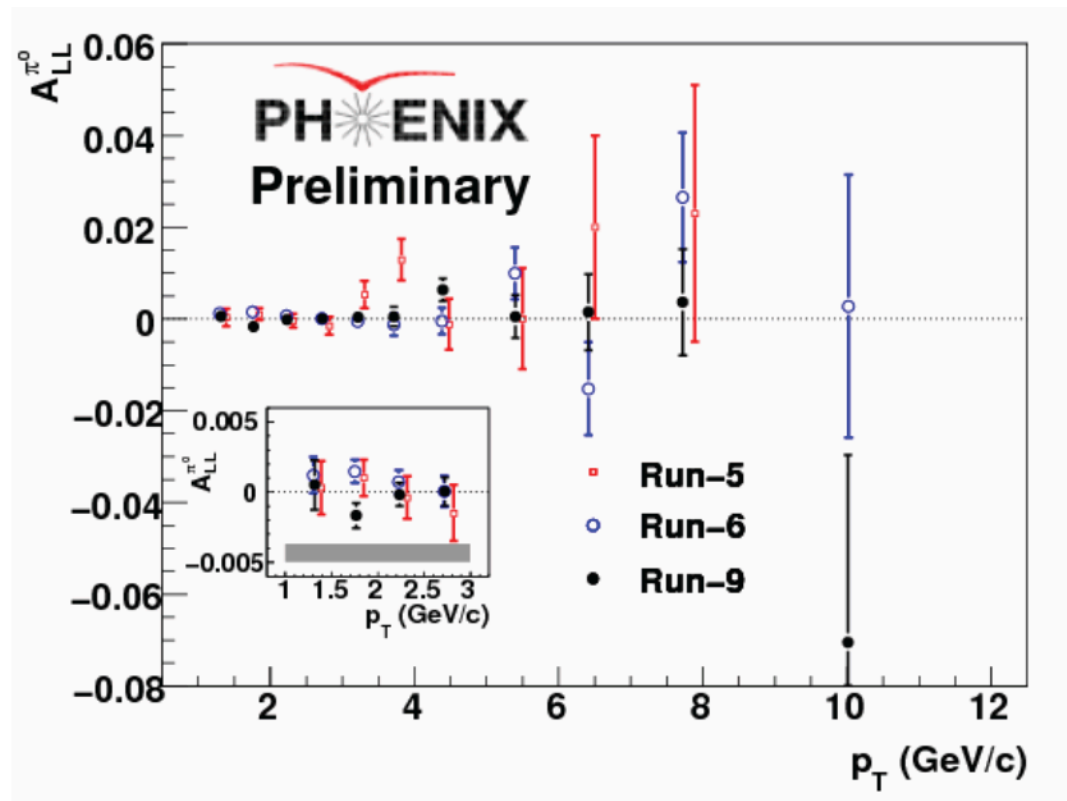


$$\int_{0.05}^{0.2} dx \Delta g = 0.006 \pm 0.06 \quad (\Delta\chi^2 = 1)$$

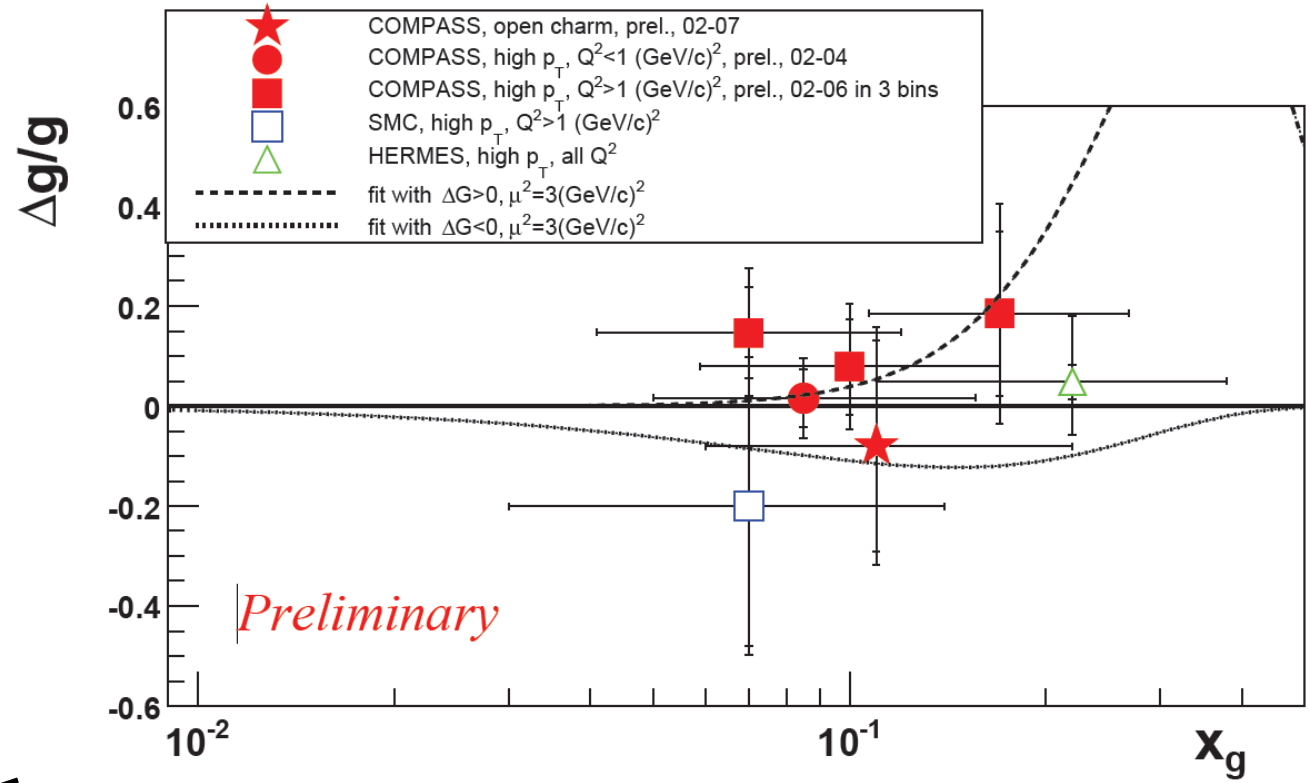
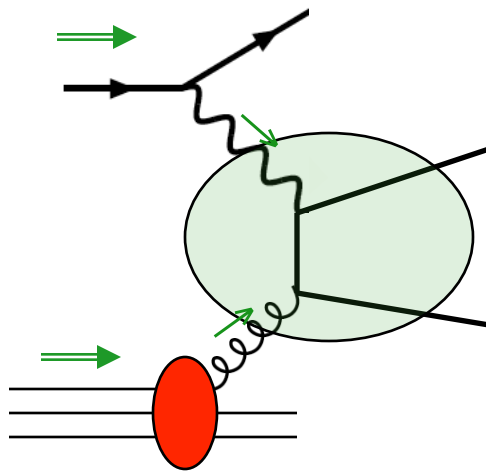
$$\int_0^1 dx \Delta g = -0.084 \pm ? \quad \frac{1}{2} \Delta\Sigma + \Delta G \approx 0 \quad ?$$

- there could still be significant contribution to proton spin
- constraints become better with new data

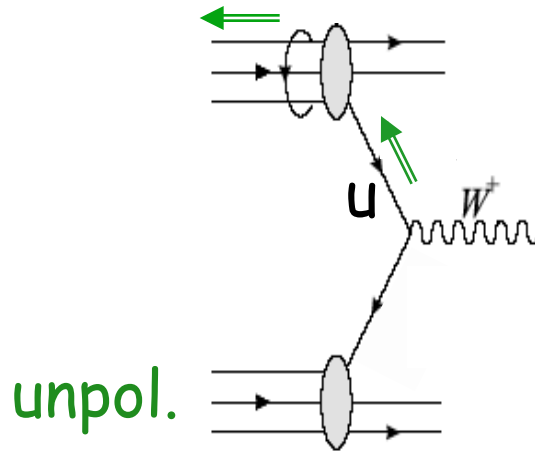
Most recent data:



Recent COMPASS data:



- **W production in pp:**



Parity violation:

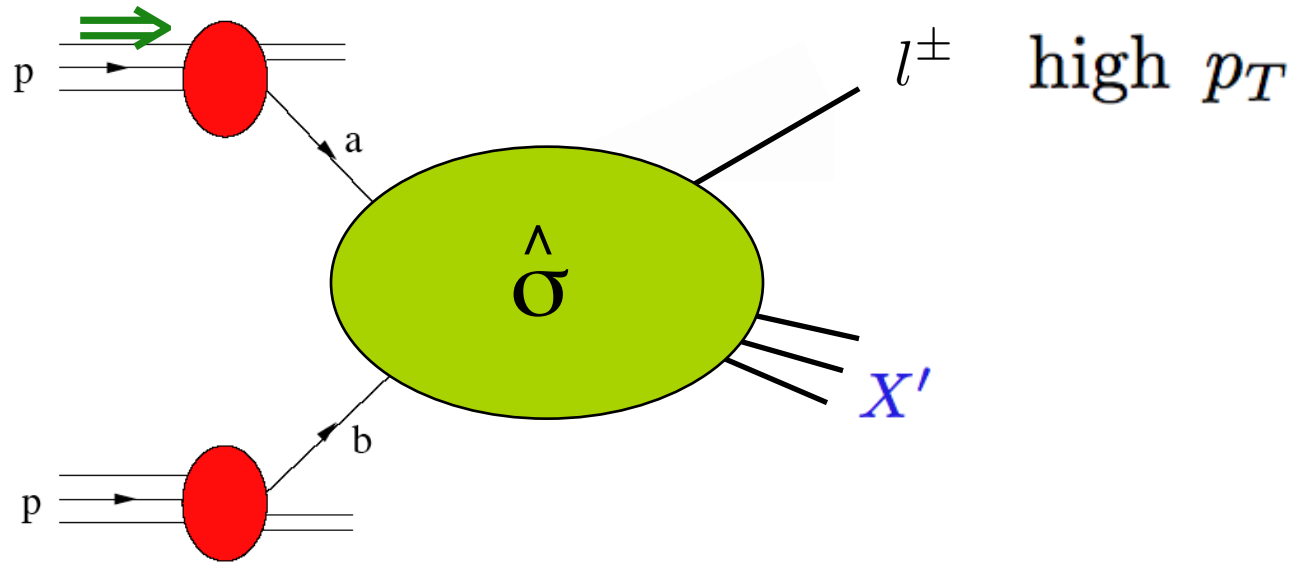
$$A_L = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \neq 0$$

$$A_L^{W^+} \approx - \frac{\Delta u(x_1) \bar{d}(x_2) - \Delta \bar{d}(x_1) u(x_2)}{u(x_1) \bar{d}(x_2) + \bar{d}(x_1) u(x_2)}$$

$$x_{1,2} = \frac{M_W}{\sqrt{S}} e^{\pm y_W}$$

- **large scale $Q \sim M_W$: pQCD**

In practice:



$$d\sigma^+ - d\sigma^- = \sum_{a,b} \int dx_a dx_b \Delta f_a(x_a, \mu) f_b(x_b, \mu) [d\hat{\sigma}^+ - d\hat{\sigma}^-]_{ab \rightarrow W \rightarrow l} + \text{power corr.}$$

smear out
 x_a, x_b

perturbative QCD

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_s \hat{\sigma}^{(1)} + \dots$$

- **new NLO** for polarized case: **de Florian, WV**

channels at NLO

$$\Delta \bar{q} q \rightarrow e \bar{\nu}_e$$

$$\Delta q \bar{q} \rightarrow e \bar{\nu}_e$$

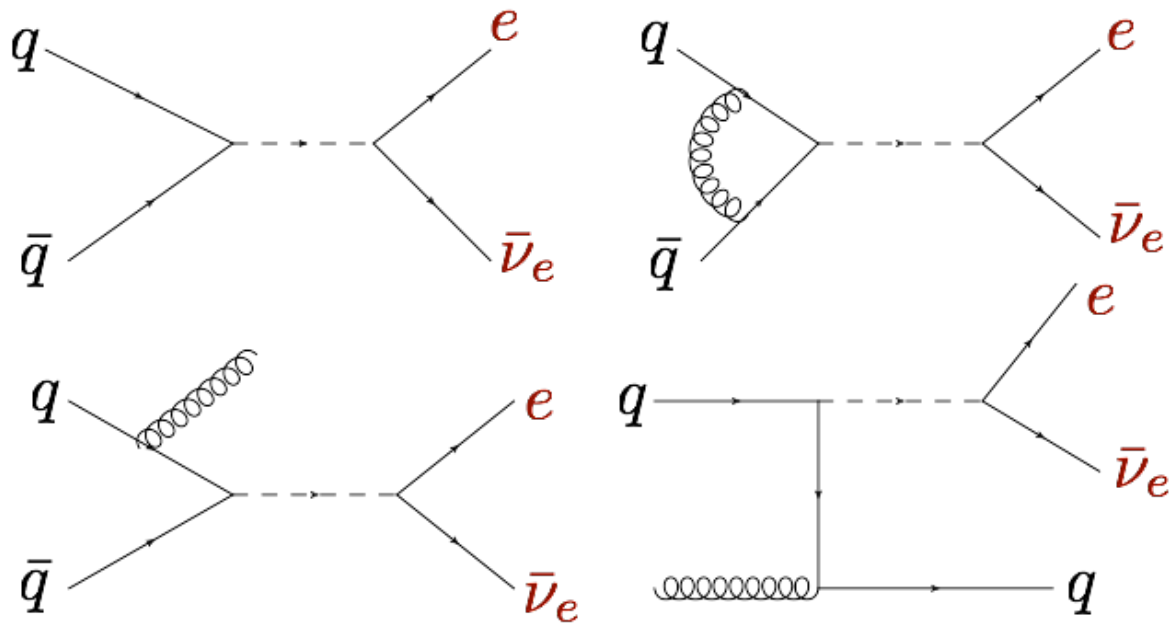
$$\Delta \bar{q} g \rightarrow e \bar{\nu}_e \bar{q}$$

$$\Delta g \bar{q} \rightarrow e \bar{\nu}_e \bar{q}$$

$$\Delta q g \rightarrow e \bar{\nu}_e g$$

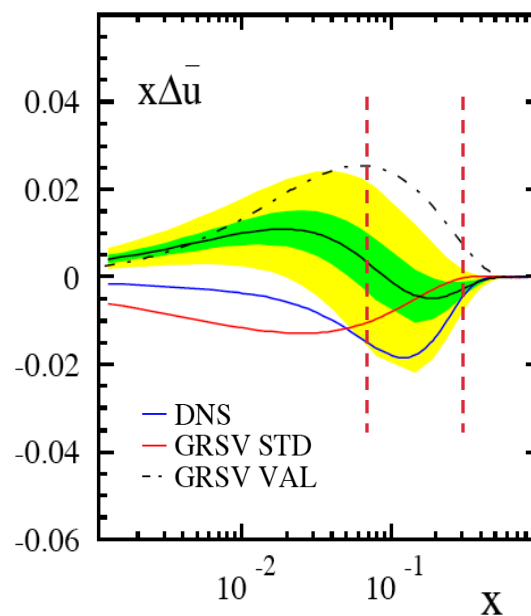
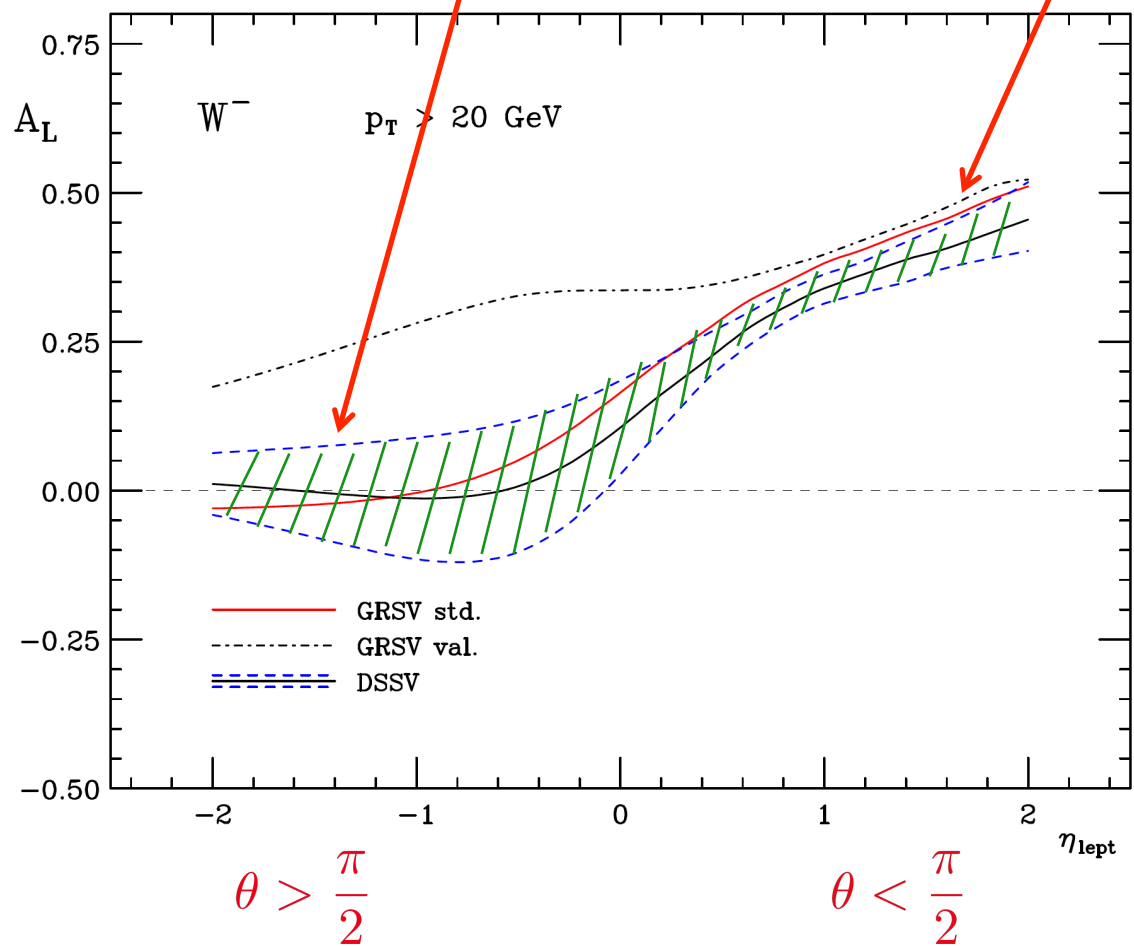
$$\Delta g q \rightarrow e \bar{\nu}_e g$$

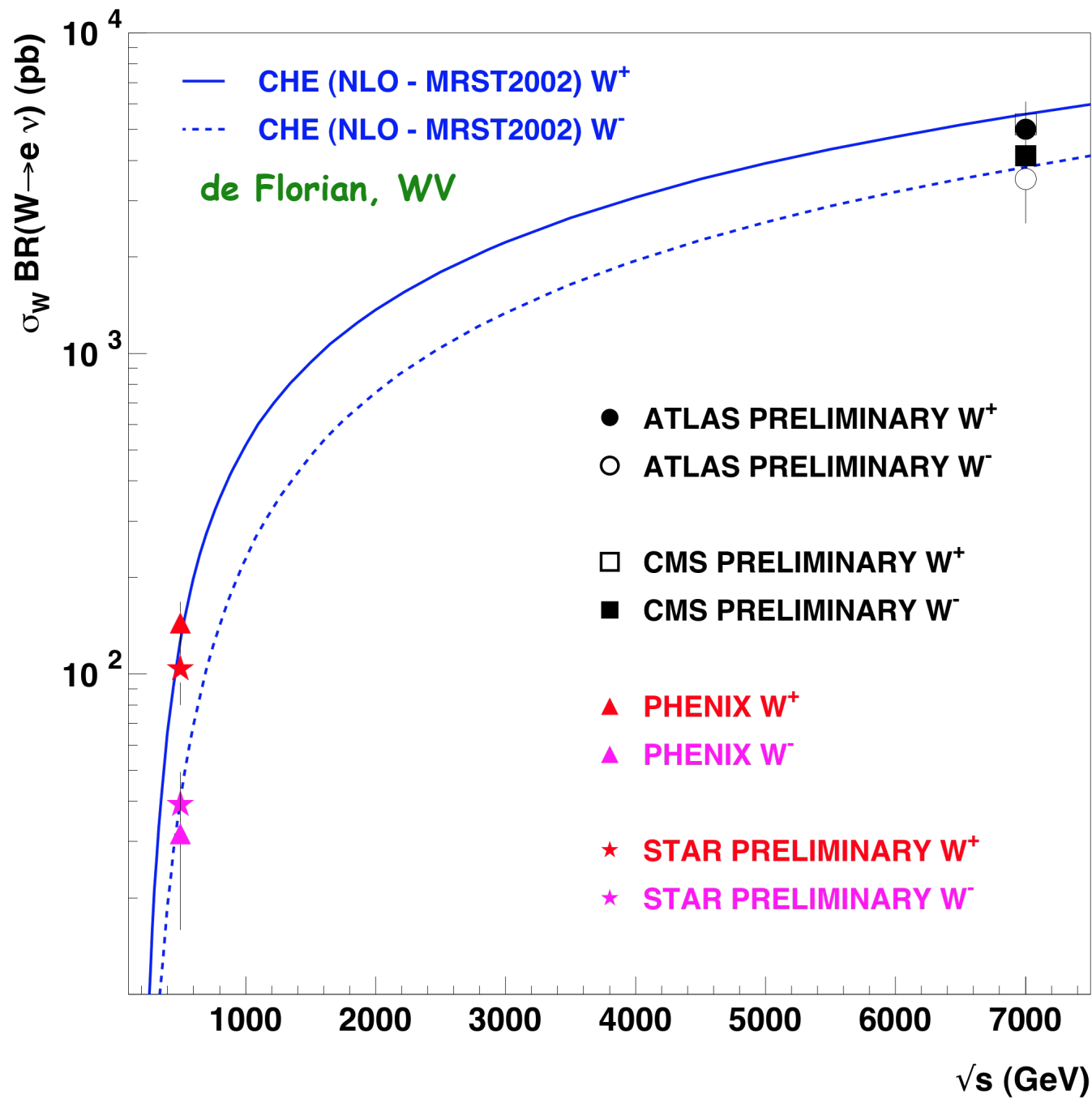
Some diagrams





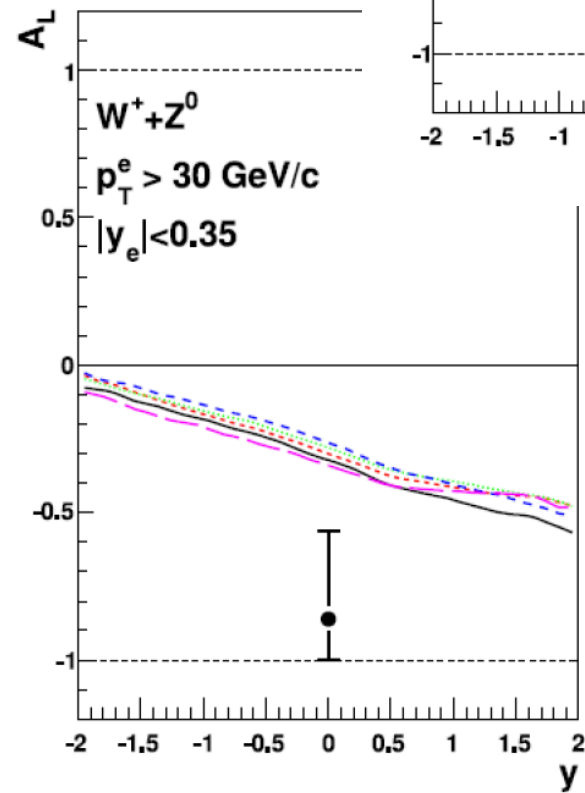
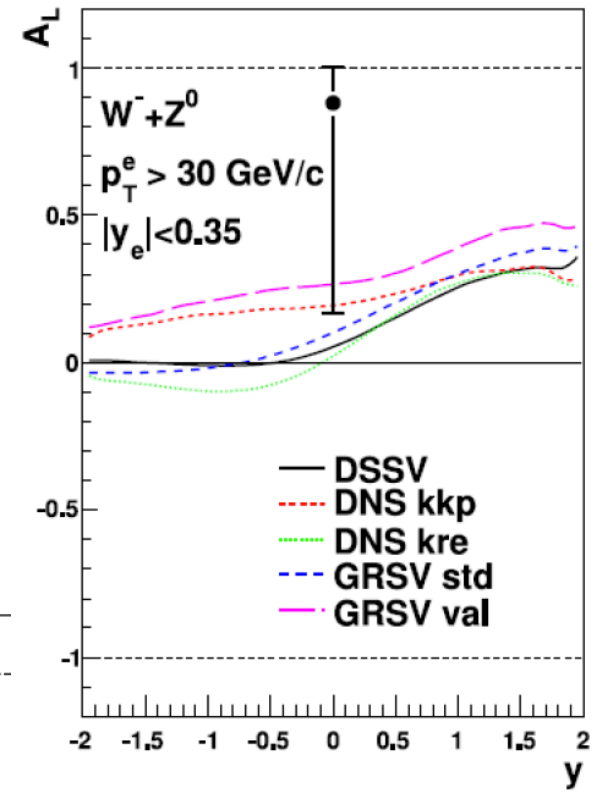
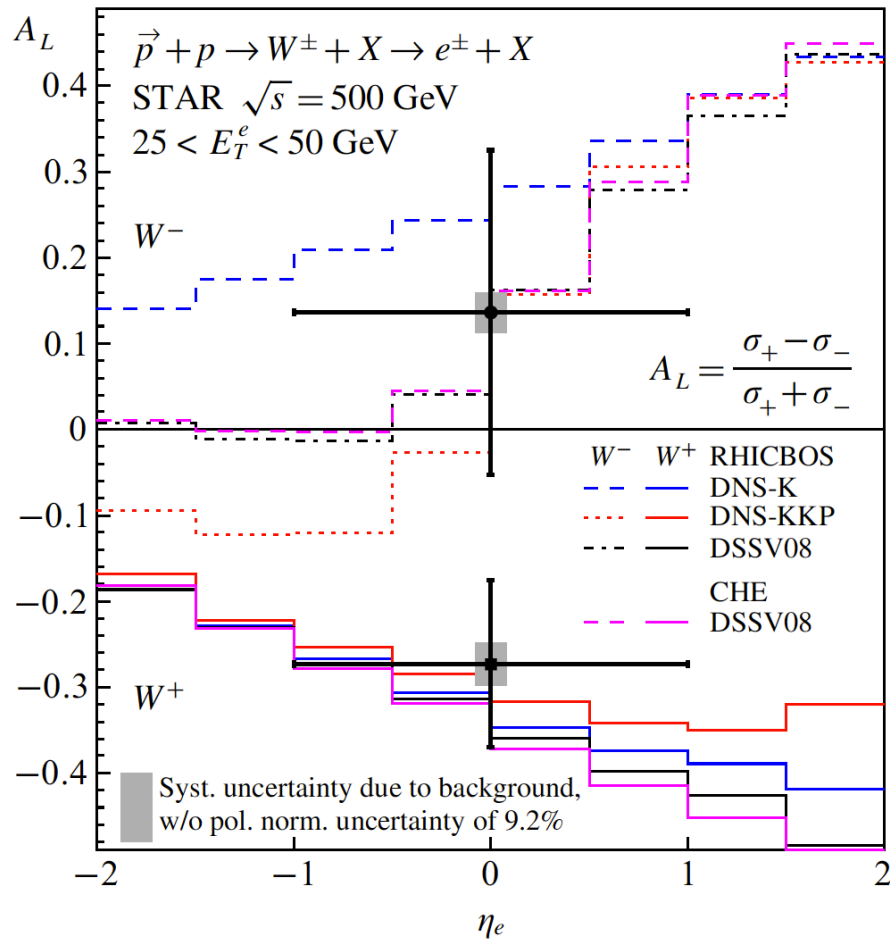
$$A_L^{e^-} \approx \frac{\int_{\otimes(x_1, x_2)} [\Delta \bar{u}(x_1) d(x_2) (1 - \cos \theta)^2 - \Delta d(x_1) \bar{u}(x_2) (1 + \cos \theta)^2]}{\int_{\otimes(x_1, x_2)} [\bar{u}(x_1) d(x_2) (1 - \cos \theta)^2 + d(x_1) \bar{u}(x_2) (1 + \cos \theta)^2]}$$





B. Surrow
(STAR)

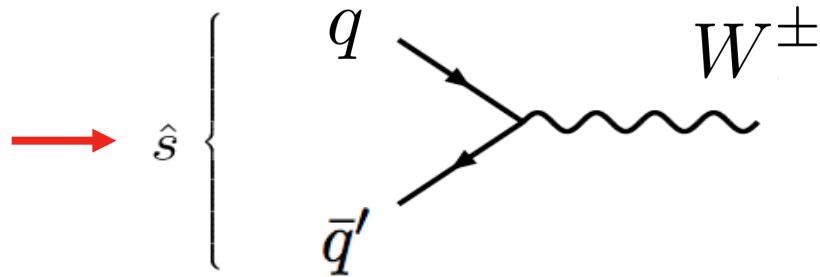
STAR



Phenix

Applications of QCD resummation

"Threshold resummation"



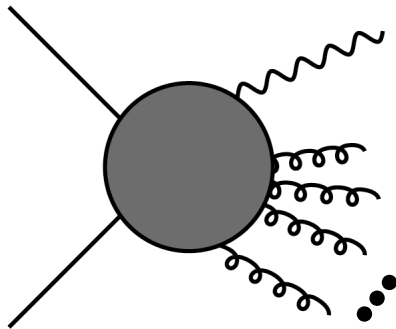
inv. mass M_W

$$\hat{s} = M_W^2, \quad z \equiv \frac{M_W^2}{\hat{s}} = 1$$

- partonic cross section :

$$\hat{\sigma}_{q\bar{q}}^{(0)} \propto \delta(1 - z)$$

- higher orders :



$$\hat{\sigma}_{q\bar{q}}^{(k)} \propto \alpha_s^k \left[\frac{\ln^{2k-1}(1-z)}{1-z} \right]_+ + \dots$$

"Threshold logarithms"

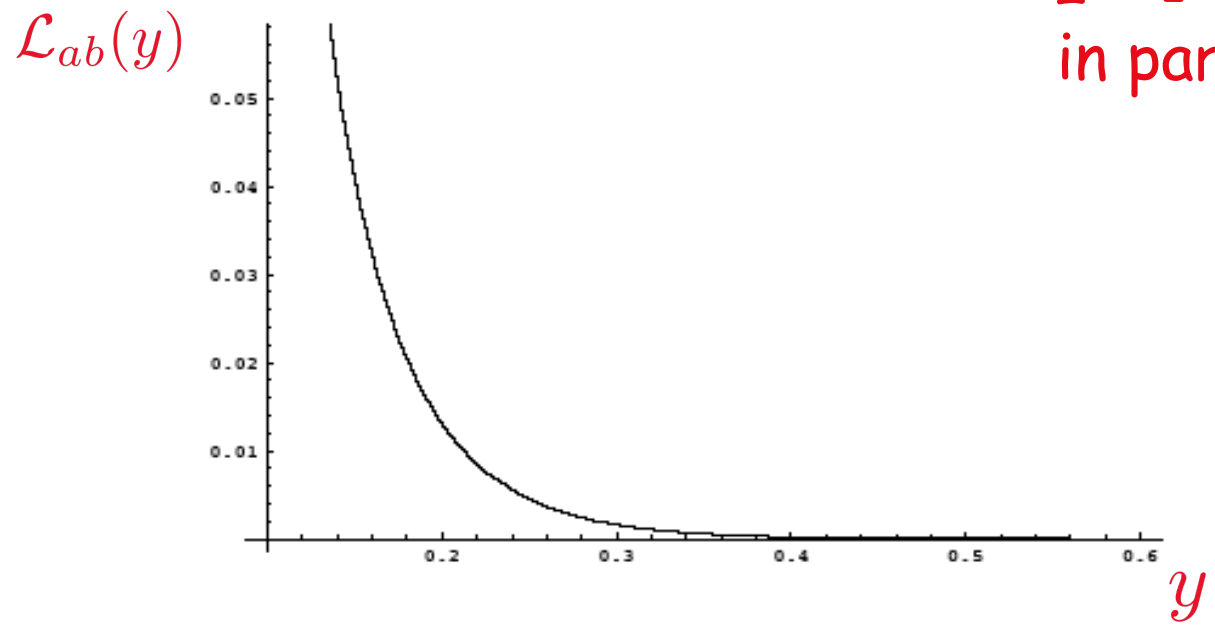
- interplay with parton distributions : $\tau = M_W^2/s$

$$\sigma^W \propto \sum_{a,b} \int_{\tau}^1 \frac{dx_a}{x_a} f_a(x_a) \int_{\tau/x_a}^1 \frac{dx_b}{x_b} f_b(x_b) \hat{\sigma}_{ab}(z = \tau/(x_a x_b))$$

$$= \sum_{a,b} \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ab}\left(\frac{\tau}{z}\right) \hat{\sigma}_{ab}(z)$$



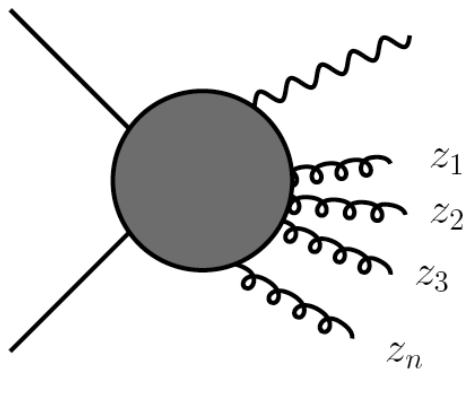
$z = 1$ emphasized,
in particular as $\tau \rightarrow 1$



Large logs can be resummed to all orders

Sterman; Catani, Trentadue; ...

- factorization of matrix elements
- and of phase space when Mellin transform is taken:



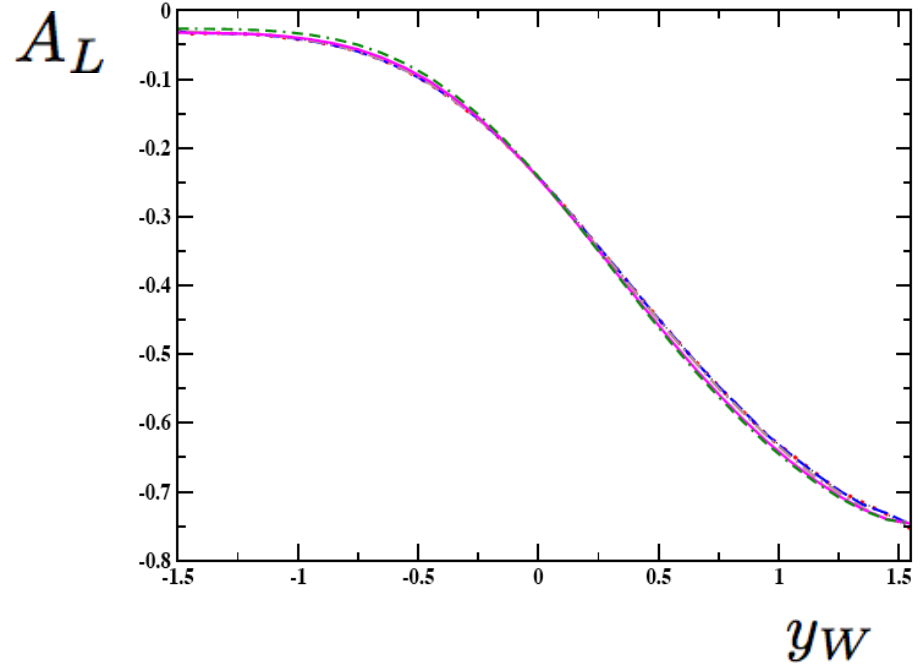
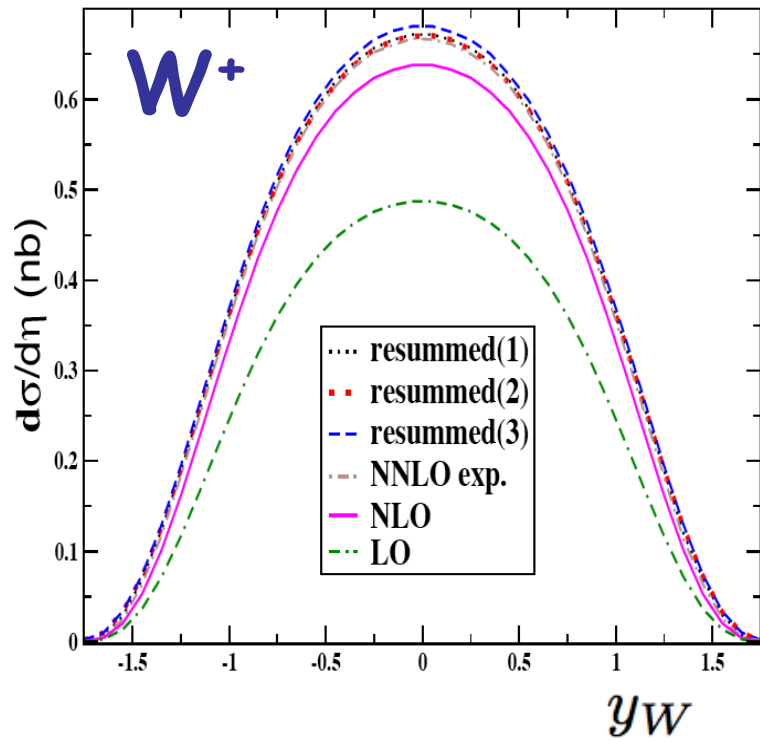
$$\delta \left(1 - z - \sum_{i=1}^n z_i \right) = \frac{1}{2\pi i} \int_C dN e^{N(1-z-\sum_{i=1}^n z_i)}$$

$$\hat{\sigma}_{q\bar{q}} \propto \exp \left[2 \int_0^1 dy \frac{y^N - 1}{1-y} \int_{\mu_F^2}^{Q^2(1-y)^2} \frac{dk_{\perp}^2}{k_{\perp}^2} A_q(\alpha_s(k_{\perp}^2)) + \dots \right]$$

$$A_q(\alpha_s) = C_F \left\{ \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{C_A}{2} \left(\frac{67}{18} - \zeta(2) \right) - \frac{5}{9} T_R n_f \right] \right\}$$

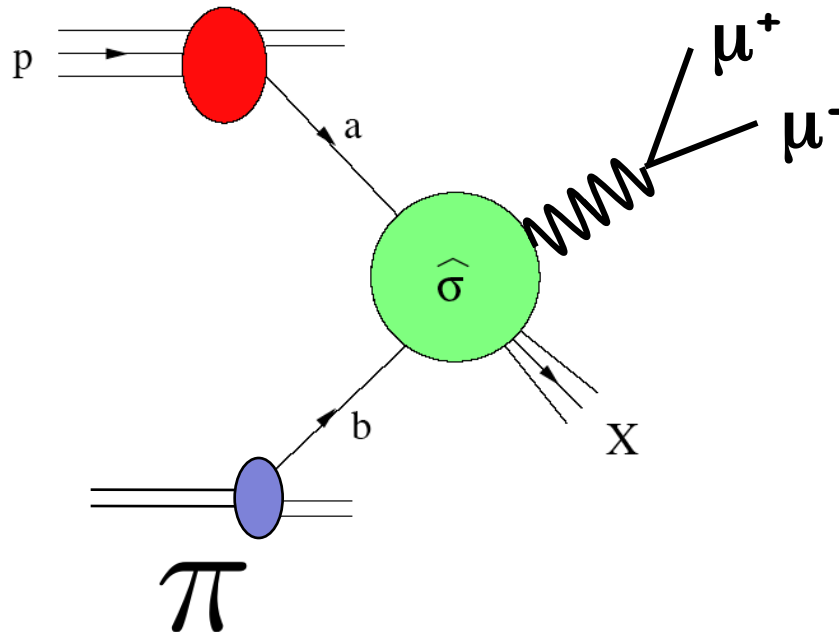
- they enhance cross sec. ! $\hat{\sigma}_{q\bar{q}} \propto \exp \left[+ \frac{2C_F}{\pi} \alpha_s \ln^2(N) \right] > 1$

Mukherjee, WV



- Drell-Yan process has been main source of information on pion structure:

E615, NA10

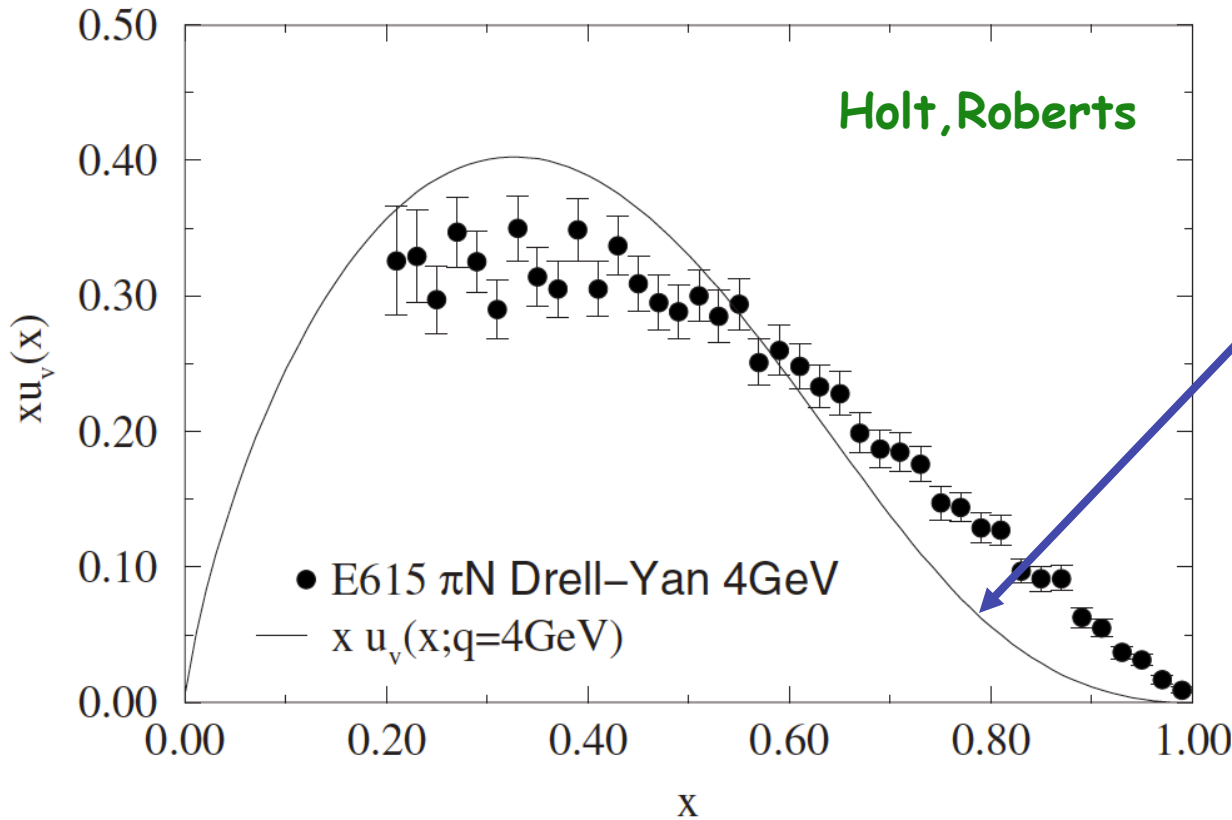


partonic hard scatt.
perturbative QCD

$$d\sigma = \sum_{ab} \int dx_a \int dx_b f_a^\pi(x_a, \mu) f_b(x_b, \mu) d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu)$$

- Kinematics such that data mostly probe valence region:
~200 GeV pion beam on fixed target

- LO extraction of u_v from E615 data:



Holt, Roberts

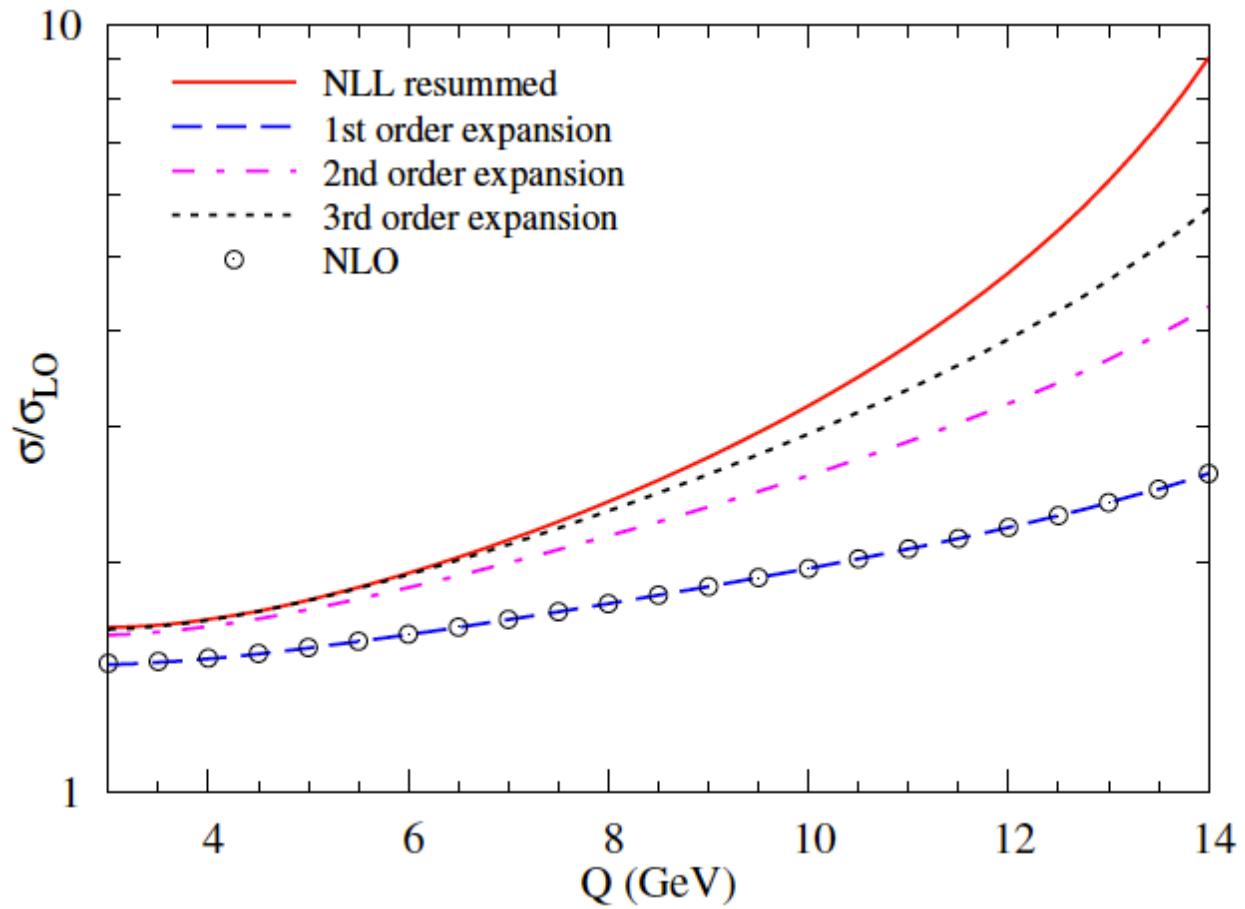
$$\sim (1 - x)^2$$

QCD counting rules

Farrar, Jackson;
Berger, Brodsky; Yuan

Dyson-Schwinger

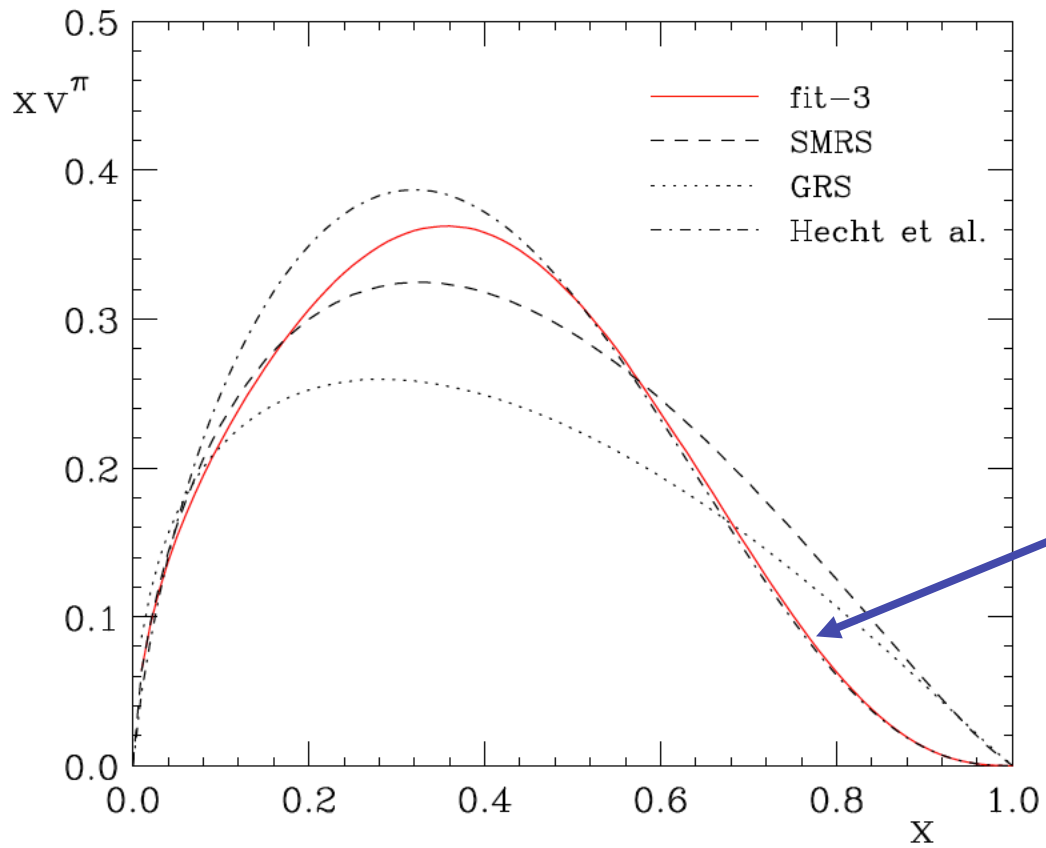
Hecht et al.



Aicher, Schäfer, WV

(Compass kinematics)

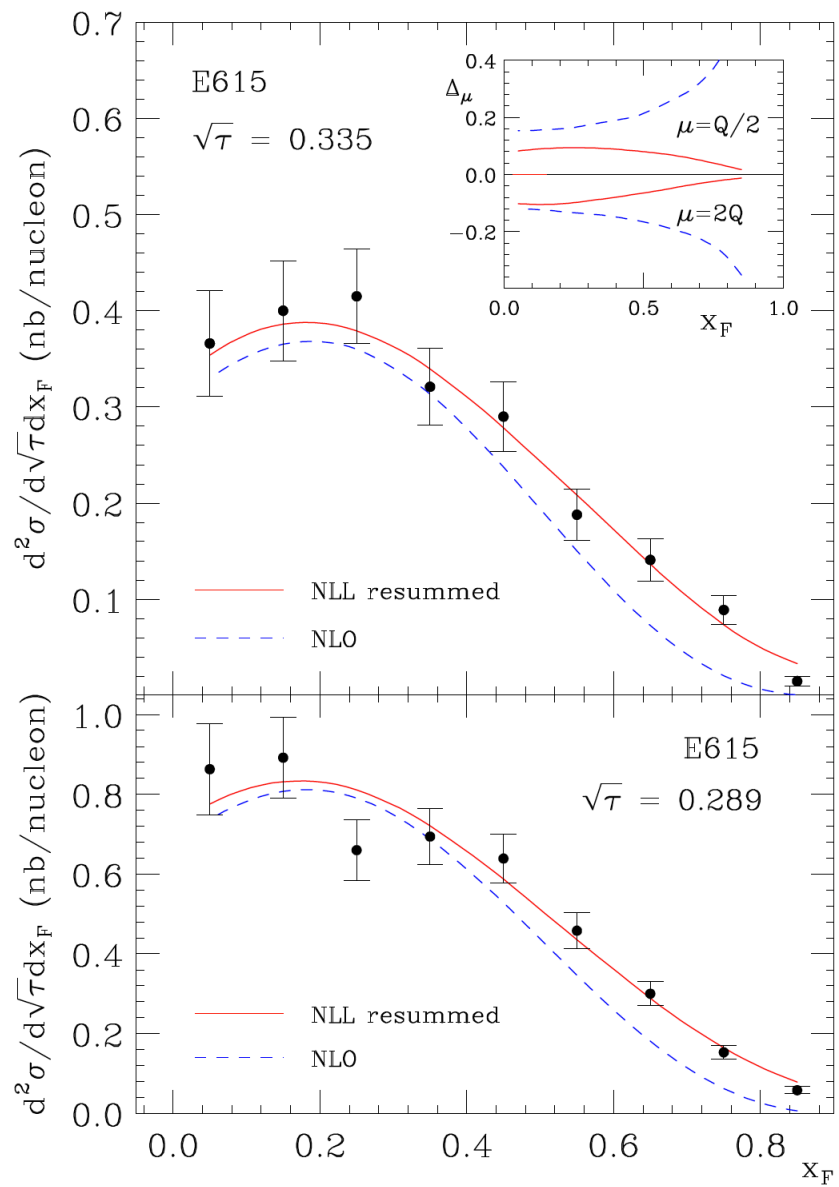
$$\sqrt{S} = 17 \text{ GeV}$$



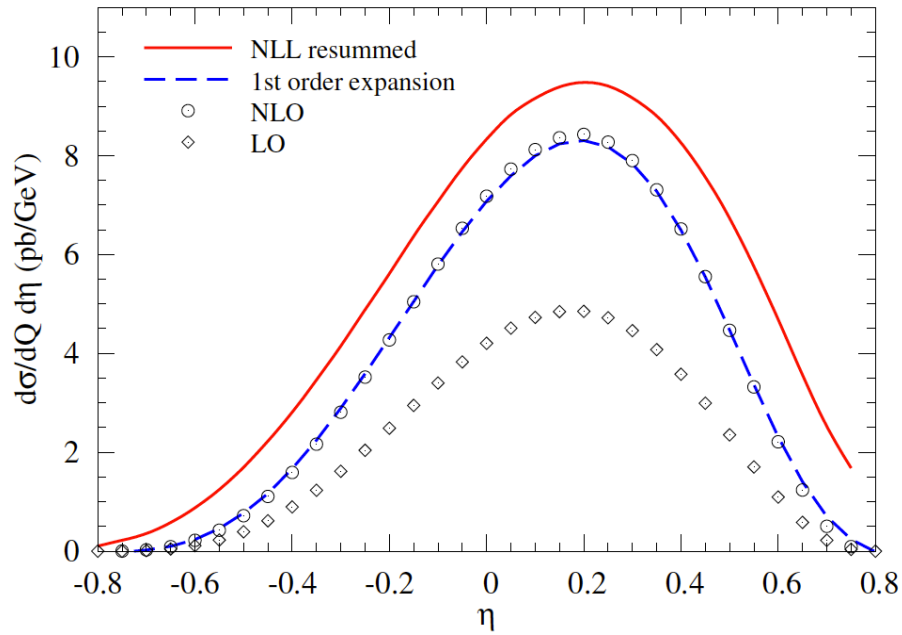
$Q = 4 \text{ GeV}$

$\sim (1-x)^{2.34}$

Aicher, Schäfer, WV

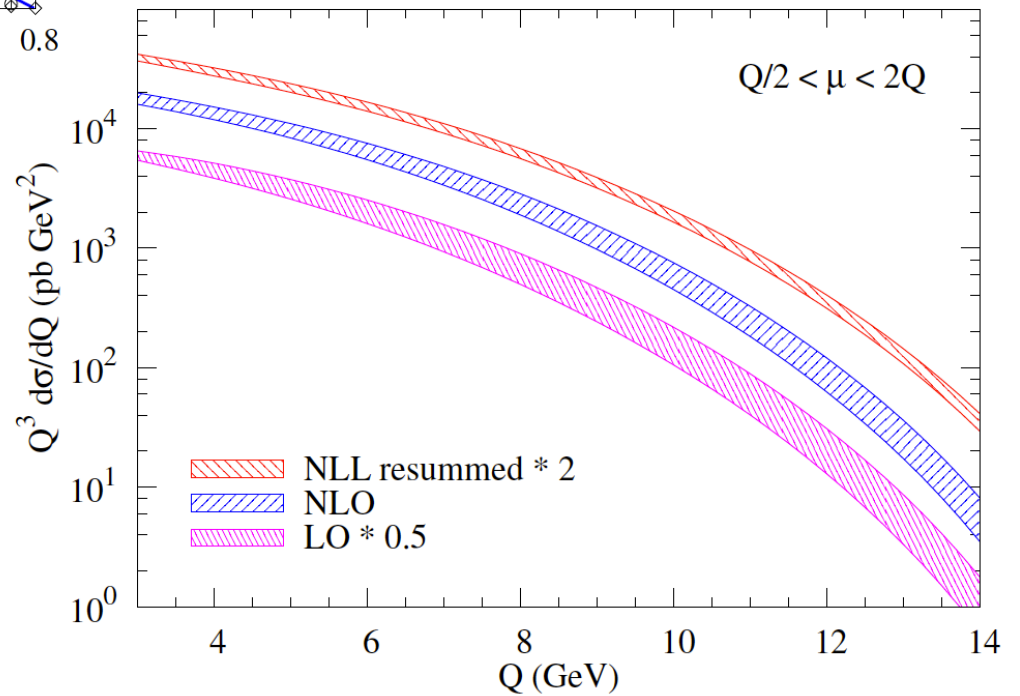


Equally relevant for COMPASS:

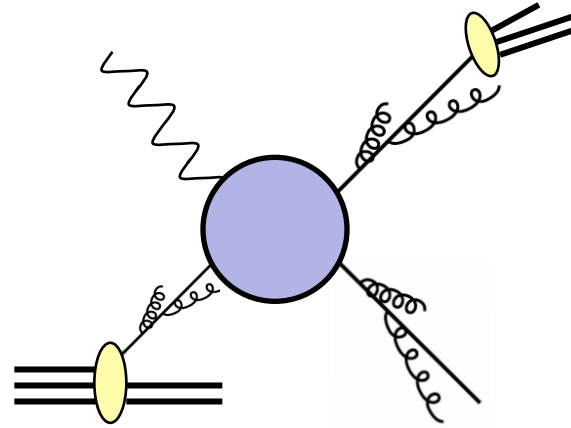


$$\frac{Q}{\sqrt{S}} = 0.45$$

Aicher, Schäfer, WV



Logarithms also present in
high- p_T processes:



$$p_T^3 \frac{d\hat{\sigma}_{ab}}{dp_T} = p_T^3 \frac{d\hat{\sigma}_{ab}^{\text{Born}}}{dp_T} \left[1 + \underbrace{\mathcal{A}_1 \alpha_s \ln^2(1 - \hat{x}_T^2) + \mathcal{B}_1 \alpha_s \ln(1 - \hat{x}_T^2)}_{\text{NLO}} + \dots + \mathcal{A}_k \alpha_s^k \ln^{2k}(1 - \hat{x}_T^2) + \dots \right] + \dots$$

$\hat{x}_T \equiv \frac{2p_T}{\sqrt{\hat{s}}}$

“threshold” logarithms

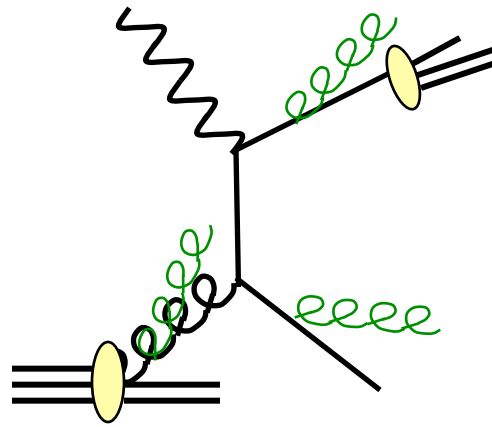
$\hat{x}_T \rightarrow 1$: only soft/collinear gluons allowed

All-order resummation:

Laenen, Oderda, Sterman; Catani et al.;
Kidonakis, Sterman; Bonciani et al.;
de Florian, WV;
Almeida, Sterman, WV

- soft-gluon effects exponentiate :

$$\gamma g \rightarrow qg$$

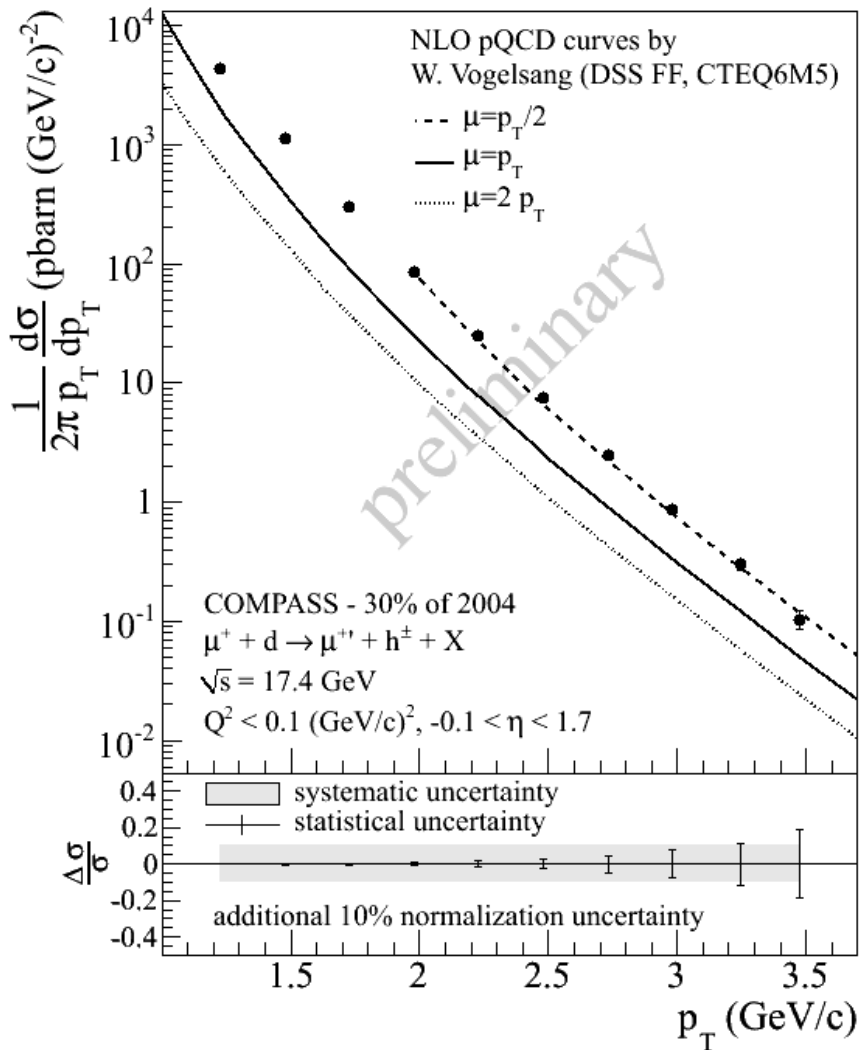


Leading logarithms:

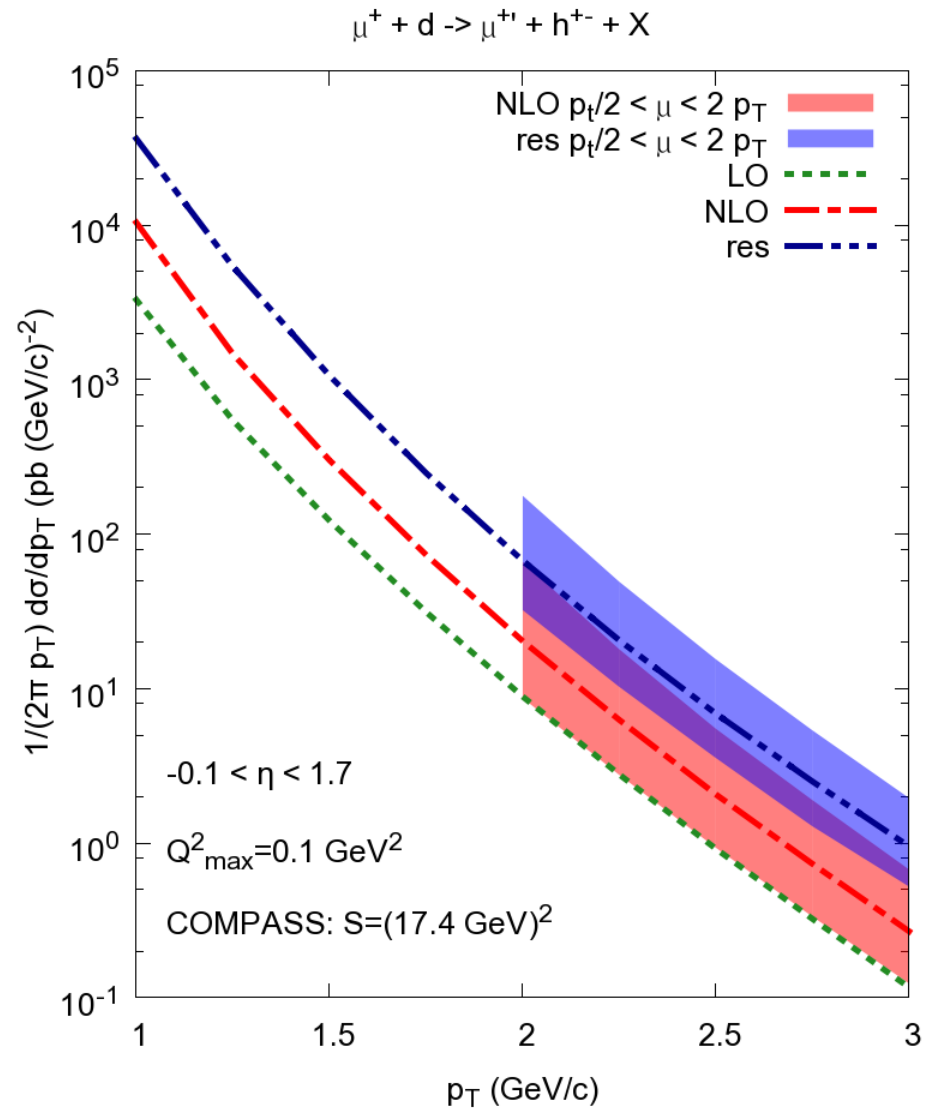
$$\sigma_{\text{res}}^{\gamma g} \sim \exp \left[\left(C_A + C_F - \frac{1}{2} C_F \right) \frac{\alpha_s}{\pi} \ln^2 N \right]$$

Mellin moment
in \hat{x}_T^2

(NLL far more complicated, but known)



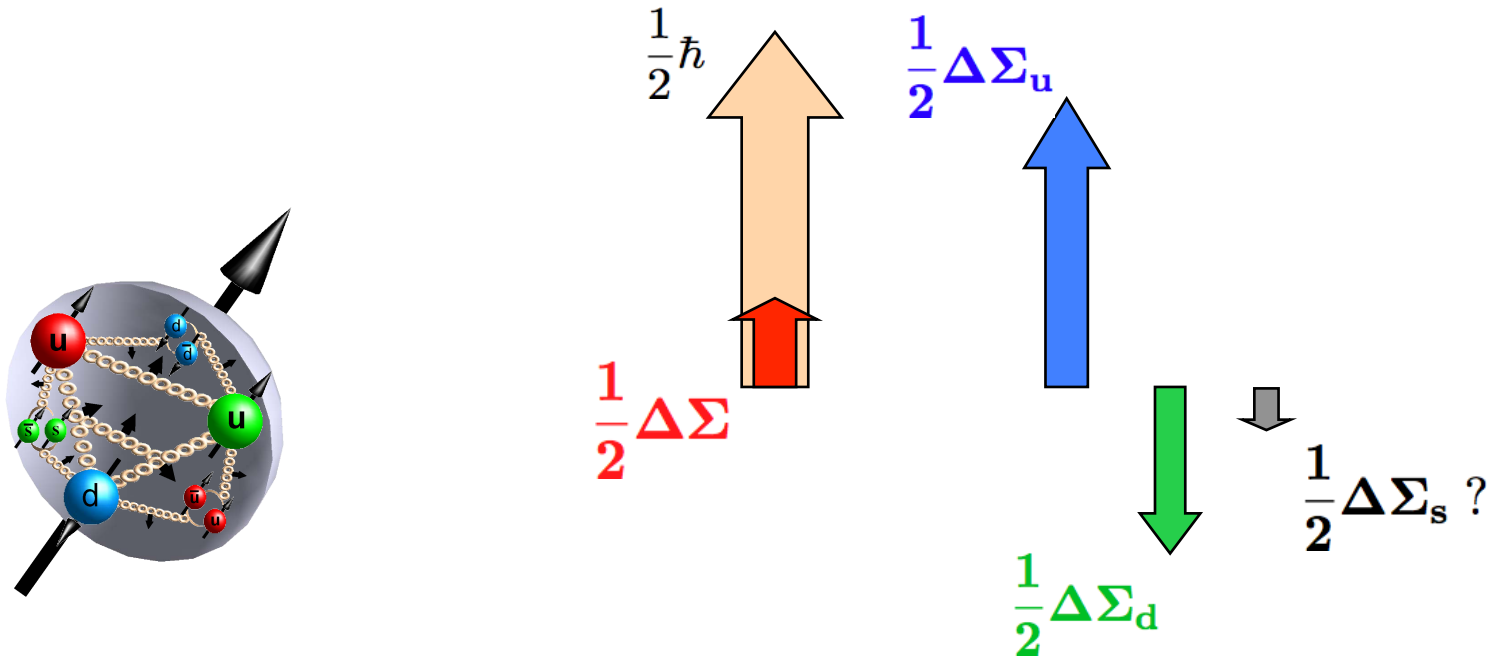
COMPASS



de Florian, Pfeuffer,
Schäfer, WV (prel.)

Conclusions:

- first global QCD analysis of DIS, SIDIS, RHIC data



- **RHIC (& HERMES, COMPASS)** closing in on Δg : small in accessible x-region. Small overall ?
- flavor asymmetry $\Delta \bar{u} - \Delta \bar{d} > 0$? Strangeness puzzle?
- many applications of QCD resummation