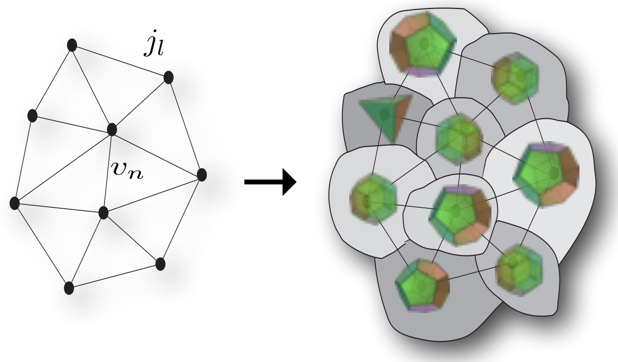
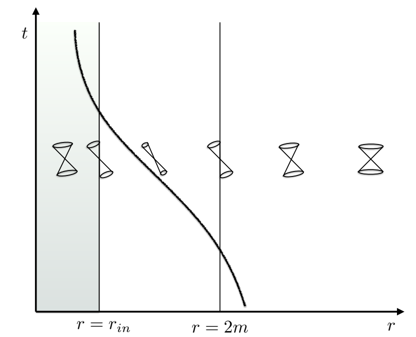
Loop Quantum Gravity and Planck Stars

Carlo Rovelli



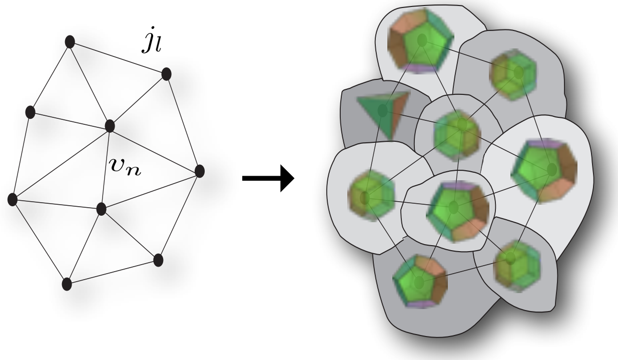


Saclay, march 2014

Loop Quantum Gravity and Planck Stars

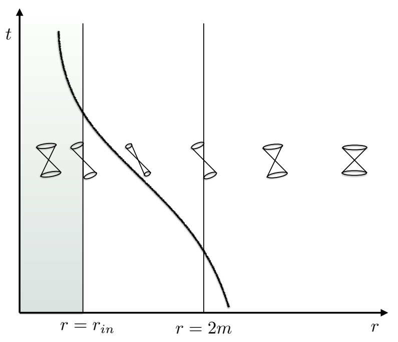
i. Loop quantum gravity: the theory

- Loop cosmology



ii. Planck Stars [F Vidotto, CR, 2014]

- Observations



no uncontrollable
infinities

existence,
not uniqueness

Is there a consistent quantum theory

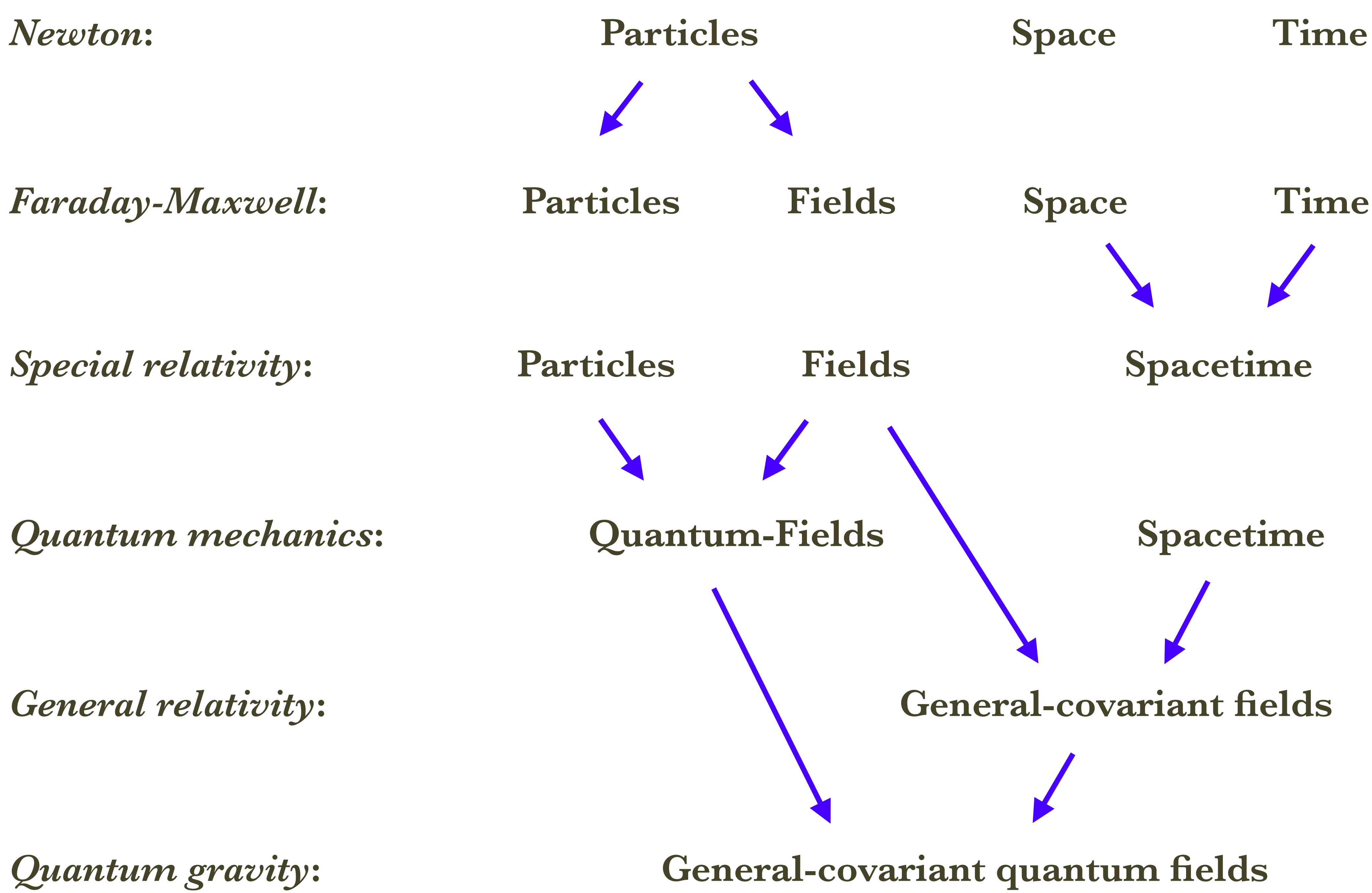
whose classical limit is general relativity,

in 4 lorentzian dimensions, with its standard matter couplings?

unification
not addressed

What are its physical consequences?

Can we observe some specific effects?

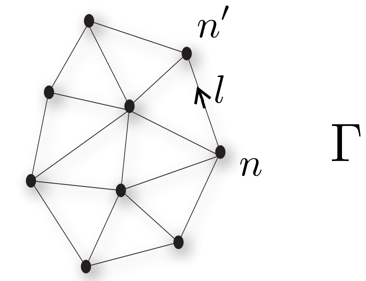

Newton:
Particles
Space
Time
Faraday-Maxwell:
Particles
Fields
Space
Time
Special relativity:
Particles
Fields
Spacetime
Quantum mechanics:
Quantum-Fields
Spacetime
General relativity:
General-covariant fields
Quantum gravity:
General-covariant quantum fields

Loop Quantum Gravity

Kinematics

State space $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]$

Operators: $\vec{L}_l = \{L_l^i\}, i = 1,2,3$ where $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$

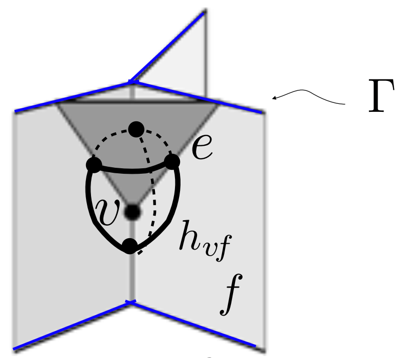


Graph
(modes, links)

Dynamics

Transition amplitudes $W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$

Vertex amplitude $A(h_f) = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_e \prod_f (2j_f+1)\ Tr_j[h_f Y_\gamma^\dagger g_e g_{e'}^{-1} Y_\gamma]$



2-complex $\mathcal{C}$
(vertices, edges, faces)

Simplicity map $Y_\gamma : \mathcal{H}_j \to \mathcal{H}_{j,\gamma j}$

$|j;m\rangle \mapsto |j,\gamma(j+1);j,m\rangle$

$$h_f = \prod_v h_{vf}$$

With a cosmological constant $\Lambda > 0$:

Amplitude: SL(2,C) ➞ SL(2,C)_q network evaluation.

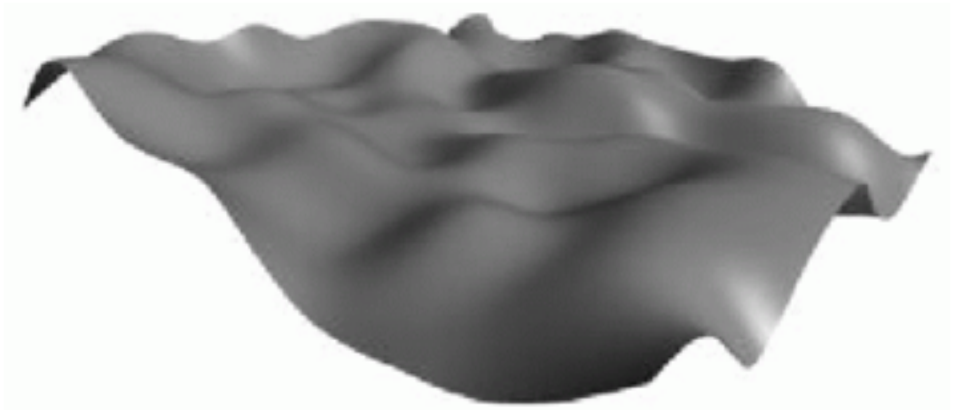
Main results

1. The amplitudes with positive cosmological constant are UV and IR finite: $W_{\mathcal{C}}^{q} < \infty$
   (Han, Fairbairn-Meusburger, 2011).

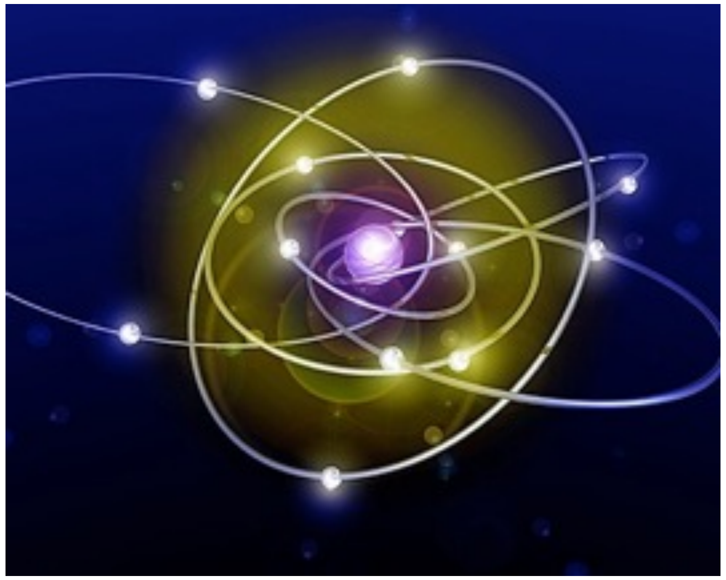
2. The classical limit of the vertex amplitude converges (appropriately) to the Regge Hamilton function (with cosmological constant).
   (Barrett *et al*, Conrady-Freidel, Bianchi-Perini-Magliaro, Engle, Han..., 2009-2012).

3. The boundary states represent classical 3d geometries.
   (Canonical LQG 1990', Penrose spin-geometry theorem 1971).

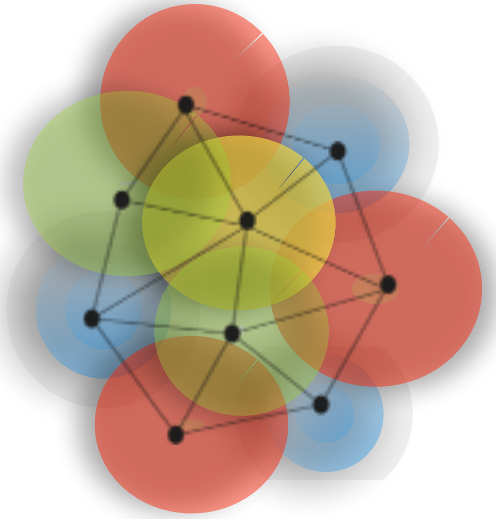
4. Boundary geometry operators have discrete spectra.
   (Canonical LQG main results, 1990').



General relativity :
Spacetime is a field (like electric field)



Quantum theory :
All fields are formed by “quanta” (like photons).

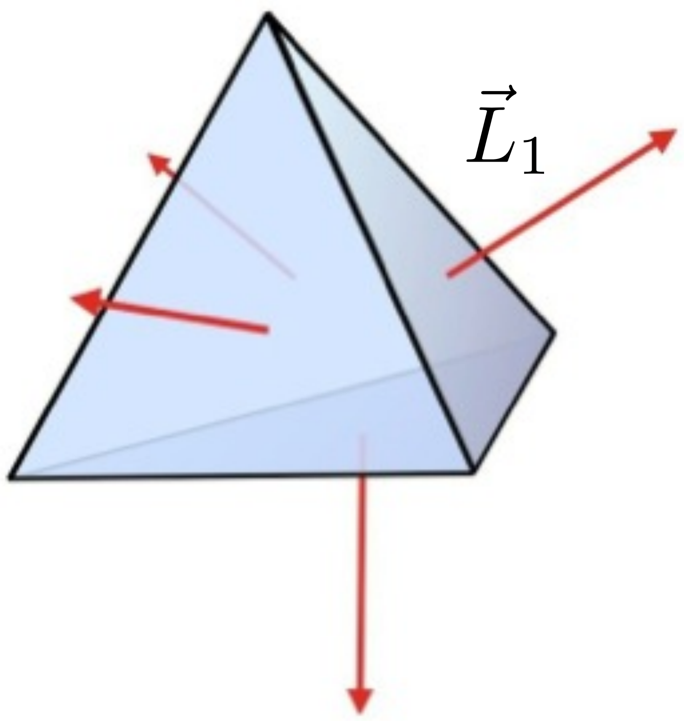


Quantum gravity :
Space is formed by “quanta”:
it is formed by elementary “bricks”

Discreteness of Geometry.

Fundamental length (like strings).

Geometry + quantum theory



Geometry: - 6 lengths
or, 4 normals (up to rotations)

$$\vec{L}_a, \qquad a = 1...4 \qquad\qquad i, j = 1, 2, 3$$

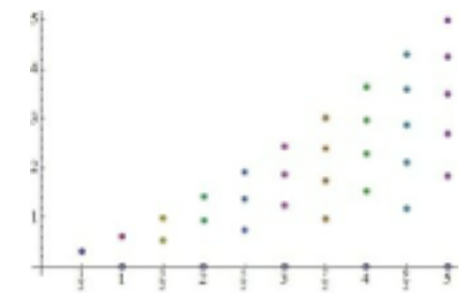
Closure constraint: $\sum_a \vec{L}_a = 0$

Volume:
$$V^2 = \frac{9}{2}\, \vec{L}_1 \cdot (\vec{L}_2 \times \vec{L}_3)$$
$$A_a = |\vec{L}_a|$$

Quantum geometry:

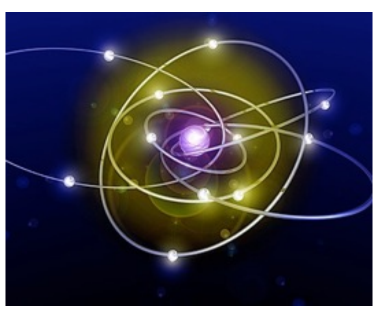
$$[L_a^i, L_b^j] = l_P^2\ \delta_{ab}\ \epsilon^{ij}{}_k\ L_a^k$$

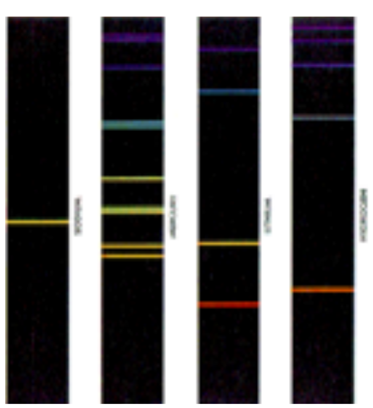
Complete set of commuting operators A_a, V

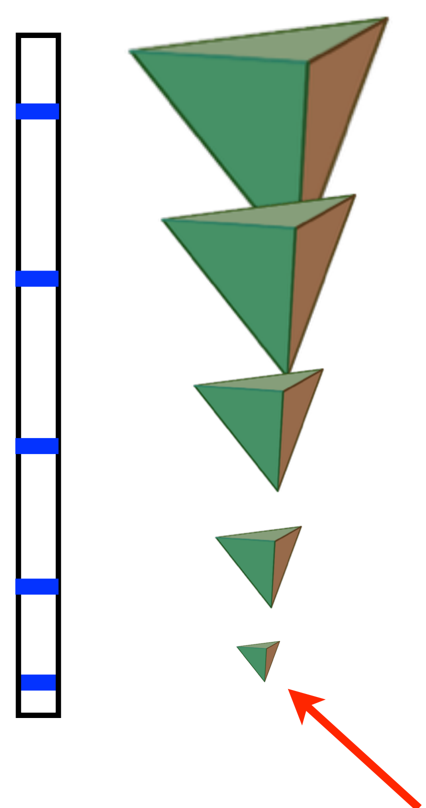


Spectrum is discrete $A = 8\pi\gamma\hbar G\,\sqrt{j_l(j_l+1)}$

Basis that diagonalises A_a, V: $|j_a, v\rangle$, $j_a \in \frac{\mathbb{N}}{2}$

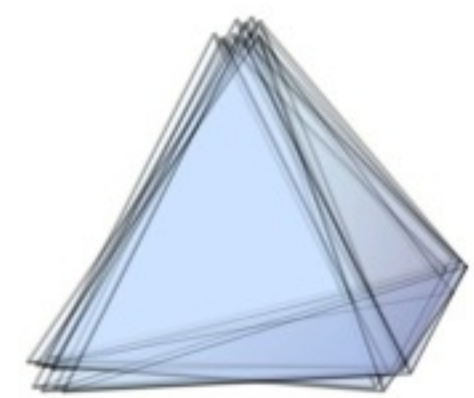


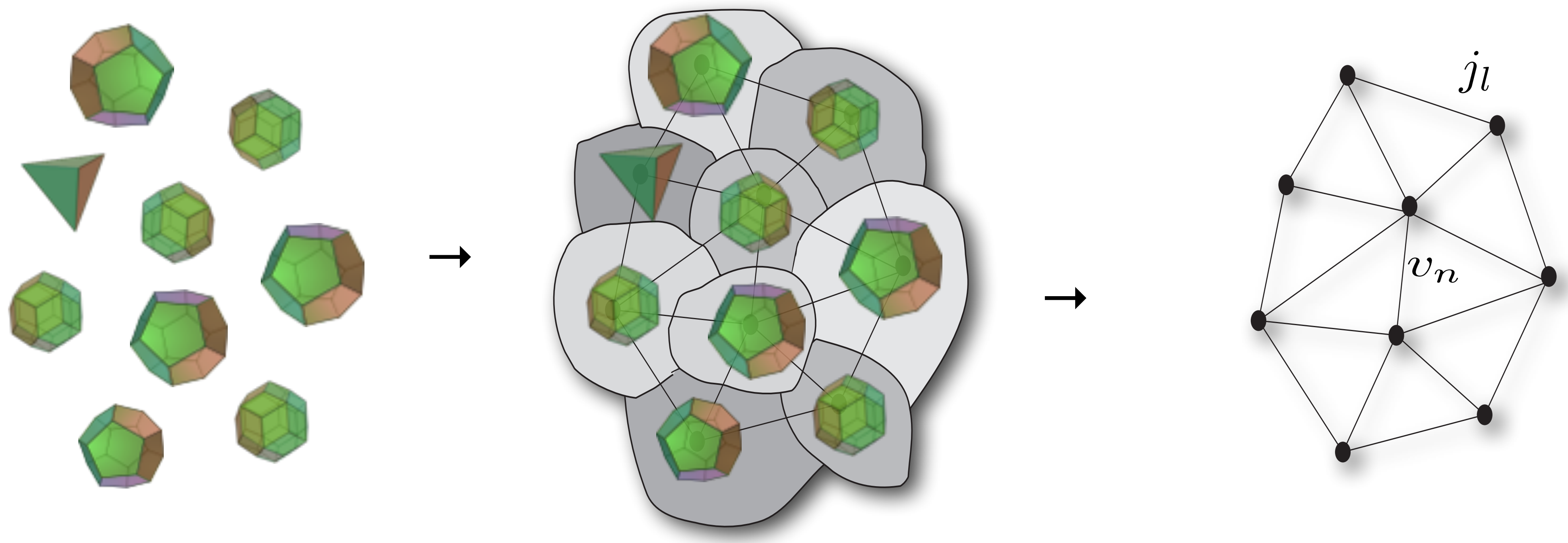




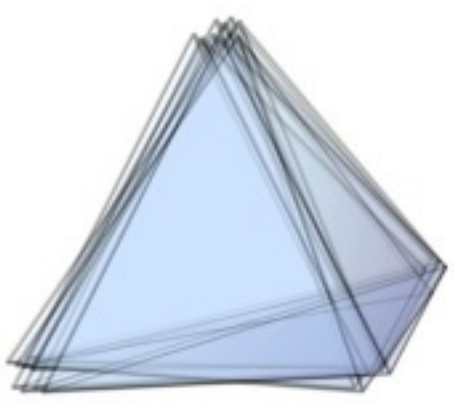
Minimal volume

A_a, V are only five quantities: geometry is never sharp





Spin network states $|j_l, v_n\rangle$

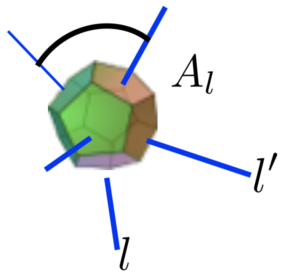


Basic states of Loop Quantum Gravity

Boundary geometry: spinfoams

State space $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]$

Operator: $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ triad (metric)

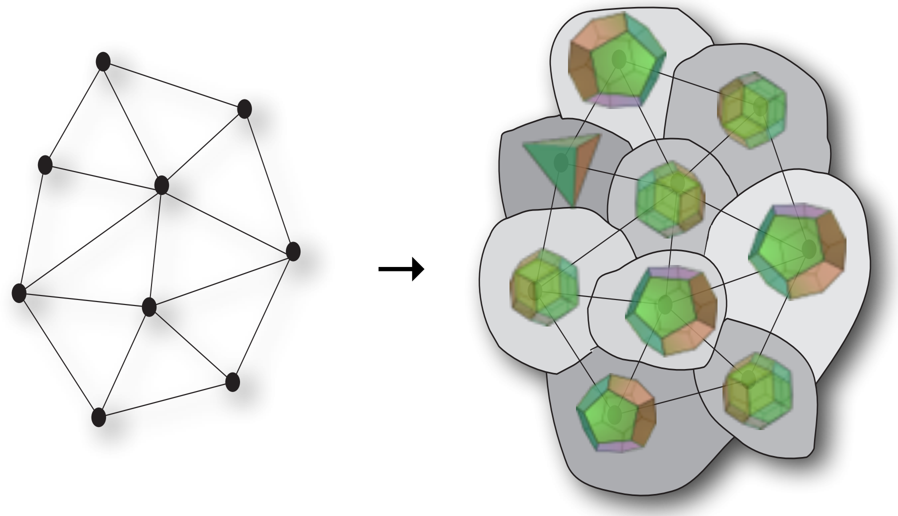


Polyhedron

- Area and volume form a complete set of commuting observables and have discrete spectra

  Nodes: discrete quanta of volume ("quanta of space")

  Links: discrete quanta of area.



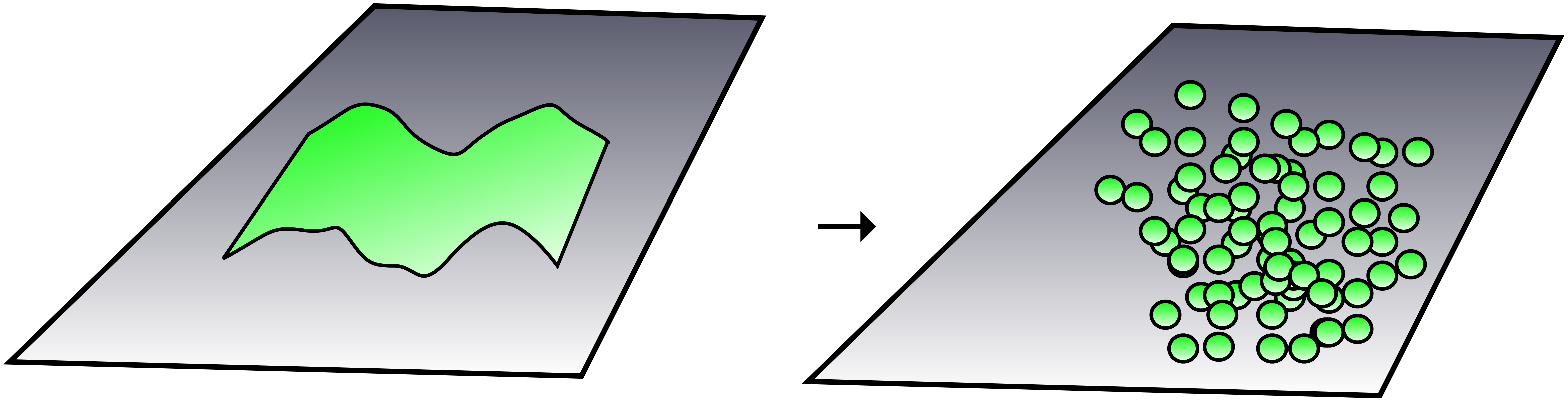
Geometry is quantized:

(i) eigenvalues are discrete
(ii) operators do not commute

coherent states theory $\rightarrow$ semiclassical spacetime

- **States describe quantum geometries:**
  not quantum states <u>in</u> spacetime but rather quantum states <u>of</u> spacetime

Quantum field theory



The quanta of a field are particles (Dirac)
(Discreteness of the spectrum of the energy of each mode)

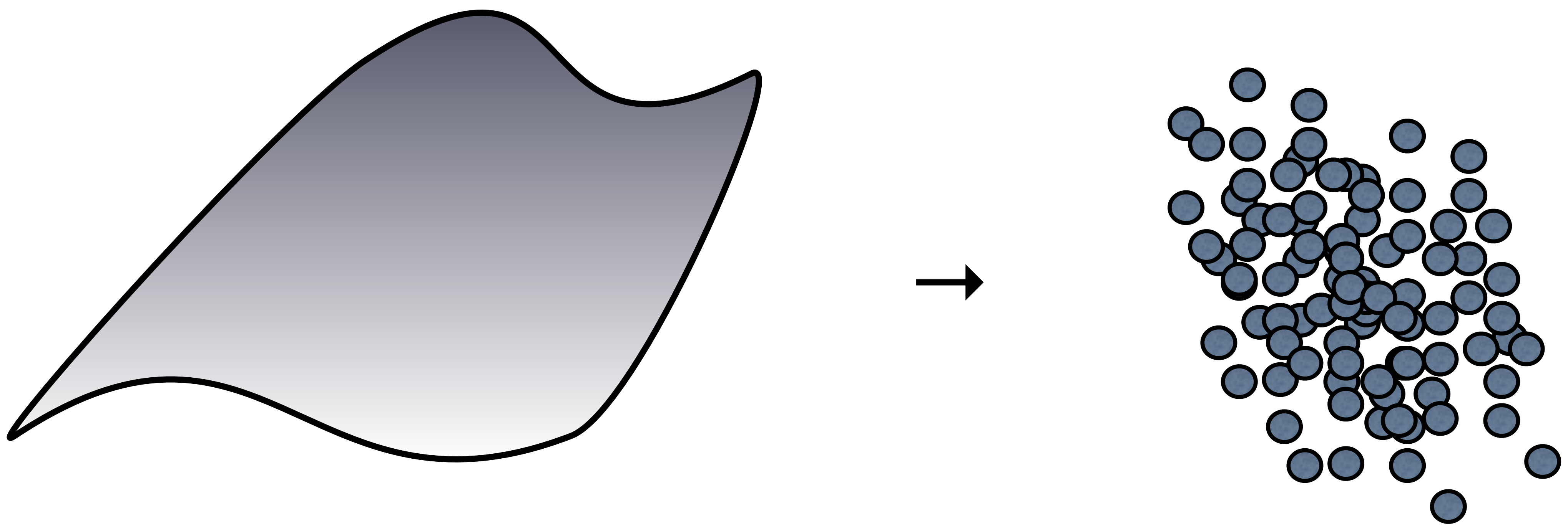
$$(\mathcal{F}, \mathcal{A}, W)$$

$$\mathcal{F} \ni |p_1 ... p_n\rangle$$

$$\mathcal{A} \ni a(k), a^\dagger(k)$$

$$W \to Feynman\ rules$$

Quantum gravity



Quantum granularity of spacetime (1995)
(Discreteness of the spectrum of geometrical operators such as volume)

$$(\mathcal{F}, \mathcal{A}, W)$$

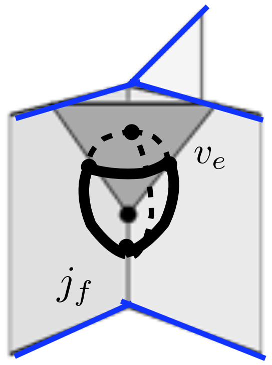
$$\mathcal{F} \ni |\Gamma, j_l, v_n\rangle$$

$$\mathcal{A} \ni \vec{L}_l$$

$$W \to Transition\ amplitudes$$

Large distance limit

A "spinfoam": a two-complex colored with spins on faces and intertwiners on edges.



Theorem : For a 5-valent vertex [Barrett, Pereira, Hellmann, Gomes, Dowdall, Fairbairn 2010]

$$A(j_f, v_e) \underset{j\gg1}{\sim} e^{iS_{\rm Regge}} + e^{-iS_{\rm Regge}}$$

[Freidel Conrady 2008, Bianchi, Satz 2006, Magliaro Perini, 2011]

$$W_{\mathcal{C}} \xrightarrow[j\gg1]{} e^{iS_\Delta} \qquad Z_{\mathcal{C}} \xrightarrow[C\to\infty]{} \int Dg \;\; e^{iS[g]}$$

Theorem : For a 5-valent vertex [Han 2012]

$$A^q(j_f, v_e) \underset{j\gg1,\, q\sim1}{\sim} e^{iS^\Lambda_{\rm Regge}} + e^{-iS^\Lambda_{\rm Regge}} \qquad q = e^{\Lambda\hbar G}$$

$$
\begin{tikzcd}
\text{Loop-gravity transition amplitudes} \arrow[r, "j\to\infty"] \arrow[d, "\mathcal{C}\to\infty"'] & \text{Regge theory} \arrow[d, "\Delta\to\infty"'] \\
\text{Exact transition amplitudes} \arrow[r, "j\to\infty"] & \text{General relativity}
\end{tikzcd}
$$

$$
\downarrow \ \text{Continuum limit} \quad L \ll L_{\text{curvature}}
$$

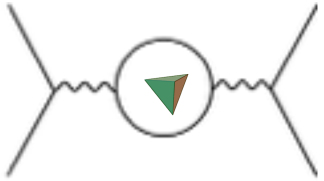
$$
\xrightarrow[L \gg L_{\text{Planck}}]{\text{Classical limit}}
$$

- Physical QFT's are constructed via a truncation of the *d.o.f.* (cfr: QED: particles, QCD Lattice).
- Physical calculation are performed within a truncation.

Results: I. Theory

1. The amplitudes (with positive cosmological constant) are UV and IR finite:
(Han, Fairbairn-Moesburger, 2011).

$$W_{\mathcal{C}}^{q} < \infty$$



3. The classical limit of the vertex amplitude converges to the Regge Hamilton function.
(Barrett *et al*, Conrady-Freidel, Bianchi-Perini-Magliaro, Engle, Han..., 2009-2013).

$$A_v \underset{j\gg 1,\, q\sim 1}{\sim} e^{iS^{\Lambda}_{\rm Regge}} + e^{-iS^{\Lambda}_{\rm Regge}}, \qquad q = e^{\Lambda\hbar G}$$

Results: II. Applications

1. Scattering
   - n-point functions (CR, Alesci, Bianchi, Perini, Magliaro, Ding, Zhang, Han)

2. Black hole thermodynamics
   - microscopic derivation of Bekenstein-Hawking entropy.

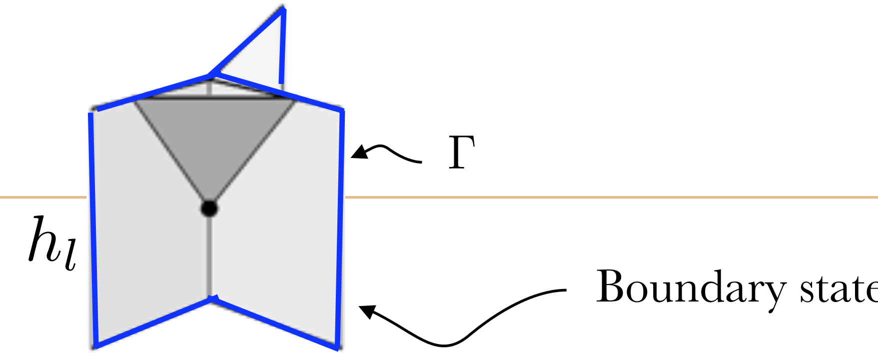
     (Krasnov, CR, Ashtekar, et al, Bianchi 2012).

$$S = k\frac{Ac^3}{4\hbar G}$$

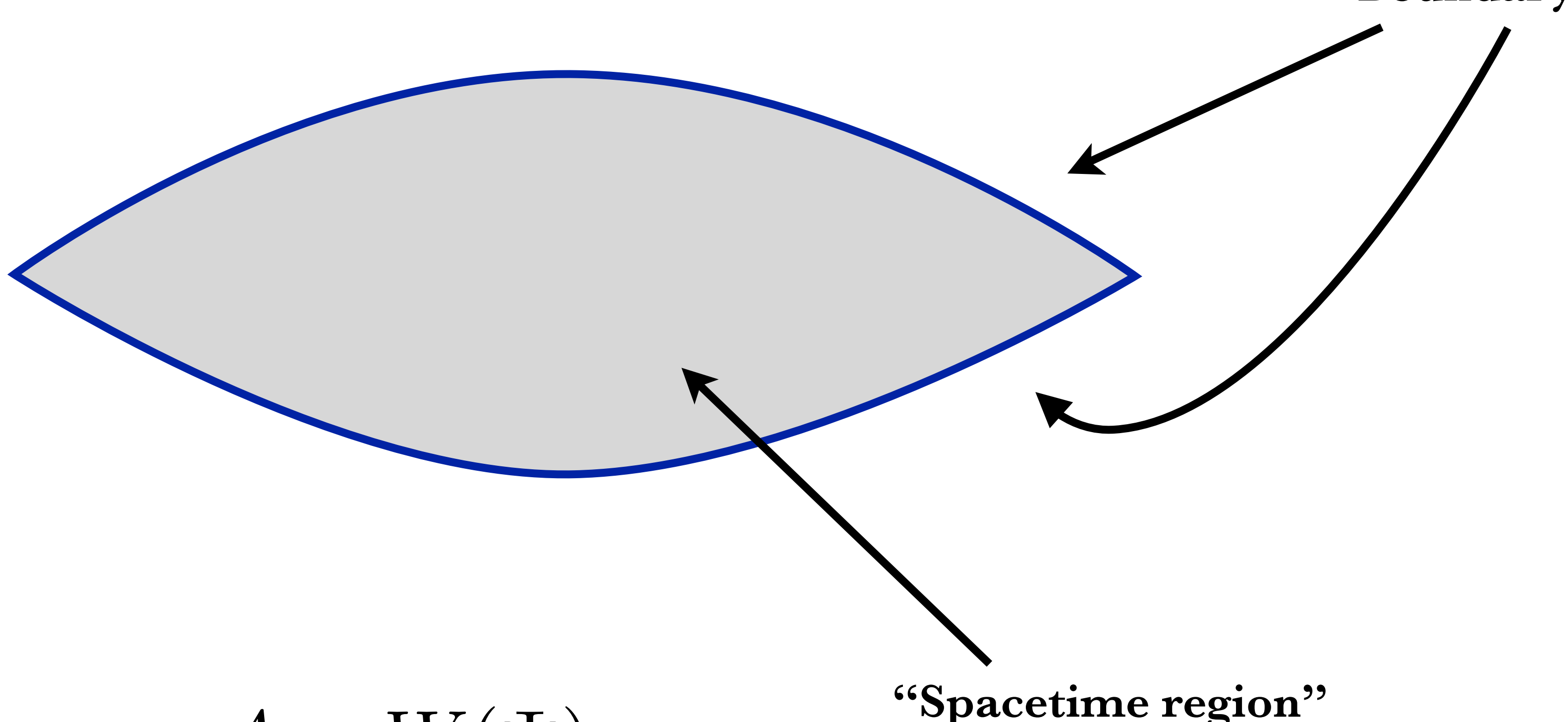
3. Quantum cosmology
   - Singularity resolution (Bojowald, Ashtekar, Lewandowski, Singh)
   - Bounce?
   - Spinfoam cosmology (Vidotto)

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\,\rho\,\left(1 - \frac{\rho}{\rho_{Pl}}\right)$$

A process and its amplitude



Boundary state $\Psi = \psi_{in} \otimes \psi_{out}$

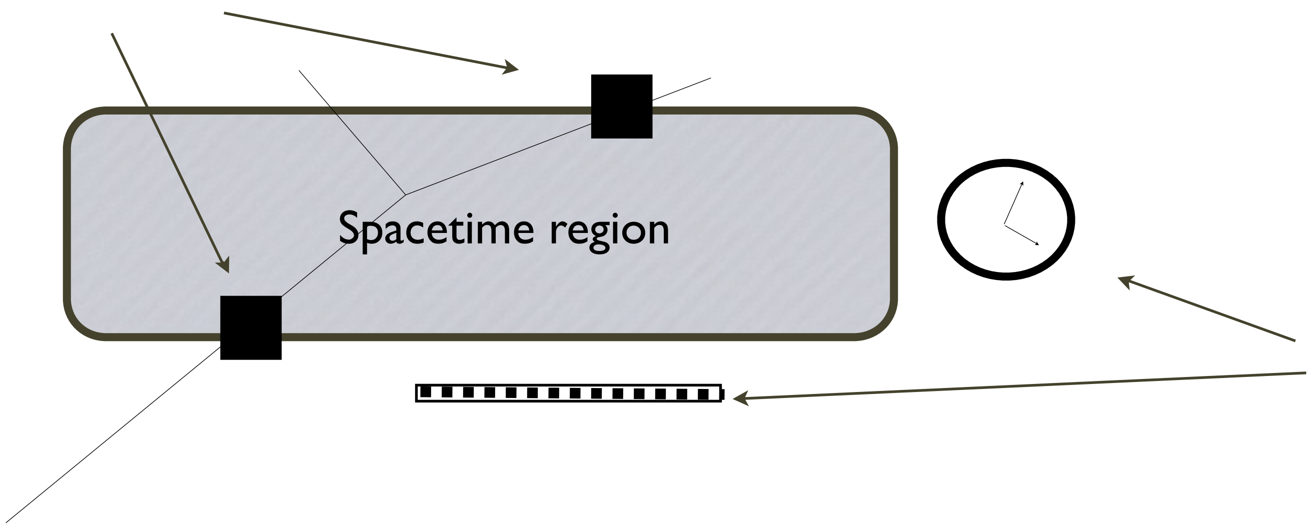


Amplitude of the process $A = W(\Psi)$

Loop Quantum Gravity gives a mathematical definition of the state of space, the boundary observables, and the amplitude of the process.

Boundary values of the gravitational field = geometry of box surface
= distance and time separation of measurements

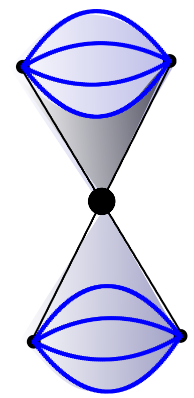
Particle detectors = field measurements



Distance and time measurements
= gravitational field measurments

In GR, distance and time measurements
are field measurements like the other ones:
they are part of the **boundary data** of the problem.

Loop quantum cosmology (Covariant)



$$W(z_i, z_f) = \int_{SO(4)^4} dG_1^i\, dG_2^i\, dG_1^f\, dG_2^f \prod_{l^i} P_t(H_l(z_i)\,,\, G_1^i G_2^{i\,-1}) \prod_{l^f} P_t(H_l(z_f)\,,\, G_1^f G_2^{f\,-1})$$

$$P_t(H,G) = \sum_j (2j+1) e^{-2t\hbar j(j+1)} tr\left[D^{(j)}(H) Y^\dagger D^{(j^+,j^-)}(G) Y\right].$$

$$e^{\frac{i}{\hbar}\frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a'^3 - a^3)} = e^{\frac{i}{\hbar}S(a,a')}$$

The expanding Friedmann dynamics is recovered

[Bianchi, Vidotto, Krajewski CR 2010]

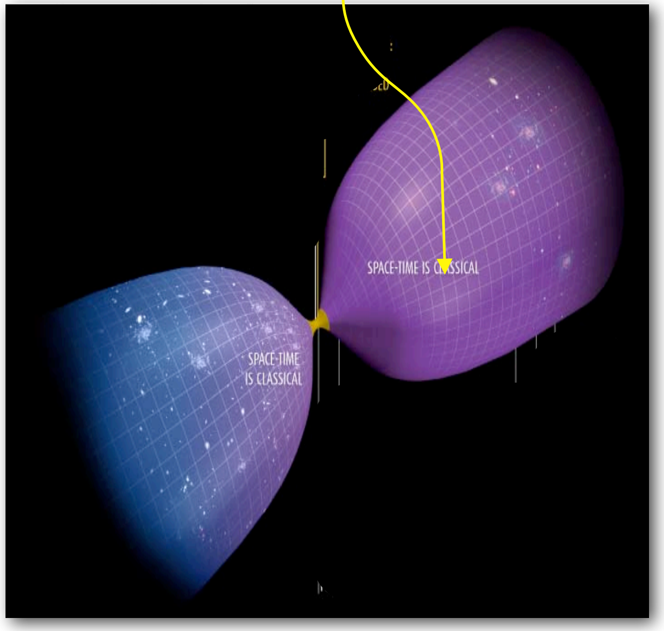
Loop quantum cosmology (Canonical)

Result:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\,\rho\,\left(1 - \frac{\rho}{\rho_{Pl}}\right)$$

$$v" - \left(1 - 2\frac{\rho}{\rho_{Pl}}\right)\nabla^2 v - \frac{z"}{z} v = 0$$

Bounce



Generic prediction of a bounce, followed by an inflationary phase

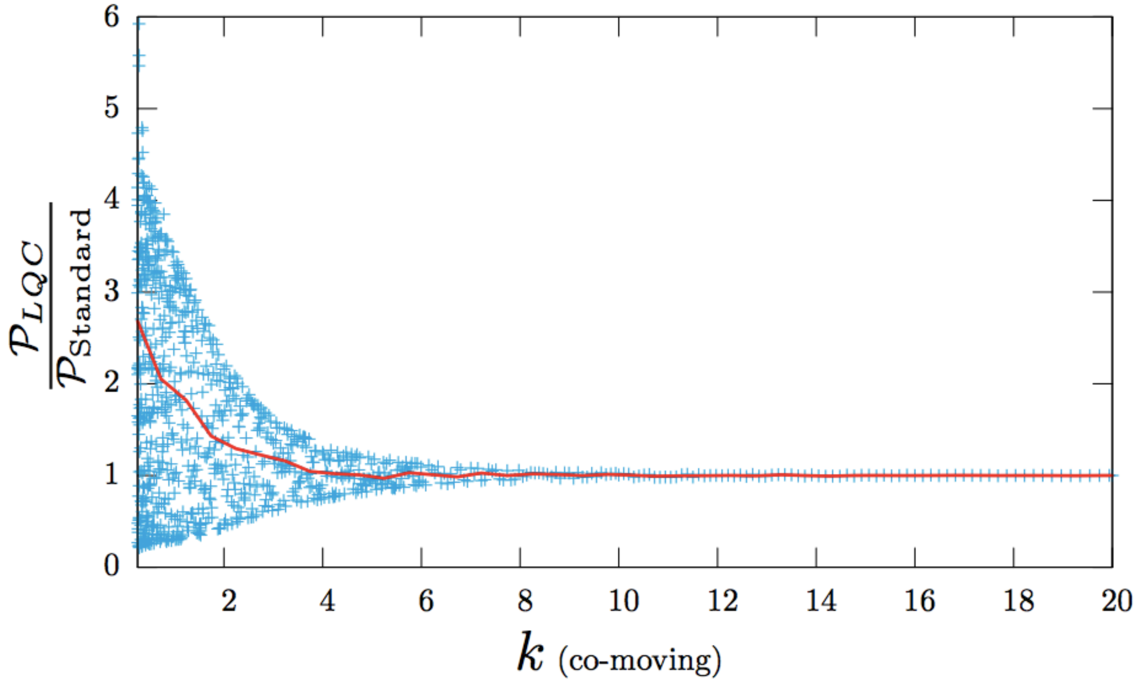
[Bojowald, Ashtekar...,]

[Barrau, Cailleteau, Grain, Vidotto, 2011]

$$\rho_{Pl} = \left(\frac{8\pi}{3}\gamma\hbar G^2\right)^{-1}$$

Corrections to the CMB

[Agullo, Ashtekar, Nelson, 2012]



Main physical lesson from loop quantum cosmology:

When the matter reaches the Planck density,
a ***strong repulsive force*** of quantum gravitation origin develops.

Can we use this to understand what happens in the interior of a black hole?

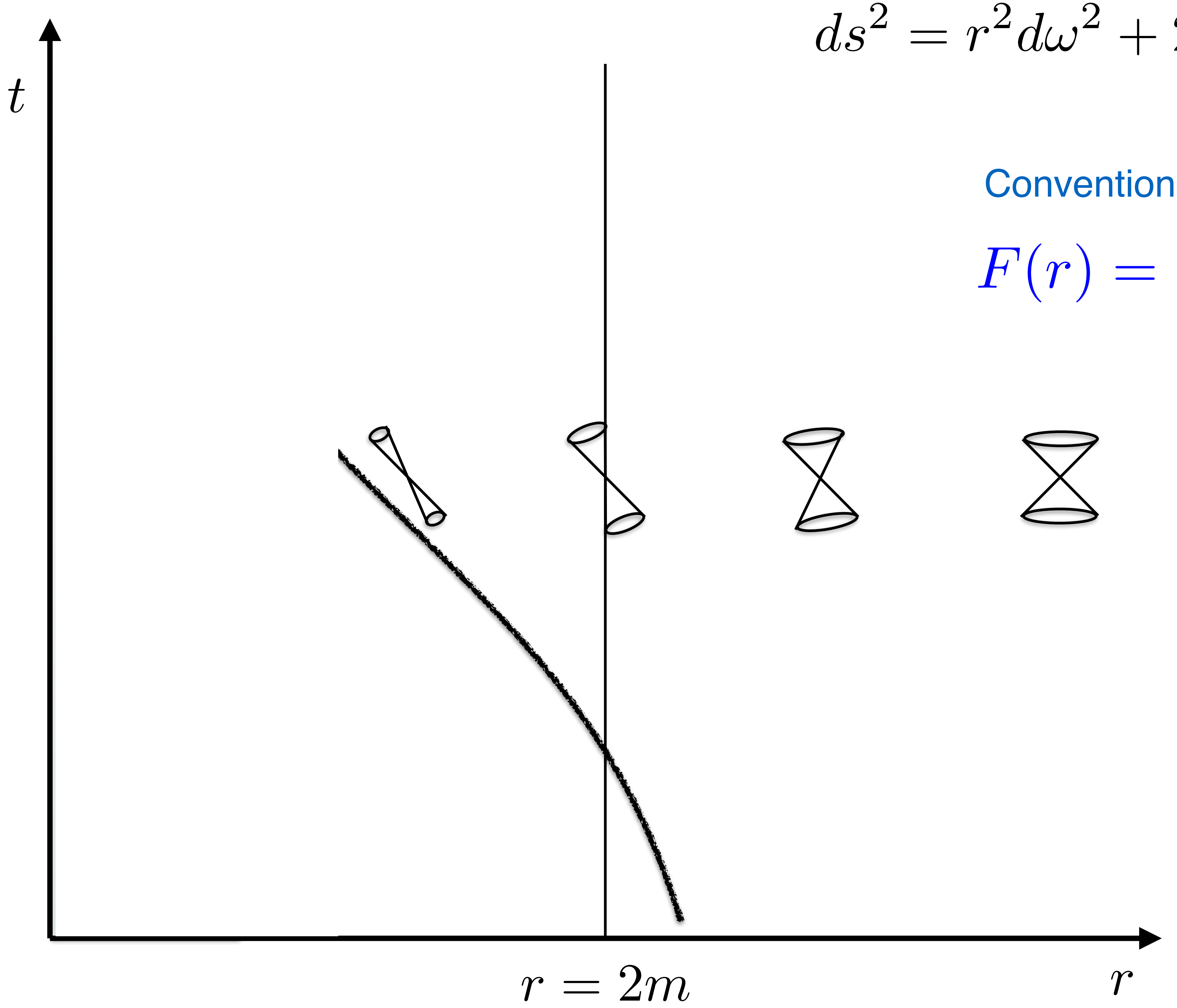
→ ***Planck Stars***

Black hole in Eddington-Finkelstein coordinates

$$ds^2 = r^2 d\omega^2 + 2dv\,dr - F(r,t)du^2.$$

Conventional static black hole

$$F(r) = 1 - 2m/r$$

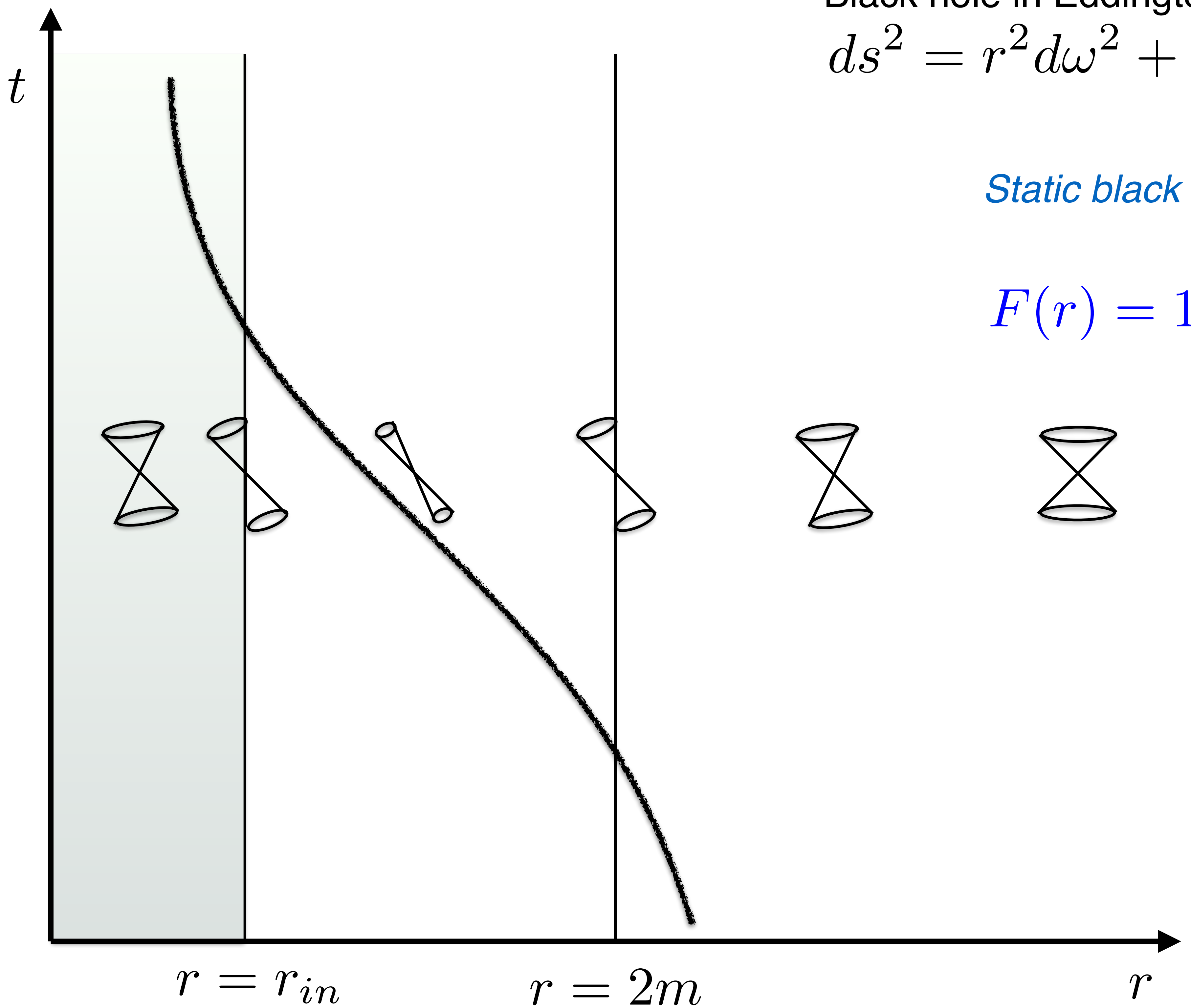


Black hole in Eddington-Finkelstein coordinates

$$ds^2 = r^2 d\omega^2 + 2dv\,dr - F(r,t)du^2.$$

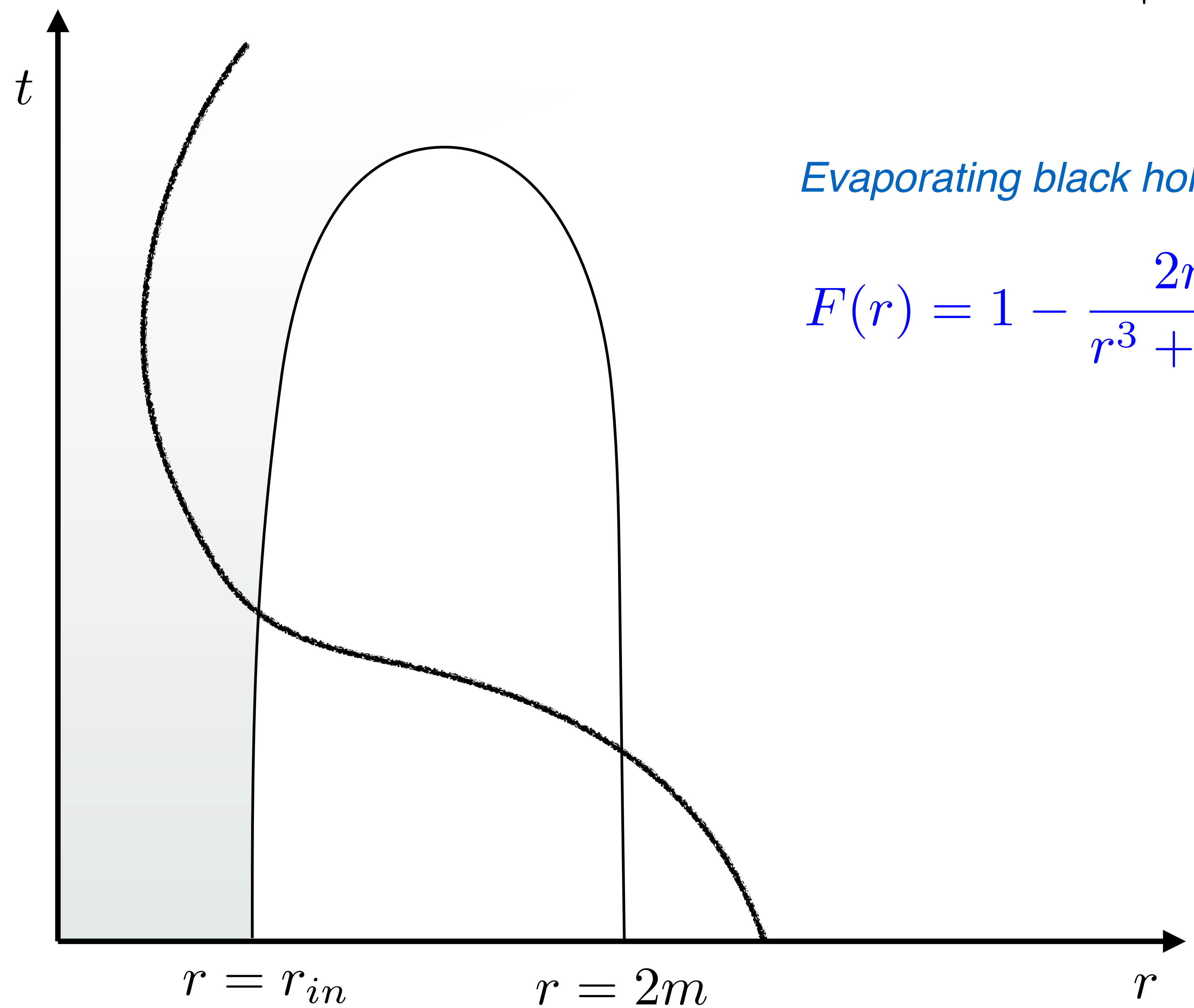
Static black hole with Planck star *[Haywards]*

$$F(r) = 1 - \frac{2mr^2}{r^3 + 2\alpha^2 m}$$



Black hole in Eddington-Finkelstein coordinates

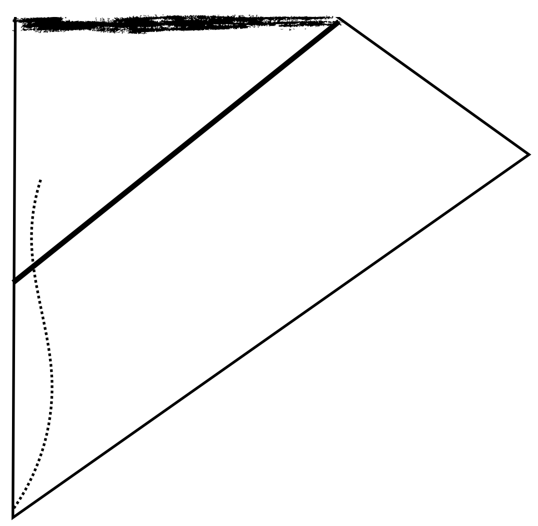
$$ds^2 = r^2 d\omega^2 + 2dv\, dr - F(r,t) du^2.$$



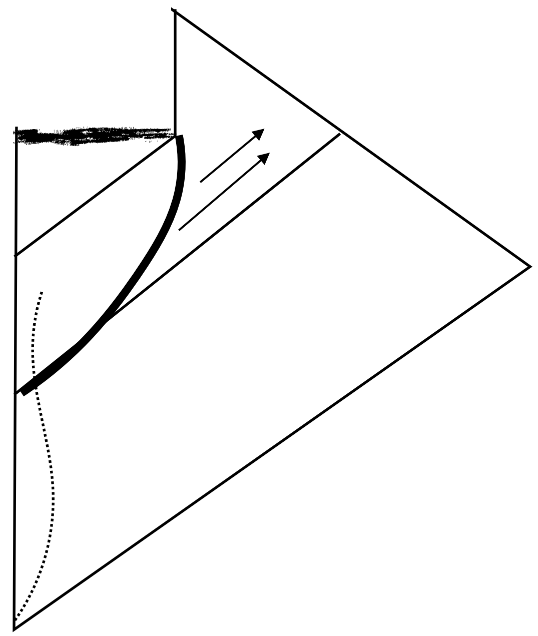
Evaporating black hole and bouncing Planck star

$$F(r) = 1 - \frac{2m(t)r^2}{r^3 + 2\alpha(t)^2 m}$$

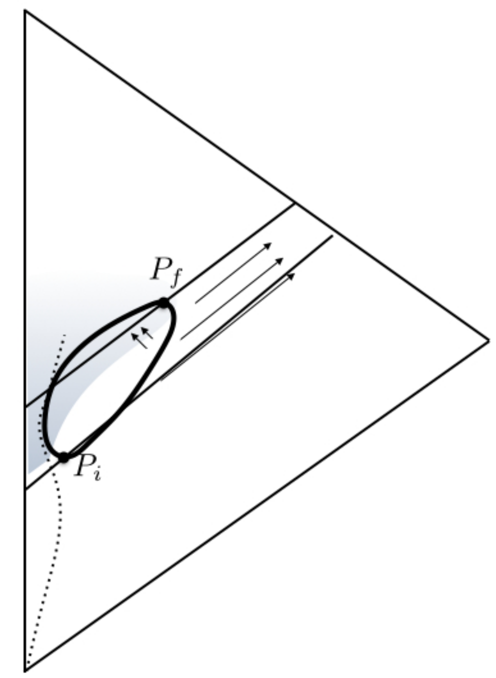
Collapsing star (Penrose diagram)



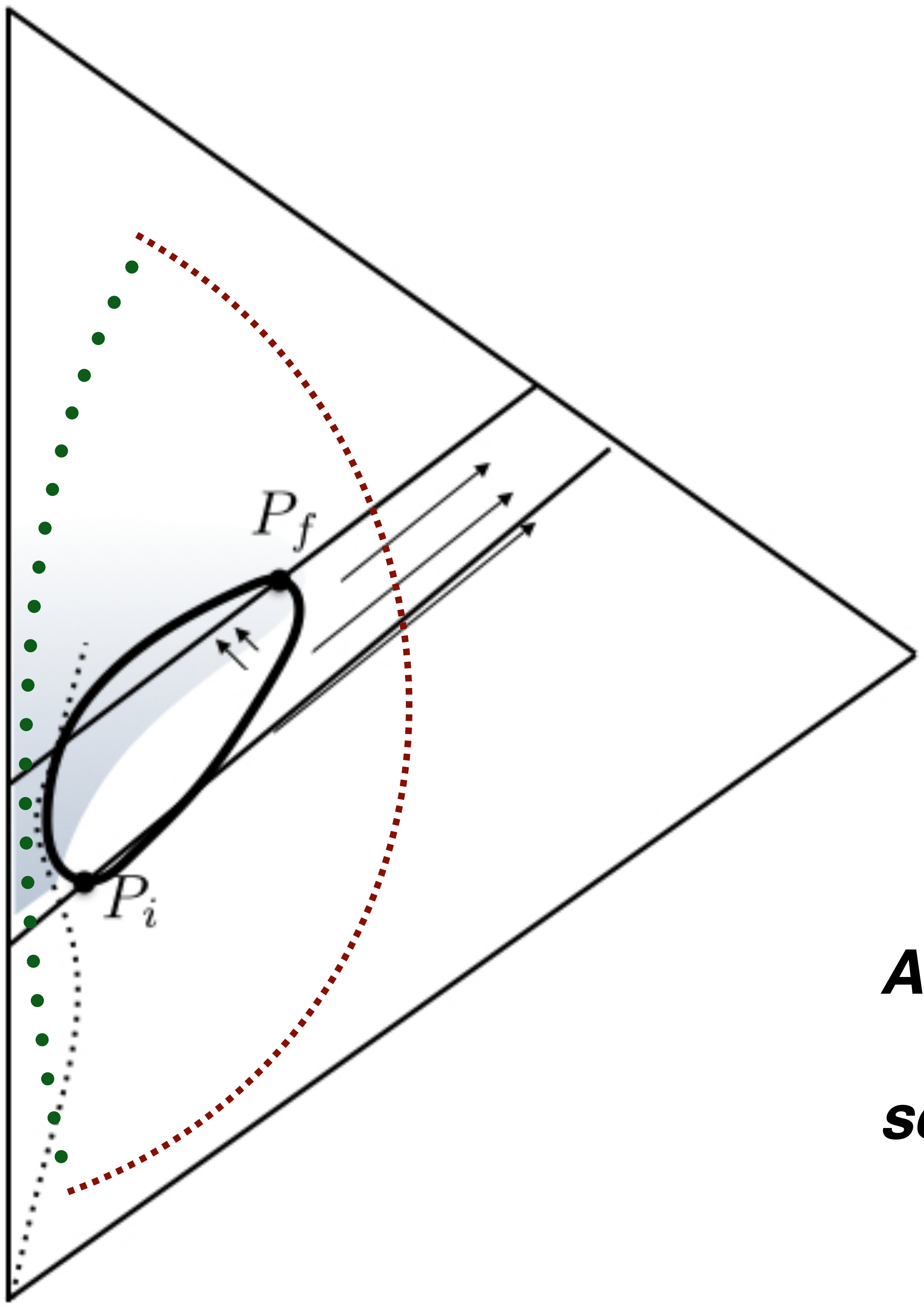
Non-evaporating
black hole



Evaporating
black hole



Planck star



Long proper time ~ m³

Short proper time ~ m

A black hole is a bouncing star

seen in slow motion

Final stage of the evaporation can be at a radius larger than L_{Planck} by a factor

$$\left(\frac{m}{m_{Planck}}\right)^n$$

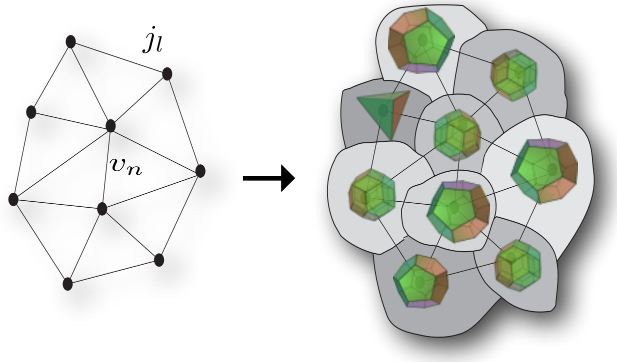
Avoiding firewalls gives n=1

This gives a final size for a primordial black hole with a size 10^{-14} cm.

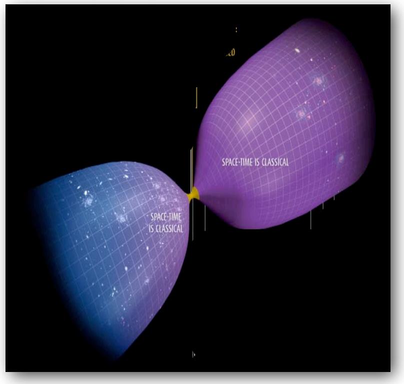
$$r = \sqrt[3]{\frac{t_H}{348\pi\ t_P}}\ l_P \sim 10^{-14}\ cm$$

A signal at this wavelength (~ GeV) ?

Summary

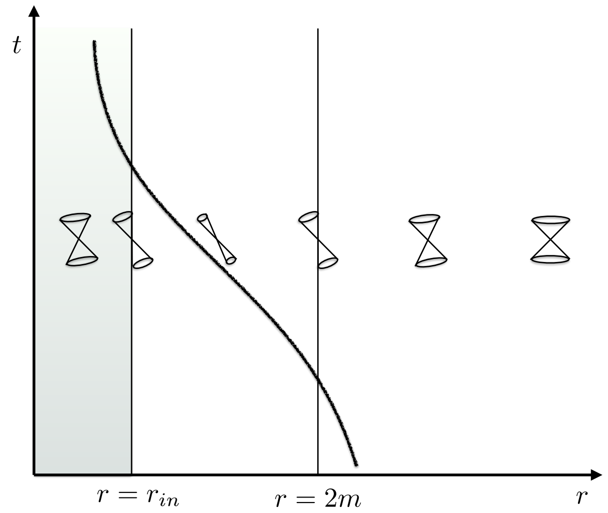


1. ***Loop gravity*** provides a tentative theory of quantum gravity, which is finite and has the correct classical limit.



2. ***Space is discrete*** and quantised at the Planck scale.

3. A ***strong repulsive force*** develops when the matter energy density reaches the Planck scale.



4. ***Bounce*** at the Big Bang? Effects on the CMB?

5. ***Planck Stars***? Observable in the gamma rays?