REDSHIFT SPACE DISTORTIONS:
From nuisance to PhD subject!

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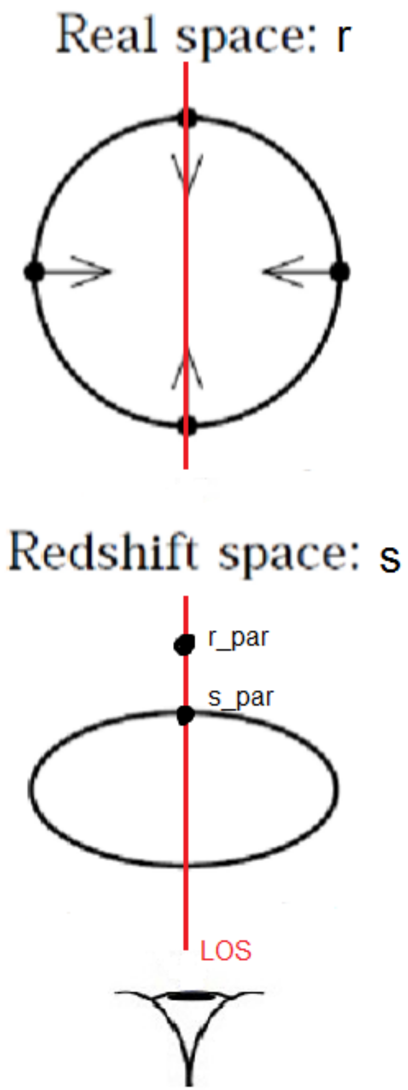
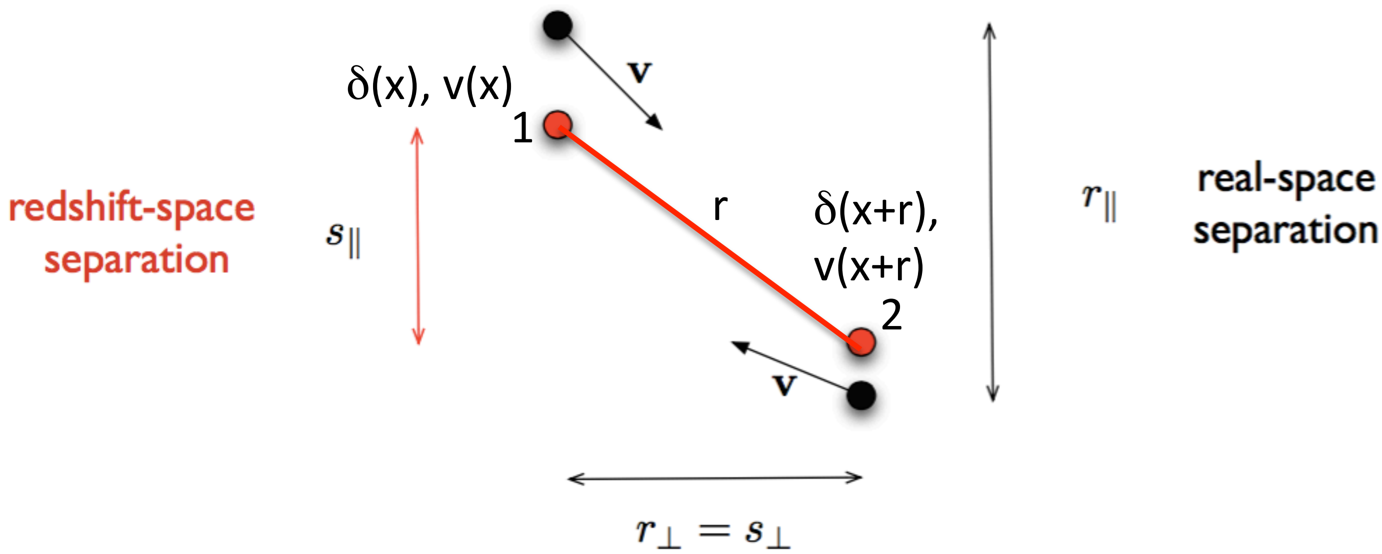
CosmoClub Tuesday 9th and 16th, February 2016

RSD: What



Relationship between real and redshift-space clustering:



→ Mean infall velocity $v_{12}(r) = (v_2 - v_1).r$

RSD: Why

- What you observe in a redshift survey is the density field in redshift space
  - Combination of density and velocity fields

- Tests of Gravitational Instability
  - Structure growth driven by motion of matter and inhibited by expansion

- Constraints on General Relativity
  - GR provides prediction for growth rate of structure f

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Dynamics of gravitational instability

- Mass conservation (continuity equation)

$$\frac{\partial \delta}{\partial \tau} + \vec{\nabla}.\left[(1+\delta)\vec{v}\right] = 0$$

- Momentum conservation (Euler equation)

$$\frac{\partial \vec{v}}{\partial \tau} + H\vec{v} + (\vec{v}.\vec{\nabla}).\vec{v} = -\vec{\nabla}\phi$$

- Matter-gravitational potential relation (Poisson equation)

$$\nabla^2\phi = 4\pi G\bar{\rho}\delta = \frac{3}{2}\Omega_m H^2 \delta$$

→ On large scales, where perturbations are small, one can linearize the equations and obtain: $\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta = 0$

→ $\delta(t,k) = \delta_+(k)D_+(t) + \delta_-(k)D_-(t)$ and $f(z) = dlnD(z))/dln(z)$

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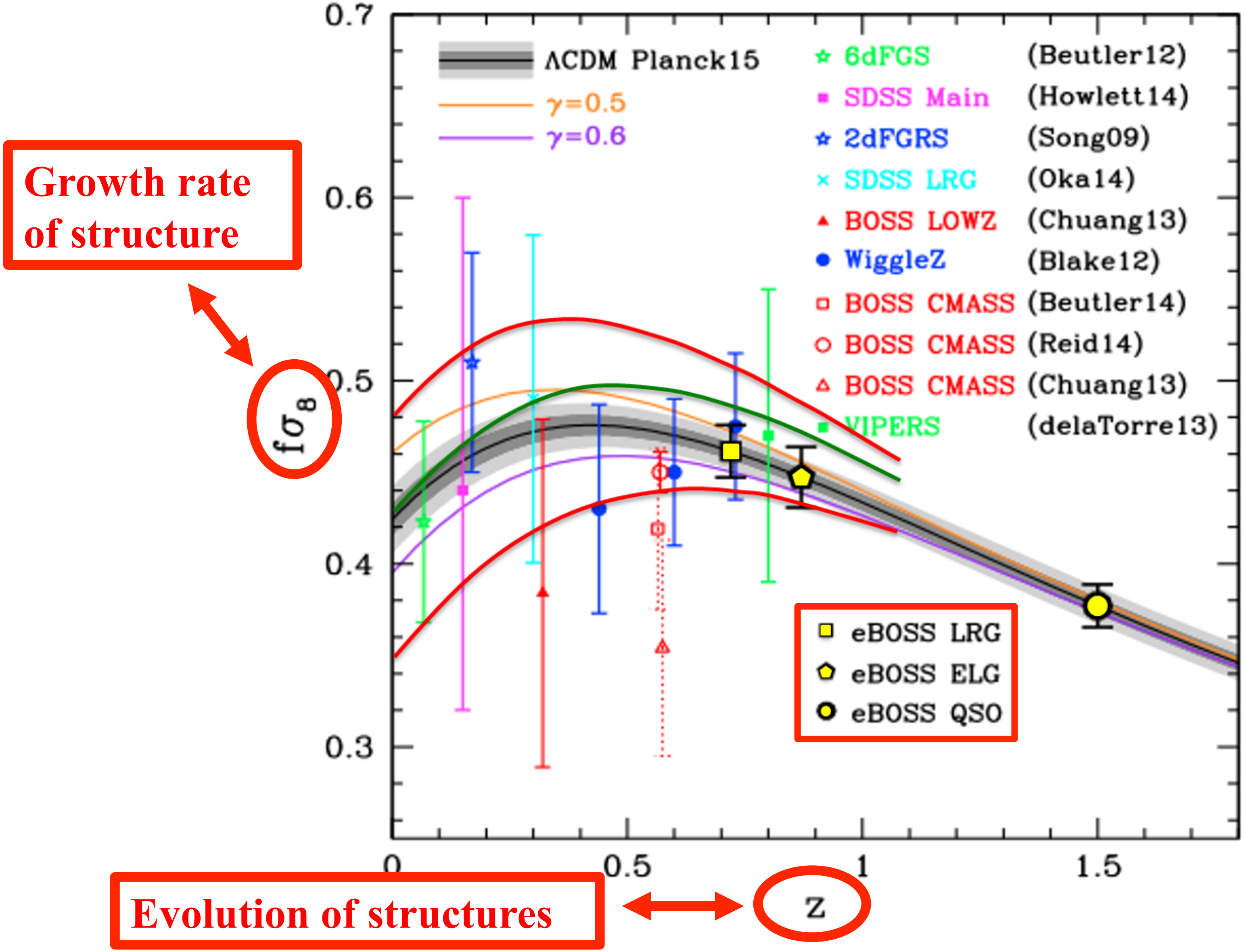
    $f(a) = \Omega_m(a)^\gamma$, with γ=0.55

Extract $f\sigma_8$ from the 2-point correlation function



2 origins for non-linearities

- Real-to-redshift space mapping

→How to express $\xi(s)$ as a function of $\xi(r)$

Where $\xi(r) = < \delta(x)\delta(x+r) >$

- Non-linear evolution of matter density and velocity fields

→ Next cosmoclub, for now we assume $\delta=\delta_{\text{Linear}}$

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History of RSD models

Linear theory

- Kaiser (1987)
- Gaussian streaming model (Fisher, 1995) with density-velocity and velocity-velocity couplings

Beyond linear theory

- Reid & White 2011
- Matsubara 2008
- Carlson, Reid & White 2012
- Wang, Reid & White 2013

Kaiser, 1987

- The redshift-space position of a galaxy differs from its real-space position due to its peculiar velocity v (in h^{-1} Mpc): $s = r + v_z(x).z = r\,[1 + \frac{v_z}{z}]$ $\boxed{v_z(\mathbf{x}) \equiv u_z(\mathbf{x})/(aH)}$

- The exact Jacobian for the real-space to redshift-space transformation is obtained using:

$$d^3s = s^2 ds\, sin^2\theta\, d\theta\, d\varphi$$
$$d^3r = r^2 dr\, sin^2\theta\, d\theta\, d\varphi$$

$\rightarrow$ $$\frac{d^3s}{d^3r} = \left(1 + \frac{v_z}{z}\right)^2 \left(1 + \frac{dv_Z}{dZ}\right)$$

- The galaxy overdensity field in the redshift space can be obtained by imposing mass conservation:

$$(1 + \delta_g^s)d^3s = (1 + \delta_g^r)d^3r$$

Kaiser, 1987

- Distant-observer (or plane-parallel) approximation

→$v_z/z \ll 1$ where the LOS direction of each object is taken to be the fixed direction z

$$1 + \delta_g^S = (1 + \delta_g^r)\left(1 + \frac{dv_z}{dz}\right)^{-1} \qquad (1)$$

- Irrotational velocity field assumption

→Potential flow: $\theta = -\nabla . v$ so $v_Z = -\frac{d}{dz}\nabla^{-2}\theta$

$$(d/dz)^2 . \nabla^{-2} = (k_z/k)^2 = \mu_k^2 \text{ which gives } \frac{dv_z}{dz} \to -\mu_k^2\, \theta(k)$$

(1) becomes

$$\delta_g^s(k) = \delta_g^r(k) + \mu_k^2\theta(k) + \mu_k^2[\delta_g^r \otimes \theta](k) + \mu_k^4[\theta \otimes \theta](k) + \cdots$$

Kaiser, 1987

- Linear coupling: $\theta(k) = f\delta_m(k)$ where $f = \frac{d\ln D}{d\ln a} \cong \Omega_m^{0.55}(z)$

- Bias matter tracers: $\delta_g = b\delta_m$

Monopole

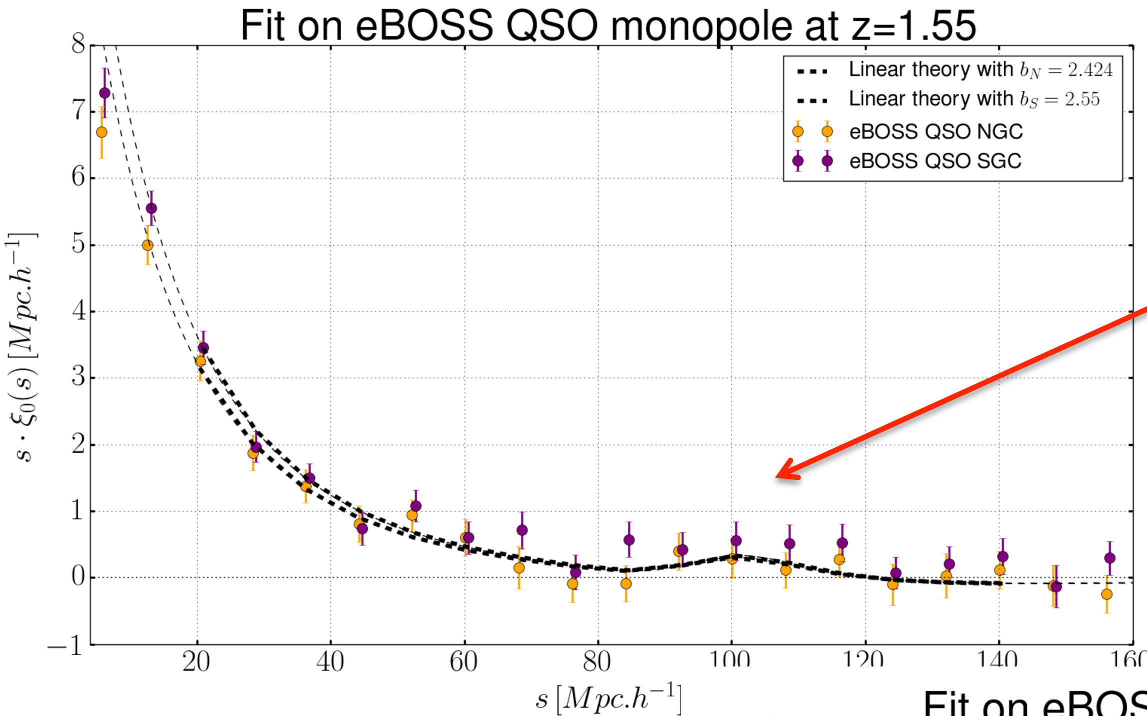
$$\xi_0^s = (b^2 + \frac{2}{3}fb + \frac{1}{5}f^2)\xi_m^r(r) = b^2[1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2]\xi_m^r(r)$$

Quadrupole

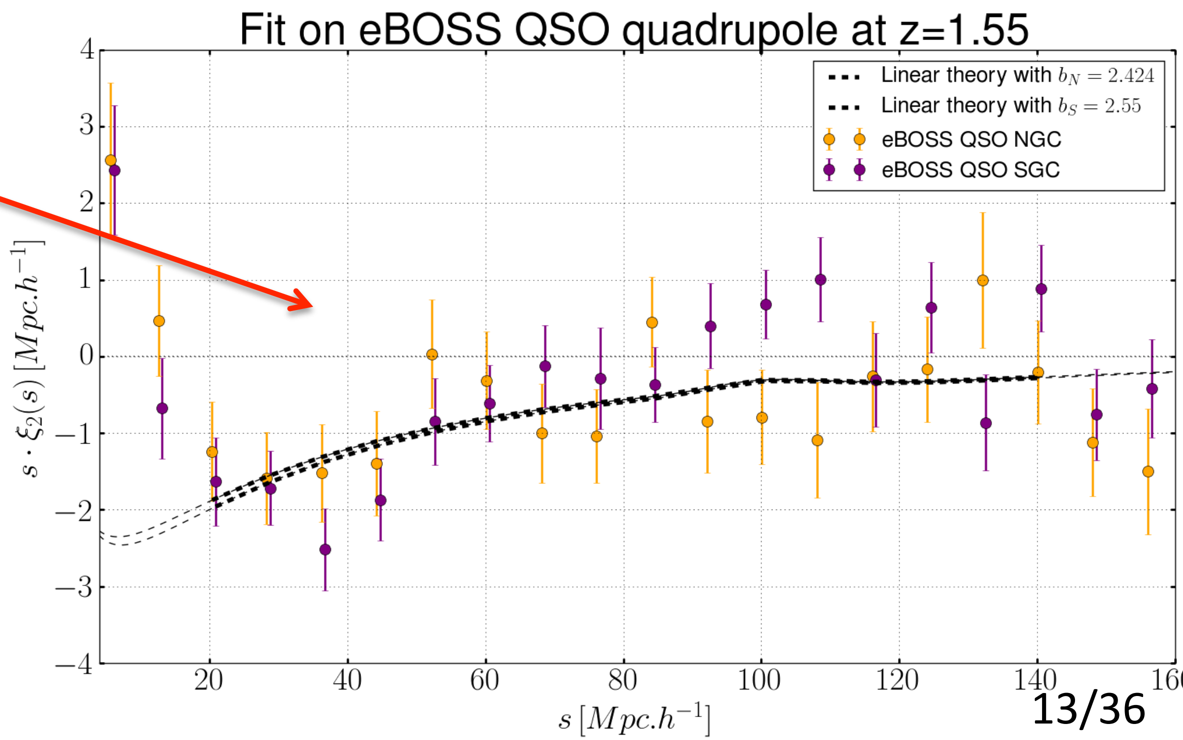
$$\xi_2^s = b^2(\frac{4}{3}\beta + \frac{4}{7}\beta^2)[\xi_m^r(r) - \bar{\xi}_m(r)]$$

$\boxed{\beta = f/b}$

$$\bar{\xi}_m(r) = 3r^{-3}\int_0^r \xi_m(r')r'^2 dr'$$



Monopole related to b

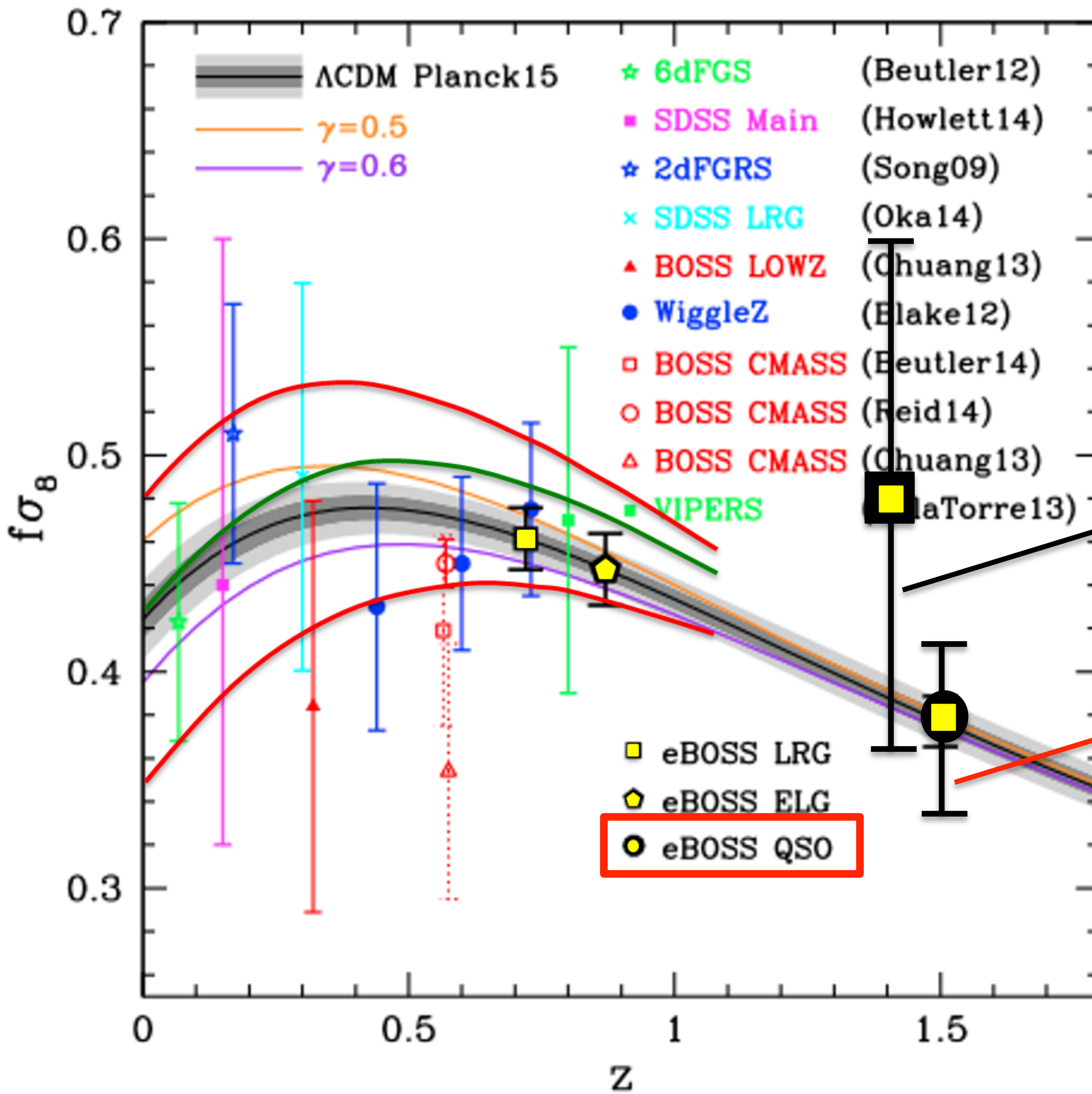


Quadrupole directly related to fb

→If no RSD, no anisotropic clustering

→ $\xi_2 = 0$

eBOSS expected values for fσ_8



Okumura et al. (2015) with Subaru FMOS

Current precision with eBOSS

Fisher, 1995

There have been two models used in the literature to describe $\xi(r_\sigma, r_\pi)$. The first and perhaps most well known model expresses $\xi(r_\sigma, r_\pi)$ as a convolution of the real-space correlation function with the probability distribution for velocities along the line of sight,[1]

$$\boxed{\xi(r_\sigma, r_\pi) = \int_{-\infty}^{+\infty} dy\, \xi(r) F_v\left[r_\pi - y - \frac{y}{r} v_{12}(r)\right],} \tag{1}$$

where $r^2 = y^2 + r_\sigma^2$ and $v_{12}(r)$ is the mean relative peculiar velocity of two galaxies separated by r (cf., Peebles 1980, § 76; Peebles 1993, p. 478). $F_v(V)$ is the probability distribution of relative velocities about $v_{12}(r)$ which is usually assumed to be isotropic. By "isotropic" we mean that the tensor second moment of the velocity distribution, $\langle v_i v_j \rangle$, is assumed to be of the form, $\sigma^2 \delta_{ij}^{\mathrm{K}}$ (δ_{ij}^{K} denotes the usual Kronecker delta symbol). Throughout this paper, the model for $\xi(r_\sigma, r_\pi)$ given in equation (1) with an isotropic dispersion will be referred to as the "streaming" model. The streaming model has mainly been used to estimate the galaxy pair-wise velocity dispersion from the suppression of $\xi(r_\sigma, r_\pi)$ on small scales (e.g., Davis & Peebles 1983).

For a Gaussian random field:

→ Streaming Model with a Gaussian velocity distribution and a scale-dependent velocity dispersion (pairwise velocity PDF)

$$\boxed{\xi(r_\sigma, r_\pi) = \int \frac{dy}{\sqrt{2\pi}\,\sigma_{-}(r)}\, \xi(r) \exp\left\{-\frac{1}{2}\frac{[r_\pi - y - (y/r)v_{12}(r)]^2}{\sigma_{-}^2(r)}\right\}}$$

Fisher, 1995

- Density-velocity coupling

For Gaussian fields:
$<\delta(x)v(x)>=0$

$$
\begin{aligned}
\langle \boldsymbol{v}_{12}(\boldsymbol{r}) \rangle &\equiv \langle (\boldsymbol{v}' - \boldsymbol{v})(1+\delta)(1+\delta') \rangle \\
&= \langle \boldsymbol{v}'\delta \rangle - \langle \boldsymbol{v}\delta' \rangle + \text{higher order terms} \\
&= -2\hat{\boldsymbol{r}} \frac{f\,b}{2\pi^2} \int dk\, k P(k) j_1(kr) \quad \text{(in linear theory)} \\
&\equiv v_{12}(r)\hat{\boldsymbol{r}} \,,
\end{aligned}
$$

In linear theory, it is straightforward to how that the density/velocity coupling is given by

$$
\boxed{\langle \delta \boldsymbol{v}' \rangle = -\hat{\boldsymbol{r}} \frac{f\,b}{2\pi^2} \int dk\, k P(k) j_1(kr) \,,}
$$

If it is not straightforward for you, as it was not for me, see next slide!

Fourier convention: $\delta(x) = \int \frac{d^3k}{(2\pi)^3} e^{ikx} \delta(k)$

$$< \delta_g(x) v(x+r) >.r = \int \frac{d^3k'}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} e^{i(k+k').x} e^{ik.r} < \delta_g(k) v(k') >.r$$

$$\boxed{v(k) = \frac{i\mathbf{k}}{k^2}\theta(k) = \frac{if\mathbf{k}}{k^2}\delta_m(k)}$$

$$= \int \frac{d^3k'}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} e^{i(k+k').x} e^{ik.r} \frac{ifb\,\mathbf{k.r}}{k^2} < \delta_m(k)\delta_m(k') >$$

$$= \int \frac{d^3k}{(2\pi)^3} e^{ik.r} \frac{ifb\,k.r}{k^2} P_m(k)$$

Isotropic case: $d^3k = d\varphi\, cos\theta d\theta\, k^2 dk$ and $k.r = krcos\theta$

$$< \delta_g(x) v(x+r) >.r = \int_0^{2\pi} d\varphi \int_0^1 cos\theta\, d\theta\, e^{ikrcos\theta} cos\theta \int \frac{k^2 dk}{(2\pi)^3} kr \frac{ifb}{k^2} P_m(k)$$

$$\boxed{2d(cos\theta)e^{ikrcos\theta}}$$

We set $\mu = cos\theta$

$$< \delta_g(x) v(x+r) >.r = r \int \frac{dk}{2\pi^2} ifbkP_m(k) \int_0^1 2d\mu\, e^{ikr\mu}\mu$$

$$\boxed{2ij_1(kr)}$$

$$\boxed{< \delta(x) v(x+r) >.r = -r \frac{fb}{2\pi^2} \int dk\, k\, P_m(k) j_1(kr)}$$

Fisher, 1995

- Velocity-velocity coupling (Gorski, 1988)

In linear theory there is also a velocity/velocity coupling given by

$$\langle \boldsymbol{v}_i \boldsymbol{v}'_j \rangle = \Psi_\perp(r)\delta^{\mathrm{K}}_{ij} + [\Psi_\parallel(r) - \Psi_\perp(r)]\hat{r}_i \hat{r}_j \,,$$

where the velocity correlation functions parallel and perpendicular to the line of separation are (Górski 1988)

$$\Psi_\perp(r) = \frac{\beta^2}{2\pi^2} \int dk\, P(k)\, \frac{j_1(kr)}{kr} \,,$$

$$\Psi_\parallel(r) = \frac{\beta^2}{2\pi^2} \int dk\, P(k) \left[j_0(kr) - \frac{2j_1(kr)}{kr} \right] .$$

which gives: $\boxed{\sigma_{12}^2(r,\mu^2) = 2\left[\sigma_v^2 - \mu^2\Psi_\parallel(r) - (1-\mu^2)\Psi_\perp(r)\right]}$

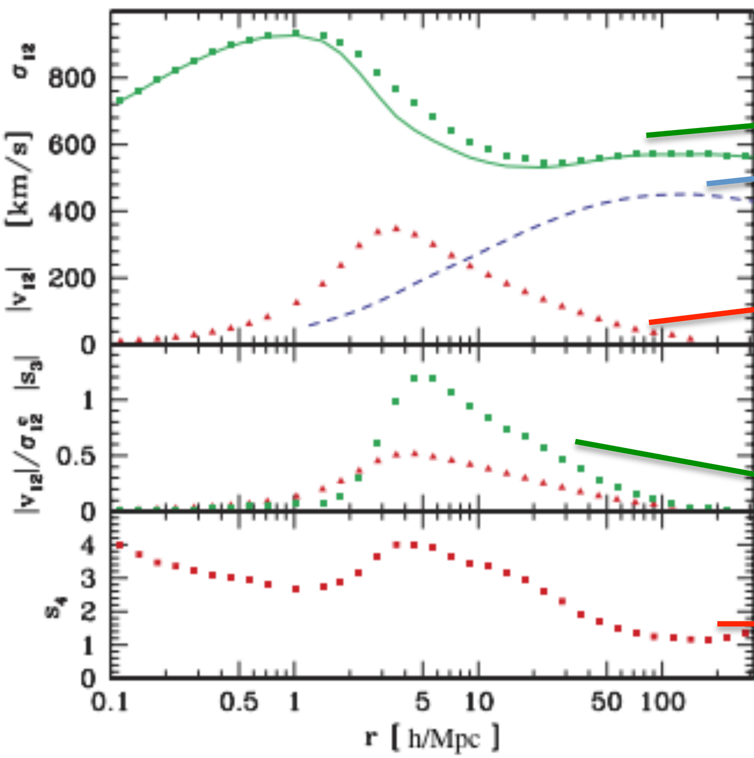
1D velocity dispersion,
⅓ <v(x)v(x)>

Linear theory: summary

Linear coupling: $\theta(k) = f\delta_m(k)$ where $f = \frac{d\ln D}{d\ln a} \cong \Omega_m^{0.55}(z)$

$$\boxed{1+\xi_g(s_\parallel, s_\perp) = \int dr_\parallel [1+\xi_g(r)] P(v_{12}, \sigma_{12}^2)}$$

R. Scoccimarro, 2004, Phys.Rev.D



Dispersion velocity σ^2_{12} prop to f^2

- From N-body simulations
- Linear prediction

Mean infall velocity v_{12} prop to fb

- From N-body simulations

Skewness

Kurtosis

Pairwise velocity PDF non-Gaussian at all scales

BEYOND LINEAR THEORY

2 origins for non-linearities

- Real-to-redshift space mapping

→How to express ξ(s) as a function of ξ(r)

Where $\xi(r) = \langle \delta(x)\delta(x+r) \rangle$

Gaussian Streaming Model:

$$\boxed{1 + \xi_g(s_\parallel, s_\perp) = \int dr_\parallel [1 + \xi_g(r)] P(v_{12}, \sigma_{12}^2)}$$

Linear theory:

- v_{12} prop to fb
- σ^2_{12} prop to f^2

2 origins for non-linearities

- Real-to-redshift space mapping

→How to express $\xi(s)$ as a function of $\xi(r)$

Where $\xi(r) = < \delta(x)\delta(x+r) >$

- Non-linear evolution of matter density and velocity fields

→Let go back to the dynamics of gravitational instability to understand how non-linear evolution comes

Dynamics of gravitational instability

- Mass conservation (continuity equation)

$$\frac{\partial \delta}{\partial \tau} + \vec{\nabla}.\left[(1+\delta)\vec{v}\right] = 0 \qquad (1)$$

- Momentum conservation (Euler equation)

$$\frac{\partial \vec{v}}{\partial \tau} + H\vec{v} + \left(\vec{v}.\vec{\nabla}\right).\vec{v} = -\vec{\nabla}\phi \qquad (2)$$

- Matter-gravitational potential relation (Poisson equation)

$$\nabla^2\phi = 4\pi G\bar{\rho}\delta = \frac{3}{2}\Omega_m H^2\delta \qquad (3)$$

Next slide:

1. ∇(2) + use (3) to replace $\nabla^2\Phi$
2. Fourier transformation

Dynamics of gravitational instability

- Mass conservation

$$\frac{\partial \delta(\boldsymbol{k})}{\partial \tau} + \theta(\boldsymbol{k}) = -\int \frac{d^3q_1 d^3q_2}{(2\pi)^3} \delta_D(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{k}) \frac{\boldsymbol{k}\cdot\boldsymbol{q}_1}{q_1^2} \theta(\boldsymbol{q}_1)\delta(\boldsymbol{q}_2)$$

- Momentum conservation

$$\frac{\partial \theta(\boldsymbol{k})}{\partial \tau} + \mathcal{H}\theta(\boldsymbol{k}) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta(\boldsymbol{k}) = -\int \frac{d^3q_1 d^3q_2}{(2\pi)^3} \delta_D(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{k}) \frac{k^2(\boldsymbol{q}_1\cdot\boldsymbol{q}_2)}{2q_1^2 q_2^2} \theta(\boldsymbol{q}_1)\theta(\boldsymbol{q}_2)$$

Fundamental mode couplings

$$\alpha(\boldsymbol{q}_1, \boldsymbol{q}_2) = \frac{\boldsymbol{k}\cdot\boldsymbol{q}_1}{q_1^2} \quad , \quad \beta(\boldsymbol{q}_1, \boldsymbol{q}_2) = \frac{k^2(\boldsymbol{q}_1\cdot\boldsymbol{q}_2)}{2q_1^2 q_2^2}$$

→ Encode the non-linearity of the evolution (mode coupling) and come from the non-linear terms in the continuity equation and in the Euler equation.

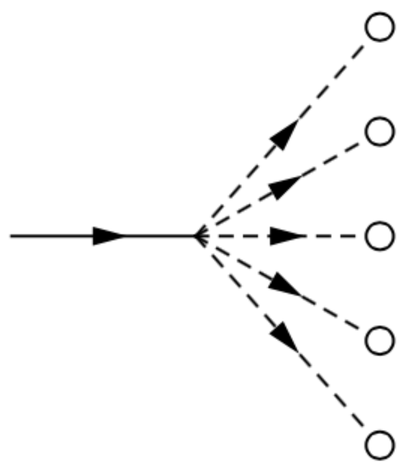
→ Non-linear evolution of δ(k,τ) and θ(k,τ) is determined by the mode coupling of the fields at all pairs of wavevectors whose sum is k

Beyond linear term: perturbative expansion

Standard Perturbation Theory

$$\delta(\boldsymbol{k};\tau) = \sum^{\infty} a^n(\tau)\delta_n(\boldsymbol{k}), \qquad \theta(\boldsymbol{k};\tau) = -\mathcal{H}(\tau)\sum_{n=1}^{\infty} a^n(\tau)\theta_n(\boldsymbol{k})$$

$\delta_n(\boldsymbol{k}) =$



Diagrammatic representation of the n^{th} order contribution to $\delta(\boldsymbol{k})$.

Mode coupling functions constructed from α and β

$$\boxed{\begin{pmatrix}\delta_n(\boldsymbol{k})\\ \theta_n(\boldsymbol{k})\end{pmatrix} = \int \frac{d^3q_1\ldots d^3q_n}{(2\pi)^{3n}}(2\pi)^3\delta_D\left(\sum \boldsymbol{q}_i - \boldsymbol{k}\right)\begin{pmatrix}F_n(\{\boldsymbol{q}_i\})\\ G_n(\{\boldsymbol{q}_i\})\end{pmatrix}\delta_0(\boldsymbol{q}_1)\ldots\delta_0(\boldsymbol{q}_n).}$$

History of RSD models

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Beyond linear theory

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Assumption: the pairwise velocity PDF is defined by its 2 first moments, v_{12} and σ_{12}

→ Scale-dependent Gaussian Streaming Model

$$\boxed{1+\xi_g(s_\parallel, s_\perp) = \int dr_\parallel [1+\xi_g(r)] P(v_{12}, \sigma_{12}^2)}$$

→ Which perturbation theory to calculate $\xi(r)$, $v_{12}(r)$ and $\sigma^2_{12}(r,\mu)$?

Standard Perturbation Theory

$$\mathbf{s} = \mathbf{x} + v_z(\mathbf{x})\widehat{\mathbf{z}},$$

where $v_z(\mathbf{x}) \equiv u_z(\mathbf{x})/(aH)$

$$(1+\delta_g^s)d^3s = (1+\delta_g^r)d^3r$$

$$\delta(\boldsymbol{k};\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(\boldsymbol{k}),$$

$$\theta(\boldsymbol{k};\tau) = -\mathcal{H}(\tau)\sum_{n=1}^{\infty} a^n(\tau)\theta_n(\boldsymbol{k}),$$

Lagrangian Perturbation Theory

$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t),$$

where Ψ is the displacement field

$$[1+\delta_m(\mathbf{x},t)]\ \mathrm{d}^3x = [1+\delta_m(\mathbf{q},0)]\ \mathrm{d}^3q = \mathrm{d}^3q$$

$$\boldsymbol{\Psi}(\boldsymbol{q},t) = \boldsymbol{\Psi}^{(1)}(\boldsymbol{q},t) + \boldsymbol{\Psi}^{(2)}(\boldsymbol{q},t) + \boldsymbol{\Psi}^{(3)}(\boldsymbol{q},t) + \cdots$$

where $\Psi^{(1)}$ is the Zel'dovich solution

Local Lagrangian bias model

Matsubara, 2008

Biased tracers

In this paper, we assume locality of the bias in Lagrangian space: the Lagrangian density field $\rho^{\rm L}_{\rm obj}(\boldsymbol{q})$ is assumed to be a function of a smoothed linear overdensity at the same Lagrangian position,

$$\delta_R(\boldsymbol{q}) = \int d^3q'\, W_R(|\boldsymbol{q}-\boldsymbol{q}'|)\, \delta_{\rm L}(\boldsymbol{q}'),$$

where W_R is a smoothing kernel of size R, and $\delta_{\rm L}(\boldsymbol{q})$ is the (unsmoothed) linear overdensity. We call such biasing scheme a "local Lagrangian bias" in this paper.

Lagrangian bias function F(δ)

$$\boxed{1+\delta(\mathbf{q}) = F[\delta_{m,R}(\mathbf{q})]}$$

Link between Eulerian and Lagrangian bias

In Eulerian space: on large scales one expects a linear bias such that $\delta_g = b\delta_m$

In Lagrangian space: on large scales, one expects 1+<F'>=b where <F'> is the 1st Lagrangian bias parameter (see slide 30)

ξ(r) using CLPT
Carlson, Reid & White 2013

- Tracer density field (from (1) and (2))

$$1 + \delta_X(\boldsymbol{x}, t) = \left|\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{q}}\right|^{-1} F[\delta_R(\boldsymbol{q})] = \int d^3q \, F[\delta_R(\boldsymbol{q})]\delta_D\left[\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{\Psi}(\boldsymbol{q}, t)\right]$$

- Correlation function

$$1 + \xi_X(\boldsymbol{r}) = \int d^3q_1 \, d^3q_2 \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} e^{i\boldsymbol{k}_1\cdot(\boldsymbol{x}_1-\boldsymbol{q}_1)} e^{i\boldsymbol{k}_2\cdot(\boldsymbol{x}_2-\boldsymbol{q}_2)}$$
$$\times \int \frac{d\lambda_1}{2\pi} \frac{d\lambda_2}{2\pi} \tilde{F}(\lambda_1)\tilde{F}(\lambda_2) \left\langle e^{i[\lambda_1\delta_1+\lambda_2\delta_2-\boldsymbol{k}_1\cdot\boldsymbol{\Psi}_1-\boldsymbol{k}_2\cdot\boldsymbol{\Psi}_2]}\right\rangle$$

ξ=<$\delta(x_1)\delta(x_2)$>
+ Fourier transformation

Change of coordinates: $(q_1,q_2) \rightarrow (\, q=q_2-q_1,\ Q=(q_1+q_2)/2\,)$

$$1+\xi_X(\boldsymbol{r}) = \int d^3q \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{r})} \int \frac{d\lambda_1}{2\pi} \frac{d\lambda_2}{2\pi} \tilde{F}_1\tilde{F}_2 \, K(\boldsymbol{q}, \boldsymbol{k}, \lambda_1, \lambda_2),$$

where we have defined

$$\boxed{K(\boldsymbol{q}, \boldsymbol{k}, \lambda_1, \lambda_2) = \left\langle e^{i(\lambda_1\delta_1+\lambda_2\delta_2+\boldsymbol{k}\cdot\boldsymbol{\Delta})}\right\rangle} \qquad (4)$$

$\xi(r)$ using CLPT

Carlson, Reid & White 2013

- Cumulant expansion theorem: $\langle e^{iX}\rangle = \exp\left[\sum_{N=1}^{\infty}\frac{i^N}{N!}\langle X^N\rangle_c\right],$

Since Ψ is Gaussian, only second cumulant survives.

$$\left\langle(\lambda_1\delta_1 + \lambda_2\delta_2 + \boldsymbol{k}\cdot\boldsymbol{\Delta})^2\right\rangle_c = (\lambda_1^2+\lambda_2^2)\sigma_R^2 + A_{ij}k_ik_j + 2\lambda_1\lambda_2\xi_R + 2(\lambda_1+\lambda_2)U_ik_i,$$

where we have defined

$$\sigma_R^2 = \langle\delta_1^2\rangle_c = \langle\delta_2^2\rangle_c, \qquad \xi_R(\boldsymbol{q}) = \langle\delta_1\delta_2\rangle_c,$$
$$A_{ij}(\boldsymbol{q}) = \langle\Delta_i\Delta_j\rangle_c, \qquad U_i(\boldsymbol{q}) = \langle\delta_1\Delta_i\rangle_c = \langle\delta_2\Delta_i\rangle_c.$$

Thus (4) becomes:

$$K = \exp\left[-\frac{1}{2}(\lambda_1^2+\lambda_2^2)\sigma_R^2 - \frac{1}{2}A_{ij}k_ik_j - \lambda_1\lambda_2\xi_R - (\lambda_1+\lambda_2)U_ik_i\right]$$

→ Go to higher order: perturbative expansion of K

→ Non-perturbative resummation of σ^2_R and A_{ij} (since σ^2_R and A_{ij} do not vanish when q tends to infinity (large-scale limit)

ξ(r) using CLPT

Carlson, Reid & White 2013

- Perform integrations

$$1+\xi_X(\boldsymbol{r}) = \int d^3q \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{r})} \int \frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi} \tilde{F}_1\tilde{F}_2 K(\boldsymbol{q},\boldsymbol{k},\lambda_1,\lambda_2),$$

$$L(\boldsymbol{q},\boldsymbol{k}) \equiv \int \frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi} \tilde{F}(\lambda_1)\tilde{F}(\lambda_2) K(\boldsymbol{q},\boldsymbol{k},\lambda_1,\lambda_2)$$

$$M(\boldsymbol{r},\boldsymbol{q}) \equiv \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{r})} L(\boldsymbol{q},\boldsymbol{k})$$

Mapping K->L

$$\int \frac{d\lambda}{2\pi} \tilde{F}(\lambda)\,(i\lambda)^n\, e^{-\frac{1}{2}\lambda^2\sigma_R^2} = \int \frac{d\delta}{\sqrt{2\pi}\sigma_R} e^{-\delta^2/2\sigma_R^2} \frac{d^nF}{d\delta^n} \equiv \left\langle F^{(n)} \right\rangle$$

$$\boxed{\xi = \xi_0 + <F'>\xi_{10} + <F''>\xi_{01} + <F'>^2\xi_{20} + <F'><F''>\xi_{11} + <F''>^2\xi_{02}}$$

Clustering and velocity statistics in CLPT

Wang, Reid & White 2014

- Velocity moments

$$\mathbf{u}(\mathbf{x}_2) - \mathbf{u}(\mathbf{x}_1) = a\left(\dot{\mathbf{x}}_2 - \dot{\mathbf{x}}_1\right) = a\dot{\mathbf{\Delta}}.$$

Time-independent approximation: $\mathbf{\Psi}^{(k)} \propto D^k$

$$\dot{\mathbf{\Psi}}^{(k)} = kHf\mathbf{\Psi}^{(k)}$$

$$v_z(\mathbf{x}) \equiv u_z(\mathbf{x})/(aH)$$

$$\mathbf{v}_n(\mathbf{x}_2) - \mathbf{v}_n(\mathbf{x}_1) = \sum_k kf\mathbf{\Delta}_n^{(k)} = \frac{\dot{\mathbf{\Delta}}_n}{H}$$

- Velocity generating function:

$$Z(\mathbf{r}, \mathbf{J}) = \int \mathrm{d}^3q \int \frac{\mathrm{d}^3k}{(2\pi)^3} \mathrm{e}^{i\mathbf{k}\cdot(\mathbf{q}-\mathbf{r})} \int \frac{\mathrm{d}\lambda_1}{2\pi}\frac{\mathrm{d}\lambda_2}{2\pi} \times \tilde{F}(\lambda_1)\tilde{F}(\lambda_2)\left\langle \mathrm{e}^{i\left(\lambda_1\delta_1+\lambda_2\delta_2+\mathbf{k}\cdot\mathbf{\Delta}+\mathbf{J}\cdot\dot{\mathbf{\Delta}}/H\right)}\right\rangle$$

Clustering and velocity statistics in CLPT

Wang, Reid & White 2014

- ξ(r) in CLPT

$$1 + \xi(\mathbf{r}) = \int \mathrm{d}^3 q \; M_0(\mathbf{r}, \mathbf{q})$$

- v_{12}(r) in CLPT

$$v_{12,n}(\mathbf{r}) = [1 + \xi(r)]^{-1} \int \mathrm{d}^3 q \; M_{1,n}(\mathbf{r}, \mathbf{q})$$

- σ_{12}(r) in CLPT

$$\sigma^2_{12,nm}(\mathbf{r}) = [1 + \xi(r)]^{-1} \int \mathrm{d}^3 q \; M_{2,nm}(\mathbf{r}, \mathbf{q})$$

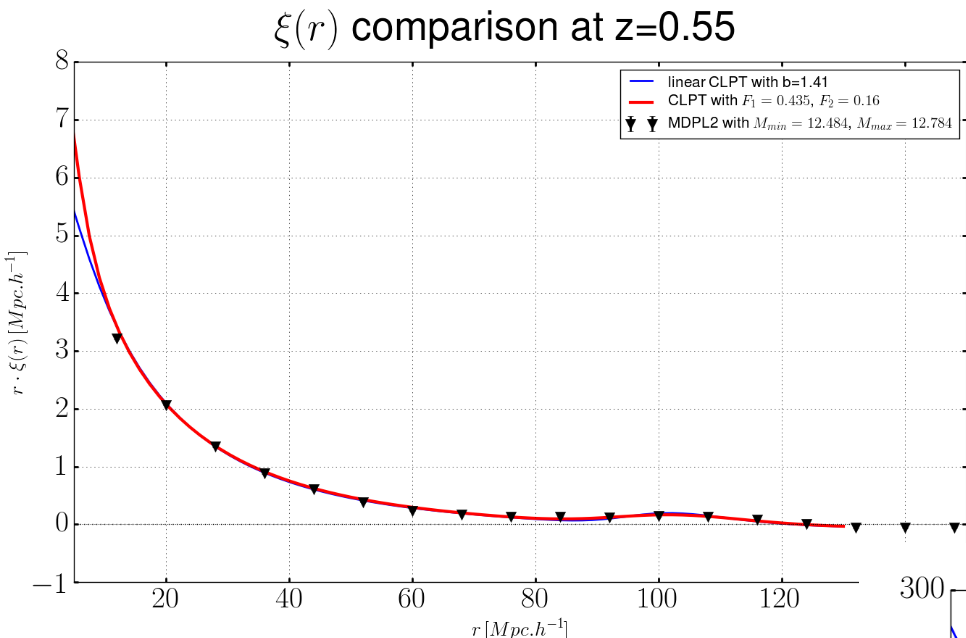
CLPT code:

(input: $P_L(k)$)

$$\boxed{\begin{aligned} A = A_0 &+ \langle F' \rangle A_{10} + \langle F'' \rangle A_{01} + \langle F' \rangle^2 A_{20} \\ &+ \langle F' \rangle \langle F'' \rangle A_{11} + \langle F'' \rangle^2 A_{02} \end{aligned}}$$

Analysis at z=0.55 for BOSS CMASS galaxies

Wang, Reid & White (2014)

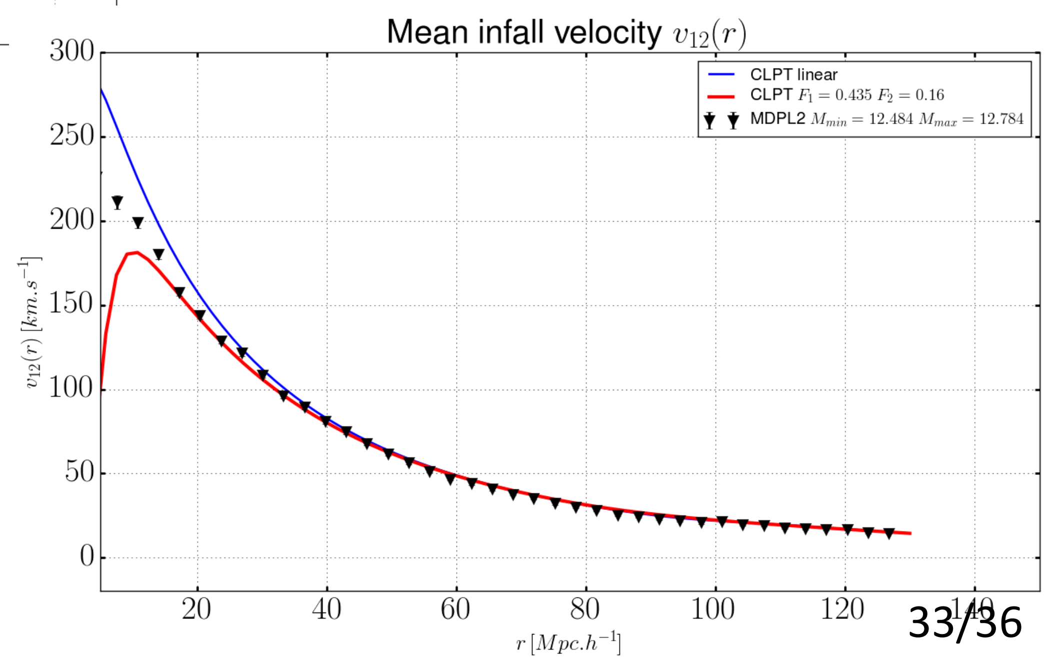


12.484 < log(M/M$_{sun}$) < 12.784

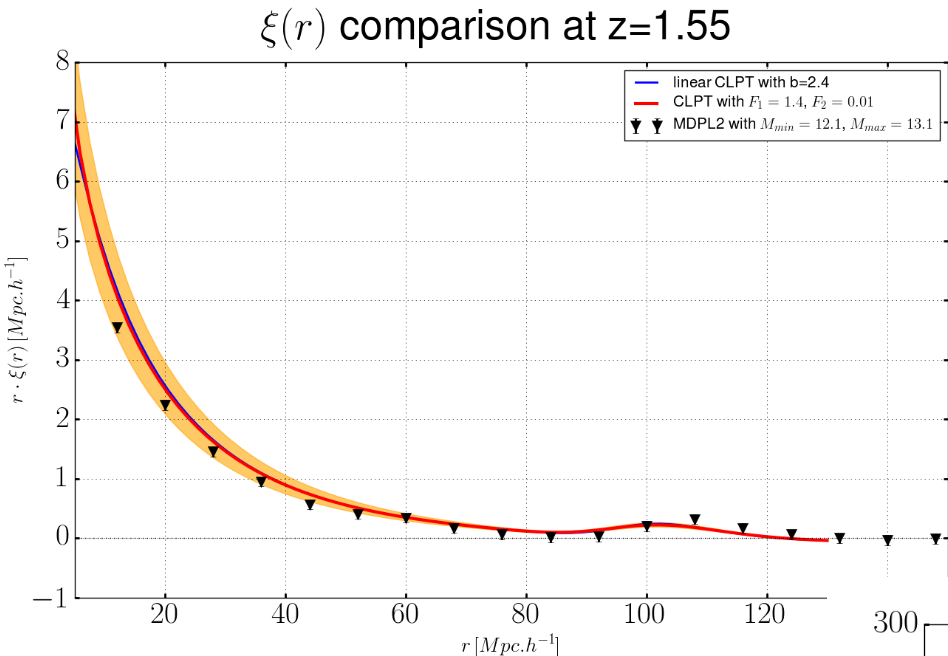
<F'> = 0.435 +/- 0.004

<F''> = 0.16 +/- 0.06

- MultiDark N-body simulations: MDPL2 http://www.cosmosim.org/



Analysis at z=1.55 for eBOSS QSO sample

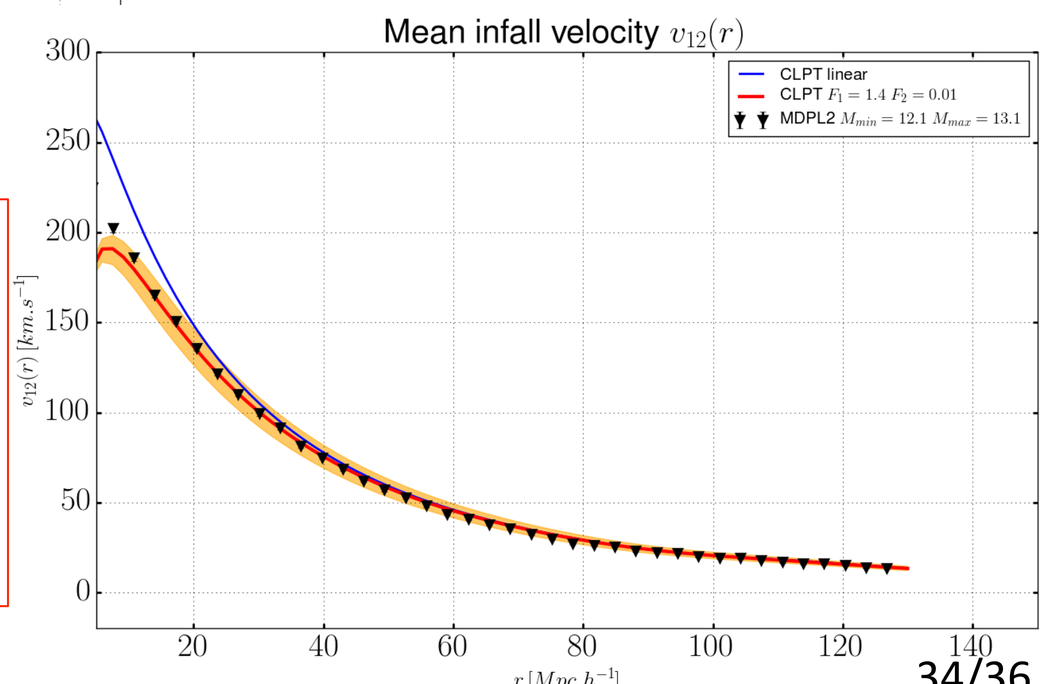


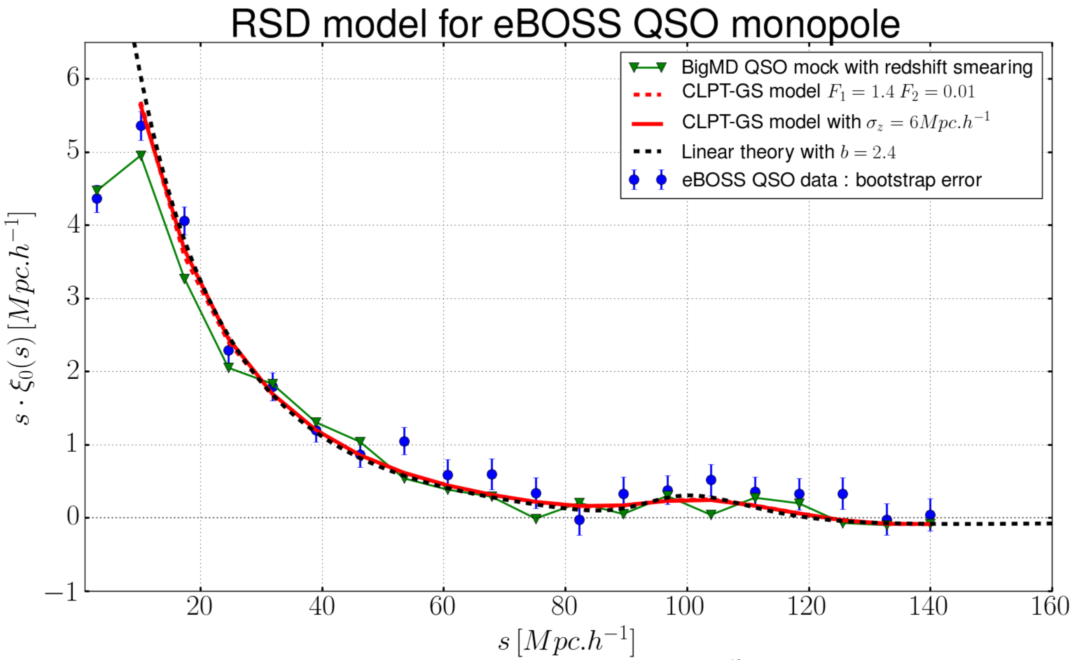
$12.1 < \log(M/M_{sun}) < 13.1$

<F'> = 1.4 +/- 0.21

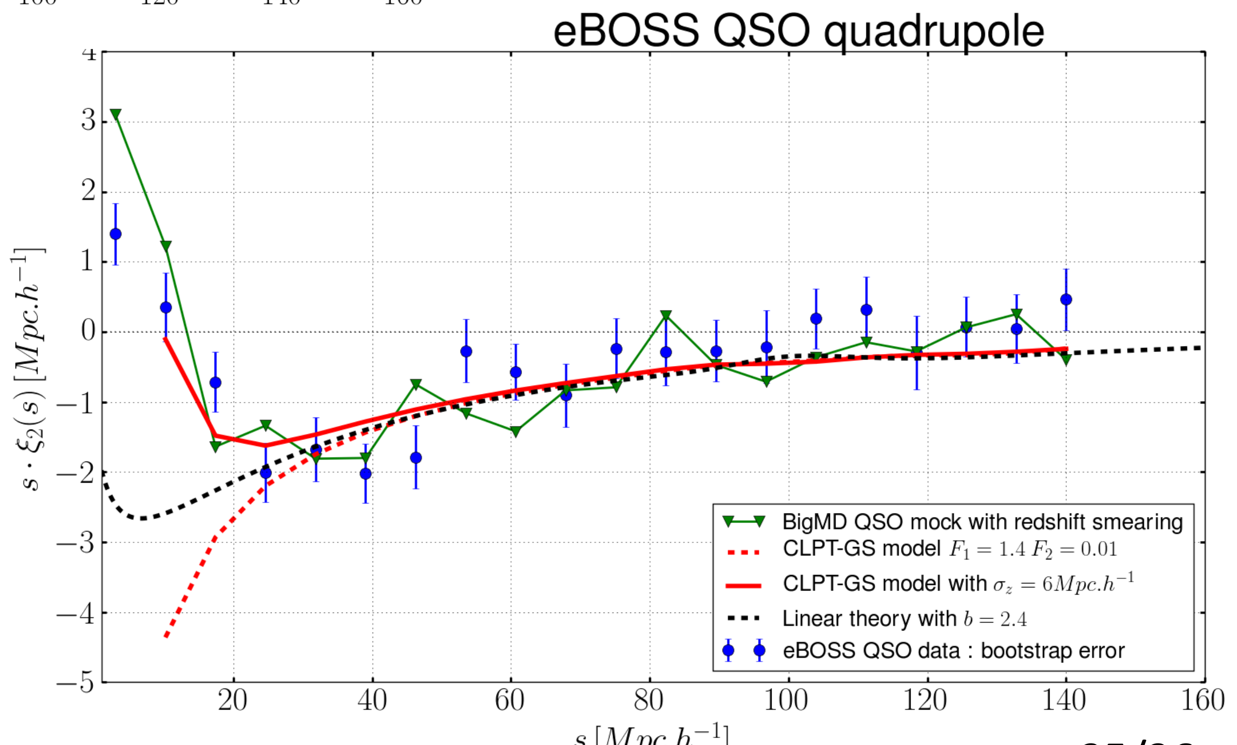
<F''> = 0.01

→ To be checked: CLPT predictions agreement with N-body simulations at z~1.5 for scales above 20 h^{-1}.Mpc and for the full halo mass range





RSD-GS model prediction for monopole and quadrupole



Modeling RSD

?	Perturbation theory	Linear theory

Evolution of density and velocity fields

Galaxy-halo connection
- scale-dependent bias
- velocity bias
- galaxy assembly
- FoG

Non-Gaussianity

Perturbative expansion about δ_{LINEAR} in SPT

$$\delta(\boldsymbol{k};\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(\boldsymbol{k}),$$

Perturbative expansion about $\Psi^{(1)}$ in LPT

$$\boldsymbol{\Psi}(\boldsymbol{q},t) = \boldsymbol{\Psi}^{(1)}(\boldsymbol{q},t) + \boldsymbol{\Psi}^{(2)}(\boldsymbol{q},t) + \boldsymbol{\Psi}^{(3)}(\boldsymbol{q},t) + \cdots$$

From halos to galaxies: bias definition
- Local Lagrangian bias (Matsubara 2008)
- Non-local Lagrangian bias (Matsubara 2011, White 2014)

Linear order:

$\delta = \delta_{\mathrm{LINEAR}} = \delta^{(1)}$

Linear coupling:

$$\theta(k) = f\delta_m(k)$$

Linear power spectrum:

$P_{\delta\delta} = P_{\delta\theta} = P_{\theta\theta} = P_m$

Scale-independent and linear bias: $\delta_g = b\delta_m$

Real-to-redshift space mapping

Include skewness (3^{rd} moment) in the expression of P

Gaussian assumption: P is defined by its 2 first moments

$$\boxed{1 + \xi_g(s_\parallel, s_\perp) = \int dr_\parallel [1 + \xi_g(r)] P(v_{12}, \sigma_{12}^2)}$$

Small scales	20-30	Intermediate scales	60-80	Large scales	h^{-1} Mpc