INSTITUT D'ASTROPHYSIQUE DE PARIS

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Dark Matter at Galactic Scales & Alternatives

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PHENOMENOLOGY OF DARK MATTER

Evidence for dark matter in Astrophysics

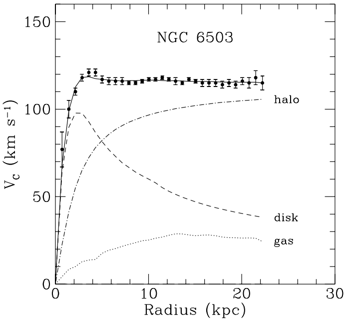
1. Oort [1932] noted that the sum of observed mass in the vicinity of the Sun falls short of explaining the vertical motion of stars in the Milky Way

2. Zwicky [1933] reported that the velocity dispersion of galaxies in galaxy clusters is far too high for these objects to remain bound for a substantial fraction of cosmic time

3. Ostriker & Peebles [1973] showed that to prevent the growth of instabilities in cold self-gravitating disks like spiral galaxies, it is necessary to embed the disk in the quasi-spherical potential of a huge halo of dark matter

4. Bosma [1981] and Rubin [1982] established that the rotation curves of galaxies are approximately flat, contrarily to the Newtonian prediction based on ordinary baryonic matter

Rotation curves of galaxies are approximately flat



- For a circular orbit we expect

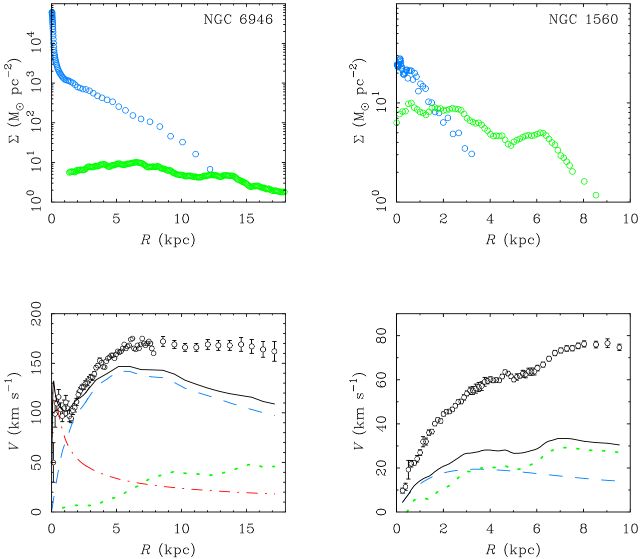
$$v(r) = \sqrt{\frac{G\,M(r)}{r}}$$

- The fact that $v(r)$ is constant implies that beyond the optical disk

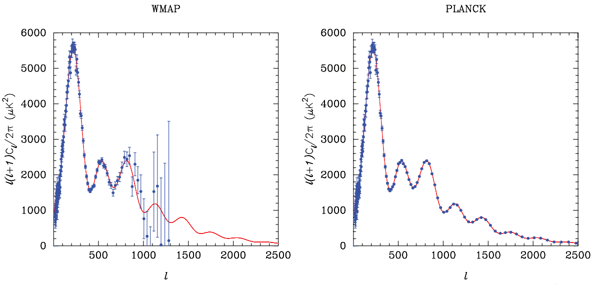
$$\boxed{M_{\text{halo}}(r) \simeq r \qquad \rho_{\text{halo}}(r) \simeq \frac{1}{r^2}}$$

HSB versus LSB galaxies

High Surface Brightness | Low Surface Brightness



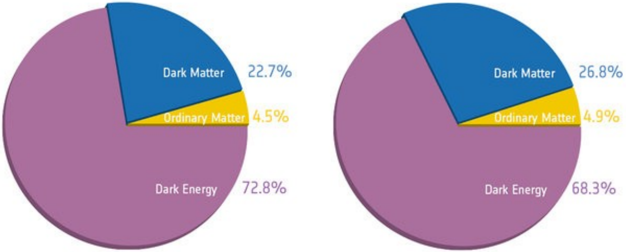
The cosmological concordance model Λ-CDM



This model brilliantly accounts for

- The mass discrepancy between the dynamical and luminous masses of clusters of galaxies
- The precise measurements of the anisotropies of the cosmic microwave background (CMB)
- The formation and growth of large scale structures as seen in deep redshift and weak lensing surveys
- The fainting of the light curves of distant supernovae

Problem of the dark constituents of the Universe

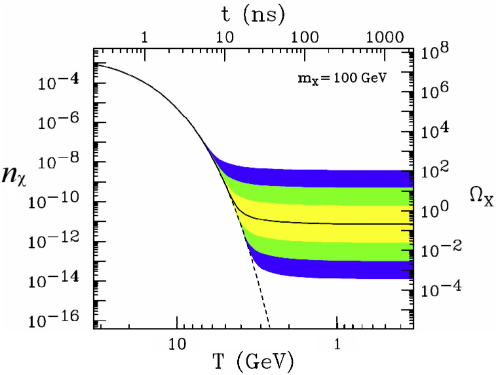


Before Planck — After Planck

Λ-CDM assumes GR is correct but

- No known particle in the standard model of particle physics could be the particle of dark matter
- Extensions of the standard model of particle physics provide well-motivated but yet to be discovered candidates
- The numerical value of the cosmological constant Λ looks un-natural from a quantum field perspective

The WIMP miracle [Lee & Weinberg 1977, Kolb & Turner 1988]



1. A new (heavy) particle X is initially in thermal equilibrium. Its relic density is

$$\Omega_X \propto \frac{1}{\langle \sigma v \rangle} \sim \frac{m_X^2}{g_X^4}$$

2. With $m_X \sim 100$ Gev and $g_X \sim 0.6$ (electroweak scale)

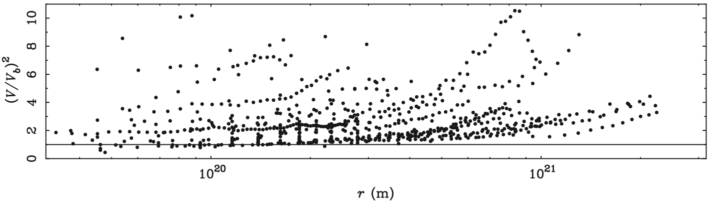
$$\boxed{\Omega_X \sim 0.1}$$

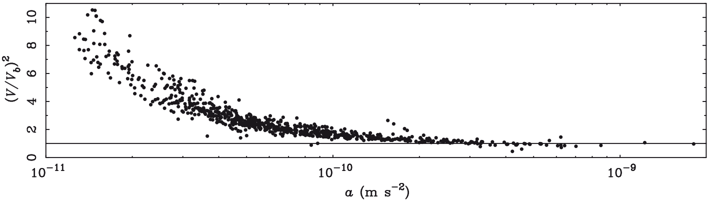
Challenges with CDM at galactic scales

The CDM paradigm faces severe challenges when compared to observations at galactic scales [McGaugh & Sanders 2004, Famaey & McGaugh 2012]

1. Unobserved predictions
   - Numerous but unseen satellites of large galaxies
   - Phase-space correlation of galaxy satellites
   - Generic formation of dark matter cusps in galaxies
   - Tidal dwarf galaxies dominated by dark matter
2. Unpredicted observations
   - Correlation between mass discrepancy and acceleration
   - Surface brightness of galaxies and the Freeman limit
   - Flat rotation curves of galaxies
   - Baryonic Tully-Fisher relation for spirals
   - Faber-Jackson relation for ellipticals

Mass discrepancy versus acceleration [Milgrom 1983]





A critical acceleration scale $a_0 \simeq 1.2 \times 10^{-10}$m/s^2 is present in the data

The critical acceleration scale a_0

- The mass discrepancy is given by

$$\frac{M_{\mathsf{dyn}}}{M_{\mathsf{b}}} \simeq \left(\frac{V}{V_{\mathsf{b}}}\right)^2 \sim \sqrt{\frac{a_0}{g}}$$

- The scale a_0 gives the transition between galaxies with high and low central surface brightness

$$\begin{cases} \Sigma \gtrsim \dfrac{a_0}{G} & \text{for HSB galaxies (baryon dominated)} \\ \Sigma \lesssim \dfrac{a_0}{G} & \text{for LSB galaxies (DM dominated)} \end{cases}$$

- The measured value of a_0 is very close (mysteriously enough) to typical cosmological values

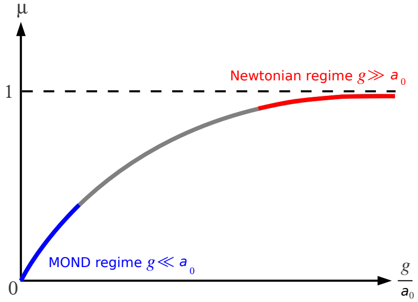
$$\boxed{a_0 \simeq 1.3\, a_\Lambda \quad \text{with} \quad a_\Lambda = \frac{c^2}{2\pi}\sqrt{\frac{\Lambda}{3}}}$$

The MOND formula [Milgrom 1983, Bekenstein & Milgrom 1984]

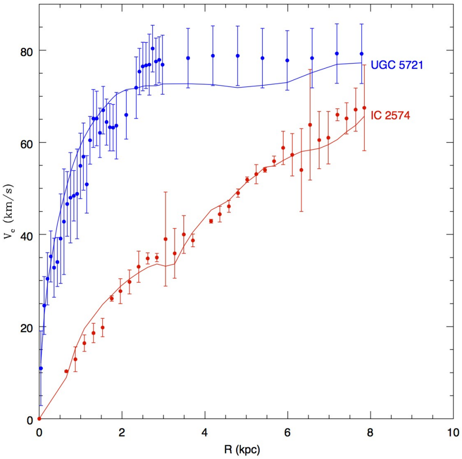
The previous challenges are mysteriously solved by the MOND empirical formula

$$\boxed{\nabla \cdot \left[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \right] = -4\pi \, G \, \rho_{\text{baryon}} \quad \text{avec} \quad \boldsymbol{g} = \nabla U}$$

- The Newtonian regime is recoved when $g \gg a_0$
- In the MOND regime $g \ll a_0$ we have $\boxed{\mu \simeq g/a_0}$



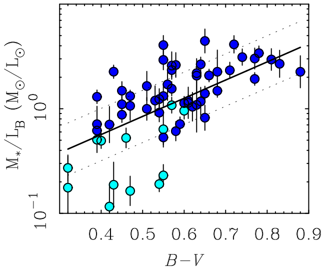
MOND fit of rotation curves of galaxies

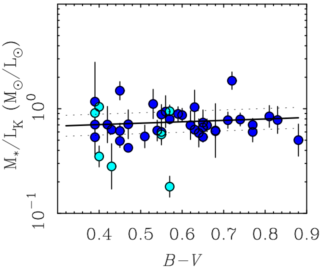


Milgrom's formula provides a mapping between the observed baryons and the observed rotation curve, including the bumps and wiggles

Fit of the mass-to-luminosity ratio

The MOND fit of rotation curves is actually a one-parameter fit as the mass-to-luminosity ratio M/L of each galaxy is adjusted





The best-fit M/L shows the same trend with colour as is implied by models of stellar population synthesis [Bell *et al.* 2003]

Flat rotation curve up to infinity?

- An isolated galaxy in MOND has a logarithmic potential

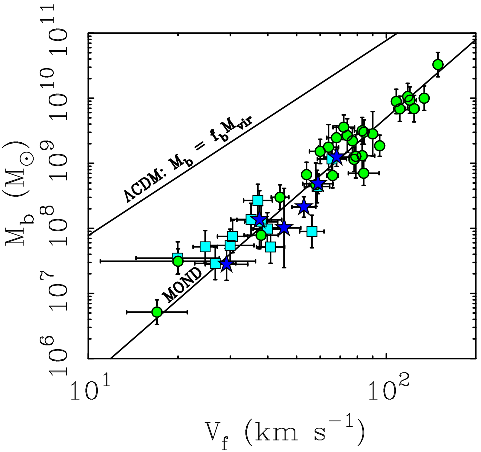
$$U = \sqrt{GMa_0}\,\ln\left(\frac{r}{r_0}\right) \qquad \text{when} \qquad r \gtrsim \overbrace{r_0 = \sqrt{\frac{GM}{a_0}}}^{\text{MOND transition radius}}$$

- In particular, there would be no escape velocity from that galaxy and the rotation curve would be constant up to infinity
- However the galaxy is embedded into the external field g_e of other galaxies and the MOND potential then becomes asymptotically Newtonian

$$U = \frac{GM/\mu_\mathsf{e}}{r} \qquad \text{when} \qquad r \gtrsim r_0\frac{a_0}{g_\mathsf{e}}$$

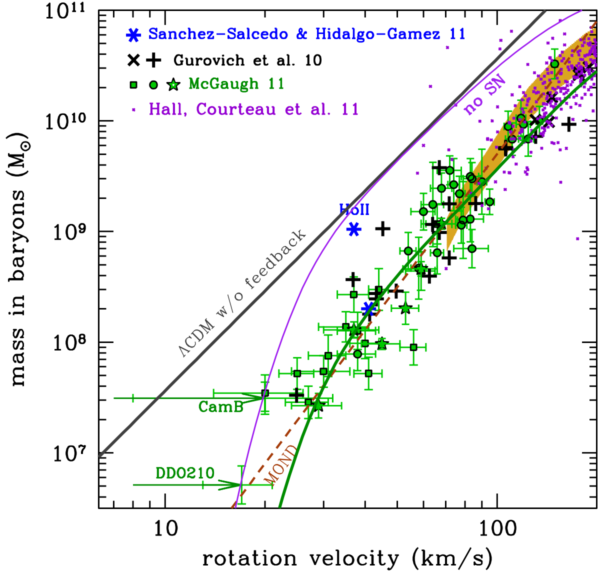
- This is a particular case of the external field effect in MOND [Milgrom 1983]
- An interesting test of MOND has been made by measuring the velocity of isolated stars escaping our galaxy [Famaey, Bruneton & Zhao 2007]

Baryonic Tully-Fisher relation [Tully & Fisher 1977, McGaugh 2011]

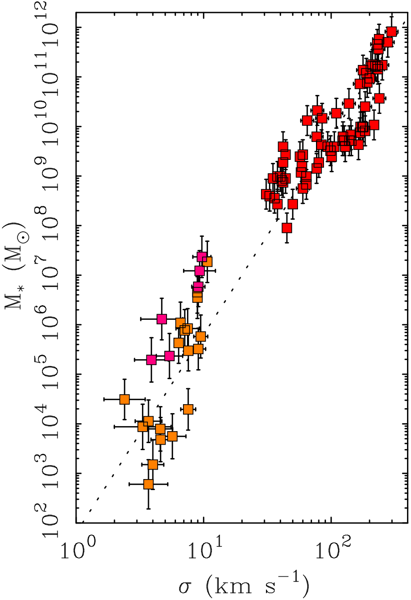


The asymptotic flat rotation velocity is approximately $V_\mathrm{f} \simeq \left(G\, M_\mathrm{b}\, a_0\right)^{1/4}$

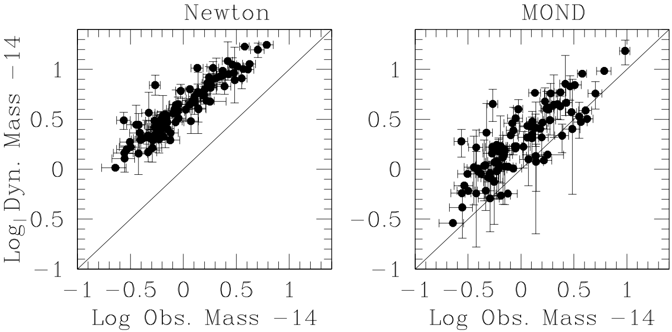
BTF relation fitted with Λ-CDM [Silk & Mamon 2012]



Mass velocity dispersion relation [Faber & Jackson 1976]

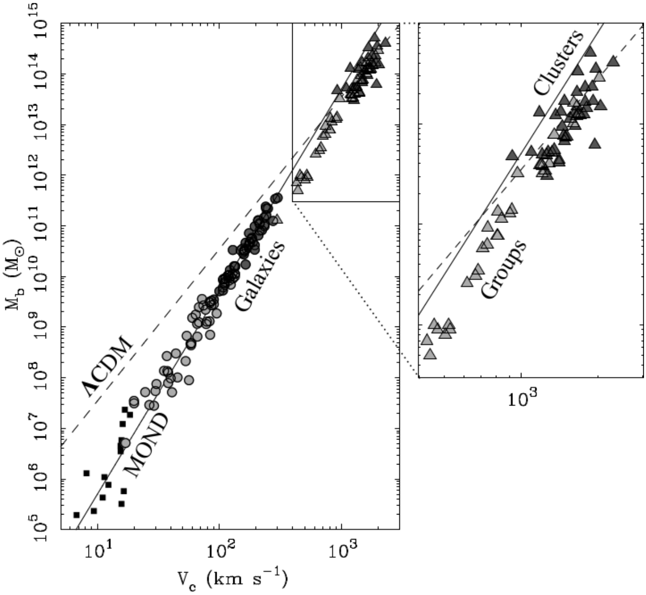


Problem with galaxy clusters [Gerbal, Durret *et al.* 1992, Sanders 1999]



- The mass discrepancy is $\approx 4-5$ with Newton and ≈ 2 with MOND
- The bullet cluster and more generally X-ray emitting galaxy clusters can be fitted with MOND only with a component of baryonic dark matter and/or hot/warm neutrinos [Angus, Famaey & Buote 2008]

Galactic versus cosmological scales



TEST OF MOND IN THE SOLAR SYSTEM

What about the Solar System scale?

- In spherical symmetry the MOND equation becomes

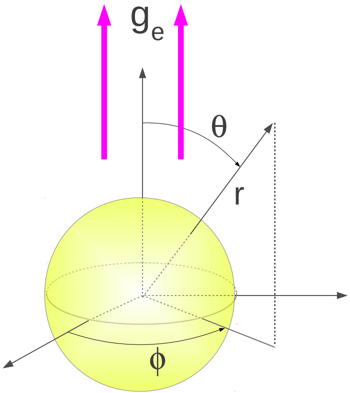
$$\mu\left(\frac{g}{a_0}\right) g = \frac{GM_\odot}{r^2}$$

- With $r_0 = \sqrt{\frac{GM_\odot}{a_0}} \approx 7100\,\text{AU}$ (MOND transition radius for the Sun)

$$\boxed{g = \frac{GM_\odot}{r^2} + k\,a_0 \left(\frac{r}{r_0}\right)^n}$$

- The case $n = 0$ corresponds to a "Pioneer" anomaly and is excluded by planetary ephemerides at the level $5 \times 10^{-13}\text{m/s}^2$ [Fienga, Laskar *et al.* 2009]

EFE in the inner Solar System [Milgrom 2009, Blanchet & Novak 2011]



The MOND field of the Sun, in the presence of the external field of the Galaxy, is deformed along the direction of the Galactic center

$$U = \boldsymbol{g}_e \cdot \mathbf{x} + \frac{GM_\odot/\mu_e}{r\sqrt{1+\lambda_e \sin^2\theta}} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

This effect influences the motion of inner planets of the Solar System

EFE in the inner Solar System [Milgrom 2009, Blanchet & Novak 2011]

1. The effect involves an abnormal quadrupole moment Q_{ij}

$$U_\odot = \frac{GM_\odot}{r} + \frac{1}{2}x^i x^j Q_{ij}$$

2. This yields a suplementary precession of the semi-major axis of planets

$$\begin{aligned}
\left\langle \frac{\mathrm{d}e}{\mathrm{d}t} \right\rangle &= \frac{5Q_2 e\sqrt{1-e^2}}{4n}\sin(2\tilde{\omega}) \\
\left\langle \frac{\mathrm{d}\ell}{\mathrm{d}t} \right\rangle &= n - \frac{Q_2}{12n}\Big[7 + 3e^2 + 15(1+e^2)\cos(2\tilde{\omega})\Big] \\
\left\langle \frac{\mathrm{d}\tilde{\omega}}{\mathrm{d}t} \right\rangle &= \frac{Q_2\sqrt{1-e^2}}{4n}\Big[1 + 5\cos(2\tilde{\omega})\Big]
\end{aligned}$$

Contraints from Solar System ephemerides

Predicted values for the orbital precession

Quadrupolar precession rate in mas/cy						
	Mercury	Venus	Earth	Mars	Jupiter	Saturn
μ_1	0.04	0.02	0.16	-0.16	-1.12	5.39
μ_2	0.02	0.01	0.09	-0.09	-0.65	3.12
μ_5	7×10^{-3}	3×10^{-3}	0.03	-0.03	-0.22	1.05
μ_{20}	2×10^{-3}	10^{-3}	9×10^{-3}	-9×10^{-3}	-0.06	0.3

Best published residuals for orbital precession

Postfit residuals for the precession rates in mas/cy						
	Mercury	Venus	Earth	Mars	Jupiter	Saturn
[Pitjeva 2005]	-3.6 ± 5	-0.4 ± 0.5	-0.2 ± 0.4	0.1 ± 0.5	-	-6 ± 2
[Fienga *et al.* 2009]	-10 ± 30	-4 ± 6	0 ± 0.016	0 ± 0.2	142 ± 156	-10 ± 8
[Fienga *et al.* 2010]	0.4 ± 0.6	0.2 ± 1.5	-0.2 ± 0.9	0 ± 0.1	-41 ± 42	0.2 ± 0.7

MOND seems to be marginally excluded by planetary ephemerides

MODIFIED GRAVITY THEORIES

Modified gravity theories without DM

1. Generalized Tensor-Scalar theory (RAQUAL) [Bekenstein & Sanders 1994]
2. Tensor-Vector-Scalar theory (TeVeS) [Bekenstein 2004]
3. Generalized Einstein-Æther theory [Zlosnik *et al.* 2007, Halle *et al.* 2008]
4. Khronometric theory [Blanchet & Marsat 2011, Bonetti & Barausse 2015]
5. Bimetric theory (BIMOND) [Milgrom 2012]

Generalized scalar-tensor (RAQUAL) [Bekenstein & Sanders 1994]

1. Formalism of scalar-tensor theories [Jordan 1946, Brans & Dicke 1961]

$$\boxed{\overbrace{g_{\mu\nu}}^{\text{Einstein metric}} \quad \overbrace{\phi}^{\text{scalar field}}}$$ describe the gravitational field

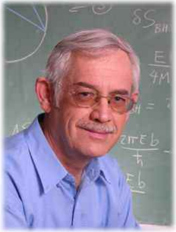
$$\boxed{\overbrace{\tilde{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}}^{\text{physical metric}}}$$ coupling of matter to physical metric

2. A function F is introduced into the action which will reduce to the MOND function μ in the limit $c \to +\infty$

$$\underbrace{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}_{\text{standard kinetic term}} \Longrightarrow \underbrace{a_0^2 F\left(\frac{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{a_0^2}\right)}_{\text{aquadratic kinetic term}}$$

3. However light signals do not feel the presence of the scalar field so the theory does not explain the dark matter seen by gravitational lensing

Tensor-vector-scalar (TeVeS) [Bekenstein 2004]



1. The scalar-tensor part is similar to RAQUAL
2. To obtain the right light deflection we introduce a vector field V_μ and modify the matter coupling

$$\boxed{\tilde{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu} + \overbrace{\left(e^{2\phi} - e^{-2\phi}\right) V_\mu V_\nu}^{\text{vector field contribution}}}$$

3. A kinetic term for the vector field is added to the action

$$\boxed{\underbrace{g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\mu\nu}}_{\text{kinetic term}} + \lambda \underbrace{\left(g^{\mu\nu} V_\mu V_\mu + 1\right)}_{\text{Lagrange constraint}}}$$

Tentative conclusions on modified gravity theories

1. Complicated theories modifying GR with *ad-hoc* extra fields
2. No physical explanation for the origin of the MOND effect
3. Non-standard kinetic terms depending on an arbitrary function which is linked *in fine* to the MOND function
4. Stability problems associated with the fact that the Hamiltonian is not bounded from below [Clayton 2001, Bruneton & Esposito-Farèse 2007]
5. Generic problems to recover the cosmological model Λ-CDM at large scales and in particular the spectrum of CMB anisotropies [Skordis, Mota *et al.* 2006]

DIELECTRIC ANALOGY OF MOND

A remarkable analogy [Blanchet 2006]

- In electrostratics the Gauss equation is modified by the polarization of the dielectric (dipolar) material

$$\nabla \cdot \Big[\underbrace{(1+\chi_e)\boldsymbol{E}}_{\boldsymbol{D}\ \text{field}}\Big] = \frac{\rho_e}{\varepsilon_0} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{E} = \frac{\rho_e + \rho_e^{\text{polar}}}{\varepsilon_0}$$

- Similarly MOND can be viewed as a modification of the Poisson equation by the polarization of some dipolar medium

$$\nabla \cdot \left[\mu\left(\frac{g}{a_0}\right)\boldsymbol{g}\right] = -4\pi\, G\, \rho_{\mathsf{b}} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{g} = -4\pi\, G \left(\rho_{\mathsf{b}} + \underbrace{\rho^{\text{polar}}}_{\text{dark matter}}\right)$$

- The MOND function can be written $\mu = 1 + \chi$ where χ appears as a susceptibility coefficient of some “dipolar dark matter” medium

Microscopic description of DDM?

- The DM medium by individual dipole moments $\boldsymbol{p}$ and a polarization field $\boldsymbol{P}$

$$\boldsymbol{P} = n\,\boldsymbol{p} \qquad \text{with} \qquad \boldsymbol{p} = m\,\boldsymbol{\xi}$$

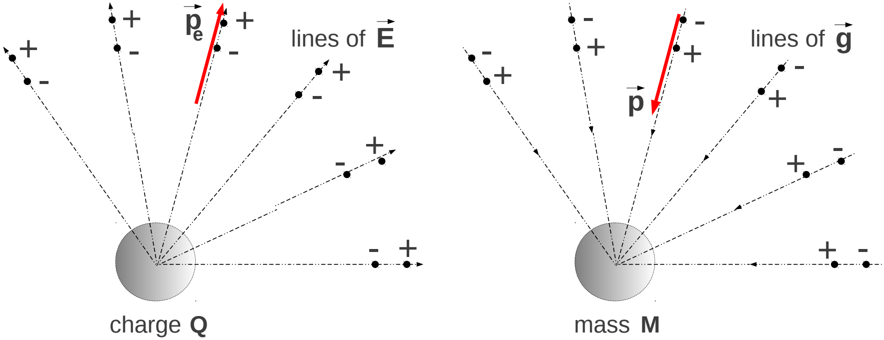
- The polarization is induced by the gravitational field of ordinary masses

$$\boldsymbol{P} = -\frac{\chi}{4\pi\,G}\,\boldsymbol{g} \qquad \boxed{\rho_{\text{DDM}} = -\boldsymbol{\nabla}\cdot\boldsymbol{P}}$$

- Because like masses attract and unlike ones repel we have anti-screening of ordinary masses by polarization masses in agreement with DM and MOND

$$\boxed{\chi < 0}$$

Anti-screening by polarization masses



Screening by polarization charges

$\boxed{\chi_e > 0}$

Anti-screening by polarization masses

$\boxed{\chi < 0}$

Need of a non-gravitational internal force

- The constituents of the dipole will repel each other so we need a non-gravitational force

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{\nabla}\big(U+\phi\big) \qquad \frac{\mathrm{d}\boldsymbol{v}'}{\mathrm{d}t} = -\boldsymbol{\nabla}\big(U+\phi\big)$$

  and look for an equilibrium when $\boldsymbol{\nabla}\big(U+\phi\big) = \mathbf{0}$

- The internal force is generated by the gravitational charge *i.e.* the mass

$$\Delta\phi = -\frac{4\pi G}{\chi}\big(\rho-\rho'\big)$$

- The DM medium appears as a polarizable plasma (similar to e^+e^- plasma) oscillating at the natural plasma frequency

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} + \omega^2\boldsymbol{\xi} = 2\boldsymbol{g} \qquad \text{with} \qquad \omega = \sqrt{-\frac{8\pi\, G\, \rho_0}{\chi}}$$

DIPOLAR DARK MATTER THEORIES

DDM in standard GR [Blanchet & Le Tiec 2008; 2009]

The action in standard general relativity is

$$S_{\mathrm{DDM}} = \int \mathrm{d}^4x \sqrt{-g} \Big[\overbrace{-\rho}^{\text{CDM}} + \underbrace{J^\mu \dot{\xi}_\mu - V(P_\perp)}_{\text{self-interaction with dipole moment}} \Big]$$

PROS

- Simple model
- Recovers Λ-CDM at first-order cosmological perturbation and is therefore in agreement with CMB anisotropies

CONS

- MOND is recovered at the price of an hypothesis of weak clustering hypothesis of DDM in cosmology
- The equation of evolution of the dipole moment involves an instability

DDM via a bimetric extension of GR [Bernard & Blanchet 2014]

1. To describe relativistically some microscopic DM particles with positive or negative gravitational masses one needs two metrics
   - $g_{\mu\nu}$ obeyed by ordinary particles (including baryons)
   - $f_{\mu\nu}$ obeyed by "dark" particles
2. In addition the DM particles forming the dipole moment should interact via a non-gravitational force field, *e.g.* a (spin-1) "graviphoton" vector field $\mathcal{A}_\mu$ with field strength $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ [Scherk 1968]
3. One needs to introduce into the action the kinetic terms for all these fields, and to define the interaction between the two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$

DDM via a bimetric extension of GR [Bernard & Blanchet 2014]

The action of the model involves three sectors

$$S = \int \mathrm{d}^4x \Big\{ \overbrace{\sqrt{-g}\left(\frac{R_g}{32\pi} - \rho_{\text{bar}} - \rho_g\right)}^{\text{ordinary sector}} + \overbrace{\sqrt{-f}\left(\frac{R_f}{32\pi} - \rho_f\right)}^{\text{dark sector}}$$
$$+ \underbrace{\sqrt{-\mathcal{G}_{\text{eff}}}\left[\frac{\mathcal{R}_{\text{eff}}}{16\pi\varepsilon} + (\mathcal{J}_g^\mu - \mathcal{J}_f^\mu)\mathcal{A}_\mu + \frac{a_0^2}{8\pi}\mathcal{W}(\mathcal{X})\right]}_{\text{interaction sector}} \Big\}$$

The two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$ interact *via* the Ricci scalar associated with a particular auxiliary metric $\mathcal{G}^{\text{eff}}_{\mu\nu}$ built out of the two

A phenomenological but non viable model

PROS

- The cosmological model Λ-CDM and its successes at cosmological scales and notably the fit of the CMB are recovered
- The phenomenology of MOND is "explained" by a physical mechanism of gravitational polarization
- The dark matter appears as a diffuse polarizable medium undergoing stable plasma-like oscillations
- The model is viable in the Solar System as its PPN parameters are the same as in general relativity

CONS

- Very complicated model
- The interaction between the two metrics implies the presence of ghosts in the gravitational sector [Blanchet & Heisenberg 2015]

Search for a massive gravity theory

1. Unique linear theory without ghosts [Fierz & Pauli 1939]

$$S_{\mathsf{FP}} = \frac{1}{16\pi}\int \mathrm{d}^4x\Big[\underbrace{\partial_\mu h_{\nu\rho}\partial^\mu \overline{h}^{\nu\rho} - H_\mu H^\mu}_{\text{linear Einstein-Hilbert action}} + \underbrace{m^2\big(h_{\mu\nu}h^{\mu\nu} - h^2\big)}_{\text{mass term}}\Big]$$

2. Massless limit of massive gravity differs from GR and is invalidated in the Solar System [Van Dam, Veltman & Zhakharov 1970]

$$\gamma_{\mathsf{PPN}} = \frac{1}{2} \quad (?)$$

3. Nonlinear terms restaure the continuity of the massless limit and GR is recovered for [Vainshtein 1972; Babichev, Deffayet & Ziour 2009]

$$r \lesssim r_{\mathsf{Vainshtein}} \simeq \left(\frac{M}{m^4}\right)^{1/5}$$

4. Instability appears at quadratic order due to 6 degrees of freedom for the massive spin-2 field instead of 5 [Boulware & Deser 1972]

Nonlinear ghost-free massive (bi-)gravity

[de Rham, Gabadadze & Tolley 2011; Hassan & Rosen 2012]

The theory (called dRGT) is defined non-perturbatively as

$$\boxed{S = \int \mathrm{d}^4x \left[\frac{M_g^2}{2}\sqrt{-g}R_g + \overbrace{m^2\sqrt{-g}\sum_{n=0}^{4}\alpha_n e_n(X)}^{\text{ghost-free interaction mass term}} + \frac{M_f^2}{2}\sqrt{-f}R_f \right]}$$

where the interaction between the two metrics is algebraic and defined from the elementary symmetric polynomials $e_n(X)$ of the square root matrix $X = \sqrt{g^{-1}f}$

DDM via massive bigravity theory [Blanchet & Heisenberg 2015]

1. The matter sector is the same as in the previous model
2. The gravitational sector of the model is based on massive bigravity theory

$$S = \int \mathrm{d}^4x \Bigg\{ \sqrt{-g}\left(\frac{M_g^2}{2}R_g - \rho_{\text{bar}} - \rho_g\right) + \sqrt{-f}\left(\frac{M_f^2}{2}R_f - \rho_f\right) + \sqrt{-g_{\text{eff}}}\left[\frac{m^2}{4\pi} + \mathcal{A}_\mu \left(j_g^\mu - \frac{\alpha}{\beta} j_f^\mu\right) + \frac{a_0^2}{8\pi}\mathcal{W}(X)\right]\Bigg\}$$

3. The ghost-free potential interactions take the form of the square root of the determinant of the effective metric [de Rham, Heisenberg & Ribeiro 2014]

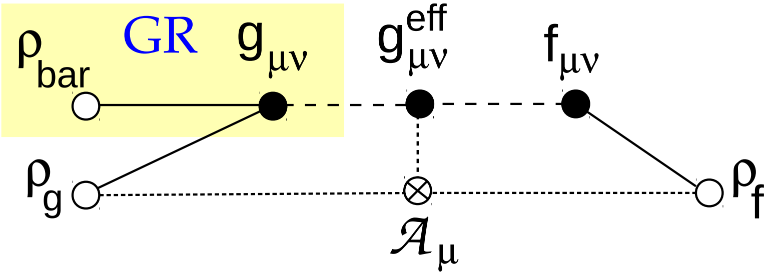
$$\boxed{g_{\mu\nu}^{\text{eff}} = \alpha^2 g_{\mu\nu} + 2\alpha\beta\, g_{\mu\rho}X^\rho_\nu + \beta^2 f_{\mu\nu}}$$

with the square-root matrix $X = \sqrt{g^{-1}f}$

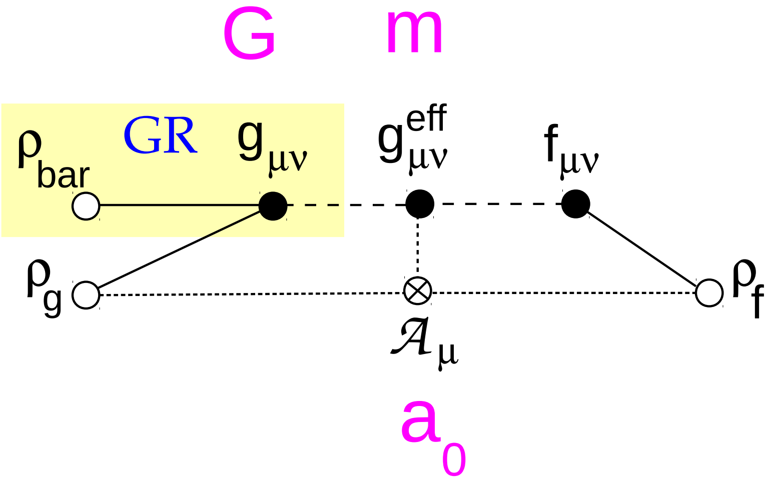
4. The graviphoton $\mathcal{A}_\mu$ is coupled to the effective metric $g_{\mu\nu}^{\text{eff}}$

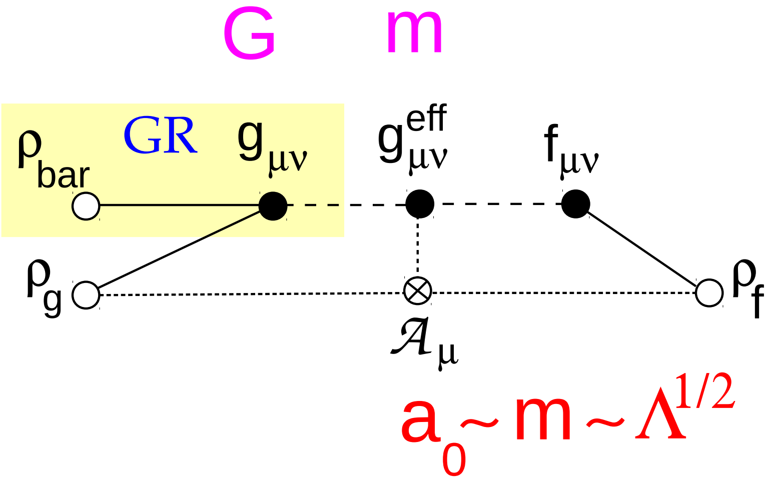
General structure of the model

General structure of the model



General structure of the model




G
m
GR
ρ_{bar}
$g_{\mu\nu}$
$g^{eff}_{\mu\nu}$
$f_{\mu\nu}$
ρ_g
ρ_f
$\mathcal{A}_\mu$
$a_0 \sim m \sim \Lambda^{1/2}$

Gravitational polarization & MOND

1. Equations of motion of DM particles in the non-relativistic limit $c \to \infty$

$$\frac{\mathrm{d}\boldsymbol{v}_g}{\mathrm{d}t} = \boldsymbol{\nabla}\big(U_g + \phi\big) \qquad \frac{\mathrm{d}\boldsymbol{v}_f}{\mathrm{d}t} = \boldsymbol{\nabla}\big(U_f - \frac{\alpha}{\beta}\phi\big)$$

2. With massive bigravity the two g and f sectors are linked together by a constraint equation coming from the Bianchi identities

$$\boxed{\boldsymbol{\nabla}\big(\alpha U_g + \beta U_f\big) = 0}$$

   showing that α/β is the ratio between gravitational and inertial masses of f particles with respect to g metric

3. The DM medium is at equilibrium when the Coulomb force annihilates the gravitational force, $\boldsymbol{\nabla} U_g + \boldsymbol{\nabla}\phi = 0$, at which point the polarization is aligned with the gravitational field

$$\boxed{\boldsymbol{P} = \frac{1}{4\pi}\,\mathcal{W}'\,\boldsymbol{\nabla} U_g}$$

Gravitational polarization & MOND

1. From the massless combination of the two metrics combined with the Bianchi identity we get a Poisson equation for the ordinary Newtonian potential U_g

$$\Delta U_g = -4\pi\Big(\rho_{\text{bar}} + \underbrace{\rho_g - \frac{\alpha}{\beta}\rho_f}_{\text{DDM}}\Big)$$

2. With the plasma-like solution for the internal force and the mechanism of gravitational polarization this yields the MOND equation

$$\boxed{\nabla\cdot\Big[\underbrace{\left(1-\mathcal{W}'\right)}_{\text{MOND function}}\nabla U_g\Big] = -4\pi\rho_{\text{bar}}}$$

3. Finally the DM medium undergoes stable plasma-like oscillations in linear perturbations around the equilibrium

Conclusions

1. An important challenge is to reproduce within a single relativistic framework
   - The concordance cosmological model Λ-CDM and its tremendous successes at cosmological scales and notably the fit of the CMB
   - The phenomenology of MOND which is a basic set of phenomena relevant to galaxy dynamics and DM distribution at galactic scales
2. While Λ-CDM meets severe problems when extrapolated at the scale of galaxies, MOND fails on galaxy cluster scales and seems also to be marginally excluded by planetary ephemerides in the Solar System
3. Pure modified gravity theories (extending GR with extra fields) do not seem to be able to reproduce Λ-CDM and the CMB anisotropies at large scales
4. An hybrid approach (DM *à la* MOND) called DDM, defined within the beautiful framework of the massive bigravity theory, could meet the challenges posed by MOND *versus* Λ-CDM