Cosmological production of “Sterile” neutrinos featuring a Diatribe against neutrino oscillations

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March, 2018

Outline

- Neutrino oscillations: the standard schpiel
- When oscillations are nearly irrelevant:
  Neutrino absorption
- Approaching thermal equilibrium
- Three examples of cosmological "sterile neutrinos"
  Reactor-anomaly neutrinos
  Dodelson-Widrow neutrinos
  Resonant neutrinos
- A diatribe against neutrino oscillations

diatribe: a bitter and abusive speech or piece of writing (Webster)

ν_e-ν_s mixing $\Rightarrow$ ν_e-ν_s oscillations

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle \qquad |\nu_s\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

ν_1 and ν_2 have masses m_1 and m_2.
Oscillation probability at a distance L from production:

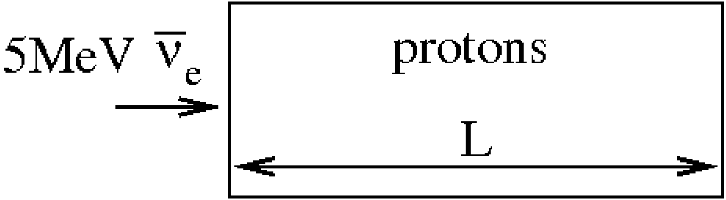
$$P(\nu_e \to \nu_s) = 2\cos^2\theta\sin^2\theta\left[\frac{\sin^2 L/L_{osc}}{1/2}\right]$$

$$P(\nu_e \to \nu_e) = \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos 2L/L_{osc}$$

where $L_{osc} = 4E_\nu/\Delta m^2$.

Probability to see $\bar{\nu}p \to e^+n$ at a distance L is proportional to $P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_s)$.

A $\bar{\nu}_e$ and a box of protons (1)



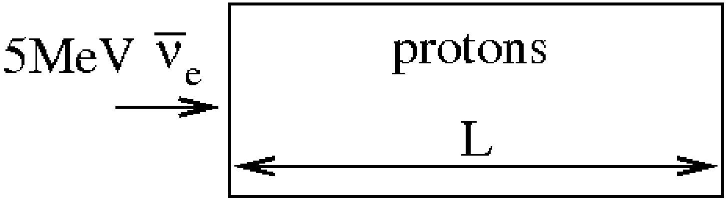
The $\bar{\nu}_e$ can be absorbed via $\bar{\nu}_e p \to e^+ n$, cross section σ
$\Rightarrow$ absorption length $L_{abs} = (\rho\sigma)^{-1}$ $\quad (\rho = \text{protons/volume})$

Question:
What is the probability that the neutrino passes through the box?

Answer if $\theta = 0$ (no mixing):

$$P = \exp(-L\rho\sigma)$$

A $\bar{\nu}_e$ and a box of protons (2)



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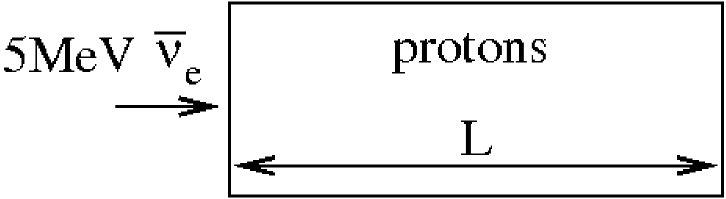
Question:
What is the probability that the neutrino passes through the box?

Answer if $\theta \neq 0$ and $L_{osc} \gg L$:

$$P = \exp(-L\rho\sigma)$$

A $\bar{\nu}_e$ and a box of protons (3)



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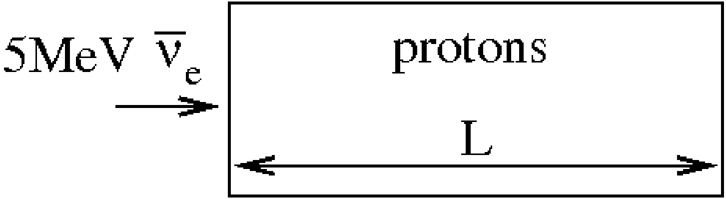

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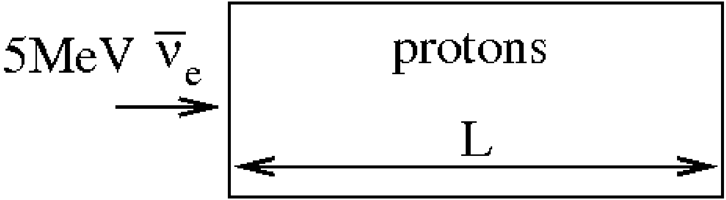
Answer if $\theta \neq 0$ and $L_{osc} \ll L$:

$$P = \cos^2\theta \exp(-L\rho\sigma\cos^2\theta) + \sin^2\theta \exp(-L\rho\sigma\sin^2\theta)$$

A $\bar{\nu}_e$ and a box of protons (3)



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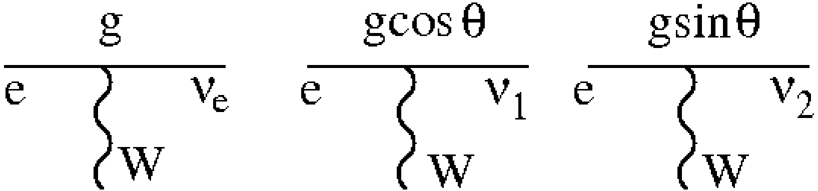
$=$ (Probability $\bar{\nu}_e$ is a $\bar{\nu}_1$) $\times$ (Probability that $\bar{\nu}_1$ passes through box)
$+$ (Probability $\bar{\nu}_e$ is a $\bar{\nu}_2$) $\times$ (Probability that $\bar{\nu}_2$ passes through box)

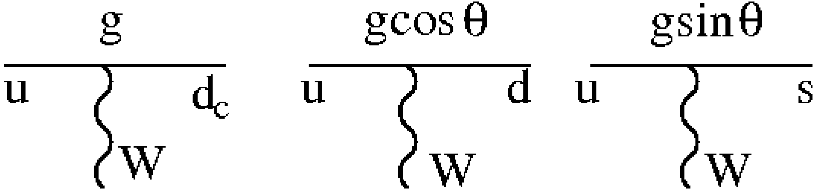
People who immediately find the right answer (1)

- People who have never heard of neutrino oscillations.
  Mixing angles just give branching ratios and cross-sections (like the Cabibbo angle)

ν_1 and ν_2 couplings



Just like Cabibbo quark mixing:



ν_e-ν_s mixing $\Rightarrow$ ν_e-ν_s oscillations

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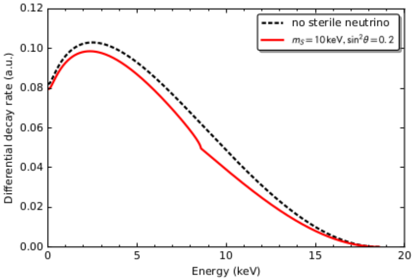
$$P(\nu_e \to \nu_e) = \cos^2\theta\cos^2\theta + \sin^2\theta\sin^2\theta + 2\cos^2\theta\sin^2\theta\cos\frac{2L}{L_{osc}}$$

where $L_{osc} = 4E_\nu/\Delta m^2$.

L-averaged probabilities are naive Cabibbo probabilities

People who immediately find the right answer (2)

- People who have never heard of neutrino oscillations.
  Mixing angles just give branching ratios and cross-sections (like the Cabibbo angle)
- People who search for sterile neutrinos via discontinuities in beta-decay spectra.



Electron spectrum is incoherent sum of spectra for ${}^3\mathrm{H} \rightarrow {}^3\mathrm{He}e^-\nu_1$ and ${}^3\mathrm{H} \rightarrow {}^3\mathrm{He}e^-\nu_2$

Branching ratio to $\nu_2 = \sin^2\theta$
$\Rightarrow$ discontinuity in electron energy spectrum at $(E_{max} - m_2)$.

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- People who have never heard of neutrino oscillations.
  Mixing angles just give branching ratios and cross-sections (like the Cabibbo angle)
- People who search for sterile neutrinos via discontinuities in beta-decay spectra.
  Branching ratio to $\nu_2 = \sin^2\theta$
  $\Rightarrow$ discontinuity in electron energy spectrum at $(E_{max} - m_2)$.
- People who think neutrinos are produced as wave packets.
  ν_1 and ν_2 wave packets quickly separate and become independent, non-interfering objects.
  $L_{sep} \sim L_{osc} \times (E/\Delta E)$
- People who are virtuosi of two-state formalism
  absorption $\Rightarrow$ imaginary part of $\langle\nu_e|H|\nu_e\rangle$

Some conclusions

- For mean absorption, oscillations are usually irrelevant:
  $L \ll L_{osc}$ no time to oscillate
  $L \gg L_{osc}$ oscillations averaged over
- For $L \gg L_{osc}$ best to think of mixing as a Cabibbo problem.
  Use propagation (mass) eigenstates, not flavor eigenstates.
  "Decoherence" of ν_1 and ν_2 which act like "normal particles"
- For $L \gg L_{abs}$ only ν_2 emerge (not ν_s).
  "Of course, such mixing renders these species
  not truly sterile" (Abazajian et al.)
  (We need a word for "nearly sterile mass eigenstate")
- For $\sin^2\theta \ll 1$ ν_2 are hard to produce,
  but they are equally hard to destroy
  $\Rightarrow \rho_1 \sim \rho_2$ is possible in early universe.

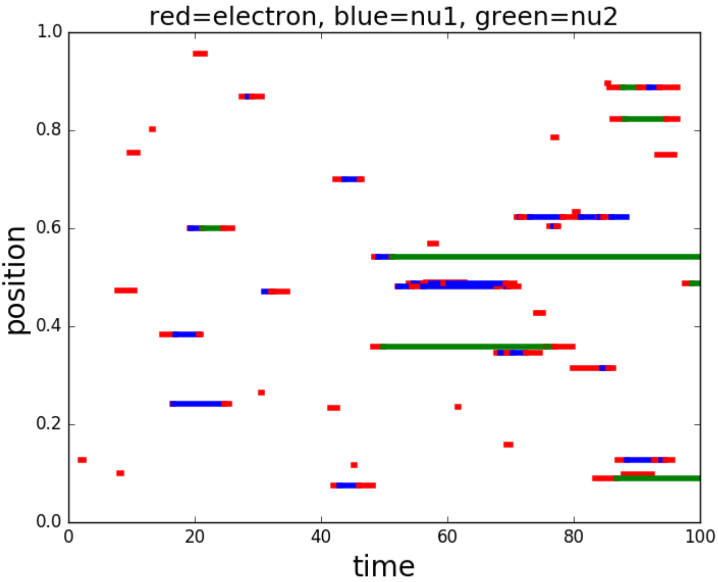
Reaching thermal equilibrium in a static universe

Starting with a thermal mixture of (γ, e^+, e^-, p, n), neutrinos (ν_1, ν_2) are produced: ν_1 rapidly and ν_2 slowly.

ν_1 abundance rapidly increases until creation rate equal destruction rate (i.e. thermal equilibrium)

ν_2 abundance slowly increases until creation rate equal destruction rate (i.e. thermal equilibrium with same density as ν_1 if $m_1 \sim m_2$)

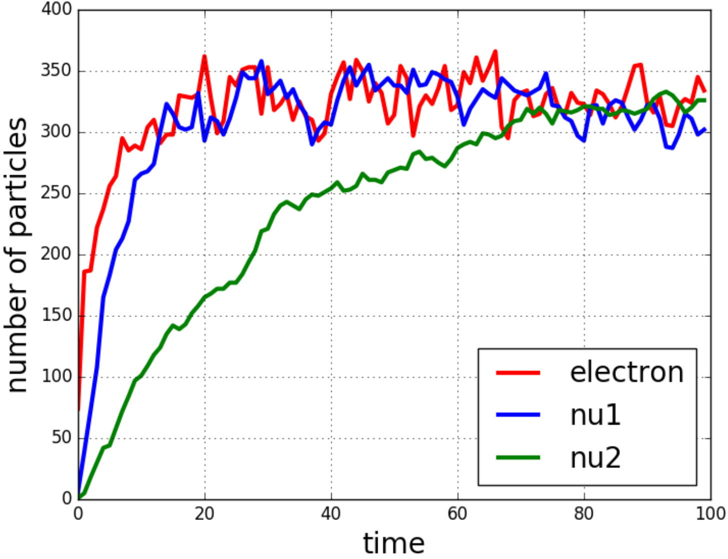
Simulation:Thermal Bath $\leftrightarrow e \leftrightarrow \nu_{1,2}$



$\theta^2 \sim 0.15$

ν_2 rarely produced but live a long time

Simulation: ν_2 reach equilibrium



Reaching equilibrium in an expanding universe

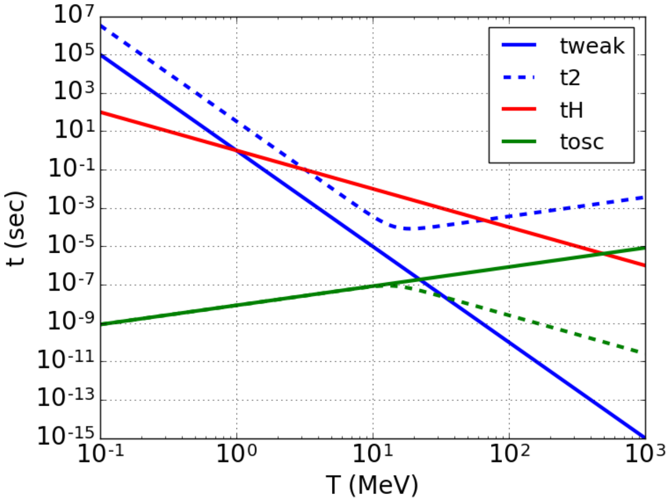
Four time scales:

- $t_H \sim 1/\sqrt{GT^4}$ (Hubble time)
  Time for temperature or density to change significantly
- $t_{weak} \sim 1/(G_F T^2 \times T^3)$
  Absorption, collision, production time for full-strength ν
- $t_2 \equiv t_{weak}/sin^2\theta$
  time for $e \rightarrow \nu_2$ via charged current
- $t_{osc} \sim T/\Delta m^2$

Thermal abundance of ν_2 obtained if at some epoch

- $t_{osc} < t_{weak}$
  $\Rightarrow$ charged-currents give ν_1 and ν_2 rather than ν_e
- $t_2 < t_H$
  $\Rightarrow$ each electron has enough time to produce a ν_2

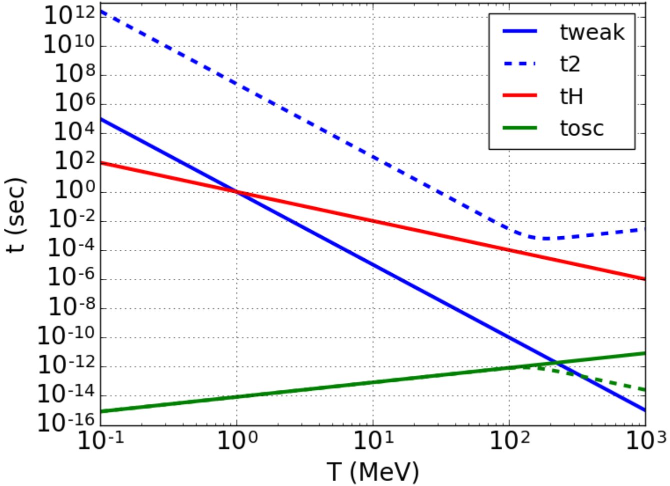
Reactor-anomaly ν: $m_2 \sim 1eV$, $\theta^2 \sim 0.03$



$T > 30$MeV:
$t_{weak} < t_{osc}$
$\Rightarrow e \rightarrow \nu_e$ only

$3 < T < 30$:
$t_2 < t_H$
$\Rightarrow \nu_2$ reaches equilibrium abundance

Warm DM neutrinos: $m_2 \sim 1keV$, $\theta^2 \sim 4 \times 10^{-8}$



Dodelson-Widrow, arXiv:hep-ph/9303287

$T > 300\text{MeV}$:
$t_{weak} < t_{osc}$
$\Rightarrow e \to \nu_e$ only

$t_2 > t_H$
$\Rightarrow \nu_2$ never reaches equilibrium abundance but high mass compensates to give $\Omega_\nu \sim 0.3$

Mixing in Matter (1)

Matter potential, V, modifies mixing angle and oscillation length:

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\sin^2 2\theta + [\cos 2\theta - V/\Delta]^2} \qquad \Delta = \frac{\Delta m^2}{2E_\nu}$$

$$l_{osc,m} = l_{osc} \frac{\sin 2\theta_m}{\sin 2\theta}$$

Matter-antimatter symmetric universe:

$V < 0$ and $\propto T^5 \Rightarrow V/\Delta \sim t_{osc}/t_{weak}$
$\Rightarrow$ mixing angle decreases but only at temperatures where there are no oscillations.

Mixing in Matter (2)

Matter potential, V, modifies mixing angle and oscillation length:

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\sin^2 2\theta + [\cos 2\theta - V/\Delta]^2} \qquad \Delta = \frac{\Delta m^2}{2E_\nu}$$

$$l_{osc,m} = l_{osc} \frac{\sin 2\theta_m}{\sin 2\theta}$$

Lepton-antilepton asymmetric universe:

Abazajian, Fuller and Patel: arXiv:astro-ph/0101524

V can be positive or negative: $\propto (\Delta L) T^3$

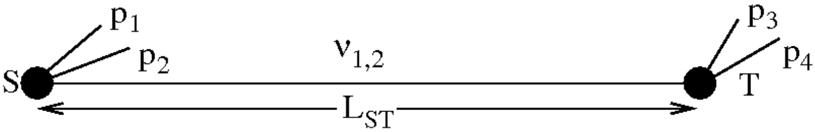
$\Rightarrow$ mixing angle can become large at certain T, E_ν

$\Rightarrow$ non-thermal spectrum of ν_2

Unlike solar neutrinos, level-crossings not generally adiabatic.

“2-state” systems beloved of teachers of QM

- NH_3 molecule
  Tunnelling between two classically degenerate configurations
- K_0-$\bar{K}_0$ system
  Particle and antiparticle share a common decay mode $(\pi\pi)$
- ν_e-ν_μ system
  ν_1 and ν_2 nearly same mass
  $\Rightarrow$ $^{98}\mathrm{X} \rightarrow {}^{98}\mathrm{Y} e^- \bar{\nu}_1$ and $^{98}\mathrm{X} \rightarrow {}^{98}\mathrm{Y} e^- \bar{\nu}_2$ nearly indistinguishable.
  $\Rightarrow$ $\bar{\nu}_1 p \rightarrow ne^+$ and $\bar{\nu}_2 p \rightarrow ne^+$ nearly indistinguishable
  $\Rightarrow$ Amplitudes for $\bar{\nu}_1$ and $\bar{\nu}_2$ scattering interfere.
  $\Rightarrow$ Amplitude is oscillating function of source-target distance



Neutrino Oscillations?

- ν_e-ν_μ system

  ν_1 and ν_2 nearly same mass

  $\Rightarrow {}^{98}\mathrm{X} \rightarrow {}^{98}\mathrm{Y} e^- \bar{\nu}_1$ and ${}^{98}\mathrm{X} \rightarrow {}^{98}\mathrm{Y} e^- \bar{\nu}_2$ nearly indistinguishable.

  $\Rightarrow \bar{\nu}_1 p \rightarrow n e^+$ and $\bar{\nu}_2 p \rightarrow n e^+$ nearly indistinguishable

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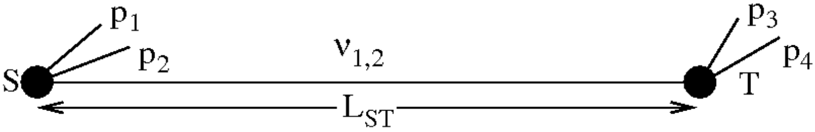

Comment

- The ν_e-ν_μ system lacks the symmetry of NH_3, $K_0 - \bar{K}_0$.
- The two-state formalism hides interesting questions, do the two neutrinos share a common energy or a common momentum?

or Scattering Amplitude oscillations?

What we generally call neutrino oscillations result from an interference between amplitudes for ν_1 scattering and ν_2 scattering. Such interference is very fragile and can be destroyed by:

- averaging over source-target distance
- averaging over "neutrino energy"

Fragility increases with time and distance from source.

$\Rightarrow$ oscillations can only be seen near a neutrino target.

Cosmological neutrinos to not oscillate because:

- Neutrinos don't oscillate (amplitudes oscillate)
  (Adapted from R. Glauber)
- No amplitude oscillations because L_{ST} not well defined, and
- Massive neutrinos have differing trajectories