Beam optics correction and non-linear magnet calibration from resonant driving terms

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Outlines

- Resonance Driving Terms Introductory (15')
- Linear optics errors (10')
    1. RDTs Vs beta-beating and phase advance error
    2. RDTs measurements & correction
    3. Accuracy and precision analysis
- Betatron coupling (10')
    1. RDTs measurements & correction
    2. Hadron Vs lepton machines
- Nonlinear lattice error (model) (15')
    1. Localization & detection of nonlinearities via RDTs
    2. RDTs Vs chromatic functions and orbit feed-downs

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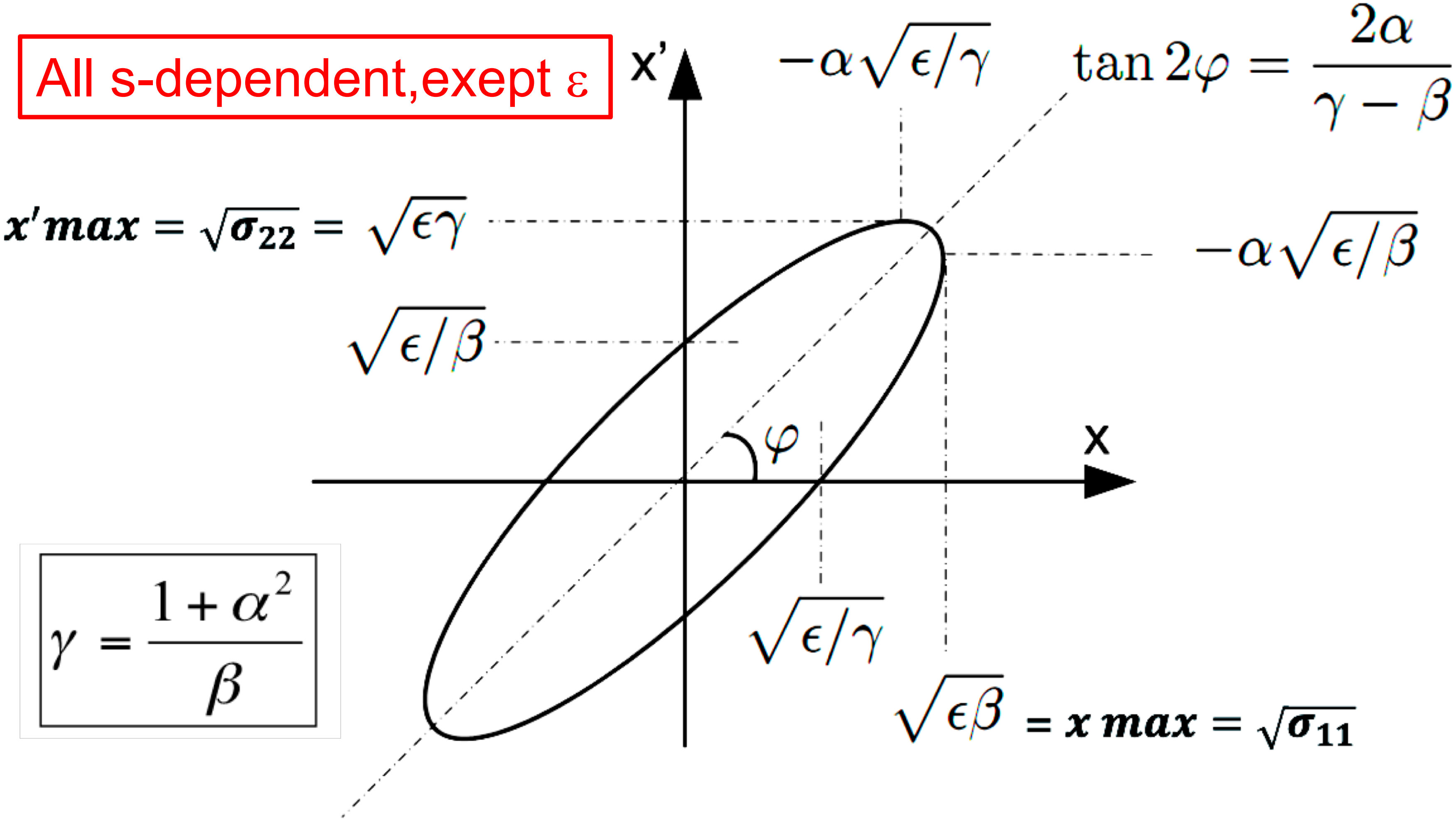
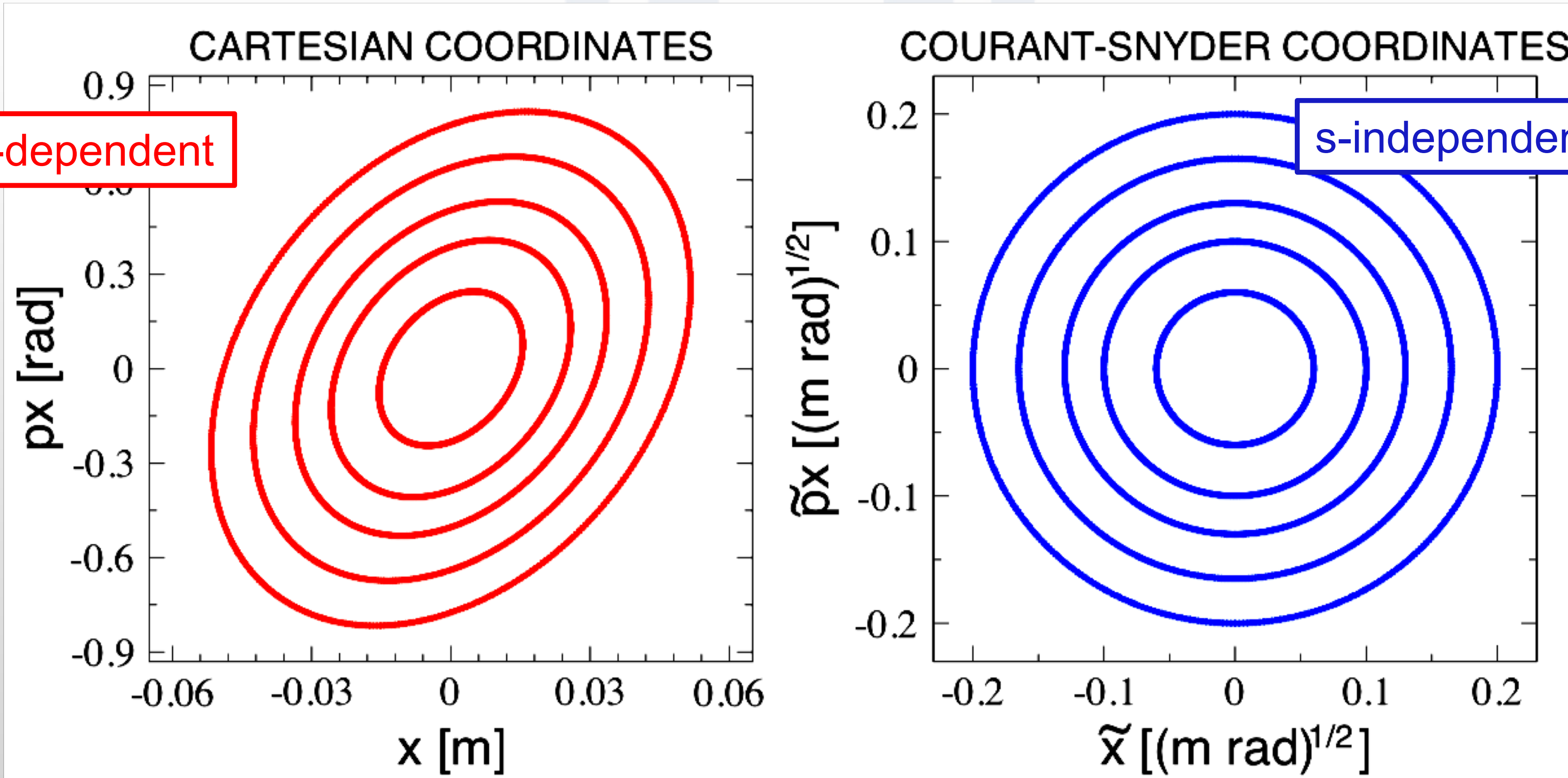
Linear lattice phase space (Cartesian coordinates) & physical meaning of the Twiss parameters

All s-dependent, exept ε



$$\gamma = \frac{1+\alpha^2}{\beta}$$

Linear lattice phase space (@ “small” amplitudes)

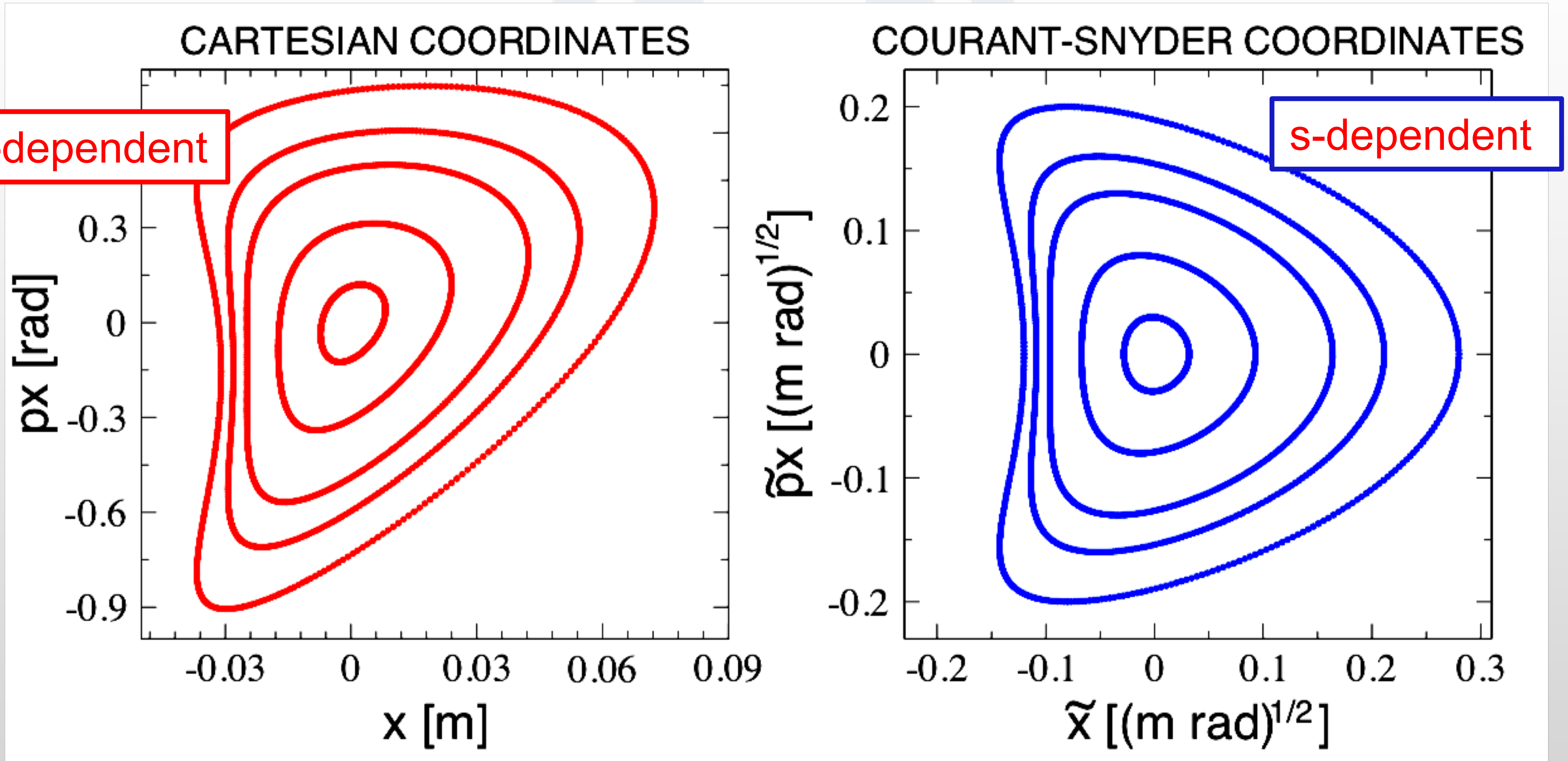


$$\begin{pmatrix} \tilde{x} \\ \tilde{p}_x \end{pmatrix} = \begin{pmatrix} (\beta_x)^{-1/2} & 0 \\ \alpha_x(\beta_x)^{-1/2} & (\beta_x)^{1/2} \end{pmatrix} \begin{pmatrix} x \\ p_x \end{pmatrix}$$

s-dependent

tune (i.e. phase-space rotation R) is amplitude independent R

Nonlinear lattice phase space (@ “large” amplitudes)

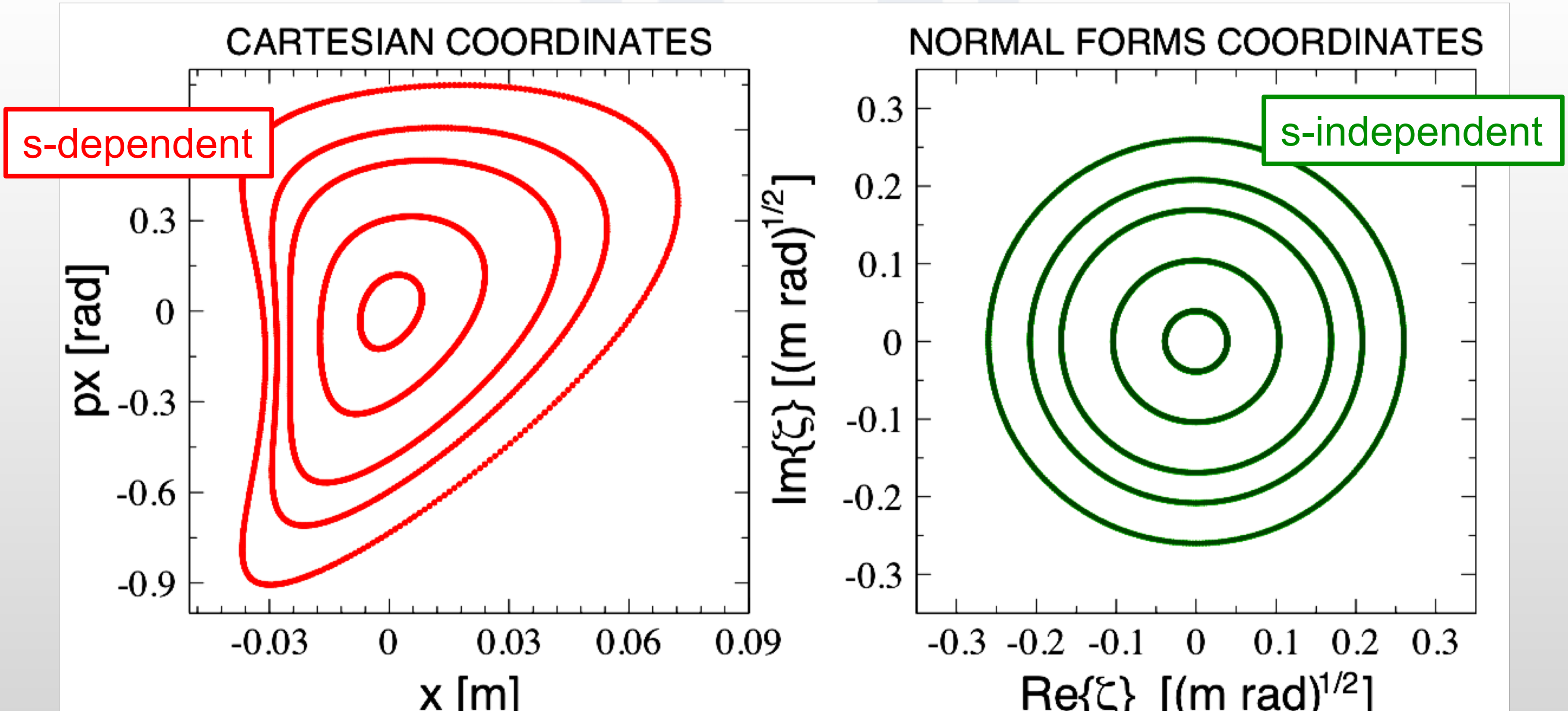


$$\begin{pmatrix} \tilde{x} \\ \tilde{p}_x \end{pmatrix} = \begin{pmatrix} (\beta_x)^{-1/2} & 0 \\ \alpha_x (\beta_x)^{-1/2} & (\beta_x)^{1/2} \end{pmatrix} \begin{pmatrix} x \\ p_x \end{pmatrix}$$

s-dependent

C-S transformation not sufficient to retrieve s-independent circles

Nonlinear lattice phase space (@ “large” amplitudes)



Stable (nonlinear) motion => closed phase-space curves => it must exist a transformation that converts (x,px) distorted curves into circles: Normal Forms transformation whose coefficients are the Resonance Driving Terms (i.e. generalization of Twiss parameters)

Nonlinear lattice phase space (@ "large" amplitudes)

	linear case	nonlinear case
1. Cartes. phase space	ellipse	closed distorted
2. C-S transformation	Twiss parameters (real)	
3. C-S phase space	circle	closed distorted
4. Nor. For. transformat.		RDTs (complex)
5. Nor. For. phase space		circle
6. tune	ampltitude indep.	amplitude dep.

Stable (nonlinear) motion => closed phase-space curves => it must exist a transformation that converts (x,px) distorted curves into circles: Normal Forms transformation whose coefficients are the Resonance Driving Terms (i.e. generalization of Twiss parameters)

Resonance Driving Terms Introductory

Twiss parameters (β, α, ϕ, γ)

- describe and account for linear focusing lattice
- transform phase space elliptic orbits into circles (tune amplitude independent)
- are measurable and deviation from model values reveal errors in quadrupoles

RDTs f_{jklm}

- describe and account for coupling & nonlinear focusing lattice
- transform phase space distorted orbits into circles (tune amplitude dependent)
- are (most of the time) measurable and deviation from model values reveal errors in sextupoles, octupoles, etc …

Stable (nonlinear) motion => closed phase-space curves => it must exist a transformation that converts (x,px) distorted curves into circles: Normal Forms transformation whose coefficients are the Resonance Driving Terms (i.e. generalization of Twiss parameters)

How are RDTs f_{jklm} selected and computed?

multipole kind	n	potential term	index relations
norm. quad. x	2	x^2	$j+k=2 \quad m+l=0$
norm. quad. y	2	y^2	$j+k=0 \quad m+l=2$
skew quad.	2	xy	$j+k=1 \quad m+l=1$
norm. sext. 1	3	x^3	$j+k=3 \quad m+l=0$
norm. sext. 2	3	xy^2	$j+k=1 \quad m+l=2$
skew sext. 1	3	y^3	$j+k=0 \quad m+l=3$
skew sext. 2	3	x^2y	$j+k=2 \quad m+l=1$

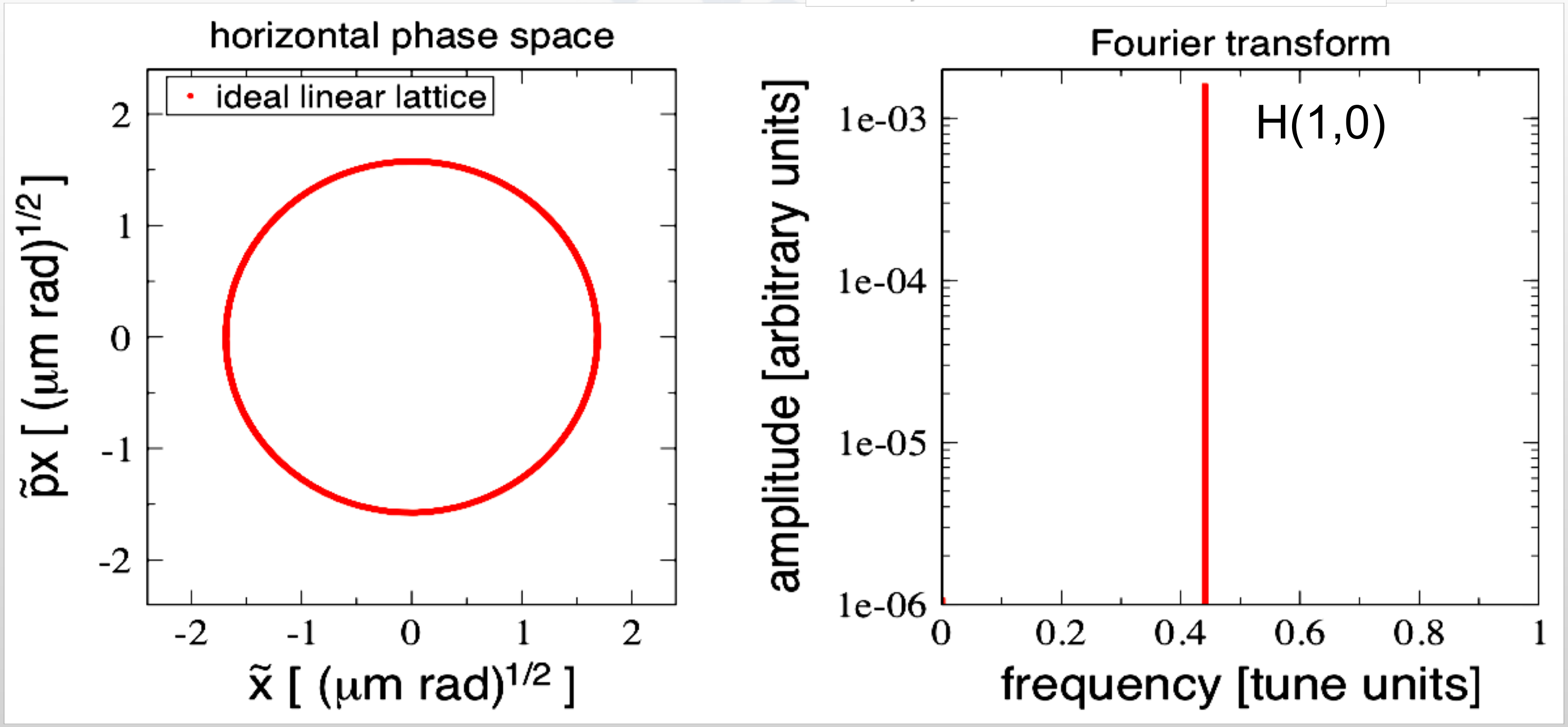
How are RDTs f_{jklm} selected and computed?

RDT	resonance and magnetic term
$f^{(1)}_{1001} = \dfrac{\sum_w J_{w,1}\sqrt{\beta^w_x\beta^w_y}\, e^{i(\Delta\phi_{w,x}-\Delta\phi_{w,y})}}{4\left[1-e^{2\pi i(Q_x-Q_y)}\right]}$	(1,-1) skew quadrupole
$f^{(1)}_{1010} = \dfrac{\sum_w J_{w,1}\sqrt{\beta^w_x\beta^w_y}\, e^{i(\Delta\phi_{w,x}+\Delta\phi_{w,y})}}{4\left[1-e^{2\pi i(Q_x+Q_y)}\right]}$	(1, 1) skew quadrupole

RDT	resonance and magnetic term
$f^{(1)}_{3000} = -\dfrac{\sum_w K_{w,2}(\beta^w_x)^{3/2} e^{i(3\Delta\phi_{w,x})}}{48\left[1-e^{2\pi i(3Q_x)}\right]}$	(3,0) normal sextupole
$f^{(1)}_{1200} = -\dfrac{\sum_w K_{w,2}(\beta^w_x)^{3/2} e^{i(-\Delta\phi_{w,x})}}{16\left[1-e^{2\pi i(-Q_x)}\right]}$	(1,0) normal sextupole
$f^{(1)}_{1020} = \dfrac{\sum_w K_{w,2}\sqrt{\beta^w_x}\beta^w_y\, e^{i(\Delta\phi_{w,x}+2\Delta\phi_{w,y})}}{16\left[1-e^{2\pi i(Q_x+2Q_y)}\right]}$	(1,2) normal sextupole
$f^{(1)}_{0120} = \dfrac{\sum_w K_{w,2}\sqrt{\beta^w_x}\beta^w_y\, e^{i(-\Delta\phi_{w,x}+2\Delta\phi_{w,y})}}{16\left[1-e^{2\pi i(-Q_x+2Q_y)}\right]}$	(1,-2) normal sextupole
$f^{(1)}_{0111} = \dfrac{\sum_w K_{w,2}\sqrt{\beta^w_x}\beta^w_y\, e^{i(-\Delta\phi_{w,x})}}{8\left[1-e^{2\pi i(-Q_x)}\right]}$	(1,0) normal sextupole

- scale linearly with (all) magnet strengths J_1, K_2, ..
- small denominators depend on tune resonant condition $(j\text{-}k)Q_x+(l\text{-}m)Q_y=M$ => name Resonance Driving Terms
- scale differently on beta functions

How are RDT f_{jklm} measured? Harmonic analysis of TbT BPM (complex) data $h_{x,\pm} = \tilde{x} \pm i\tilde{p}_x$



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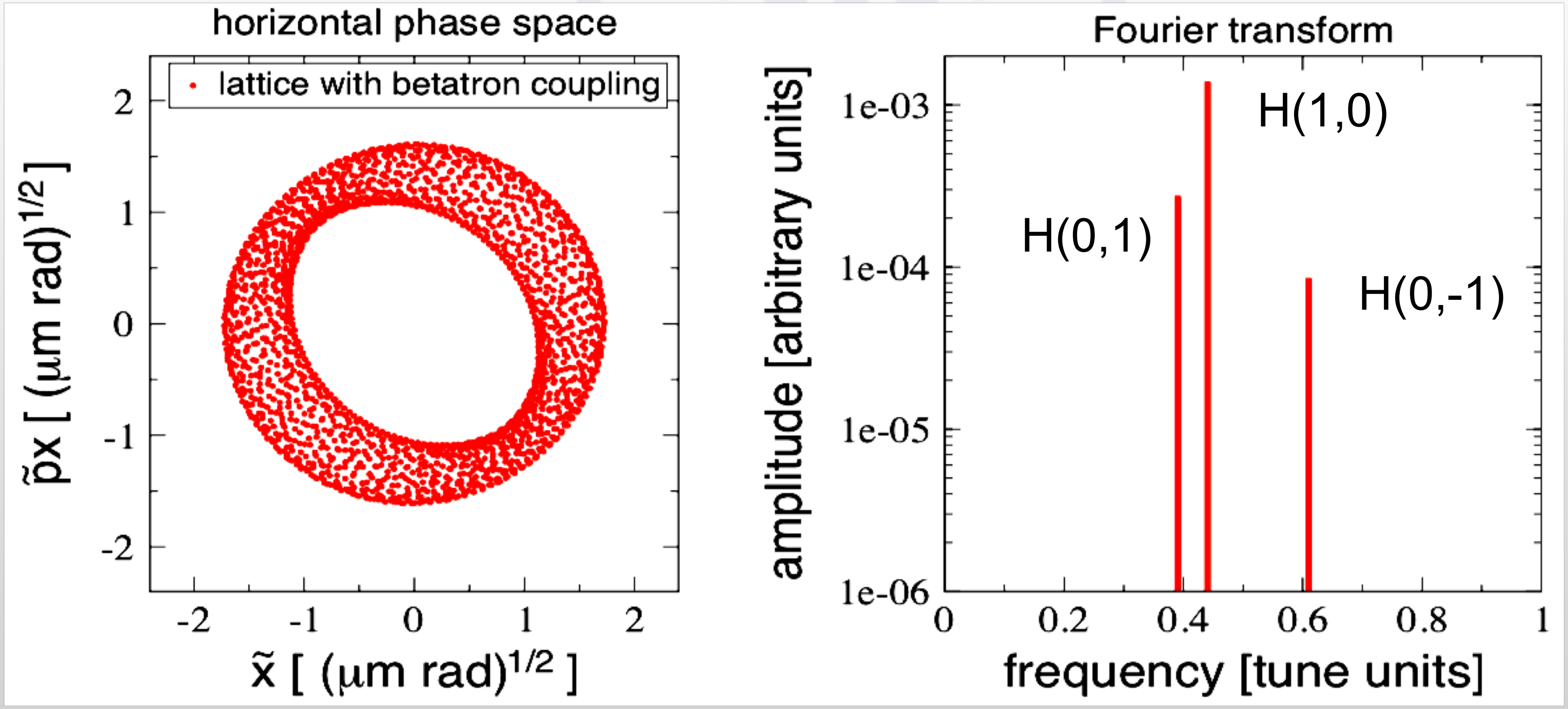

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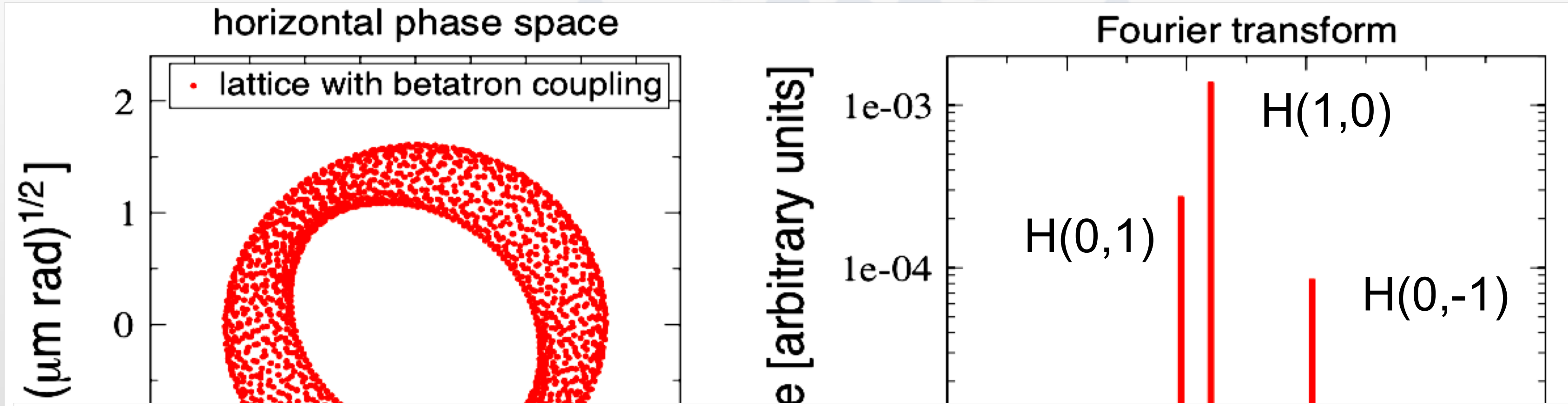
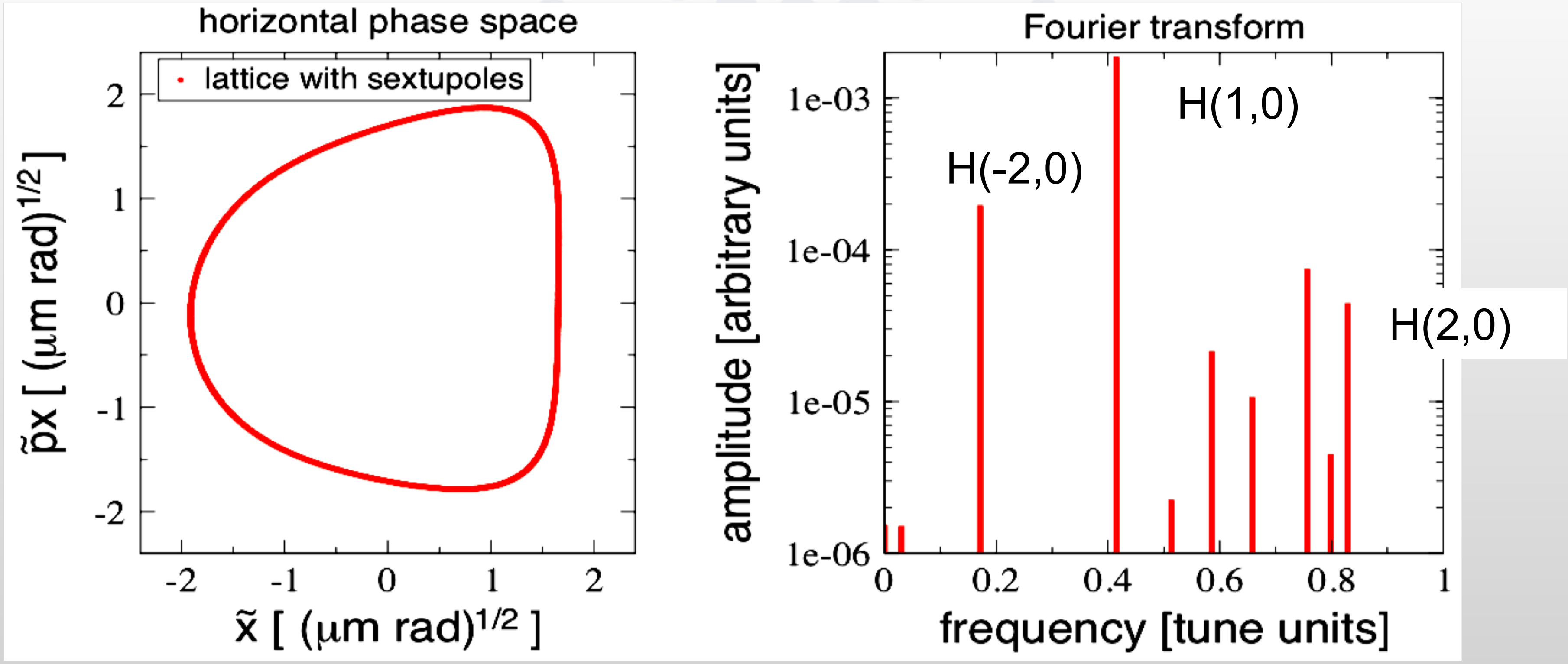


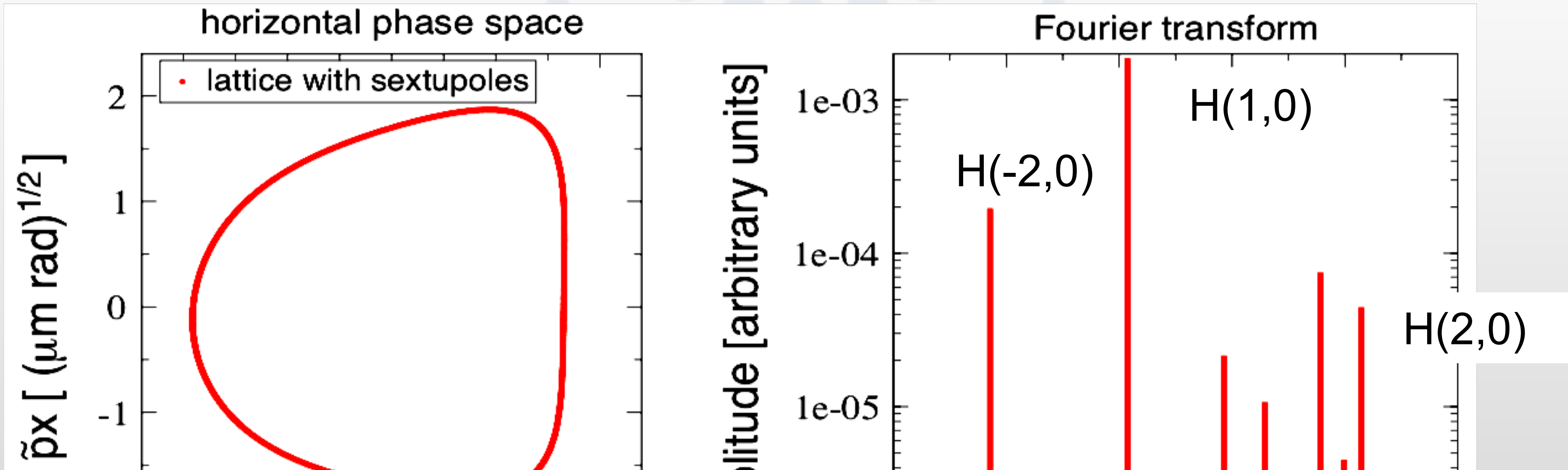
$jklm$	resonance	H-line	V-line	$\lvert a_{jklm}\rvert$	ϕ^a_{jklm}
0110	(1,-1)		$(1,0)$	$2\lvert f_{0110}\rvert(2I_x)^{1/2}$	$\phi^f_{0110}+\psi_{x,0}-\frac{\pi}{2}$
1001	(1,-1)	$(0,1)$		$2\lvert f_{1001}\rvert(2I_y)^{1/2}$	$\phi^f_{1001}+\psi_{y,0}-\frac{\pi}{2}$
1010	(1,1)	$(0,-1)$	$(-1,0)$	H: $2\lvert f_{1010}\rvert(2I_y)^{1/2}$ V: $2\lvert f_{1010}\rvert(2I_x)^{1/2}$	H: $\phi^f_{1010}-\psi_{y,0}-\frac{\pi}{2}$ V: $\phi^f_{1010}-\psi_{x,0}-\frac{\pi}{2}$

How are RDT f_{jklm} measured? Harmonic analysis of TbT BPM (complex) data $h_{x,\pm} = \tilde{x} \pm i\tilde{p}_x$



Resonance Driving Terms Introductory

How are RDT f_{jklm} measured? Harmonic analysis of TbT BPM (complex) data $h_{x,\pm} = \tilde{x} \pm i\tilde{p}_x$



$jklm$	resonance	H-line	V-line	$\lvert a_{jklm}\rvert$	ϕ^a_{jklm}
1200	(1,0)	$(2,0)$		$2\lvert f_{1200}\rvert(2I_x)$	$\phi^f_{1200} + 2\psi_{x,0} - \frac{\pi}{2}$
2100	(1,0)	$(0,0)$		$4\lvert f_{2100}\rvert(2I_x)$	$\phi^f_{2100} - \frac{\pi}{2}$
3000	(3,0)	$(-2,0)$		$6\lvert f_{3000}\rvert(2I_x)$	$\phi^f_{3000} - 2\psi_{x,0} - \frac{\pi}{2}$

not observable

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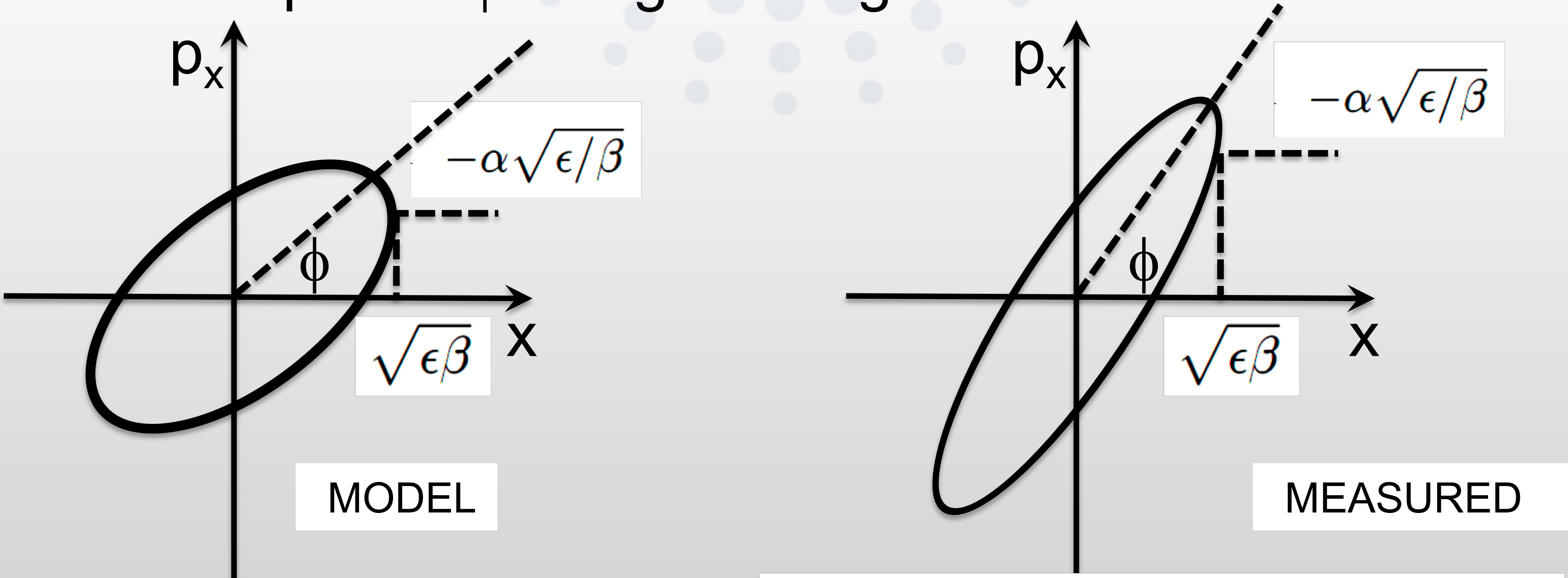
Linear optics errors

Focusing (quadrupole) errors induce deformation of phase space ellipse (i.e. of Twiss β and α) **and** betatron phase ϕ along the ring



$$\frac{\Delta\beta}{\beta} = \frac{\beta^{(meas)} - \beta^{(mod)}}{\beta^{(mod)}}$$

$$\delta\phi_{ij} = \Delta\phi_{ij}^{(meas)} - \Delta\phi_{ij}^{(mod)}$$

$$\Delta\phi_{ij} = \phi_i - \phi_j$$ @ BPMs i & j

Linear optics errors

Focusing (quadrupole) errors δK_1 excite half-integer resonance and the corresponding RDT f_{2000} (closely related to β-beating and phase errors)

$$f_{2000,j} = \frac{\sum_{w}^{W} \beta_{x,w}^{(mod)} \delta K_{w,1} e^{2i\Delta\phi_{x,wj}^{(mod)}}}{8(1-e^{4\pi i Q_x})} + O(\delta K_1^2)$$

$$\left(\frac{\Delta\beta_x}{\beta_x}\right)_j = 8\Im\{f_{2000,j}\} + O(|f_{2000}|^2)$$

$$\Delta\phi_{x,ij} \simeq \Delta\phi_{x,ij}^{(mod)} - 2h_{x,ij} + 4\Re\{f_{2000,i} - f_{2000,j}\}$$

Remark # 1: By minimizing RDT f_{2000} @ BPMs, β-beating is automatically corrected, though not necessarily the phase errors, because detuning term $h_{x,ij}$ is not observable

$$h_{x,ij} = -\frac{1}{4}\sum_{j<w<i} \beta_{x,w}^{(mod)}\delta K_{w,1} + O(\delta K_1^2)$$

$$\left(\frac{\Delta\beta_x}{\beta_x}\right)_j = 8\Im\left\{f_{2000,j}\right\} + O(|f_{2000}|^2)$$

$$\Delta\phi_{x,ij} \simeq \Delta\phi_{x,ij}^{(mod)} - 2h_{x,ij} + 4\Re\left\{f_{2000,i} - f_{2000,j}\right\}$$

Linear optics errors

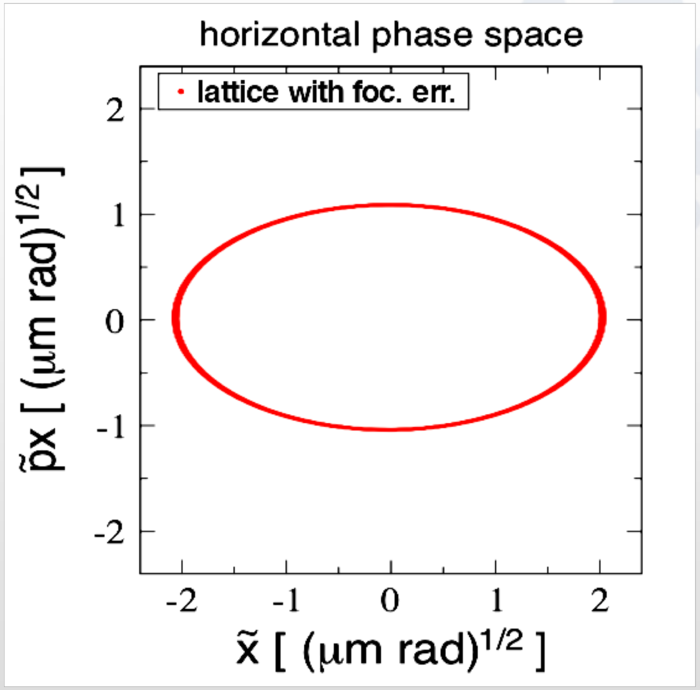
Remark # 2: It is not true that BPM phase advance shift depends on the quad errors between BPMs only (i.e. on $h_{x,ij}$). It depends also on quad errors δK_1 elsewhere via the RDT f_{2000}

$$h_{x,ij} = -\frac{1}{4}\sum_{j<w<i}\beta_{x,w}^{(mod)}\delta K_{w,1} + O(\delta K_1^2)$$

$$\left(\frac{\Delta\beta_x}{\beta_x}\right)_j = 8\Im\left\{f_{2000,j}\right\} + O(|f_{2000}|^2)$$

$$\Delta\phi_{x,ij} \simeq \Delta\phi_{x,ij}^{(mod)} - 2h_{x,ij} + 4\Re\left\{f_{2000,i} - f_{2000,j}\right\}$$

Measuring RDT f_{2000} (TbT BPM data)



complex C-S TbT signal

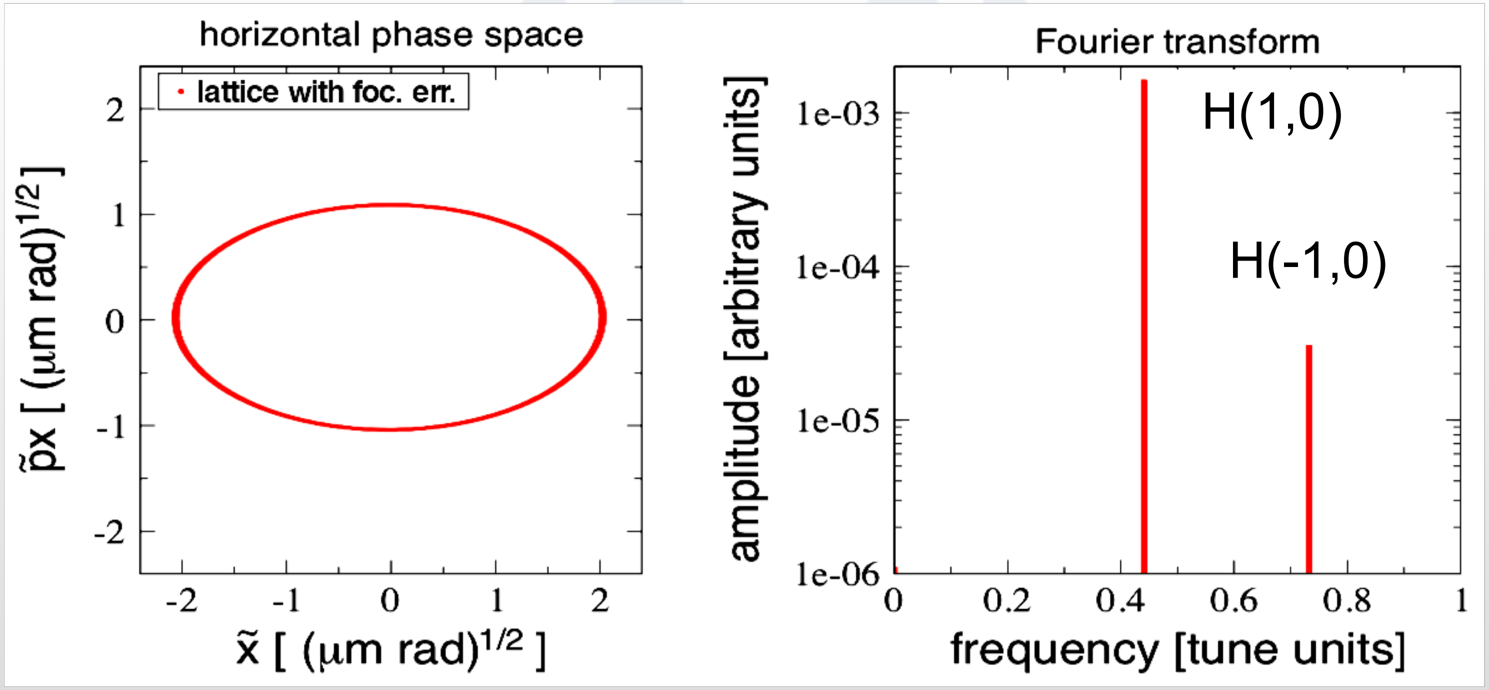
$$h_x = \tilde{x} - i\tilde{p}_x$$

$$\widetilde{p}_{i,x} = (\widetilde{x}_{i+1} - \widetilde{x}_i \cos\Delta\phi_x) / \sin\Delta\phi_x$$

$$\widetilde{p}_{i,x} = (-\widetilde{x}_{i-1} + \widetilde{x}_i \cos\Delta\phi_x) / \sin\Delta\phi_x$$

assumption: no nonlinear magnets between BPMs or “low” amplitude !!

Measuring RDT f_{2000} (TbT BPM data)



complex C-S TbT signal

$$h_x = \tilde{x} - i\tilde{p}_x$$

$$|f_{2000}| = \frac{1}{4}\,\text{arctanh}\,\frac{|H_h(-1,0)|}{|H_h(1,0)|}$$

$$q_{2000} = \Phi_{H_h(-1,0)} + \Phi_{H_h(1,0)} + \frac{\pi}{2}$$

Measuring RDT f_{2000} (orbit BPM data)

$$\begin{pmatrix} \vec{O}_x \\ \vec{O}_y \end{pmatrix} = \mathbf{ORM} \begin{pmatrix} \vec{\Theta}_x \\ \vec{\Theta}_y \end{pmatrix}, \quad \mathbf{ORM} = \begin{pmatrix} \mathbf{O}^{(\mathbf{xx})} & \mathbf{O}^{(\mathbf{xy})} \\ \mathbf{O}^{(\mathbf{yx})} & \mathbf{O}^{(\mathbf{yy})} \end{pmatrix}$$

$$O_{wj}^{(xx)} = \frac{\partial O_{x,j}}{\partial \Theta_{x,w}}, \quad O_{wj}^{(xy)} = \frac{\partial O_{x,j}}{\partial \Theta_{y,w}}, \quad 1 < j < N_B$$

$$O_{wj}^{(yx)} = \frac{\partial O_{y,j}}{\partial \Theta_{x,w}}, \quad O_{wj}^{(yy)} = \frac{\partial O_{y,j}}{\partial \Theta_{y,w}}, \quad 1 < w < N_S$$

$$\delta\mathbf{ORM} = \mathbf{ORM}^{(\mathbf{meas})} - \mathbf{ORM}^{(\mathbf{ideal})} .$$

pseudo-inverted (SVD)

$$\begin{pmatrix} \delta\vec{O}^{(xx)} \\ \delta\vec{O}^{(yy)} \\ \delta\vec{D}_x \end{pmatrix} = \mathbf{N} \begin{pmatrix} \delta\vec{K}_1 \\ \delta\vec{K}_0 \end{pmatrix}$$

$$\begin{pmatrix} \delta\vec{O}^{(xy)} \\ \delta\vec{O}^{(yx)} \\ \delta\vec{D}_y \end{pmatrix} = \mathbf{S} \begin{pmatrix} \vec{J}_1 \\ \vec{J}_0 \end{pmatrix} .$$

$$f_{2000,j} = \frac{\sum_{w}^{W} \beta_{x,w}^{(mod)} \delta K_{w,1} e^{2i\Delta\phi_{x,wj}^{(mod)}}}{8(1 - e^{4\pi i Q_x})}$$

Correcting RDT f_{2000} (and dispersion)

$$\begin{pmatrix} a_1 \vec{f}_{2000} \\ a_1 \vec{f}_{0020} \\ a_2 \delta \vec{D}_x \end{pmatrix}_{\text{meas}} = -\mathbf{N} \vec{K}_c$$

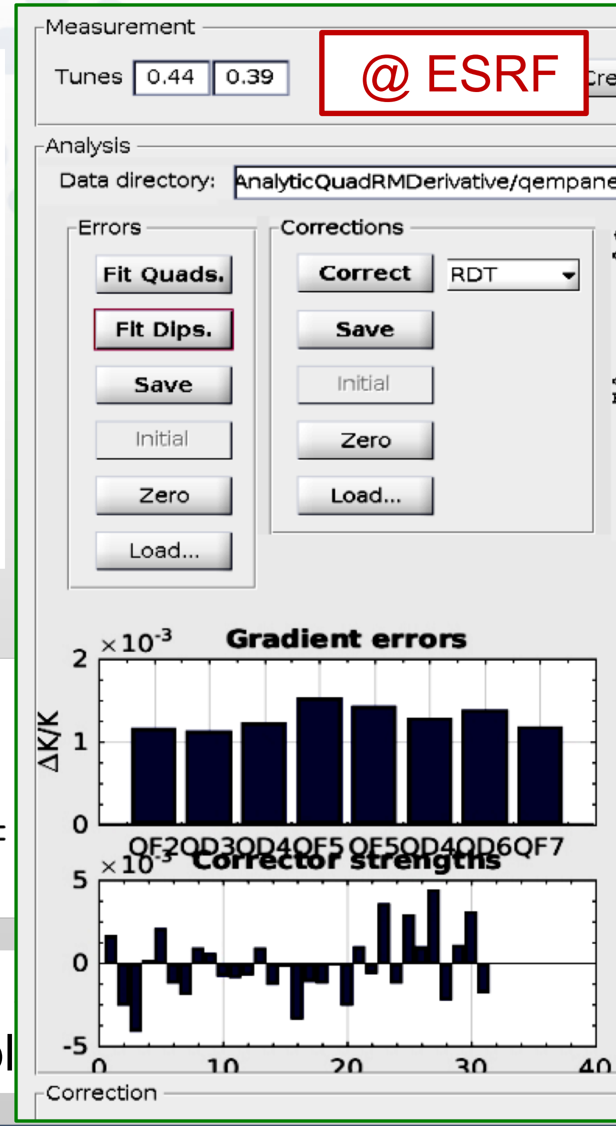
to be pseudo-inverted (SVD). Output is a vector containing the trim strengths of the (corrector) quadrupoles

$$f_{2000,j} = \frac{\sum_{w}^{W} \beta_{x,w}^{(mod)} \delta K_{w,1} e^{2i\Delta\phi_{x,wj}^{(mod)}}}{8(1 - e^{4\pi i Q_x})}$$

f_{2000} from same formula in vertical plane

Correcting RDT f_{2000} (and dispersion)

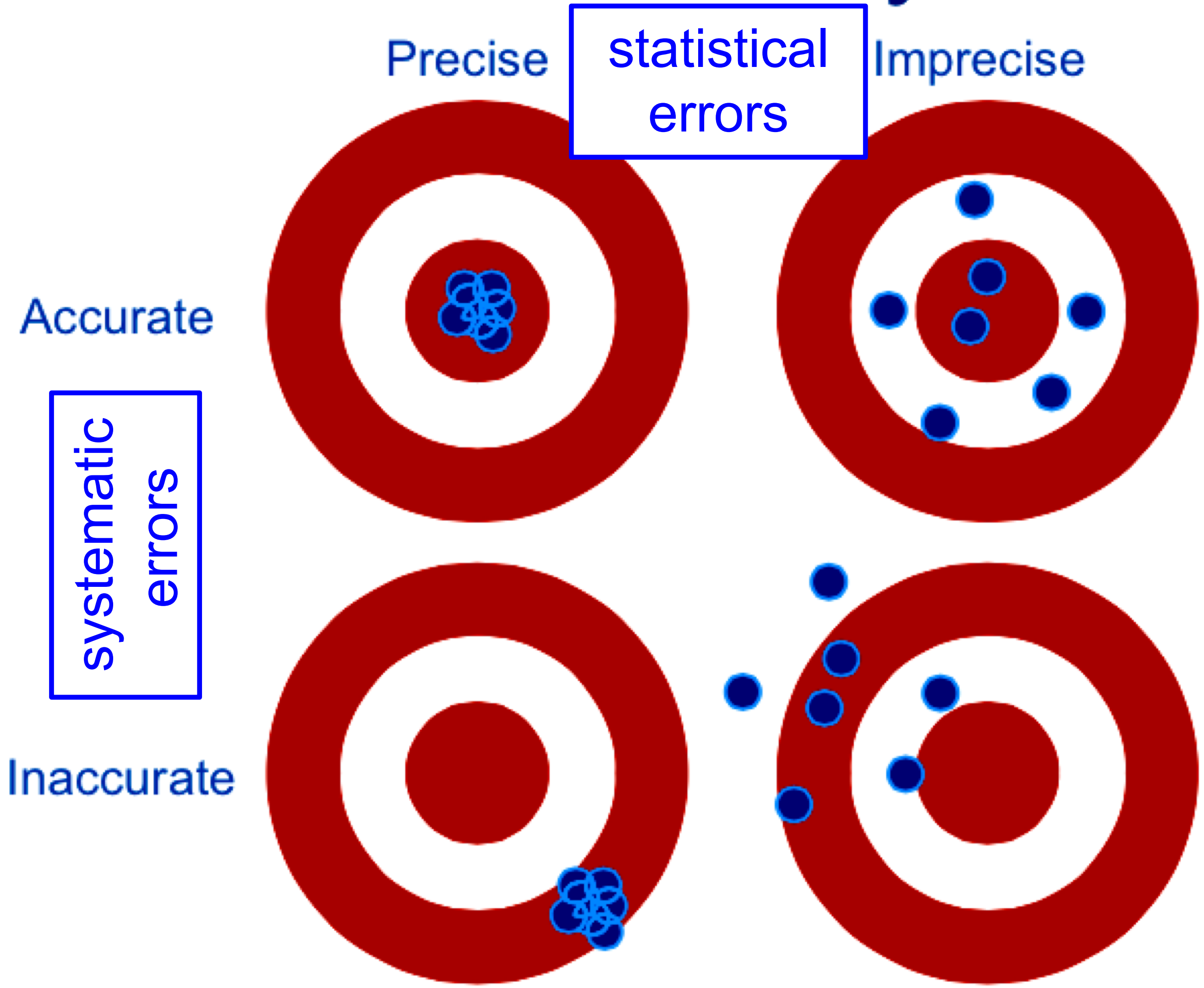
$$\begin{pmatrix} a_1 \vec{f}_{2000} \\ a_1 \vec{f}_{0020} \\ a_2 \, \delta \vec{D}_x \end{pmatrix}_{\text{meas}} = -\mathbf{N}\vec{K}_c$$



$f_{2000,j} =$ ... $od)$ ... wj

f_{2000} from vertical pl

Precision / Accuracy



Linear optics errors

Error contribution to rms β-beating @ ESRF (in ‰)

- Statistical errors (precision) the most significant (machine vibrations, orbit drifts [@ ESRF 15 μm rms => 5‰], …)
- Systematic (accuracy): SVD on ORM: 3‰ (simulations over ten sets, w/wo 10 nm BPM noise)
- Reproducibility (precision): 5‰ (H) & 2‰(V) over 5 consecutive ORM measurements (orbit corrected within 2μm rms)
- Lattice non-linearities polluting TbT tune line (from simulations): 1-2‰ accuracy @ lowest kick amplitude
- BPM noise and harmonic analysis of TbT data: depends on methods

Method / Mean error	β_x-beating precision [‰]	β_y-beating precision [‰]
TbT @ ESRF	4	4
ORM @ ESRF	6	4

L. Malina et al. , PRAB 20, 082802 (2017)

Linear optics errors

Error contribution to rms β-beating @ ESRF (in ‰)

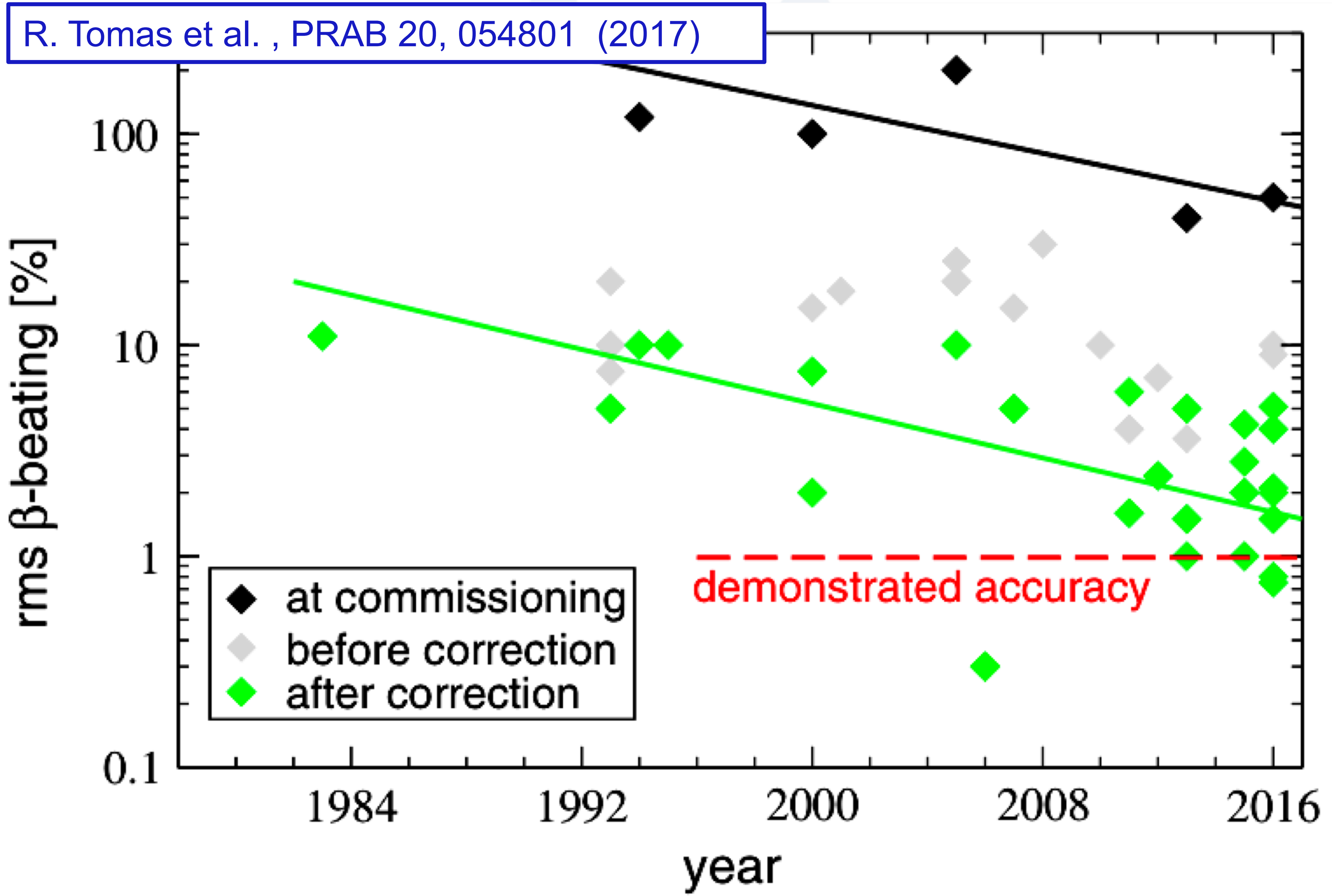
- Statistical errors (precision) the most significant (machine vibrations, orbit drifts [@ ESRF 15 μm rms => 5‰], …)
- Sy… ets, w…
- R… RM m…
- La… -2‰ accuracy @ lowest kick amplitude
- BPM noise and harmonic analysis of TbT data: depends on methods

β-beating from TbT and ORM analysis differs by ~1% rms (demonstrated accuracy @ ESRF, ALBA, …)

Method / Mean error	β_x-beating precision [‰]	β_y-beating precision [‰]
TbT @ ESRF	4	4
ORM @ ESRF	6	4

Linear optics errors

R. Tomas et al. , PRAB 20, 054801 (2017)



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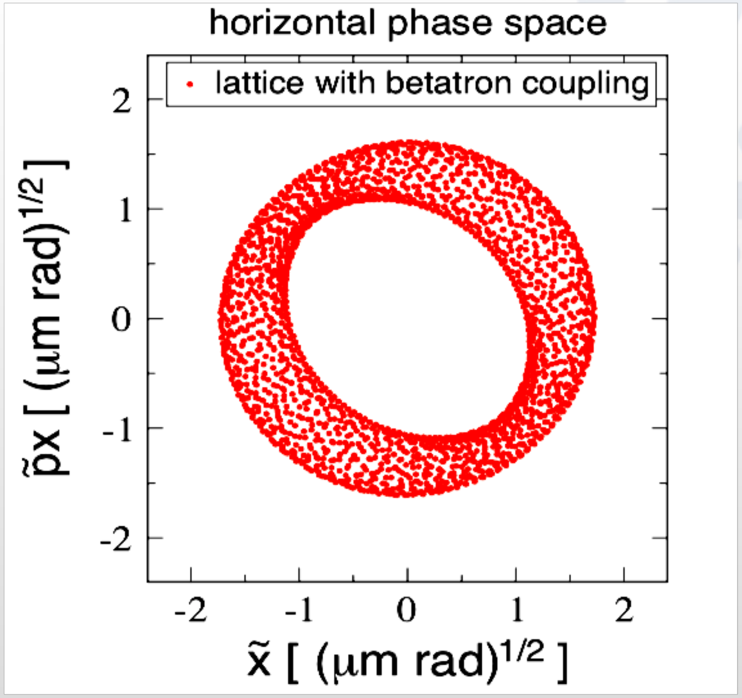
Measuring RDTs f_{1001} & f_{1010} (TbT BPM data)



complex C-S TbT signal

$$h_x = \tilde{x} - i\tilde{p}_x$$

$$\widetilde{p}_{i,x} = (\widetilde{x}_{i+1} - \widetilde{x}_i \cos\Delta\phi_x) / \sin\Delta\phi_x$$

$$\widetilde{p}_{i,x} = (-\widetilde{x}_{i-1} + \widetilde{x}_i \cos\Delta\phi_x) / \sin\Delta\phi_x$$

assumption: no nonlinear magnets between BPMs or "low" amplitude !!

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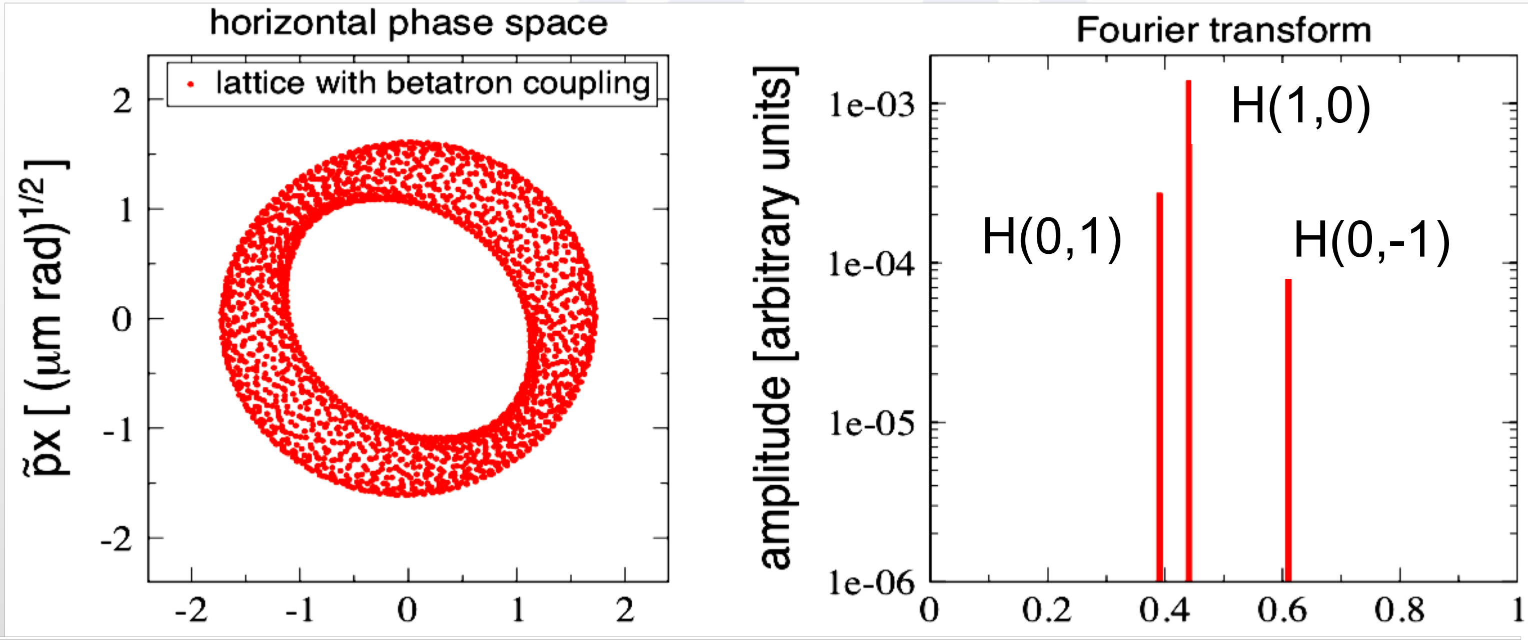
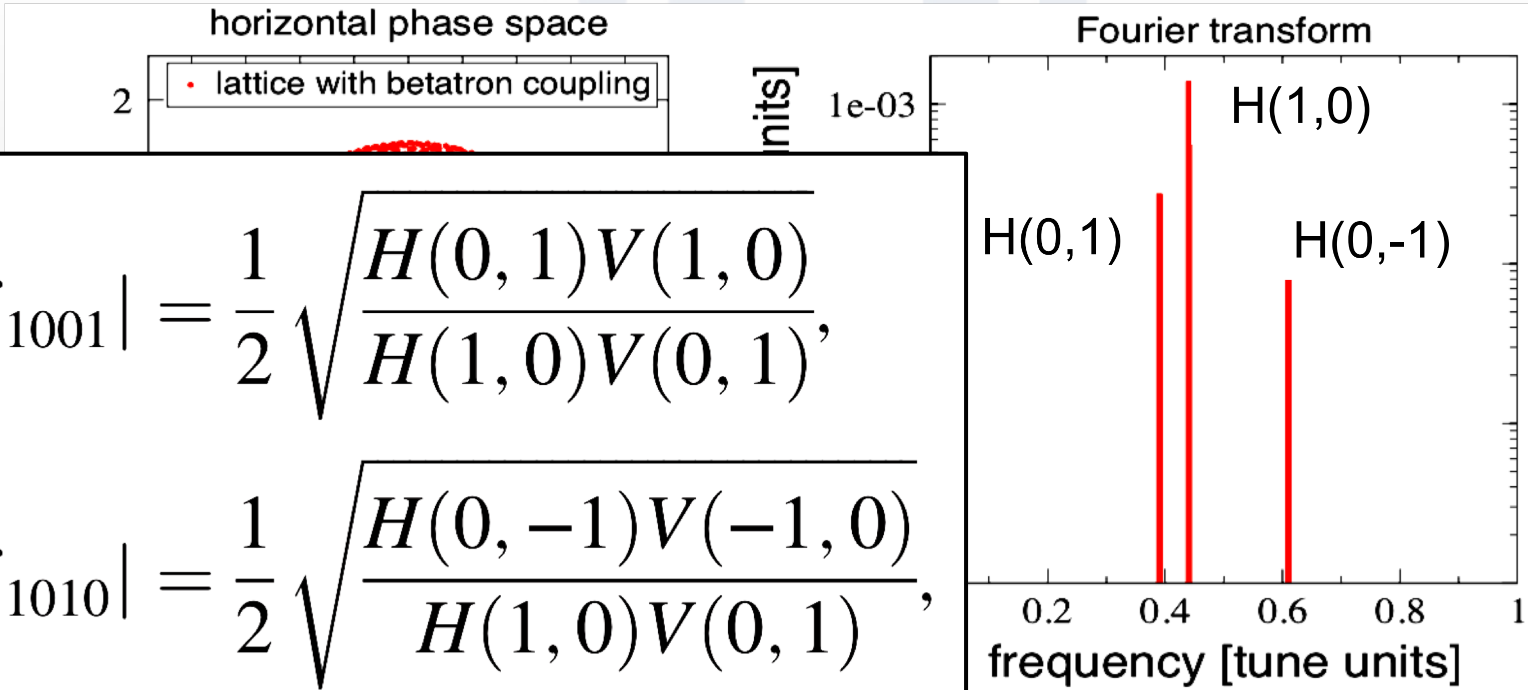


$jklm$	resonance	H-line	V-line	$\|a_{jklm}\|$	ϕ^a_{jklm}
0110	(1,-1)		$(1,0)$	$2\|f_{0110}\|(2I_x)^{1/2}$	$\phi^f_{0110}+\psi_{x,0}-\frac{\pi}{2}$
1001	(1,-1)	$(0,1)$		$2\|f_{1001}\|(2I_y)^{1/2}$	$\phi^f_{1001}+\psi_{y,0}-\frac{\pi}{2}$
1010	(1,1)	$(0,-1)$	$(-1,0)$	H: $2\|f_{1010}\|(2I_y)^{1/2}$ V: $2\|f_{1010}\|(2I_x)^{1/2}$	H: $\phi^f_{1010}-\psi_{y,0}-\frac{\pi}{2}$ V: $\phi^f_{1010}-\psi_{x,0}-\frac{\pi}{2}$

Measuring RDTs f_{1001} & f_{1010} (TbT BPM data)



$$|f_{1001}| = \frac{1}{2}\sqrt{\frac{H(0,1)V(1,0)}{H(1,0)V(0,1)}},$$

$$|f_{1010}| = \frac{1}{2}\sqrt{\frac{H(0,-1)V(-1,0)}{H(1,0)V(0,1)}},$$

$$q_{1001} = \frac{1}{2}\left(\phi_{H(0,1)} + \phi_{H(1,0)} - \phi_{V(1,0)} - \phi_{V(0,1)}\right)$$

$$q_{1010} = \frac{1}{2}\left(\phi_{H(0,-1)} + \phi_{H(1,0)} + \phi_{V(-1,0)} + \phi_{V(0,1)} + \pi\right)$$

Measuring RDTs f_{1001} & f_{1010} (orbit BPM data)

$$\begin{pmatrix}\vec{O}_x\\ \vec{O}_y\end{pmatrix} = \mathbf{ORM}\begin{pmatrix}\vec{\Theta}_x\\ \vec{\Theta}_y\end{pmatrix}, \quad \mathbf{ORM} = \begin{pmatrix}\mathbf{O^{(xx)}} & \mathbf{O^{(xy)}}\\ \mathbf{O^{(yx)}} & \mathbf{O^{(yy)}}\end{pmatrix}$$

$$O^{(xx)}_{wj} = \frac{\partial O_{x,j}}{\partial \Theta_{x,w}}, \quad O^{(xy)}_{wj} = \frac{\partial O_{x,j}}{\partial \Theta_{y,w}}, \quad 1 < j < N_B$$

$$O^{(yx)}_{wj} = \frac{\partial O_{y,j}}{\partial \Theta_{x,w}}, \quad O^{(yy)}_{wj} = \frac{\partial O_{y,j}}{\partial \Theta_{y,w}}, \quad 1 < w < N_S$$

$$\delta\mathbf{ORM} = \mathbf{ORM^{(meas)}} - \mathbf{ORM^{(ideal)}} .$$

pseudo-inverted (SVD)

$$\begin{pmatrix}\delta\vec{O}^{(xx)}\\ \delta\vec{O}^{(yy)}\\ \delta\vec{D}_x\end{pmatrix} = \mathbf{N}\begin{pmatrix}\delta\vec{K}_1\\ \delta\vec{K}_0\end{pmatrix}$$

$$\begin{pmatrix}\delta\vec{O}^{(xy)}\\ \delta\vec{O}^{(yx)}\\ \delta\vec{D}_y\end{pmatrix} = \mathbf{S}\begin{pmatrix}\vec{J}_1\\ \vec{J}_0\end{pmatrix} .$$

$$f_{\substack{1001\\1010},\,j} = \frac{\sum_w^W J_{w,1}\sqrt{\beta_{w,x}\beta_{w,y}}\,e^{i(\Delta\phi_{x,wj}\mp\Delta\phi_{y,wj})}}{4(1-e^{2\pi i(Q_u\mp Q_v)})}$$

Correcting RDTs f_{1001} & f_{1010} (& vertical dispersion)

$$\begin{pmatrix} a_1\vec{f}_{1001} \\ a_1\vec{f}_{1010} \\ a_2\vec{D}_y \end{pmatrix}_{\mathrm{meas}} = -\mathbf{M}\vec{J}_c$$

to be pseudo-inverted (SVD). Output is a vector containing the trim strengths of the corrector skew quadrupoles

$$f_{\substack{1001\\1010},j} = \frac{\sum_{w}^{W} J_{w,1}\sqrt{\beta_{w,x}\beta_{w,y}}\,e^{i(\Delta\phi_{x,wj}\mp\Delta\phi_{y,wj})}}{4(1-e^{2\pi i(Q_u\mp Q_v)})}$$

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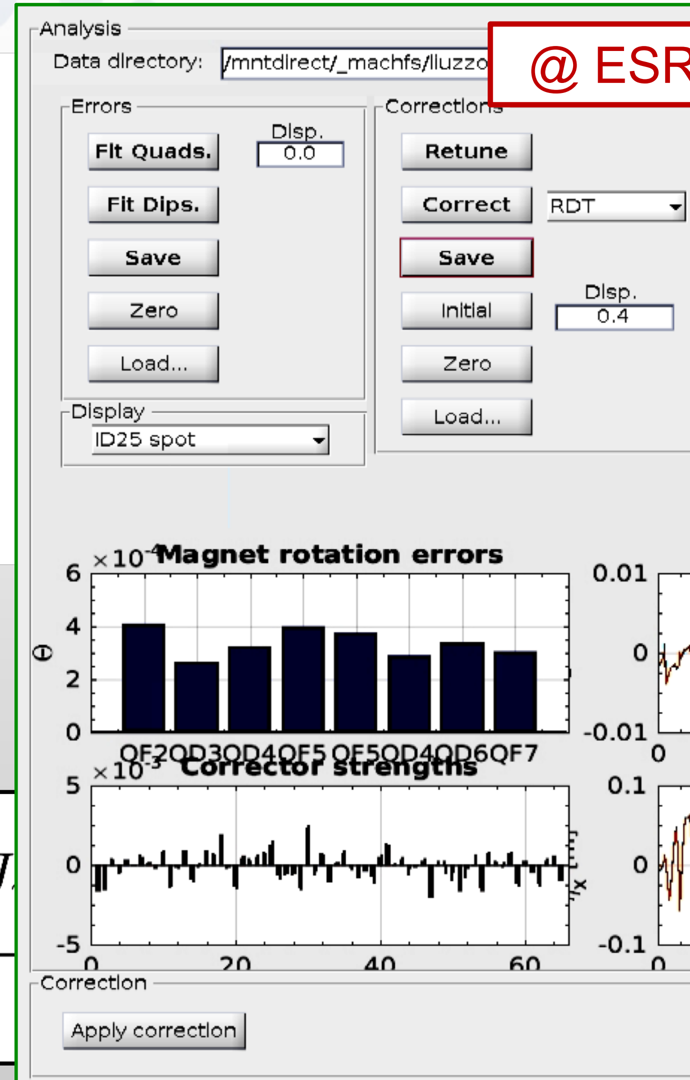
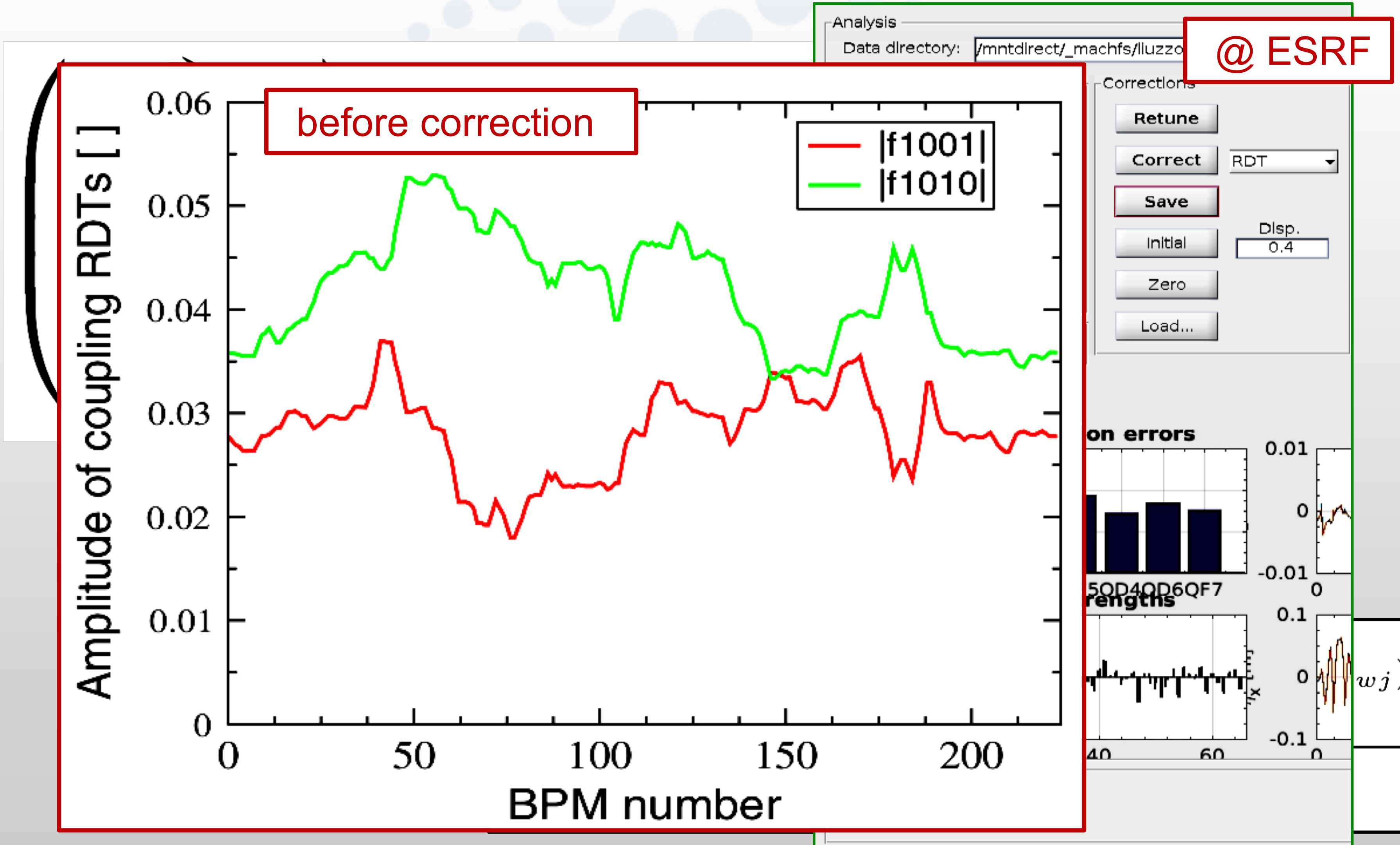
$$f_{\substack{1001\\1010},\,j} = \frac{\sum_{w}^{W} J \ldots}{\ldots} \ldots w j)$$

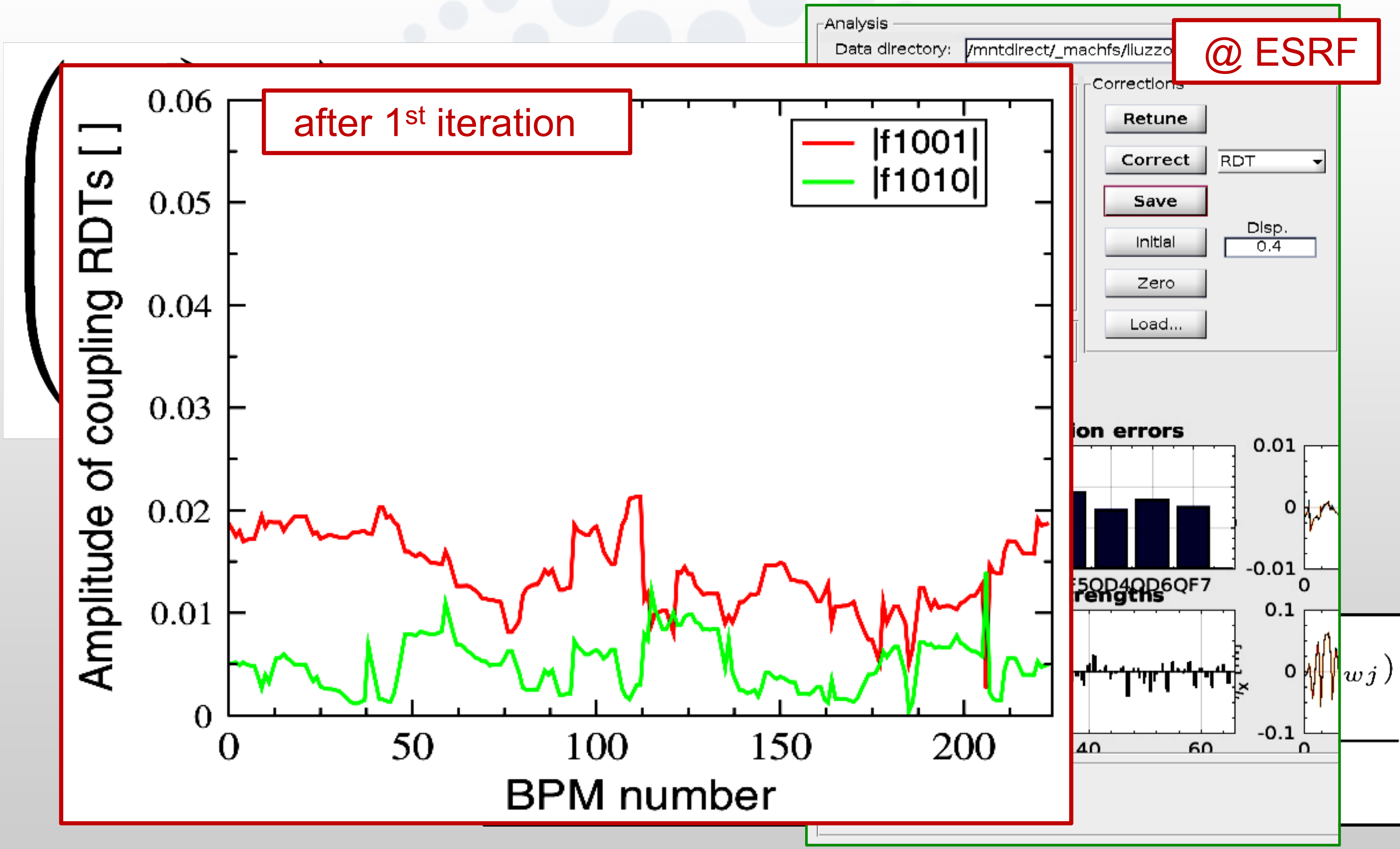


@ ESRF

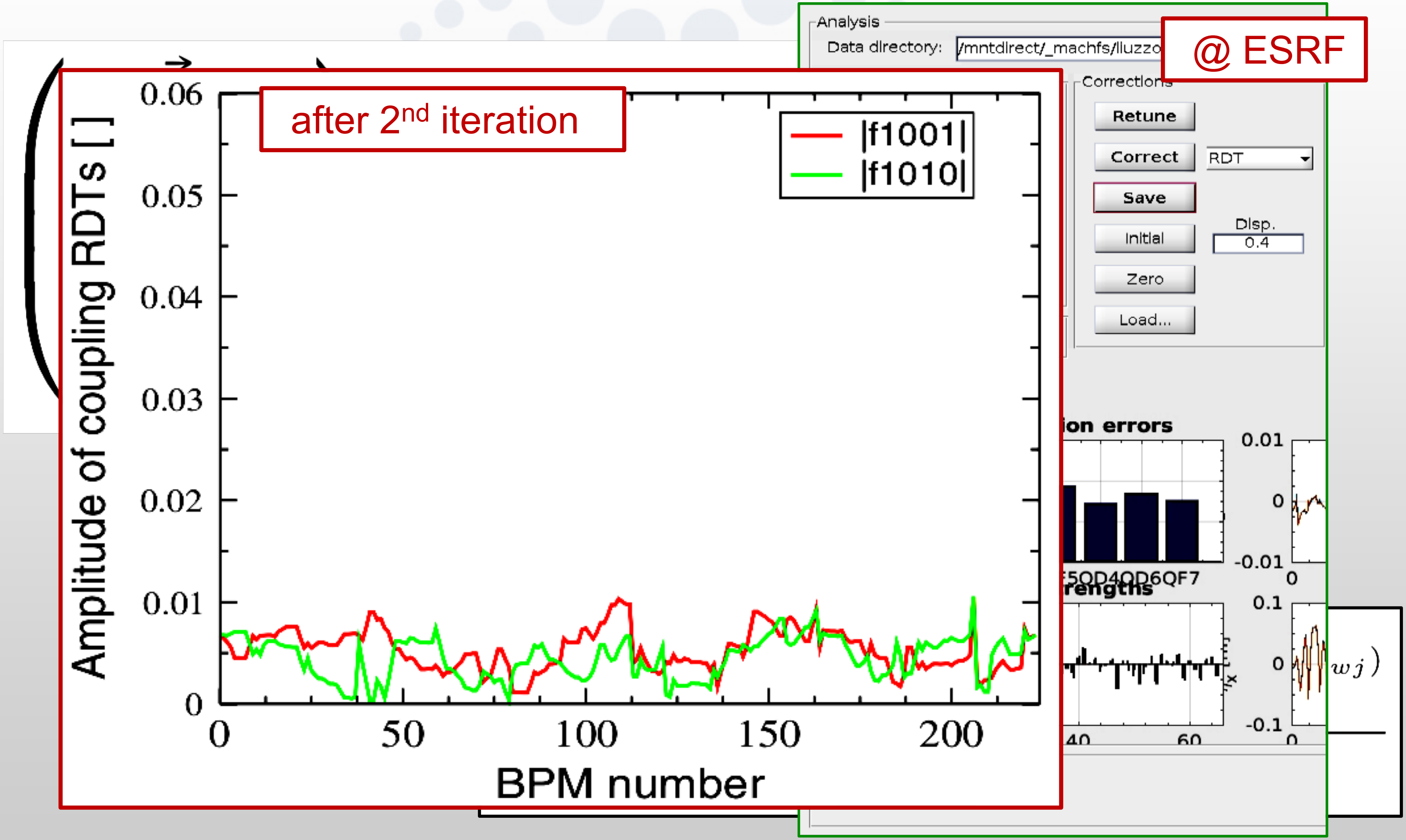
Correcting RDTs f_{1001} & f_{1010} (& vertical dispersion)



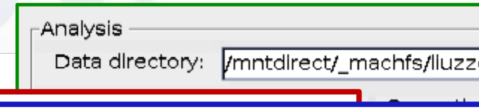
Correcting RDTs f_{1001} & f_{1010} (& vertical dispersion)



Correcting RDTs f_{1001} & f_{1010} (& vertical dispersion)



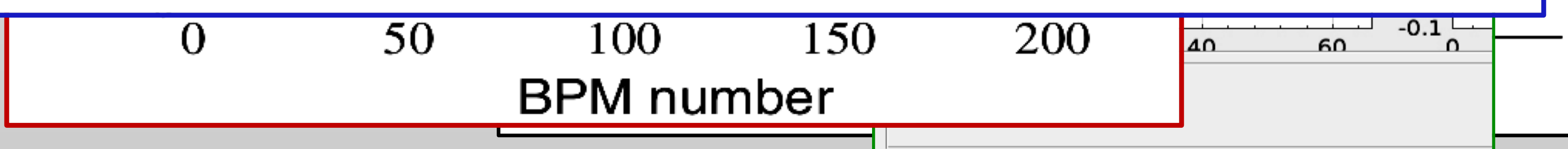
Correcting RDTs f_{1001} & f_{1010} (& vertical dispersion)



@ ESRF

TABLE I. Summary table of the first correction with RDTs of January 16, 2010. The corresponding horizontal emittance is $\epsilon_x \simeq \mathbb{E}_x \simeq \mathcal{E}_u \simeq 4$ nm. Beta beat refers to the peak value.

Condition	$\bar{\epsilon}_y \pm \delta\epsilon_y$ [pm]	β beat [%]
With 2009 correction	46 ± 18	5
All correctors OFF	237 ± 122	50
After 1st correction	23.6 ± 6.3	8
After 2nd correction	11.5 ± 4.3	5

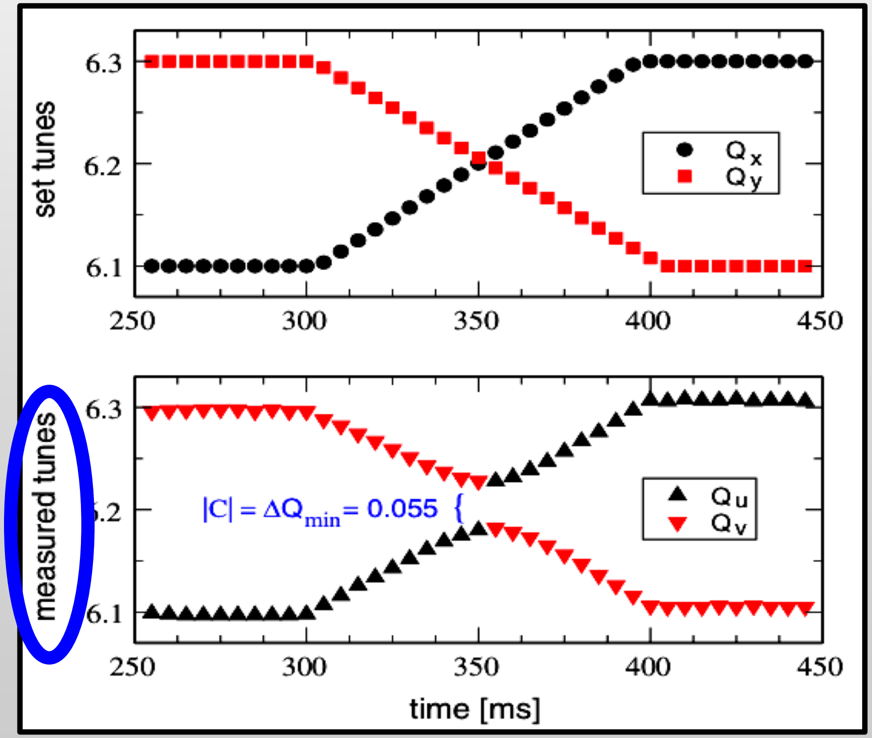


Hadron Vs lepton machines

~ hadrons

$$Q_x - Q_y$$

$$\Delta Q_{\min} = \left| \frac{1}{2\pi} \oint \mathrm{d}s\, j(s) \sqrt{\beta_x \beta_y} e^{-i(\phi_x - \phi_y) + i(\hat{Q}_x - \hat{Q}_y)s/R} \right|,$$



Hadron Vs lepton machines

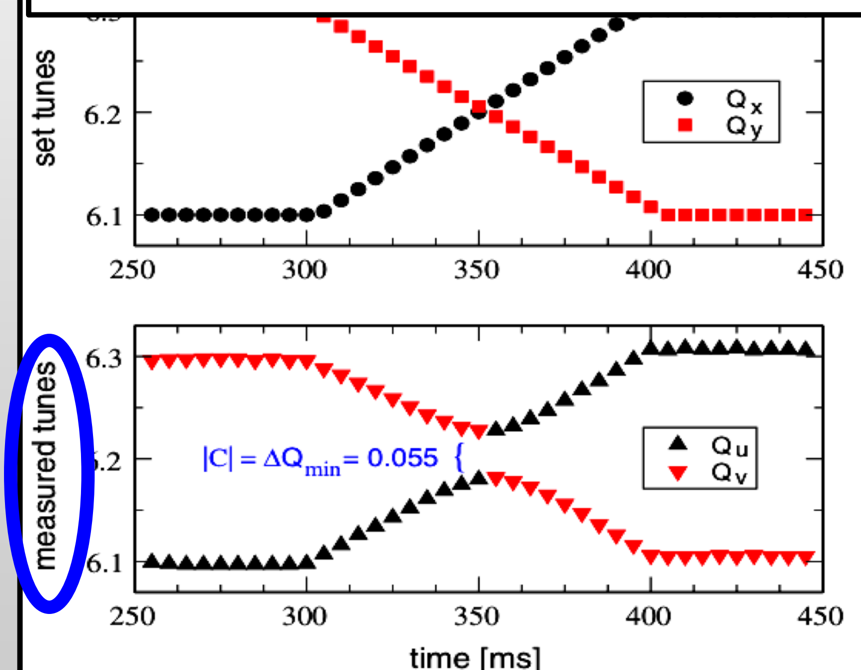
$Q_x - Q_y$

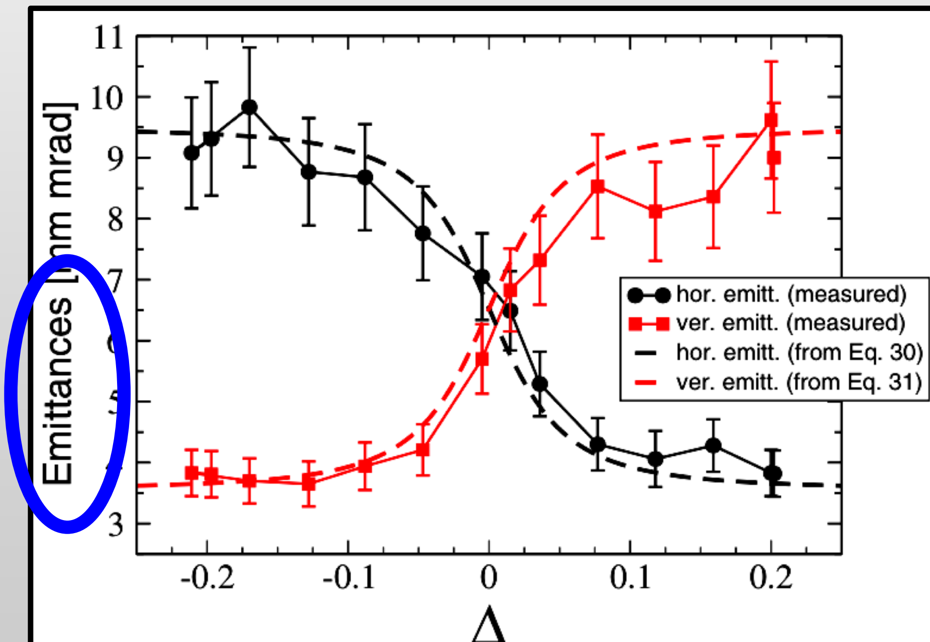
$$\Delta Q_{\min} = \left| \frac{1}{2\pi} \oint \mathrm{d}s\, j(s) \sqrt{\beta_x \beta_y} e^{-i(\phi_x - \phi_y) + i(\hat{Q}_x - \hat{Q}_y)s/R} \right| ,$$

~ hadrons

~ leptons

$\varepsilon_y/\varepsilon_x \approx 1$ fully coupled $\varepsilon_y/\varepsilon_x \approx 1‰$ ultra-low coupling





Hadron Vs lepton machines

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$$\Delta Q_{\min} = \left| \frac{1}{2\pi} \oint \mathrm{d}s\, j(s) \sqrt{\beta_x \beta_y} e^{-i(\phi_x - \phi_y) + i(\hat{Q}_x - \hat{Q}_y)s/R} \right|,$$

~ leptons

$\varepsilon_y/\varepsilon_x \approx 1$ fully coupled $\varepsilon_y/\varepsilon_x \approx 1‰$ ultra-low coupling

$$Q_x \approx Q_y \Rightarrow |f_{1010}| \ll |f_{1001}|$$

$$\Delta Q_{\min} \approx \left| 4(\hat{Q}_x - \hat{Q}_y) \overline{f_{1001} e^{-i(\phi_x - \phi_y)}} \right| \lesssim 4 |\hat{Q}_x - \hat{Q}_y| \left| \overline{f_{1001}} \right|.$$

round beams usually ($\varepsilon_y/\varepsilon_x \approx 1$) from injection even with ultra-low coupling (ΔQmin @ LHC ~2×10^{-4})

Hadron Vs lepton machines

$$\Delta Q_{\min} = \left| \frac{1}{2\pi} \oint ds\, j(s) \sqrt{\beta_x \beta_y} e^{-i(\phi_x - \phi_y) + i(\hat{Q}_x - \hat{Q}_y)s/R} \right|,$$

$Q_x - Q_y$

~ hadrons

~ leptons

$\varepsilon_y/\varepsilon_x \approx 1$ fully coupled $\varepsilon_y/\varepsilon_x \approx 1‰$ ultra-low coupling

$$Q_x \gg Q_y \Rightarrow |f_{1010}| \approx |f_{1001}|$$

$$\mathcal{E}_x = \mathcal{C}^2 \mathcal{E}_u + [\mathcal{S}_-^2 + \mathcal{S}_+^2 - 2\mathcal{S}_-\mathcal{S}_+ \cos(q_+ + q_-)]\mathcal{E}_v$$

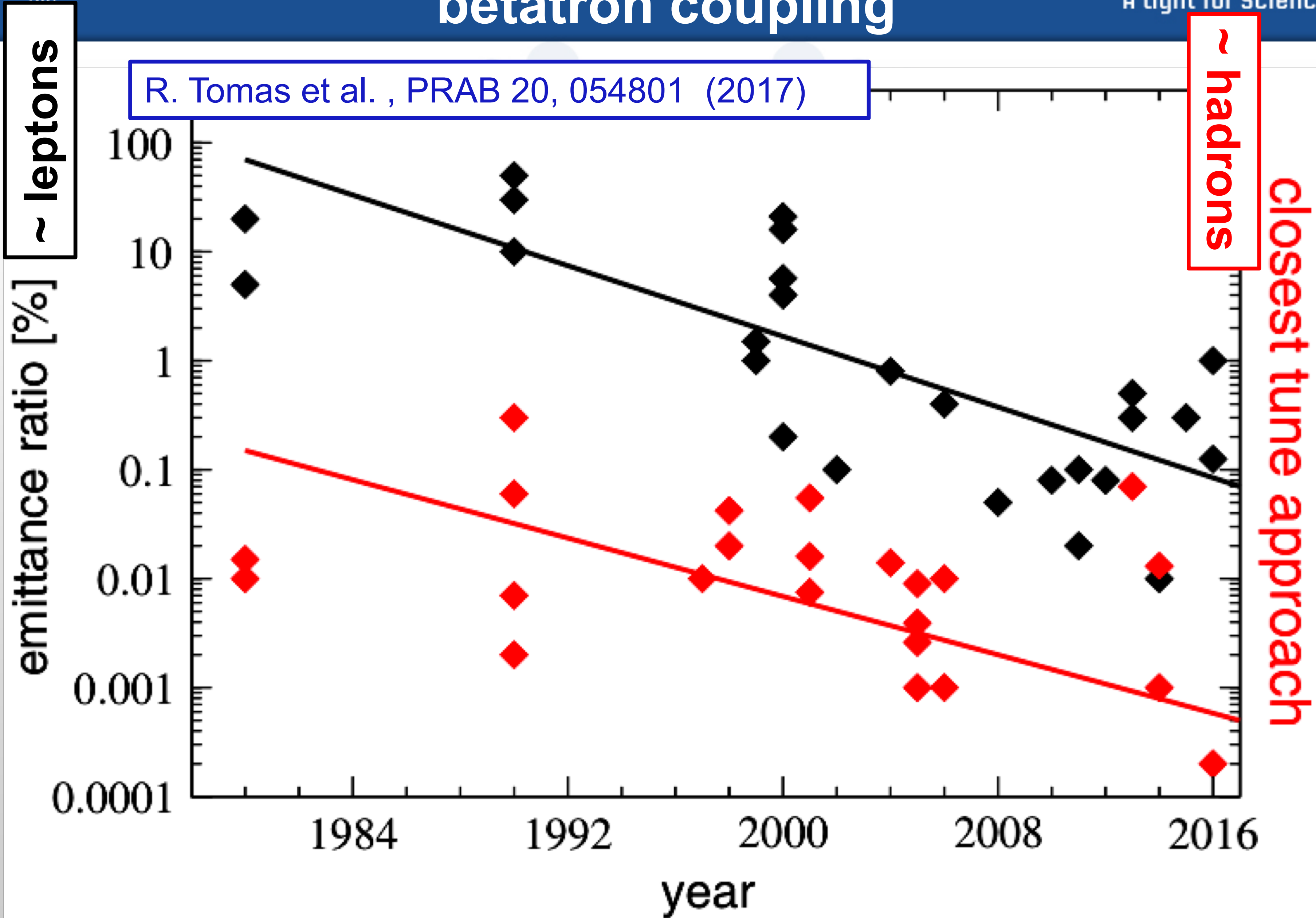
$$\mathcal{E}_y = \mathcal{C}^2 \mathcal{E}_v + [\mathcal{S}_-^2 + \mathcal{S}_+^2 - 2\mathcal{S}_-\mathcal{S}_+ \cos(q_+ - q_-)]\mathcal{E}_u$$

$$\mathcal{C} = \cosh(2\mathcal{P})$$

$$\mathcal{S}_\pm = \frac{\sinh(2\mathcal{P})}{\mathcal{P}} |f_{\substack{1001\\1010}}|$$

$$\mathcal{P} = \sqrt{-|f_{1001}|^2 + |f_{1010}|^2}$$

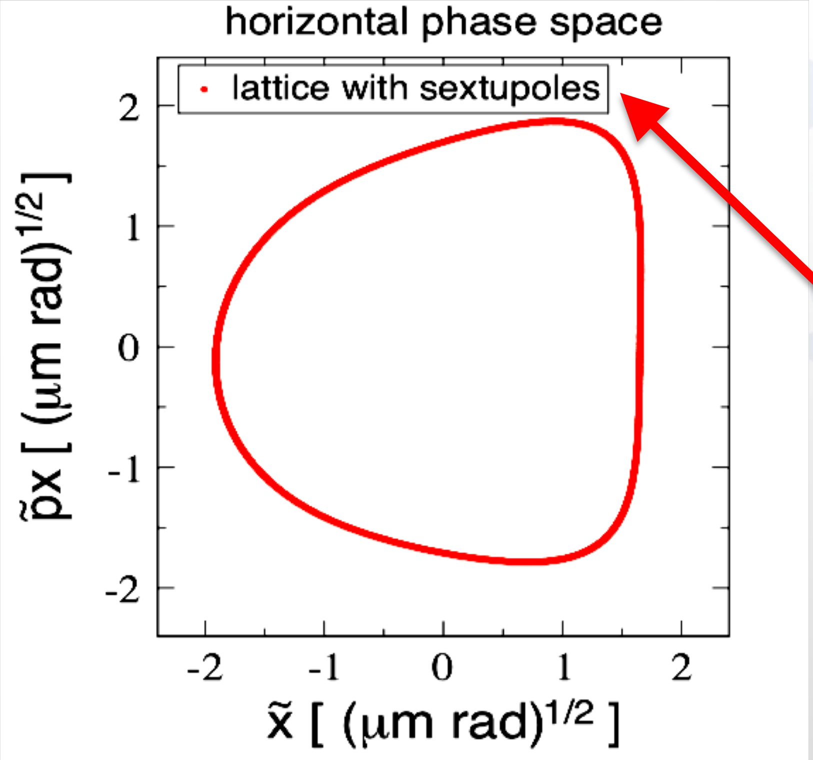
betatron coupling



Outlines

- Resonance Driving Terms Introductory (15')
- Linear optics errors (10')
  1. RDTs Vs beta-beating and phase advance error
  2. RDTs measurements & correction
  3. Accuracy and precision analysis
- Betatron coupling (10')
  1. RDTs measurements & correction
  2. Hadron Vs lepton machines
- Nonlinear lattice error (model) (15')
  1. Localization & detection of nonlinearities via RDTs
  2. RDTs Vs chromatic functions and orbit feed-downs

Nonlinear lattice error (model)



1st caveat: measuring nonlinear RDTs from complex C-S signal is affected by a systematic error in the reconstruction of p_x

complex C-S TbT signal

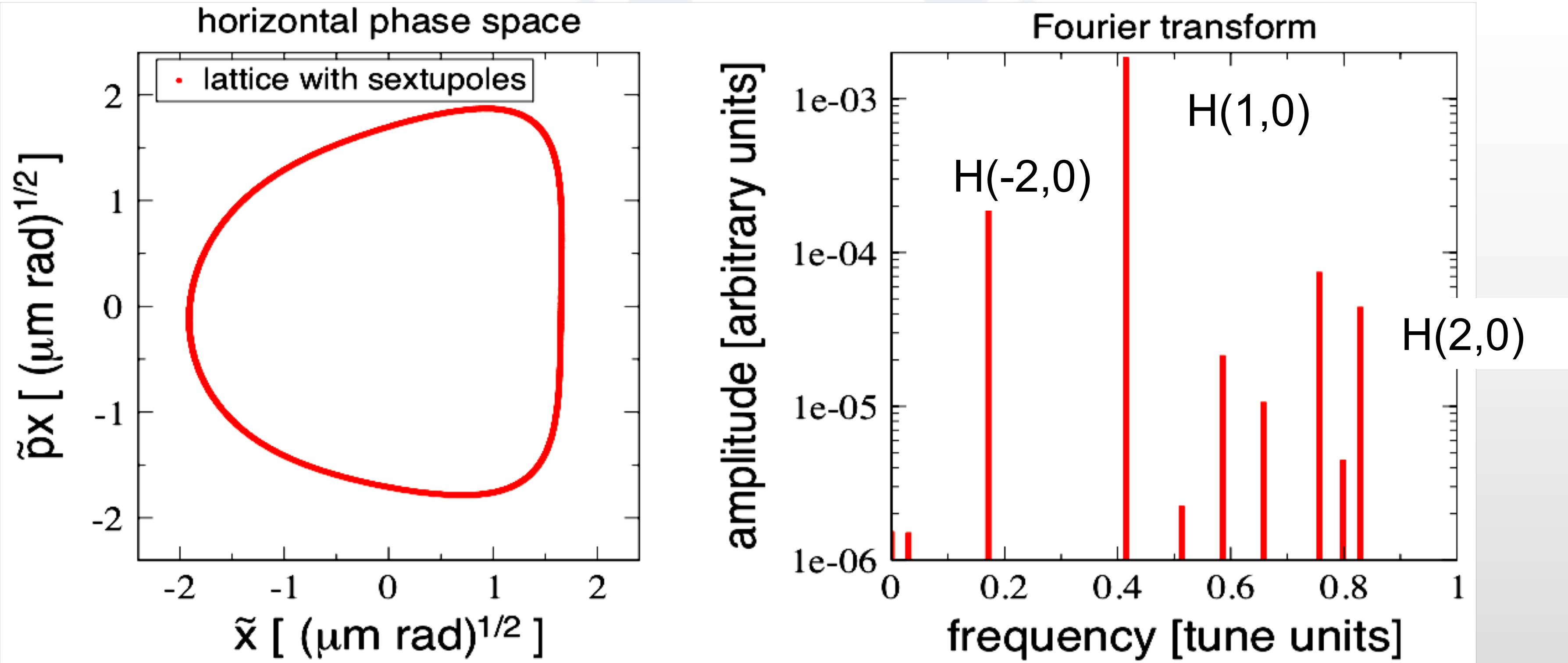
$$h_x = \tilde{x} - i\tilde{p}_x$$

$$\tilde{p}_{i,x} = (\tilde{x}_{i+1} - \tilde{x}_i \cos\Delta\phi_x)/\sin\Delta\phi_x$$

$$\tilde{p}_{i,x} = (-\tilde{x}_{i-1} + \tilde{x}_i \cos\Delta\phi_x)/\sin\Delta\phi_x$$

assumption: no nonlinear magnets between BPMs or "low" amplitude !!

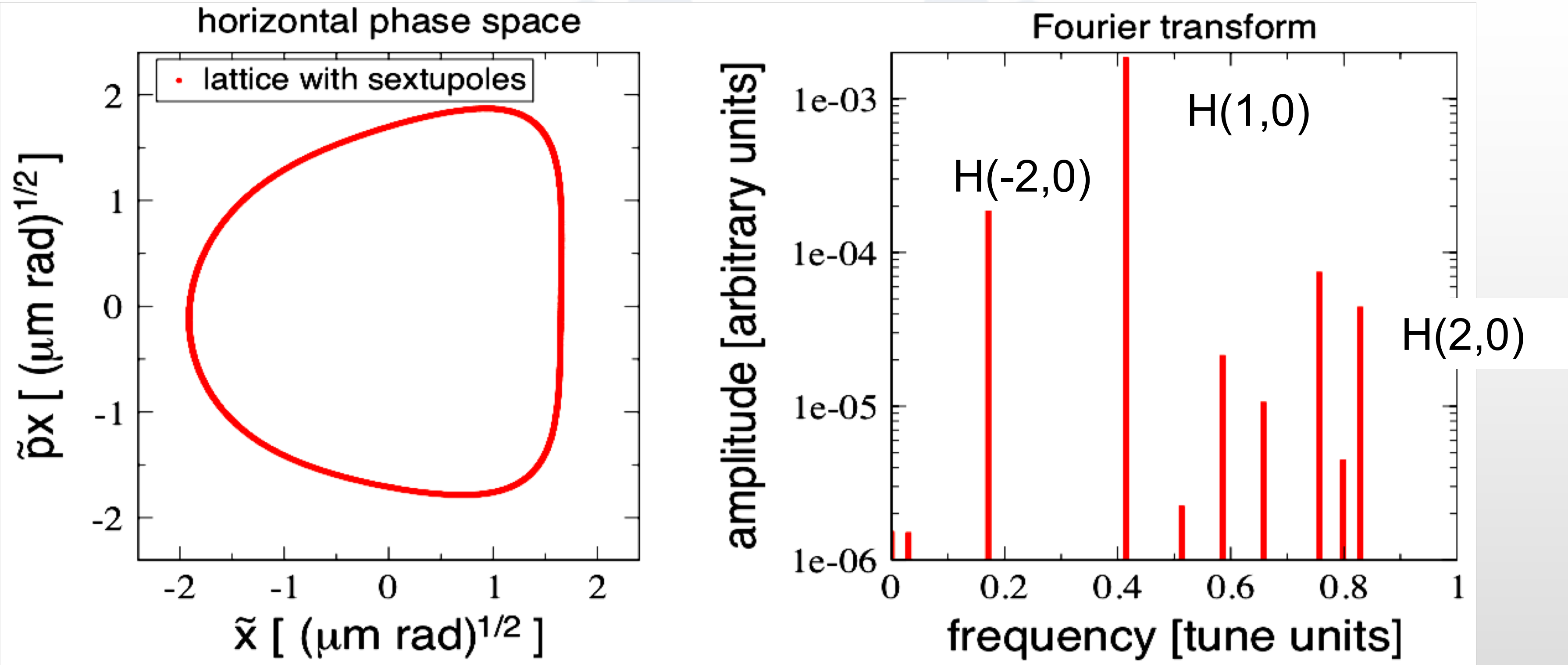
Nonlinear lattice error (model)



$jklm$	resonance	H-line	V-line	$\lvert a_{jklm} \rvert$	ϕ^{a}_{jklm}
1200	(1,0)	$(2,0)$		$2\lvert f_{1200}\rvert(2I_x)$	$\phi^{f}_{1200} + 2\psi_{x,0} - \frac{\pi}{2}$
2100	(1,0)	$(0,0)$		$4\lvert f_{2100}\rvert(2I_x)$	ϕ^{f}_{2100}
3000	(3,0)	$(-2,0)$		$6\lvert f_{3000}\rvert(2I_x)$	$\phi^{f}_{3000} - 2\psi_{x,0} - \frac{\pi}{2}$

let's focus on this

Nonlinear lattice error (model)



$$|f_{3000}| = \frac{H(-2,0)}{6H(1,0)^2}$$

2nd caveat: measured RDTs affected by BPM calibration error (if unknown)

$$q_{1001} = \phi_{H(-2,0)} + 2\phi_{H(1,0)} + \frac{\pi}{2}$$

Nonlinear lattice error (model)

$$h_{w,3000}\, e^{-i3\phi_x^w} = f_{3000}^{(w)} e^{-i3\Delta\phi_x^{w,w-1}} - f_{3000}^{(w-1)}$$

$$h_{w,3000} = K_{w,2}\beta_{w,x}^{3/2}/16$$

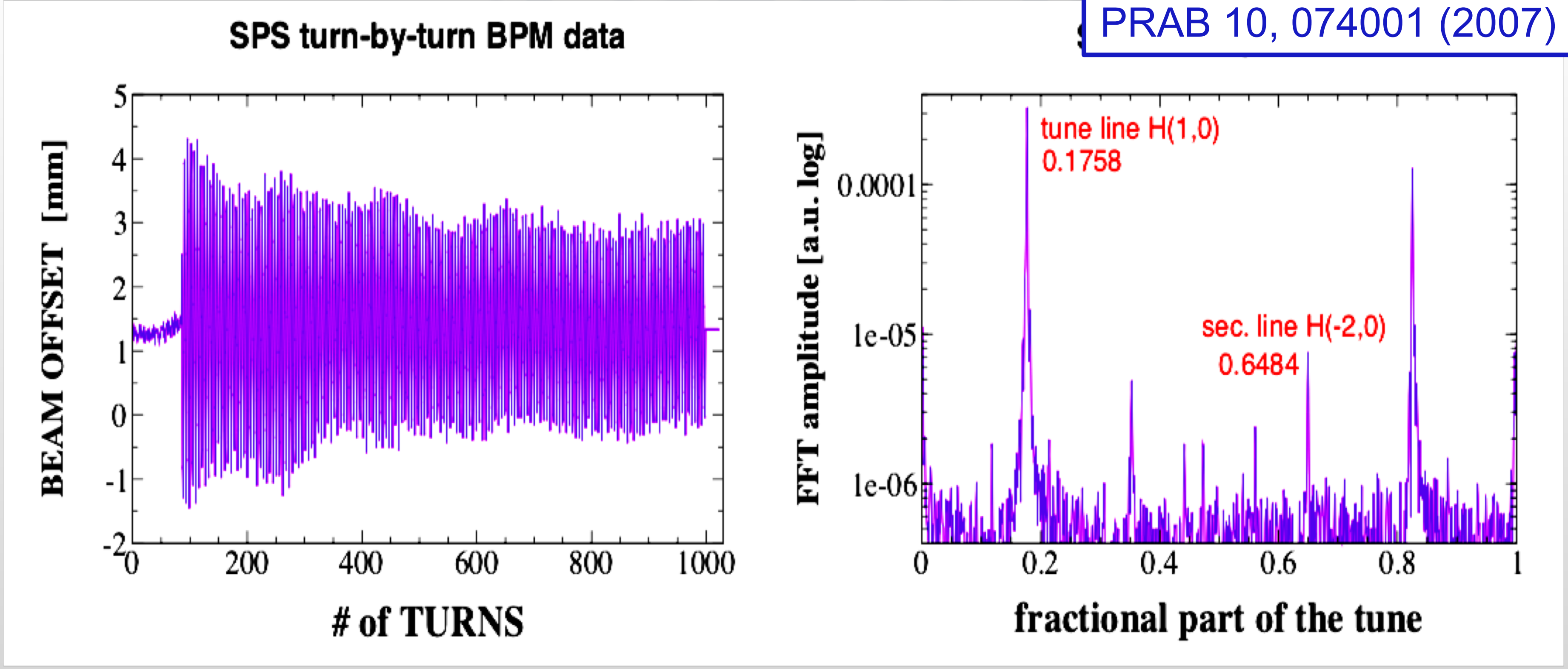
The RDT amplitude between two BPMs (w-1) & (w) changes only if there is at least one corresponding magnet (sextupole in the case of f_{3000}) within the two BPMs => This can be used to localize and calibrate magnets !!!

$$|f_{3000}| = \frac{H(-2,0)}{6H(1,0)^2}$$

$$q_{1001} = \phi_{H(-2,0)} + 2\phi_{H(1,0)} + \frac{\pi}{2}$$

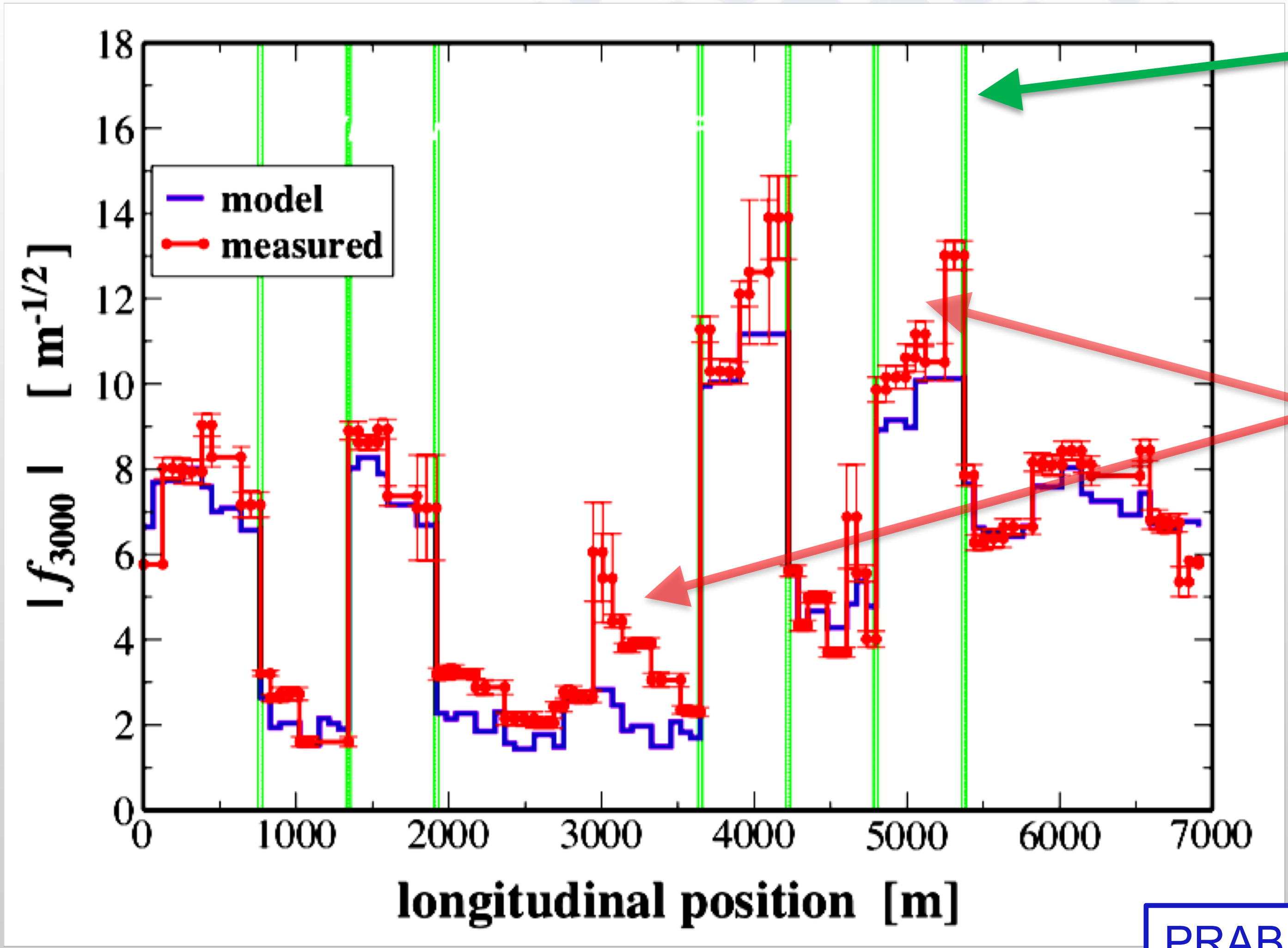
f_{3000} and sextupole strength measurement @ SPS

PRAB 10, 074001 (2007)



3rd caveat: sextupole (and higher-order) spectral lines are close to the background noise for operational setting. @ SPS and ESRF sextupoles were modified to enhance them.

f_{3000} and sextupole strength measurement @ SPS



7 strong slow-extraction sextupoles

artificial jumps from wrong p_x reconstruction due to (weak) chromatic sextupoles

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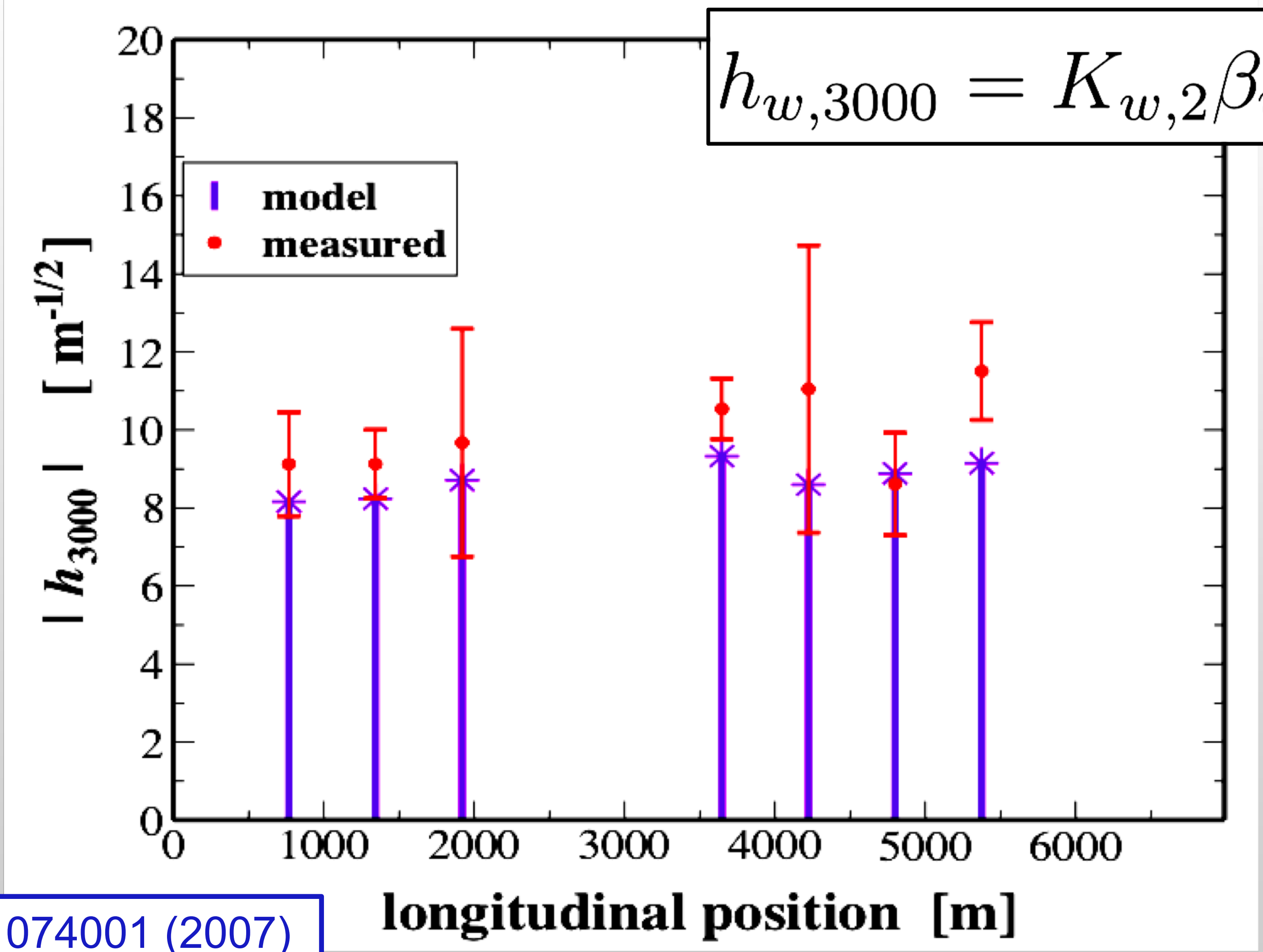
Nonlinear lattice error (model)

f_{3000} and sextupole strength measurement @ SPS



$$h_{w,3000} = K_{w,2}\beta_{w,x}^{3/2}/16$$

PRAB 10, 074001 (2007)

f_{3000} and sextupole strength measurement @ SPS

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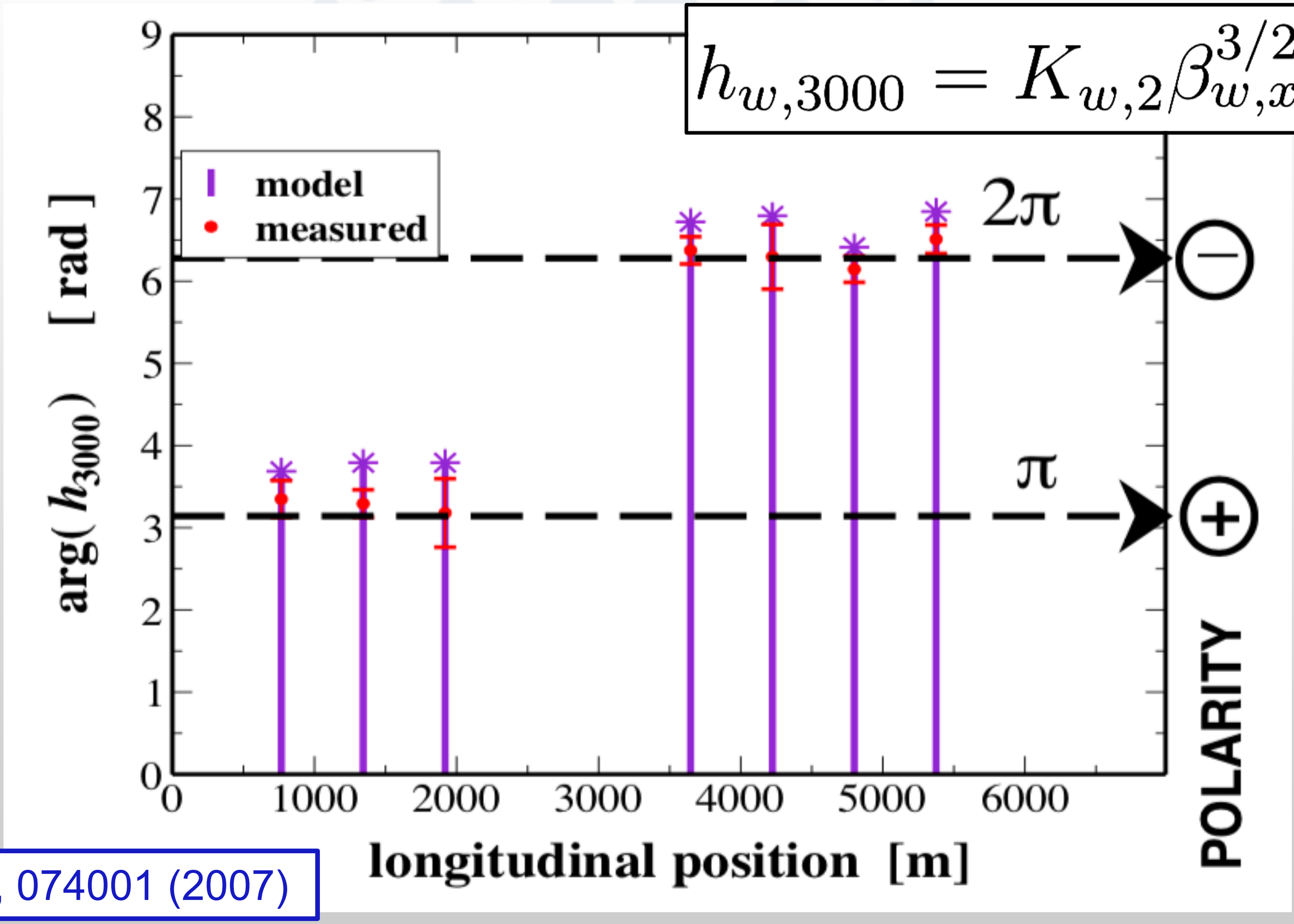

PRAB 10, 074001 (2007)

Caveats & cures when measuring nonlinear RDTs

1. measuring nonlinear RDTs from complex C-S signal is affected by a systematic error in the reconstruction of p_x

$$h_x = \tilde{x} - i\tilde{p}_x$$

$$\widetilde{p}_{i,x} = (\widetilde{x}_{i+1} - \widetilde{x}_i \cos\Delta\phi_x) / \sin\Delta\phi_x$$

assumption: no nonlinear magnets between BPMs or low amplitude

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assumption: no nonlinear magnets between BPMs or low amplitude

solution 1 (SPS, RHIC): combine signal from 3 BPMs to cancel out error in the reconstruction of p_x. No good when BPM phase advance multiple of 90° (IRs)

Caveats & cures when measuring nonlinear RDTs

1. measuring nonlinear RDTs from complex C-S signal is affected by a systematic error in the reconstruction of p_x

$$h_x = \tilde{x} - i\tilde{p}_x \qquad \tilde{p}_{i,x} = (\tilde{x}_{i+1} - \tilde{x}_i \cos\Delta\phi_x)/\sin\Delta\phi_x$$

assumption: no nonlinear magnets between BPMs or low amplitude

solution 2 (ESRF): FFT on real C-S signal $\tilde{x}$ only. Combined RDTs (CRDTs) can be measured

$$F_{NS3} = 3f_{3000} - f^*_{1200} = \sum_w \frac{K_{w,2}\beta^{3/2}_{w,x}}{16}\left[\frac{e^{-3i\Delta\phi_{x,bw}}}{1-e^{i6\pi Q_x}} - \frac{e^{i\Delta\phi_{x,bw}}}{1-e^{i2\pi Q_x}}\right]$$

$$\vec{F}_{NS,\mathrm{meas}} - \vec{F}_{NS,\mathrm{mod}} = \mathbf{M}_{\mathbf{NS}}\Delta\vec{K}_2$$

pseudo-inverted (SVD)

Caveats & cures when measuring nonlinear RDTs

2. Measured RDTs affected by BPM calibration errors (if unknown)

$$|f_{3000}| = \frac{H(-2,0)}{6H(1,0)^2}$$

Caveats & cures when measuring nonlinear RDTs

2. Measured RDTs affected by BPM calibration errors (if unknown)

$$|f_{3000}| = \frac{H(-2,0)}{6H(1,0)^2}$$

$$\tilde{H}(-2,0) = \frac{H(-2,0)}{H(1,0)} = 6|f_{3000}|\sqrt{2I_x}$$

solution 1 (SPS, RHIC): measure calibration-free $\tilde{H}$, repeat measurement @ different kicker strengths, i.e. $\sqrt{2I_x}$ and infer $|f_{3000}|$ from slope of linear fit Vs $\sqrt{2I_x}$, i.e. tune line amplitude H(1,0).

Caveats & cures when measuring nonlinear RDTs

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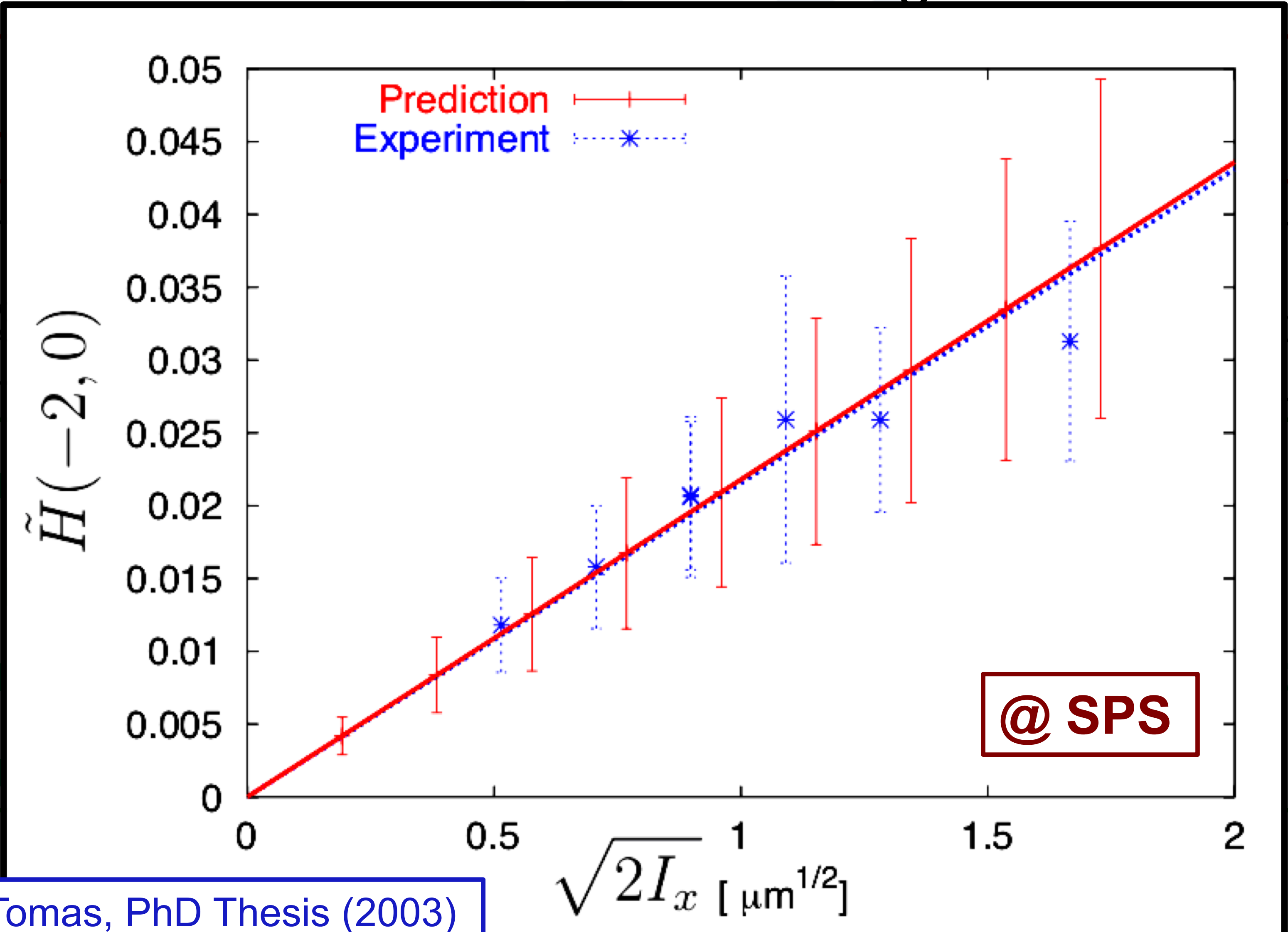


R. Tomas, PhD Thesis (2003)

Caveats & cures when measuring nonlinear RDTs

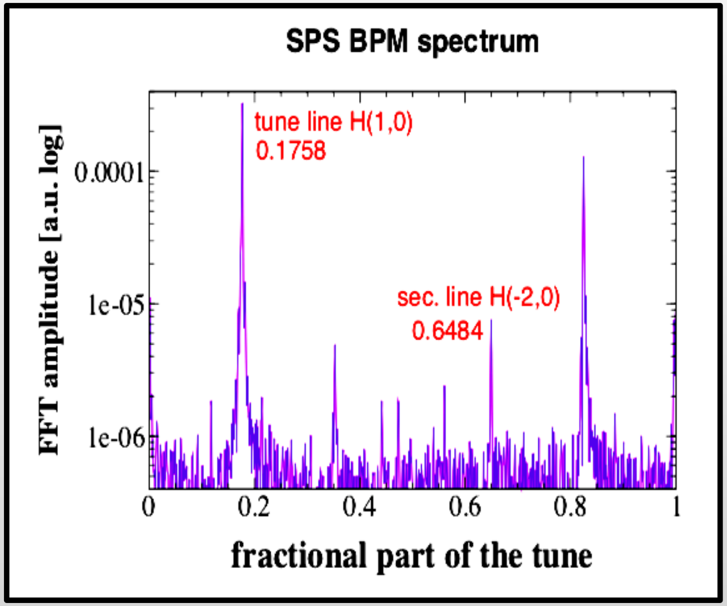
2. Measured RDTs affected by BPM calibration errors (if unknown)

$$|f_{3000}| = \frac{H(-2,0)}{6H(1,0)^2}$$

solution 2 (ESRF): infer BPM calibration errors from ORM fit and use them on TbT data. Remark # 1: not significant impact @ ESRF (errors ~ 1-2%). Remark # 2: is assumes that DC calibration errors are frequency independent

Caveats & cures when measuring nonlinear RDTs

3. sextupole (and higher-order) spectral lines are close to the background noise for operational setting.

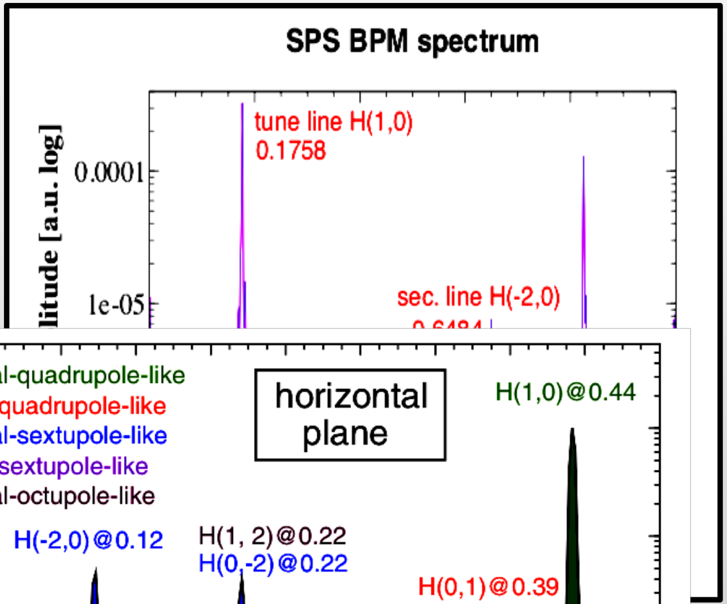


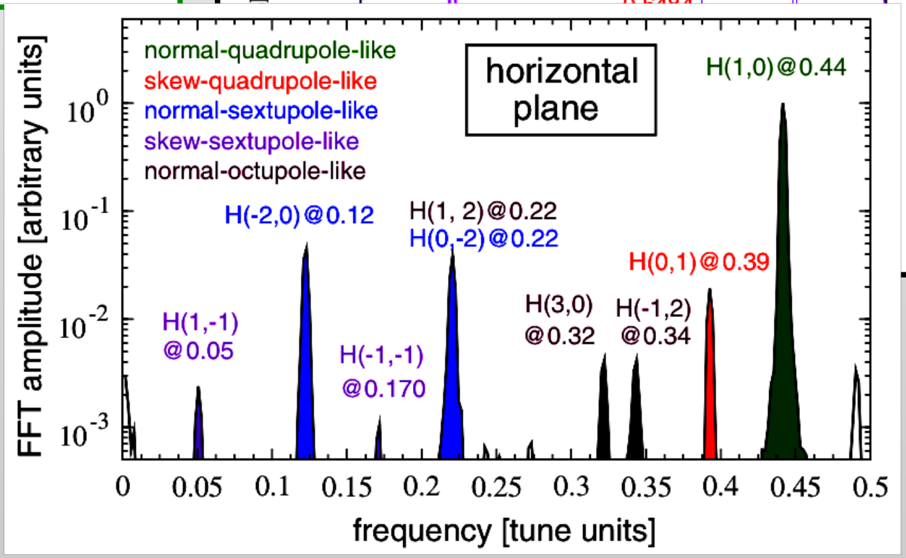
Caveats & cures when measuring nonlinear RDTs

3. sextupole (and higher-order) spectral lines are close to the background noise for operational setting.

solution (SPS & ESRF): Use modified (non-operational) sextupole setting to carry RDT measurement

impractical @ LHC





Caveats & cures when measuring nonlinear RDTs

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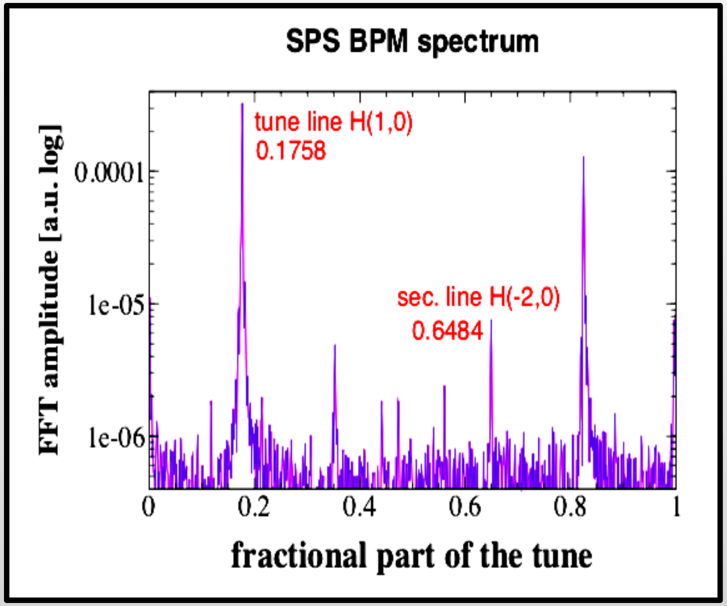
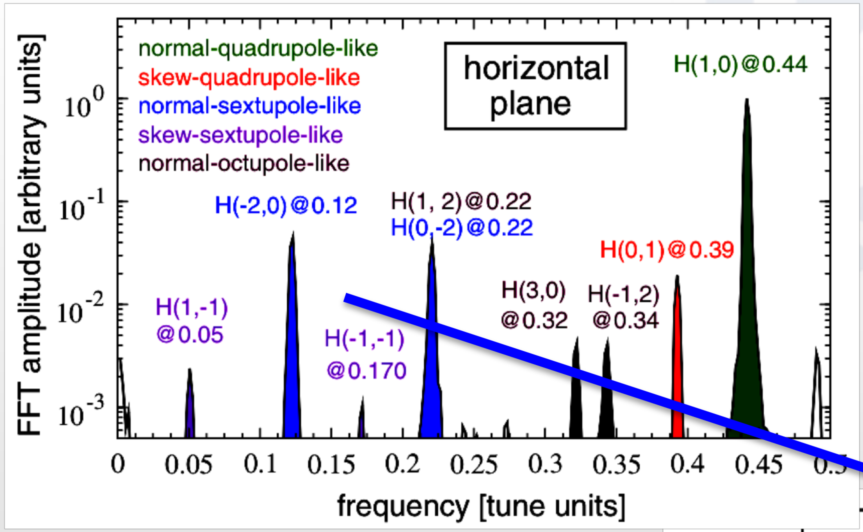


4. @ LHC & RHIC AC dipole is used instead of pulsed kicker: AC dipole alter the content of sextupole (and higher-order) spectral lines, theory not yet avaliable

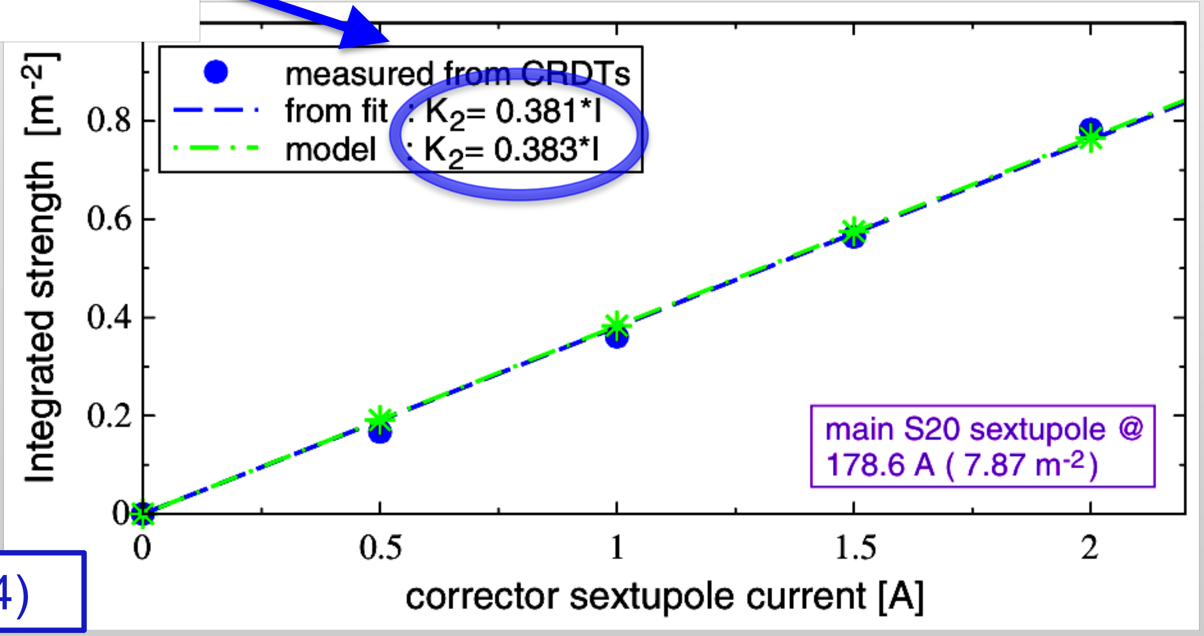
Measurement of nonlinear CRDTs @ ESRF



beam-based
sextupole error model

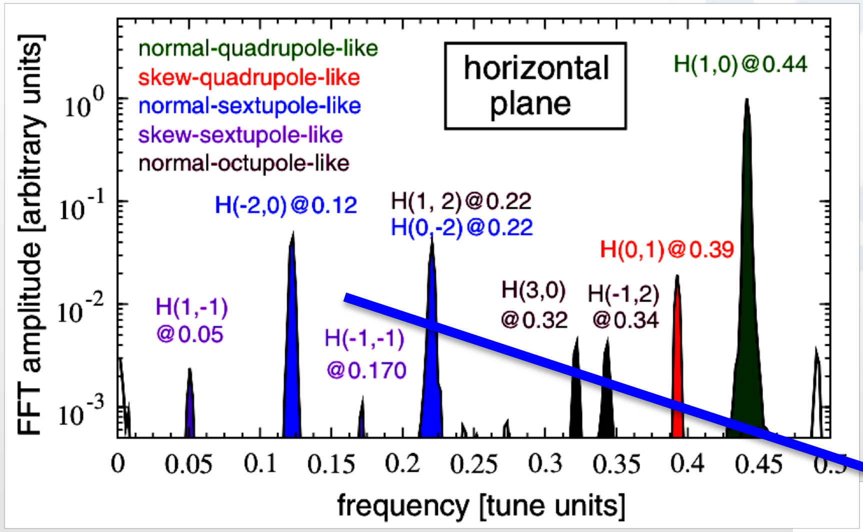
FFT on real C-S signal $\tilde{x}$ only.

$$F_{NS3} = 3f_{3000} - f^*_{1200}$$



PRAB, 17, 074001 (2014)

Measurement of nonlinear CRDTs @ ESRF



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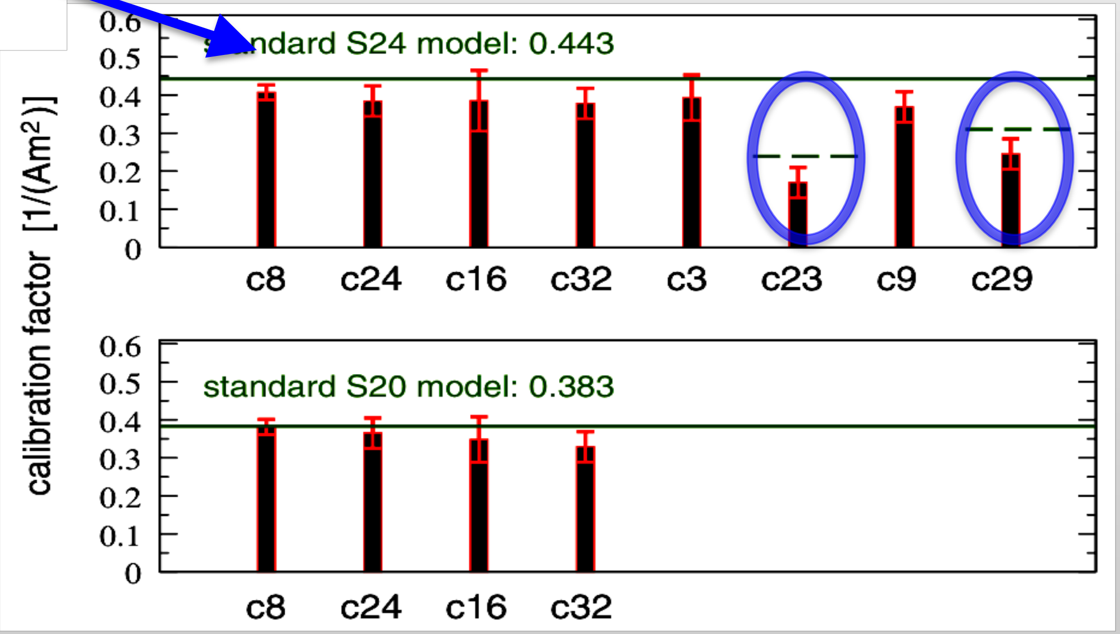
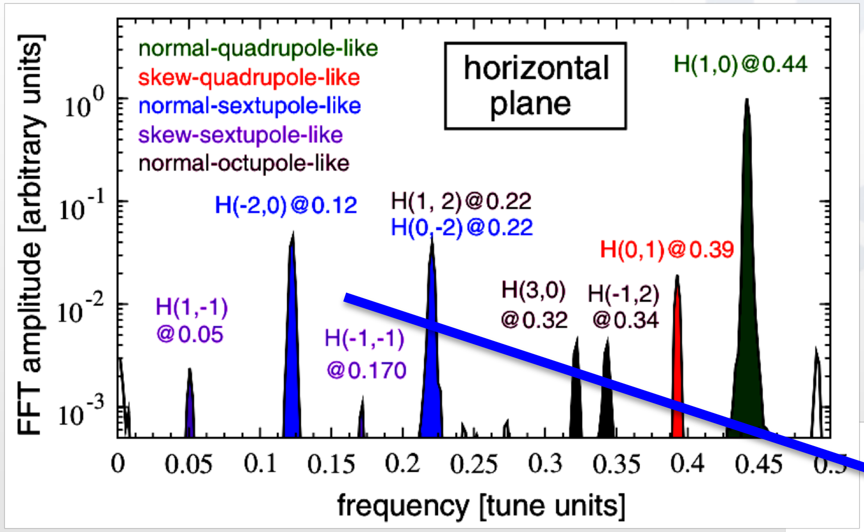
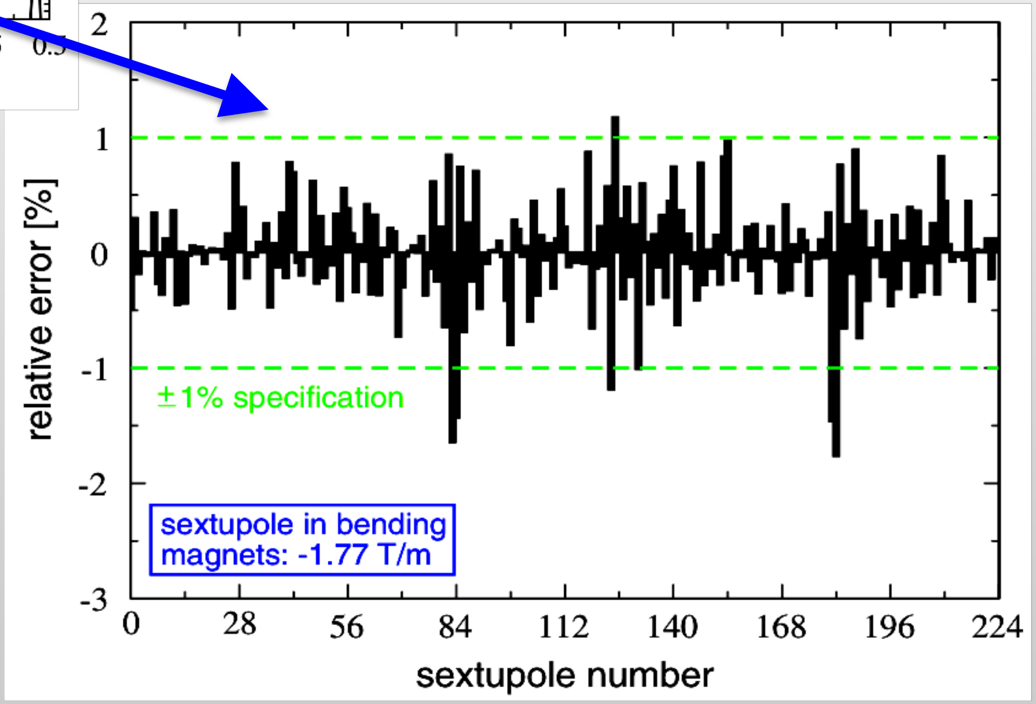


PRAB, 17, 074001 (2014)

Measurement of nonlinear CRDTs @ ESRF

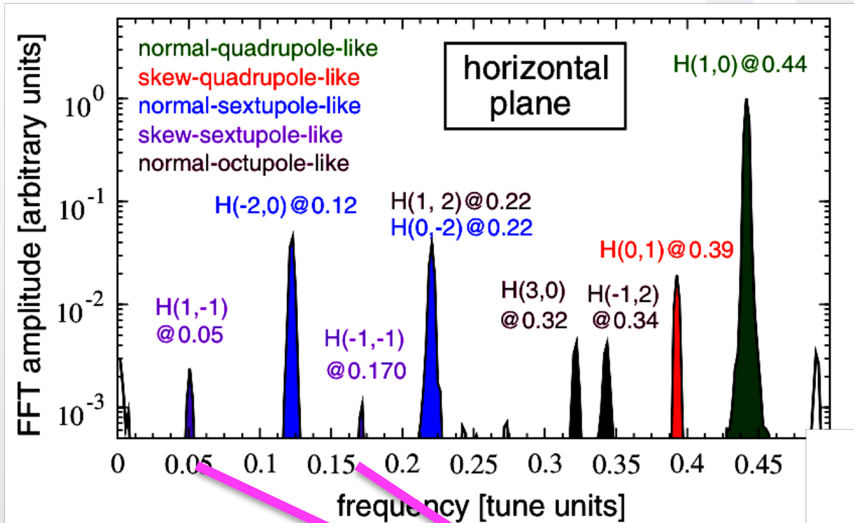


beam-based sextupole error model (strength)

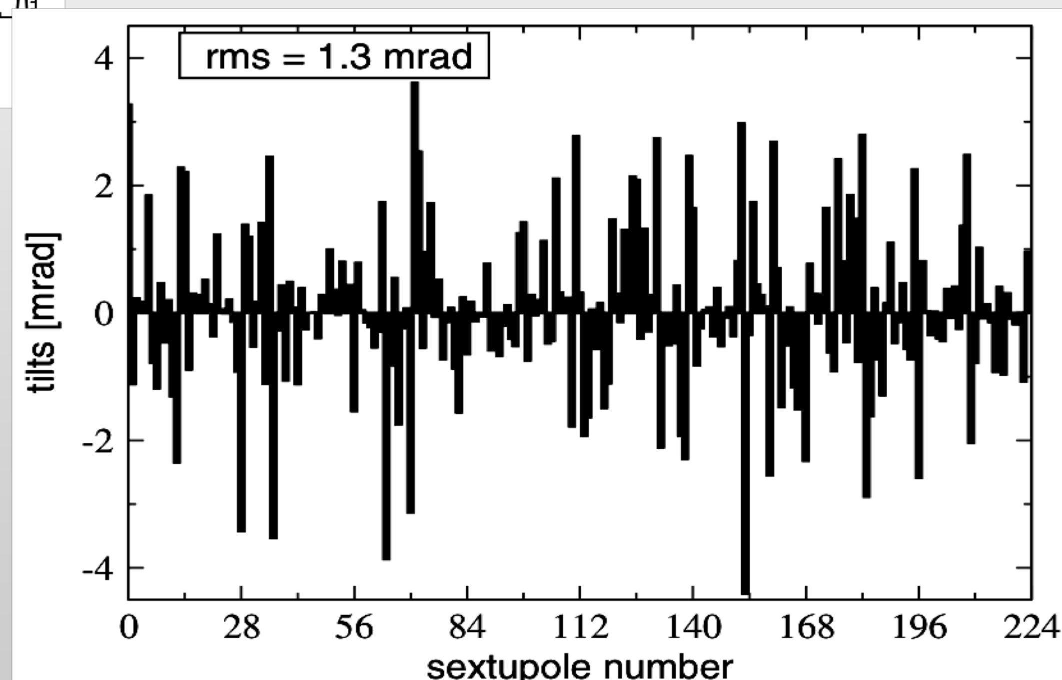


PRAB, 17, 074001 (2014)

Measurement of nonlinear CRDTs @ ESRF

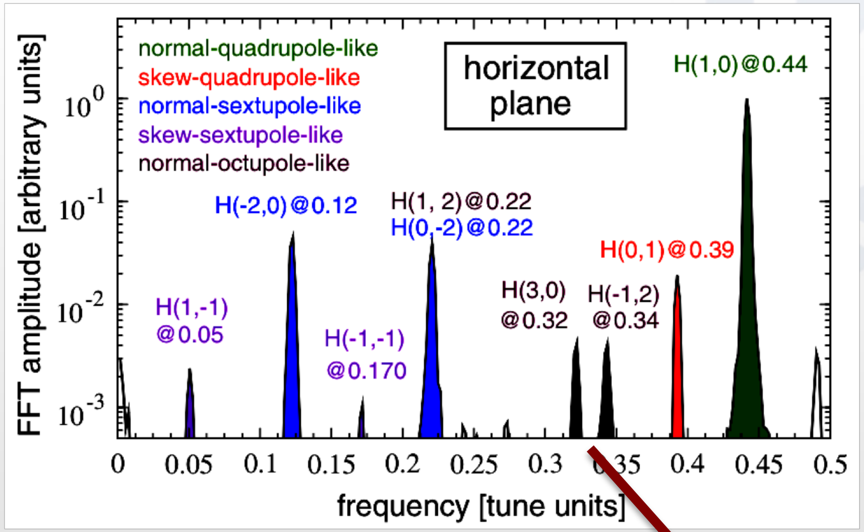


beam-based sextupole error model (tilt)



PRAB, 17, 074001 (2014)

Measurement of nonlinear CRDTs @ ESRF



beam-based octupolar model (in quads)

Quad family	K_3 $[m^{-3}]$ (average ± rms)
QF2	4.4 ± 1.6
QD3	-6.9 ± 0.7
QD4	2.5 ± 0.8
QF5	-2.2 ± 1.4
QD6	-0.4 ± 2.4
QF7	4.0 ± 3.3

PRAB, 17, 074001 (2014)

Nonlinear lattice error (model)

RDTs Vs chromatic functions and orbit feed-downs

The strengths of nonlinear magnets (K_2, J_2, K_3, …) can be inferred "directly" from the (C)RDTs via TbT BPM data, though with all aforementioned caveats.

Nonlinear lattice error (model)

RDTs Vs chromatic functions and orbit feed-downs

The strengths of nonlinear magnets (K_2, J_2, K_3, …) can be inferred "directly" from the (C)RDTs via TbT BPM data, though with all aforementioned caveats.

Alternatively, their strengths can be inferred from a linear analysis (focusing & coupling) after making the beam cross the nonlinear magnets off-axis (Δx, Δy), via feed-down fields:

$$\delta K_1 = 2(K_2 \Delta x + J_2 \Delta y) \qquad \delta J_1 = 2(J_2 \Delta x + K_2 \Delta y)$$

RDTs Vs chromatic functions and orbit feed-downs

The strengths of nonlinear magnets (K_2, J_2, K_3, …) can be inferred "directly" from the (C)RDTs via TbT BPM data, though with all aforementioned caveats.

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$$\delta K_1 = 2(K_2 \Delta x + J_2 \Delta y) \qquad \delta J_1 = 2(J_2 \Delta x + K_2 \Delta y)$$

(Δx, Δy) can be generated by going off energy $\delta p/p$ (via dispersion $\Delta x = D_x \delta p/p$), with orbit bumps or controlled distortion, with crossing angle orbit @ IRs

Nonlinear lattice error (model)

RDTs Vs chromatic functions and orbit feed-downs

$$\delta K_1 = 2(K_2 \Delta x + J_2 \Delta y) \qquad \delta J_1 = 2(J_2 \Delta x + K_2 \Delta y)$$

$(\Delta x, \Delta y)$ generated by going **off energy** $\delta p/p$ (via dispersion $\Delta x = D_x \delta p/p$).

- $\delta p/p$ error from $\delta f_{RF}/f_{RF}$ (very accurate!) and momentum compaction (1% w.r.t. model @ ESRF)
- D_x @ sextupole not observable, fit needed from data @ BPMs
- effective analysis only for chromatic sextupoles with large dispersion, not for harmonic.
- dependence of Twiss upon $\delta p/p$, i.e. chromatic functions and coupling as observables.

Nonlinear lattice error (model)

RDTs Vs chromatic functions and orbit feed-downs

$$\frac{\partial \beta_x(j)}{\partial \delta} \simeq -\beta_x(j) + \frac{\beta_x(j)}{2\sin(2\pi Q_x)} \sum_{m=1}^{M} (K_{m,1} - K_{m,2} D_{m,x})\, \beta_{m,x} \cos(2\Delta\phi_{x,mj} - 2\pi Q_x)$$

$$\frac{\partial \beta_y(j)}{\partial \delta} \simeq -\beta_y(j) - \frac{\beta_y(j)}{2\sin(2\pi Q_y)} \sum_{m=1}^{M} (K_{m,1} - K_{m,2} D_{m,x})\, \beta_{m,y} \cos(2\Delta\phi_{y,mj} - 2\pi Q_y)$$

$$D'_x(j) = -2D_x(j) + \frac{\sqrt{\beta_{j,x}}}{\sin(\pi Q_x)} \sum_{m=1}^{M} \left[K_{m,1} - \frac{1}{2} K_{m,2} D_{m,x}\right] D_{m,x} \sqrt{\beta_{m,x}} \cos(\Delta\phi_{x,mj} - \pi Q_x))$$

D'_y is a bit more complicated expression

@ ESRF

[green box] from meas. & fit of standard on-energy ORM

[red box] from meas. & fit of 1 or 2 off-energy ORM
the dispersive off-axis orbit across sextupoles introduces additional focusing ($d\beta/d\delta$) and dispersion (D').

Nonlinear lattice error (model)

RDTs Vs chromatic functions and orbit feed-downs

$$\frac{\partial \beta_x(j)}{\partial \delta} \simeq -\beta_x(j) + \frac{\beta_x(j)}{2\sin(2\pi Q_x)} \sum_{m=1}^{M} (K_{m,1} - K_{m,2} D_{m,x})\, \beta_{m,x} \cos(2\Delta\phi_{x,mj} - 2\pi Q_x)$$

$$\frac{\partial \beta_y(j)}{\partial \delta} \simeq \ldots {}_{,mj} - 2\pi Q_y)$$

$$D'_x(j) = \ldots \phi_{x,mj} - \pi Q_x) \,)$$

D'_y is a ... @ ESRF

to be pseudo-inverted (SVD)

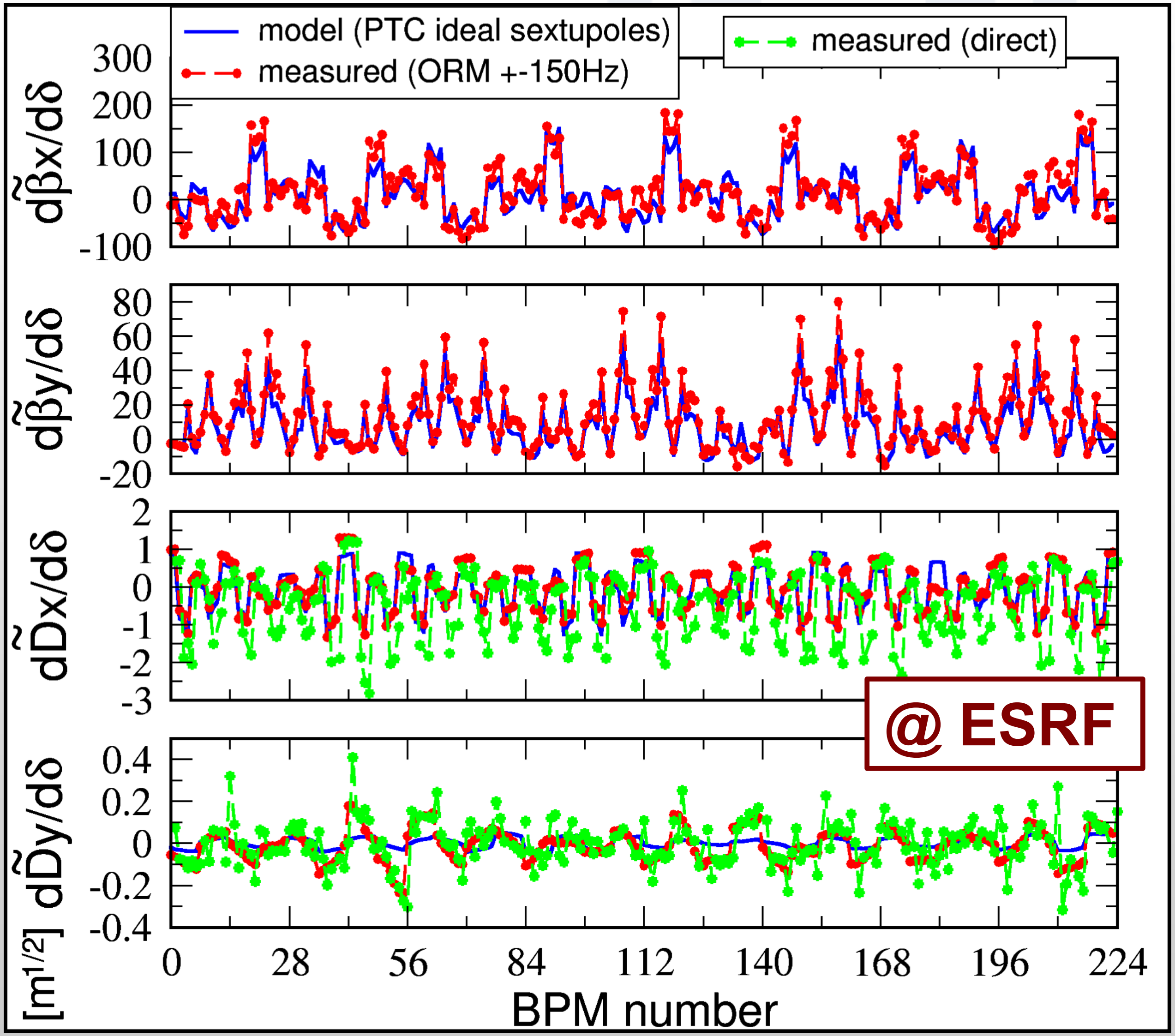
$$\begin{pmatrix} \partial\vec{\beta}/\partial\delta \\ \partial\vec{D}/\partial\delta \\ \partial\vec{F}_{xy}/\partial\delta \end{pmatrix}^{(fit)} \Rightarrow \begin{pmatrix} \vec{\beta}\,' \\ \vec{D}\,' \\ \vec{F}_{xy}\,' \end{pmatrix}^{(fit)} - \begin{pmatrix} \vec{\beta}\,' \\ \vec{D}\,' \\ \vec{F}_{xy}\,' \end{pmatrix}^{(model)} = S \begin{pmatrix} \delta\vec{K}_2^{(sext)} \\ \vec{\theta}^{(sext)} \end{pmatrix}$$

(green) from meas. & fit of standard on-energy ORM

(red) from meas. & fit of 1 or 2 off-energy ORM
the dispersive off-axis orbit across sextupoles introduces additional focusing ($d\beta/d\delta$) and dispersion (D').

Nonlinear lattice error (model)

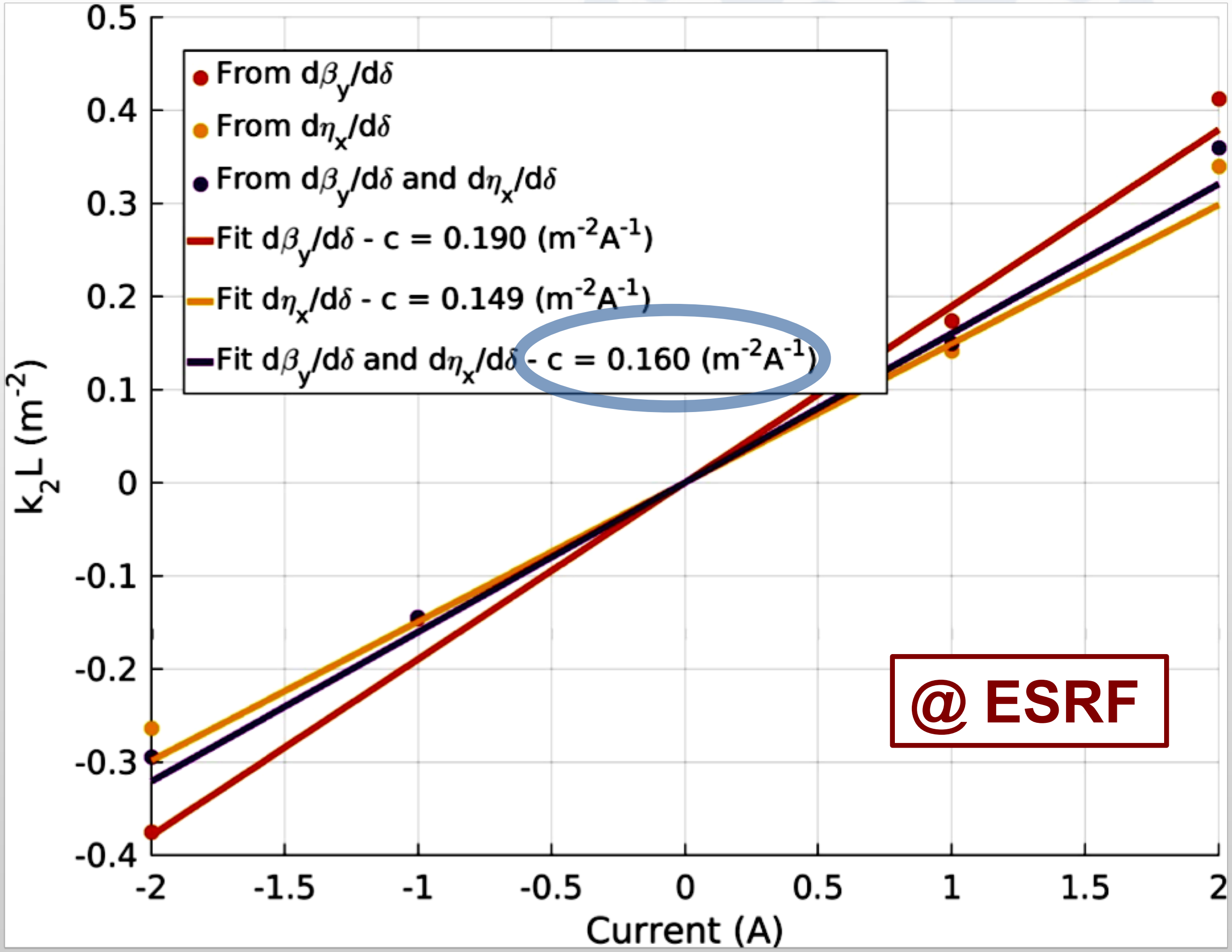
RDTs Vs chromatic functions and orbit feed-downs



Sextupole Calibration from Chromatic functions (off-energy ORMs)

Nonlinear lattice error (model)

RDTs Vs chromatic functions and orbit feed-downs

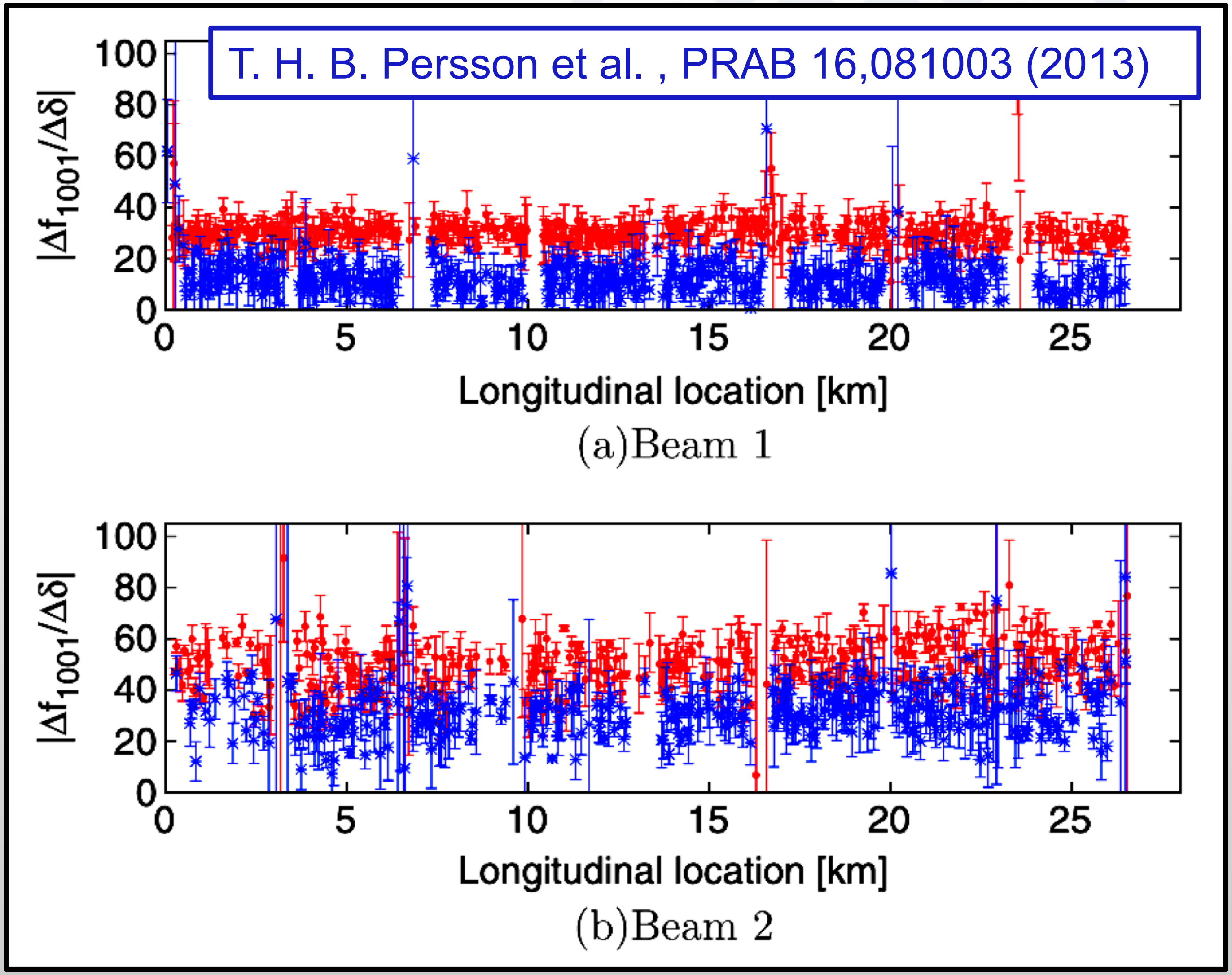


Sextupole Calibration from Chromatic functions (off-energy ORMs)

The calibration factor from magnetic measurements is 0.1569 m^{-2} A^{-1}

Nonlinear lattice error (model)

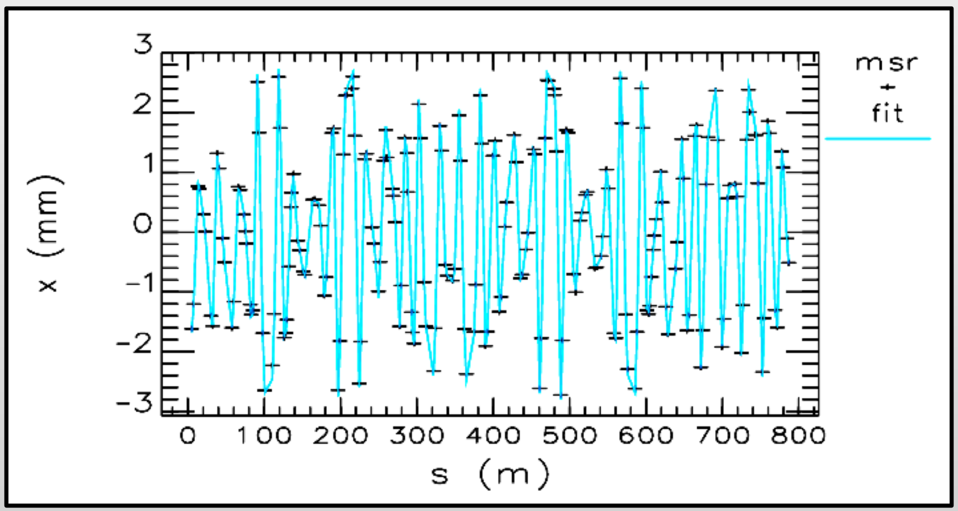
RDTs Vs <u>chromatic functions</u> and orbit feed-downs

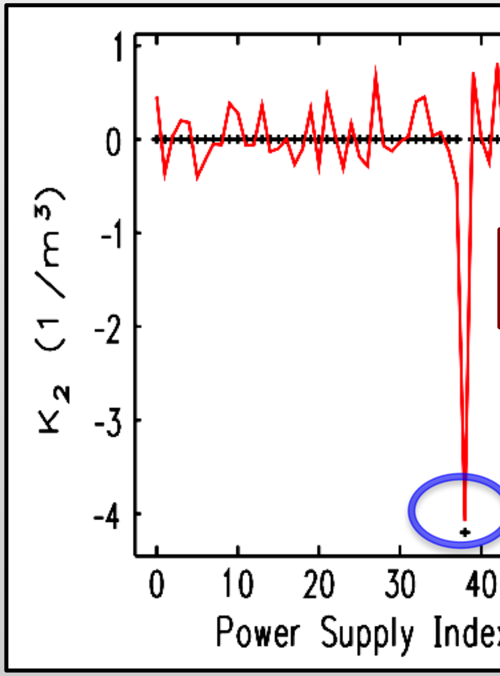


measurem. & correction of chromatic coupling via $\Delta f_{1001}/\Delta\delta$ & skew sextupoles @ LHC (TbT BPM data)

RDTs Vs chromatic functions and orbit feed-downs

- tune response to bump on sextupoles (G. Franchetti et al., PRSTAB, 11, 094001, 2008) [*]
- betatron phase advance response to off-axis orbits @ sextupoles (W. Guo et al, PRAB 21, 081001, 2018) [*]





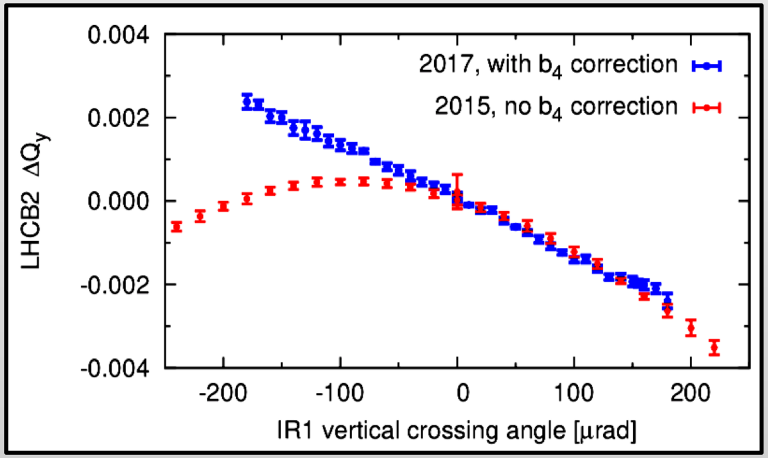
@ NLS II

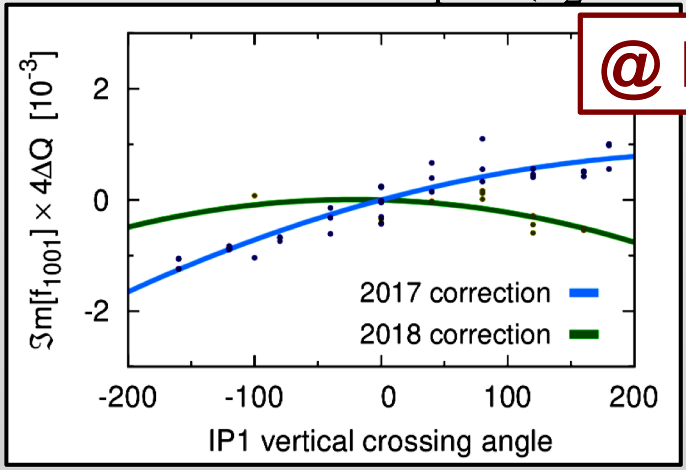
[*] these methods may suffer from degeneracy issues (several sextupoles within an orbit bump) and no knowledge of orbit offsets (Δx, Δy) @ sexts

Nonlinear lattice error (model)

RDTs Vs chromatic functions and orbit feed-downs

- response to off-axis orbits (Δx, Δy) on sextupoles & octupoles generated by crossing-angle schemes @ colliders' IRs (E.H. Maclean et al., CERN-ACC-2019-0029) [^]
- tune Vs crossing angle $\delta K_1=2(K_2\Delta x+ J_2\Delta y) +3K_3(\Delta x^2+\Delta y^2)$
- ΔQ_{min} & f_{1001} Vs crossing angle $\delta J_1=2(J_2\Delta x+ K_2\Delta y)$





@ LHC

[^] goal is to correct nonlinearities @ IRs with operational β* optics , not possible to use strong magnet to enhance RDTs (not used)

Outlines

- Resonance Driving Terms Introductory (15’)
- Linear optics errors (10’)
    1. RDTs Vs beta-beating and phase advance error
    2. RDTs measurements & correction
    3. Accuracy and precision analysis
- Betatron coupling (10’)
    1. RDTs measurements & correction
    2. Hadron Vs lepton machines
- Nonlinear lattice error (model) (15’)
    1. Localization & detection of nonlinearities via RDTs
    2. RDTs Vs chromatic functions and orbit feed-downs

EXTRA SLIDES

EXTRA SLIDES

Nonlinear one-turn map (OTM from turn *N* to *N+1*)

$$h_{x,\pm} = \tilde{x} \pm i\tilde{p}_x = \sqrt{2J_x}e^{\mp i(\phi_x + \phi_{x,0})}$$

complex C-S coordinates

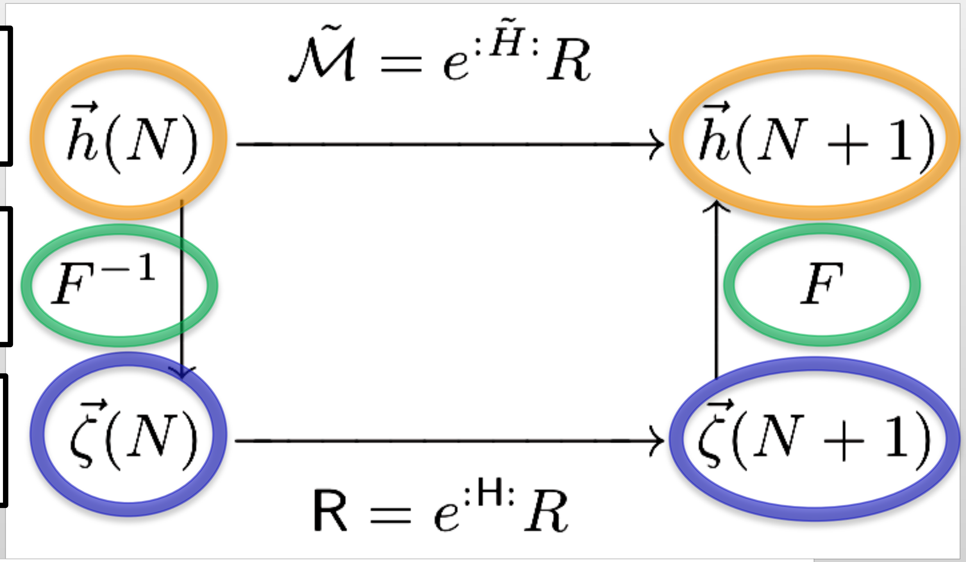
$$\zeta_{q,\pm} = \sqrt{2I_q}e^{\mp i(\psi_q + \psi_{q,0})}$$

normal forms coordinates

$\tilde{\mathcal{M}}$ — s-dependent OTM

$e^{:\tilde{H}:}$ — phase-space deformation

R — phase-space linear rotation



s-independent OTM (rotation) — R

s-independent detuning with amplitude — $e^{:\mathrm{H}:}$

$$F = \sum_{n\geq 2} \sum_{jklm}^{n=j+k+l+m} f_{jklm}\zeta_{x,+}^{j}\zeta_{x,-}^{k}\zeta_{y,+}^{l}\zeta_{y,-}^{m}$$

s-dependent normal forms transformation

Measuring β-beating from BPM phase advance (FFT of TbT data)

BPM phase advance = difference between the phase of the tune spectral line measured @ 2 BPMs

Castro's formula (@LEP), assumes no error between 2 BPMs

$$\beta_1^{(meas)} = \beta_1^{(mod)} \frac{\cot \Delta\phi_{21}^{(meas)} - \cot \Delta\phi_{31}^{(meas)}}{\cot \Delta\phi_{21}^{(mod)} - \cot \Delta\phi_{31}^{(mod)}}$$

Measurement error depends thus on the unknown quadrupole error. Iterations are needed

Measuring β-beating from BPM phase advance (FFT of TbT data)

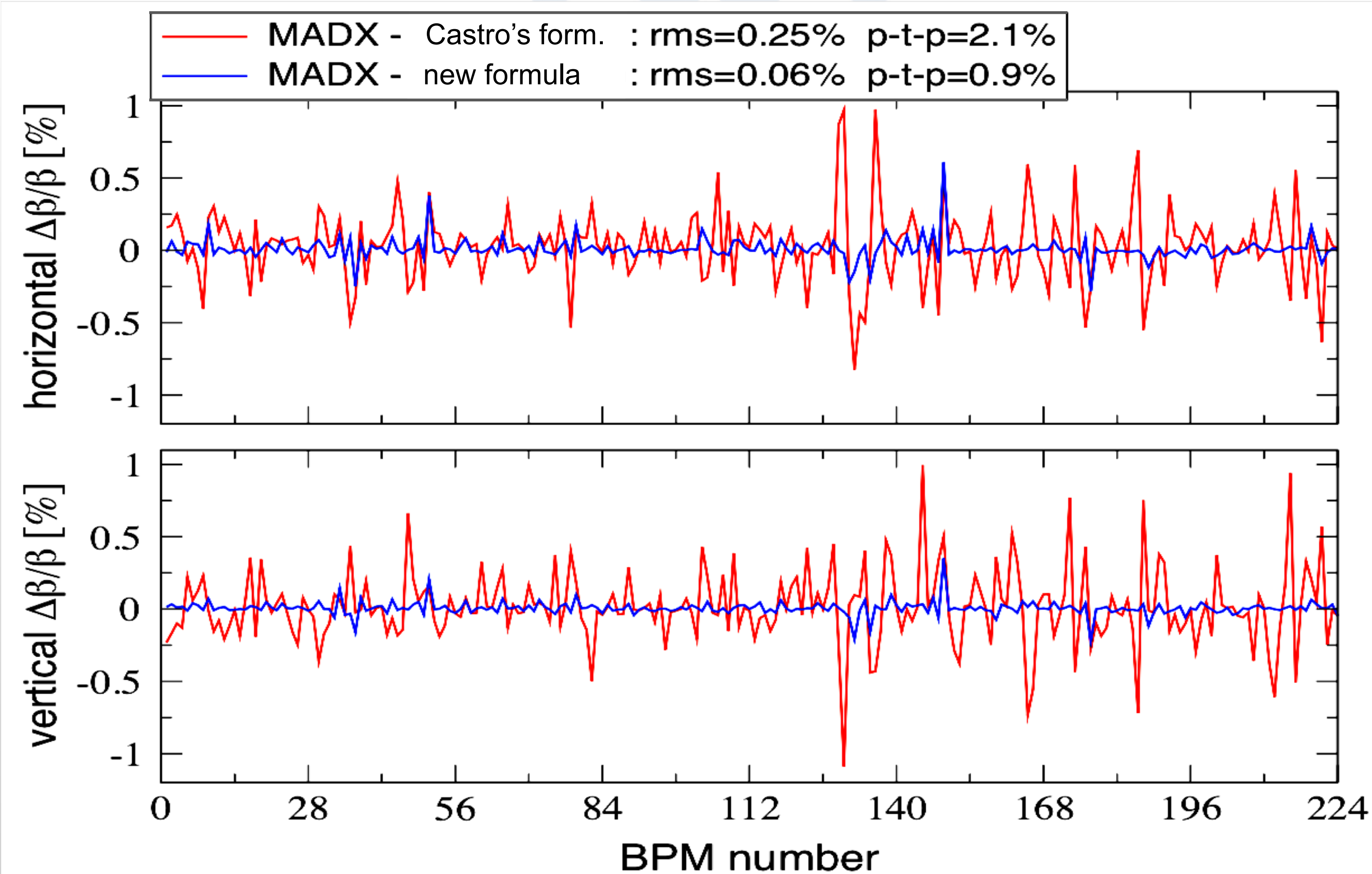
Modified Castro's formula (@ESRF, LHC) accounts for quadrupole error between 2 BPMs

$$\beta_1^{(meas)} = \beta_1^{(mod)} \frac{\cot \Delta\phi_{21}^{(meas)} - \cot \Delta\phi_{31}^{(meas)}}{\cot \Delta\phi_{21}^{(mod)} - \cot \Delta\phi_{31}^{(mod)} + (\bar{h}_{21} - \bar{h}_{31})} + O(\delta K_1^2)$$

$$\bar{h}_{ij} = \mp \frac{\sum_{j<w<i} \beta_w^{(mod)} \delta K_{w,1} \sin^2 \Delta\phi_{iw}^{(mod)}}{\sin^2 \Delta\phi_{ij}^{(mod)}}$$

not observable, though useful in simulations (HL-LHC) and to estimate measurement error

Measuring β-beating from BPM phase advance (FFT of TbT data)



increasing precision in measuring β function

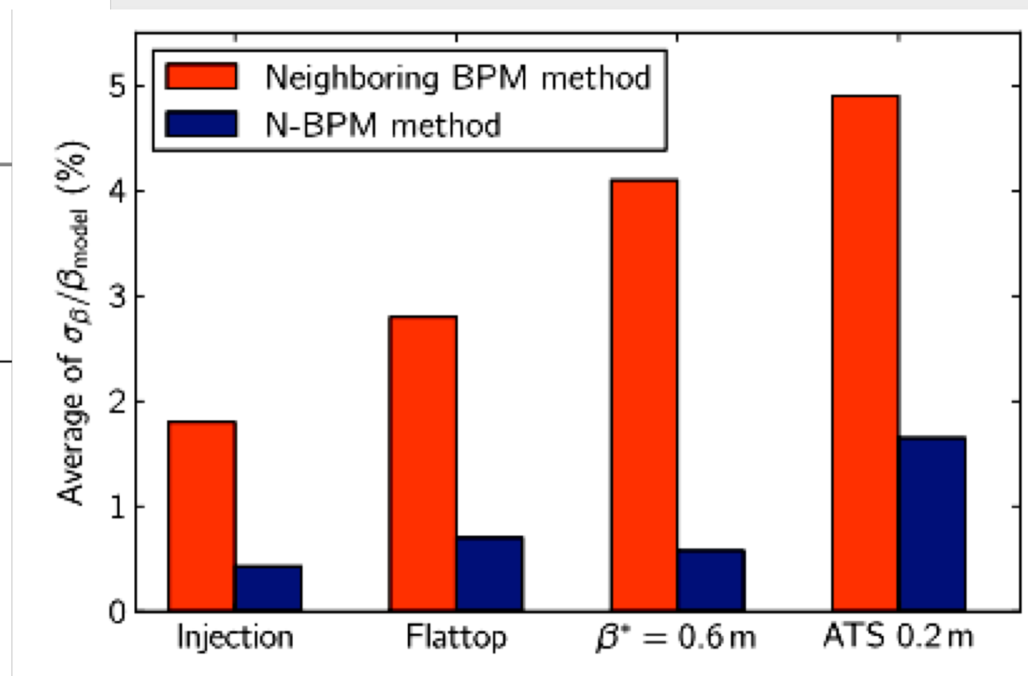
The N-BPM method

$$\beta_1^{(meas)} = \beta_1^{(mod)} \frac{\cot\Delta\phi_{12}^{(meas)} - \cot\Delta\phi_{13}^{(meas)}}{\cot\Delta\phi_{12}^{(mod)} - \cot\Delta\phi_{13}^{(mod)}}$$

TABLE III. Systematic error of the measured β-function at arc BPMs for using different BPM combinations. The phase advance between consecutive BPMs is approximately $\pi/4$.

BPM combination (▲: probed, ▲: used, ▲: unused)	Systematic error (%)
unused, used, probed, used, unused	0.3
used, used, probed, unused, unused	0.4
used, unused, probed, used, unused	1.0
used, unused, probed, unused, used	7.1
used, unused, used, probed, unused, unused, unused	1.1
used, unused, unused, probed, unused, unused, used	1.4
used, used, unused, probed, unused, unused, unused	1.7
unused, used, unused, probed, unused, unused, used	1.8
used, unused, unused, probed, used, unused, unused	7.9
used, used, unused, unused, unused, probed, unused, unused, unused, unused, unused	22.3
used, unused, used, unused, unused, probed, unused, unused, unused, unused, unused	1.3
used, unused, unused, used, unused, probed, unused, unused, unused, unused, unused	1.9
used, unused, unused, unused, used, probed, unused, unused, unused, unused, unused	6.1
used, unused, unused, unused, unused, probed, used, unused, unused, unused, unused	1.0
used, unused, unused, unused, unused, probed, unused, used, unused, unused, unused	3.0
used, unused, unused, unused, unused, probed, unused, unused, used, unused, unused	4.5
used, unused, unused, unused, unused, probed, unused, unused, unused, used, unused	5.2
used, unused, unused, unused, unused, probed, unused, unused, unused, unused, used	1.6

LHC 2016



Measuring β-beating from BPM phase advance or tune amplitude (FFT of TbT data)

$$\beta_1^{(meas)} = \beta_1^{(mod)} \frac{\cot \Delta\phi_{21}^{(meas)} - \cot \Delta\phi_{31}^{(meas)}}{\cot \Delta\phi_{21}^{(mod)} - \cot \Delta\phi_{31}^{(mod)}}$$

$$\Delta\Phi_{ij} = \Delta\phi_{ij} + \delta\phi_{ij}^{(tim)}$$

model & BPM synchronization * dependent

BPM calibration ε independent

$$\beta_{x,j}^{(meas)} = \beta_{x,j}^{(mod)} \left(\frac{|H(1,0)_j|}{< |H(1,0)| >} \right)^2 + O(\mathcal{E}_x, |f_{2000}|^2)$$

model & BPM synchronization independent

BPM calibration ε dependent

*: @ ESRF 0.1 μs p-t-p over 2.82 μs revolution time

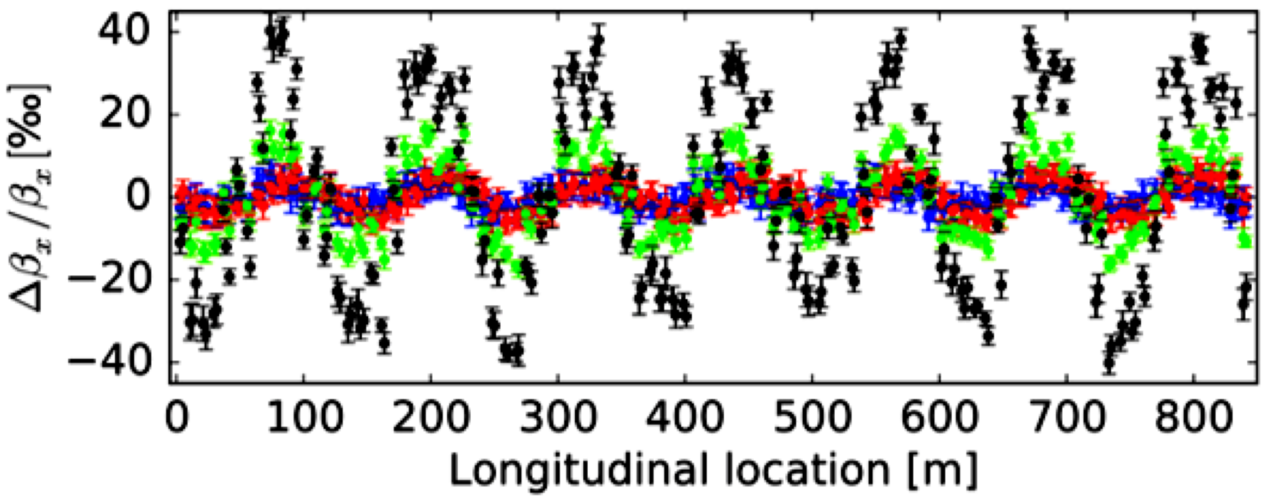
Kick Amplitude

artificial β-beating from TbT data

- Limited by non-linearities – simulation

$$\beta_1^{(meas)} = \beta_1^{(mod)} \frac{\cot \Delta\phi_{12}^{(meas)} - \cot \Delta\phi_{13}^{(meas)}}{\cot \Delta\phi_{12}^{(mod)} - \cot \Delta\phi_{13}^{(mod)}}$$

The measured BPM phase advance = betatron BPM phase advance + non-linear terms



Simulated kicks @ β = 38 m

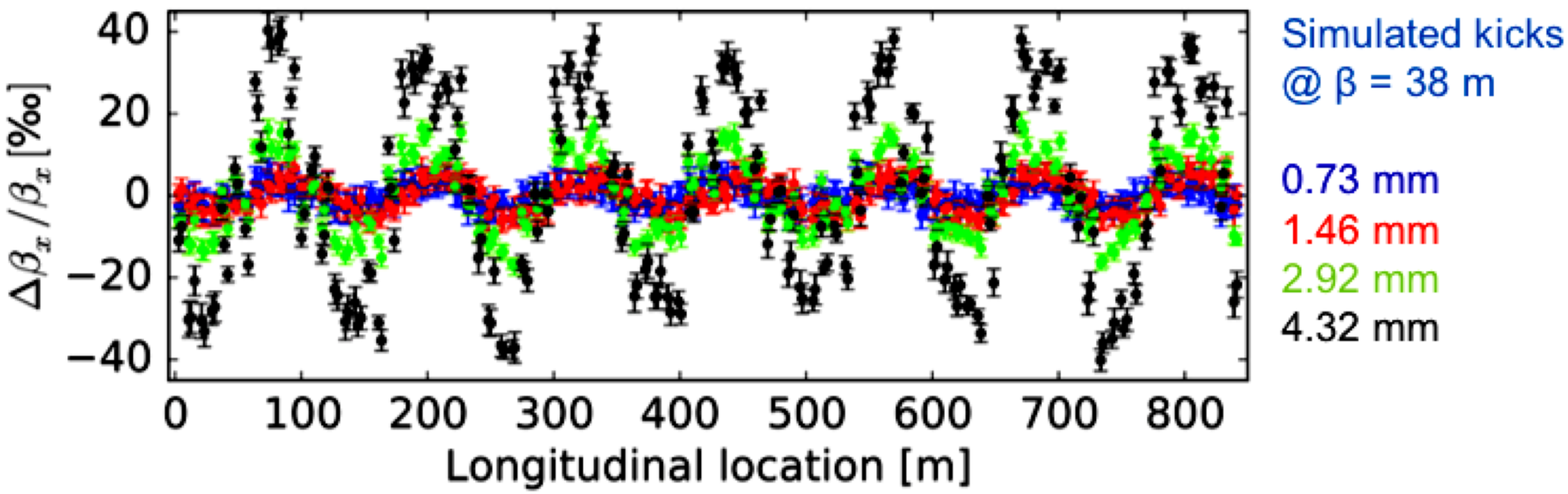
0.73 mm
1.46 mm
2.92 mm
4.32 mm

Linear optics errors

Kick Amplitude

artificial β-beating from TbT data

- Limited by non-linearities – simulation
- However has to be high enough
  - with respect to BPM resolution
  - Decoherence and number of turns



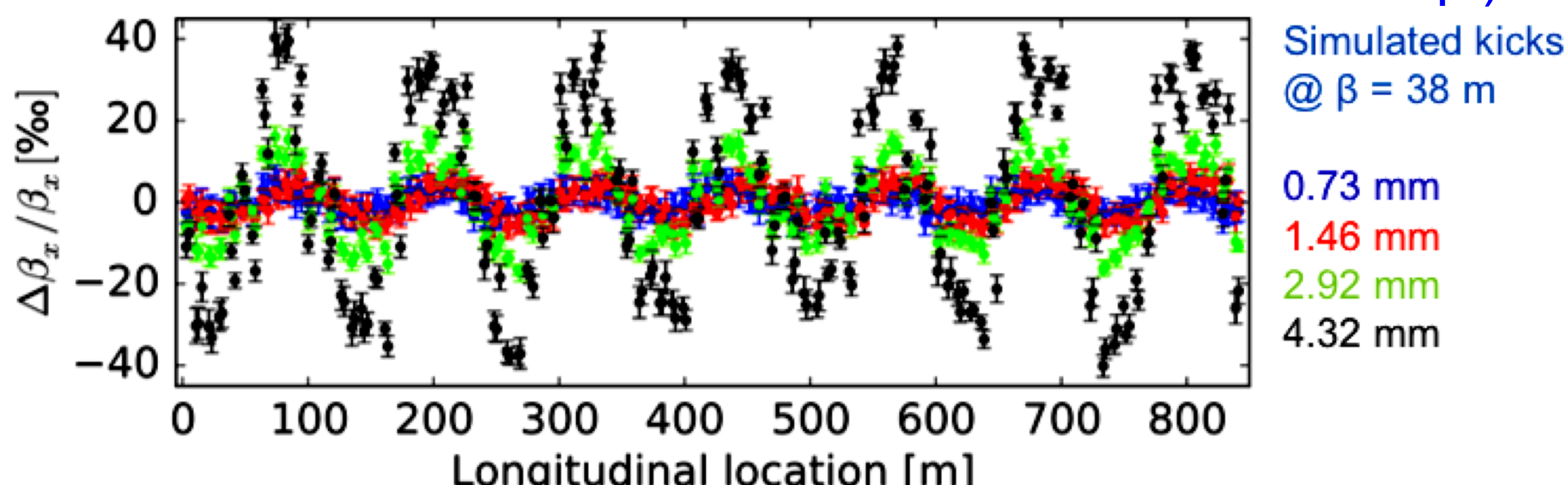
Kick Amplitude

artificial β-beating from TbT data

- Limited by non-linearities – simulation
- However has to be high enough
  - with respect to BPM resolution
  - Decoherence and number of tu

Libera BPMs

~10 μm/√Hz (TbT, 353 kHz)
Vs
~10 nm/√Hz (ORM, 10 Hz aqn)

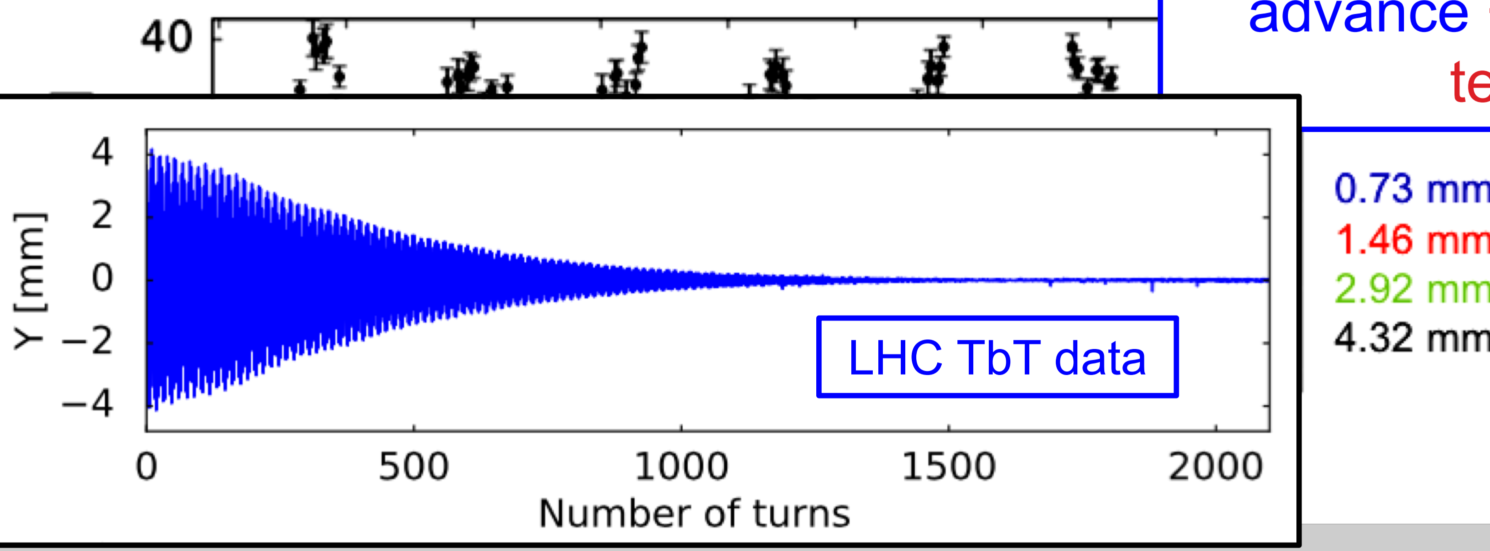


Kick Amplitude

artificial β-beating from TbT data

- Limited by non-linearities – simulation
- However has to be high enough
  - with respect to BPM resolution
  - Decoherence and number of turns

The measured BPM phase advance
=
betatron BPM phase advance + non-linear terms

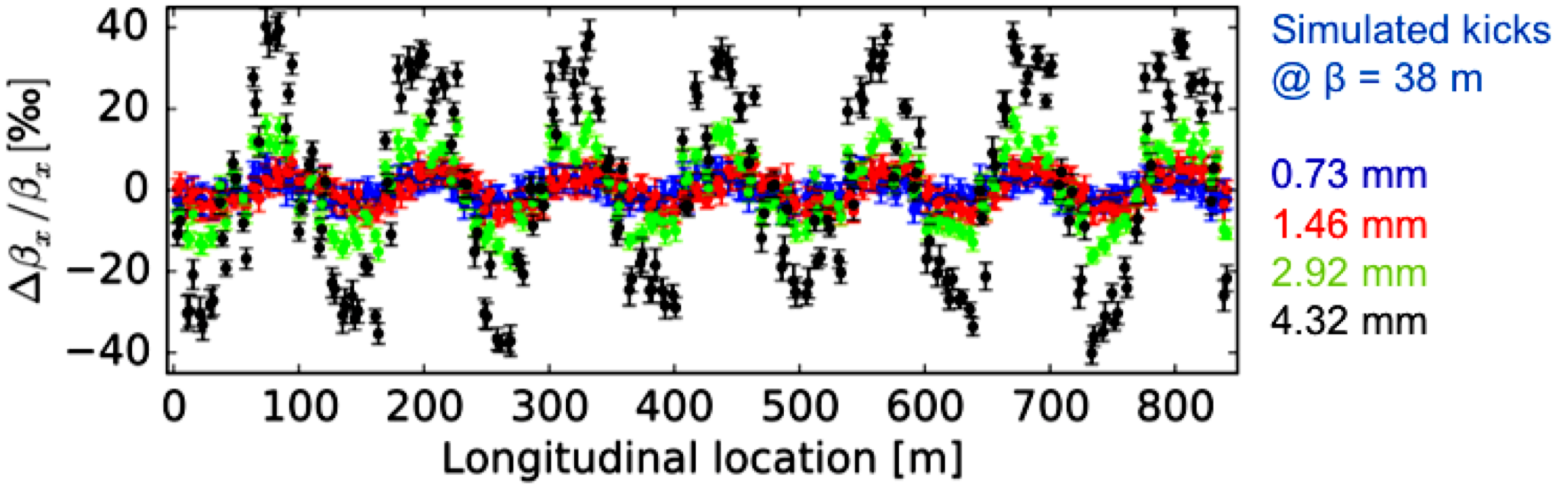


Kick Amplitude

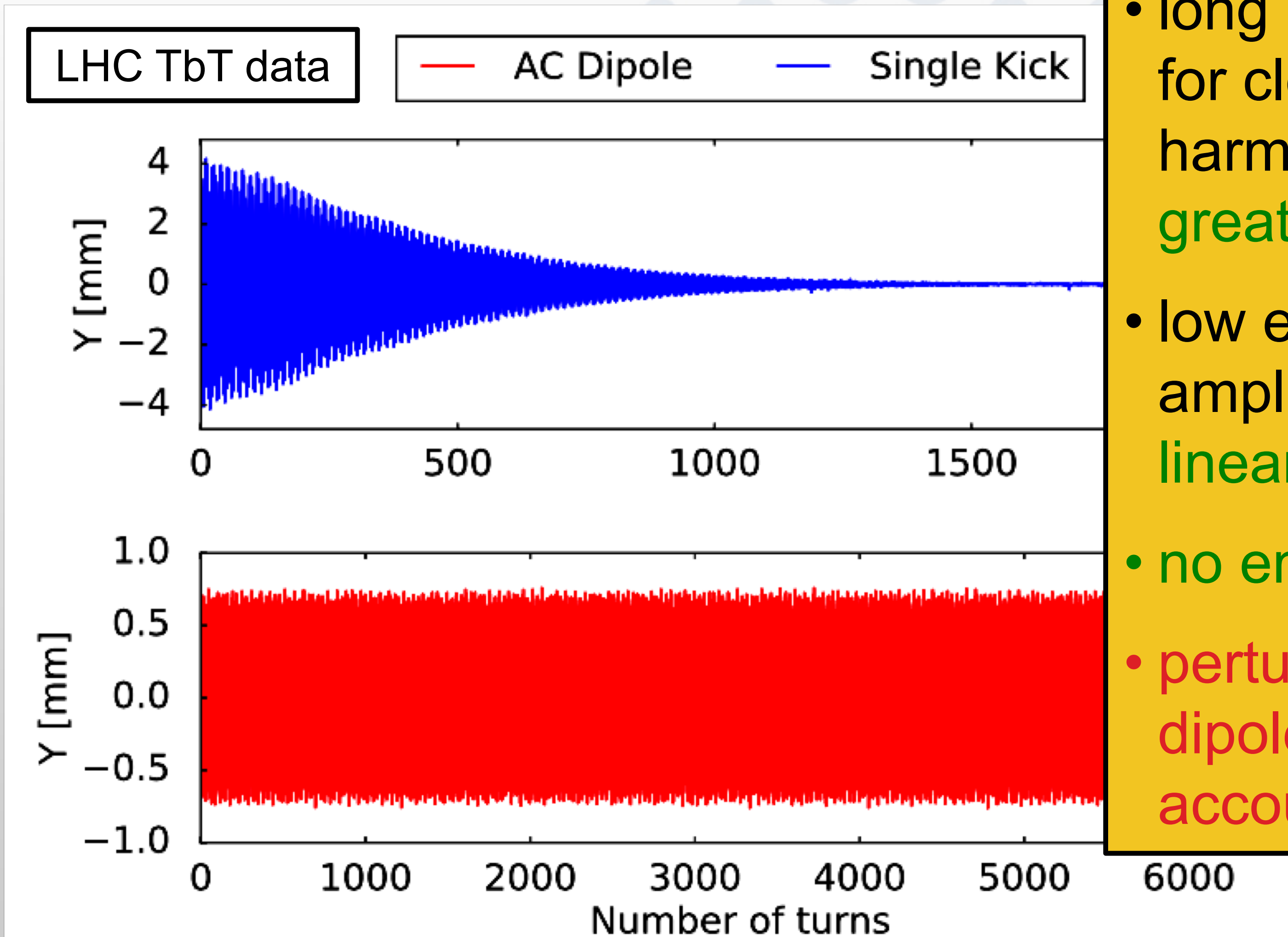
artificial β-beating from TbT data

- Limited by non-linearities – simulation
- However has to be high enough
  - with respect to BPM resolution
  - Decoherence and number of turns

$\sim 1/N^2$ (TbT)



a possible way out: the AC dipole



- long TbT signal good for cleaning before harmonic analysis => greater spectral res.
- low excitation amplitude => non-linearities avoided
- no emittance blowup
- perturbations from AC dipole to be accounted for

momentum reconstruction