

2018 Blaise Pascal Chair Lecture 10

*Dark Energy and the  
Cosmological Constant Problem*

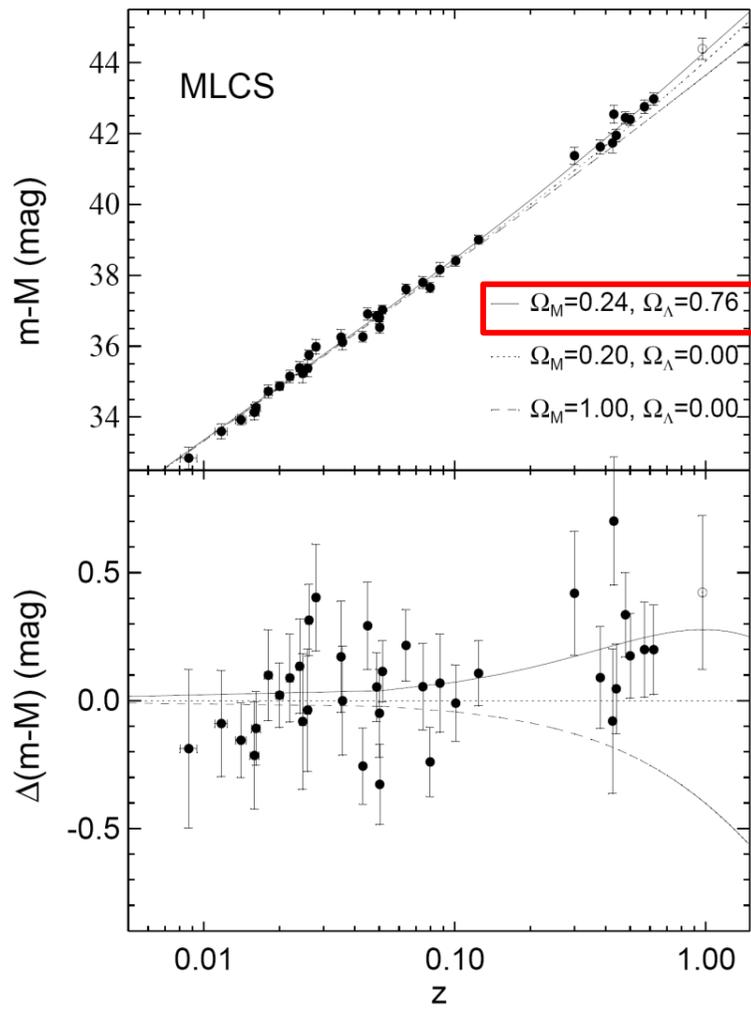
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CEA-Saclay DPhP, February 2020

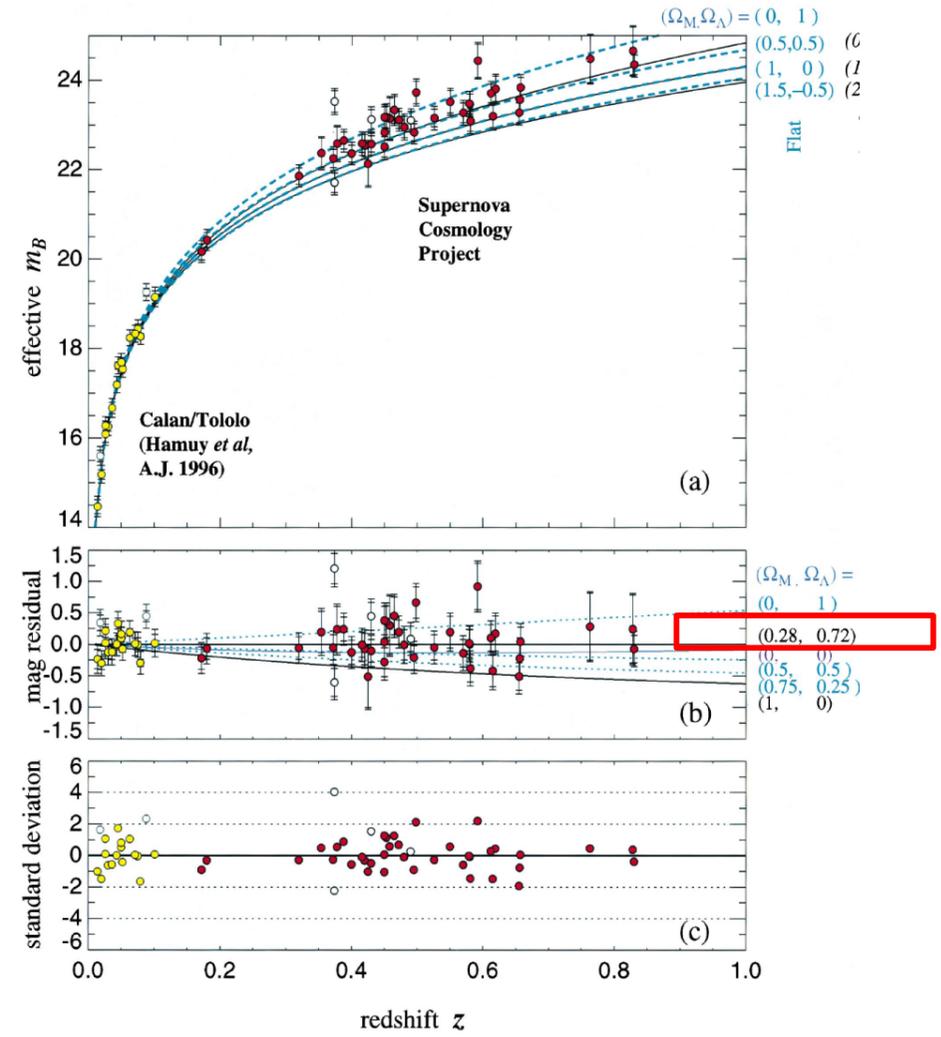
# Cosmic Acceleration: SN-Ia as Standard Candle

## 1998 Discovery!



**Riess et al, 1998**

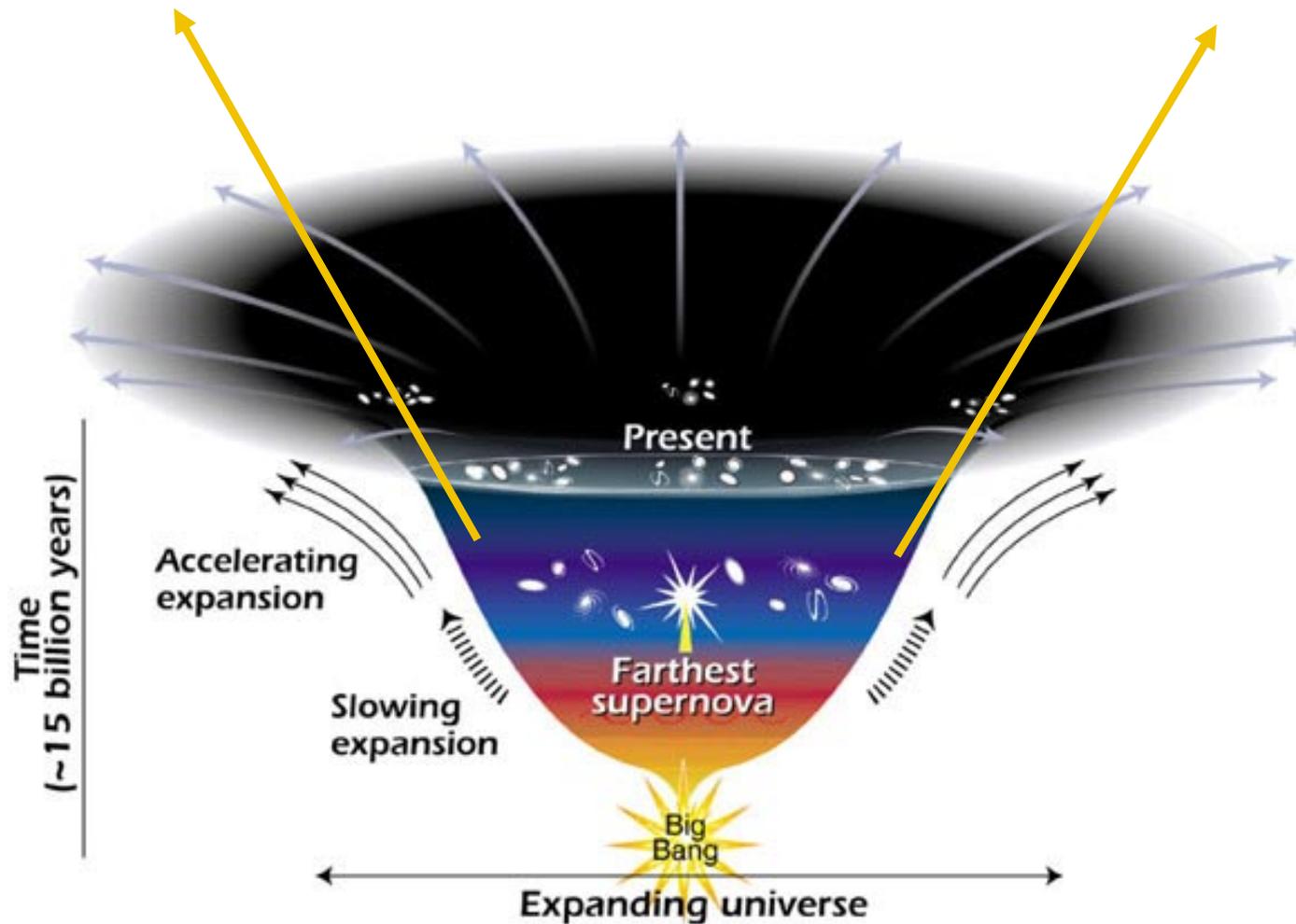
**High z Supernova Team**



**Perlmutter et al, 1999**

**Supernova Cosmology Project**

# Accelerating Expansion (1998): One of the biggest surprises in science!



# 2011 Nobel Prize for Physics



Photo: Roy Kaltschmidt. Courtesy:  
Lawrence Berkeley National Laboratory

**Saul Perlmutter**



Photo: Belinda Pratten, Australian  
National University

**Brian P. Schmidt**



Photo: Homewood Photography

**Adam G. Riess**

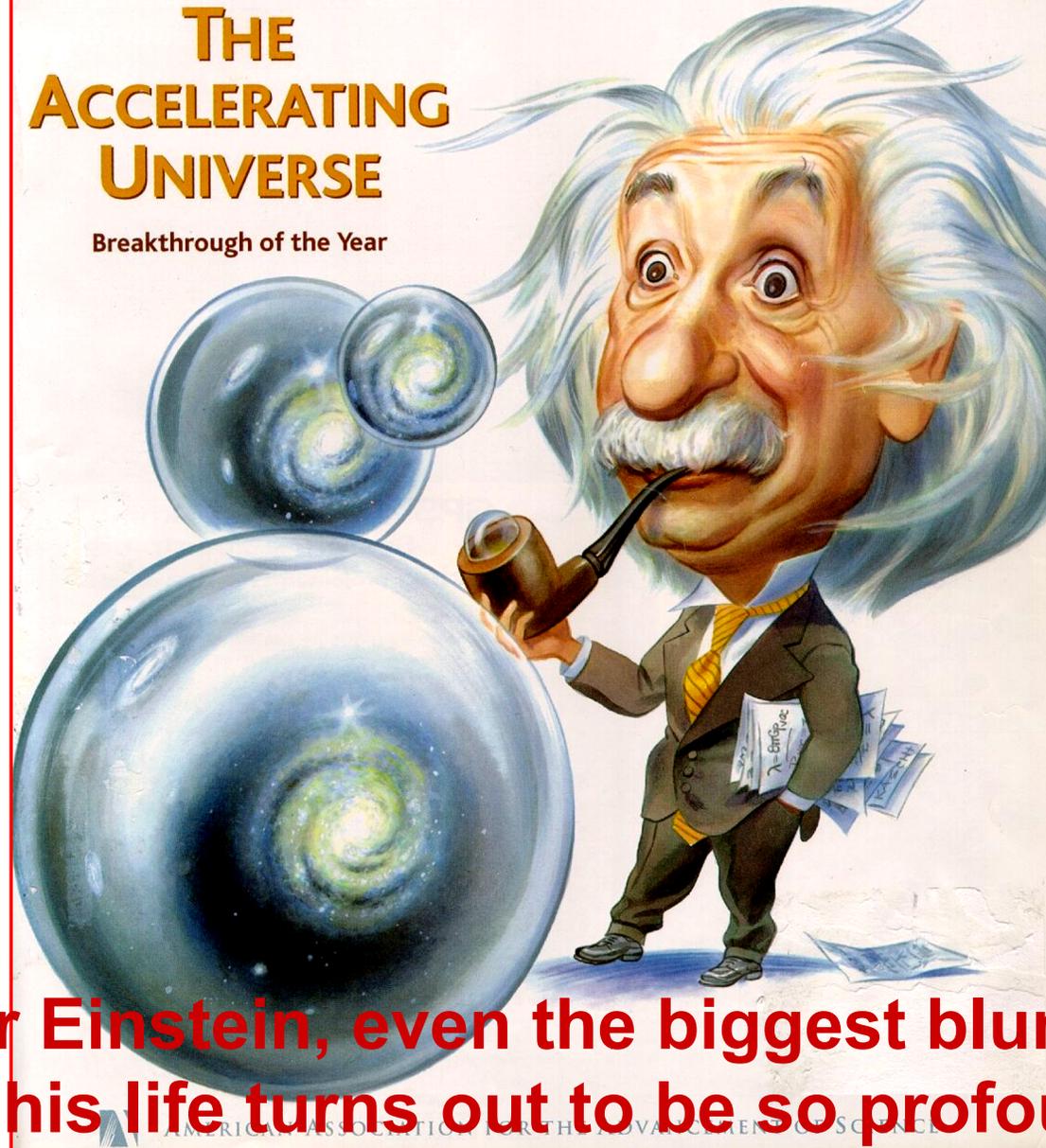
# Science

18 December 1998

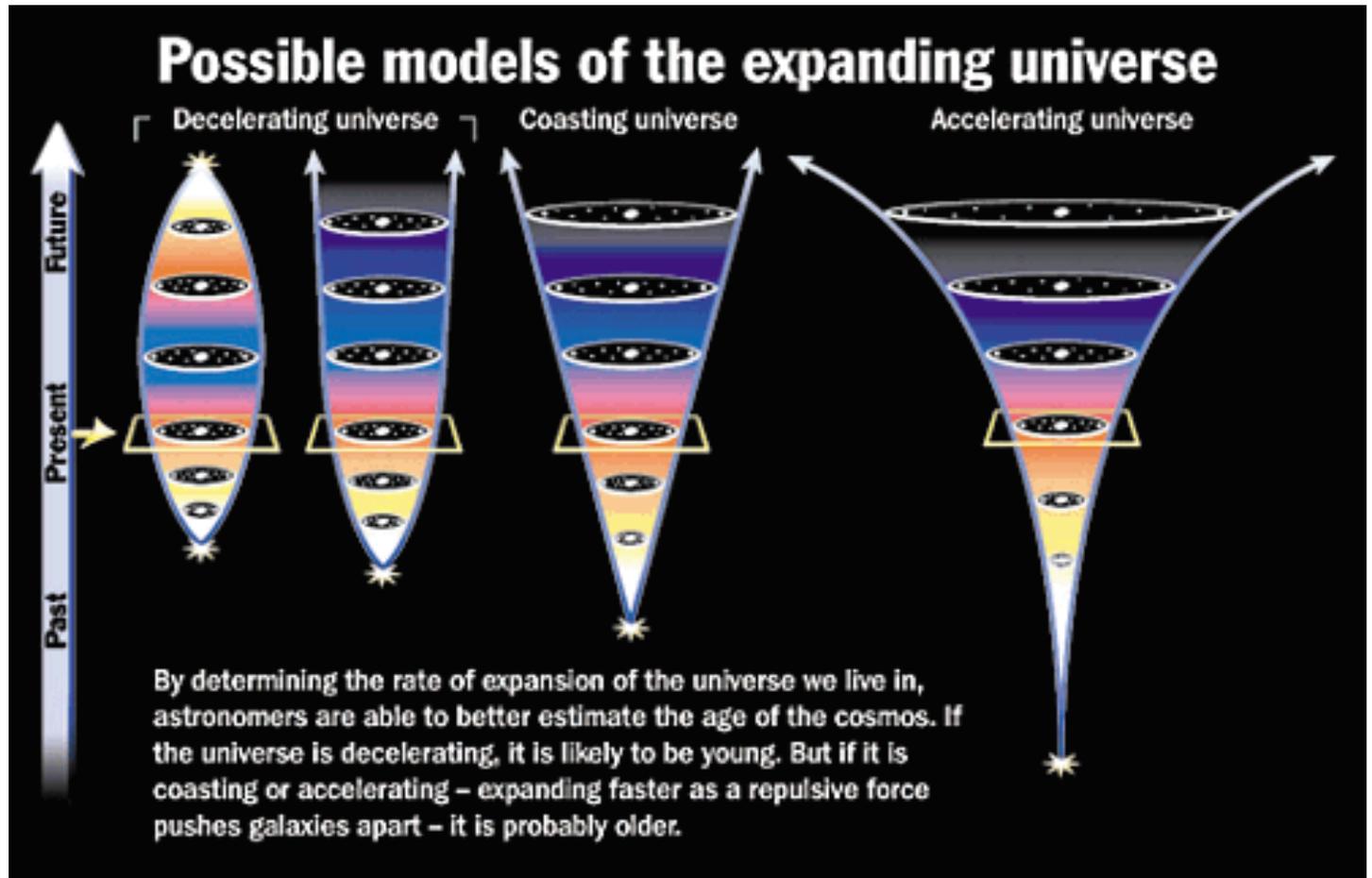
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## THE ACCELERATING UNIVERSE

Breakthrough of the Year



**For Einstein, even the biggest blunder  
in his life turns out to be so profound!**



Newton's constant

Expansion of Universe governed by the Friedmann eqs.,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3}$$

curvature signature

cosmological constant

- Critical density:  $\rho_c = \frac{3H^2}{8\pi G}$
- Total and fractional densities  $\Omega = \rho_{total} / \rho_c$ 
  - radiation:  $\Omega_r = \rho_r / \rho_c$
  - matter (baryon & cold):  $\Omega_m = \rho_m / \rho_c$
  - cosmology constant:  $\Omega_\Lambda = \rho_\Lambda / \rho_c$
- Friedmann eq. again: Dominant at early times

$$H^2 = H_0^2 \left[ \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_{de} (1+z)^{3(1+w)} + \Omega_k (1+z)^2 \right]$$

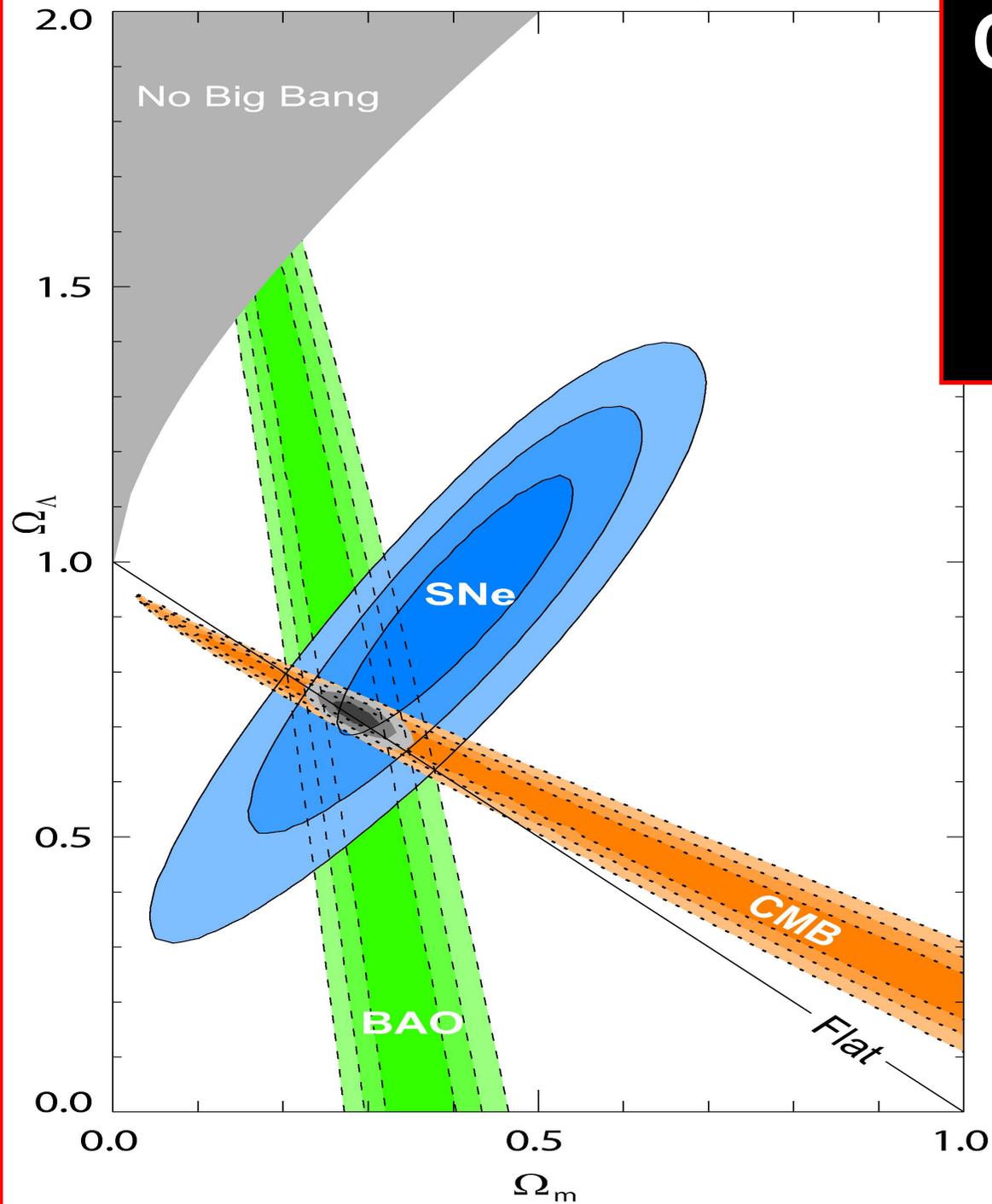
Dominant at late times

- Equation of state  $\rho = w\rho$  For accel. universe,  $w < -1/3$   
 For cosmological constant,  $w = -1$   $w = w_0 + w_a(1-a)$   
 In general,  $w$  can depend on  $a$ , e.g.,

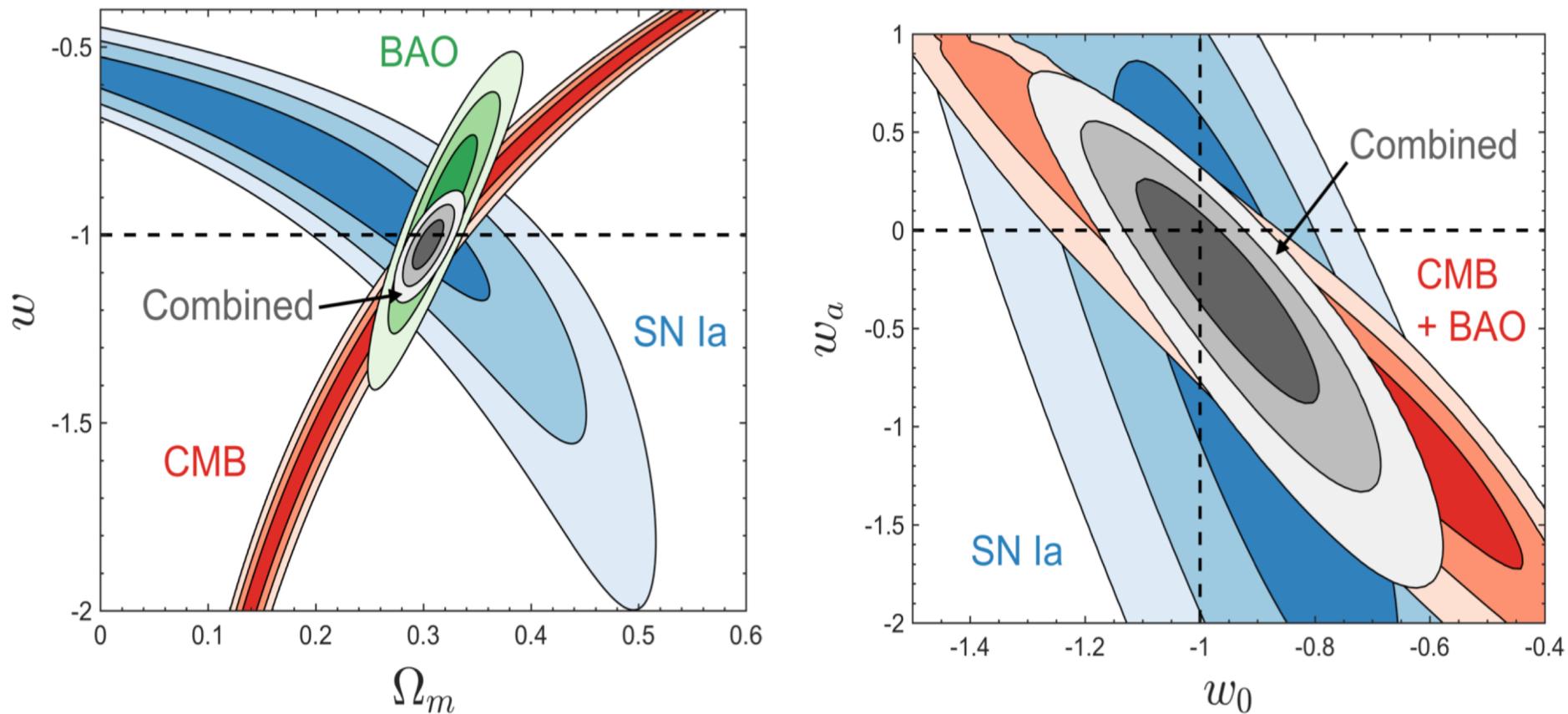
# “Extraordinary claims requires extraordinary evidence.”

To constrain the nature of dark energy we need to be able to measure the expansion rate of the Universe and there are three main approaches:

- **Standard candles:** which measure the **luminosity distance** as a function of redshift.
- **Standard rulers:** which measure the **angular diameter distance** and expansion rate as a function of redshift.
- **Growth of fluctuations.**



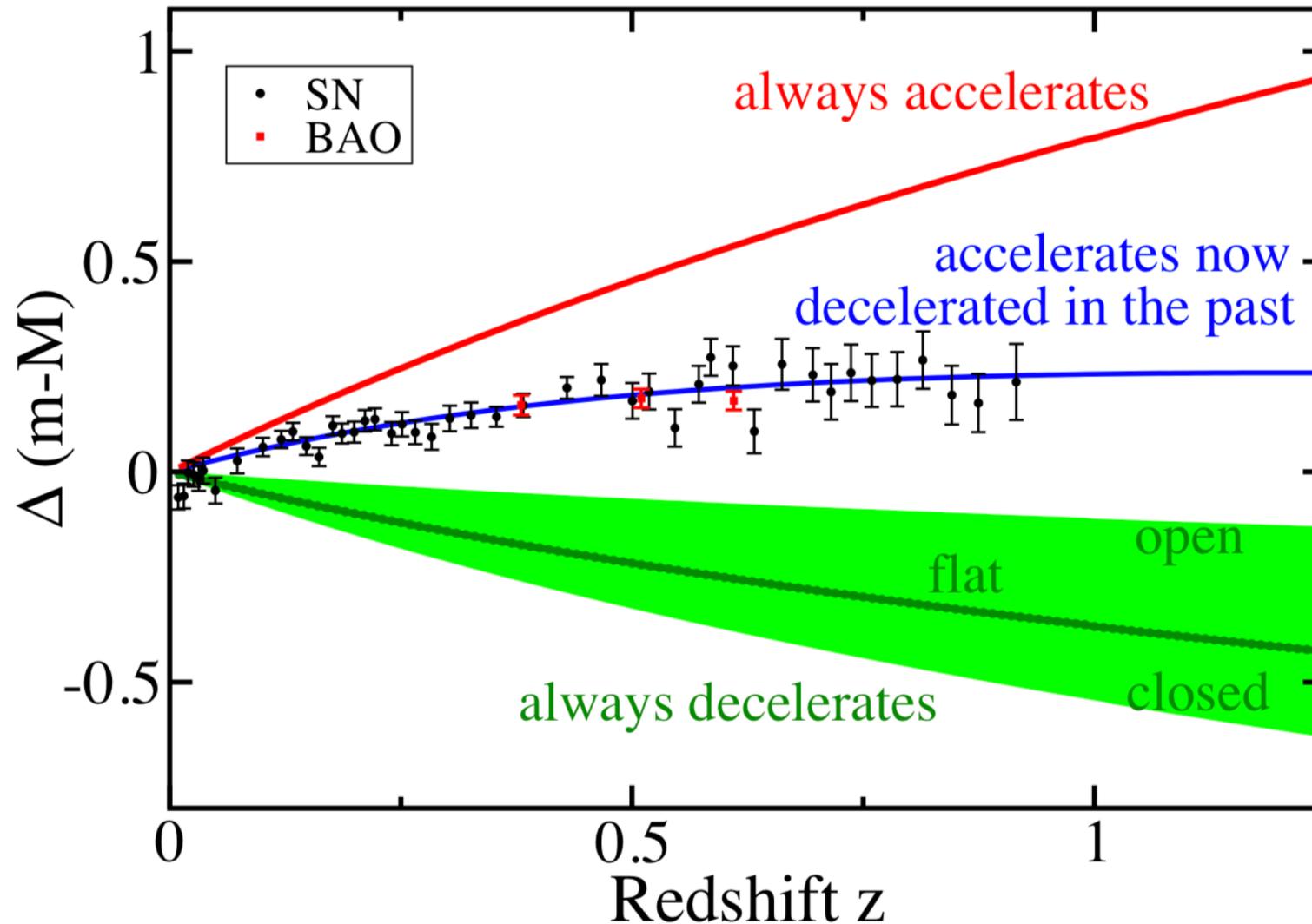
**Consistent with  
all  
observations:**



**Figure 9.** Constraints on cosmological parameters from our analysis of current data from three principal probes: SN Ia (JLA [203]; blue), BAO (BOSS DR12 [30]; green), and CMB (*Planck* 2015 [74]; red). We show constraints on  $\Omega_m$  and constant  $w$  (left panel) and on  $w_0$  and  $w_a$  in the parametrization from (17), marginalized over  $\Omega_m$  (right panel). The contours contain 68.3%, 95.4%, and 99.7% of the likelihood, and we assume a flat universe in both cases.

Huterer-Shafer (2018)

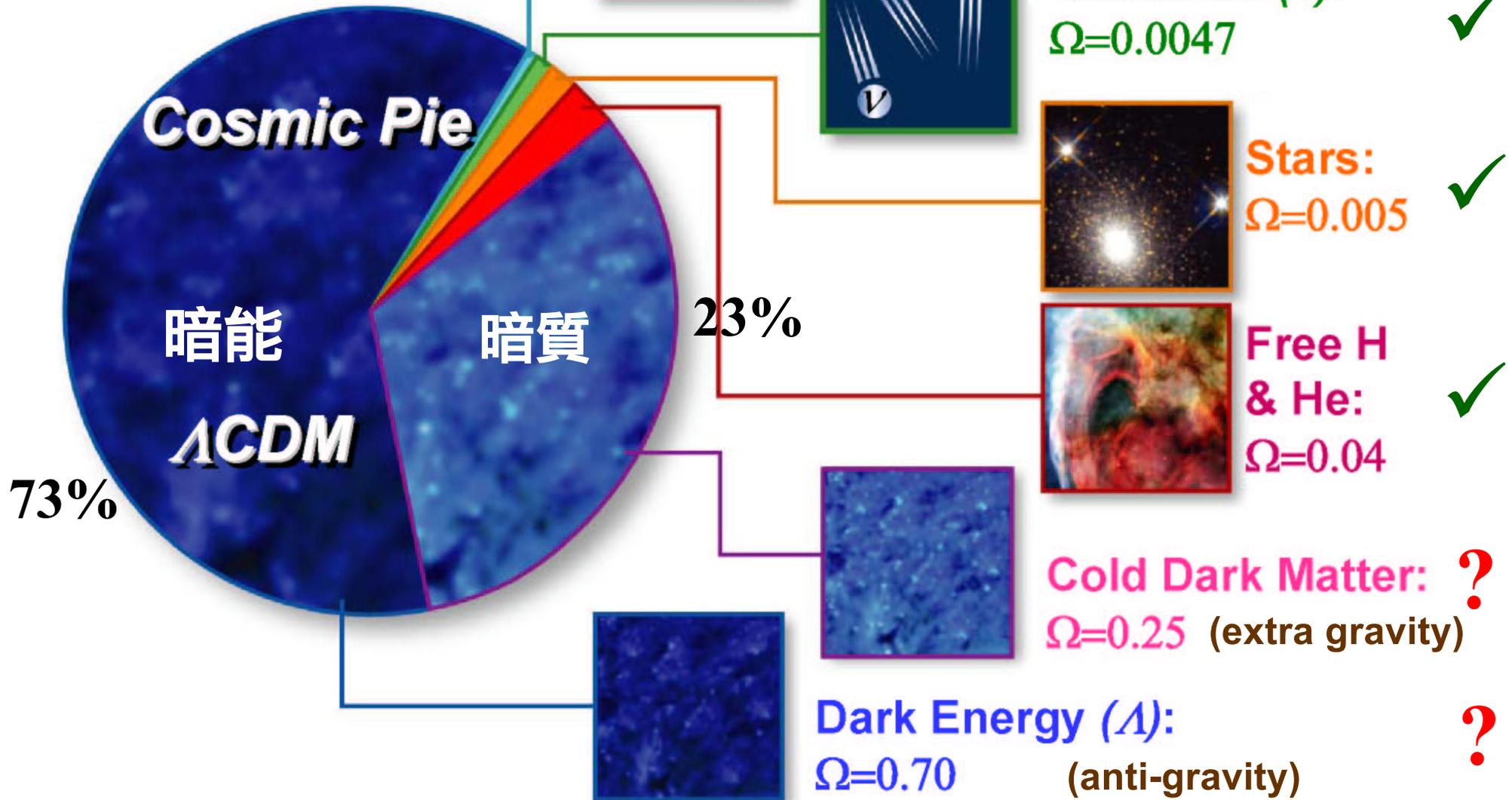
# Evidence for past deceleration: Important reality check



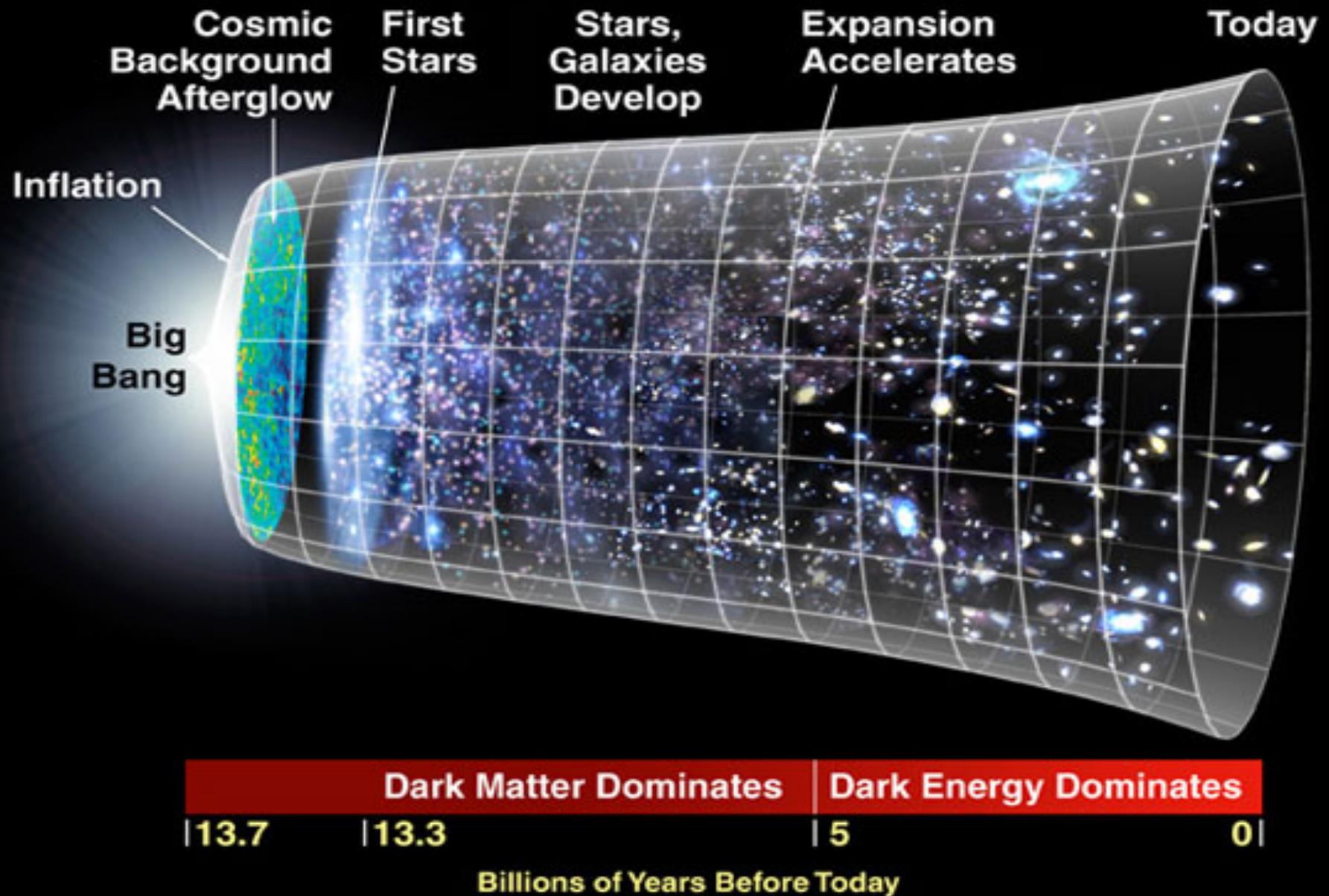
*SUPERCAL*, Scolnic et al. (2015); BOSS-DR12, Alam et al. (2016);  
D. Huterer, D. Safer, "Dark energy two decades after" (2018).

$$\Omega_i \equiv \rho_i / \rho_{\text{CRITICAL}}$$

$$\Omega_{\text{TOTAL}} = 1$$



# THE EXPANDING UNIVERSE: A CAPSULE HISTORY



# What we understand

- Smooth, very elastic, non-particulate (medium)
- Extremely weak interaction with ordinary matter
- Insignificant at small scales, important at large scales
- Insignificant at early times, important at late times
- Isotropic and homogeneous (apparently)

# Dark Energy is a profound mystery because it touches so many other important puzzles

- Vacuum energy/cosmological constant
- Destiny of the Universe
- Related to Dark Matter, Inflation, Neutrino Mass?
- Connections to SUSY/Superstrings/Extra dimensions?
- Signal of new gravitational physics?
- Hole in the Universe?
- Connection to the hierarchy problem?

# Theoretical Attempts

- Assume General Relativity (GR) is correct.

Einstein equation:

Einstein tensor

energy-momentum tensor

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

geometry

gravity

Introduce (anti-)gravity

- simplest model: cosmological constant

$$\Lambda = 8\pi G \rho_{vac} \quad w = -1$$

Commonly associated with vacuum energy.

- dynamical models: rolling scalar field with potential (quintessence, phantom, etc.)

$$w < -1/3$$

# Theoretical Attempts

- Quintessence
  - Accelerating expansion caused by the potential energy of a scalar field.
  - It must be very light (large Compton wavelength) so it won't clump or form structures.

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \quad \begin{cases} \rho_\phi = \dot{\phi}^2/2 + V(\phi) \\ p_\phi = \dot{\phi}^2/2 - V(\phi) \end{cases}$$

$$H^2 = H_0^2 \left[ \Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_\phi \left( 3 \int_0^z [1 + w_\phi(z')] \frac{dz'}{1+z'} \right) \right].$$

# Theoretical Attempts

- Assume General Relativity (GR) is correct.

Einstein equation:

Einstein tensor

energy-momentum tensor

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

geometry

gravity

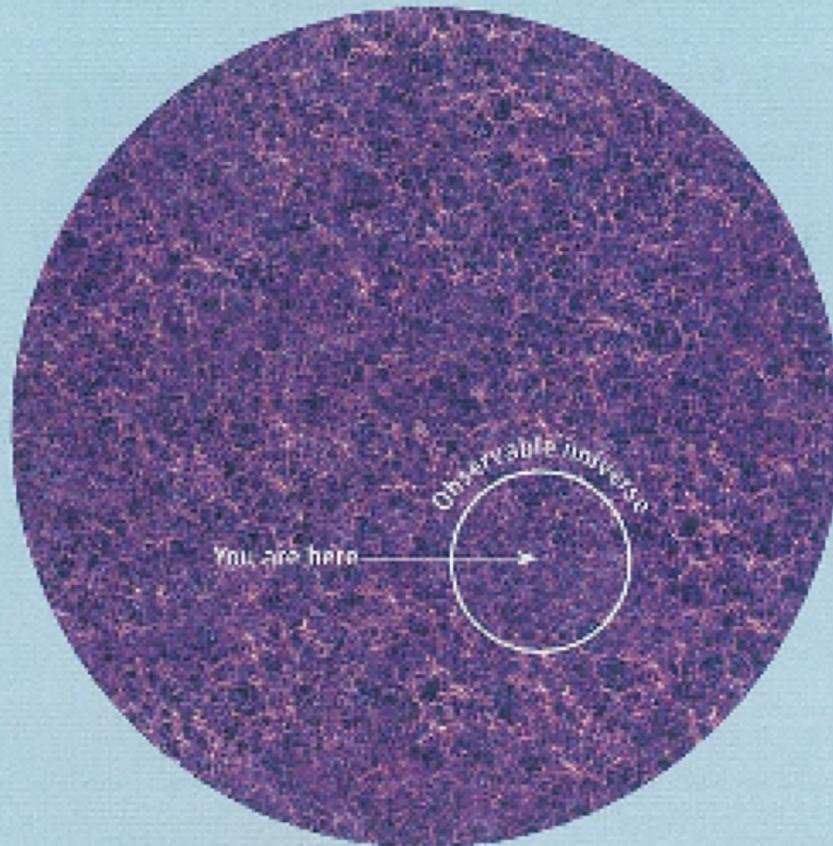
Special place in the universe

- Anti-Copernican Principle: “We are special”.
- No need for  $\Lambda$ .

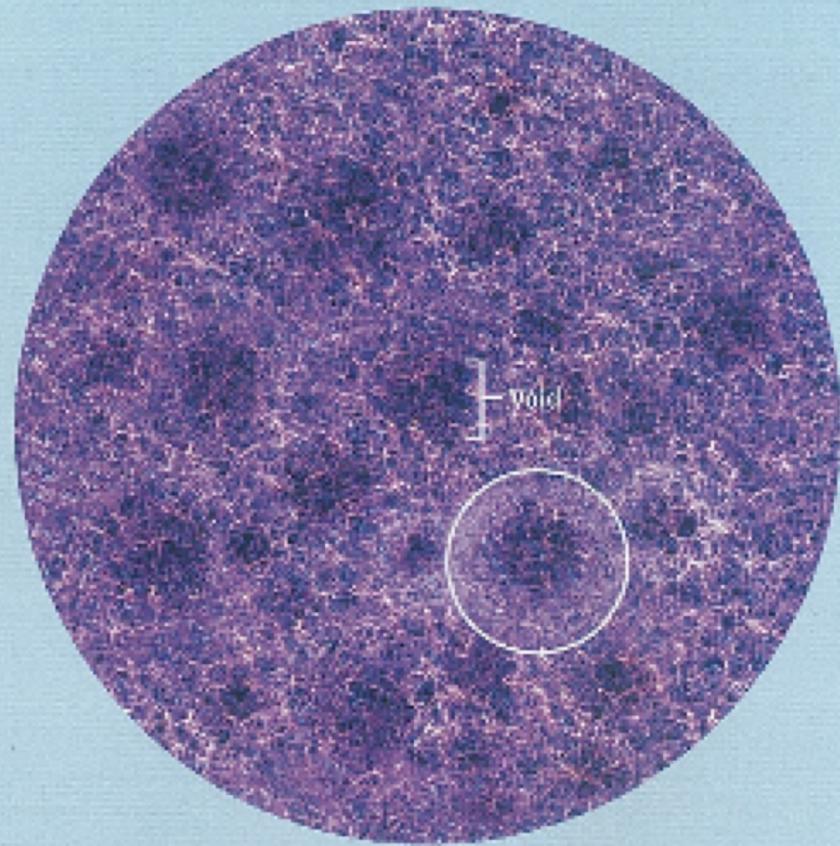
## A Special Place for Us

In his *Hitchhiker's Guide to the Galaxy* series of novels, Douglas Adams imagines a torture device that drives people insane by showing them the utter insignificance of their place in the universe. One would-be victim emerges

unscathed when it turns out that the universe does, in fact, revolve around him. In a case of life imitating art, many cosmologists are investigating whether our planet indeed has a special place within the grand scheme of things.



HOMOGENEOUS UNIVERSE: OUR LOCATION IS TYPICAL



INHOMOGENEOUS UNIVERSE: OUR LOCATION IS SPECIAL

# Theoretical Attempts

- **General Relativity (GR) is the problem**

- **Modify Einstein-Hilbert action**

$$S_{EH} = \frac{1}{8\pi G} \int d^4x \sqrt{-g} R$$

Ricci Scalar



to something more general:

$$S_{MG} = \frac{1}{8\pi G} \int d^4x \sqrt{-g} [R + f(R)].$$

- **DGP model**

string-theory-inspired IR modification of GR

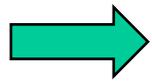
- **Emergent gravity**

GR as an effective theory emerging from quantum theory of gravity

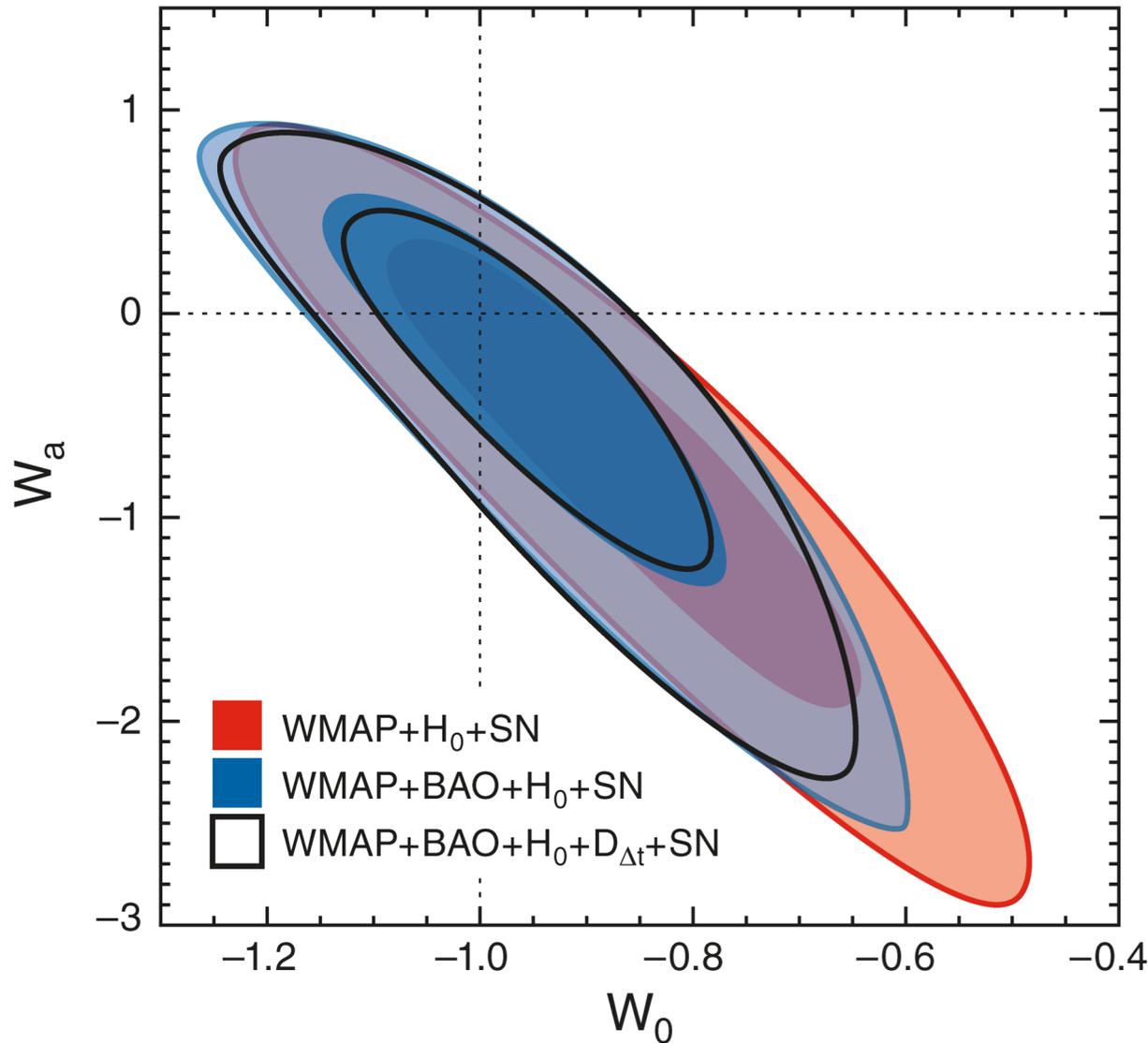
# Some Challenges

- **Cosmological constant:** simple and natural. But what makes it so small?
- **Quintessence:** Who ordered it? Renormalized scalar field often acquires large mass.
- **“Anti-Copernican Principle”:** hard to modify enough to accommodate cosmic acceleration and satisfy other constraints (importance of dynamical tests)
- **Modified GR:** hard to satisfy short-distance constraints while providing significant departure at large distance.

# Observations show that $w = -1$



## Dark Energy = CC?



$$\rho = w\rho$$

$$w = w_0 + w_a(1 - a)$$

**2009 data:**

$$w_0 = -0.97^{+0.12}_{-0.07}$$

$$w_a = 0.03^{+0.26}_{-0.75}$$

# The Cosmological Constant Problem

- Cosmological constant has long been a problem in theoretical physics during most of the 20<sup>th</sup> century.
- Since 1998, it has further become one of the most challenging issues in astrophysics in the new century.
- Several excellent review articles:  
S. Weinberg (1989), Carroll (2000), Sahni & Starobinsky (2000, 2006), Peebles & Ratra (2002), Padmanabhan (2003)...

More than 1000 papers in arXiv that has 'cosmological constant' in the title. Can't possibly cover all ideas.

# What is the Problem?

- **History**

After completing his formulation of general relativity (GR), Einstein (1917) introduced a cosmological constant (CC) to his eq. for the universe to be static:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = -8\pi GT_{\mu\nu}.$$

As is well-known, he gave up this term after Hubble's discovery of cosmic expansion.

Unfortunately, not so easy to drop it.

In GR, anything that contributes to the energy density of the vacuum acts like a CC.

# The Old CC Problem

## The old (< 1998):

- Lorentz invariance, upon which QFT is based, tells us that in the vacuum the energy-momentum tensor must take the form

$$\langle T_{\mu\nu} \rangle = -\langle \rho \rangle g_{\mu\nu}.$$

This is equivalent to adding a term to CC:

$$\Lambda_{\text{eff}} = \Lambda + 8\pi G \langle \rho \rangle. \quad \longleftrightarrow \quad \rho_V = \langle \rho \rangle + \Lambda / 8\pi G = \Lambda_{\text{eff}} / 8\pi G.$$

- Quantum vacuum (zero point) energies with cutoff at Planck scale gives

$$\rho_V \sim M_{\text{Pl}}^4 \sim 10^{112} \text{ eV}^4.$$

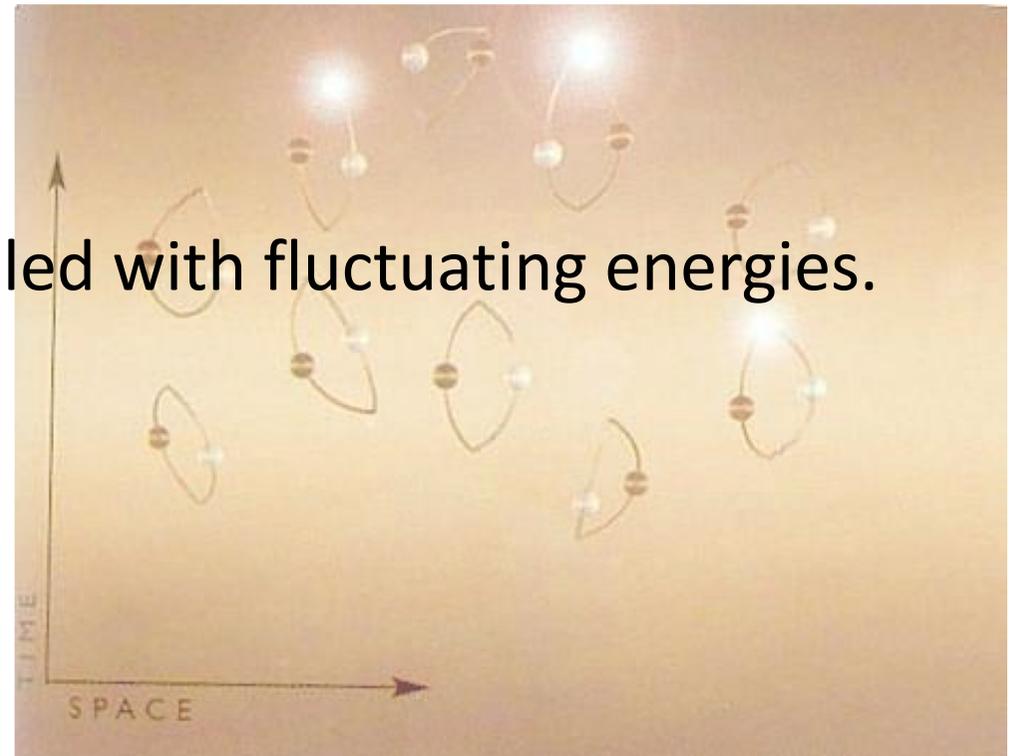
# What is Quantum vacuum Energy?

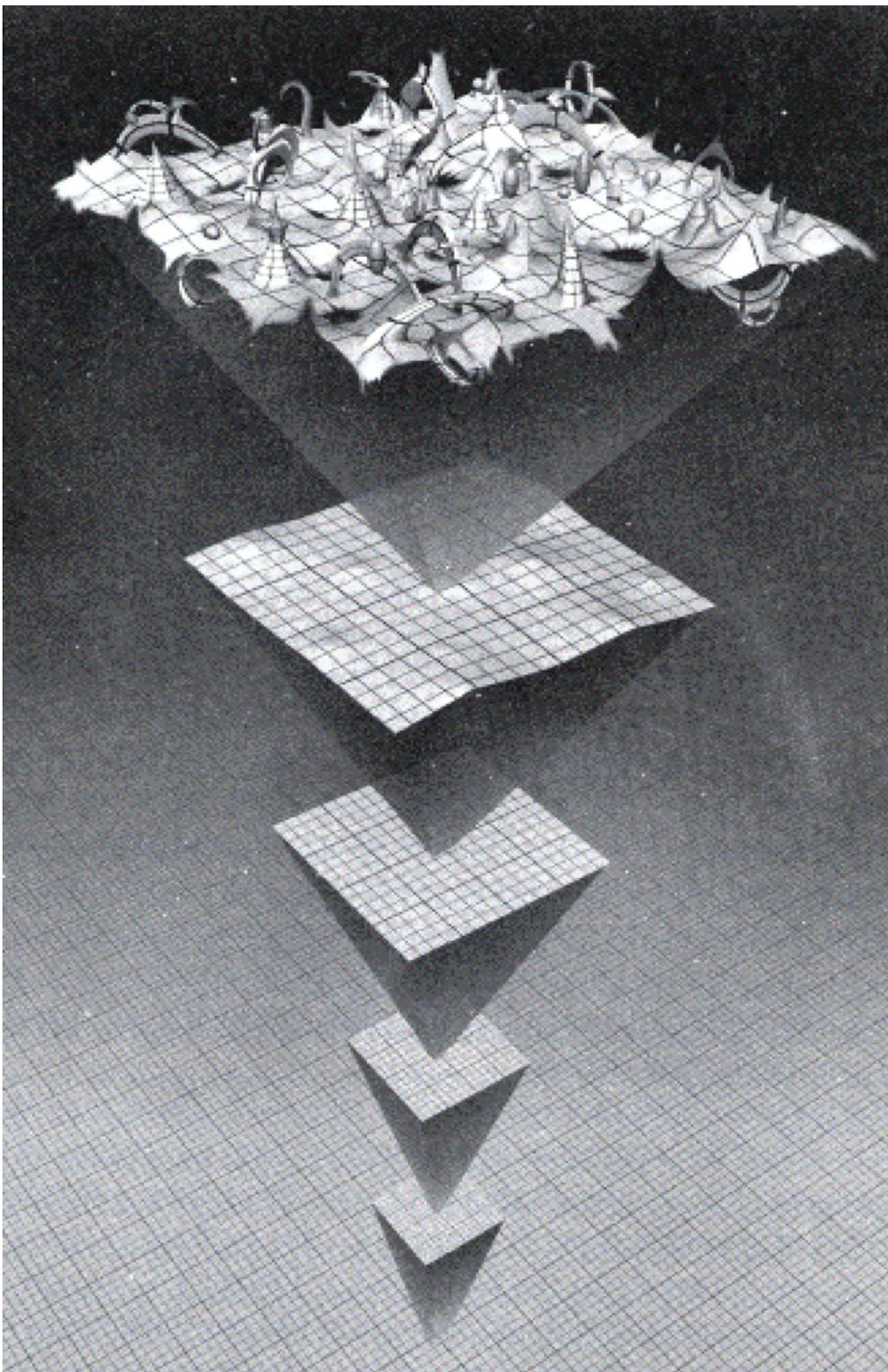
- Heisenberg Uncertainty Principle:

$$\Delta x \Delta p \geq \frac{h}{2\pi} \quad \longrightarrow \quad \Delta t \Delta E \geq \frac{h}{2\pi}$$



Vacuum is not empty, but filled with fluctuating energies.





# Quantization of Spacetime

Planck scale:

$$l_p = \sqrt{\frac{hG}{2\pi c^3}} \approx 1.6 \times 10^{-35} \text{ m}$$

$$E_p = \sqrt{\frac{hc}{2\pi G}} \approx 1.2 \times 10^{19}$$

[GeV/c<sup>2</sup>]

Ultimate vacuum energy:

$$E_p^4 : (10^{19} \text{ GeV})^4$$

$$= 10^{112} [\text{eV}]^4$$

- Astrophysics, however, demands that it must be smaller than the critical density of the universe:

$$\rho_V \leq \rho_{cr} \sim 10^{-12} \text{ eV}^4.$$

This is 124 orders of magnitude in discrepancy!

- Evidently QVE should not gravitate. Otherwise our universe would not have survived until now.
- This conflict between GR and quantum theory is the essence of the longstanding CC problem, which clearly requires a resolution. In short,

**Why doesn't quantum vacuum energy gravitate?**

We shall call this the “old” CC problem.

# The New CC Problem, or the Dark Energy Puzzle

## The new (>1998)

- The dramatic discovery of the accelerating expansion of the universe ushers in a new chapter of the CC problem.
- The substance responsible for it is referred to as the dark energy (DE), described by its equation of state

$$p = w\rho, \quad p: \text{pressure}, \rho: \text{density}$$

- According to GR, accelerating expansion can happen if  $w < -1/3$ . Einstein's CC corresponds to  $w = -1$ .

- DE=CC remains the simplest and most likely answer.
- New challenge: after finding a way, hopefully, to cancel the CC to 124 decimal points, how do we reinstate 1 to the last digit and keep it tiny? That is,

**Why is CC nonzero but tiny?**

- We shall call this the “new” CC problem, or the DE puzzle.

# Attempt 1: Casimir Energy in Extra-Dimensions

*If dark energy never changes in space and time,  
then it must be associated with the fundamental  
properties of spacetime!*

Observations  $\rightarrow M_{CC} ; \rho_{DE}^{1/4} : 10^{-3} eV$

Why much smaller than standard model scale?

$$\frac{\rho_{DE}^{1/4}}{M_{SM}} : 10^{-15}!$$

# FUNDAMENTAL PARTICLES AND INTERACTIONS

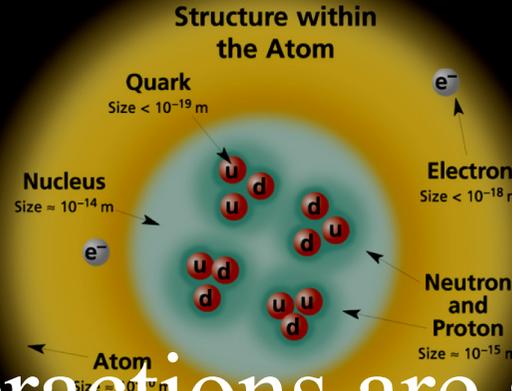
The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

### FERMIONS

matter constituents  
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2		
Flavor	Mass GeV/c <sup>2</sup>	Electric charge
$\nu_e$ electron neutrino	<1×10 <sup>-8</sup>	0
e electron	0.000511	-1
$\nu_\mu$ muon neutrino	<0.0002	0
$\mu$ muon	0.106	-1
$\nu_\tau$ tau neutrino	<0.02	0
$\tau$ tau	1.7771	-1

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c <sup>2</sup>	Electric charge
u up	0.003	2/3
d down	0.006	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	175	2/3
b bottom	4.3	-1/3



### BOSONS

force carriers  
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c <sup>2</sup>	Electric charge
$\gamma$ photon	0	0
W <sup>-</sup>	80.4	-1
W <sup>+</sup>	80.4	+1
Z <sup>0</sup>	91.187	0

Strong (color) spin = 1		
Name	Mass GeV/c <sup>2</sup>	Electric charge
g gluon	0	0

#### Color Charge

Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and W and Z bosons have no strong interactions and hence no color charge.

#### Quarks Confined in Mesons and Baryons

One cannot isolate quarks and gluons; they are confined in color-neutral particles called hadrons. This confinement results from multiple exchanges of gluons among the color-charged constituents. Color-charged particles (quarks and gluons) move apart, the energy of the color-force field between them increases, and this energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: mesons  $q\bar{q}$  and baryons  $qqq$ .

The strong binding in nuclei, protons, and neutrons to form nuclei is due to residual strong interaction, also called the nuclear force. This force can be viewed as the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

# 3 gauge interactions are at TeV scale, why is gravity so much weaker, at Planck scale?

**Spin** is the intrinsic angular momentum of particles. Spin is given in units of which is the quantum unit of angular momentum, where  $\hbar = 1.0545718 \times 10^{-34}$  J·s.

**Electric charges** are given in units of the proton's charge. In SI units the electric charge of the proton is  $1.60 \times 10^{-19}$  coulombs.

The energy of a particle physics scale is given in units of electron volts (eV). The electron volt is a unit of energy, and a potential difference of one volt will give a unit mass of one electron volt (eV) of energy ( $E = mc^2$ ), where  $1 \text{ eV} = 1.60 \times 10^{-19}$  joules. The mass of a proton is  $1.67 \times 10^{-27}$  kg.

### PROPERTIES OF THE INTERACTIONS

Baryons $qqq$ and Antibaryons $\bar{q}\bar{q}\bar{q}$					
Baryons are fermionic hadrons. There are about 120 types of baryons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c <sup>2</sup>	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	-1	0.938	1/2
n	neutron	udd	0	0.940	1/2
$\Lambda$	lambda	uds	0	1.116	1/2
$\Omega^-$	omega	sss	-1	1.672	3/2

Property	Interaction	Gravitational	Weak (Electroweak)	Electromagnetic	Strong	
	Acts on:	Mass - Energy	Flavor	Electric Charge	Fundamental	Residual
Particles experiencing:		All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:		Graviton (not yet observed)	W <sup>+</sup> W <sup>-</sup> Z <sup>0</sup>	$\gamma$	Gluons	Mesons
Strength relative to electromag. for two u quarks at:						
for two u quarks at:	10 <sup>-18</sup> m	10 <sup>-41</sup>	0.8	1	25	Not applicable to quarks
	3×10 <sup>-17</sup> m	10 <sup>-41</sup>	10 <sup>-4</sup>	1	60	
for two protons in nucleus		10 <sup>-36</sup>	10 <sup>-7</sup>	1	Not applicable to hadrons	

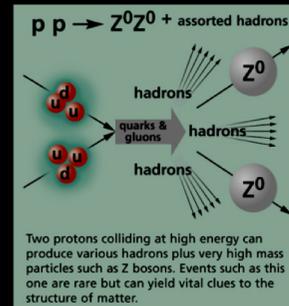
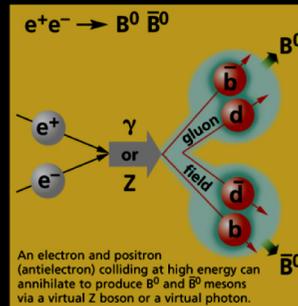
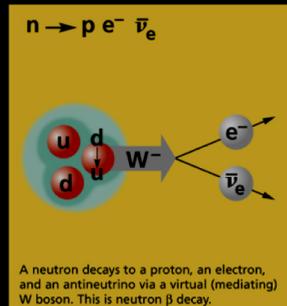
Mesons $q\bar{q}$					
Mesons are bosonic hadrons. There are about 140 types of mesons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c <sup>2</sup>	Spin
$\pi^+$	pion	$u\bar{d}$	+1	0.140	0
$K^-$	kaon	$s\bar{u}$	-1	0.494	0
$\rho^+$	rho	$u\bar{d}$	+1	0.770	1
$B^0$	B-zero	$d\bar{b}$	0	5.279	0
$\eta_c$	eta-c	$c\bar{c}$	0	2.980	0

#### Matter and Antimatter

For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or - charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z<sup>0</sup>,  $\gamma$ , and  $\eta_c = c\bar{c}$ , but not K<sup>0</sup> = d $\bar{s}$ ) are their own antiparticles.

#### Figures

These diagrams are an artist's conception of physical processes. They are not exact and have no meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.



#### The Particle Adventure

Visit the award-winning web feature *The Particle Adventure* at <http://ParticleAdventure.org>

This chart has been made possible by the generous support of:

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- U.S. National Science Foundation
- Lawrence Berkeley National Laboratory
- Stanford Linear Accelerator Center
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# A Numerical Coincidence

- A remarkable numerical coincidence, a ‘gravity fine structure constant’:

$$\frac{M_{CC}}{M_{SM}} \simeq \frac{M_{SM}}{M_{Pl}} \equiv \alpha_G$$

- Perhaps not accidental but implies a deeper connection:

$$M_{CC} \simeq \frac{M_{SM}}{M_{pl}} M_{SM} = \left( \frac{M_{SM}}{M_{Pl}} \right)^2 M_{Pl} = \alpha_G^2 M_{Pl}.$$

- **Caution: Unlike the 1<sup>st</sup> hierarchy that links 4 fundamental interaction strengths, DE must be a secondary, derived quantity.**

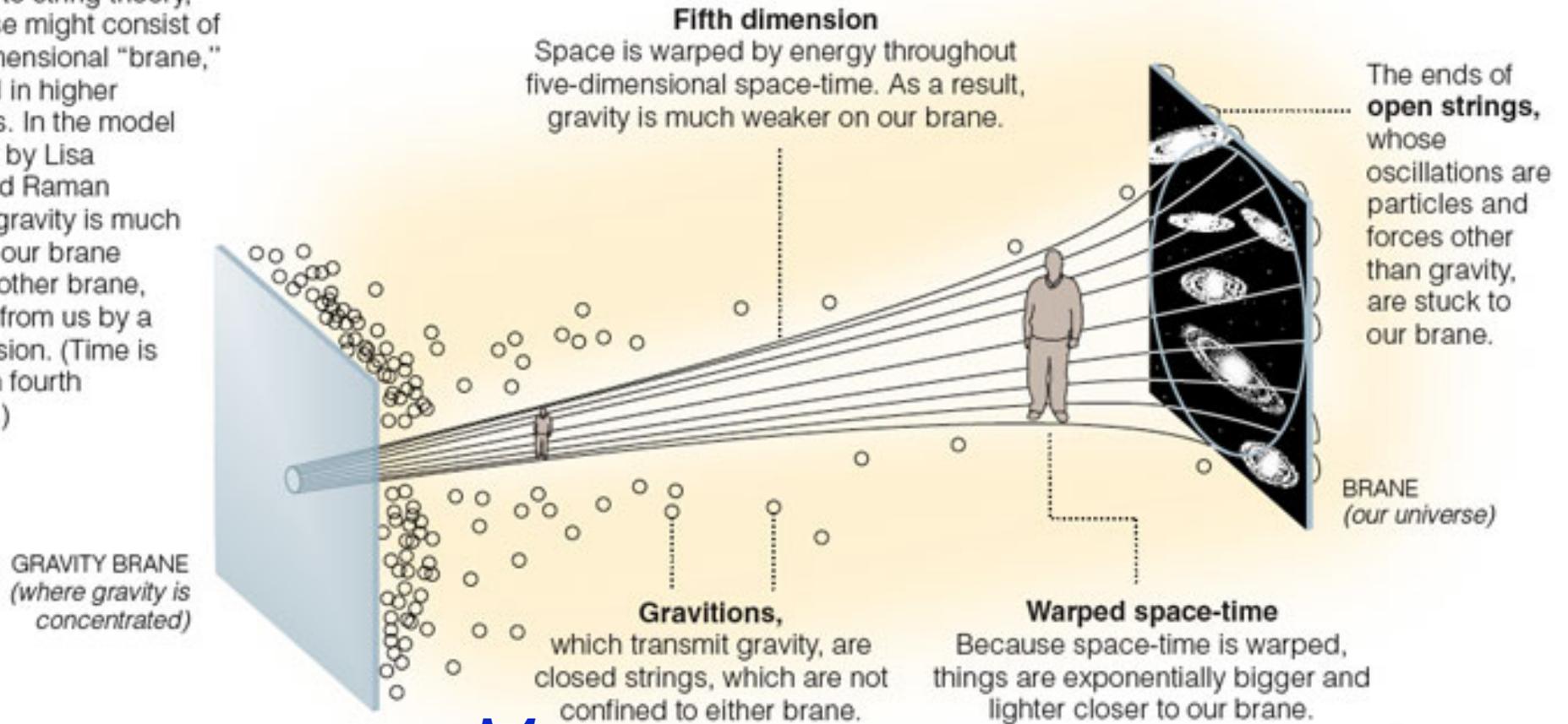
# Analogy in Atomic Physics

- Bohr atom
- Fundamental energy scale in Schrödinger equation:  $m_e$
- Ground state energy suppressed by 2 powers of fine structure constant
- Dark energy
- Fundamental energy scale in quantum gravity:  $M_{Pl}$
- Dark energy suppressed by 2 powers of “gravity fine structure constant”

# Randall-Sundrum Model to bridge the hierarchy between SM and gravity scales

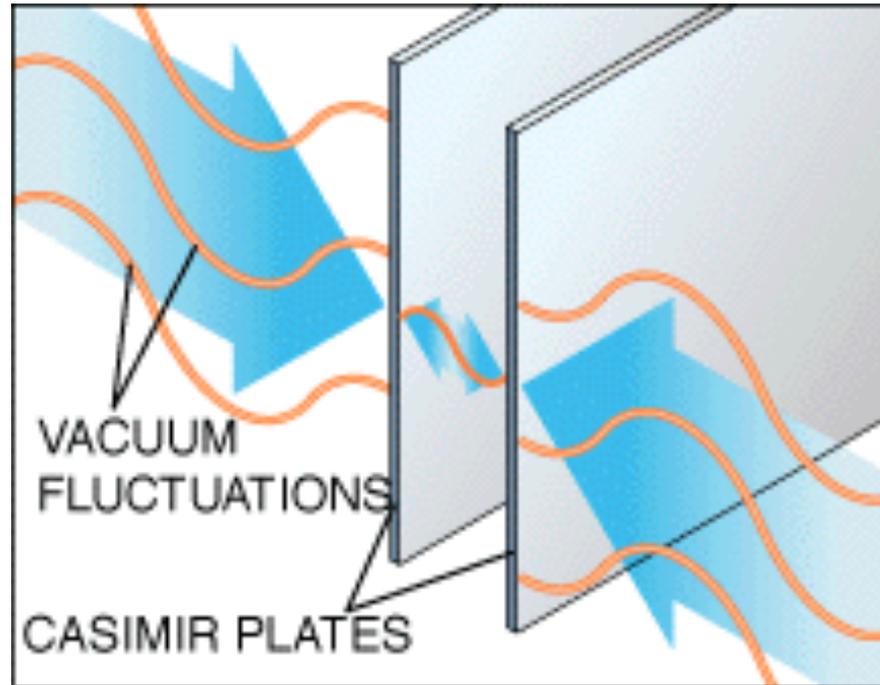
## Island Universes in Warped Space-Time

According to string theory, our universe might consist of a three-dimensional "brane," embedded in higher dimensions. In the model developed by Lisa Randall and Raman Sundrum, gravity is much weaker on our brane than on another brane, separated from us by a fifth dimension. (Time is the unseen fourth dimension.)



$$\frac{M_{SM}}{M_{Pl}} = e^{-\pi kR} \sim 10^{-16}$$

# Casimir Energy: Evidence of vacuum fluctuations



Casimir energy induced by fields in the RS bulk on the TeV brane gives rise to DE as we set out to look for:

$$M_{Casimir} ; \alpha_G^2 M_{Pl} ; 10^{-3} eV ; M_{CC}.$$

This may solve the New CC Problem.

**It, however, still does not address the Old CC Problem...**

## Attempt 2:

# Boundary Condition of the Universe

- “Gauge Theory of Gravity with de Sitter Symmetry”  
(PC, MPLA (2009) [arXiv:1002.4275])

1. In GR Einstein equation is a 2<sup>nd</sup> order differential eq. and thus the nature of CC is undetermined:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

1. If the field eq. is higher (e.g., 3<sup>rd</sup>) order instead, then the CC term is not allowed. To recover the well tested GR, one should integrate it once. Then CC is recovered through the constant of integration determined by the boundary condition of the universe.

- **Motivations for Gauge theory of gravity (GG)**

- To reformulate gravity as a gauge theory
- To hopefully quantize gravity theory
- To substantiate the ‘constant of integration’ approach as a means to solve the CC problem.

C. N. Yang (1983): “In [ ] I proposed that the gravitational equation should be changed to a third order equation. I believe today, even more than 1974, that this is a promising idea, because the third order equation is more natural than the second order one and because quantization of Einstein’s theory leads to difficulties.”

# Here's how it goes

- In GG, the **gauge potential** (affine connection) is the dynamical variable, which determines the curvature tensor

$$R^{\alpha}_{\beta\mu\nu} = \partial_{\mu}\Gamma^{\alpha}_{\beta\nu} - \partial_{\nu}\Gamma^{\alpha}_{\beta\mu} + \Gamma^{\alpha}_{\tau\mu}\Gamma^{\tau}_{\beta\nu} + \Gamma^{\alpha}_{\tau\nu}\Gamma^{\tau}_{\beta\mu}.$$

- In close analogy with Maxwell theory, the action for gravity reads (Cook 09)

$$S_G = \kappa \int dx^4 \sqrt{-g} \left( R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} + 16\pi J^{\mu}_{\alpha\beta} \Gamma^{\alpha\beta}_{\mu} \right),$$

where the “gravitational current” ( $\nabla_{\alpha}$  = covariant deriv.)

$$\text{and } \bar{T}^{\mu}_{\beta} = T^{\mu}_{\beta} - \frac{1}{2} \delta^{\mu}_{\beta} T, \quad T = T^{\mu}_{\mu},$$
$$J^{\mu}_{\alpha\beta} = \frac{2G}{c^4} \left[ \nabla_{\alpha} \bar{T}^{\mu}_{\beta} - \nabla_{\beta} \bar{T}^{\mu}_{\alpha} \right],$$

# Field Equations

- Varying  $S_G$  against  $\Gamma_{\alpha\beta}^{\mu}$ , we arrive at the field eq.

$$\nabla_{\nu} R_{\alpha\beta}^{\mu\nu} = -4\pi J_{\alpha\beta}^{\mu}.$$

This and the Bianchi identity,

$$\nabla_{\lambda} R_{\alpha\beta\mu\nu} + \nabla_{\nu} R_{\alpha\beta\lambda\mu} + \nabla_{\mu} R_{\alpha\beta\nu\lambda} = 0,$$

together determine the curvature tensor.

- Now we recall that  $\Gamma_{\alpha\mu\nu} = \frac{1}{2} [\partial_{\nu} g_{\alpha\mu} + \partial_{\mu} g_{\alpha\nu} - \partial_{\alpha} g_{\mu\nu}]$ , and that covariant divergence of  $g_{\mu\nu}$  is identically 0.

Therefore the field eq. of GG removes the CC term by construction.

- Integrating this eq. once, we recover the Einstein eq. with a **constant of integration** which is associated with the **boundary condition of the universe**.

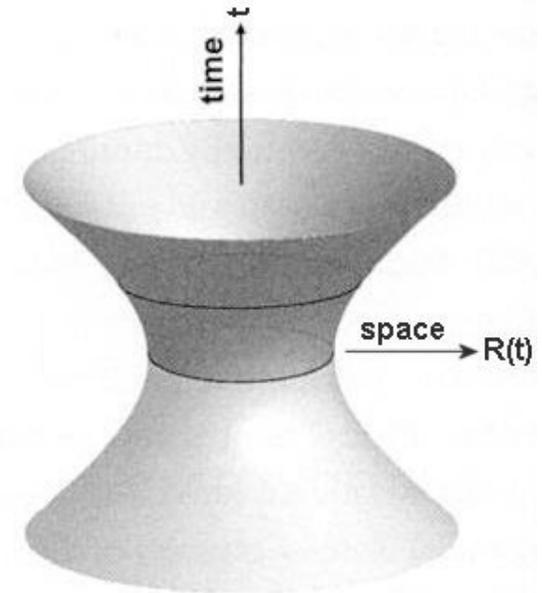
# de Sitter Universe as Asymptotic Limit of Hubble Expansion

- Now we invoke our second assumption, that the universe is inherently de Sitter, where the 4-spacetime is a hyperboloid of a 5-d Minkowski space with the constraint

$$-x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = l_{dS}^2$$

where  $l_{dS}$  is the radius of curvature of dS.

- dS universe as asymptotic limit of Hubble expansion.
- Observation gives  $\Omega_{DE} = \rho_{DE} / \rho_{cr} ; 0.75$ ,  
so we find  $l_{dS} ; 1.33H_0 \sim 1.5 \times 10^{28} \text{ cm}$



**But why did our universe choose such a geometry?!**

An aerial photograph of a rugged mountain range with several lakes. The mountains are brown and rocky, with some green vegetation in the valleys. The sky is blue with white clouds. The text is overlaid on the top half of the image.

**Anthropic Principle & Multiverse argument:**  
**“Our Universe is one that is suitable for intelligent habitat.”**

**String theory allows for a “landscape” of universes ( $10^{500}$ !)**

# Back to experiment:

## Is DE dynamical or a CC?

**A: Dynamical DE would induce anisotropy in cosmic expansion.**

C.-T. Chen, P. Chen, “Dark energy induced anisotropy in cosmic expansion”, EPJC (2019)

### Assumption:

- Dynamical DE, e.g., quintessence, already exists prior to the inflation. (A reasonable assumption.)

### Consequence:

- Quantum fluctuations of DE during inflation would leave some footprints in the anisotropy of the late-time accelerated expansion.

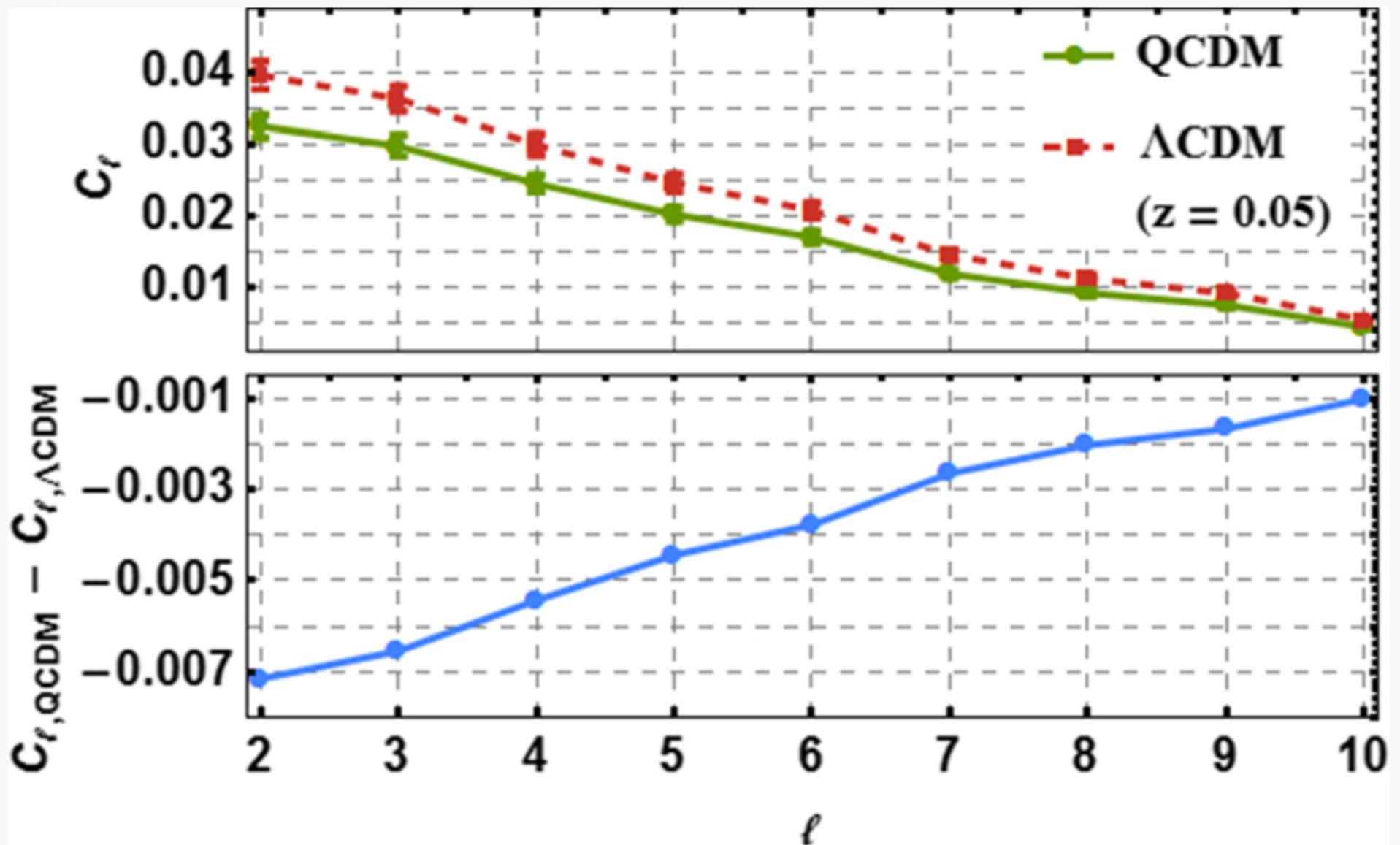


Fig. 5

Top: luminosity distance power spectrum  $C_\ell$  at  $z = 0.05$  with integration ranging from  $k = 10^{-4}$  to  $0.1 \text{ Mpc}^{-1} h$  for each  $\ell = 2$  to  $10$ . Bottom: residue of the luminosity distance power spectrum between QCDM and  $\Lambda$ CDM

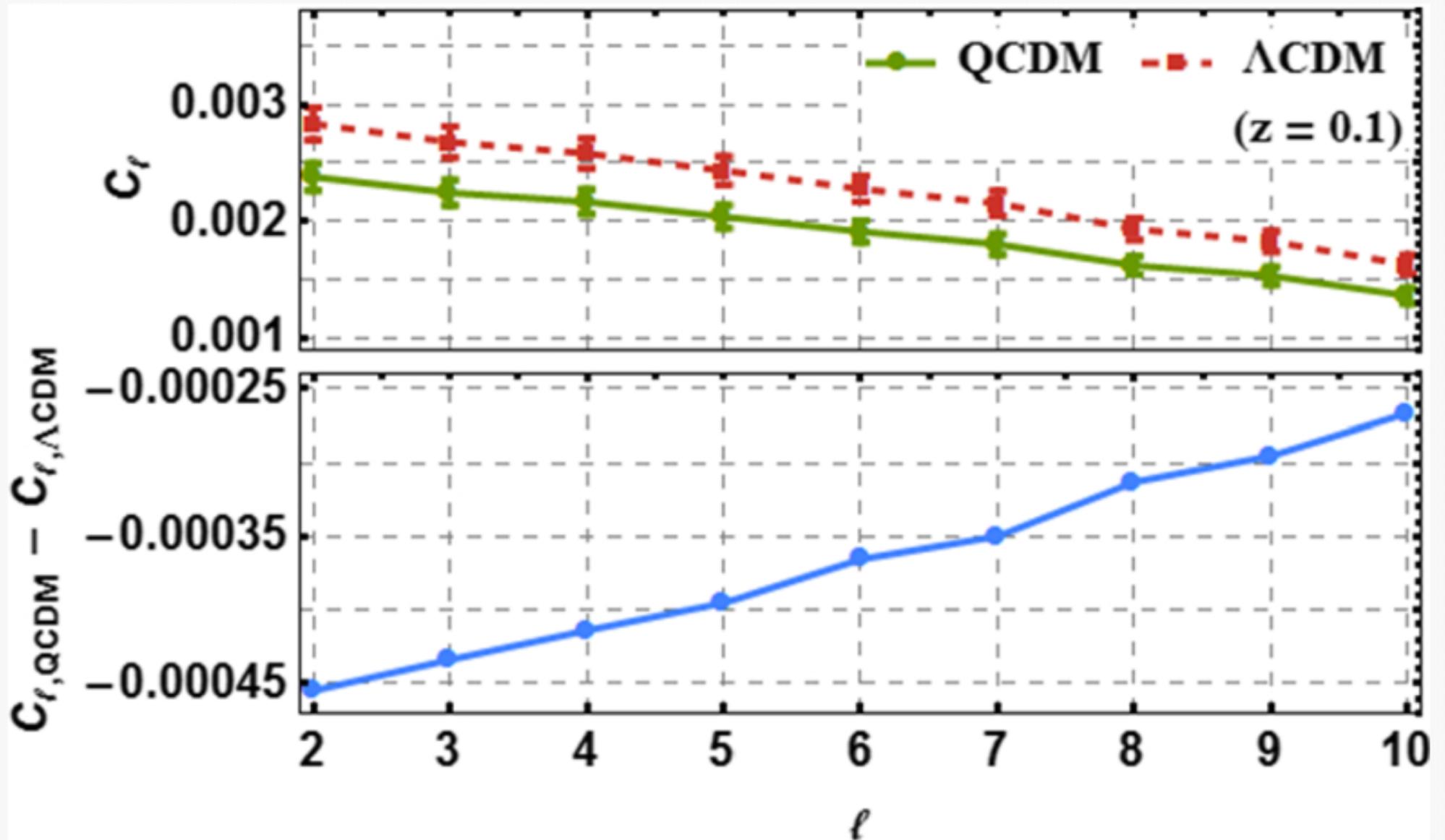


Fig. 6

Top: luminosity distance power spectrum  $C_\ell$  at  $z = 0.1$  with integration ranging from  $k = 10^{-4}$  to  $0.1 \text{ Mpc}^{-1} \text{ h}$  for each  $\ell = 2$  to 10. Bottom: residue of the luminosity distance power spectrum between QCDM and  $\Lambda$ CDM

# *Reflections and Prospects*

At the turn of the 20<sup>th</sup> century, two “dark clouds” in physics (a la Lord Kelvin): the Michelson-Morley experiment and the blackbody radiation, had later developed into revolutionary storms of Relativity and Quantum Mechanics.

At the turn of the 21<sup>st</sup> century, a new dark cloud – the dark energy, appears above the horizon. Will the history repeat itself and turn this into another revolutionary storm in physics, a storm that would clean up the conflict between QM and GR?