2018 Blaise Pascal Chair Lecture 10

Dark Energy and the Cosmological Constant Problem

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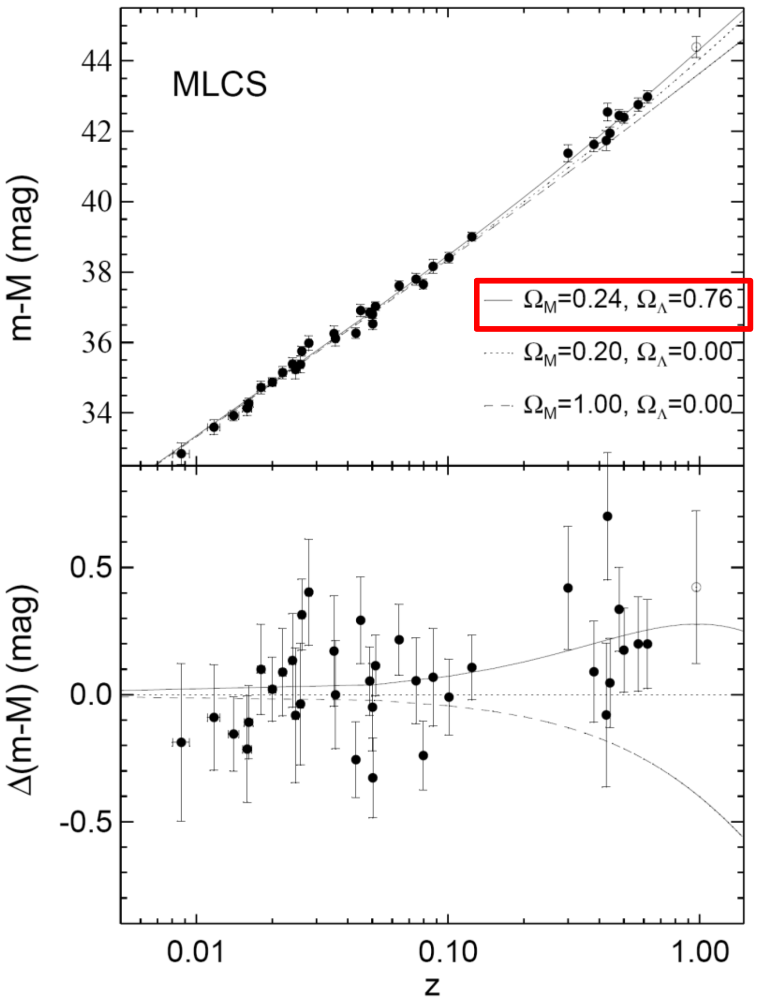
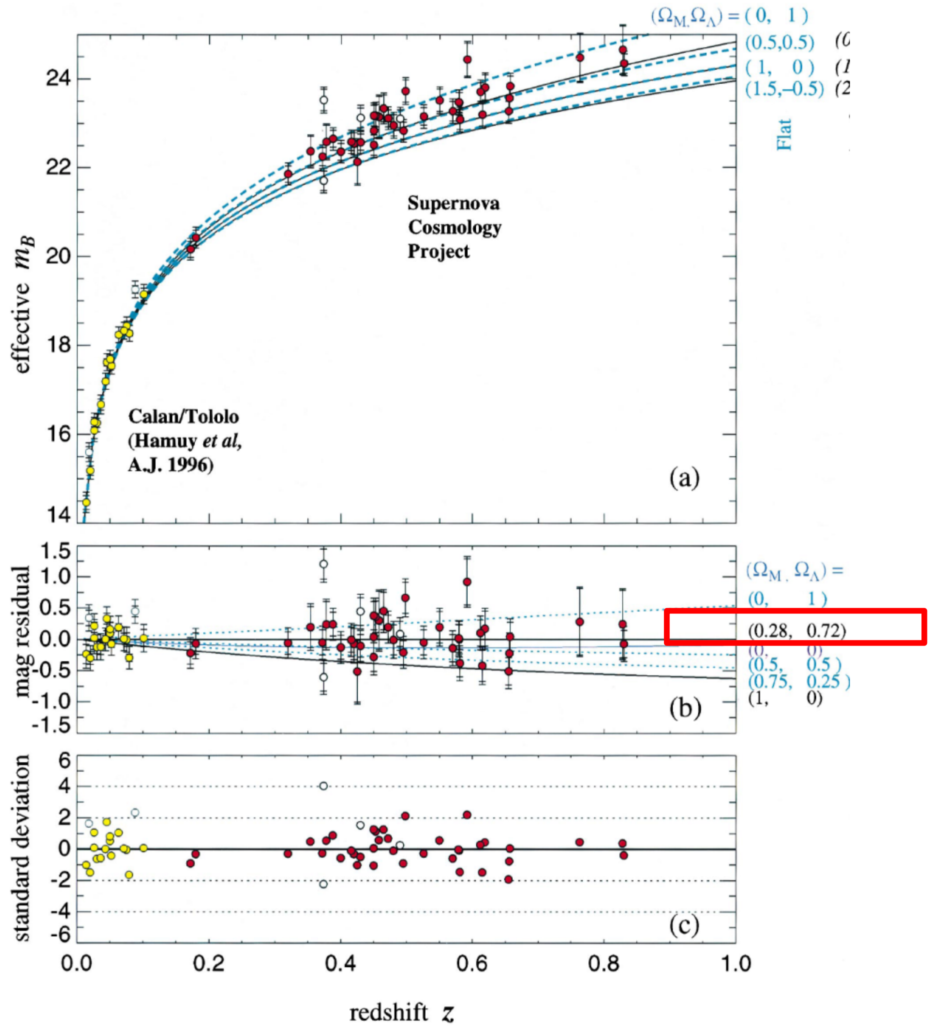
Cosmic Acceleration: SN-Ia as Standard Candle

1998 Discovery!





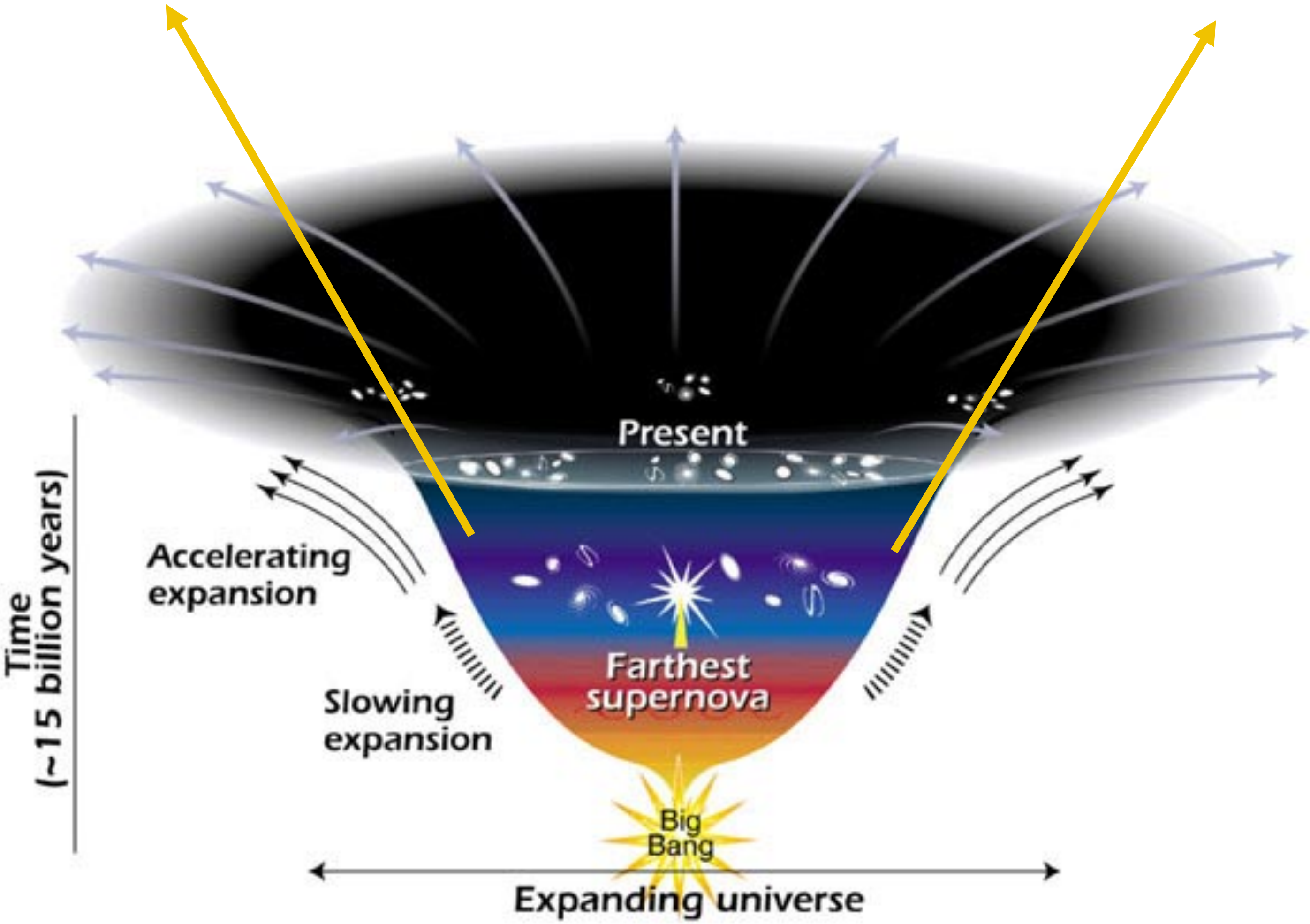
Riess et al, 1998

High z Supernova Team

Perlmutter et al, 1999

Supernova Cosmology Project

Accelerating Expansion (1998):
One of the biggest surprises in science!



2011 Nobel Prize for Physics

Photo: Roy Kaltschmidt. Courtesy: Lawrence Berkeley National Laboratory

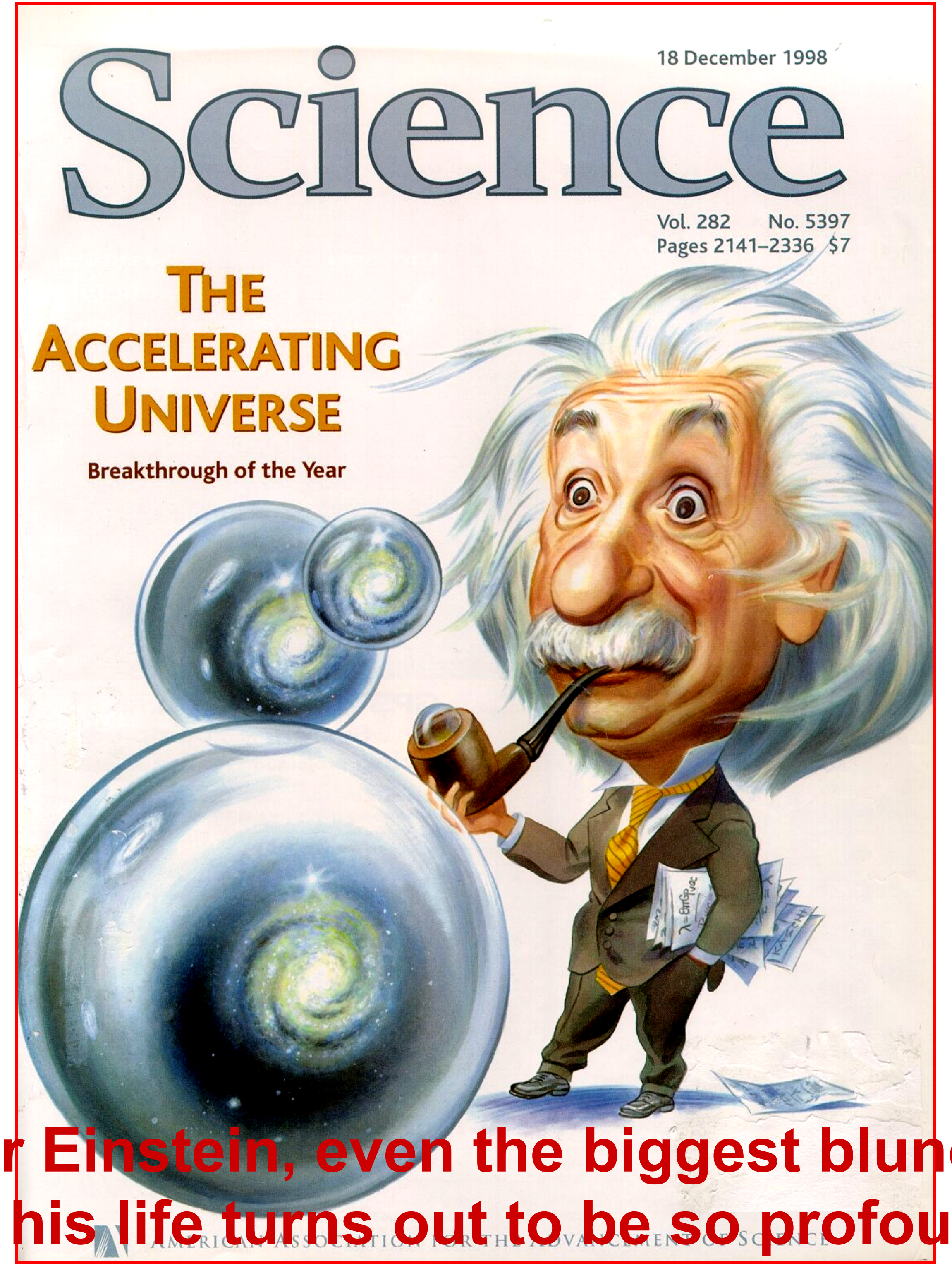
Saul Perlmutter

Photo: Belinda Pratten, Australian National University

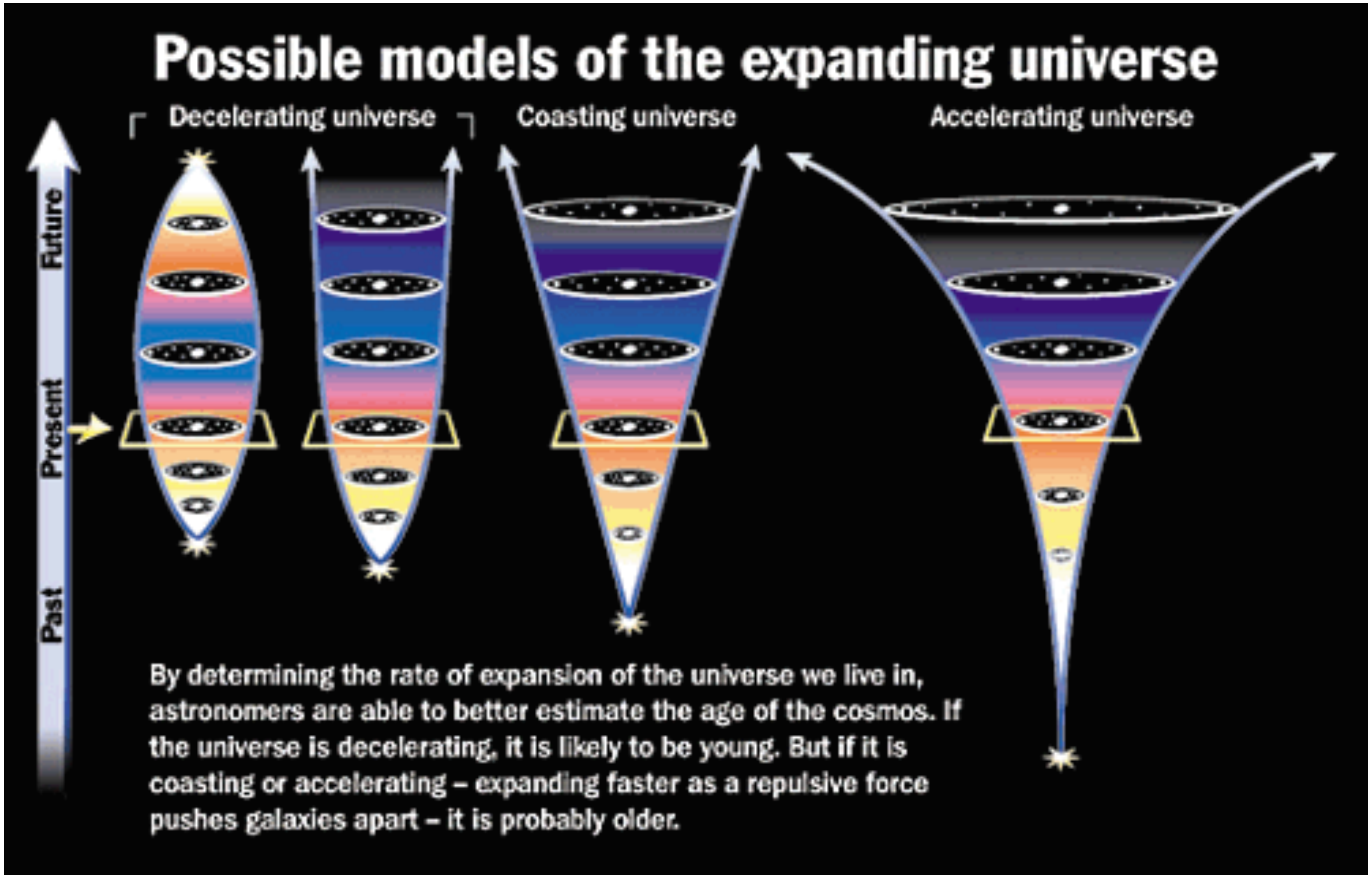
Brian P. Schmidt

Photo: Homewood Photography

Adam G. Riess



For Einstein, even the biggest blunder in his life turns out to be so profound!



Newton's constant

Expansion of Universe governed by the Friedmann eqs.,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\,\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

curvature signature

cosmological constant

- Critical density: $\rho_c = \frac{3H^2}{8\pi G}$

- Total and fractional densities $\Omega = \rho_{total} / \rho_c$

radiation: $\Omega_r = \rho_r / \rho_c$

matter (baryon & cold): $\Omega_m = \rho_m / \rho_c$

cosmology constant: $\Omega_\Lambda = \rho_\Lambda / \rho_c$

- Friedmann eq. again:

Dominant at early times

$$H^2 = H_0^2 \Big[\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_{\rm de} (1+z)^{3(1+w)} + \Omega_k (1+z)^2 \Big]$$

Dominant at late times

- Equation of state $p = w\rho$ For accel. universe, $w < -1/3$

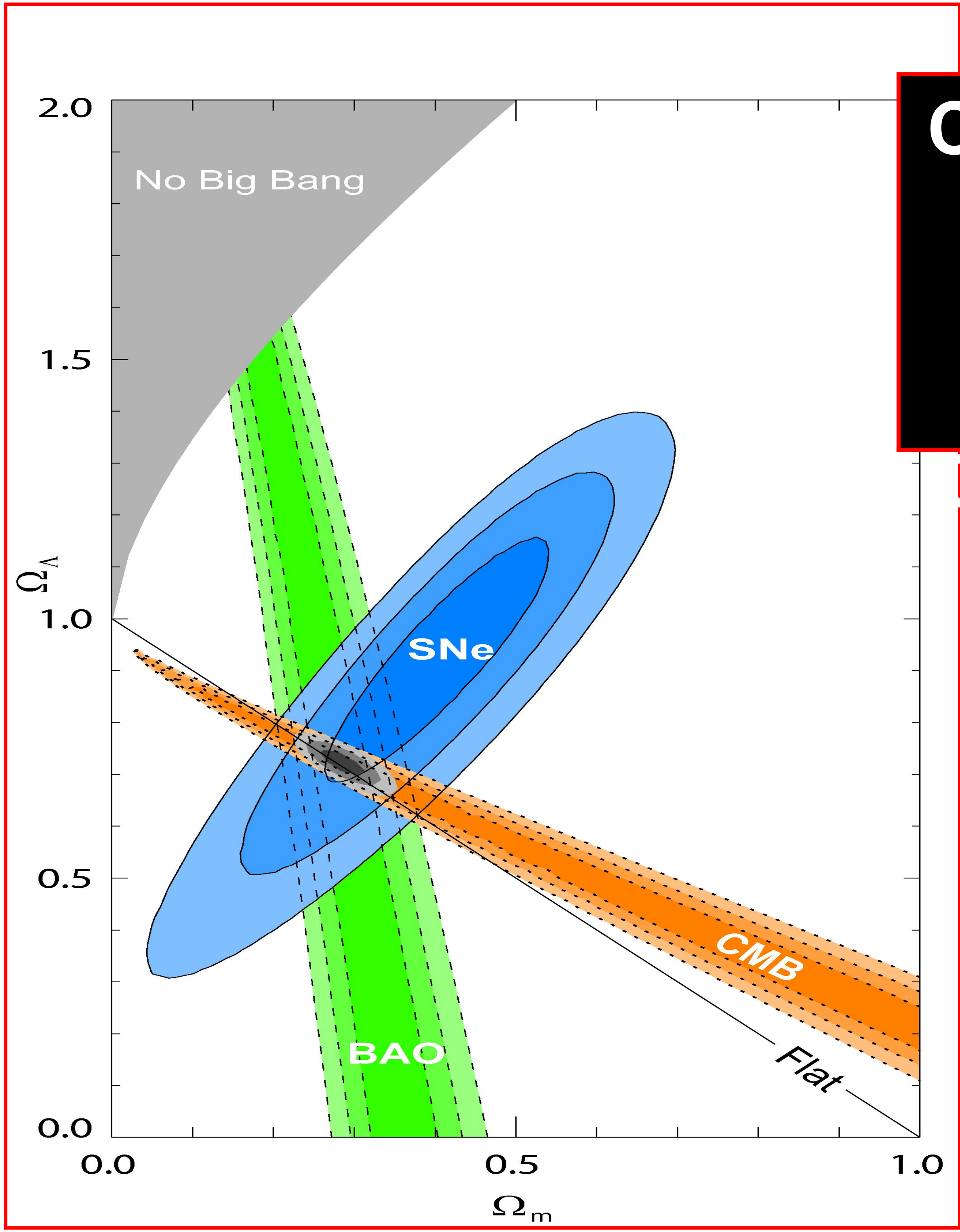
For cosmological constant, $w = -1$ $w = w_0 + w_a(1-a)$

In general, w can depend on a, e.g.,

“Extraordinary claims requires extraordinary evidence.”

To constrain the nature of dark energy we need to be able to measure the expansion rate of the Universe and there are three main approaches:

- Standard candles: which measure the luminosity distance as a function of redshift.
- Standard rulers: which measure the angular diameter distance and expansion rate as a function of redshift.
- Growth of fluctuations.



Consistent with all observations:

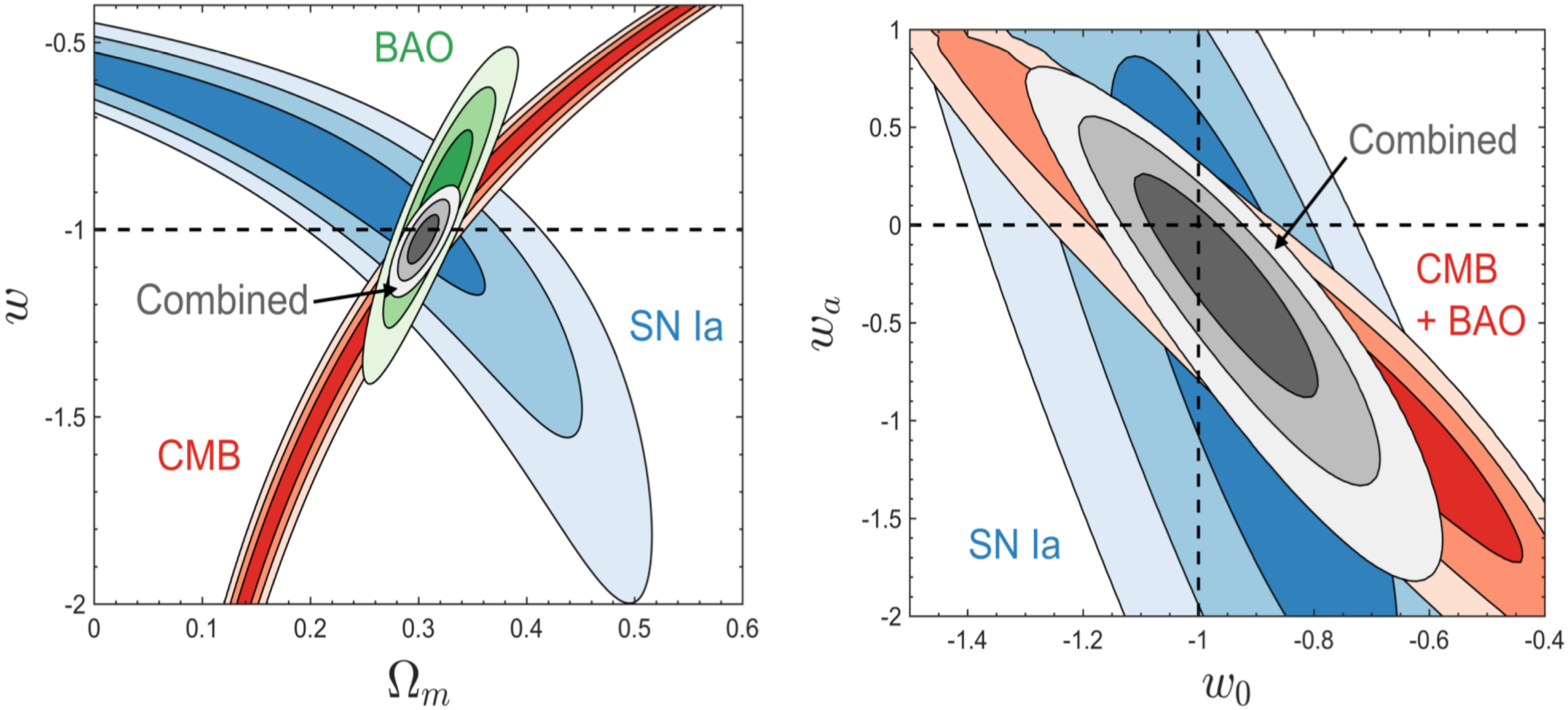
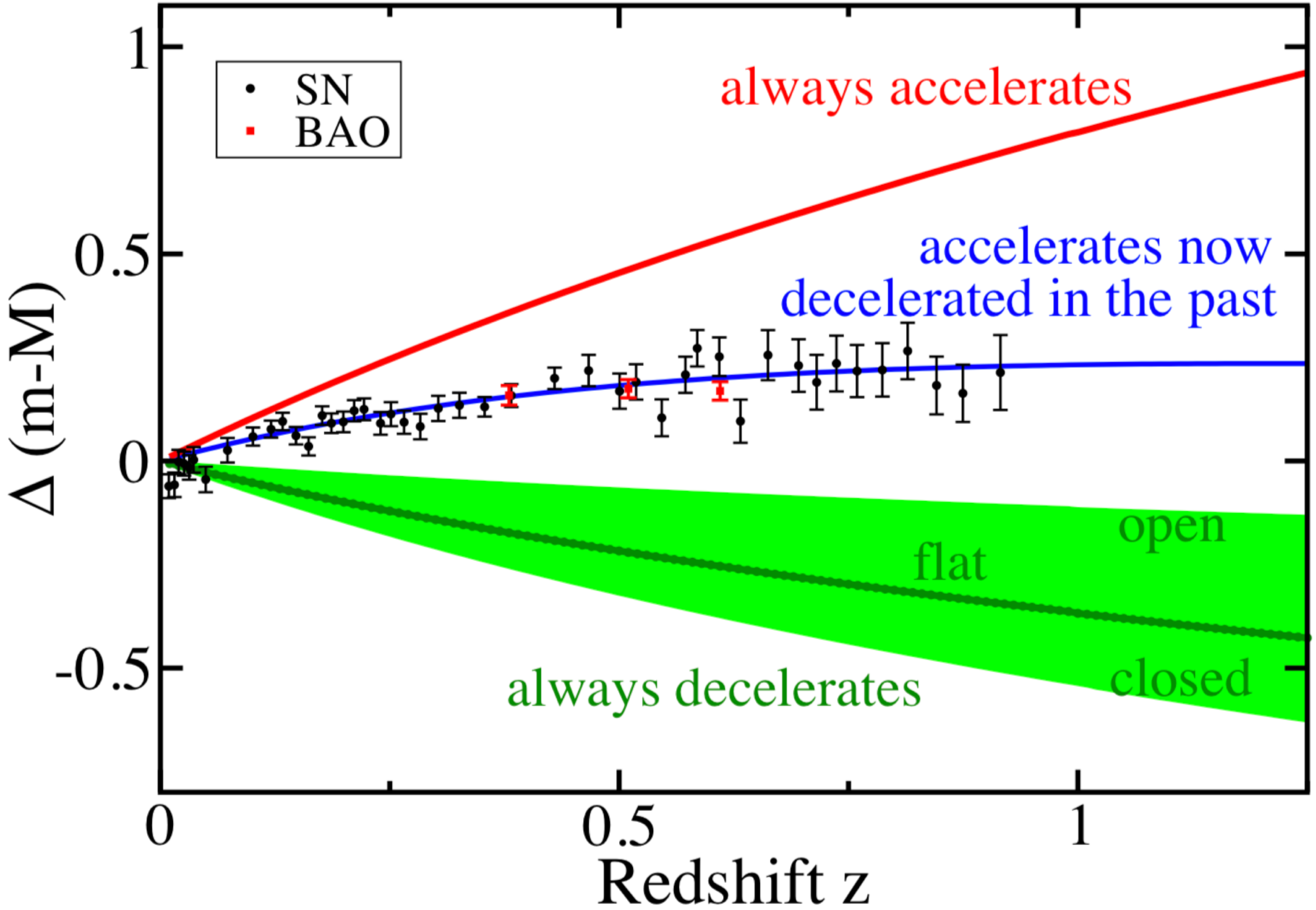


Figure 9. Constraints on cosmological parameters from our analysis of current data from three principal probes: SN Ia (JLA [203]; blue), BAO (BOSS DR12 [30]; green), and CMB (*Planck* 2015 [74]; red). We show constraints on Ω_m and constant w (left panel) and on w_0 and w_a in the parametrization from (17), marginalized over Ω_m (right panel). The contours contain 68.3%, 95.4%, and 99.7% of the likelihood, and we assume a flat universe in both cases.

Huterer-Shafer (2018)

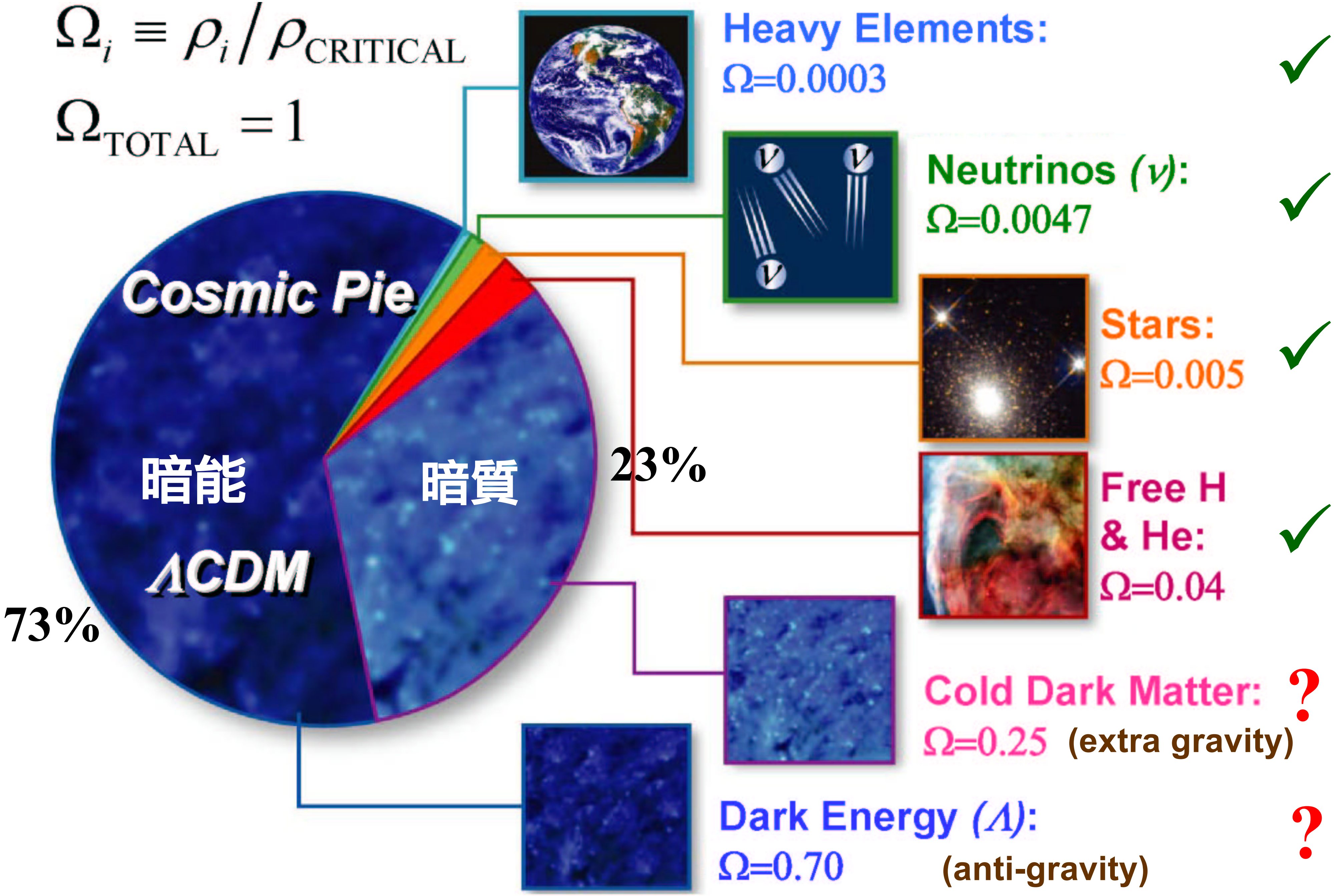
Evidence for past deceleration: Important reality check

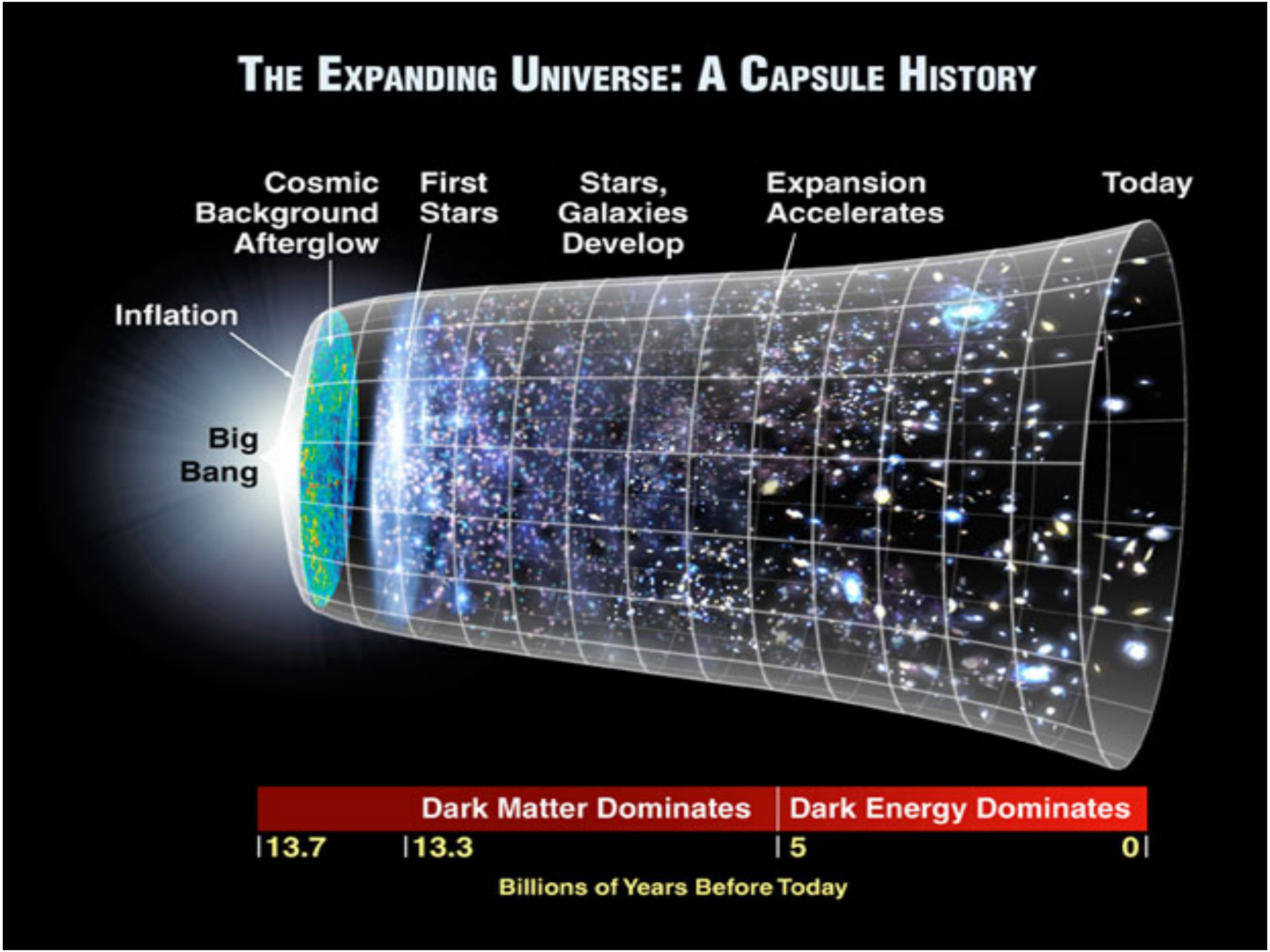


SUPERCAL, Scolnic et al. (2015); BOSS-DR12, Alam et al. (2016); D. Huterer, D. Safer, “Dark energy two decades after” (2018).

$$\Omega_i \equiv \rho_i / \rho_{\mathrm{CRITICAL}}$$

$$\Omega_{\mathrm{TOTAL}} = 1$$




THE EXPANDING UNIVERSE: A CAPSULE HISTORY
Cosmic Background Afterglow
First Stars
Stars, Galaxies Develop
Expansion Accelerates
Today
Inflation
Big Bang
Dark Matter Dominates
Dark Energy Dominates
13.7
13.3
5
0
Billions of Years Before Today

What we understand

- Smooth, very elastic, non-particulate (medium)
- Extremely weak interaction with ordinary matter
- Insignificant at small scales, important at large scales
- Insignificant at early times, important at late times
- Isotropic and homogeneous (apparently)

Dark Energy is a profound mystery

because it touches so many other important puzzles

- Vacuum energy/cosmological constant
- Destiny of the Universe
- Related to Dark Matter, Inflation, Neutrino Mass?
- Connections to SUSY/Superstrings/Extra dimensions?
- Signal of new gravitational physics?
- Hole in the Universe?
- Connection to the hierarchy problem?

Theoretical Attempts

- **Assume General Relativity (GR) is correct.**

Einstein equation:

Einstein tensor → $G_{\mu\nu}$; energy-momentum tensor → $T_{\mu\nu}$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

geometry ($G_{\mu\nu}$) gravity ($8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$)

Introduce (anti-)gravity

- simplest model: cosmological constant

$$\Lambda = 8\pi G \rho_{vac} \qquad w = -1$$

Commonly associated with vacuum energy.

- dynamical models: rolling scalar field with potential (quintessence, phantom, etc.) $w < -1/3$

Theoretical Attempts

- Quintessence
  - Accelerating expansion caused by the potential energy of a scalar field.
  - It must be very light (large Compton wavelength) so it won't clump or form structures.

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \qquad \begin{cases} \rho_\phi = \dot{\phi}^2/2 + V(\phi) \\ p_\phi = \dot{\phi}^2/2 - V(\phi) \end{cases}$$

$$H^2 = H_0^2\left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\phi\left(3\int_0^z [1+w_\phi(z')]\frac{dz'}{1+z'}\right)\right].$$

Theoretical Attempts

- **Assume General Relativity (GR) is correct.**

Einstein equation:

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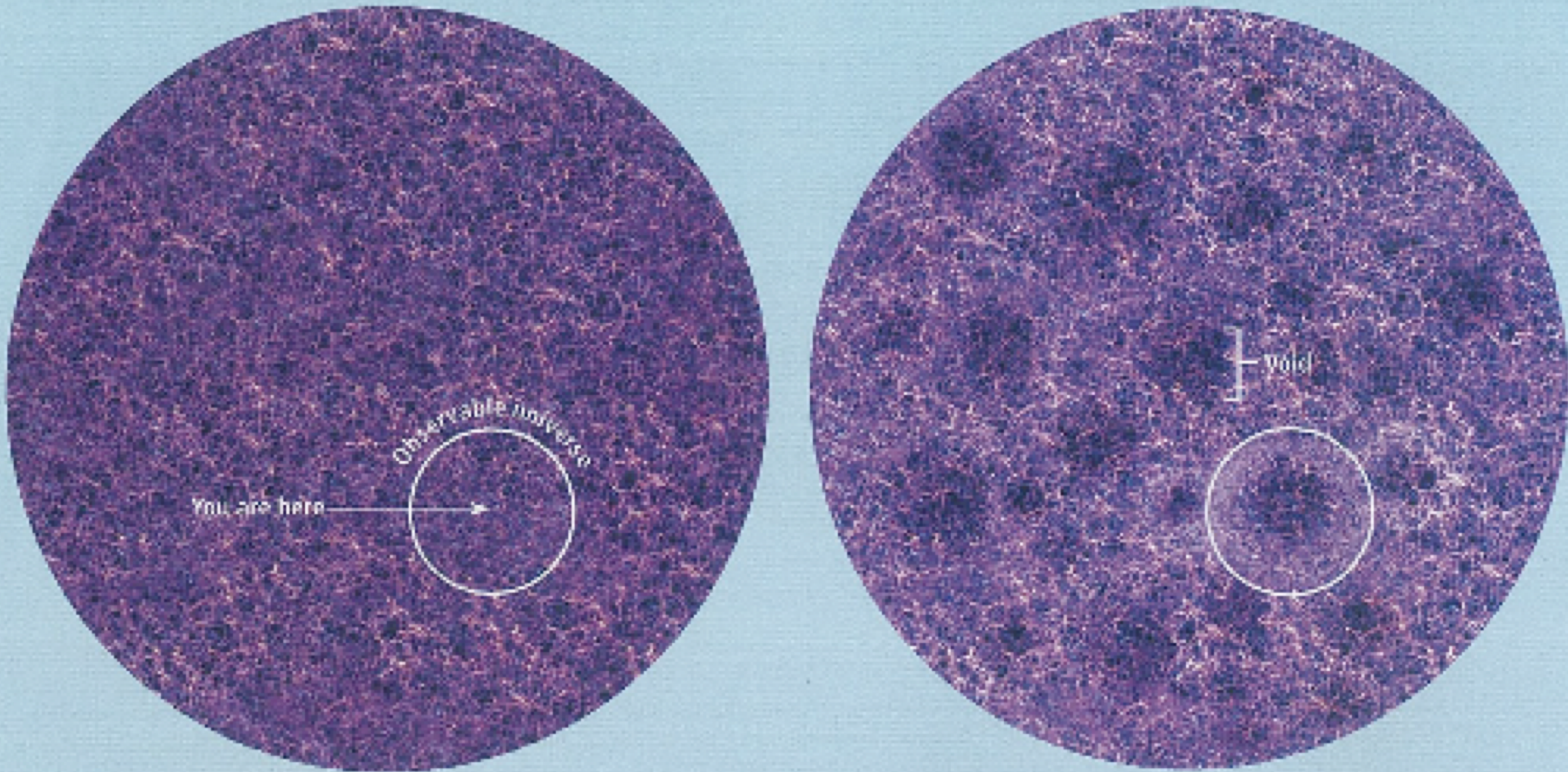
geometry ($G_{\mu\nu}$) gravity ($8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$)

Special place in the universe

- Anti-Copernican Principle: "We are special".
- No need for Λ.

A Special Place for Us

In his *Hitchhiker's Guide to the Galaxy* series of novels, Douglas Adams imagines a torture device that drives people insane by showing them the utter insignificance of their place in the universe. One would-be victim emerges unscathed when it turns out that the universe does, in fact, revolve around him. In a case of life imitating art, many cosmologists are investigating whether our planet indeed has a special place within the grand scheme of things.



HOMOGENEOUS UNIVERSE: OUR LOCATION IS TYPICAL

INHOMOGENEOUS UNIVERSE: OUR LOCATION IS SPECIAL

Theoretical Attempts

- **General Relativity (GR) is the problem**

  - Modify Einstein-Hilbert action

Ricci Scalar

$$S_{EH} = \frac{1}{8\pi G}\int d^4x\sqrt{-g}R$$

to something more general:

$$S_{MG} = \frac{1}{8\pi G}\int d^4x\sqrt{-g}\left[R + f(R)\right].$$

  - DGP model

  string-theory-inspired IR modification of GR

  - Emergent gravity

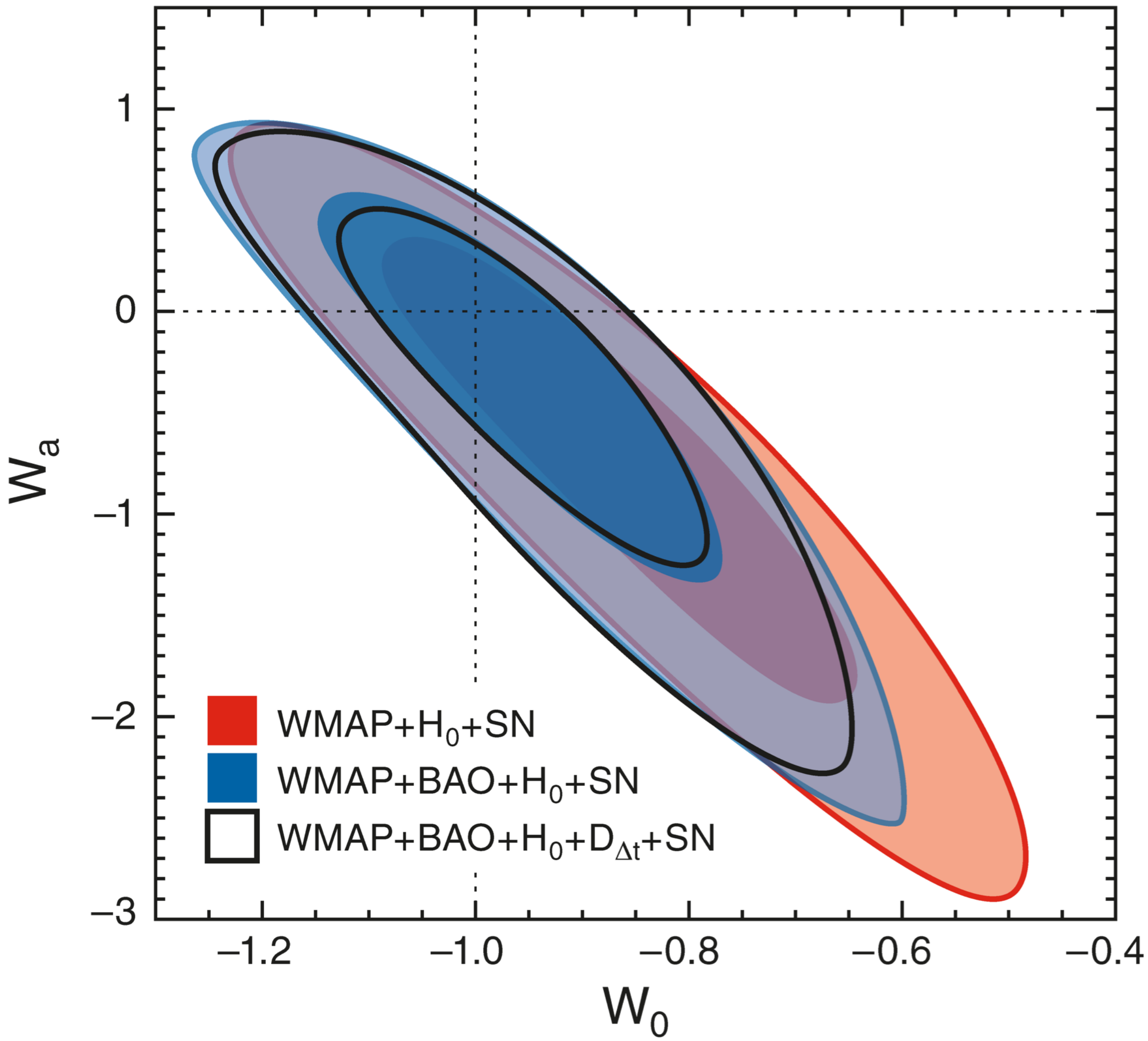
  GR as an effective theory emerging from quantum theory of gravity

Some Challenges

- Cosmological constant: simple and natural. But what makes it so small?
- Quintessence: Who ordered it? Renormalized scalar field often acquires large mass.
- "Anti-Copernican Principle": hard to modify enough to accommodate cosmic acceleration and satisfy other constraints (importance of dynamical tests)
- Modified GR: hard to satisfy short-distance constraints while providing significant departure at large distance.

Observations show that $w = -1$

→ Dark Energy = CC?



$$p = w\rho$$

$$w = w_0 + w_a(1 - a)$$

2009 data:

$$w_0 = -0.97^{+0.12}_{-0.07}$$

$$w_a = 0.03^{+0.26}_{-0.75}$$

The Cosmological Constant Problem

- Cosmological constant has long been a problem in theoretical physics during most of the 20th century.
- Since 1998, it has further become one of the most challenging issues in astrophysics in the new century.
- Several excellent review articles:
  S. Weinberg (1989), Carroll (2000), Sahni & Starobinsky (2000, 2006), Peebles & Ratra (2002), Padmanabhan (2003)…

More than 1000 papers in arXiv that has ‘cosmological constant’ in the title. Can’t possibly cover all ideas.

What is the Problem?

- **History**

After completing his formulation of general relativity (GR), Einstein (1917) introduced a cosmological constant (CC) to his eq. for the universe to be static:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}.$$

As is well-known, he gave up this term after Hubble's discovery of cosmic expansion.

Unfortunately, not so easy to drop it.

In GR, anything that contributes to the energy density of the vacuum acts like a CC.

The Old CC Problem

The old (< 1998):

- Lorentz invariance, upon which QFT is based, tells us that in the vacuum the energy-momentum tensor must take the form

$$\langle T_{\mu\nu} \rangle = -\langle \rho \rangle g_{\mu\nu}.$$

This is equivalent to adding a term to CC:

$$\Lambda_{eff} = \Lambda + 8\pi G \langle \rho \rangle. \Longleftrightarrow \rho_V = \langle \rho \rangle + \Lambda / 8\pi G = \Lambda_{eff} / 8\pi G.$$

- Quantum vacuum (zero point) energies with cutoff at Planck scale gives

$$\rho_V \sim M_{Pl}^4 \sim 10^{112} eV^4.$$

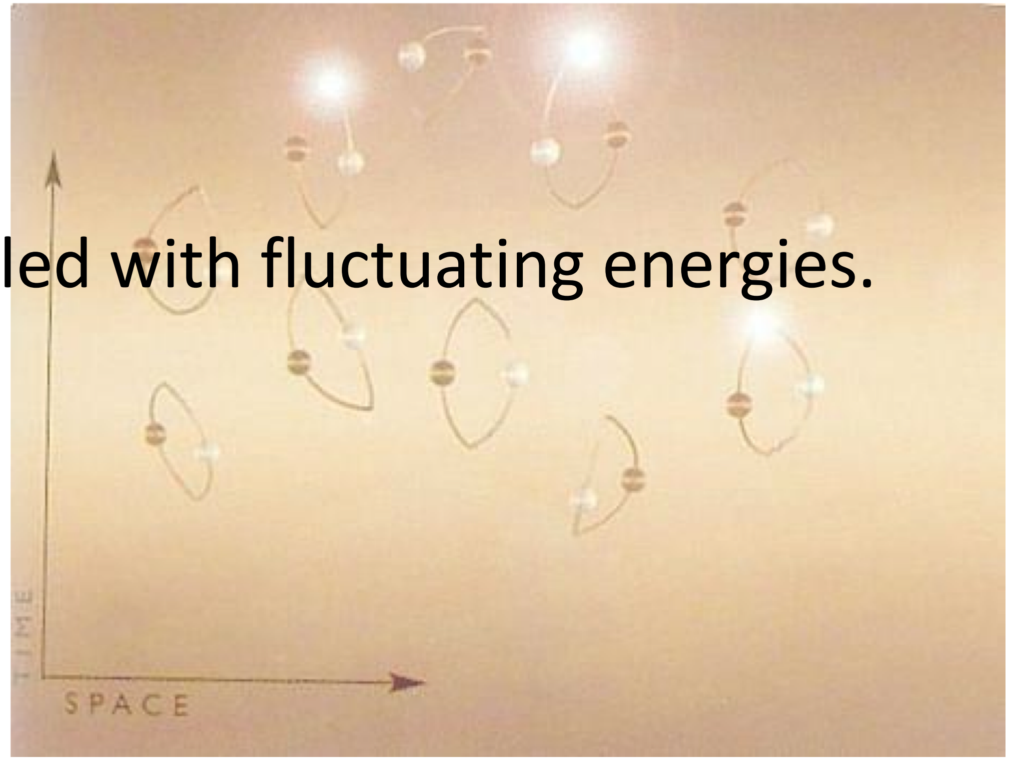
What is Quantum vacuum Energy?

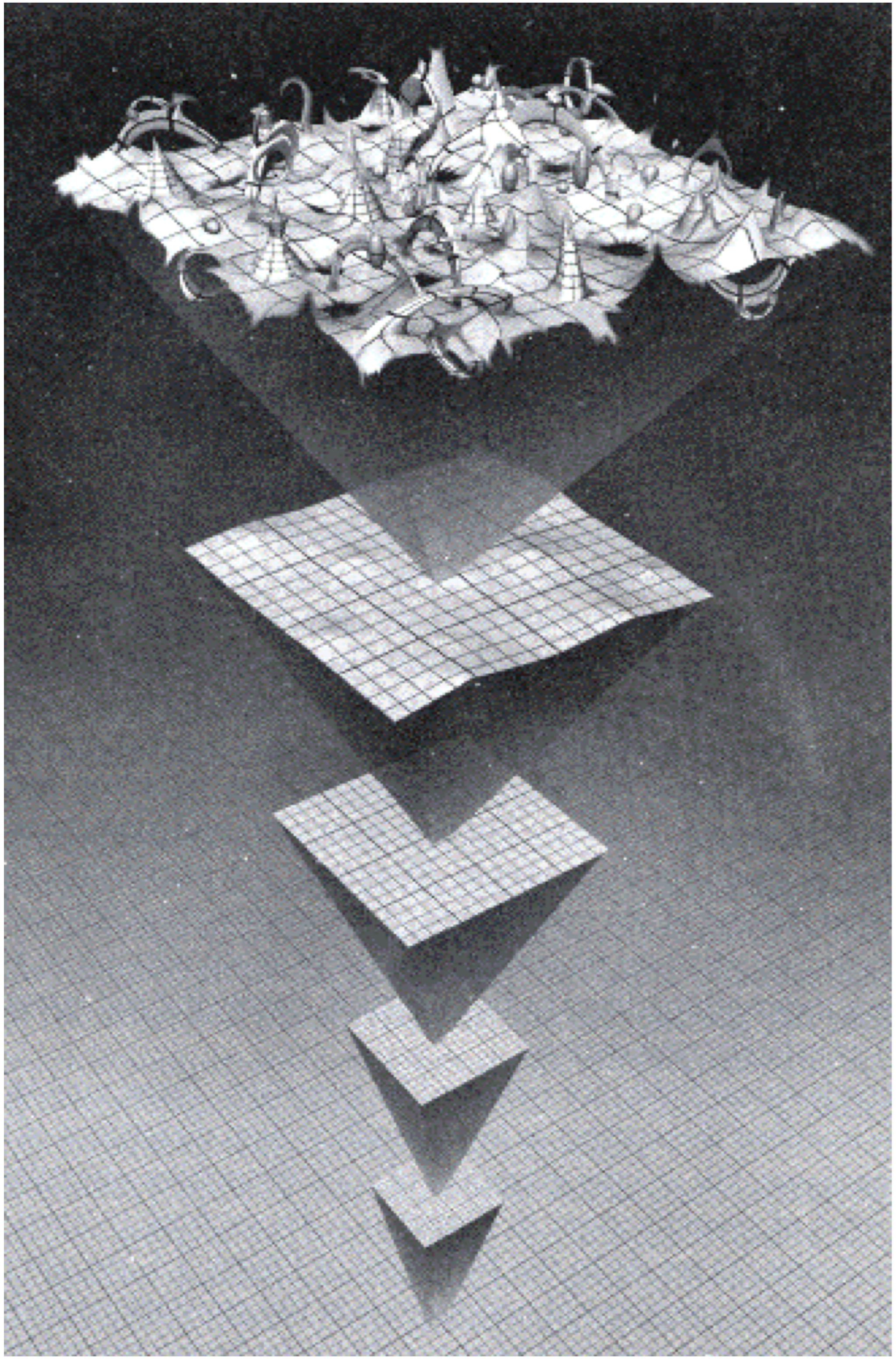
- Heisenberg Uncertainty Principle:

$$\Delta x \Delta p \geq \frac{h}{2\pi} \Rightarrow \Delta t \Delta E \geq \frac{h}{2\pi}$$

$\Rightarrow$

Vacuum is not empty, but filled with fluctuating energies.





Quantization of Spacetime

Planck scale:

$$l_P = \sqrt{\frac{hG}{2\pi c^3}} \approx 1.6\times 10^{-35} m$$

$$E_P = \sqrt{\frac{hc}{2\pi G}} \approx 1.2\times 10^{19}$$

$$[GeV/c^2]$$

Ultimate vacuum energy:

$$E_P^4 : (10^{19} GeV)^4$$

$$= 10^{112} [eV]^4$$

- Astrophysics, however, demands that it must be smaller than the critical density of the universe:

$$\rho_V \leq \rho_{cr} \sim 10^{-12} eV^4.$$

This is 124 orders of magnitude in discrepancy!

- Evidently QVE should not gravitate. Otherwise our universe would not have survived until now.
- This conflict between GR and quantum theory is the essence of the longstanding CC problem, which clearly requires a resolution. In short,

Why doesn't quantum vacuum energy gravitate?

We shall call this the "old" CC problem.

The New CC Problem, or the Dark Energy Puzzle

The new (>1998)

- The dramatic discovery of the accelerating expansion of the universe ushers in a new chapter of the CC problem.

- The substance responsible for it is referred to as the dark energy (DE), described by its equation of state

$$p = w\rho,$$ p: pressure, ρ: density

- According to GR, accelerating expansion can happen if $w < -1/3$. Einstein's CC corresponds to $w = -1$.

- DE=CC remains the simplest and most likely answer.
- New challenge: after finding a way, hopefully, to cancel the CC to 124 decimal points, how do we reinstate 1 to the last digit and keep it tiny? That is,

Why is CC nonzero but tiny?

- We shall call this the “new” CC problem, or the DE puzzle.

Attempt 1:
Casimir Energy in Extra-Dimensions

If dark energy never changes in space and time, then it must be associated with the fundamental properties of spacetime!

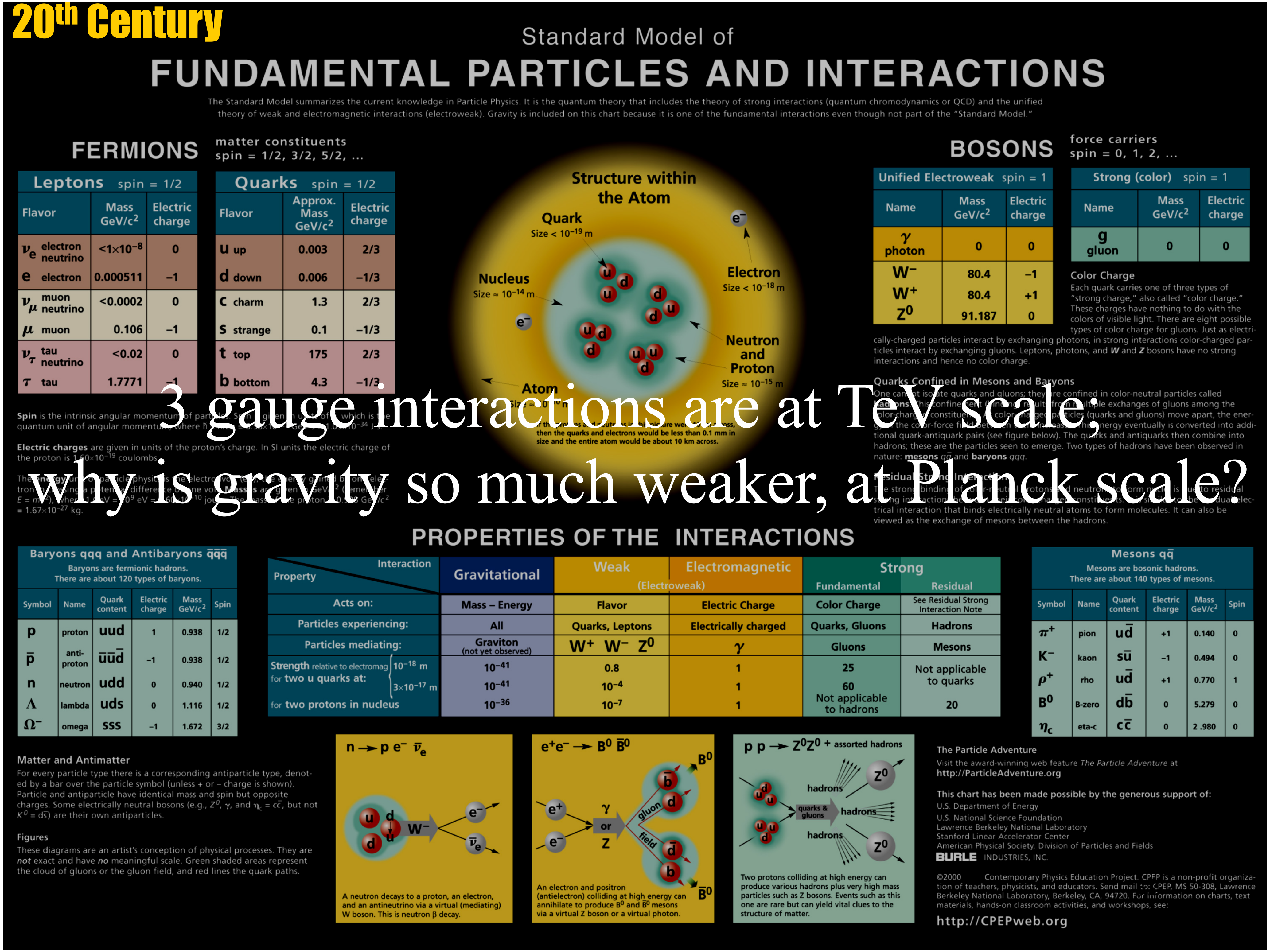
Observations → $M_{CC} \; ; \; \rho_{DE}^{1/4} \; : \; 10^{-3} eV$

Why much smaller than standard model scale?

$$\frac{\rho_{DE}^{1/4}}{M_{SM}} \; : \; 10^{-15}!$$

PC, *Nucl. Phys. Proc. Suppl.*173, 137 (2007).
PC and J-A. Gu, *Mod. Phys. Lett*. A22, 1995 (2007); arXiv:0712.2441

20th Century



3 gauge interactions are at TeV scale;

why is gravity so much weaker, at Planck scale?

A Numerical Coincidence

- A remarkable numerical coincidence, a '**gravity fine structure constant**':

$$\frac{M_{\rm CC}}{M_{\rm SM}} \simeq \frac{M_{\rm SM}}{M_{\rm Pl}} \equiv \alpha_G$$

- Perhaps not accidental but implies a deeper connection:

$$M_{\rm CC} \simeq \frac{M_{\rm SM}}{M_{\rm pl}} M_{\rm SM} = \left(\frac{M_{\rm SM}}{M_{\rm Pl}}\right)^2 M_{\rm Pl} = \alpha_G^2 M_{\rm Pl}.$$

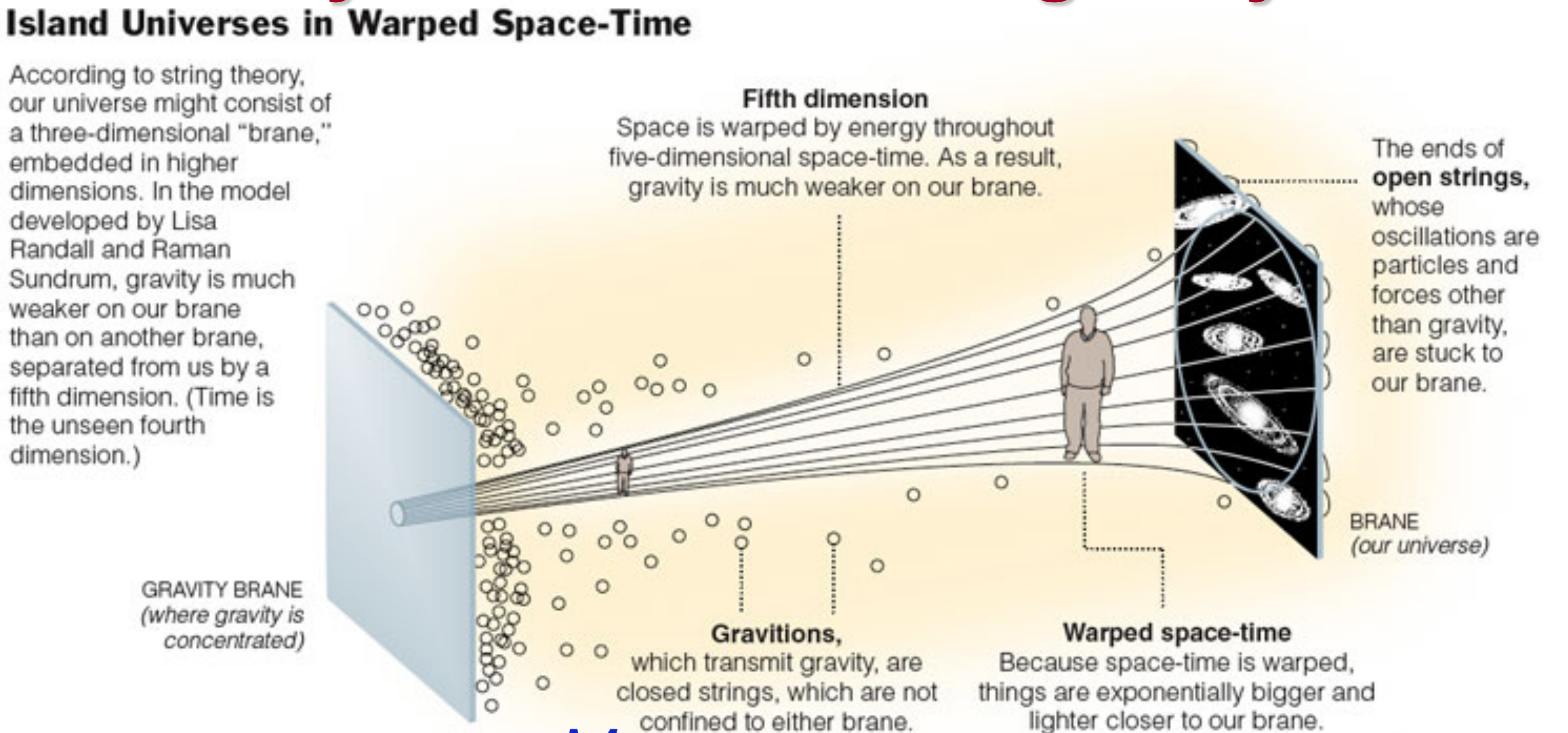
- Caution: Unlike the 1st hierarchy that links 4 fundamental interaction strengths, DE must be a secondary, derived quantity.

Analogy in Atomic Physics

- Bohr atom
- Fundamental energy scale in Schrödinger equation: m_e
- Ground state energy suppressed by 2 powers of fine structure constant

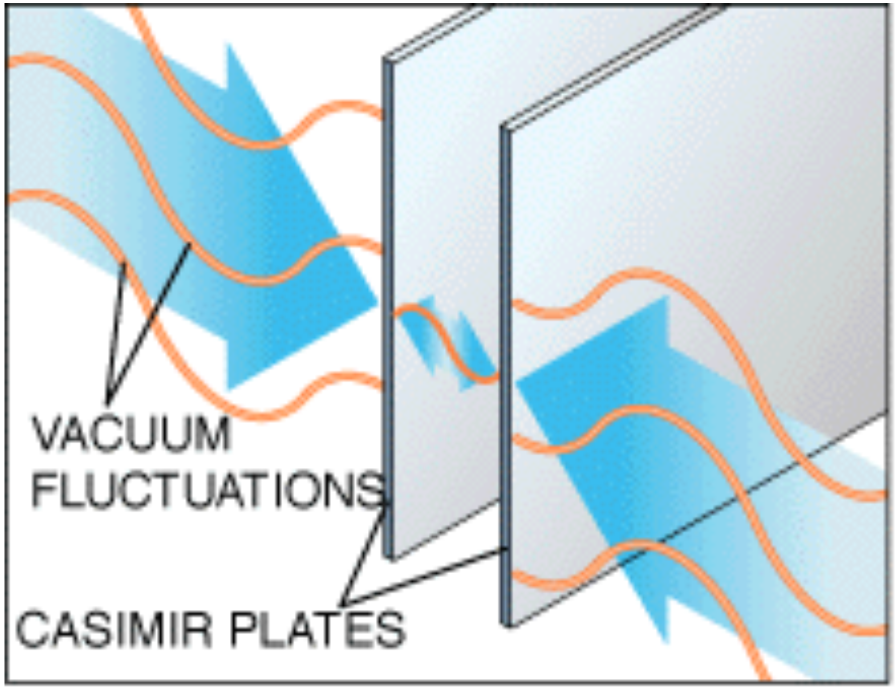
- Dark energy
- Fundamental energy scale in quantum gravity: M_{Pl}
- Dark energy suppressed by 2 powers of "gravity fine structure constant"

Randall-Sundrum Model to bridge the hierarchy between SM and gravity scales



$$\frac{M_{SM}}{M_{Pl}} = e^{-\pi kR} \sim 10^{-16}$$

Casmir Energy: Evidence of vacuum fluctuations



Casimir energy induced by fields in the RS bulk on the TeV brane gives rise to DE as we set out to look for:

$$M_{Casimir} \sim \alpha_G^2 M_{Pl} \sim 10^{-3} eV \sim M_{CC}.$$

This may solve the New CC Problem.

It, however, still does not address the Old CC Problem…

Attempt 2:
Boundary Condition of the Universe

- "Gauge Theory of Gravity with de Sitter Symmetry" (PC, MPLA (2009) [arXiv:1002.4275])

1. **In GR Einstein equation is a 2nd order differential eq. and thus the nature of CC is undetermined:**

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

1. **If the field eq. is higher (e.g., 3rd) order instead, then the CC term is not allowed. To recover the well tested GR, one should integrate it once. Then CC is recovered through the constant of integration determined by the boundary condition of the universe.**

- **Motivations for Gauge theory of gravity (GG)**

  - To reformulate gravity as a gauge theory
  - To hopefully quantize gravity theory
  - To substantiate the 'constant of integration' approach as a means to solve the CC problem.

C. N. Yang (1983): "In [ ] I proposed that the gravitational equation should be changed to a third order equation. I believe today, even more than 1974, that this is a promising idea, because the third order equation is more natural than the second order one and because quantization of Einstein's theory leads to difficulties."

Here's how it goes

- In GG, the gauge potential (affine connection) is the dynamical variable, which determines the curvature tensor

$$R^{\alpha}_{\beta\mu\nu} = \partial_{\mu}\Gamma^{\alpha}_{\beta\nu} - \partial_{\nu}\Gamma^{\alpha}_{\beta\mu} + \Gamma^{\alpha}_{\tau\mu}\Gamma^{\tau}_{\beta\nu} + \Gamma^{\alpha}_{\tau\nu}\Gamma^{\tau}_{\beta\mu}.$$

- In close analogy with Maxwell theory, the action for gravity reads (Cook 09)

$$S_G = \kappa \int dx^4 \sqrt{-g}\left(R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} + 16\pi J^{\mu}_{\alpha\beta}\Gamma^{\alpha\beta}_{\mu}\right),$$

where the "gravitational current" (∇_{α} = covariant deriv.)

$$J^{\mu}_{\alpha\beta} = \frac{2G}{c^4}\left[\nabla_{\alpha}\overline{T}^{\mu}_{\beta} - \nabla_{\beta}\overline{T}^{\mu}_{\alpha}\right],$$

and $\overline{T}^{\mu}_{\beta} = T^{\mu}_{\beta} - \frac{1}{2}\delta^{\mu}_{\beta}T, \;\; T = T^{\mu}_{\mu}.$

Field Equations

- Varying S_G against $\Gamma^{\mu}_{\alpha\beta}$, we arrive at the field eq.

$$\nabla_{\nu} R^{\mu\nu}_{\alpha\beta} = -4\pi J^{\mu}_{\alpha\beta}.$$

  This and the Bianchi identity,

$$\nabla_{\lambda} R_{\alpha\beta\mu\nu} + \nabla_{\nu} R_{\alpha\beta\lambda\mu} + \nabla_{\mu} R_{\alpha\beta\nu\lambda} = 0,$$

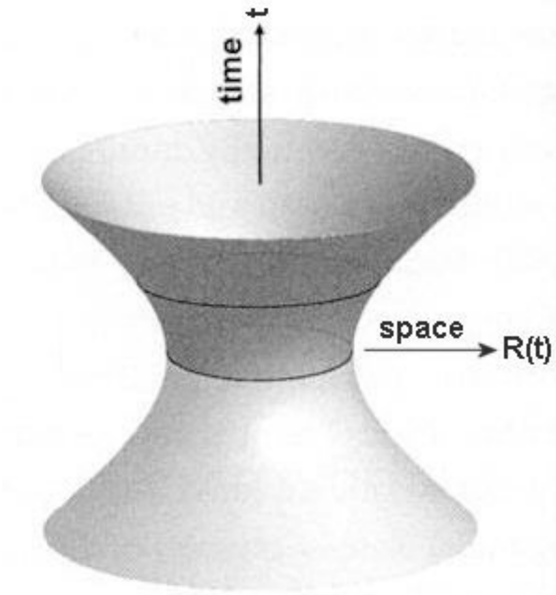
  together determine the curvature tensor.
- Now we recall that $\Gamma_{\alpha\mu\nu} = \frac{1}{2}\left[\partial_{\nu} g_{\alpha\mu} + \partial_{\mu} g_{\alpha\nu} - \partial_{\alpha} g_{\mu\nu}\right]$,
  and that covariant divergence of $g_{\mu\nu}$ is identically 0.
  Therefore the field eq. of GG removes the CC term by construction.
- Integrating this eq. once, we recover the Einstein eq. with a constant of integration which is associated with the boundary condition of the universe.

de Sitter Universe as Asymptotic Limit of Hubble Expansion

- Now we invoke our second assumption, that the universe is inherently de Sitter, where the 4-spacetime is a hyperboloid of a 5-d Minkowski space with the constraint

$$-x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = l_{dS}^2$$

where l_{dS} is the radius of curvature of dS.



- dS universe as asymptotic limit of Hubble expansion.
- Observation gives $\Omega_{DE} = \rho_{DE}/\rho_{cr} ; 0.75$, so we find $l_{dS} ; 1.33H_0 \sim 1.5 \times 10^{28} cm$

But why did our universe choose such a geometry?!

Anthropic Principle & Multiverse argument: "Our Universe is one that is suitable for intelligent habitat."

String theory allows for a "landscape" of universes (10^{500}!)

Back to experiment:
Is DE dynamical or a CC?
A: Dynamical DE would induce anisotropy in cosmic expansion.

C.-T. Chen, P. Chen, “Dark energy induced anisotropy in cosmic expansion”, EPJC (2019)

Assumption:

- Dynamical DE, e.g., quintessence, already exists prior to the inflation. (A reasonable assumption.)

Consequence:

- Quantum fluctuations of DE during inflation would leave some footprints in the anisotropy of the late-time accelerated expansion.

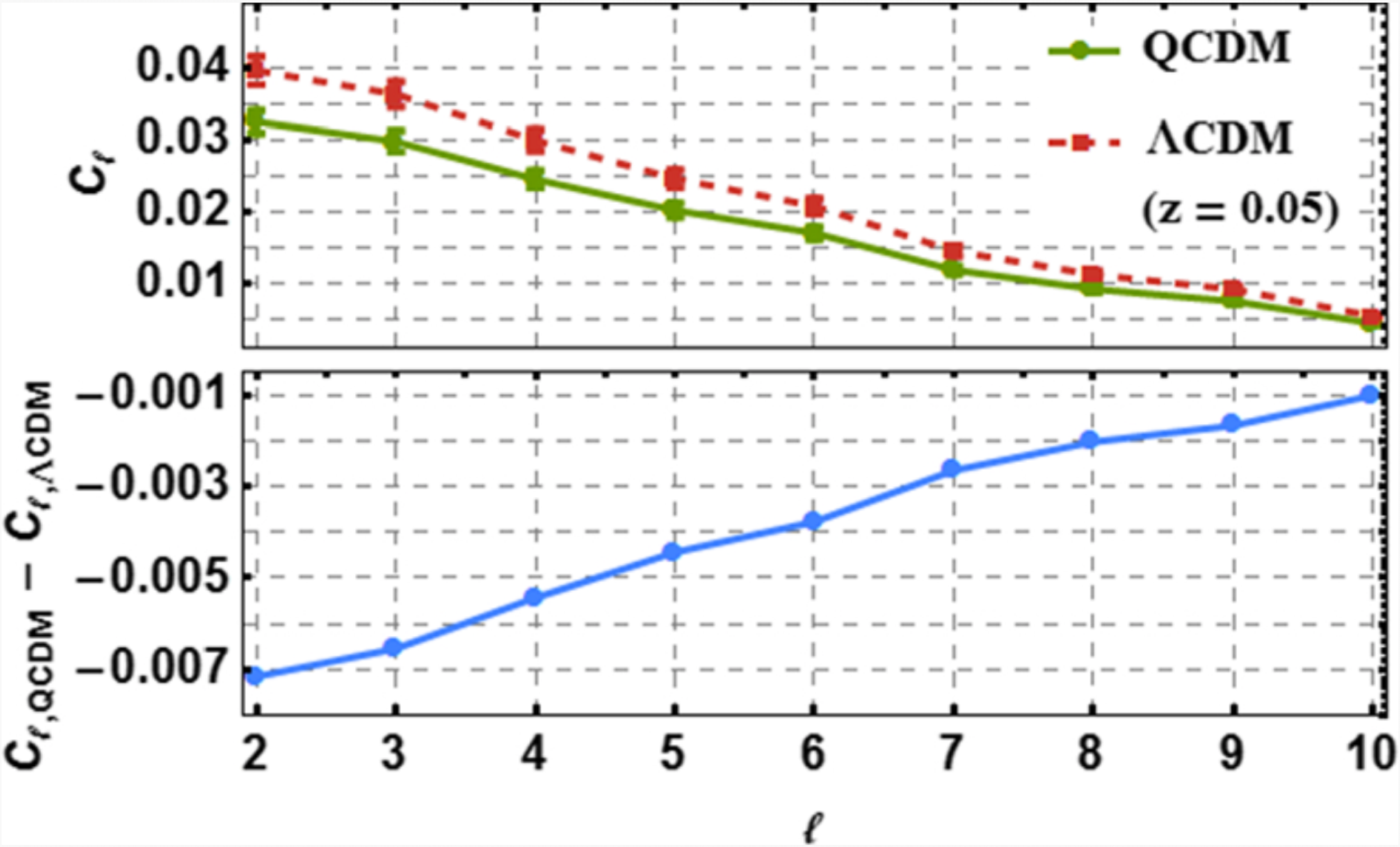


Fig. 5

Top: luminosity distance power spectrum C_ℓ at $z = 0.05$ with integration ranging from $k = 10^{-4}$ to 0.1 $\mathrm{Mpc}^{-1}\mathrm{h}$ for each $\ell = 2$ to 10. Bottom: residue of the luminosity distance power spectrum between QCDM and ΛCDM

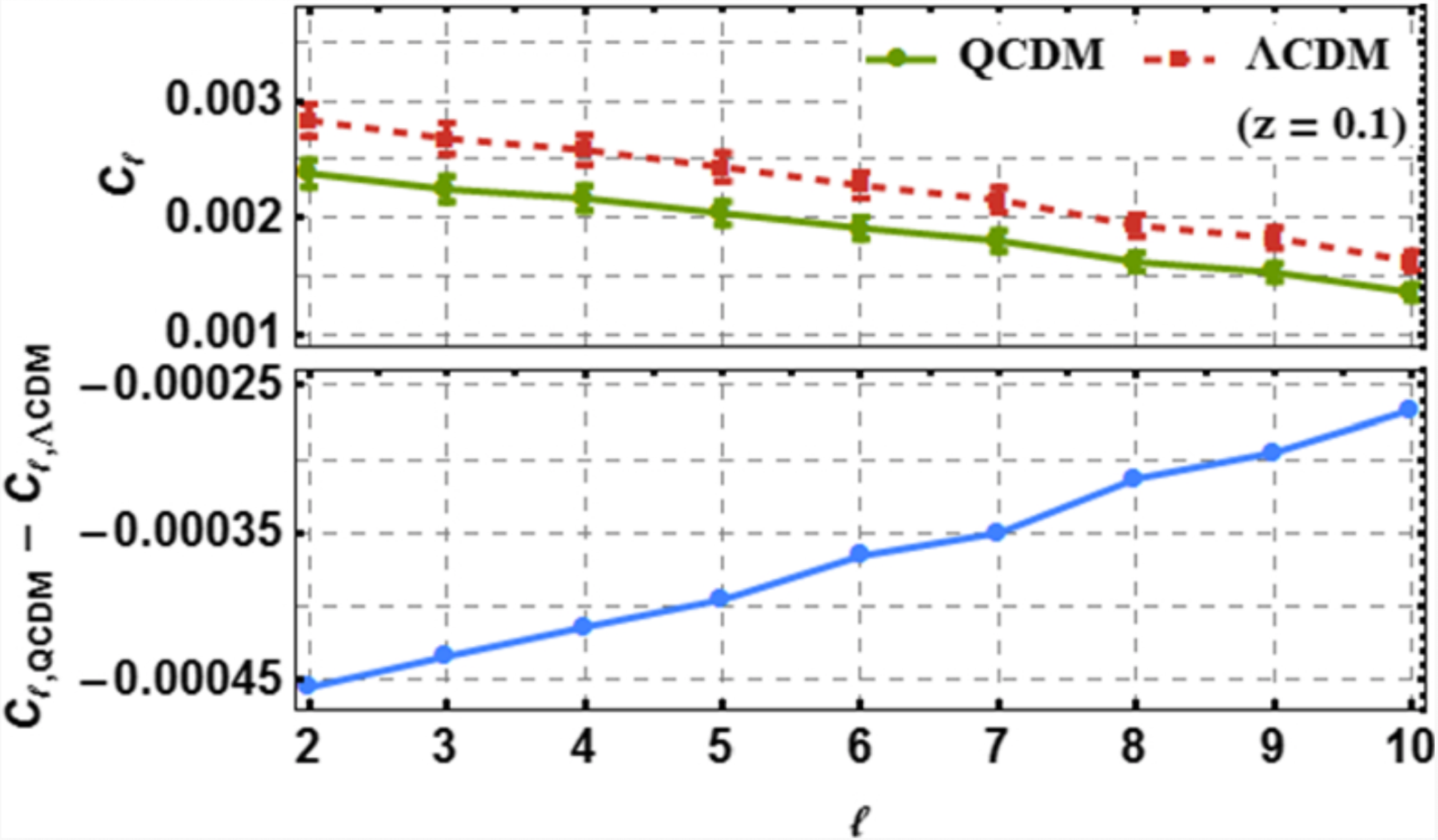


Fig. 6

Top: luminosity distance power spectrum C_ℓ at $z = 0.1$ with integration ranging from $k = 10^{-4}$ to 0.1 Mpc^{-1}h for each $\ell = 2$ to 10. Bottom: residue of the luminosity distance power spectrum between QCDM and ΛCDM

Reflections and Prospects

At the turn of the 20th century, two “dark clouds” in physics (a la Lord Kelvin): the Michelson-Morley experiment and the blackbody radiation, had later developed into revolutionary storms of Relativity and Quantum Mechanics.

At the turn of the 21st century, a new dark cloud – the dark energy, appears above the horizon. Will the history repeat itself and turn this into another revolutionary storm in physics, a storm that would clean up the conflic between QM and GR?