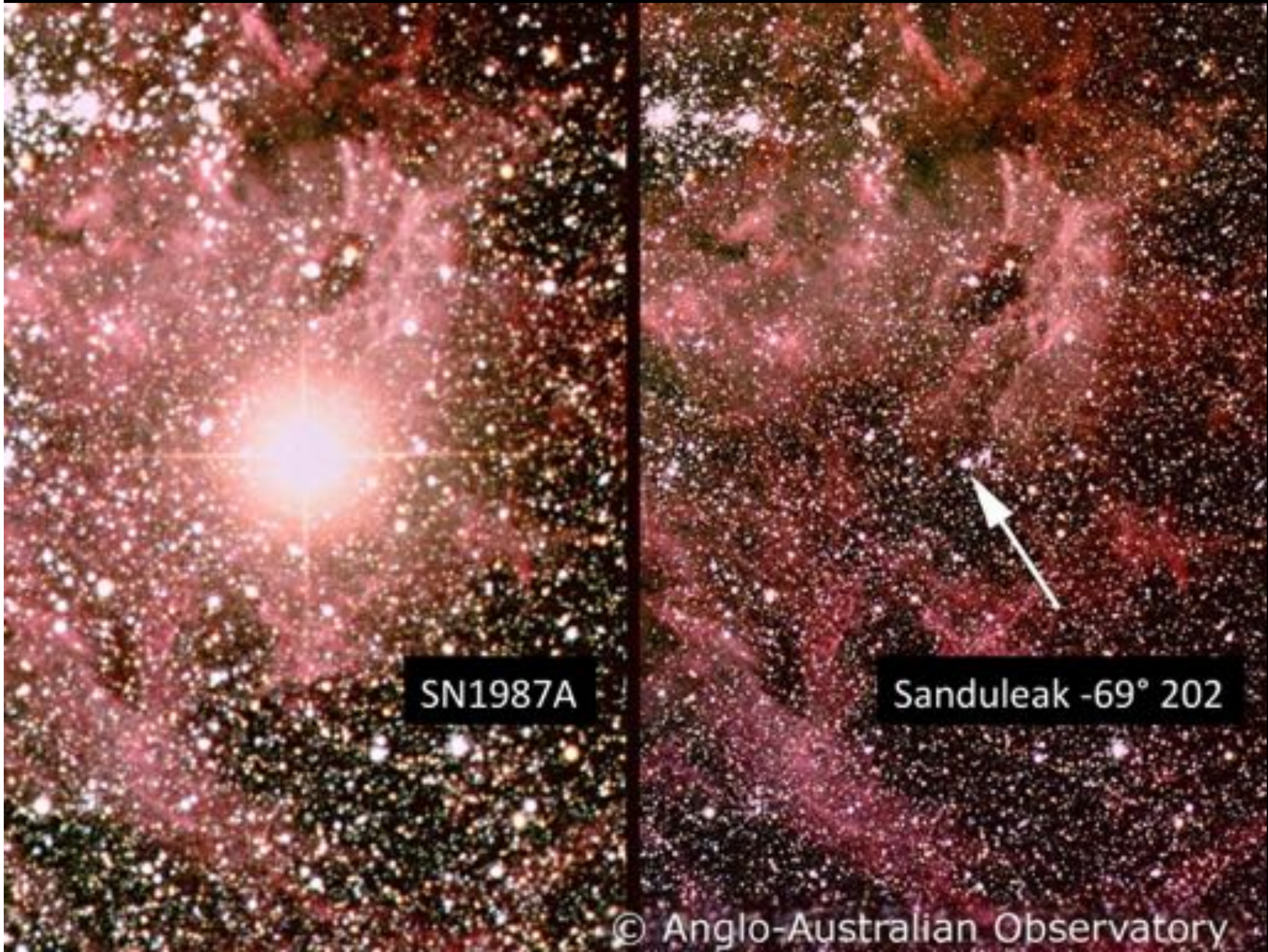
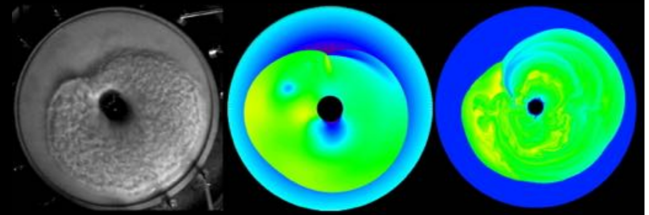




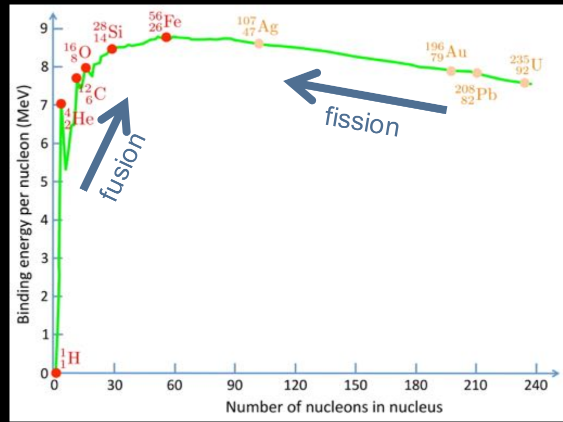
5 lectures on

The Physics of Core-Collapse Supernovae

Supernova physics can be made simple*

*at least what we understand of it

e.g. some of its hydrodynamical properties



The "supernova fountain"
in Paris science museum
"Palais de la Découverte"
December 2013 - February 2014

PRIX 2014
LE GOÛT DES SCIENCES

Outline

1 Introduction to supernovae: following our common sense

The framework of delayed neutrino driven explosions

2 Some observational clues and puzzles

3 The basics of hydrodynamical instabilities

Neutrino driven convection

4 The Standing Accretion shock instability

5 Impact on the explosion & new ideas

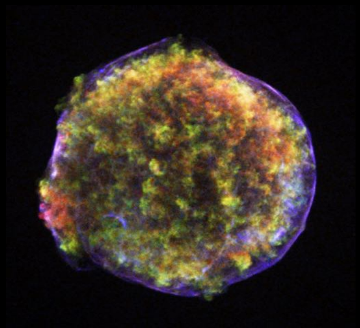
Outline of lecture 1

Introduction to supernovae: following our common sense

- why study supernovae ?
- the basics of the Chandrasekhar limit
- the maximum mass of neutron stars

Supernova remnants

SN 1006

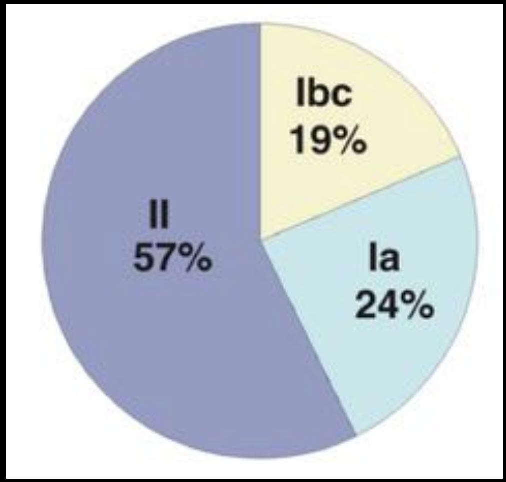


Tycho (1572)

Kepler (1604)

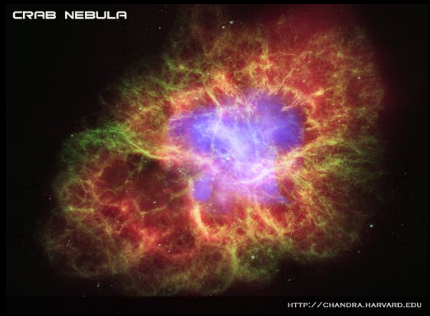
thermonuclear
supernovae
Ia

gravitational
supernovae
II, Ibc



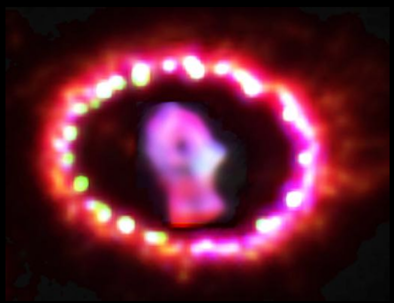
Volume distribution

(Li+11)



Crab (1054)

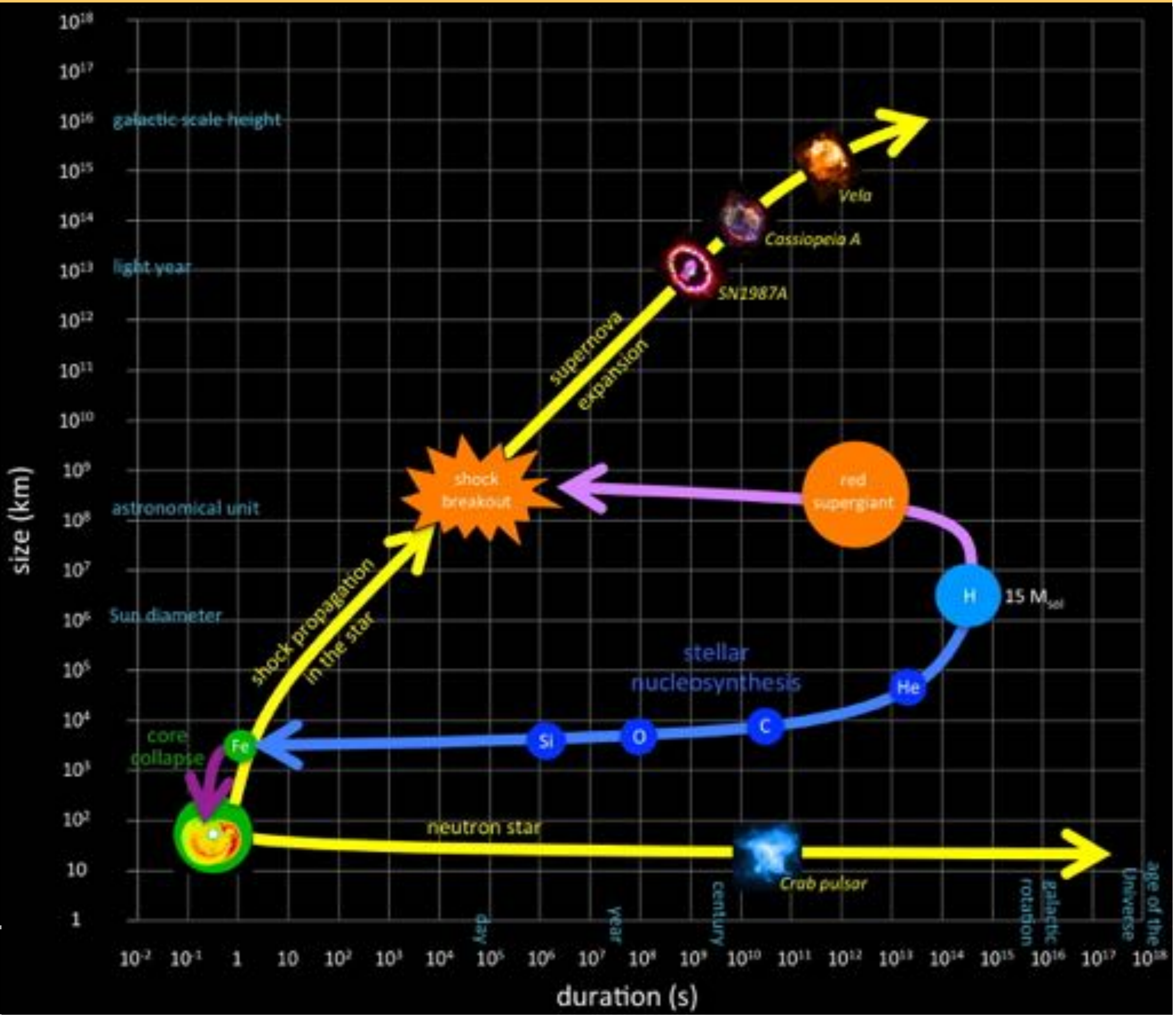
Cassiopeia A (~1680)



SN1987A

a key process in stellar evolution

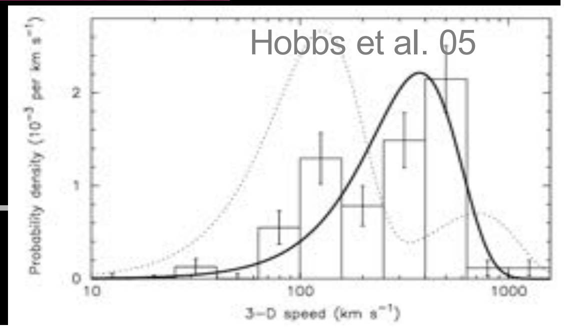
the physical puzzle
takes place
during 1 second
within a 100km radius



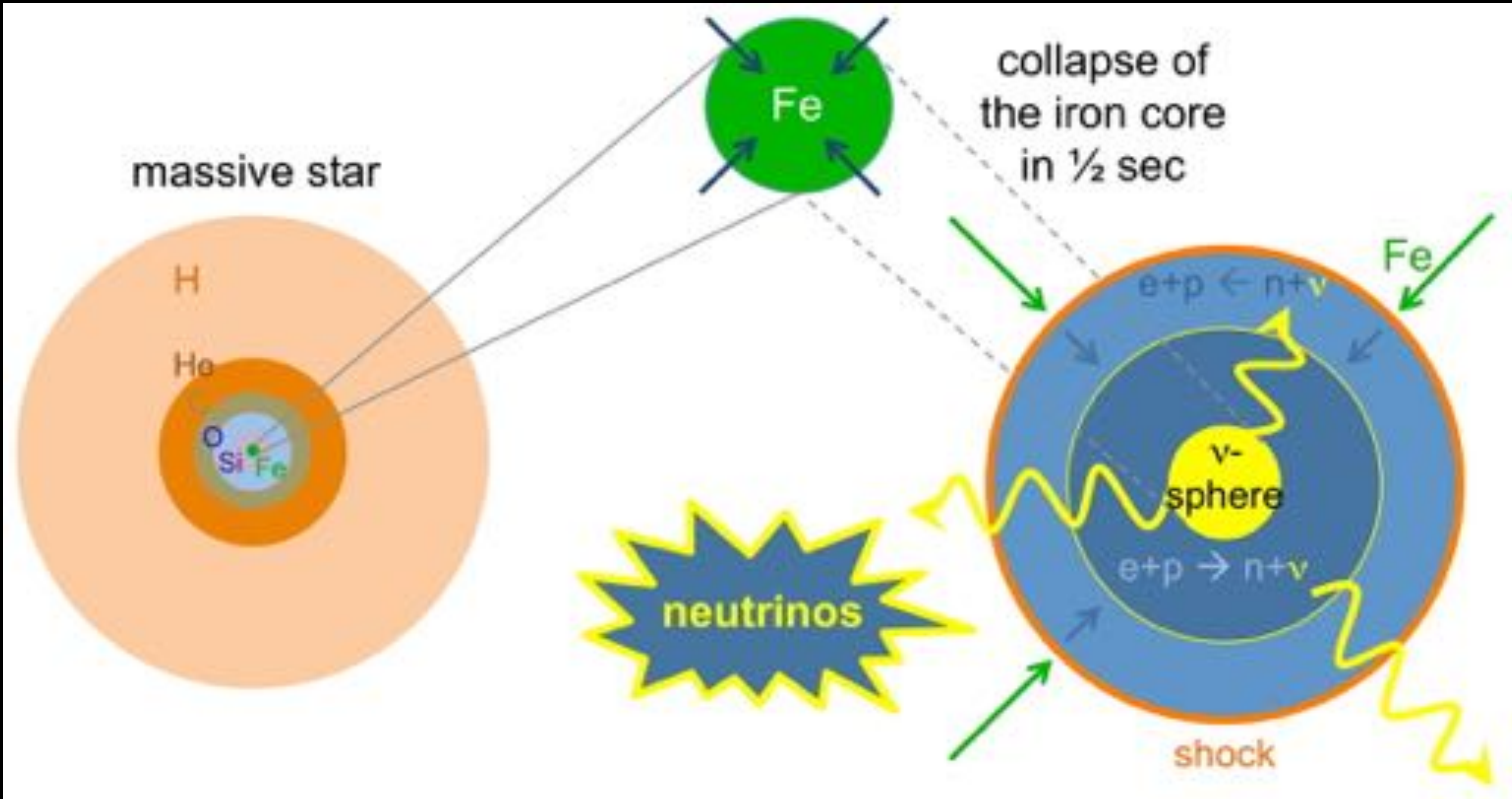
The high velocities of neutron stars suggest an asymmetric supernova explosion

Chatterjee & Cordes+02

pulsar in the guitar nebula: >1000km/s



The framework of neutrino-driven delayed explosions



Why should we care?

A long standing physical puzzle, still unsolved

massive stars are expected to collapse, but why do they explode ?

do we miss a physical process?

VOLUME 90, NUMBER 24 PHYSICAL REVIEW LETTERS week ending 20 JUNE 2003

Improved Models of Stellar Core Collapse and Still No Explosions: What Is Missing?

R. Buras, M. Rampp, H.-Th. Janka, and K. Kifonidis

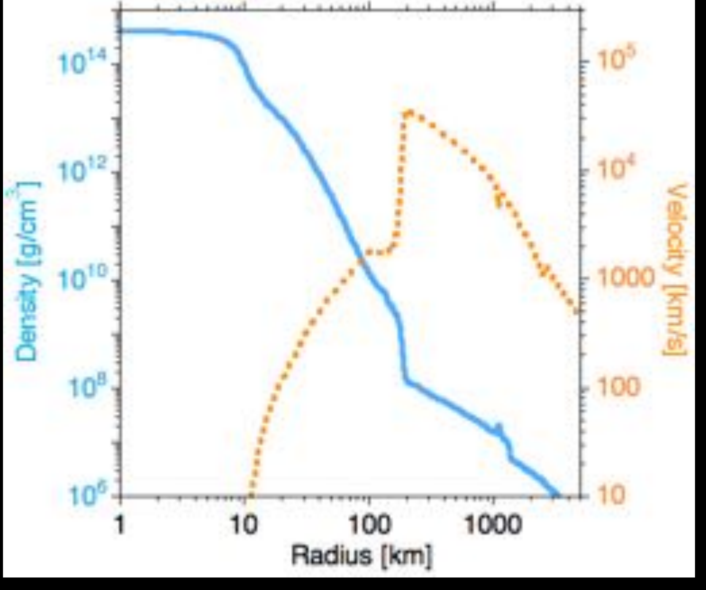
Why should we care?

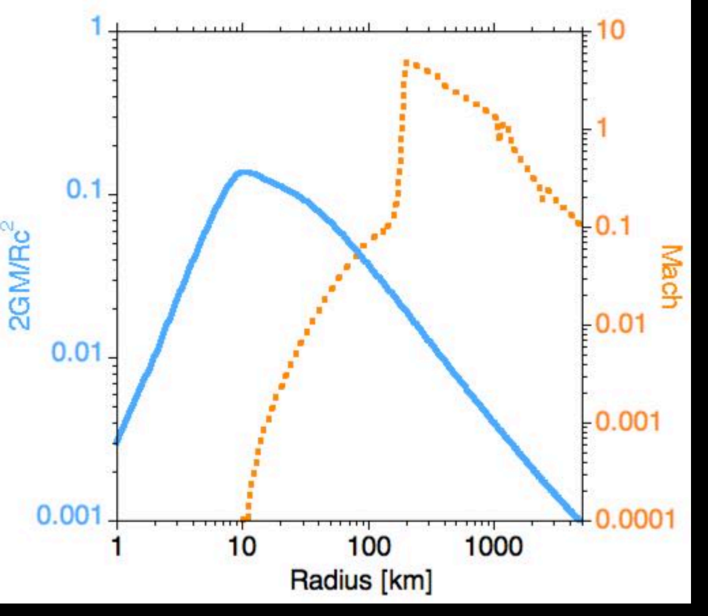
A long standing physical puzzle, still unsolved

massive stars are expected to collapse, but why do they explode ?
do we miss a physical process?

A laboratory for extreme physical conditions

nuclear physics from 10^6 to 10^{15} g/cm^3
special & general relativity and black hole formation
shock dynamics
neutrino interactions
magnetic fields





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A decisive astrophysical process

a milestone in stellar evolution and population synthesis:	mass range, missing RSG, binarity
a signature of stellar structure:	compactness, angular momentum, B, turbulence
the birth of a neutron star or a black hole:	mass/kick/spin/B ?
the dissemination of stellar nucleosynthesis:	which elements? fallback?
a site for explosive nucleosynthesis:	which sites for the r-process?
a tracer of star formation:	which bias? mass loss ?
a source of neutrinos:	direct insight, mass hierarchy, oscillations
a source of gravitational waves:	direct insight, progenitor of NS mergers
a clue to the transient sky:	connection to GRB, hypernovae, SLSN...

also, a site for dust production
the injection of kinetic energy in the ISM
the birth of a remnant=cosmic ray accelerator

Observed kinetic energy: 10^{51}erg

Reference energies:

solar mass annihilation:

neutron star gravitational energy:

kicked neutron star kinetic energy:

spinning neutron star kinetic energy:

O→Fe nuclear binding energy (SNIa):

$$Mc^2 = 1.8\times10^{54}\left(\frac{M}{M_{\rm sol}}\right)\,{\rm erg},$$

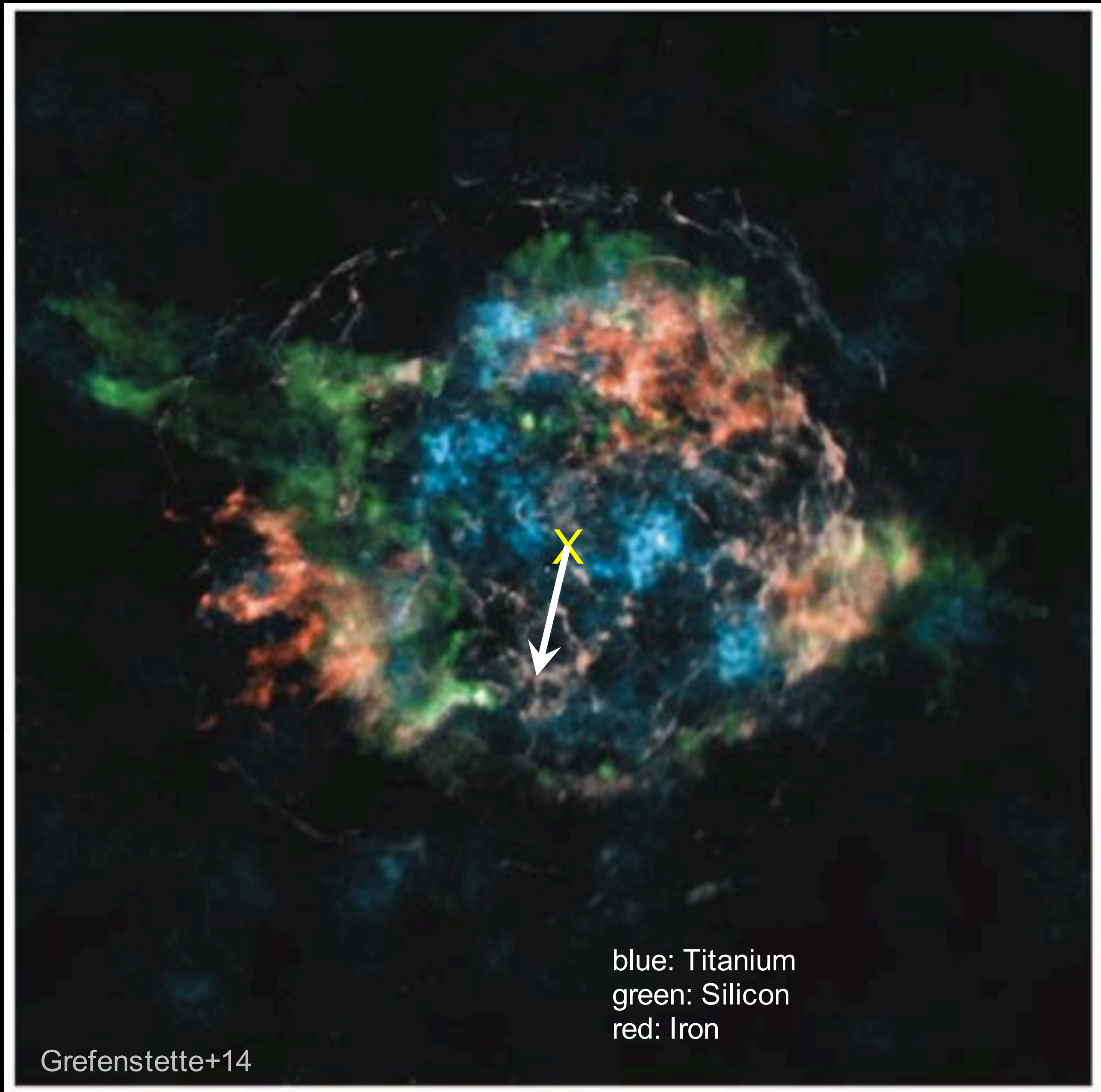
$$\frac{GM^2}{R} = 5.2\times10^{53}\left(\frac{M}{1.4M_{\rm sol}}\right)^2\left(\frac{R}{10{\rm km}}\right)^{-1}\,{\rm erg},$$

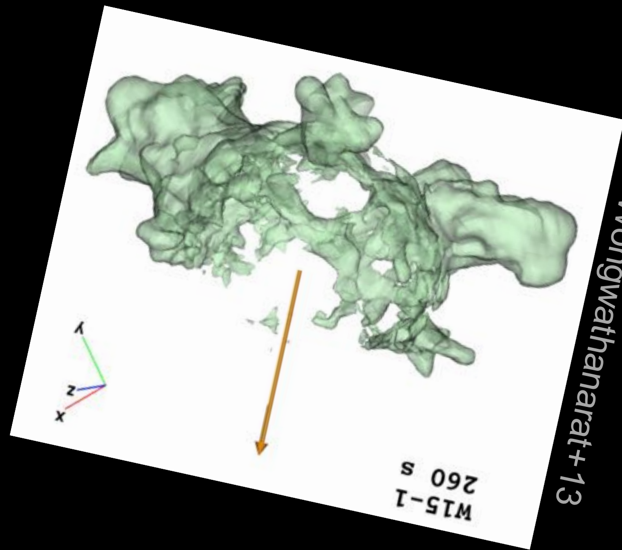
$$\frac{1}{2}Mv^2 = 1.3\times10^{48}\left(\frac{M}{1.4M_{\rm sol}}\right)\left(\frac{v}{300{\rm km/s}}\right)^2\,{\rm erg},$$

$$\frac{1}{2}MR^2\Omega^2 = 6.1\times10^{49}\left(\frac{M}{1.4M_{\rm sol}}\right)\left(\frac{R}{10{\rm km}}\right)^2\left(\frac{T}{30{\rm ms}}\right)^{-2}\,{\rm erg},$$

$$\frac{M}{m_{\rm p}}\times0.8{\rm MeV/nucleon} = 2.1\times10^{51}\left(\frac{M}{1.4M_{\rm sol}}\right)\,{\rm erg}.$$

The encouraging results of 3D modelling





Wongwathanarat+13

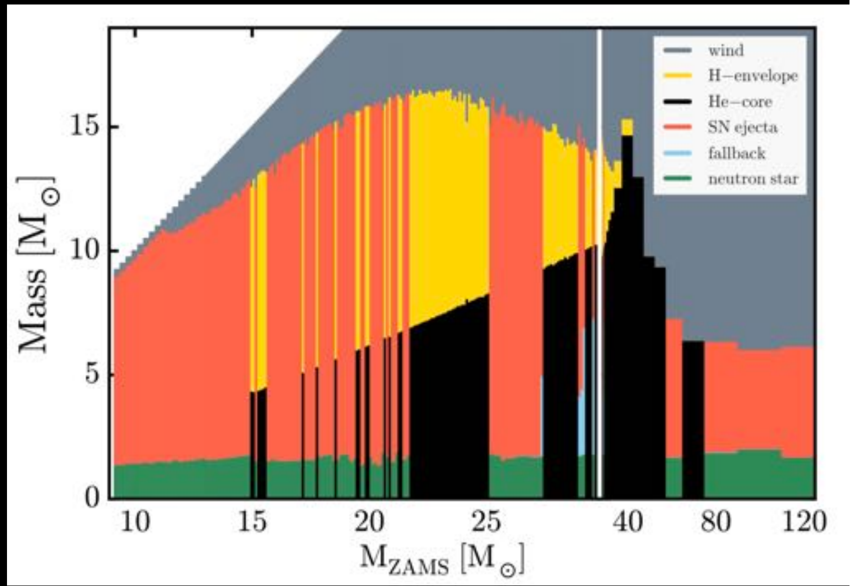
The apparent success of supernova theory

Sukhbold+16

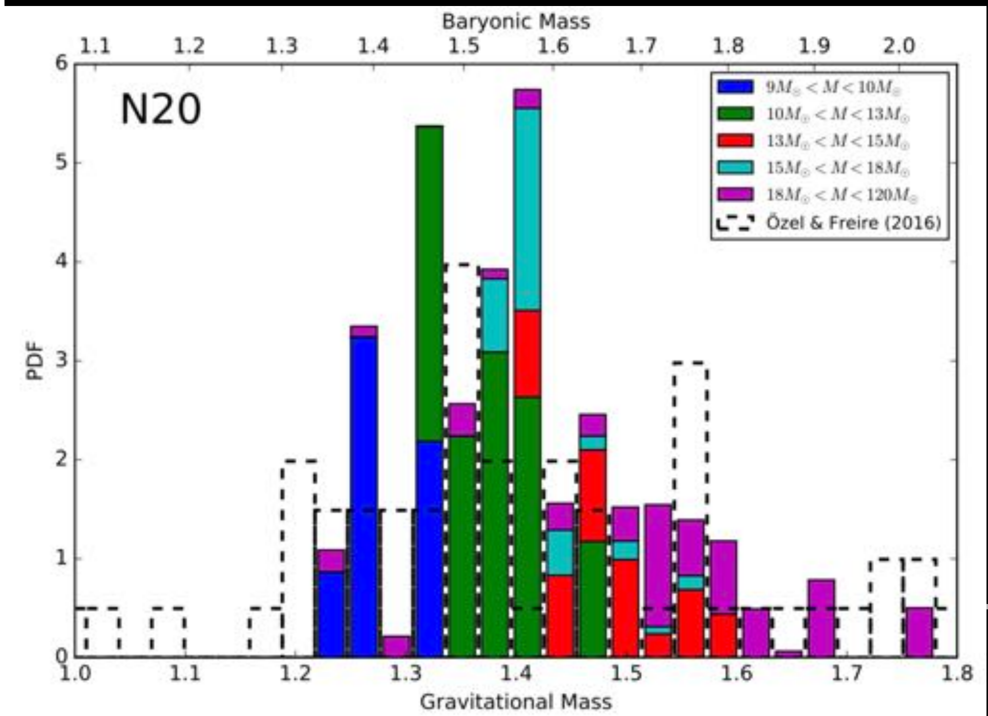
« Islands of explodability in a sea of black hole formation »

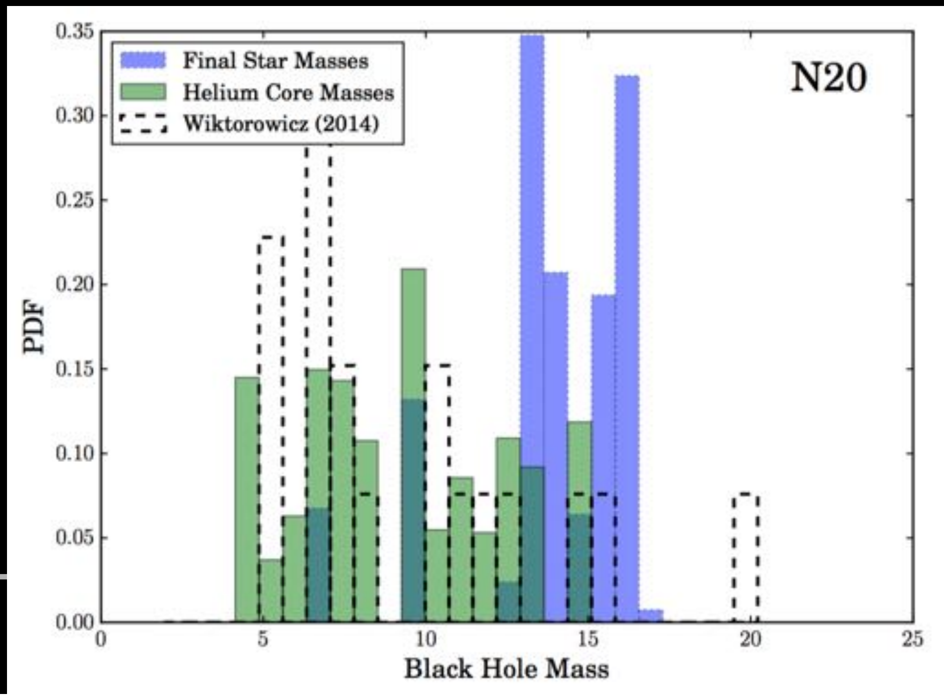
1-D models calibrated with SN1987A (~18M_{sol}) and the Crab (~10M_{sol})

- -single star evolution: binarity is ignored (Sana+12)
- -rotation largely neglected
- -SN1987A was peculiar (Morris & Podsiadlowski 07)
- -the SASI/convective multi-D diversity is ignored



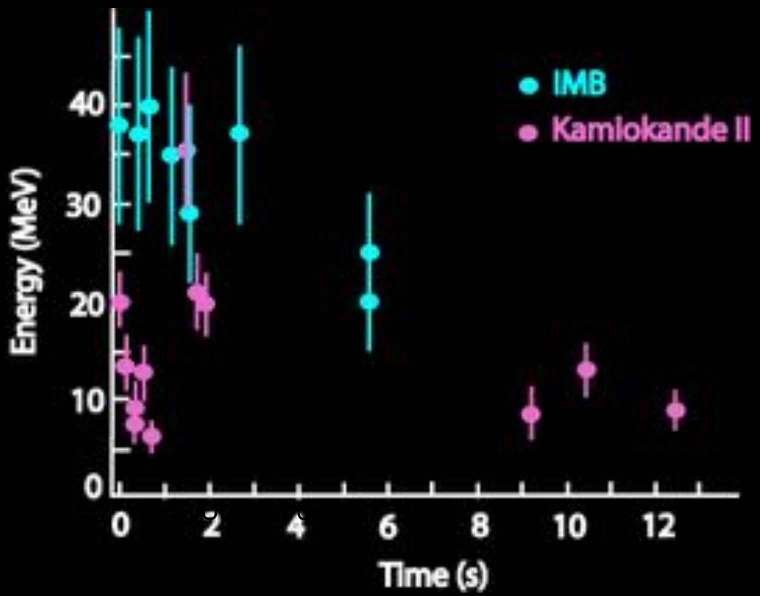
distribution of masses
of neutron stars and black holes

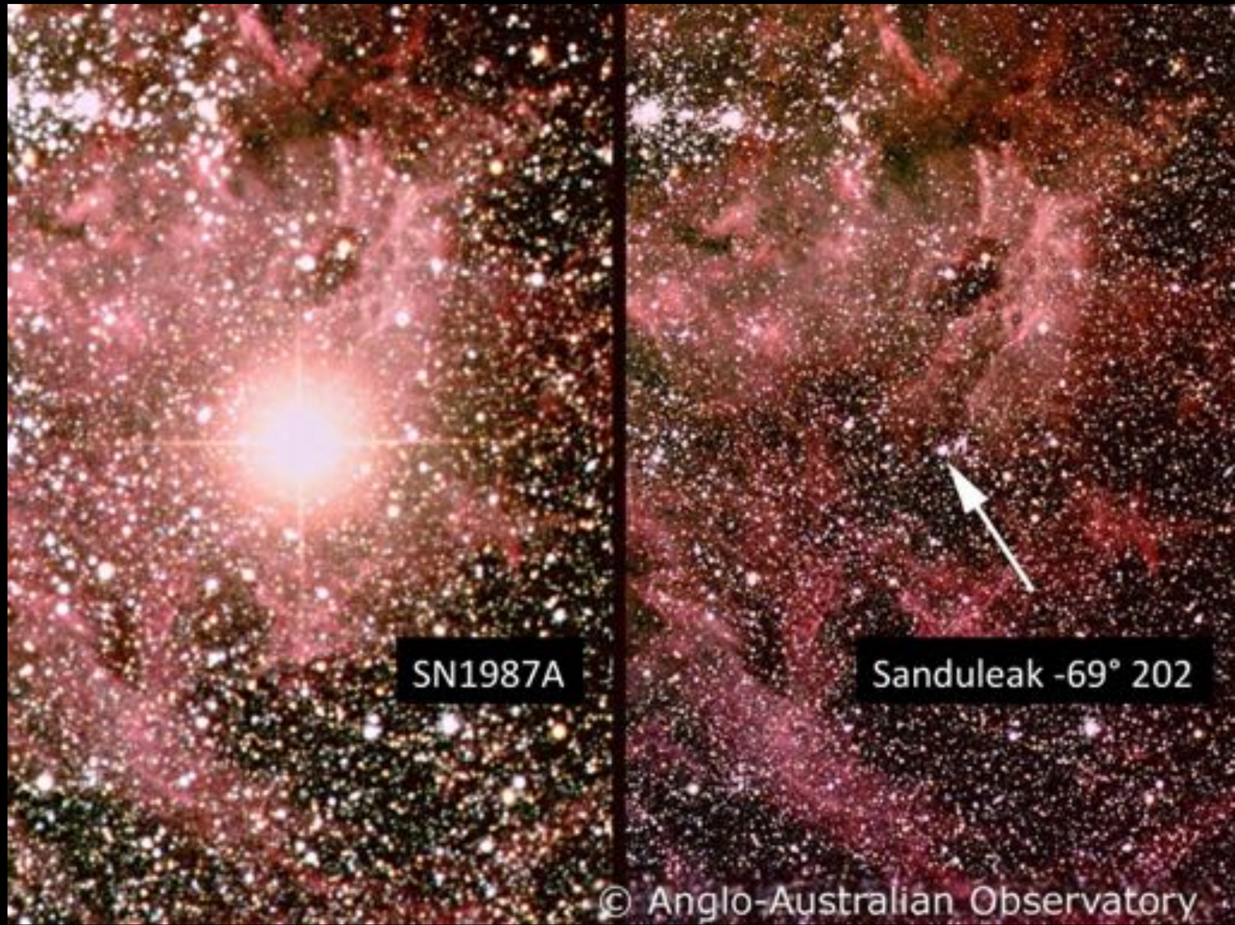




SN1987A

-identification of the massive progenitor
-detection of supernova neutrinos

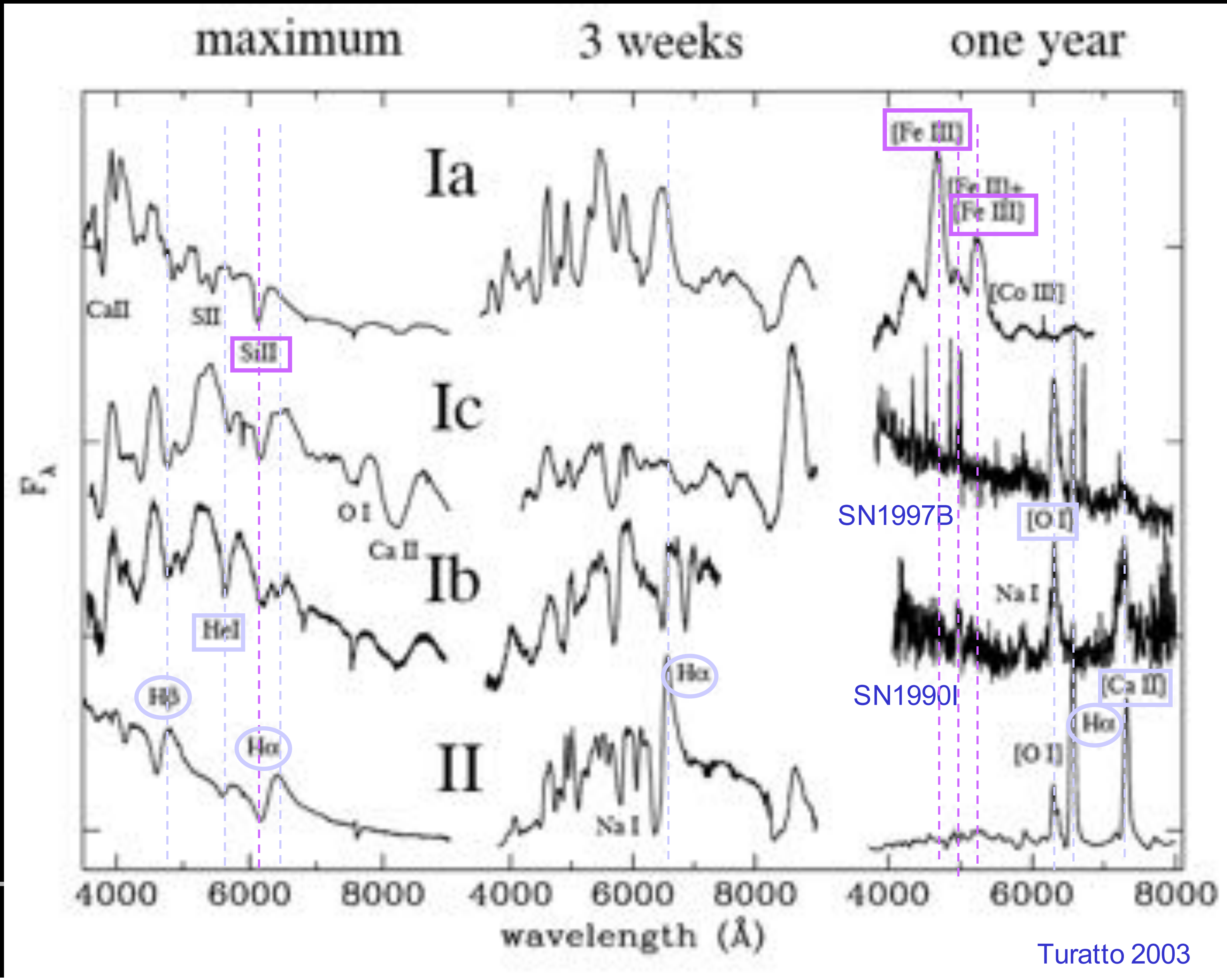




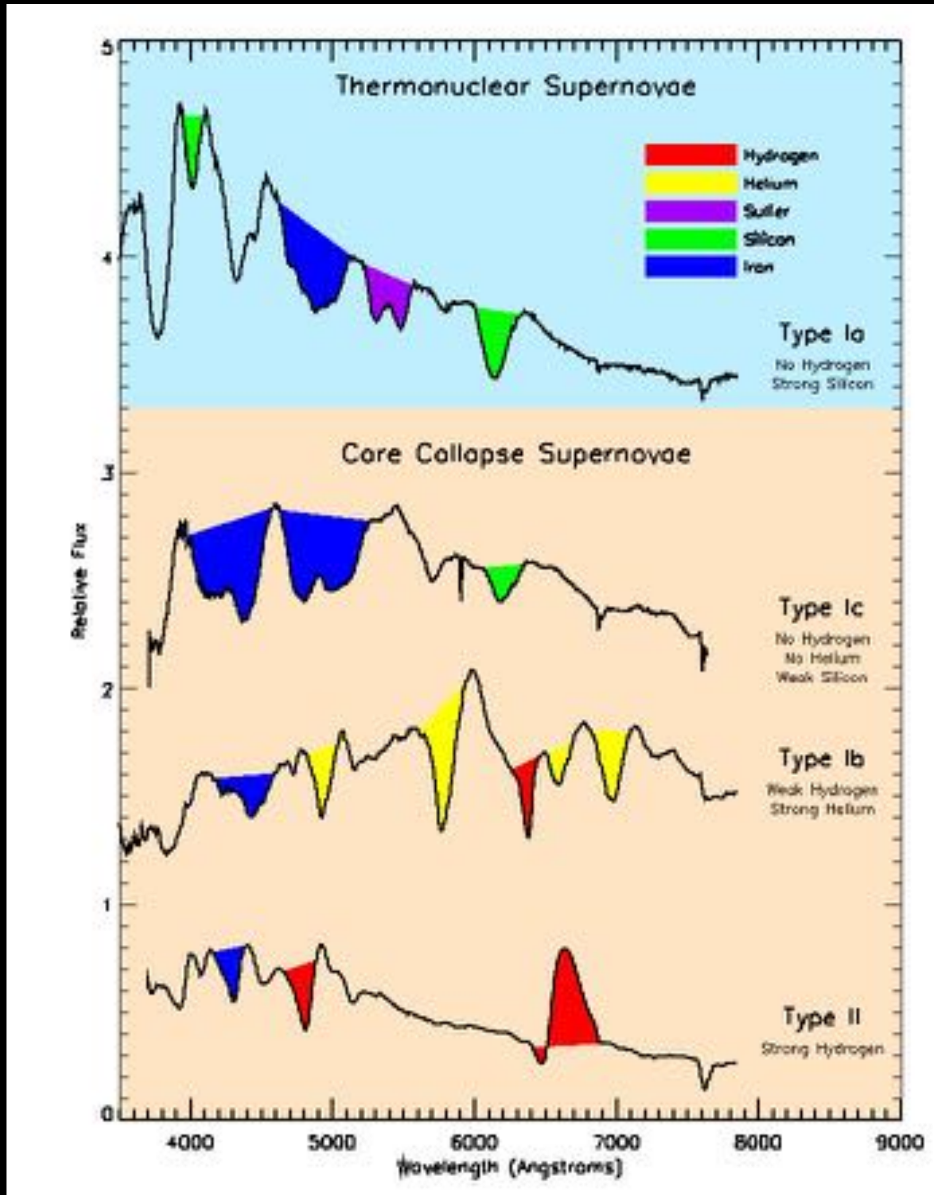
-duration of neutrino detection: 12s
→ a fast process involving dense enough material to trap neutrinos

Spectral classification

Hα 6563: II
Hβ 4861: II
SiII 6355->6150: Ia
HeI 5876: Ic

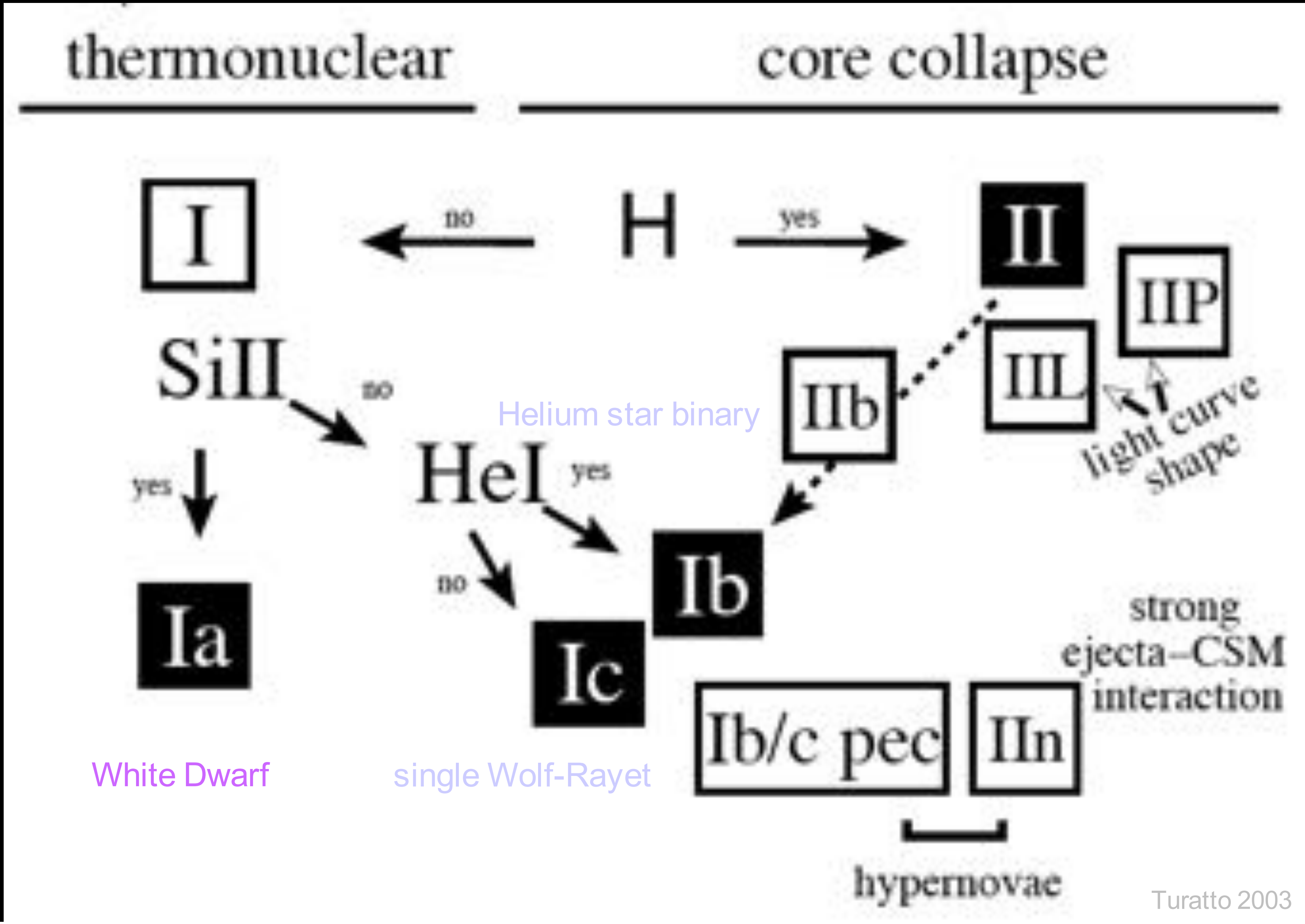


Core-collapse vs Thermonuclear supernovae

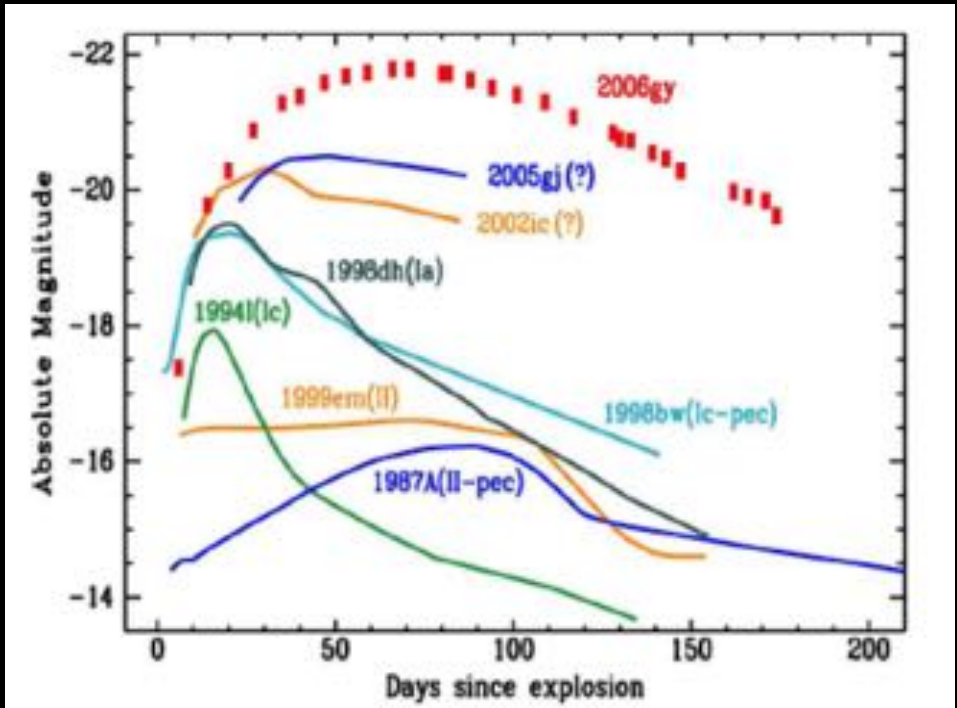


Dan Kasen, http://supernova.lbl.gov/~dnkasen/tutorial/

SN classification

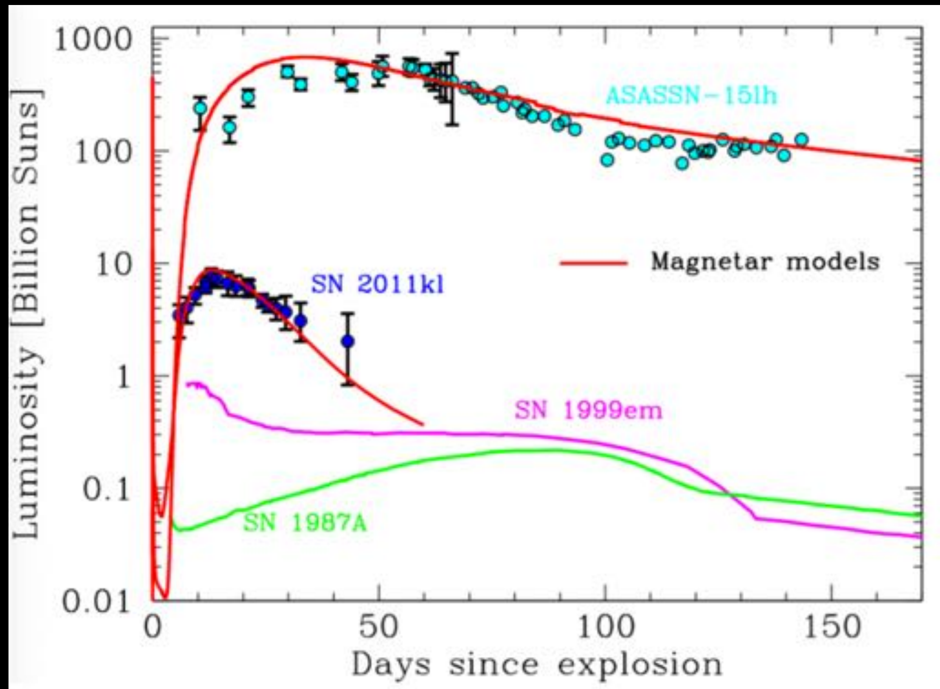


The diversity of light curves



Smith+07

timescales of the light curve:
fast rise (~days),
~100 days plateau
slow decay (~months)



Bernsten+16

-radioactive decay of ^{56}Ni

$^{56}Ni \xrightarrow{6.1d} {}^{56}Co \xrightarrow{78.5d} {}^{56}Fe$

Supernova arithmetic

Typical observations (e.g. Type IIP SN1999em@11.5Mpc, Dessart & Hillier 06)

$L_{\rm sol} \sim 3.8 \times 10^{33}$ erg s^{-1}

$M_{\rm sol} \sim 2 \times 10^{33}$ g $R_{\rm sol} \sim 7 \times 10^{10}$ cm

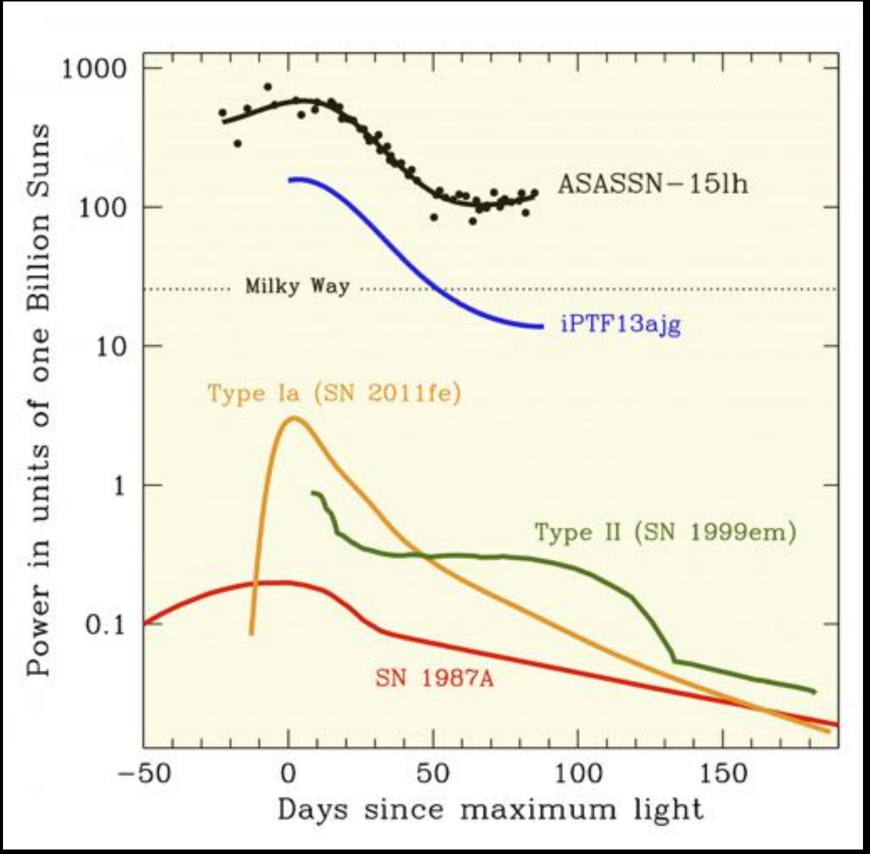
Luminosity 2.5×10^{42}erg/s = 6.6×10^{8} $L_{\rm sol}$ during 120 days (~10^7 s)

$$E_{\rm rad} \sim 2.6 \times 10^{49} {\rm erg}$$

Photospheric temperature ~0.9 $T_{\rm sol}$

$$R_{\rm rad} \sim \left(\frac{6.6 \times 10^8}{0.9^4}\right)^{\frac{1}{2}} R_{\rm sol} \sim 32 \times 10^3\ R_{\rm sol} \sim 2.2 \times 10^{15} {\rm cm}$$

32 times bigger
than the biggest supergiant star



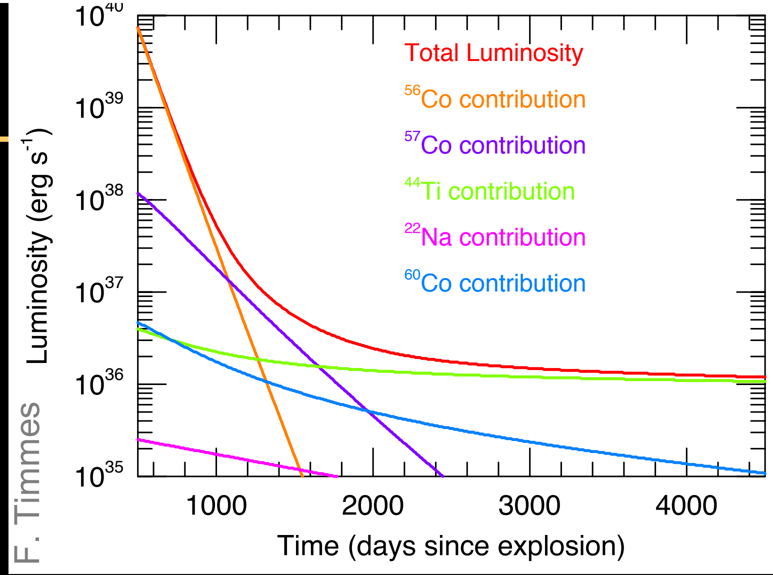
Velocities from Doppler shifts: 3×10^3 km/s

Kinetic energy

$$E_{\rm K} \equiv \frac{1}{2} M v^2 = 9.9 \times 10^{50} \left(\frac{M}{11 M_{\rm sol}}\right) \left(\frac{v}{3 \times 10^3 {\rm km\ s}^{-1}}\right)^2 {\rm erg}$$

Advanced calculation of non LTE radiation transfer: see Dessart & Hillier 11, Dessart+13

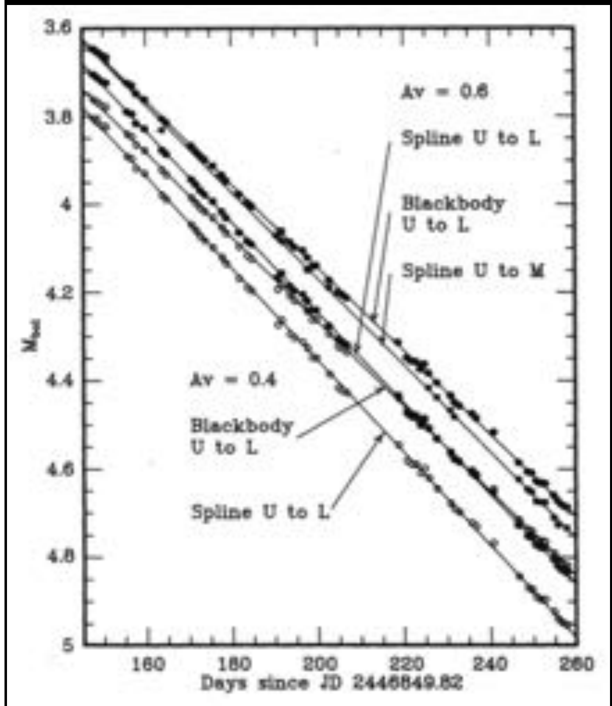
Radioactive decay of ^{56}Ni, ^{56}Co, ^{44}Ti



The early light curve is powered by the shock and the H recombination.
The mixing of ^{56}Ni powers the light curve after some weeks
The late light curve is powered by the radioactive decay of ^{56}Co (>150 days).

The decay of ^{44}Ti can be directly observed in γ-rays after >20 years

Catchpole+88



-radioactive decay of ^{56}Ni (Nadyozhin 94)

^{56}Ni → ^{56}Co → ^{56}Fe

$\tau_{1/2}$=6.1d
τ_{Ni}=8.8d

$\tau_{1/2}$=77.1d
τ_{Co}=111.3d = $\tau_{1/2}$ / log(2)

γ-rays:
E_{Ni}~1.75MeV/Ni

E_{Co}~3.6MeV/Co

Ni decay power (<10 days):

$$L_{\rm Ni} \sim \frac{M_{\rm Ni}}{56 m_{\rm p}} \frac{E_{\rm Ni}}{\tau_{\rm Ni}} \exp\left(-\frac{t}{\tau_{\rm Ni}}\right), \sim 5.5\times 10^{42}\left(\frac{M_{\rm Ni}}{0.07 M_{\rm sol}}\right)\exp\left(-\frac{t}{\tau_{\rm Ni}}\right) {\rm erg\ s^{-1}}.$$

Co decay power (>100 days):

$$L_{\rm Co} \sim \frac{M_{\rm Ni}}{56 m_{\rm p}} \frac{E_{\rm Co}}{\tau_{\rm Co}-\tau_{\rm Ni}} \exp\left(-\frac{t}{\tau_{\rm Co}}\right), \sim 9.7\times 10^{41}\left(\frac{M_{\rm Ni}}{0.07 M_{\rm sol}}\right)\exp\left(-\frac{t}{\tau_{\rm Co}}\right) {\rm erg\ s^{-1}}.$$

-radioactive decay of ^{44}Ti → ^{44}Sc → ^{44}Ca (Ahmad+06)

τ_{Ti}=85yr, E_{Ti}~1.157MeV/Ti

$$L_{\rm Ti} \sim \frac{M_{\rm Ti}}{44 m_{\rm p}} \frac{E_{\rm Ti}}{\tau_{\rm Ti}} \exp\left(-\frac{t}{\tau_{\rm Ti}}\right), \sim 5.6\times 10^{36}\left(\frac{M_{\rm Ti}}{3\times 10^{-4} M_{\rm sol}}\right)\exp\left(-\frac{t}{\tau_{\rm Ti}}\right) {\rm erg\ s^{-1}}.$$

observed in SN1987A by Integral (Grebenev+12)
(3.1+-0.8)x10^{-4} M_{sol} of ^{44}Ti

The transient universe

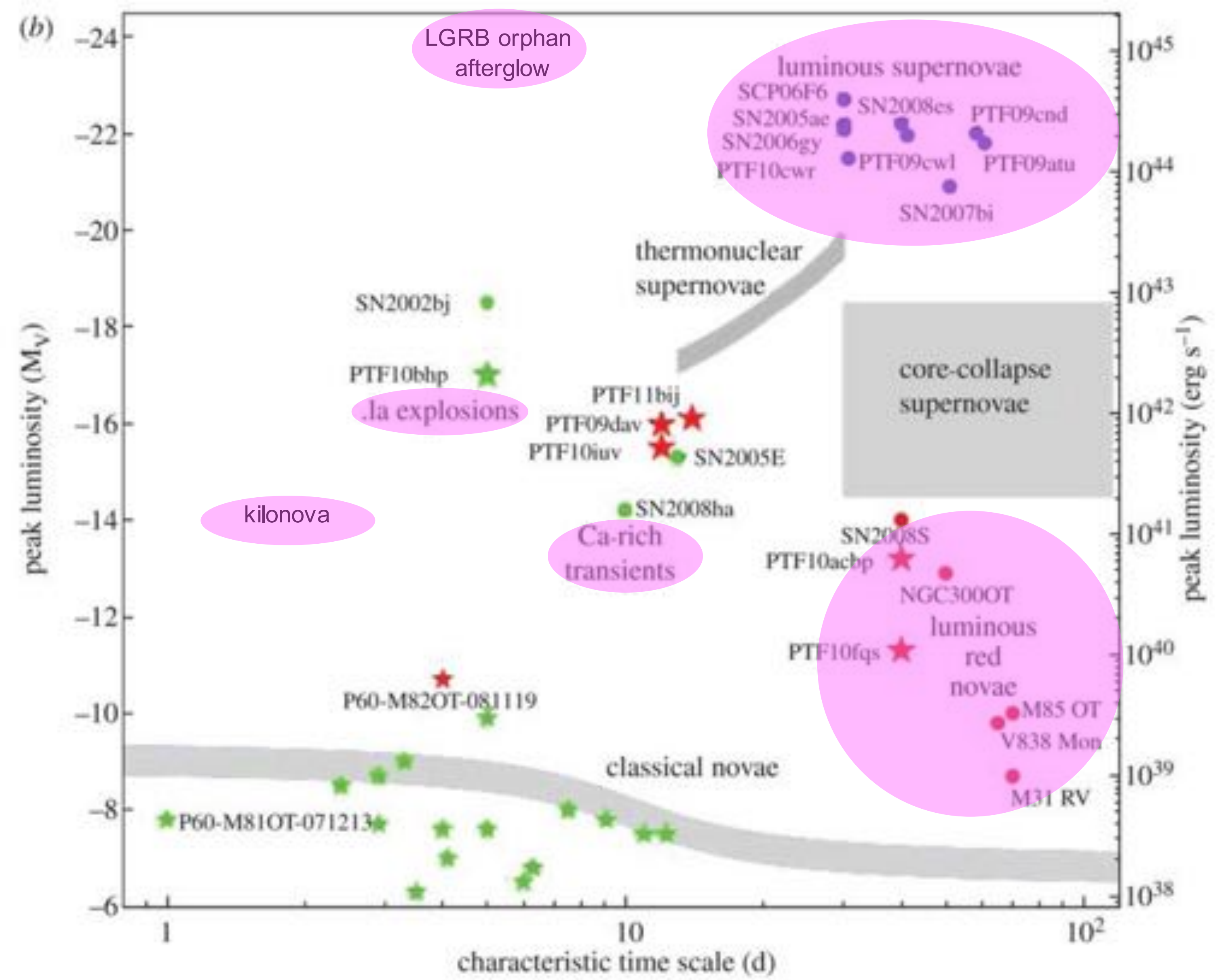
Expanding zoo:

supernovae
SNIa, **SNIax**, **SN.Ia**
SNIb, SNIc, SNIIP, SNIIL, SNIIL

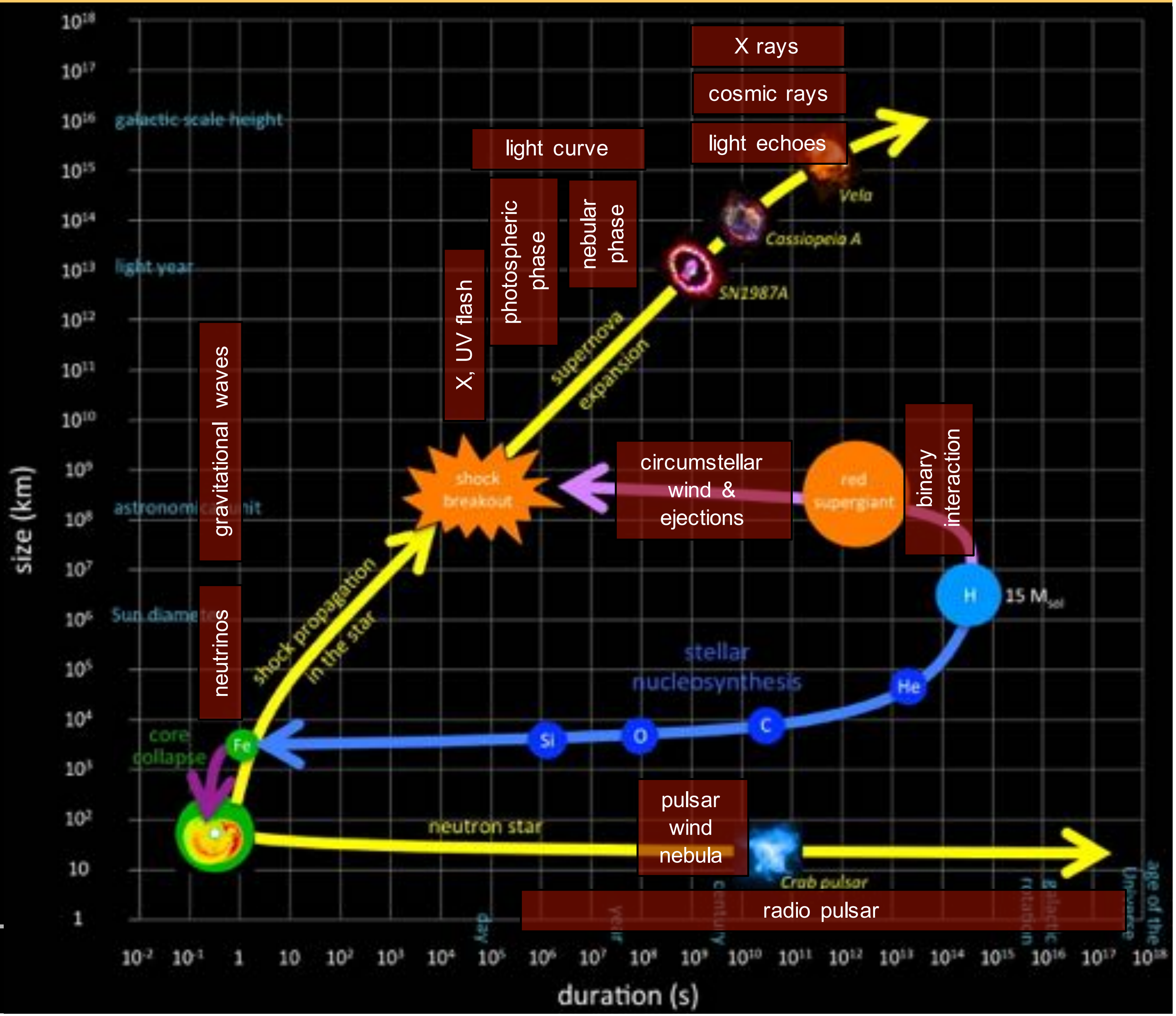
superluminous supernova
hypernova
kilonova
short/long GRB
orphan afterglow
Ca-rich transients
Fast Radio Burst
luminous red nova
(X ray burst, recurrent nova)

Diagnostics multi-λ:

light curve
spectrum
nucleosynthesis
neutrinos
gravitational waves
cosmic rays



What can be observed of a supernova?



Which fields of physics?

Stellar structure and evolution:	Newtonian gravity, 3D radiative hydrodynamics, nuclear statistical equilibrium, turbulence, dynamo, binary interactions.
Collapse of the iron core:	Newtonian gravity, quantum mechanics, special relativity, 3D hydrodynamics.
Formation of a proto-neutron star:	general relativity, nuclear equation of state, electron capture, hyperons, neutrino interactions
Stalled accretion shock:	3D radiative (magneto) hydrodynamics, neutrino interactions.
Neutrino driven wind, nucleosynthesis:	nuclear cross sections, 3D radiative hydrodynamics

The panorama of scenarios

Neutrino driven explosion	but the explosion energy seems weakish improved neutrino transport in 3D? improved 3D progenitor structure?
Fast rotation	but most of the massive stars are slow rotators. ok for a minority
Strong magnetic field	but most of the stellar cores are weakly magnetized. ok for a minority
Quark matter transition	but experimental support is missing. ad hoc ?
Jittering jet	how would the jet be efficiently formed?

Remarks on Super-Novae and Cosmic Rays

We have recently called attention to a remarkable type of giant novae.[1] As the subject of super-novae is probably very unfamiliar we give here a few more details which are not contained in our original articles.

1. Distribution of super-novae

In our calculations we made use of the assumption that on the average one super-nova appears in each galaxy every thousand years. This estimate is based on the occurrence of super-novae in the following galaxies,

Our own galaxy	in 1572
Andromeda	1885
Messier 101	1907

These three systems are located within a sphere of radius 12×10^6 light years.

In the Virgo cluster, which contains about 500 nebulae, six super-novae were found on plates taken during the last thirty years. As a curiosity we mention that in N.G.C. 4321, which is a member of Virgo, two super-novae have appeared in 1901 and 1914, respectively.

In the same interval of 30 years six additional super-novae were found in isolated nebulae.

We wish to emphasize that all of these finds are chance finds since a systematic search for super-novae has been organized only recently.

From the estimate of one super-nova per galaxy per thousand years it follows that 10^7 super-novae appear per year in the 10^{10} nebulae which are contained in a sphere of 2×10^9 years radius (critical distance derived from the red shift of nebulae). If cosmic rays come from super-novae their intensity in points far away from any individual super-nova will be essentially independent of time.

2. Comparison with the lifetime of stars

The lifetime of stars is supposed to be of the order of at least 10^{12} years. A nebula contains about 10^9 stars. These estimates, combined with the frequency of occurrence of one super-nova per galaxy per 10^3 years suggest that the super-nova process might occur to every star once in its lifetime, marking perhaps the cessation of its existence as an ordinary star. We realize that this suggestion is highly speculative in view of the possibility that the frequency of occurrence of super-novae may depend on time and in view

[1] W. Baade and F. Zwicky, Proc. Nat. Acad. Sci. May, 1934.

1934

of our complete ignorance with respect to the evolution of the universe.

3. Ions in super-novae

If super-novae are giant analogues to ordinary novae we may expect that ionized gas shells are expelled from them at great speeds. If this assumption is correct, part of the cosmic rays should consist of protons and heavier ions. Direct tests by cloud chamber experiments at high altitudes are desirable in order to test this conclusion. Also the problem suggests itself to investigate how much energy corpuscular particles lose on their long journey through space. On the picture of an expanding universe this loss has been computed by R. C. Tolman.

4. Fluctuations of cosmic rays

In our original papers we have calculated the change in intensity of cosmic rays caused by flare-ups of super-novae in nearby galaxies. The estimates given are perhaps too optimistic in view of the fact that the velocities of different particles are different. If various particles are ejected simultaneously at the time $t=0$ from a galaxy which is 10^6 L.Y. away the times t of arrival on the earth are

$t=10^6$ years for light if its velocity does not depend on the frequency.

$t_1=10^6$ years $+410$ seconds for 10^{11} volt electrons.

$t_2=$ " $+47.6$ days " 10^9 " "

$t_3=$ " $+44$ years " 10^{11} " protons.

These time lags t_i-t would tend to smear out the change of intensity caused by the flare-up of individual super-novae. Dr. R. M. Langer in one of our seminars was the first to call attention to the straggling of simultaneously ejected particles.

5. The super-nova process

We have tentatively suggested that the super-nova process represents the transition of an ordinary star into a neutron star. If neutrons are produced on the surface of an ordinary star they will "rain" down towards the center if we assume that the light pressure on neutrons is practically zero. This view explains the speed of the star's transformation into a neutron star. We are fully aware that our suggestion carries with it grave implications regarding the ordinary views about the constitution of stars and therefore will require further careful studies.

W. BAADE
F. ZWICKY

Mt. Wilson Observatory and
California Institute of Technology, Pasadena.
May 28, 1934.

1941

APRIL 1, 1941 PHYSICAL REVIEW VOLUME 59

Neutrino Theory of Stellar Collapse

G. Gamow, *George Washington University, Washington, D. C.*
M. Schoenberg,* *University of São Paulo, São Paulo, Brazil*
(Received February 6, 1941)

At the very high temperatures and densities which must exist in the interior of contracting stars during the later stages of their evolution, one must expect a special type of nuclear processes accompanied by *the emission of a large number of neutrinos*. These neutrinos penetrating almost without difficulty the body of the star, must carry away very large amounts of energy and prevent the central temperature from rising above a certain limit. This must cause *a rapid contraction of the stellar body* ultimately resulting in a *catastrophic collapse*. It is shown that energy losses through the neutrinos produced in reactions between free electrons and oxygen nuclei can cause a complete collapse of the star within the time period of half an hour. Although the main energy losses in such collapses are due to neutrino emission which escapes direct observation, the heating of the body of a collapsing star must necessarily lead to the *rapid expansion of the outer layers* and the *tremendous increase of luminosity*. It is suggested that stellar collapses of this kind are responsible for the phenomena of *novae* and *supernovae*, the difference between the two being probably due to the difference of their masses.

1965

THE HYDRODYNAMIC BEHAVIOR OF SUPERNOVAE EXPLOSIONS*

Stirling A. Colgate and Richard H. White
Lawrence Radiation Laboratory, University of California, Livermore, California
Received June 29, 1965

ABSTRACT

We regard the release of gravitational energy attending a dynamic change in configuration to be the primary energy source in supernovae explosions. Although we were initially inspired by and agree in detail with the mechanism for initiating gravitational instability proposed by Burbidge, Burbidge, Fowler, and Hoyle, we find that the dynamical implosion is so violent that an energy many times greater than the available thermonuclear energy is released from the star's core and transferred to the star's mantle in a supernova explosion. The energy released corresponds to the change in gravitational potential of the unstable imploding core; the transfer of energy takes place by the emission and deposition of neutrinos.

Why should the stellar core collapse?

Chandrasekhar mass

Crash course

Special relativity:

the velocity of electrons approaches the speed of light c

Lorentz factor $\Gamma >> 1$

$$\Gamma \equiv \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

the rest mass of electrons $m_e c^2$ is negligible compared to their kinetic energy $(\Gamma - 1) m_e c^2$

the momentum $p = \Gamma m v$ of relativistic electrons is approximately $\Gamma m c$

Quantum mechanics:

Planck constant

the Heisenberg relation $p_e \Delta x_e \sim \hbar$

the quantification of angular momentum determining the Bohr radius

the quantification of the photon energy in the photoelectric effect $E = h\nu$

the UV catastrophe in the black body spectrum

the Pauli principle: fermions cannot have the same momentum and position

Newtonian gravity:

classical gravitational force GM/r^2

Stellar nucleosynthesis in a 15 M_{sol} star

Woosley & Janka 06

Fermi momentum of electrons $p_\mathrm{F} \sim \hbar \left(\frac{\rho Y_\mathrm{e}}{m_\mathrm{p}}\right)^{\frac{1}{3}}$

Fermi energy

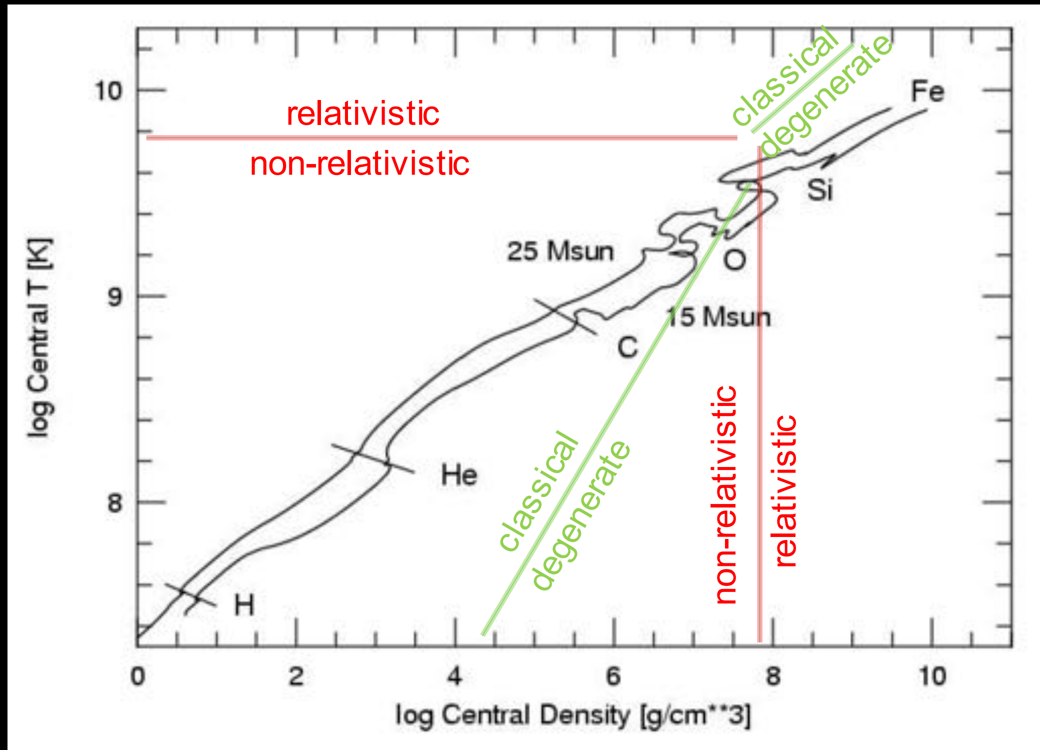
$E_F \sim p_F^2/2m_e$ (non relativistic)

$E_F \sim p_F c$ (relativistic)

electrons are degenerate if $E_F > kT$:

non relativistic: $$T < \frac{\hbar^2}{2km_\mathrm{e}}\left(\frac{\rho Y_\mathrm{e}}{m_\mathrm{p}}\right)^{\frac{2}{3}} = 3.1\times 10^8 \left(\frac{Y_\mathrm{e}\rho}{10^6\mathrm{g\ cm}^{-3}}\right)^{\frac{2}{3}}\mathrm{K}$$

relativistic: $$T < \frac{\hbar c}{k}\left(\frac{\rho Y_\mathrm{e}}{m_\mathrm{p}}\right)^{\frac{1}{3}} = 1.9\times 10^{10} \left(\frac{Y_\mathrm{e}\rho}{10^9\mathrm{g\ cm}^{-3}}\right)^{\frac{1}{3}}\mathrm{K}$$



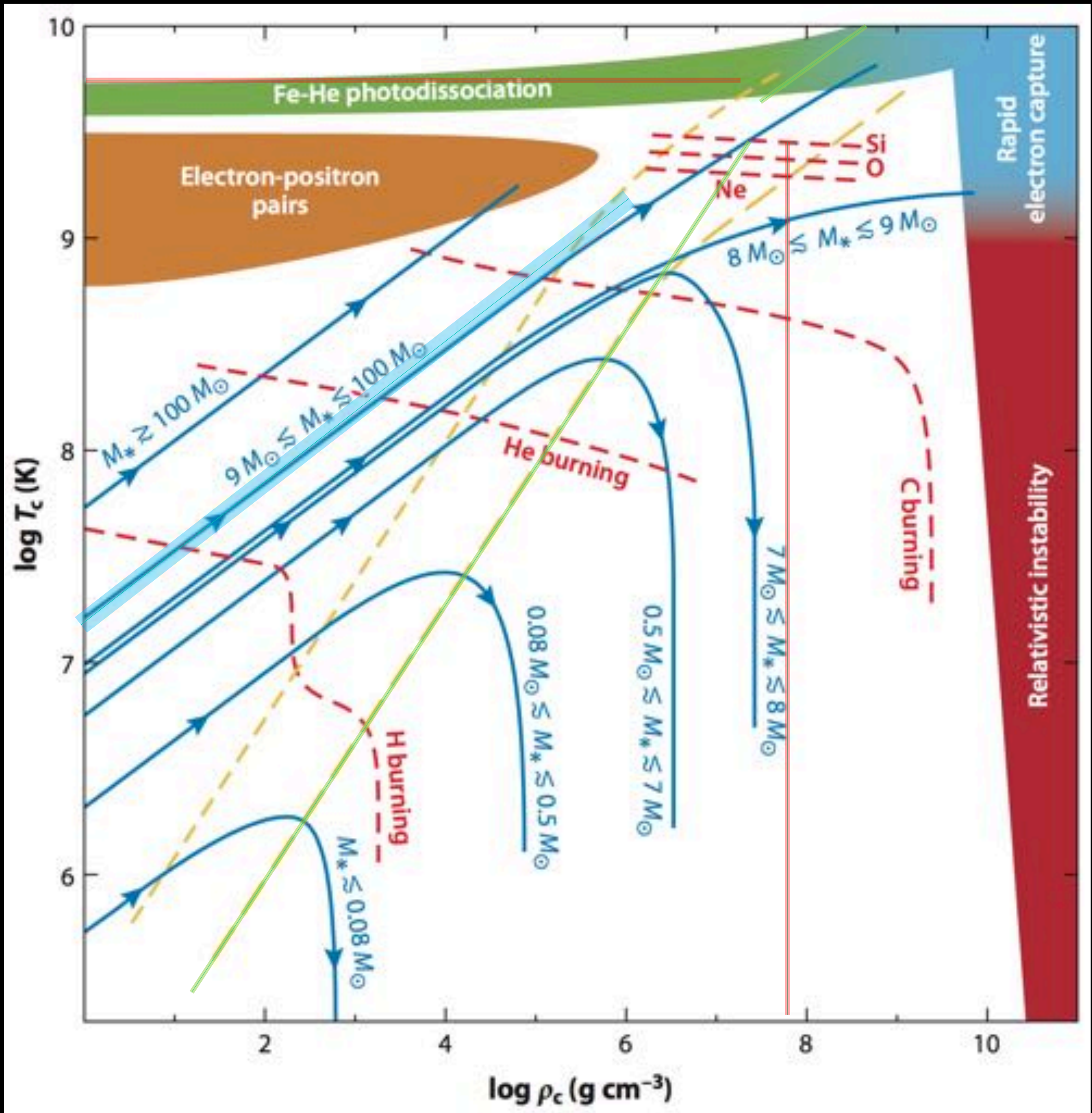
Stage	Time Scale	Fuel or Product	Ash or product	Temperature (10^9 K)	Density (gm/cm^3)	Luminosity (solar units)	Neutrino Losses (solar units)
Hydrogen	11 My	H	He	0.035	5.8	28,000	1800
Helium	2.0 My	He	C,O	0.18	1390	44,000	1900
Carbon	2000 y	C	Ne,Mg	0.81	2.8 x 10^5	72,000	3.7 x 10^5
Neon	0.7 y	Ne	O,Mg	1.6	1.2 x 10^7	75,000	1.4 x 10^8
Oxygen	2.6 y	O,Mg	Si,S,Ar, Ca	1.9	8.8 x 10^6	75,000	9.1 x 10^8
Silicon	18 d	Si,S,Ar, Ca	Fe,Ni, Cr,Ti,...	3.3	4.8 x 10^7	75,000	1.3 x 10^{11}
Iron core collapse[a]	~1 s	Fe,Ni, Cr, Ti,...	Neutron Star	> 7.1	>7.3 x 10^9	75,000	>3.6 x 10^{15}

$$\frac{p_e}{m_e c} \sim \frac{\hbar}{m_e c}\left(\frac{Y_e \rho}{m_p}\right)^{\frac{1}{3}} \sim 4.8\left(\frac{\rho}{7\times 10^9\,\mathrm{g/cm^3}}\right)^{\frac{1}{3}}$$

the thermal motion of electrons is non relativistic if

$$T < \frac{m_\mathrm{e}c^2}{k} = 5.9\times 10^9\mathrm{K}$$

The final stages of stellar evolution



Fe-He photodissociation:
nuclear binding energy E_{nuc}

$E_{nuc}(^4He)$~7MeV/nucleon

$E_{nuc}(^{56}Fe)$~8.8MeV/nucleon

→ΔE_{nuc}=1.8MeV/nucleon ~ $3.5 m_e c^2$

hydrostatic equilibrium

$kT_c \sim GM_c/R_c$

$R_c \sim (M_c/\rho_c)^{1/3}$

$$T_c \propto \rho_c^{\frac{1}{3}} M_c^{\frac{2}{3}}$$

cooling of degenerate matter (white dwarfs) at constant density

Why should the stellar iron core collapse?

Chandrasekhar 1930,
Landau 1932

As the mass M of the degenerate iron core increases,
the density ρ increases,
the electronic interspacing Δx_e decreases.

$$\Delta x_e \sim \left(\frac{m_\mathrm{p}}{\rho Y_\mathrm{e}}\right)^{\frac{1}{3}}$$

Each nucleus of ^{56}Fe contains 26 protons and 30 neutrons
The electron fraction is $Y_e=26/56\sim0.46$

The momentum p_e of electrons is deduced from the Heisenberg relation $p_\mathrm{e}\Delta x_\mathrm{e} \sim \hbar$

Electrons are relativisitic for stellar densities ρ exceeding 6×10^7 g/cm^3.

$$\frac{p_\mathrm{e}}{m_\mathrm{e}c} \sim \frac{\hbar}{m_\mathrm{e}c}\left(\frac{Y_\mathrm{e}\rho}{m_\mathrm{p}}\right)^{\frac{1}{3}} \sim 4.8\left(\frac{\rho}{7\times10^9\,\mathrm{g/cm^3}}\right)^{\frac{1}{3}}$$

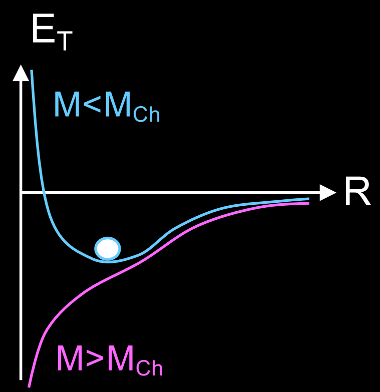
A spherical stellar core of mass M and radius R contains $N=Y_eM/m_p$ electrons
the mean density is $\rho=M/(4\pi R^3/3)$
The total energy E_T is approximated as the sum of the potential energy of the nuclei $E_p\sim-GM^2/R$
and the kinetic energy E_k of the electrons

$E_k\sim Np_e^2/2m_e$ for non-relativistic electrons:

$$E_\mathrm{T} \sim -\frac{GM^2}{R} + \left(\frac{Y_\mathrm{e}M}{m_\mathrm{p}}\right)^{\frac{5}{3}}\frac{\hbar^2}{m_\mathrm{e}R^2}$$

$E_k\sim Np_ec$ for relativistic electrons:

$$E_\mathrm{T} \sim -\frac{GM^2}{R} + \frac{\hbar c}{R}\left(\frac{Y_\mathrm{e}M}{m_\mathrm{p}}\right)^{\frac{4}{3}} = \frac{\hbar c}{R}\left(\frac{Y_\mathrm{e}M}{m_\mathrm{p}}\right)^{\frac{4}{3}}\left[1-\left(\frac{M}{M_\mathrm{Ch}}\right)^{\frac{2}{3}}\right]$$



The density is not uniform: inner regions are denser
The non relativistic energy E_T is dominated by $E_p<0$ at large radius, it increases with radius.
In the relativistic inner region, E_T decreases with radius only if $M<M_{Ch}$

$$M_\mathrm{Ch} \propto \left(\frac{Y_\mathrm{e}}{m_\mathrm{p}}\right)^2\left(\frac{\hbar c}{G}\right)^{\frac{3}{2}}$$

The exact calculation yields

$$M_\mathrm{Ch} \sim 3.0\left(\frac{Y_\mathrm{e}}{m_\mathrm{p}}\right)^2\left(\frac{\hbar c}{G}\right)^{\frac{3}{2}} \sim 1.4M_\mathrm{sol}\left(\frac{Y_\mathrm{e}}{0.5}\right)^2$$

Another hydrostatic approach

$$\Delta x_e \sim \left(\frac{m_\mathrm{p}}{\rho Y_\mathrm{e}}\right)^{\frac{1}{3}}$$

$$p_\mathrm{e}\Delta x_\mathrm{e} \sim \hbar$$

$$\frac{p_\mathrm{e}}{m_\mathrm{e}c} \sim \frac{\hbar}{m_\mathrm{e}c}\left(\frac{Y_\mathrm{e}\rho}{m_\mathrm{p}}\right)^{\frac{1}{3}} \sim 4.8\left(\frac{\rho}{7\times 10^9\,\mathrm{g/cm^3}}\right)^{\frac{1}{3}}$$

Gravity in the iron core is balanced by the pressure P_{deg} of degenerate relativistic electrons: $P_\mathrm{deg} \sim \frac{p_\mathrm{e}c}{3\Delta x_\mathrm{e}^3}$

The pressure in the stellar core dominated by degenerate relativistic electrons is thus described as a gas with an adiabatic index γ=4/3

$$\frac{P_\mathrm{deg}}{\rho^{\frac{4}{3}}} \sim \left(\frac{Y_\mathrm{e}}{m_\mathrm{p}}\right)^{\frac{4}{3}}\hbar c$$

The equilibrium of the pressure force against Newtonian gravity $\nabla P_\mathrm{deg} = -\frac{\rho G M(r)}{r^2}$

can be rewritten dimensionally $\frac{P_\mathrm{deg}}{R} \sim \frac{\rho G M}{R^2}$

using $R \sim \left(\frac{M}{4\rho}\right)^{\frac{1}{3}}$ $\quad \frac{P_\mathrm{deg}}{\rho^{\frac{4}{3}}} \sim GM^{\frac{2}{3}}$

$$M_\mathrm{Ch} \propto \left(\frac{Y_\mathrm{e}}{m_\mathrm{p}}\right)^2\left(\frac{\hbar c}{G}\right)^{\frac{3}{2}}$$

If $M<M_{Ch}$, the dominant degeneracy pressure expands the star until the density decreases to the non relativistic regime where an equilibrium is found.

If $M>M_{Ch}$, the dominant gravitational force increases the density, thus further increasing the relativistic character of the electrons, without ever reaching an equilibrium.

Why should the stellar core collapse?

-The radius of a degenerate core is a decreasing function of its mass.
For non relativistic electrons,

$$R_{\rm WD} \propto Y_{\rm e} R_0 \left(\frac{M}{M_{\rm Ch}}\right)^{-\frac{1}{3}} \sim 2\times 10^3 {\rm km} \left(\frac{M}{M_{\rm Ch}}\right)^{-\frac{1}{3}}$$

where

$$R_0 \equiv \left(\frac{1}{m_{\rm p} m_{\rm e}}\right)\left(\frac{\hbar c}{G}\right)^{\frac{3}{2}} \frac{G}{c^2} = \frac{1}{2}\left(\frac{m_{\rm p}}{m_{\rm e}}\right)\left(\frac{\hbar c}{G m_{\rm p}^2}\right)^{\frac{3}{2}} \frac{m_{\rm p}}{M_{\rm sol}} \frac{2GM_{\rm sol}}{c^2} = 4.8\times 10^3 {\rm km}$$

as the mass approach the Chandrasekhar limit, the radius shrinks due to relativistic effects

$$R_{\rm WD} \sim 3.2\, Y_{\rm e} R_0 \left(\frac{M}{M_{\rm Ch}}\right)^{-\frac{1}{3}} \left[1 - \left(\frac{M}{M_{\rm Ch}}\right)^{\frac{4}{3}}\right]^{\frac{1}{2}}$$

-The Chandrasekhar mass M_{Ch}~$1.4M_{sol}$ is a stellar mass defined from universal constants associated to

- -quantum mechanics,
- -Newtonian gravity,
- -special relativity.

$$M_{\rm Ch} \propto \left(\frac{Y_{\rm e}}{m_{\rm p}}\right)^2 \left(\frac{\hbar c}{G}\right)^{\frac{3}{2}}$$

-The reaction of electron capture decreases the pressure support, and also decrease the Chandrasekhar mass: a runaway collapse starts as the mass of the core approaches M_{Ch}

$$p + e \rightarrow n + \nu_e$$

Hydrostatic equilibrium of degenerate neutrons (neglecting GR and the strong force)

As the mass M of the degenerate iron core increases,
the density ρ increases,
the neutron interspacing Δx_n decreases.

$$\Delta x_{\rm n} \sim \left(\frac{m_{\rm n}}{\rho}\right)^{\frac{1}{3}}$$

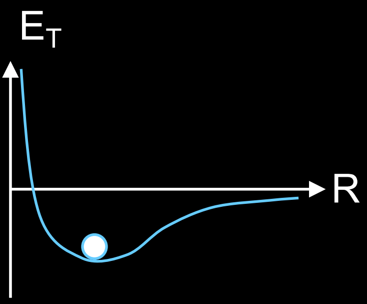
The momentum p_n of degenrate neutrons is deduced from the Heisenberg relation

$$p_{\rm n}\Delta x_{\rm n} \sim \hbar$$

Neutrons are non-relativisitic for nuclear densities <2x10^{17} g/cm^3.

$$\frac{p_{\rm n}}{m_{\rm n}c} \sim \frac{\hbar}{m_{\rm n}c}\left(\frac{\rho}{m_{\rm n}}\right)^{\frac{1}{3}} \sim 0.18\left(\frac{\rho}{10^{15}{\rm g\ cm^{-3}}}\right)^{\frac{1}{3}} \ll 1$$

A spherical stellar core of mass M and radius R contains $N=M/m_n$ neutrons
the mean density is $\rho=M/(4\pi R^3/3)$
The total energy E_T is approximated as the sum of the potential energy of the nuclei $E_p \sim -GM^2/R$
and the kinetic energy E_k of the neutrons



$E_k \sim Np_n^2/2m_n$ for non-relativistic electrons:

$$E_{\rm T} \sim -\frac{GM^2}{R} + \left(\frac{M}{m_{\rm n}}\right)^{\frac{5}{3}}\frac{\hbar^2}{m_{\rm n}R^2}$$

The radius of minimal energy would be a factor m_n/m_e ~2000 smaller than the Chandrasekhar radius

$$R_{\rm n} \equiv \left(\frac{1}{m_{\rm n}^2}\right)\left(\frac{\hbar c}{G}\right)^{\frac{3}{2}}\frac{G}{c^2} = \frac{1}{2}\left(\frac{\hbar c}{G m_{\rm n}^2}\right)^{\frac{3}{2}}\frac{m_{\rm n}}{M_{\rm sol}}\frac{2GM_{\rm sol}}{c^2} = 2.7{\rm km}$$

For such a small radius, general relativistic effects have to be taken into account.
Beside, the strong repulsive force between neutrons results in a significantly larger radius ~10km

A first glimpse into the limiting mass of neutron stars

General relativity: the Schwarzschild radius can be viewed in Newtonian gravity as the radius where the escape velocity $(2GM/R)^{1/2}$ would reach the speed of light c. It defines the horizon of a black hole of mass M.

The Schwarzschild radius of the sun is R_s~3km, it scales linearly with the mass

$$R_{\rm Sch} \equiv \frac{2GM}{c^2} = 2.95 \times \left(\frac{M}{M_{\rm sol}}\right) \ {\rm km}$$

Incompressibility of nuclear matter: Neutrons packed against each other are nearly incompressible
The incompressibility at saturation density is estimated as K=230+-40MeV (Khan+12)

The radius R of a sphere of incompressible neutrons with density ρ_{ns} scales like the power 1/3 o the mass

$$R \sim \left(\frac{M}{\frac{4\pi}{3}\rho_{\rm ns}}\right)^{\frac{1}{3}}$$

It becomes smaller than its Schwarzschild radius if its mass exceeds a threshold defined by

$$M_{\rm crit} \sim \left(\frac{3c^6}{2^5\pi G^3 \rho_{\rm ns}}\right)^{\frac{1}{2}} \sim 4.3 \left(\frac{\rho_{\rm ns}}{10^{15}{\rm g\ cm}^{-3}}\right)^{-\frac{1}{2}} M_{\rm sol}$$

The actual limit is in the range 2-3M_{sol} depending on the equation of state of dense matter, which is not determined yet.

The maximum mass of an observed neutron star is ~2M_{sol} (Demorest+12, Antoniadis+13)